JET MOMENTUM BROADENING IN VISCOUS QCD MATTER: A MOMENT EXPANSION APPROACH

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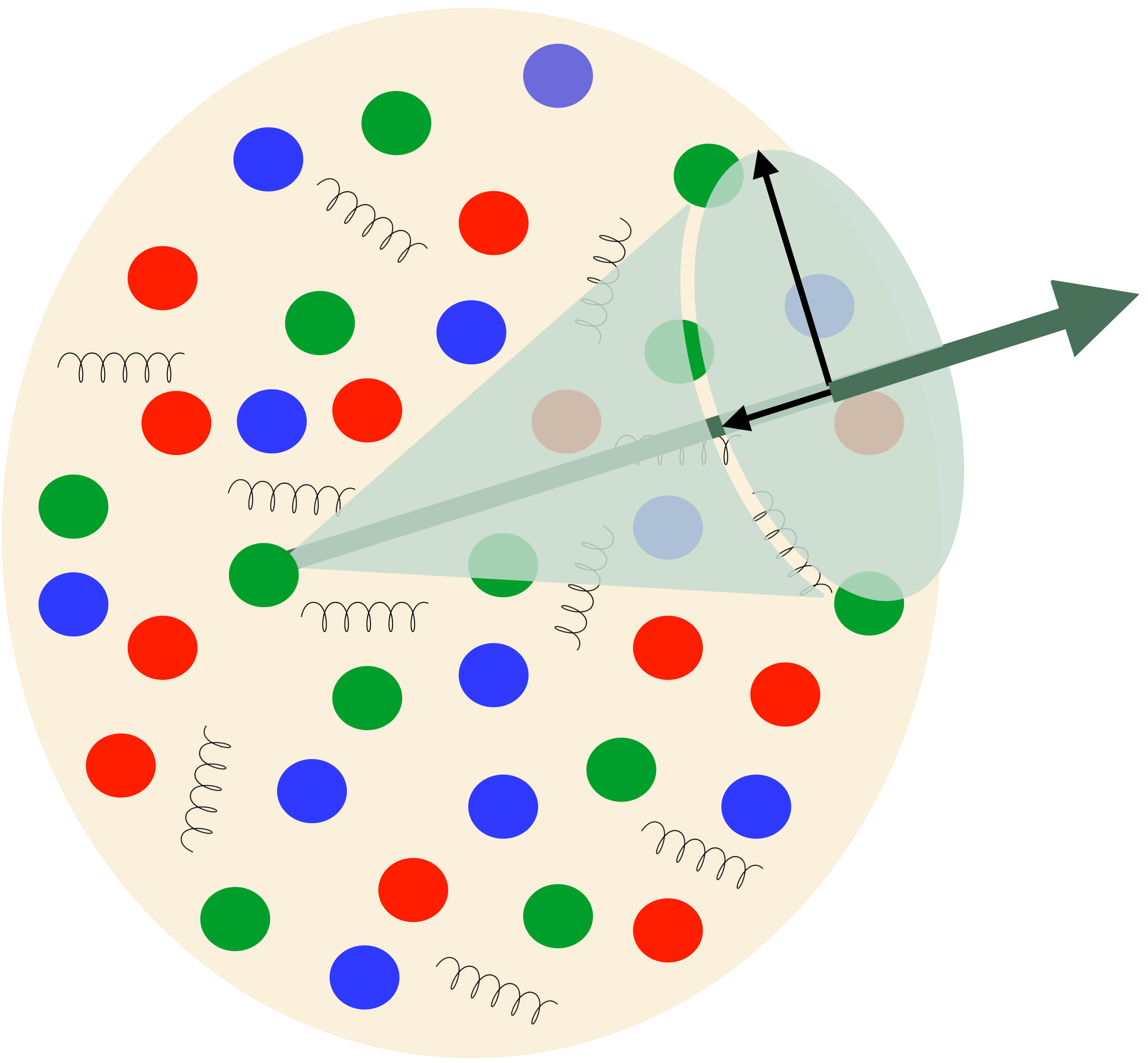
arXiv:2605.12791 [hep-ph]

