MASSACHUSETTS INSTITUTE OF TECHNOLOGY
MIT CENTER FOR THEORETICAL PHYSICS – A LEINWEBER INSTITUTE

FWF Österreichischer Wissenschaftsfonds

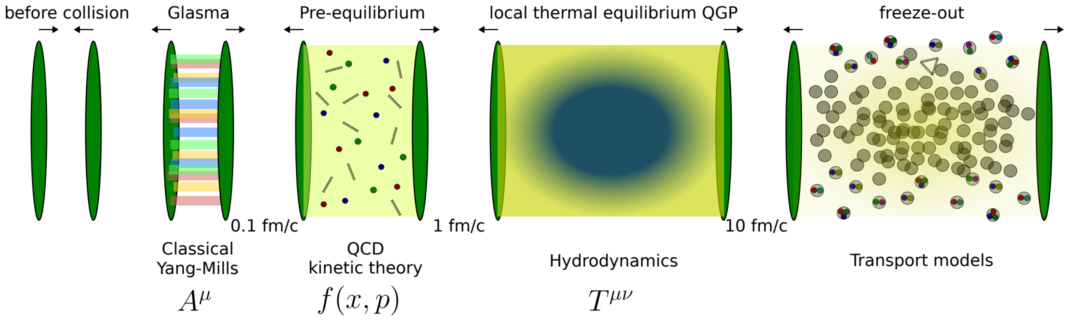
How the nonequilibrium initial stages influence medium-induced radiation

Mainly based on
Phys.Rev.D 113 (2026) 11, 114009 [Barata, Boguslavski, FL, Sadofyev]

Florian Lindenbauer (MIT Center for Theoretical Physics – a Leinweber Institute)

July 13, 2026, EMMI Workshop "Probing the early stages with jets", GSI, Darmstadt

Time-evolution of the QGP in heavy-ion collisions



Interested in pre-equilibrium stages ("Initial stages")

$\rightarrow$ **QCD out of equilibrium** $\rightarrow$ Use **QCD kinetic theory** to simulate plasma evolution

[Rev.Mod.Phys. 93 (2021) [Berges, Heller, Mazeliauskas, Venugopalan]]

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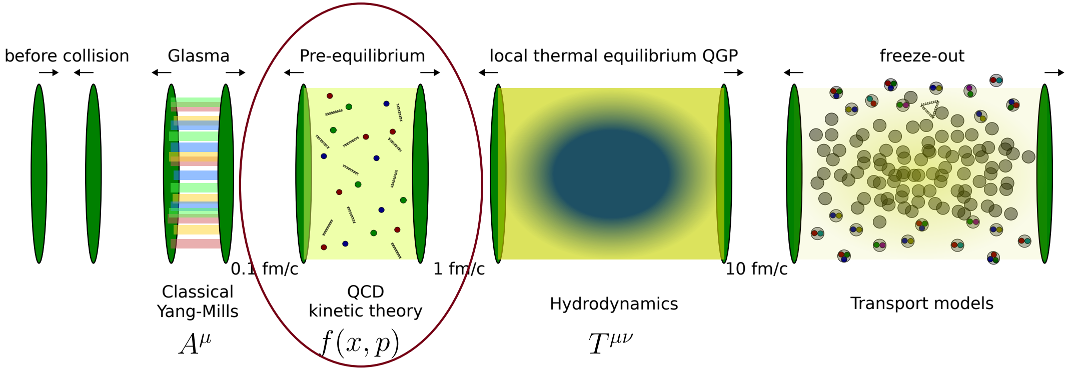

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Effective kinetic theory (EKT) description of the QGP

How many particles/gluons

- Gluons with **distribution function** $f(t, \mathbf{x}, \mathbf{p})$

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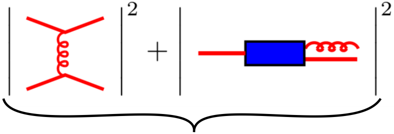
$$(\partial_t + \boldsymbol{v} \cdot \boldsymbol{\nabla}) f =$$



Collision term

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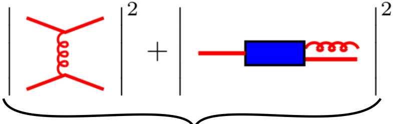
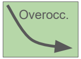
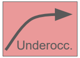


Collision term

- Consider 3 types of plasmas:



Isotropic
Initially overocc.



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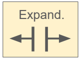

Bjorken
expanding

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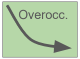
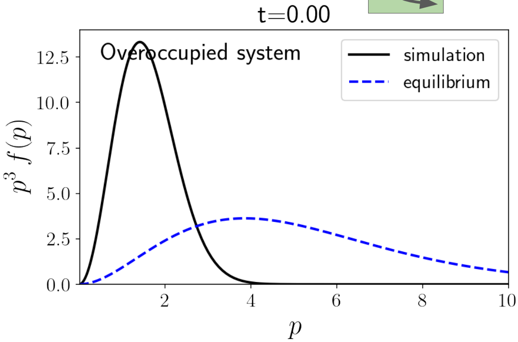
Thermalization in isotropic plasmas





- (Left): Initially over-occupied system
- (Right): Underocc. system: Soft thermal bath formed (bottom-up thermalization)

First studied in Phys.Rev.Lett. 133 (2014) [Kurkela, Lu], see also Phys.Rev.D 105 (2022) [Fu, Ghiglieri, Iqbal, Kurkela]

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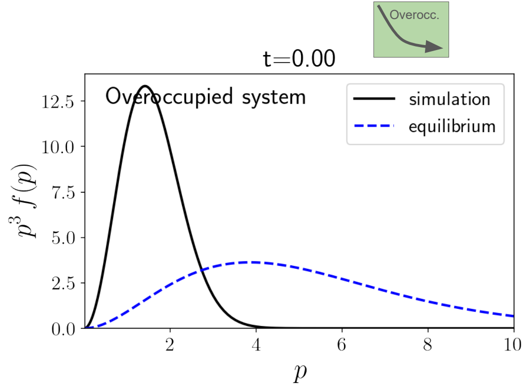

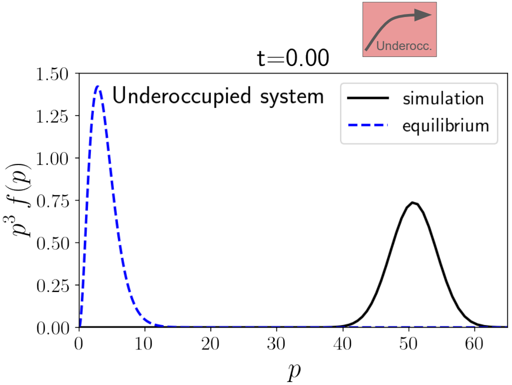

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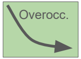

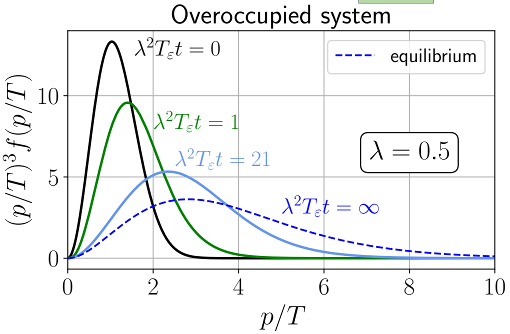

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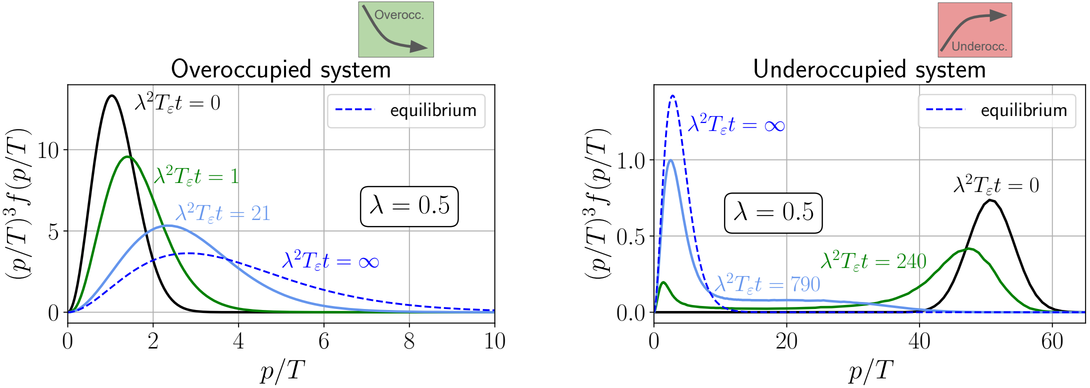

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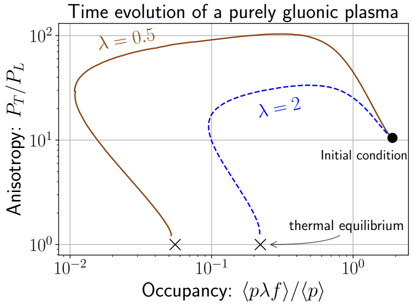
Bottom-up thermalization in heavy-ion collisions

Initial condition[2], with $\lambda = g^2 N_\mathrm{C}$:

$f(p_\perp, p_z) =$ "squeezed" $\frac{1}{\lambda} \times \exp\left(-p^2\right)/p$

Expand.



