Glasma properties from a proper time expansion

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also: Alina Czajka, Bryce Friesen, Doug Pickering, J.Y. Ollitrault . . .

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Introduction

goal: describe early time ($\tau \leq 1$ fm) dynamics of heavy-ion collisions
- evolution of system during this early stage not well understood

• initial conditions for subsequent hydro evolution

• want to understand transition btwn the early-time dynamics and hydro phase
1. microscropic theory of non-abelian gauge fields (far from equilibrium)
$\rightarrow$
2. macroscopic effective theory based on universal conservation laws
- valid close to equilibrium

• important physics takes place in the glasma phase
- *fluid-like behaviour*
- *angular momentum of the glasma*
- *contribution to jet quenching (in the next talk)*

for more details see the review
M. E. Carrington and S. Mrówczyński, Acta Phys. Polon. B **55**, 1 (2024).

method - Colour Glass Condensate (CGC) effective theory

CGC = high energy density largely gluonic matter
- associated with wavefunction of a high energy hadron
- initial state in high energy hadronic collisions

after collision CGC fields are transformed into glasma fields
- initially longitudinal colour electric and magnetic fields

method is based on a separation of scales between
1. valence partons with large nucleon momentum fraction (x)
2. gluon fields with small x and large occupation numbers

basic picture

dynamics of gluon fields determined from classical YM equation
$\rightarrow$ source provided by the valence partons

L. D. McLerran and R. Venugopalan, Phys. Rev. D **49**, 2233 (1994); **49**, 3352 (1994); **50**, 2225 (1994).
A. Kovner, L. D. McLerran and H. Weigert, Phys. Rev. D **52**, 6231 (1995).
J. Jalilian-Marian, A. Kovner, L. D. McLerran and H. Weigert, Phys. Rev. D **55**, 5414 (1997).

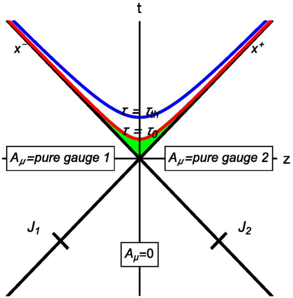
F. Gelis, E. Iancu, J. Jalilian-Marian and R. Venugopalan, Ann. Rev. Nucl. Part. Sci. **60**, 463 (2010).
T. Lappi, Int. J. Mod. Phys. E **20**, 1 (2011).

method - notation

light-cone coordinates: $x^{\pm} = (t \pm z)/\sqrt{2}$
Milne coordinates:
$\tau = \sqrt{2x^+x^-} = \sqrt{t^2 - z^2}$ and $\eta = \ln((t+z)/(t-z))$

sources: $J^\mu(x) = J_1^\mu(x) + J_2^\mu(x)$

$J_1^\mu(x) = \delta^{\mu +} g\rho_1(x^-, \vec{x}_\perp)$ and $J_2^\mu(x) = \delta^{\mu -} g\rho_2(x^+, \vec{x}_\perp)$

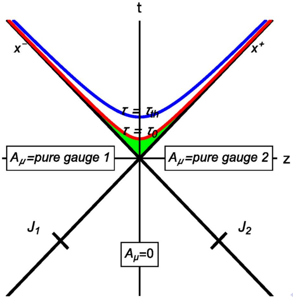


gauge : $A^{\mu}_{\mathrm{milne}} = \theta(\tau)\big(0, \alpha(\tau,\vec{x}_\perp), \vec{\alpha}_\perp(\tau,\vec{x}_\perp)\big)$

ansatz:

$$
\begin{aligned}
A^+(x) &= \Theta(x^+)\Theta(x^-)x^+\alpha(\tau,\vec{x}_\perp)\\
A^-(x) &= -\Theta(x^+)\Theta(x^-)x^-\alpha(\tau,\vec{x}_\perp)\\
A^i(x) &= \Theta(x^+)\Theta(x^-)\alpha^i_\perp(\tau,\vec{x}_\perp)\\
&+ \Theta(-x^+)\Theta(x^-)\beta^i_1(x^-,\vec{x}_\perp) + \Theta(x^+)\Theta(-x^-)\beta^i_2(x^+,\vec{x}_\perp)
\end{aligned}
$$