ILLINOIS

muses

Introduction

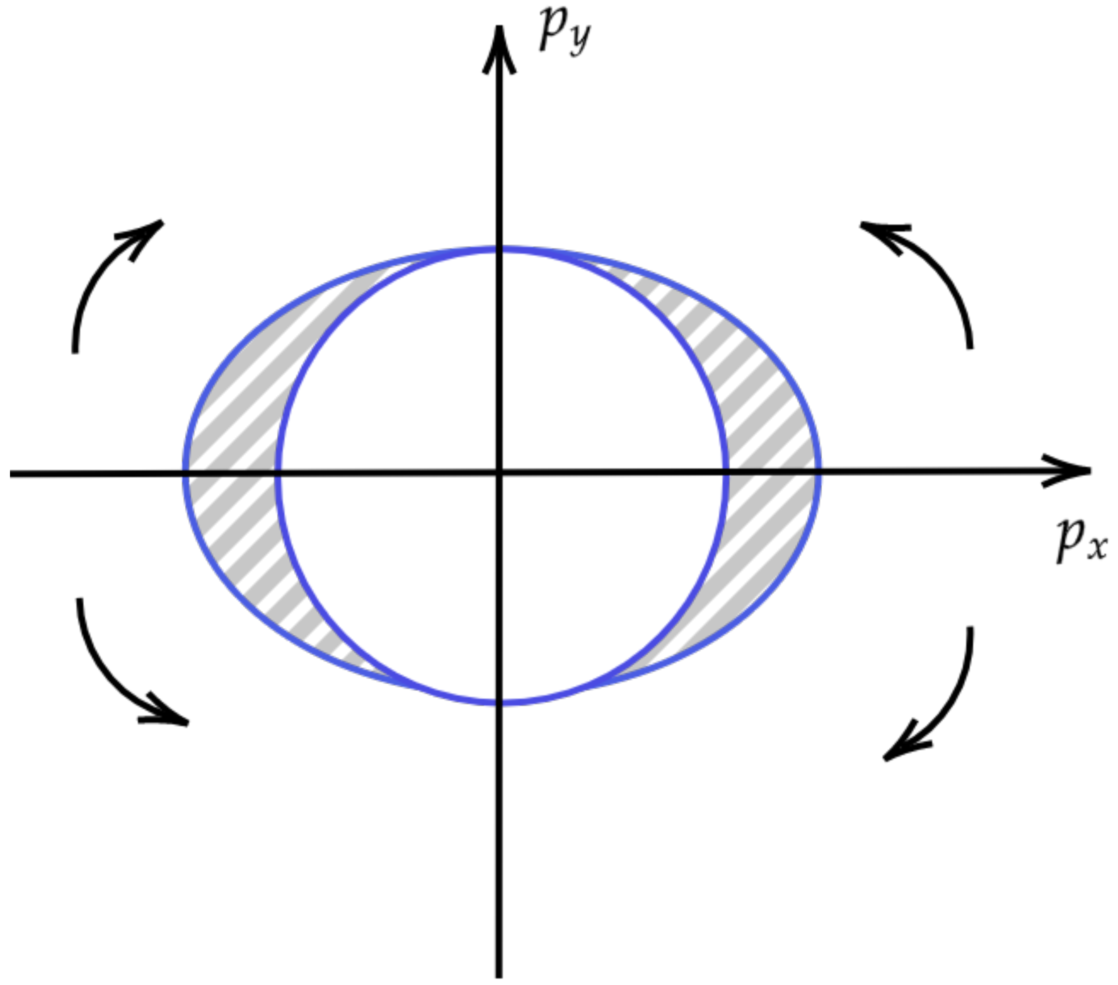
Kinetic Theory Picture



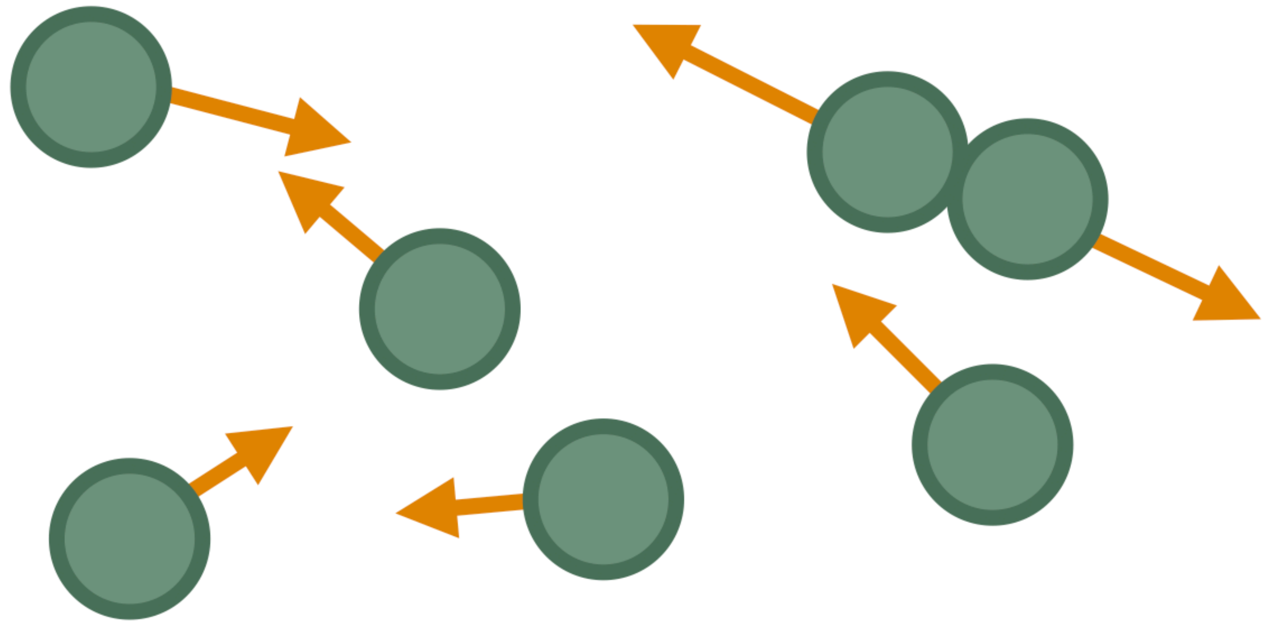
The dynamics of the system are determined from a Boltzmann equation:

$$\left[\frac{\partial}{\partial t} + \vec{v}_p \cdot \frac{\partial}{\partial \vec{x}} + \vec{F}_{\text{ext}} \cdot \frac{\partial}{\partial \vec{p}}\right] f^a(\vec{p}, \vec{x}, t) = -\mathscr{C}^a[f]$$

Describes propagation



Scatterings

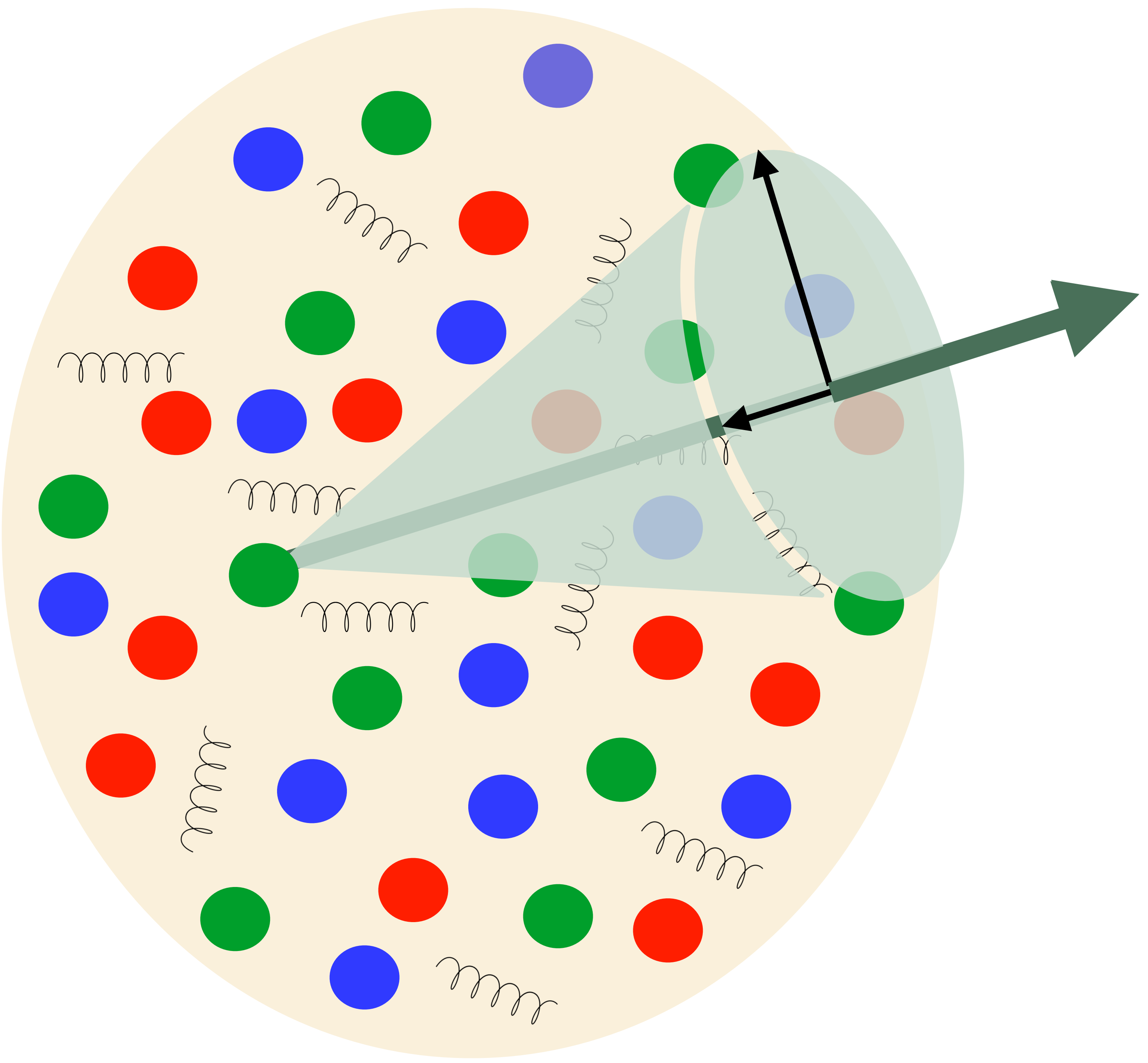


arXiv:0811.1603 [hep-ph]