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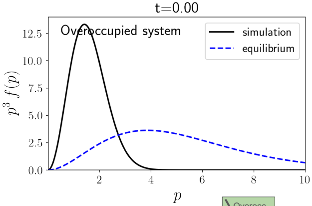
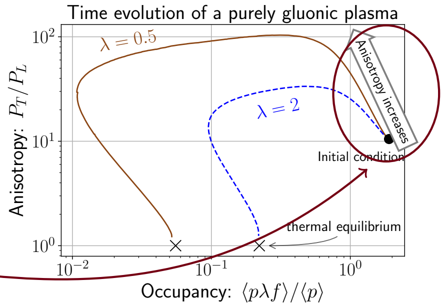


Expand.

Overocc.



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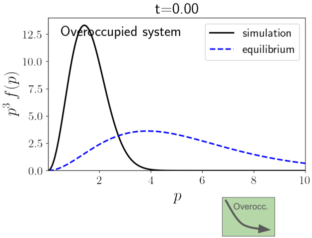

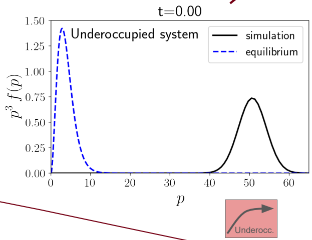

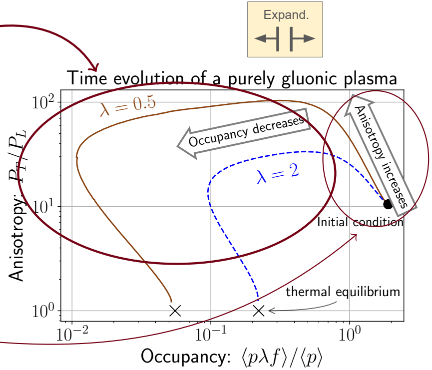

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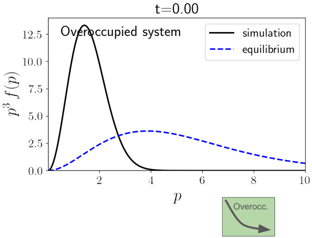

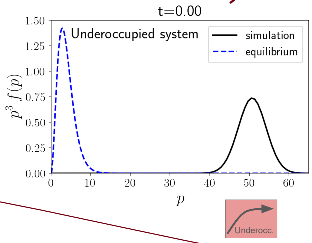

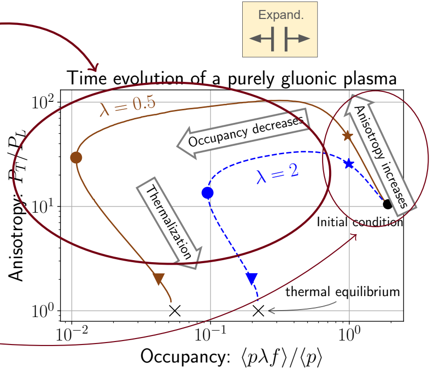

- **Markers** represent **different stages**
- thermalizes after triangle marker

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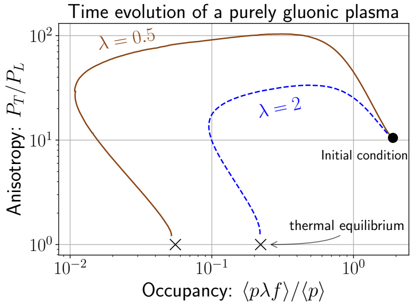

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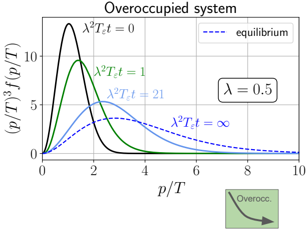

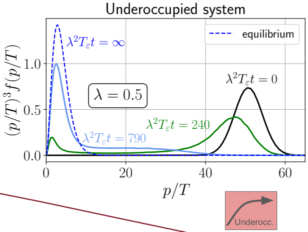

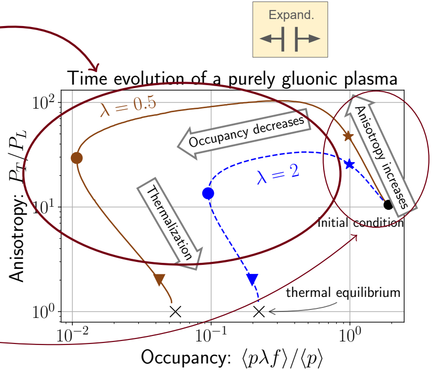

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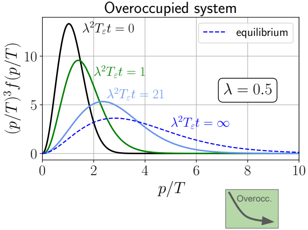

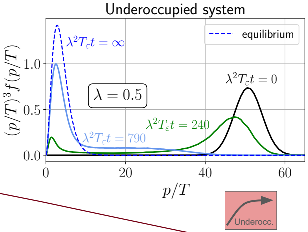

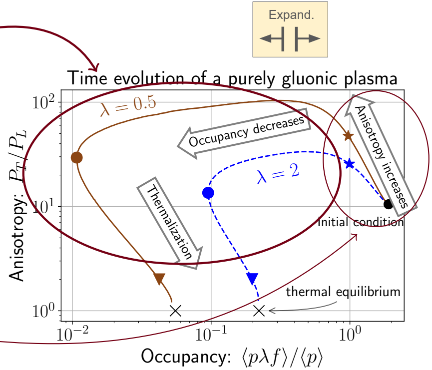

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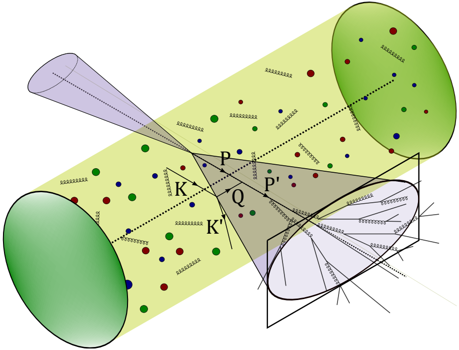
Jets in heavy-ion collisions

- Jets **interact** with nonequilibrium plasma

→ Imprints of **medium interactions**

- Many event generators use simplifying assumptions (isotropic, thermal, . . .)