A. Kovner, L. D. McLerran and H. Weigert, Phys. Rev. D **52**, 6231 (1995).



description of method:

$$\underbrace{\rho^n(x^\pm,\vec{x}_\perp)}_{\text{static valence parton sources}} \rightarrow \underbrace{\beta^n(x^\pm,\vec{x}_\perp)}_{\text{CGC pre-collision fields}} \rightarrow \underbrace{\alpha(0,\vec{x}_\perp)}_{\text{initial glasma fields (boost invariant)}} \rightarrow \underbrace{\alpha(\tau,\vec{x}_\perp)}_{\text{glasma fields}}$$

parton sources $\rightarrow$ solve YM equation in the pre-collision region

apply boundary conditions $\rightarrow$ initial glasma potentials

$$\alpha^i_\perp(0,\vec{x}_\perp) = \alpha^{i(0)}_\perp(\vec{x}_\perp) = \lim_{w\to 0}\Big(\beta^i_1(x^-,\vec{x}_\perp) + \beta^i_2(x^+,\vec{x}_\perp)\Big)$$

$$\alpha(0,\vec{x}_\perp) = \alpha^{(0)}(\vec{x}_\perp) = -\frac{ig}{2}\lim_{w\to 0}\,[\beta^i_1(x^-,\vec{x}_\perp),\beta^i_2(x^+,\vec{x}_\perp)]$$

use a proper time expansion *(dimensionless small parameter is $\tilde{\tau} = \tau Q_s$)*

$$\alpha(\tau,\vec{x}_\perp) = \alpha(0,\vec{x}_\perp) + \tau\alpha^{(1)}(\vec{x}_\perp) + \tau^2\alpha^{(2)}(\vec{x}_\perp) + \cdots$$

$\rightarrow$ find gluon potentials at finite τ

R. J. Fries, J. I. Kapusta and Y. Li, Nucl. Phys. A **774**, *861 (2006).*
K. Fukushima, Phys. Rev. C **76**, *021902 (2007).*
H. Fujii, K. Fukushima and Y. Hidaka, Phys. Rev. C **79**, *024909 (2009).*
G. Chen, R. J. Fries, J. I. Kapusta and Y. Li, Phys. Rev. C **92**, *064912 (2015).*

$\rightarrow$ colour electric and magnetic fields $\Rightarrow$ observables

next: colour charge distributions are not known
CGC: use Gaussian distributed random variables and average
$\to$ use Wick's theorem and the glasma graph approximation $\cdots$

result for correlator of 2 potentials: $(\vec{\mathcal{R}} = \frac{1}{2}(\vec{x}_\perp + \vec{y}_\perp),\ \vec{r} = \vec{x}_\perp - \vec{y}_\perp)$

$$\delta_{ab} B^{ij}(\vec{x}_\perp, \vec{y}_\perp) \equiv \lim_{\mathrm{w}\to 0} \langle \beta^i_a(x^-, \vec{x}_\perp) \beta^j_b(y^-, \vec{y}_\perp) \rangle$$

$$\lim_{r\to 0} B^{ij}(\vec{x}_\perp, \vec{y}_\perp) = \delta^{ij} g^2 \frac{\mu(\vec{\mathcal{R}})}{8\pi} \left(\ln\left(\frac{Q_s^2}{m^2} + 1\right) - \frac{Q_s^2}{Q_s^2 + m^2} \right) + \cdots$$

J. Jalilian-Marian, A. Kovner, L. McLerran, H. Weigert, Phys. Rev. D **55**, *5414 (1997).*

infra-red regulator $m \sim \Lambda_{\rm QCD} \sim 0.2$ GeV
ultra-violet regulator = saturation scale = $Q_s = 2$ GeV