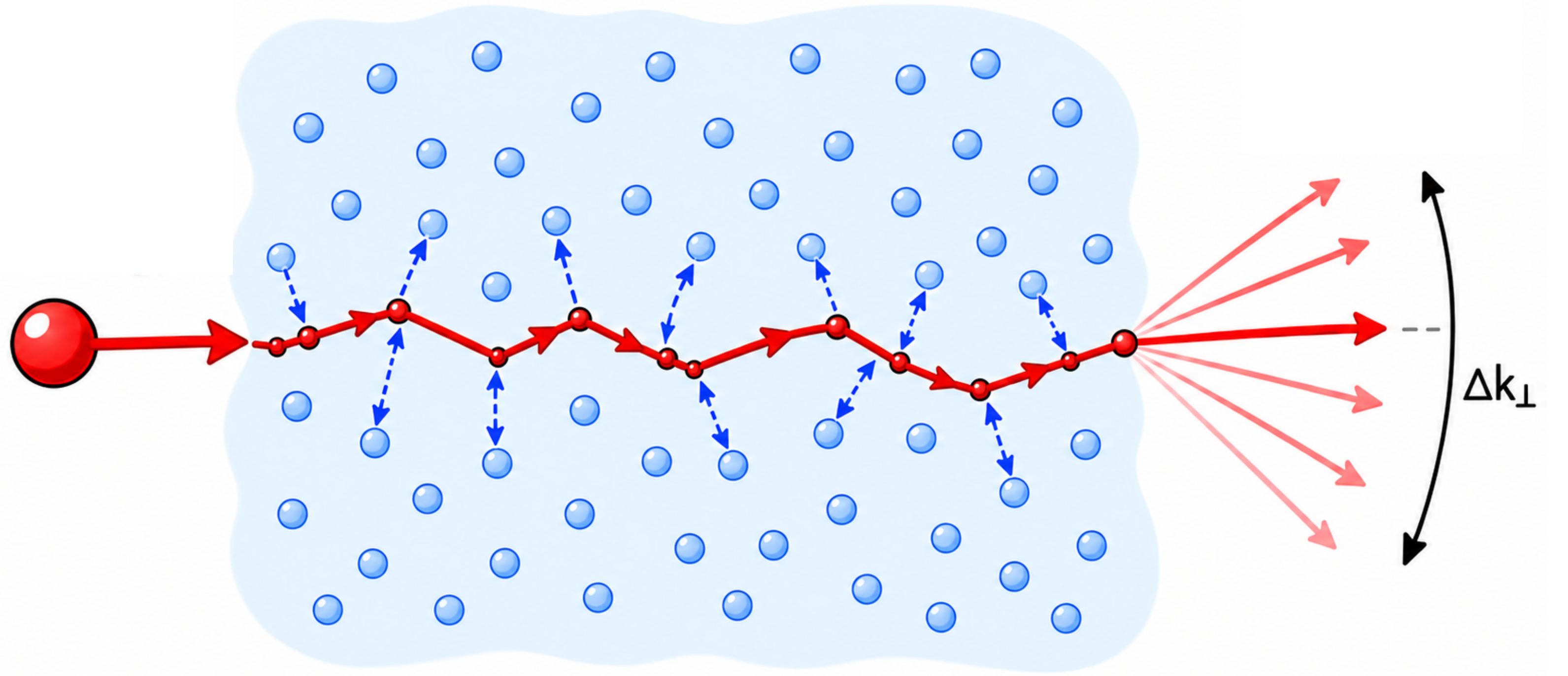
The medium is out of equilibrium, so $f^a = f_0^a + \delta f^a$

Introduction

Kinetic Theory Picture



Jet momentum broadening is generated by the cumulative effect of microscopic scatterings with medium constituents



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$$\hat{q} = \int d^2\vec{q}_\perp \, \vec{q}_\perp^{\,2} \frac{d\Gamma_{a,\mathrm{el}}}{d^2\vec{q}_\perp},$$

with the elastic scattering rate related directly to:

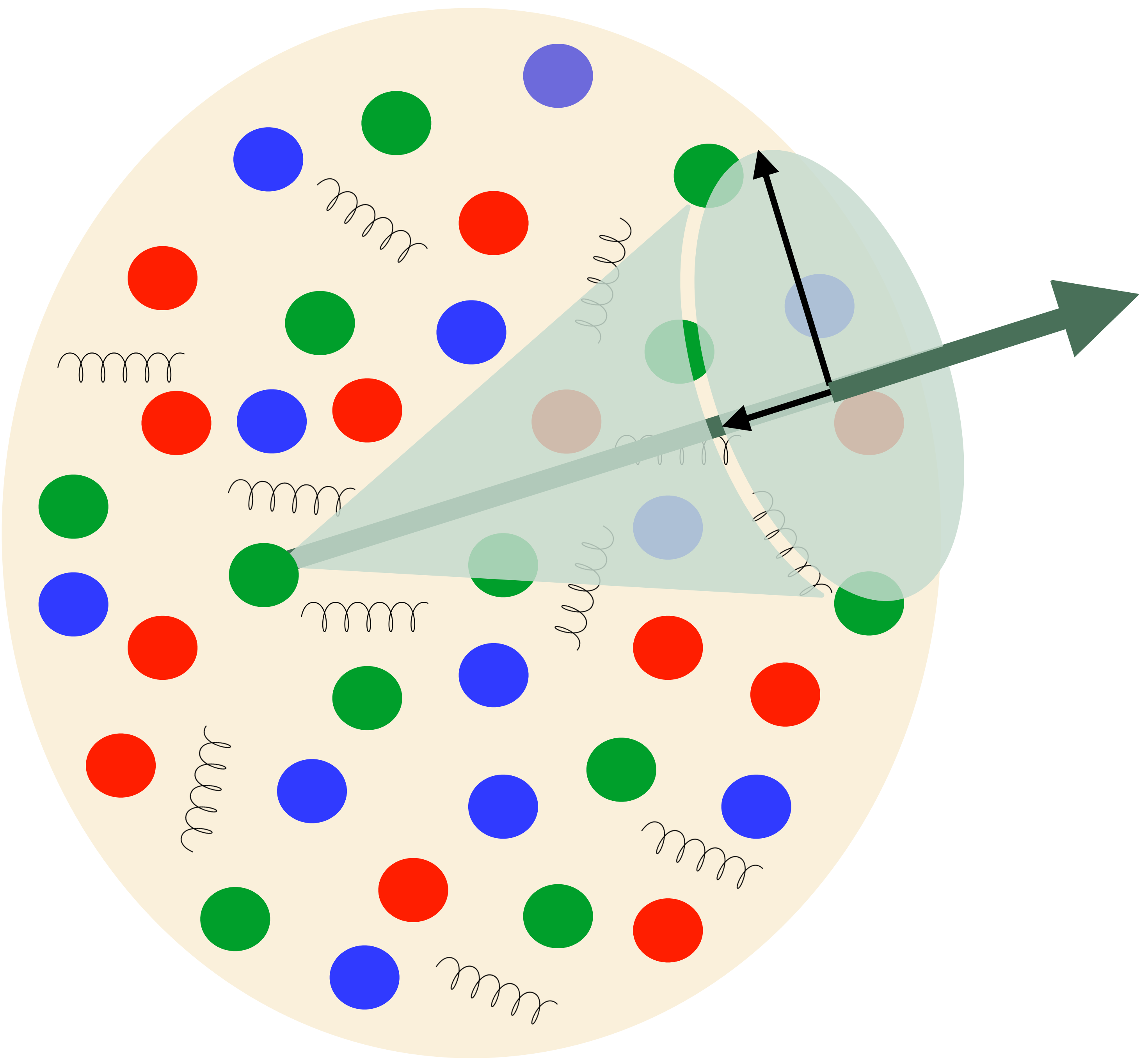
$$\Gamma_{a,\mathrm{el}}[\mathbf{k}] = \frac{1}{2}\int dP\, dK'\, dP'\, \mathscr{W}_a\, f_{\mathbf{k}'} \tilde{f}_{\mathbf{p}} \tilde{f}_{\mathbf{p}'}$$

where $\mathscr{W}_a = \mathscr{W}_{pp' \leftrightarrow kk'}$ denotes the relevant scattering kernel (rate) for the process.