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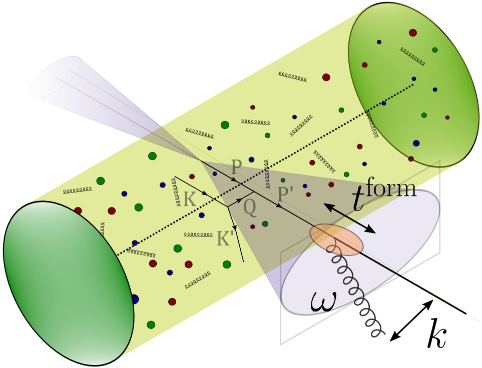

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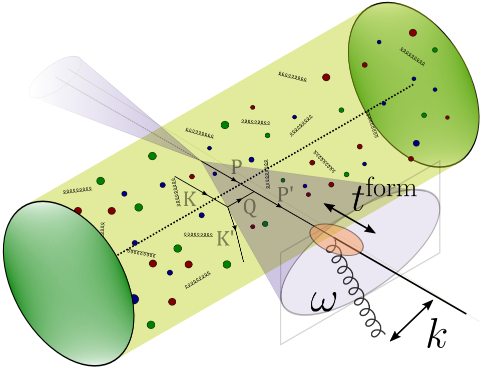
- **Energy loss** dominated by **gluon radiation**
  - $\mathrm{d}I/\mathrm{d}\omega$: **Spectrum** $=$Probability of emitting gluon with energy ω (relative to vacuum)



- ω: emitted gluon energy
- k: transverse momentum
- $t^{\text{form}} \sim \sqrt{\omega/\hat{q}}$: formation time

Obtaining the gluon emission spectrum

Phys.Rev.D 79 (2009) [Arnold]
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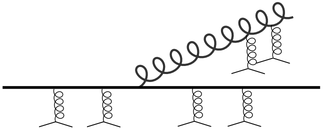

→See also: Marco Leitão, Mo 14:30

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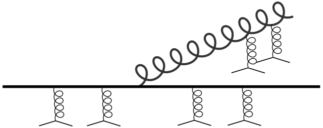

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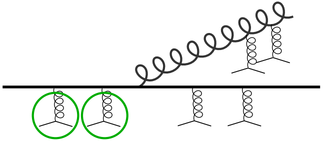

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Spectrum $\omega\frac{\mathrm{d}I}{\mathrm{d}\omega} \sim$ from solution to 2D Schrödinger equation

- Input: **dipole cross section** $C(\mathbf{x}, t)$ = potential
- **Harmonic approximation** (for jets $|\mathbf{x}| \ll 1$ important!)

$$C(\mathbf{x}, t) = \frac{1}{4}\hat{q}(t)\mathbf{x}^2, \qquad \hat{q} = \frac{\mathrm{d}\langle p_\perp^2\rangle}{\mathrm{d}L} = \frac{\mathrm{d}\langle p_\perp^2\rangle}{\mathrm{d}t} = \int \mathrm{d}^2 q_\perp\, q_\perp^2 \frac{\mathrm{d}\Gamma^{\mathrm{el}}}{\mathrm{d}^2 q_\perp}$$

scattering rate
probability of receiving
$q_\perp$ kick

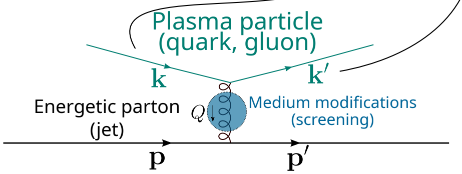
- $\hat{q}$ quantifies **momentum broadening**, "jet quenching parameter"
- Schrödinger equation for harmonic oscillator → analytic solution!

Results for $\hat{q}$: Broadening

Phys.Rev.D 110 (2024) [Boguslavski, Kurkela, Lappi, FL, Peuron]
Phys.Lett.B 850 (2024) [Boguslavski, Kurkela, Lappi, FL, Peuron]]

- Obtain **Jet quenching parameter** from kinetic theory, input $f(\mathbf{k})$,

$$\hat{q}^{ij} = \int_{\substack{q_\perp < \Lambda_\perp \\ p \to \infty}} \mathrm{d}\Gamma \, q^i q^j |\mathcal{M}|^2 f(k)(1 + f(k'))$$



→See also: Isabella Danhoni, Tue 9:00

Note: Isotropic screening to prevent plasma instabilities! Phys.Lett.B 214 (1988) [Mrowczynski], Phys.Rev.D 68 (2003) [Romatschke, Strickland]

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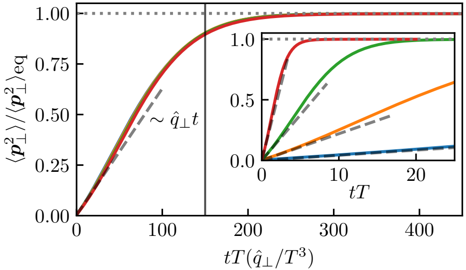
- Simulate jet equilibration

→ $\hat{q}$ accurately captures transverse momentum broadening

→ **Remarkable collapse** for different couplings $\lambda \in (2, 20)$!

Jet evolution in thermal equilibrium[4]



→talk by Fabian Zhou, Friday 9:30

[4] [JHEP 05 (2026) [Boguslavski, FL, Mazeliauskas, Takacs, Zhou]]

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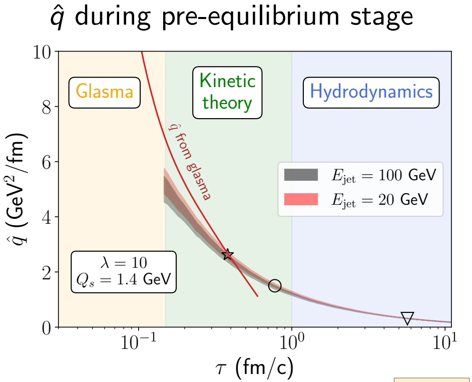
- **Bands:** Vary cutoff and initial conditions
- Little jet energy dependence
- Supports hydrodynamic values and **large values** in **Glasma**

Glasma values from Phys.Lett.B 810 (2020) [Ipp, Müller, Schuh]]



Expand.

Problems with harmonic approximation

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scattering rate

Probability of receiving kick $q_\perp$

$$\hat{q}(t) = \frac{\mathrm{d}\langle p_\perp^2 \rangle}{\mathrm{d}L} = \frac{\mathrm{d}\langle p_\perp^2 \rangle}{\mathrm{d}t} \sim \int \mathrm{d}^2 q_\perp \, q_\perp^2 \quad \underbrace{C(q_\perp, t)}_{1/q_\perp^4 \text{ for large } q_\perp} \sim \int \frac{\mathrm{d}q_\perp}{q_\perp} \quad \to \text{log divergence}$$

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Need cutoff $\Lambda_\perp$ to restrict momentum transfer $q_\perp < \Lambda_\perp$

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scattering rate

Probability of receiving kick $q_\perp$

$$\hat{q}(t) = \frac{\mathrm{d}\langle p_\perp^2\rangle}{\mathrm{d}L} = \frac{\mathrm{d}\langle p_\perp^2\rangle}{\mathrm{d}t} \sim \int \mathrm{d}^2 q_\perp\, q_\perp^2 \underbrace{C(q_\perp,t)}_{1/q_\perp^4 \text{ for large } q_\perp} \sim \int \frac{\mathrm{d}q_\perp}{q_\perp} \to \text{log divergence}$$

Need cutoff $\Lambda_\perp$ to restrict momentum transfer $q_\perp < \Lambda_\perp$

Can we do better? $\to$ **Consider full kernel** $C(\mathbf{q}_\perp,t)$ for $C(\mathbf{x},t)$

Angular dependence and contribution to $\hat{q}$

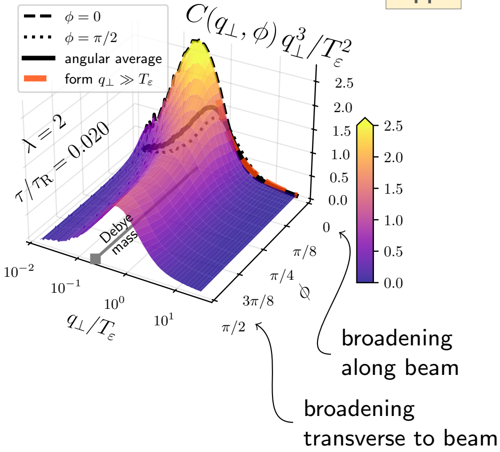
arXiv:2509.03868 [Altenburger, Boguslavski, FL]

Expand.

- Momentum broadening kernel:

$$C(\mathbf{q}_\perp) = \int \mathrm{d}\Gamma_{\rm PS}\, |\mathcal{M}|^2\, f(\mathbf{k})(1 + f(\mathbf{k}-\mathbf{q}))$$

- $C(q_\perp, \phi)$ is 2D function



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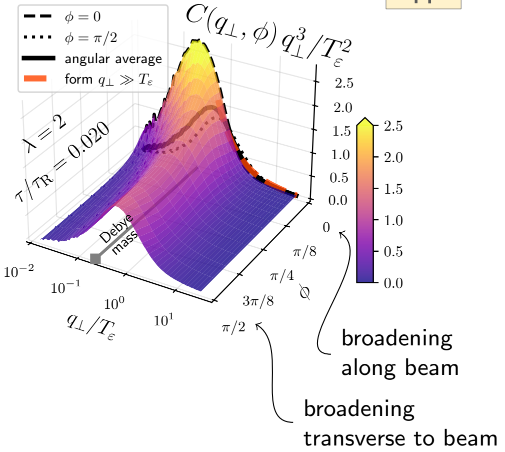
- $C(q_\perp, \phi)$ is 2D function
- More differential than $\hat{q}$,

$$\hat{q}^{ij} = \int \frac{\mathrm{d}^2\mathbf{q}_\perp}{(2\pi)^2} q^i q^j C(\mathbf{q}_\perp),$$

- Along beam ($\phi_{pq} = 0$): Much larger and different form at early times



Angular average

arXiv:2509.03868 [Altenburger, Boguslavski, FL]

Expand.