$\mu(\vec{\mathcal{R}})$ is a surface colour charge density

dots indicate kept terms to 2nd order in grad expansion of $\mu(\vec{\mathcal{R}})$

collisions with non-zero impact parameter (non-central collisions)
$\Rightarrow$ expand $\mu_{1/2}(\vec{\mathcal{R}})$ around $\vec{R} \pm \vec{b}/2$
$\vec{R}$ is distance from center of collision

surface charge density μ

must specify the form of the surface colour charge density $\mu(\vec{x}_\perp)$
- 2-dimensional projection of a Woods-Saxon distribution

$$\mu(\vec{x}_\perp) = \left(\frac{A}{207}\right)^{1/3} \frac{\bar{\mu}}{2a\log(1+e^{R_A/a})} \int_{-\infty}^{\infty} dz \frac{1}{1+\exp\left[(\sqrt{(\vec{x}_\perp)^2+z^2}-R_A)/a\right]}$$

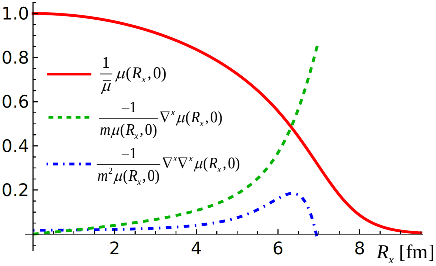
R_A and a = radius and skin thickness of nucleus mass number A
$r_0 = 1.25$ fm, $a = 0.5$ fm $\rightarrow$ when $A = 207$ gives
$R_A = r_0 A^{1/3} = 7.4$ fm

normalization: $\mu(\vec{0}) = \bar{\mu} = Q_s^2/g^4$ lead nucleus
$g^2\sqrt{\bar{\mu}}$ = McLerran-Venugopalan (MV) scale
proportional to Q_s - exact value not determined with CGC approach

** numerical results for $\mathcal{E}\ldots$ are order of magnitude estimates
ratios of different elements of the energy momentum tensor
$\rightarrow$ will have much weaker dependence on this factor

gradient expansion

a parameter that we assume small is $\delta = \dfrac{|\nabla^i \mu(\vec{R})|}{m\mu(\vec{R})}$



the curves show colour charge density and its derivatives

- gradient expansion can be trusted for $\mathcal{R} \lesssim \mathcal{R}_{\max} = 6.5$ fm

- where magnitude of derivatives $\lesssim$ 50% of charge density itself

summary of method:

YM eqn with average over Gaussian distributed valence sources

$\rightarrow$ correlators of pre-collision fields

$\rightarrow$ glasma field correlators (b. conds, sourceless YM eqn, τ exp)

$\rightarrow$ correlators of glasma chromodynamic $\vec{E}$ and $\vec{B}$ fields

$\Rightarrow$ observables

we work to order τ^8 and study

1. isotropization of transverse/longitudinal pressures
2. hydrodynamic behaviour
3. azimuthal momentum distribution and spatial eccentricity
4. angular momentum
5. momentum broadening of hard probes - talk by Stanisław Mrówczyński

comment: many numerical approaches to study initial dynamics
our method is fully analytic
- allows control over different approximations and sources of errors
- can be systematically extended
- it has limitations
- *classical / no fluctuations of positions of nucleons*

isotropization

at $\tau = 0^+$ the energy-momentum tensor has the diagonal form

$$T(\tau = 0) = \begin{pmatrix} \mathcal{E}_0 & 0 & 0 & 0 \\ 0 & -\mathcal{E}_0 & 0 & 0 \\ 0 & 0 & \mathcal{E}_0 & 0 \\ 0 & 0 & 0 & \mathcal{E}_0 \end{pmatrix}$$

$\rightarrow$ the longitudinal pressure is large and negative
- system is far from equilibrium