These two define the scalar:

$$\hat{q}[\mathbf{k}] = \frac{1}{2|\mathbf{k}|}\int dP\, dK'\, dP' \vec{q}_\perp \vec{q}_\perp \mathscr{W}_a f_{\mathbf{k}'} \tilde{f}_{\mathbf{p}} \tilde{f}_{\mathbf{p}'}$$

Introduction



Isotropic medium

QCD Kinetic Theory Picture

The jet quenching parameter $\hat{q}$ is determined by the elastic scattering rate:

$$\hat{q}^{ij} = \frac{(2\pi)^4}{4p\,\nu_a} \sum_{bcd} \int d\Gamma_{kp'k'} \delta^{(4)}(P+K-P'-K') \left| \mathcal{M}^{ab}_{cd} \right|^2 q^i q^j f_b(X,K) \tilde{f}_c(X,P') \tilde{f}_d(X,K')$$

arXiv:2303.12520 [hep-ph]

arXiv:2603.00844 [hep-ph]

$\hat{q}$ is intrinsically tied to a specific frame,

and to a specific choice of transverse plane:

Azimuthal symmetry around the beam axis:

$$\hat{q}^{12} = \hat{q}^{23} = 0$$

$$\hat{q} = \hat{q}_{11} + \hat{q}_{22}$$

$$\hat{q}^{ij} = \begin{pmatrix} q^{11} & 0 & 0 \\ 0 & q^{22} & 0 \\ 0 & 0 & q^{33} \end{pmatrix}$$

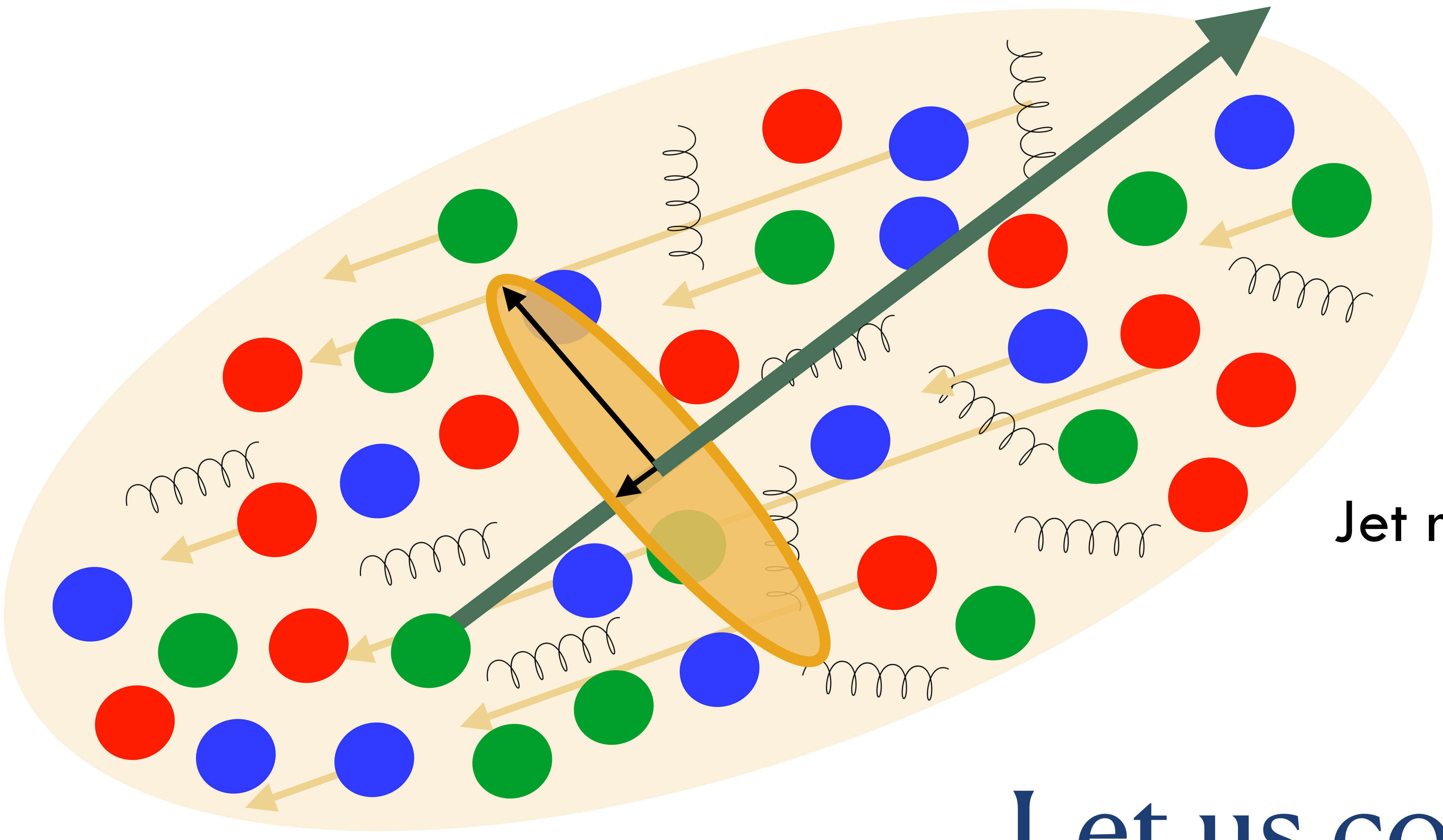
Spherical symmetry of the phase-space:

$$\hat{q}^{11} = \hat{q}^{22}$$

$$\hat{q}^{12} = \hat{q}^{13} = \hat{q}^{23} = 0$$

How do jet momentum broadening and the dissipative currents of viscous relativistic hydrodynamics connect in the QGP evolution?