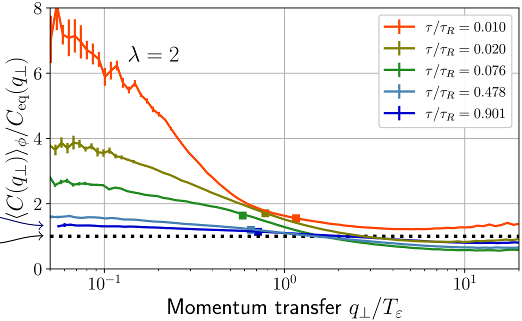
$$C(\mathbf{q}_\perp) = \int \mathrm{d}\Gamma_{\mathrm{PS}}\, |\mathcal{M}|^2 f(\mathbf{k})(1 + f(\mathbf{k}-\mathbf{q}))$$

Normalize using (Landau-matched) thermal kernel

earliest time

latest time

thermal $= 1$



□: Debye mass

Angular average

arXiv:2509.03868 [Altenburger, Boguslavski, FL]

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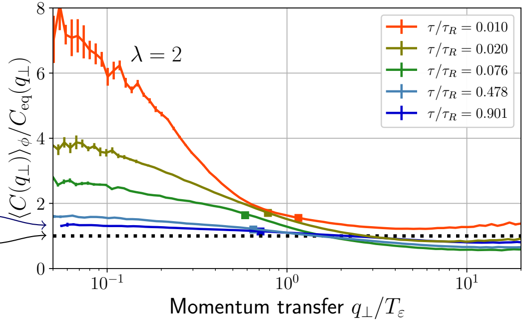
Normalize using (Landau-matched) thermal kernel

- Angular average
  $C(q_\perp) = \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi\, C(q_\perp, \phi)$

latest time

thermal $= 1$

earliest time



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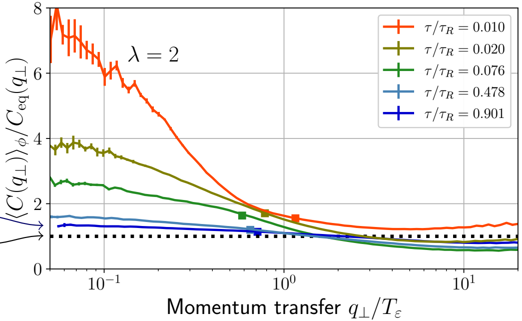
- Angular average $C(q_\perp) = \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi\, C(q_\perp, \phi)$
- Late times $\to$ thermal
- **Small-momentum transfer enhanced**

earliest time

latest time

thermal = 1



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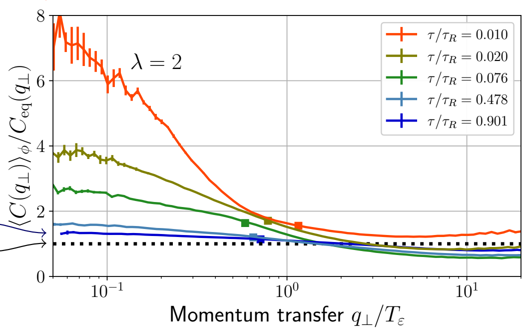
- Late times $\rightarrow$ thermal
- **Small-momentum transfer enhanced**
- What to do with jet quenching?

earliest time

latest time

thermal = 1



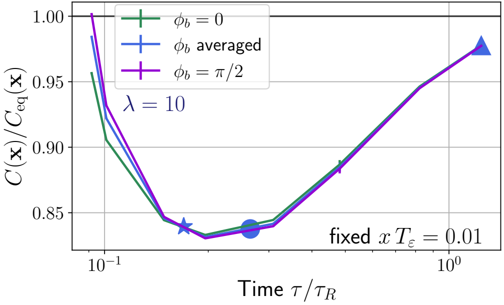
□: Debye mass

Small-distance behavior of $C(\mathbf{x})$

arXiv:2509.03868 [Altenburger, Boguslavski, FL]

Expand.

- Obtained via Fourier transform
  $C(\mathbf{x}) = \int \frac{\mathrm{d}^2\mathbf{q}_\perp}{(2\pi)^3}\left(1 - e^{i\mathbf{q}_\perp\cdot\mathbf{x}}\right)C(\mathbf{q}_\perp)$
- Small-distance behavior important for jet quenching!
- For $\lambda = 10$, always smaller than in equilibrium



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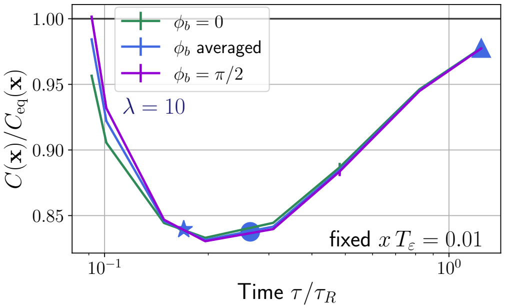
- Small-distance behavior important for jet quenching!
- For $\lambda = 10$, always smaller than in equilibrium
- Take small distance behavior and calculate jet emission spectrum

Phys.Rev.D 113 (2026) 11 [Barata, Boguslavski, FL, Sadofyev]



Small-distance form of $C(x)$

- Proper expansion for small distance:

medium input,
obtained from $\hat{q}(\Lambda_\perp)$

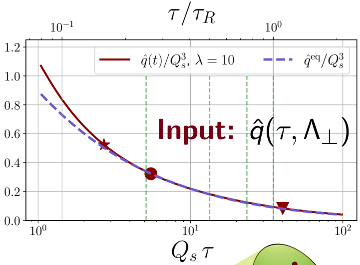
$$C(\mathbf{x},t) = \frac{1}{4}\hat{q}_0(t)\mathbf{x}^2 \log\frac{1}{\mathbf{x}^2\mu_*^2(t)}$$

- **Expand** perturbatively **around harmonic oscillator solution**
  $\rightarrow$ "improved opacity expansion" JHEP 09 (2021) [Barata, Mehtar-Tani, Soto-Ontoso, Tywoniuk]

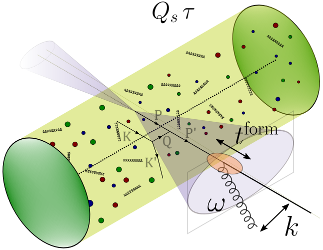
$$C(\mathbf{x},t) = \underbrace{\frac{\hat{q}_0(t)}{4}\mathbf{x}^2\log\frac{Q^2}{\mu_*^2}}_{\text{harmonic approximation}} - \underbrace{\frac{\hat{q}_0(t)}{4}\mathbf{x}^2\log(Q^2\mathbf{x}^2)}_{\text{perturbation}}$$

Obtaining the spectrum – Schematic

Phys.Rev.D 113 (2026) 11 [Barata, Boguslavski, FL, Sadofyev]



Input: $\hat{q}(\tau, \Lambda_\perp)$



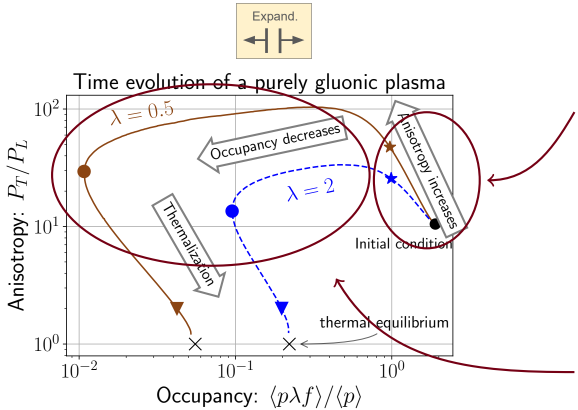
$\hat{q}_0(\tau),\ \mu_*(\tau)$

Small-distance form of potential

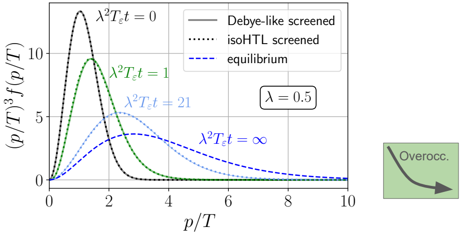
"Improved opacity expansion"