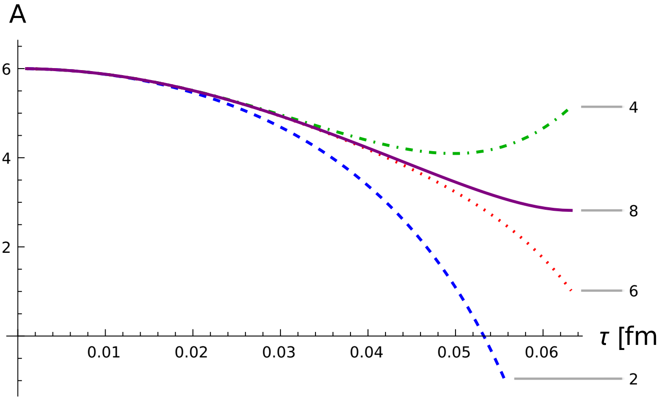
if the system approaches equilibrium as it evolves
- the longitudinal pressure must grow
- transverse pressure must decrease ($T_{\mu\nu}$ is traceless)

to study pressure isotropization

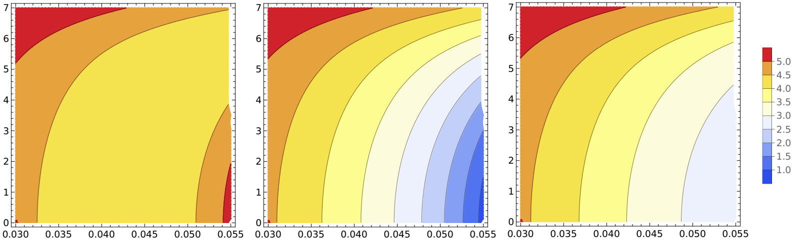
$$A_{TL} \equiv \frac{3(p_T - p_L)}{2p_T + p_L}$$

J. Jankowski, S. Kamata, M. Martinez and M. Spaliński, Phys. Rev. D **104**, 074012 (2021).

in equilibrium $(p_L = p_T = \mathcal{E}/3) \longrightarrow A_{TL} = 0$



$R = 5$ fm, $\eta = 0$ and $b = 0$



3 panels show A_{TL} at 4,6,8 order in the proper time expansion

vertical/horizontal axes are R and τ in fm

dependence on confinement and saturation scales

the correlator $\langle \beta_a^i(x^-,\vec{x}_\perp)\beta_b^j(y^-,\vec{y}_\perp)\rangle$

depends on two regulators: m (infra-red) and Q_s (ultra-violet)
- physically related to confinement / saturation scales
$\rightarrow$ constraints on how to choose them

we used: $m = 0.2$ GeV and $Q_s = 2.0$ Gev - standard choices

- want results $\approx$ independent of these numbers

- especially since the two scales are pretty close together

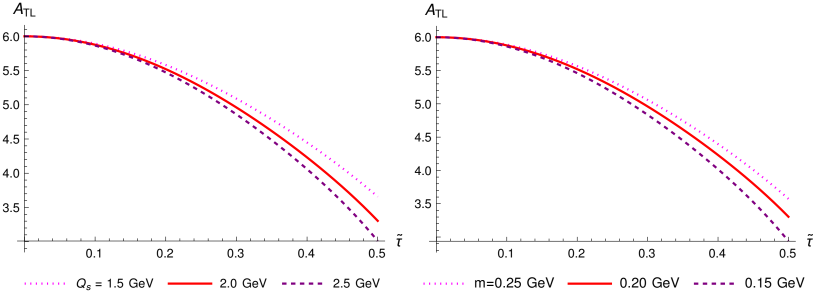
A_{TL} at order τ^6 as a function of time

3 different values of Q_s with $m = 0.2$ GeV (left)

3 different values of m with $Q_s = 2.0$ GeV (right)