Jet momentum broadening out of equilibrium

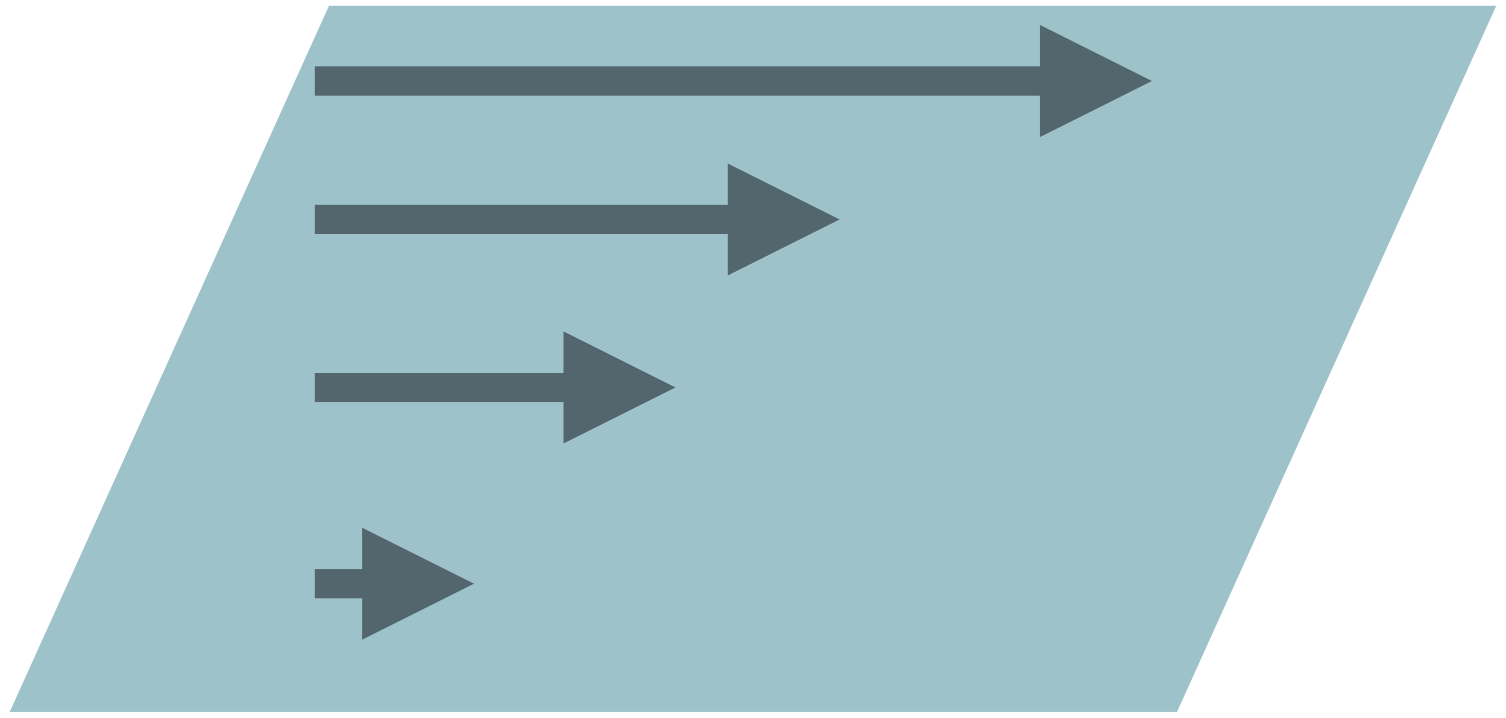


The broadening process is intrinsically direction-dependent.

There is no symmetry reason for the broadening to reduce to a single scalar

Jet momentum broadening becomes: $\hat{q}^{ij} = \begin{pmatrix} q^{11} & q^{12} & q^{13} \\ q^{12} & q^{22} & q^{23} \\ q^{13} & q^{23} & q^{33} \end{pmatrix}$ $\hat{q}^{12} \neq \hat{q}^{13} \neq \hat{q}^{23} \neq 0,$ $\hat{q}^{11} \neq \hat{q}^{22}.$

Let us consider a fluid with a shear flow!



General tensorial structure of $\delta\hat{q}^{ij}$

What symmetry constraints does this tensor have?

- $\delta\hat{q}$ is constrained by rotational symmetry in the LRF
- Symmetry under $i \leftrightarrow j$
- Linearity in the shear-stress tensor (linear response)

The structure of this tensor is fixed by symmetry and is model independent!

Only numerical factors will depend on the microscopic dynamics.

arXiv:1401.3751 [hep-ph]
arXiv:2312.00447 [hep-ph]

General tensorial structure of $\delta\hat{q}^{ij}$

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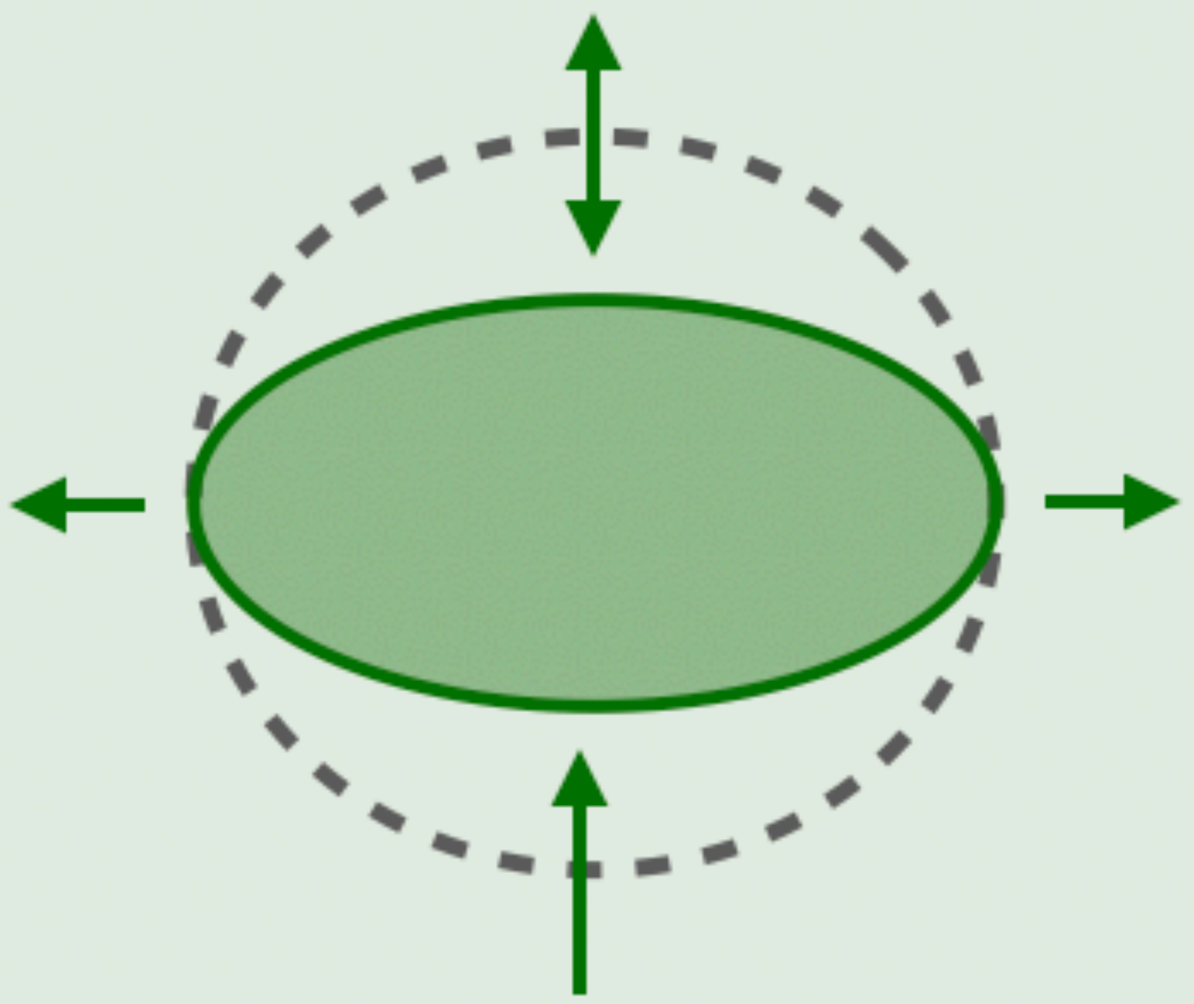
Let us begin with the simplest possible tensor structure, and write something linear in π^{ij}:

$$\delta\hat{q}^{ij} = \boxed{\alpha\, \pi^{ij}}$$

This is not enough - our system has a preferred direction $\hat{n}$, the jet direction!