Output: Spectrum $\frac{\mathrm{d}I}{\mathrm{d}\omega\,\mathrm{d}^2\mathbf{k}}$

Recall: Bottom-up thermalization



Overoccupied, initial $n > n_{therm}$



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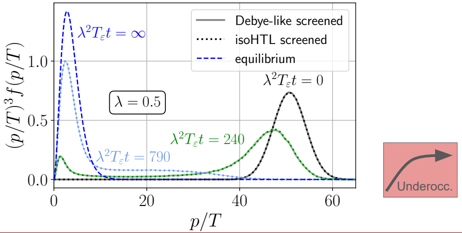
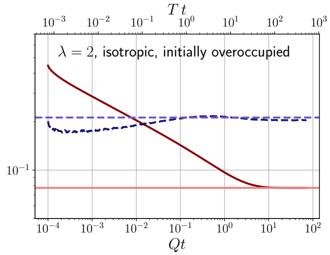
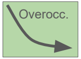
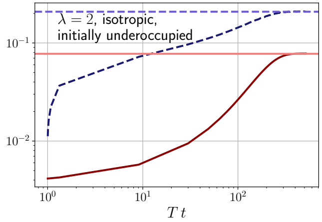


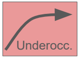
Isotropic toy models

Phys.Rev.D 113 (2026) 11 [Barata, Boguslavski, FL, Sadofyev]









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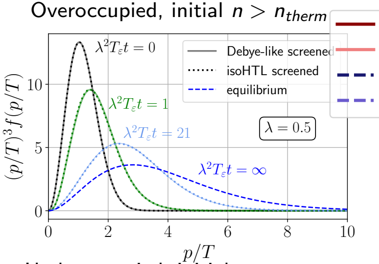

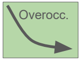

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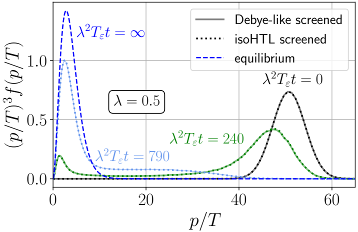
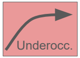
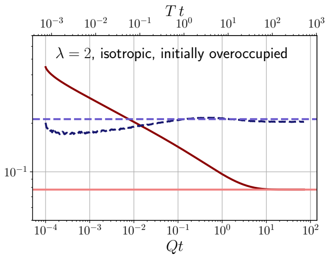
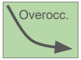


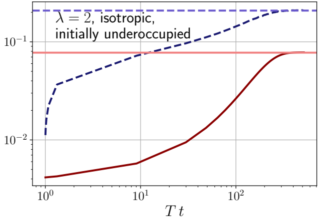


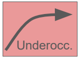
Comparison with isotropic toy models

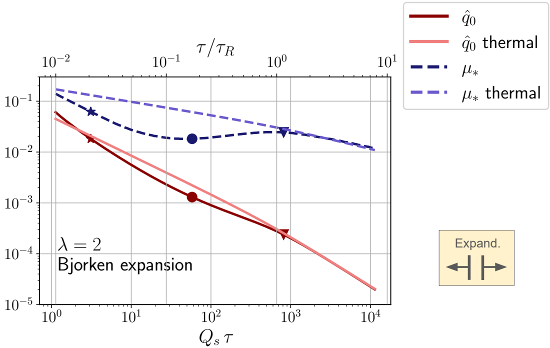
Phys.Rev.D 113 (2026) [Barata, Boguslavski, FL, Sadofyev]







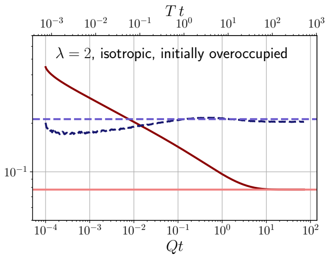


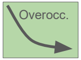


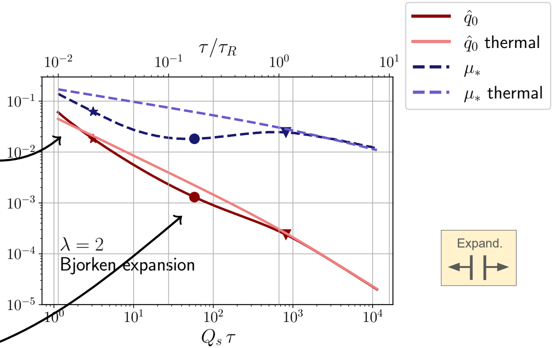
- Expanding evolution is combination of over- and underoccupied evolution
- Better visible at smaller couplings!

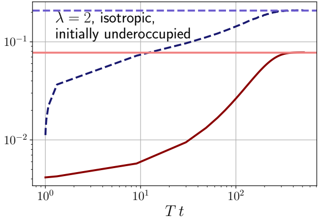
Comparison with isotropic toy models

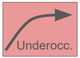
Phys.Rev.D 113 (2026) [Barata, Boguslavski, FL, Sadofyev]











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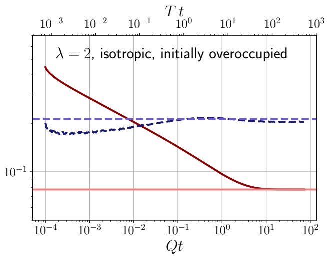

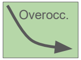

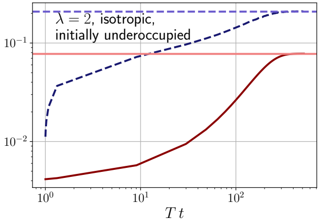

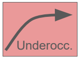

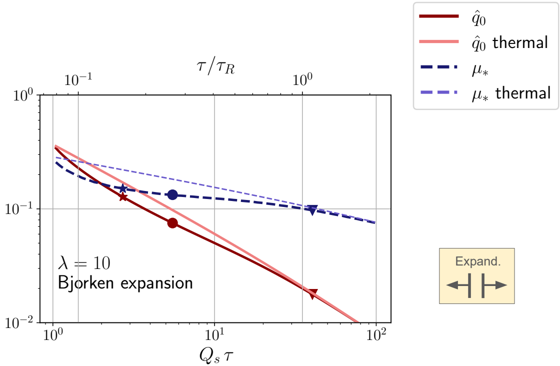

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Gluon spectrum

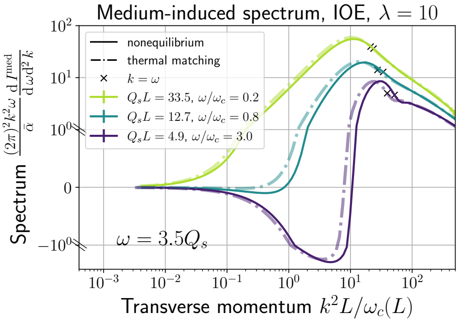
Phys.Rev.D 113 (2026) [Barata, Boguslavski, FL, Sadofyev]

Expand.

- $\hat{q}_0(t),\ \mu_*(t) \to$ spectrum $\mathrm{d}I/\mathrm{d}\omega$
- Consider different medium lengths (colors in plot: green, blue purple)

→ Set $\hat{q}_0(t) = 0$ for $t > L$

- Form of spectrum dominated by ratio ω/ω_c, $\omega_c = \int_{t_{min}}^{t_{min}+L} \mathrm{d}t\,(t - t_{min})\hat{q}(t)$
- All information, difficult to analyze



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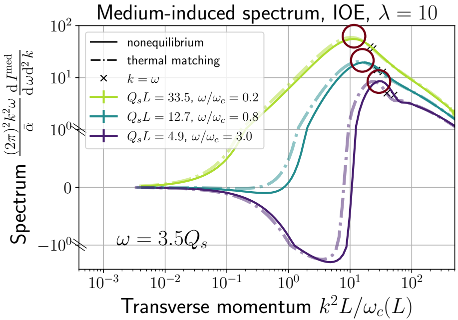


Comparison to equilibrium

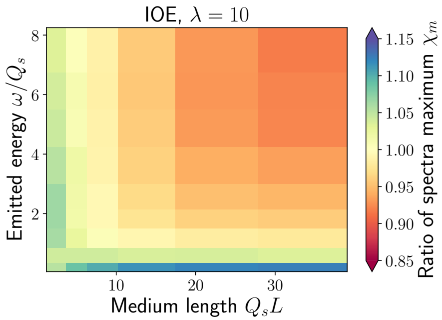
Phys.Rev.D 113 (2026) [Barata, Boguslavski, FL, Sadofyev]

Expand.