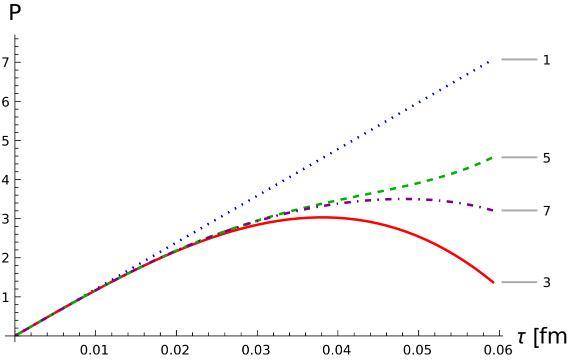
$R = 5$ fm, $b = 0$ and $\eta = 0$ at order τ^6

$\Rightarrow$ dependence on these scales is weak



Radial Flow

transverse momentum flow vector $= T_{i0}$ (trans. Poynting vector)
radial flow of the expanding glasma = radial projection $P \equiv \hat{R}_i T_{i0}$
$R = 3$ fm, $b = 1$ fm, $\phi = \pi/2$ (perpendicular to the reaction plane)



- P slows as system expands
- limit of validity of the expansion $\tau \sim 0.06$ fm

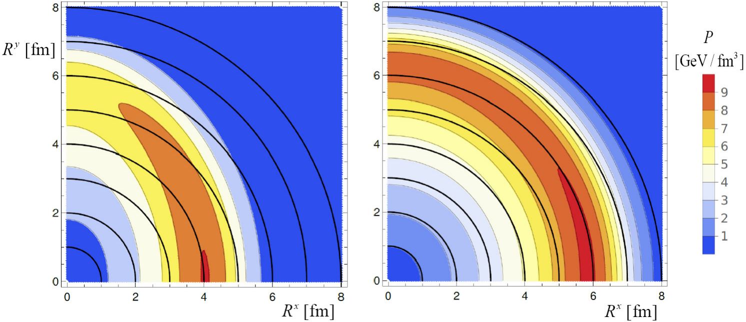
reaction plane defined by collision axis and impact parameter

$\vec{b} = b\hat{i} \rightarrow$ reaction plane is x-z

$\phi = 0$ is in reaction plane

$\phi = \pi/2$ is perpendicular to reaction plane

expect radial flow greater at $\phi = 0$



left panel $b = 6$ fm $\rightarrow R \lesssim 5$ fm

right panel $b = 2$ fm $\rightarrow R \lesssim 7$ fm

azimuthal asymmetry

in a non-central collision

- initial spatial asymmetry - moments of the energy density

$\rightarrow$ final state momentum asymmetry

physically: momentum anisotropy is generated by pressure gradients

asymmetry in momentum from Fourier coefficients of the flow

$\cdots$ write in terms of components T^{00}, T^{0x} and T^{0y}

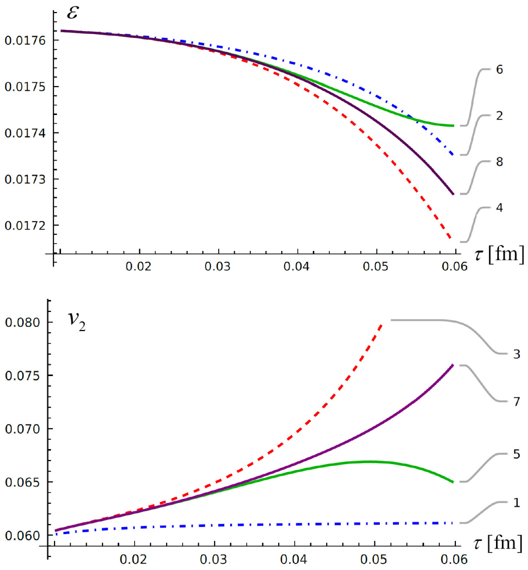
$$\varepsilon = -\frac{\int d^2R \, \frac{R_x^2 - R_y^2}{\sqrt{R_x^2 + R_y^2}} \, T^{00}}{\int d^2R \, \sqrt{R_x^2 + R_y^2} \, T^{00}} \qquad \text{and} \qquad v_2 = \frac{\int d^2R \, \frac{T_{0x}^2 - T_{0y}^2}{\sqrt{T_{0x}^2 + T_{0y}^2}}}{\int d^2R \, \sqrt{T_{0x}^2 + T_{0y}^2}}$$