Direct shear contribution

$\alpha\, \pi^{ij}$



Pure shear (traceless): Stretches in one direction and squeezes in the orthogonal with no overall size change

arXiv:1401.3751 [hep-ph]
arXiv:2312.00447 [hep-ph]

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What symmetry constraints does this tensor have?

- $\delta\hat{q}$ is constrained by rotational symmetry in the LRF
- Symmetry under $i \leftrightarrow j$
- Linearity in the shear-stress tensor (linear response)

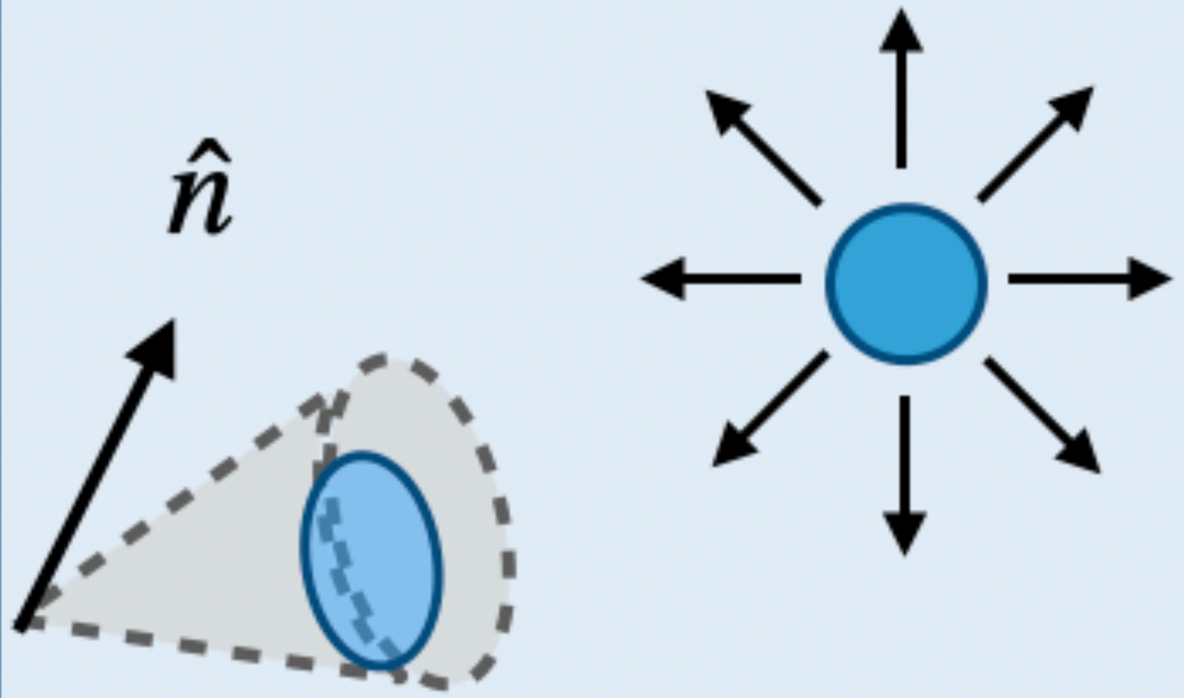
Let us now use the available tensors $\delta^{ij}$and the jet direction $\hat{n}^i$:

$$\delta\hat{q}^{ij} = \boxed{\alpha\,\pi^{ij}} + \boxed{\beta\,\delta^{ij}\,\hat{n}^a\hat{n}^b\pi_{ab}}$$

We now have the direct shear plus the isotropic projection term — but one contribution is still missing.

Isotropic contribution from projection along $\hat{n}$

$$\beta\,\delta^{ij}\,\hat{n}^a\hat{n}^b\pi_{ab}$$



Project π^{ij} along the direction $\hat{n}$ to obtain the scalars $= \hat{n}^a\hat{n}^b\pi_{ab}$, then distribute it isotropically via δ^{ij}

arXiv:1401.3751 [hep-ph]
arXiv:2312.00447 [hep-ph]

General tensorial structure of $\delta\hat{q}^{ij}$

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Let us now use the available tensors $\delta^{ij}$and the jet direction $\hat{n}^i$:

$$\delta\hat{q}^{ij} = \boxed{\alpha\,\pi^{ij}} + \boxed{\beta\,\delta^{ij}\,\hat{n}^a\hat{n}^b\pi_{ab}}$$

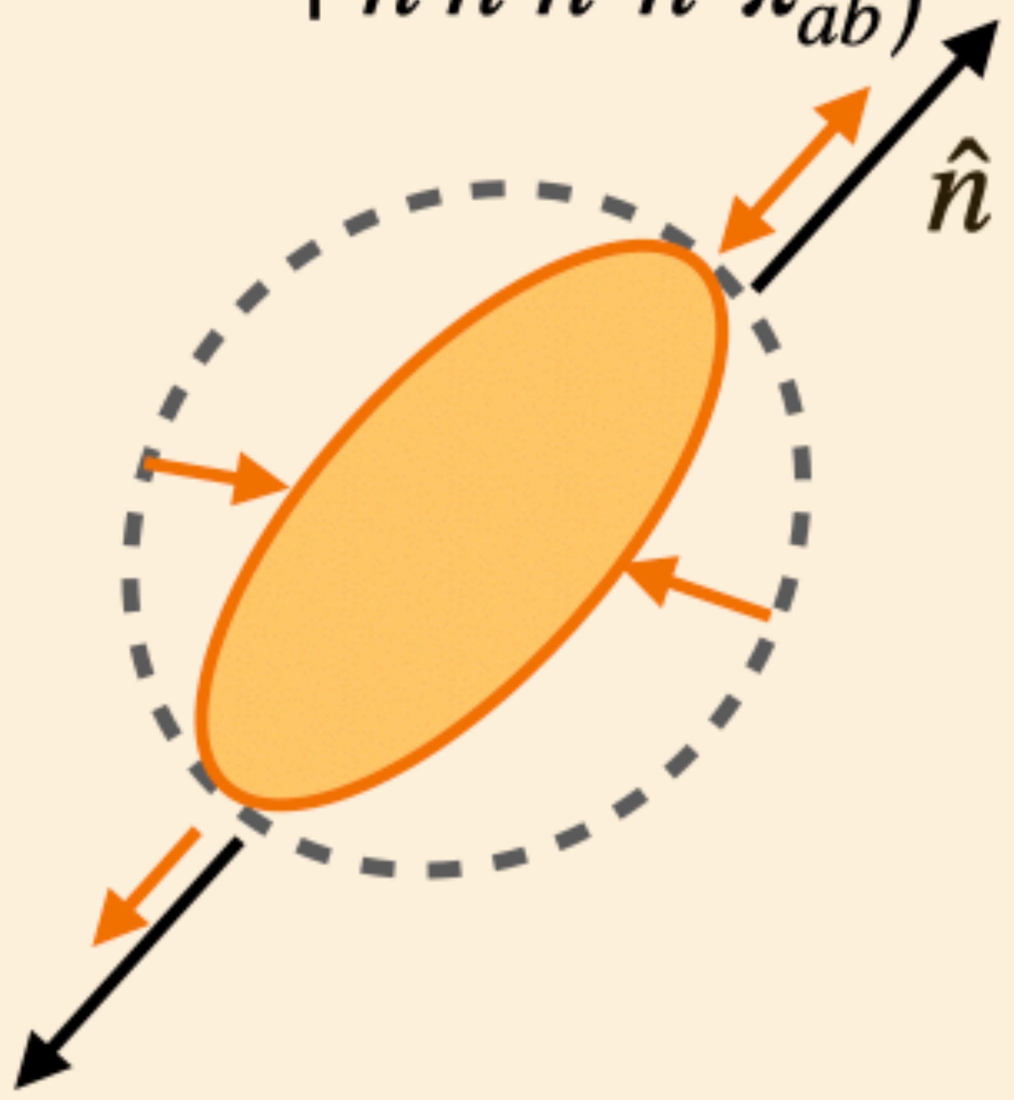
$$\boxed{+\gamma\,\left(\hat{n}^i\pi^{ja}\hat{n}_a + \hat{n}^j\pi^{ia}\hat{n}_a + \hat{n}^i\hat{n}^j\hat{n}^a\hat{n}^b\pi_{ab}\right)}$$