- Comparison to equilibrium: Maximum of spectrum plot

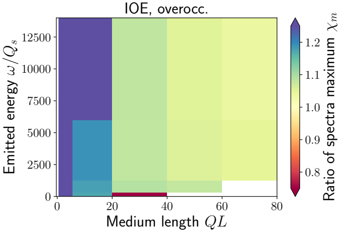
$$\chi_m = \frac{\text{Maximum spectrum nonequ.}}{\text{Maximum spectrum equ.}}$$

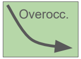
- Clear systematics visible
- Does not approach 1 even for $L \to \infty$!



Comparison to equilibrium II

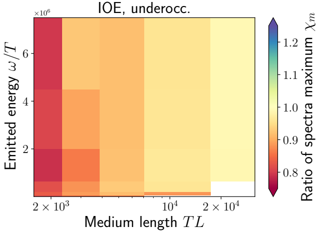
Phys.Rev.D 113 (2026) [Barata, Boguslavski, FL, Sadofyev]

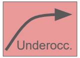




initially overoccupied

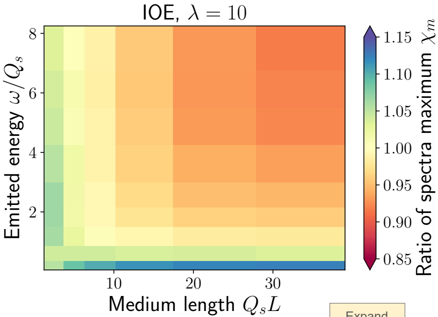
$\chi > 1$

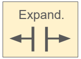




then underoccupied

$\chi < 1$





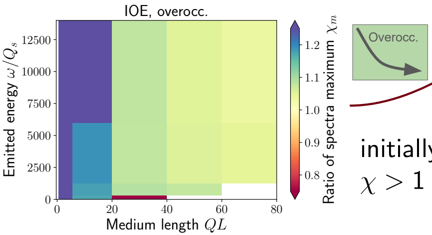
- For fixed L:

  $\chi > 1 \rightarrow \chi < 1$

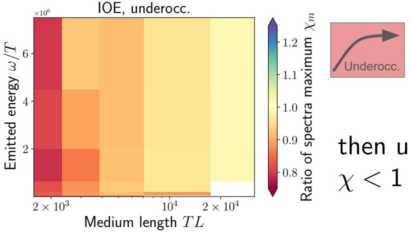
$\rightarrow$ Can be explained with toy models

Comparison to equilibrium II

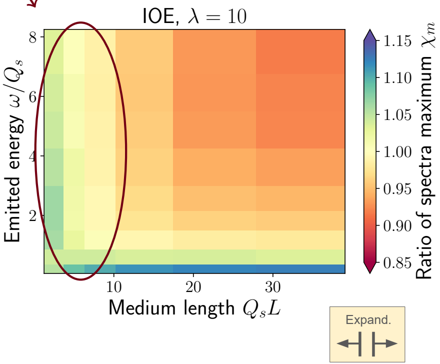
Phys.Rev.D 113 (2026) [Barata, Boguslavski, FL, Sadofyev]



initially overoccupied
$\chi > 1$



then underoccupied
$\chi < 1$



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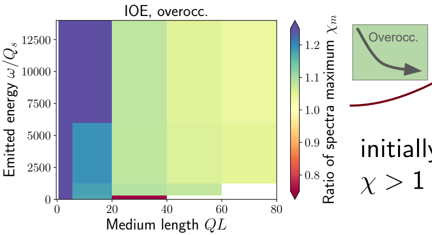

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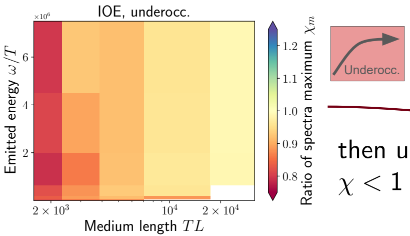

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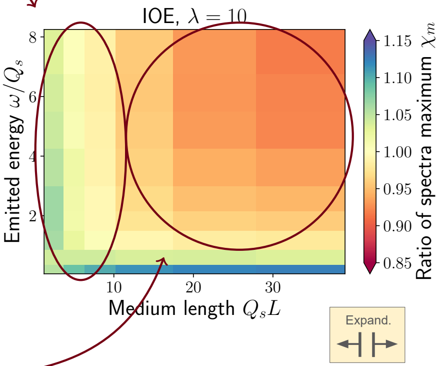

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  $\chi > 1 \rightarrow \chi < 1$

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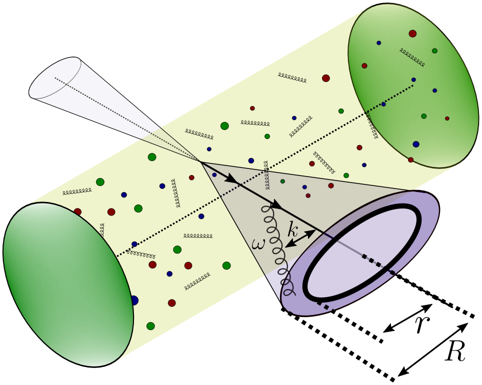
Jet shape

Phys.Rev.D 113 (2026) 11 [Barata, Boguslavski, FL, Sadofyev]

- Energy deposited in *annulus* with radii r and R

$$\frac{\mathrm{d}\rho(r,R)}{\mathrm{d}\omega} = \int_{r\omega}^{R\omega} \mathrm{d}^2\mathbf{k}\,\omega\frac{\mathrm{d}I}{\mathrm{d}\omega\,\mathrm{d}^2\mathbf{k}}$$



Jet shape

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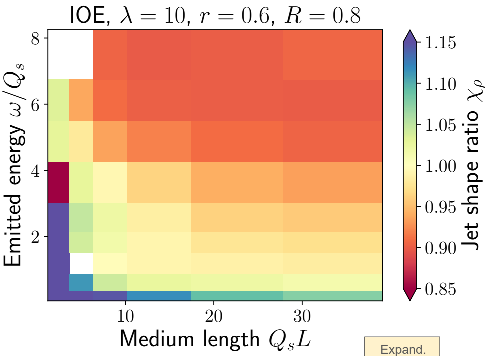
- Qualitatively same behavior as spectra maximum



Expand.

Conclusions

- Use **QCD kinetic theory** to describe initial **nonequilibrium QCD plasma** in heavy-ion collisions
- Calculate **momentum broadening of jets** $\rightarrow \hat{q}(t)$ and $C(\mathbf{q}_\perp, t)$
- Taken as input to calculate **gluon emission spectrum**
- Clear differences to thermal medium evolution, even at **large medium lengths**

Outlook

- Prediction/effects for specific more differential observables
- General structure and scaling of spectra?
- Improve QCD kinetic theory simulations of the background e.g., with machine learning Eur.Phys.J.C 85 (2025) [Barrera Cabodevila, Kurkela, FL]

and ongoing work with Barrera Cabodevila, Kurkela, Mazeliauskas, Taghavi, Zhou

Thank you very much for your attention!

Observables in kinetic theories

- Fundamental quantity: Distribution function $f(\mathbf{p})$
- **Energy-Momentum tensor**:

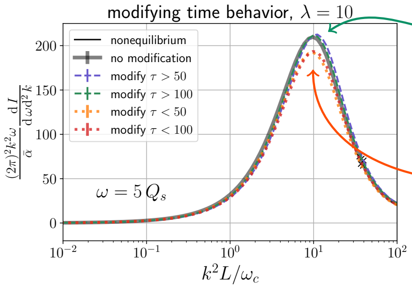
$$T^{\mu\nu} = \nu_g \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3} \frac{p^\mu p^\nu}{p} f(\mathbf{p}) \qquad \leftarrow \quad \text{“Moments of distribution function”}$$

- Longitudinal pressure $P_L = T_{zz}$
- Transverse pressure $P_T = T_{xx} = T_{yy}$
- How many ‘hard’ particles

$$\frac{\langle pf \rangle}{\langle p \rangle} = \frac{\int \mathrm{d}^3\mathbf{p}\, p\, f(\mathbf{p})^2}{\int \mathrm{d}^3\mathbf{p}\, p\, f(\mathbf{p})} \qquad \leftarrow \quad \text{“particle number weighted with energy”}$$

Effect of modifications

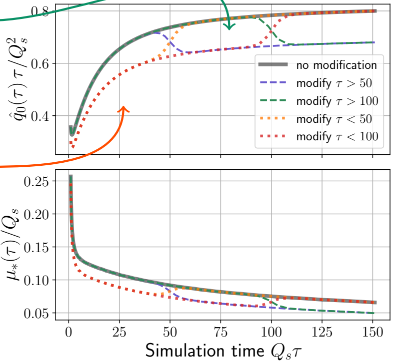
Gluon spectrum



late-time modification

early-time modification

Input $\hat{q}_0$ and μ_*



- Only modifications at early time influence spectrum!