in a relativistic collision spatial asymmetries rapidly decrease

$\rightarrow$ anisotropic momentum flow can develop only in the first fm/c

• sensitive to system properties very early in its evolution

• provides direct information about the early stages of the system



$\mathcal{E}$
0.0176
0.0175
0.0174
0.0173
0.0172
0.02
0.03
0.04
0.05
0.06
τ [fm]
6
2
8
4
v_2
0.080
0.075
0.070
0.065
0.060
0.02
0.03
0.04
0.05
0.06
τ [fm]
3
7
5
1

hydrodynamics and QGP

surprising $\rightsquigarrow$ at very early times system far from equilibrium

but we know hydro $\rightarrow$ good description of QGP at $\tau \gtrsim 1$ fm/c

different explanations for early onset of hydrodynamics:

1. non-equilibrium systems evolve along hydro-attractor trajectory

2. universal flow:

if EMT is traceless & initially diagonal; system is boost-invt
$\rightarrow$ the early behaviour governed by a universal flow equation
$\longrightarrow$ the system evolves like a fluid

J. Jankowski and M. Spaliński, Prog. Part. Nucl. Phys. **132**, 104048 (2023)
J. Vredevoogd and S. Pratt, Phys. Rev. C **79**, 044915 (2009).

hydrodynamics

M. E. Carrington, S. Mrowczynski and J. Y. Ollitrault, Phys. Rev. C **110**, 054903 (2024).

how to assess extent to which system behaves hydrodynamically ?

recall: ideal hydrodynamic assumes a specific structure of the EMT

$$T^{\mu\nu}_{\rm ideal} = (\varepsilon + p)u^\mu u^\nu - p\, g^{\mu\nu}$$

ε is the energy density
p is the pressure
u^μ is the fluid velocity

anisotropic hydrodynamics

initial longitudinal pressure is negative $\Rightarrow$

modification of ideal hydro gives better description

$$T^{\mu\nu}_{\mathrm{aniso}} = (\varepsilon + p_T)u^\mu u^\nu - p_T g^{\mu\nu} - (p_T - p_L)z^\mu z^\nu$$

p_T and p_L are the transverse and longitudinal pressures

four-vector z^μ defines the direction of the longitudinal pressure

space-like and normalized $z^\mu z_\mu = -1$

orthogonal to the fluid four-velocity $(u^\mu z_\mu = 0)$

W. Florkowski and R. Ryblewski, Phys. Rev. C **83**, 034907 (2011)

M. Martinez and M. Strickland, Nucl. Phys. A **848**, 183 (2010).

Q: to what extent can the glasma EMT be represented by $T^{\mu\nu}_{\rm aniso}$?

construct the hydro form:

find eigenvalues and eigenvectors of $T^{\mu\rho}_{\rm glasma} g_{\rho\nu} u^\nu = \lambda u^\mu$
- at each point in the transverse plane

- 1 timelike vector which we take to be the fluid velocity
- corresponding eigenvalue (the largest) is the energy
- the negative eigenvalue is the longitudinal pressure
- z^μ can be determined in two ways

- the eigenvector of the negative eigenvalue
- use fluid velocity to construct L-transf to act on $z^\mu_{\rm lrf} = (0,1,0,0)$