arXiv:2605.12791 [hep-ph]

Together, these structures span every independent tensor allowed in the LRF!

Jet direction-selective

$$\gamma\,\left(\hat{n}^i\pi^{ja}\hat{n}_a + \hat{n}^j\pi^{ia}\hat{n}_a + \hat{n}^i\hat{n}^j\hat{n}^a\hat{n}^b\pi_{ab}\right)$$

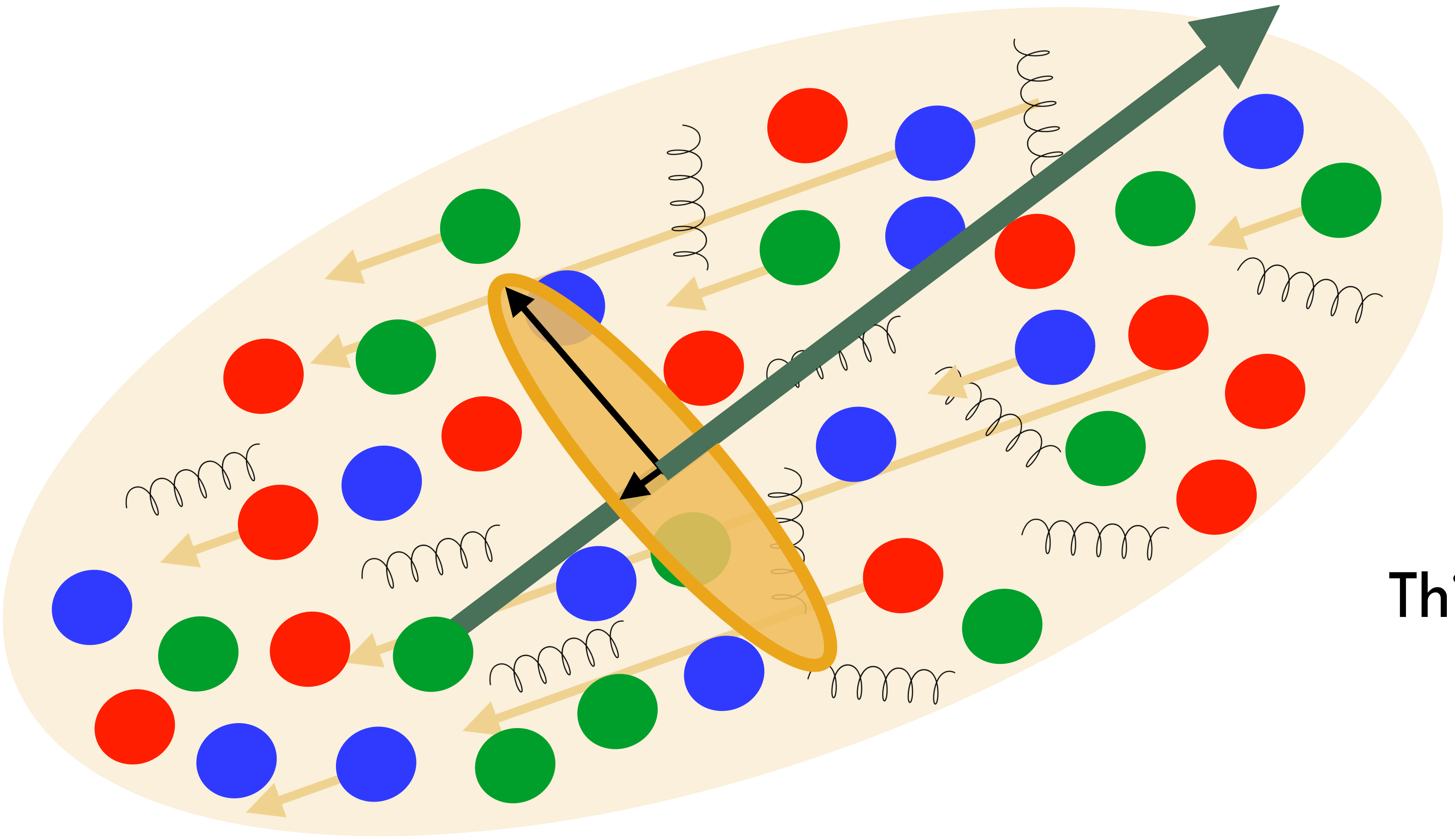


Couples shear to direction $\hat{n}$: enhances broadening along the jet direction

arXiv:1401.3751 [hep-ph]
arXiv:2312.00447 [hep-ph]

How can we obtain this tensor structure by solving the Boltzmann equation?