Spectrum more explicit

Phys.Rev.D 113 (2026) 11 [Barata, Boguslavski, FL, Sadofyev]

IOE Spectrum is obtained via

$$\frac{\mathrm{d}I^{(0)}}{\mathrm{d}\omega\,\mathrm{d}^2k} = \frac{2\bar{\alpha}}{\pi\omega}\mathrm{Re}\left\{k^{-2}\left(e^{-i\frac{k^2}{2\omega\mathrm{Cot}(L,t_0)}}-1\right)+\int_{t_0}^{L}\mathrm{d}t\,\frac{\mathrm{Cot}(t,t_0)}{\hat{P}^2(t,t_0)}e^{-\frac{k^2}{\hat{P}^2(t,t_0)}}\right\},$$

$$\begin{aligned}\frac{\mathrm{d}I^{(1)}}{\mathrm{d}\omega\,\mathrm{d}^2k} = \frac{\bar{\alpha}\pi}{k^4}\mathrm{Re}\int_{t_0}^{L}\mathrm{d}t_2\Bigg\{&\frac{i}{2\omega}\int_{t_0}^{t_2}\mathrm{d}t_1\,\frac{\hat{q}_0(t_1)}{\hat{R}_{21}^2}e^{-\frac{k^2}{\hat{P}_{21}^2}}\left[Q_s^2(L,t_2)I_a\left(\frac{k^2}{\hat{K}_{21}},\frac{k^2\hat{R}_{21}^2}{Q_r^2}\right)+2C_{12}\hat{R}_{21}k^2I_b\left(\frac{k^2}{\hat{K}_{21}},\frac{k^2\hat{R}_{21}^2}{Q_r^2}\right)\right]-\mathrm{Cot}_{20}\,Q_{s0}^2(L,t_2)\,I_a\left(\frac{k^2}{\hat{P}^2(t_2,t_0)},\frac{k^2}{Q_b^2}\right)\\&+2\,\hat{q}_0(t_2)C^2(t_2,L)I_b\left(\frac{ik^2}{2\omega C^2(t_2,L)\left(\mathrm{Cot}_{20}-\frac{1}{C(t_2,L)S(L,t_2)}-\mathrm{Cot}(t_2,L)\right)};\frac{k^2}{Q_r^2C^2(t_2,L)}\right)e^{-\frac{ik^2}{2\omega\mathrm{Cot}(L,t_2)}}\Bigg\}\end{aligned}$$

And requires solving a differential equation

medium input,
obtained from $\hat{q}(\Lambda_\perp)$

scale,
obtained self-consistently

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}t^2}+\Omega^2(t)\right]F(t,t_0)=0\,,\qquad \Omega^2(t)=-i\frac{\hat{q}_r(t)}{\omega},\qquad \hat{q}_r(t)=\hat{q}_0(t)\log\frac{Q_r^2}{\mu_*^2(t)}$$

with specific boundary conditions: $S(t_0,t_0)=\partial_t C(t,t_0)_{t=t_0}=0$, $\partial_t S(t,t_0)_{t=t_0}=C(t_0,t_0)=1$

Small-distance form of $C(x)$

Phys.Rev.D 113 (2026) 11 [Barata, Boguslavski, FL, Sadofyev]

- Introduce cutoff

$$C(\mathbf{x}) = \underbrace{\int_{|\mathbf{q}_\perp|<\Lambda} \frac{\mathrm{d}^2\mathbf{q}_\perp}{(2\pi)^2} C(\mathbf{q}_\perp) \underbrace{\left(1 - e^{i\mathbf{x}\cdot\mathbf{q}_\perp}\right)}_{\text{expand: } \sim\frac{1}{4}x^2 q_\perp^2 + \ldots}} + \underbrace{\int_{|\mathbf{q}_\perp|>\Lambda_\perp} \frac{\mathrm{d}^2\mathbf{q}_\perp}{(2\pi)^3} C(\mathbf{q}_\perp)(1 - e^{i\mathbf{x}\cdot\mathbf{q}_\perp})}$$

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- $\hat{q}_0$ is constant related to large $q_\perp$ behavior of $C(q_\perp \to \infty) \to \frac{\hat{q}_0}{2\pi}\frac{1}{q_\perp^4}$

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- **Expand** perturbatively **around harmonic oscillator solution**
  $\rightarrow$ "improved opacity expansion" JHEP 09 (2021) [Barata, Mehtar-Tani, Soto-Ontoso, Tywoniuk]

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$\hat{q}_0$ and μ_* from EKT

Phys.Rev.D 113 (2026) 11 [Barata, Boguslavski, FL, Sadofyev]

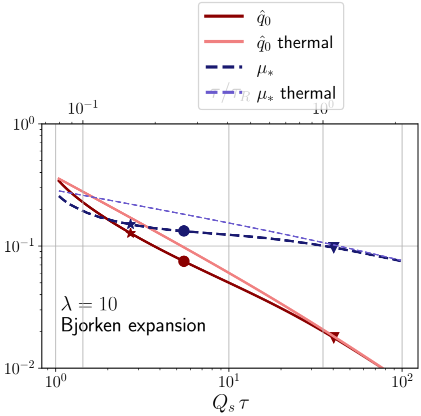
- Recall: $\hat{q}_0$, μ_* characterize small-x of $C(\mathbf{x})$,
- Get $\hat{q}_0$ from large-cutoff behavior,
  $\hat{q}(t) \approx 2\hat{q}_0(t) \log \frac{\Lambda_\perp}{Q_s} + b(t)$
- Get μ_* via

$$\mu_*^2(t) = \frac{1}{4} \exp(-b(t)/\hat{q}_0(t) + 2\gamma_E - 2)$$

- Thermal:

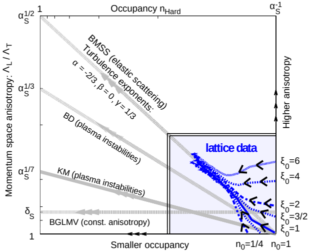
$$\hat{q}_0 = T^3 \frac{N_C C_R g^4 \zeta(3)}{2\pi^3}$$

$$\mu_*^2 = \frac{m_D^2}{4} \left(\frac{m_D}{2T}\right)^{2\zeta(2)/\zeta(3)-2} e^{2\gamma_E - 2 - \left(\frac{\zeta(2)}{\zeta(3)} - 1\right)(1-2\gamma_E) + \frac{\sigma_+}{\pi\zeta(3)}}$$



Plasma instabilities

- Anisotropic QCD plasmas generally lead to plasma instabilities Phys.Lett.B 214 (1988) [Mrowczynski], Phys.Rev.D 68 (2003) [Romatschke, Strickland]
- Soft modes grow exponentially, until perturbation theory (HTL) breaks down
- Although recent progress[4], no clear avenue for implementation in kinetic theory
- Classical statistical simulations seem to indicate that instabilities not dominant Phys.Rev.D 89 (2014) [Berges, Boguslavski, Schlichting, Venugopalan]



Phys.Rev.D 89 (2014) [Berges, Boguslavski, Schlichting, Venugopalan]

[4] [Phys.Rev.C 105 (2023) [Hauksson, Jeon, Gale]]

Momentum broadening kernel vs. jet quenching parameter

Momentum broadening kernel:

$$C(\mathbf{q}_\perp) = \int \mathrm{d}\Gamma_{\mathrm{PS}}\, |\mathcal{M}|^2 f(\mathbf{k})(1+f(\mathbf{k}-\mathbf{q}))$$