$\rightarrow$ determines $T^{\mu\nu}_{\rm aniso}$ (or $T^{\mu\nu}_{\rm ideal}$)

quantify difference between glasma EMT $T^{\mu\nu}_{\rm glasma}$ and $T^{\mu\nu}_{\rm hydro}$

$$\sigma^{\mu\nu}_{\rm hydro} = \sqrt{\frac{\sum_j \left(T^{\mu\nu}_{\rm glasma}(\vec{x}^{\,j}_\perp) - T^{\mu\nu}_{\rm hydro}(\vec{x}^{\,j}_\perp)\right)^2}{\sum_j \left(T^{\mu\nu}_{\rm glasma}(\vec{x}^{\,j}_\perp) + T^{\mu\nu}_{\rm hydro}(\vec{x}^{\,j}_\perp)\right)^2}}$$

sums are over all points in the transverse plane

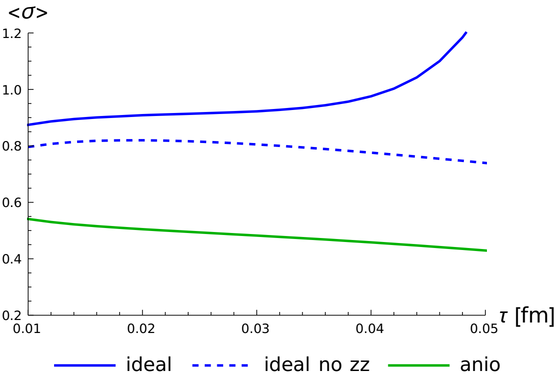


figure shows $\sigma^{\mu\nu}$ averaged over all components $(b = \eta = 0)$

$T^{\mu\nu}_{\rm anio}$ matches $T^{\mu\nu}_{\rm CGC}$ much better (as expected)

- decreases with time (system approaches anisotropic hydro)

remove T^{zz} from $T^{\mu\nu}_{\rm ideal} \to$ a much better match

angular momentum

M. E. Carrington and S. Mrowczynski, Phys. Rev. C **113**, 064910 (2026).

define tensor $M^{\mu\nu\lambda} = T^{\mu\nu} R^\lambda - T^{\mu\lambda} R^\nu$

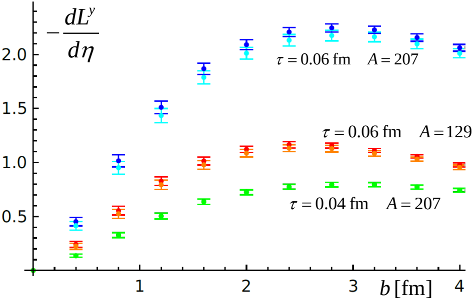
$\nabla_\mu M^{\mu\nu\lambda} = 0 \to$ conserved charges $J^{\nu\lambda} = \int_\Sigma d^3y \sqrt{|\gamma|} n_\mu M^{\mu\nu\lambda}$
- n^μ is a unit vector perpendicular to the hypersurface Σ
- γ is the induced metric on this hypersurface
- d^3y is the corresponding volume element

$n^\mu = (1, 0, 0, 0)$ in Milne coordinates
$\to$ $J^{\nu\lambda}$ defined on a hypersurface of constant τ

Pauli-Lubanski vector: $L_\mu = -\frac{1}{2} \epsilon_{\mu\alpha\beta\gamma} J^{\alpha\beta} u^\gamma$

global angular momentum per unit rapidity (symmetric collision)

$$\frac{dL^y}{d\eta} = -\tau^2 \int d^2\vec{R}\, R^x\, T^{0z}$$



sign is negative because of our collision geometry

- ion moving in $+$ve z-dirn has center displace in $+$ve x-dirn

initially increases with b

drops as ions become widely separated $\rightarrow$ interaction region $\downarrow$

important point

result is much smaller than $\vec{L}$ of initial system of colliding ions

interpretation: small fraction of initial $\vec{L}$ is transmitted to glasma

consistent with experimental results

idea: initial rotation of glasma
$\rightarrow$ could be observed via polarization of final state hadrons