Compare to $\hat{q}$,

weighted with
momentum transfer

$$\hat{q}^{ij} = \int_{\substack{q_\perp<\Lambda_\perp \\ p\to\infty}} \mathrm{d}\Gamma\, q^i q^j |\mathcal{M}|^2 f(\mathbf{k})(1+f(\mathbf{k}-\mathbf{q}))$$

different
integration measure

momentum cutoff
needed

- Similar quantity, less differential,

$$\hat{q}^{ij} = \int \frac{\mathrm{d}^2\mathbf{q}_\perp}{(2\pi)^2} q^i q^j C(\mathbf{q}_\perp),$$

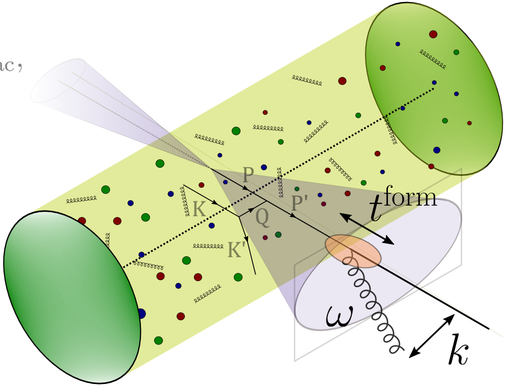
- $C(q_\perp)$ is **more general** than $\hat{q}$

Obtaining the gluon emission spectrum

Phys.Rev.D 79 (2009) [Arnold]
JETP Lett. 65 (1997) [Zakharov]
Nucl.Phys.B 483 (1997) [Baier, Dokshitzer, Mueller, Peigne, Schiff]

- **Energy loss** dominated by **gluon radiation**

$$\omega \frac{\mathrm{d}I}{\mathrm{d}\omega} \sim \mathrm{Re} \int_{t1,t2} \partial_{\mathbf{x}} \cdot \partial_{\mathbf{y}} G(\mathbf{x}, t_2; \mathbf{y}, t_1)_{\mathbf{y}=0} - G_{\mathrm{vac}},$$



- $\mathrm{d}I/\mathrm{d}\omega$: **Spectrum**
  =Probability of emitting gluon with energy ω (relative to vacuum)
- ω: emitted gluon energy
- k: transverse momentum
- $t^{\mathrm{form}} \sim \sqrt{\omega/\hat{q}}$: formation time

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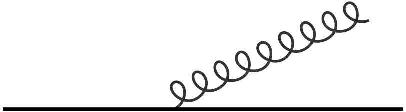

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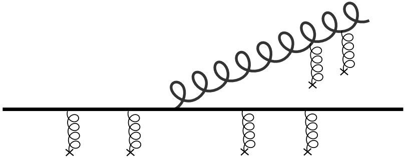

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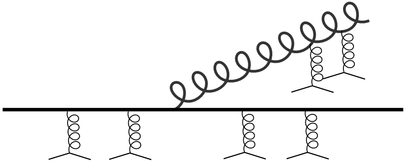

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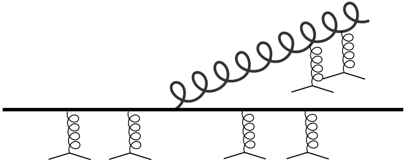

- Green's function solves

energy shift

$$(i\partial_t - \delta E + iC(\mathbf{x}, t))\, G(\mathbf{x}, t; \mathbf{y}, t_1) = \delta^2(\mathbf{x} - \mathbf{y})$$

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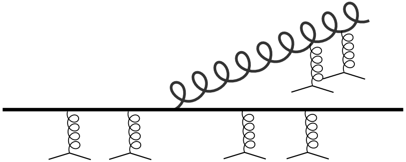

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$$C(\mathbf{x}) = \int \frac{\mathrm{d}^2\mathbf{q}_\perp}{(2\pi)^2} C(\mathbf{q}_\perp) \left(1 - e^{i\mathbf{x}\cdot\mathbf{q}_\perp}\right), \qquad C(\mathbf{q}_\perp) = (2\pi)^2 \frac{\mathrm{d}\Gamma^{\mathrm{el}}}{\mathrm{d}^2 q_\perp}$$

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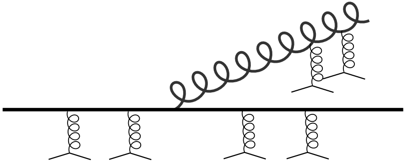

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'broadening probability'
Probability of receiving momentum kick $\mathbf{q}_\perp$

$$C(\mathbf{q}_\perp) = (2\pi)^2 \frac{\mathrm{d}\Gamma^{\mathrm{el}}}{\mathrm{d}^2 q_\perp}$$

Harmonic approximation for radiation

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- **Harmonic approximation** (for jets small-x important!)

$$C(\mathbf{x}, t) = \frac{1}{4}\hat{q}(t)\mathbf{x}^2, \qquad \hat{q} = \frac{\mathrm{d}\langle p_\perp^2\rangle}{\mathrm{d}L} = \frac{\mathrm{d}\langle p_\perp^2\rangle}{\mathrm{d}t} = \int \mathrm{d}^2 q_\perp \, q_\perp^2 \frac{\mathrm{d}\Gamma^{\mathrm{el}}}{\mathrm{d}^2 q_\perp}$$

Harmonic approximation for radiation

- Input: **dipole cross section** = potential in Schrödinger equation

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- **Harmonic approximation** (for jets small-x important!)

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scattering rate probability of receiving $q_\perp$ kick

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- $\hat{q}$ quantifies **momentum broadening**, "jet quenching parameter"
- Schrödinger equation for harmonic oscillator $\rightarrow$ analytic solution!

Small-distance form of $C(x)$

Phys.Rev.D 113 (2026) 11 [Barata, Boguslavski, FL, Sadofyev]

- Proper expansion for small distance:

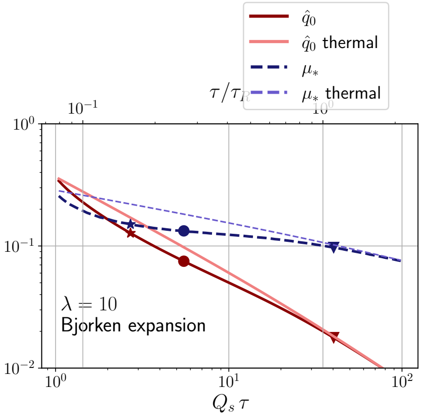
$$C(\mathbf{x},t) = \frac{1}{4}\hat{q}_0(t)\mathbf{x}^2 \log\frac{1}{\mathbf{x}^2\mu_*^2(t)} + \mathcal{O}(\hat{q}_0\mathbf{x}^4\mu_*^2)$$

- μ_* and $\hat{q}_0$ are medium-parameters uniquely determined from large-cutoff form of $\hat{q}(\Lambda_\perp)$,

$$\hat{q}(t) \approx 2\hat{q}_0(t)\log\frac{\Lambda_\perp}{Q_s} + b(t)$$

- Obtain μ_* via

$$\mu_*^2(t) = \frac{1}{4}\exp(-b(t)/\hat{q}_0(t) + 2\gamma_E - 2)$$



Problems with $\hat{q}$ for radiation

- Recall: $\hat{q}$ appeared in small-distance expansion:

expansion parameter

physical quantity, enters equations as potential

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Can we do better? $\to$ Consider full kernel $C(\mathbf{q}_\perp)$

Comparing to thermal system

Phys.Rev.D 113 (2026) 11 [Barata, Boguslavski, FL, Sadofyev]

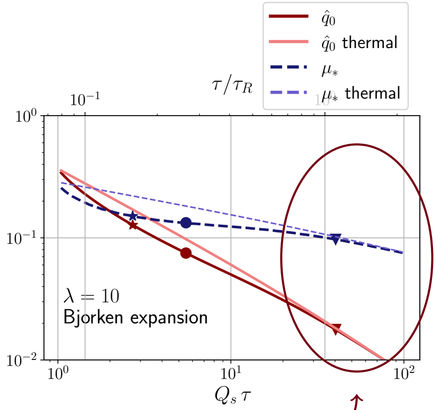
- Recall: $\hat{q}_0$, μ_* obtained from large cutoff behavior of $\hat{q}$
- Compare with an evolving thermal system

$$T_\varepsilon(t) = \left(\frac{30\epsilon(t)}{\pi^2}\right)^{1/4}$$

with energy density from kinetic theory,

$$\epsilon = \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}|\mathbf{p}|f(\mathbf{p})$$

- Thermal forms of $\hat{q}_0$, μ_* known[5]



Late time
approache to thermal

[5] [Phys.Rev.D 78 (2008) [Arnold, Xiao]]