- large $\vec{L}$ & spin-orbit coupling $\rightarrow$ alignment of spins with $\vec{L}$

- searches for polarization of produced Λ particles

- effect of a fraction of a percent has been seen

see reviews
F. Becattini, M. Buzzegoli, T. Niida, S. Pu, A. H. Tang and Q. Wang, Int. J. Mod. Phys. E **33**, 2430006 (2024).
T. Niida and S. A. Voloshin, Int. J. Mod. Phys. E **33**, 2430010 (2024).

local angular momentum in the z-direction at the point $\vec{R}_0$

$$\frac{dL^z(\vec{R}_0)}{d\eta} = -\tau \int d^2R \left[(R^y - R_0^y)\, T^{0x}(\vec{R}) - (R^x - R_0^x)\, T^{0y}(\vec{R})\right]$$

integral calculated over small region $d^2R = \Delta^2$ centered on pt $\vec{R}_0$

a related quantity is vorticity (we define: $V^i(\vec{r}) \equiv T^{0i}(\vec{r})/T^{00}(\vec{r})$)

$$\omega^z(\vec{r}) \equiv \left(\vec{\nabla} \times \vec{V}(\vec{r})\right)^z = -\left(\frac{\partial V^x(\vec{r})}{\partial y} - \frac{\partial V^y(\vec{r})}{\partial x}\right)$$

compare with hydro definition of thermal vorticity

$\omega^z = (\vec{\nabla} \times \vec{\beta}(\vec{r}))^z$

$\vec{\beta}(\vec{r})$ is the spatial part of $\beta^\mu = u^\mu(\vec{r})/T(\vec{r})$

$T(\vec{r})$ is position dependent temperature $\sim \mathcal{E}(\vec{r})^{1/4}$

results for $\omega^z(\vec{r})$ depend on spatial dependence of $T(\vec{r})$

but hydro assumes that thermodynamic variables depend weakly on $\vec{r}$ ...

quadrapole moments of vorticity and local L_z

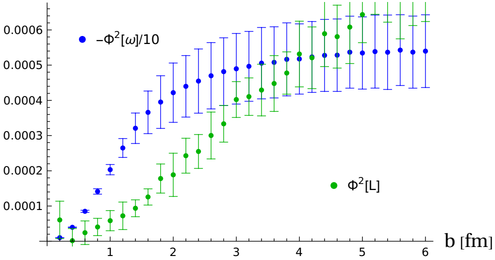
$$\Phi^2(\omega) \equiv \int_0^{r_{\rm max}} dr\, r \int_0^{2\pi} d\phi \sin(2\phi)\, \omega^z(r,\phi)$$
$$\Phi^2(L) \equiv \frac{1}{\Delta^6} \int_0^{r_{\rm max}} dr\, r \int_0^{2\pi} d\phi \sin(2\phi)\, L^z(r,\phi)$$

$r_{\rm max}$ is the largest r for which the gradient expansion is reliable

note: no direct comparison is possible

- the units are fm^{-1} for $\Phi^2(\omega)$ and fm^{-6} for $\Phi^2(L)$

local L^z is calculated in a box of length Δ
- factor of $1/\Delta^6$ removes dependence on size of this box



important point: they do not have the same sign

* the sign of local L_z agrees with experiments

hydrodynamic thermal vorticity has same sign as our vorticity result

including 'shear effects' sign changes $\rightarrow$ agrees with experiments

caution:

1. all of our results are valid only at very early times

- some hydro features of QGP in local equilib. are present in glasma at early τ

2. a quantitative way to relate L^z to polarization to be developed

message:

vorticity not equivalent to local angular momentum

conclusions

1. proper time expansions a useful to study glasma dynamics
2. fully analytic method
   - control over different approximations / sources of errors
   - can be systematically extended
3. many physical characteristics of the system are well described
   - approach to equilibrium
   - early onset of hydrodynamics
   - angular momentum of glasma
   - jet broadening