Jet momentum broadening out of equilibrium



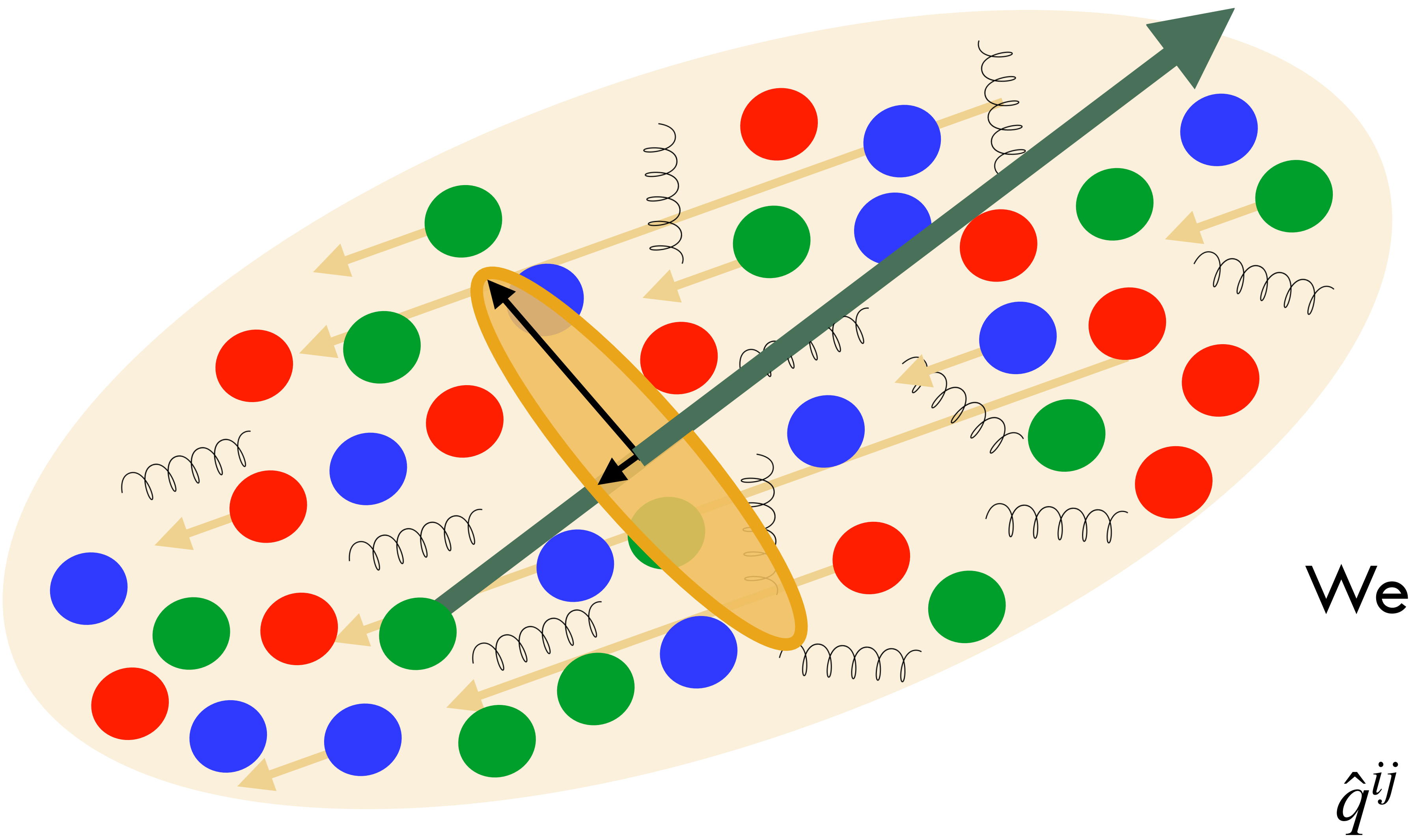
The broadening process is intrinsically direction-dependent.
No symmetry reason for momentum broadening to be a single scalar.

This scenario has been extensively studied before, see

arXiv:2511.07519 [hep-ph]. arXiv:2312.00447 [hep-ph]. arXiv:2308.01294 [hep-ph]. arXiv:2008.04928 [hep-ph]. arXiv:2309.00683 [hep-ph]. arXiv:2502.13205 [nucl-th]. arXiv:2512.11951 [hep-ph]. arXiv:2110.03590 [hep-ph]. arXiv:2202.08847 [hep-ph]. arXiv:1203.0561 [hep-th].

arXiv:2605.12791 [hep-ph]

Jet momentum broadening out of equilibrium



The broadening process is intrinsically direction-dependent.

There is no symmetry reason for the broadening to reduce to a single scalar

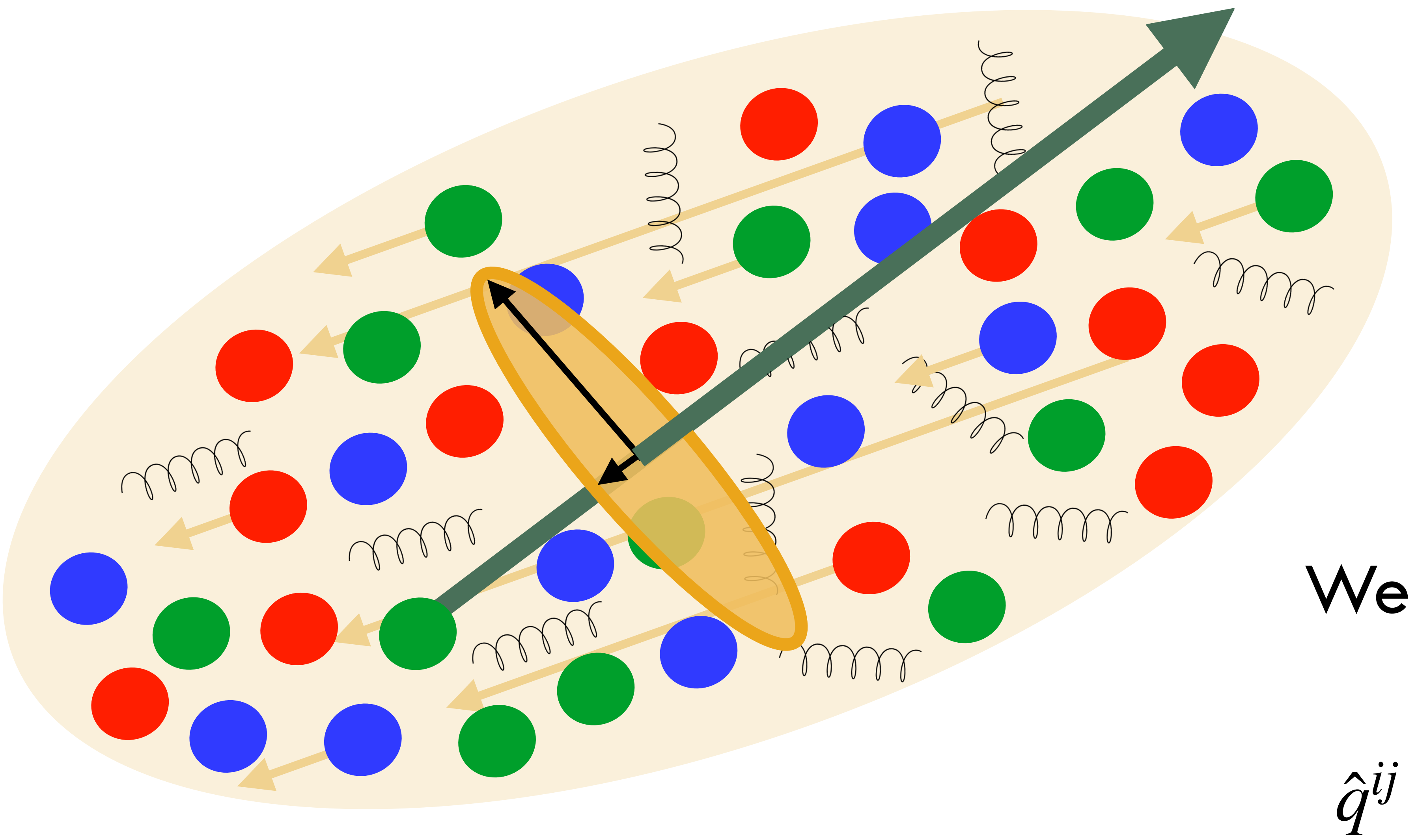
We begin with the general definition of the jet momentum broadening tensor:

$$\hat{q}^{ij} = \frac{(2\pi)^4}{4p\,\nu_a} \sum_{bcd} \int d\Gamma_{kp'k'} \delta^{(4)}(P+K-P'-K') \left| \mathcal{M}^{ab}_{cd} \right|^2 q^i q^j f_b(X,K) \tilde{f}_c(X,P') \tilde{f}_d(X,K')$$

Let us go back to the general expression and introduce non-equilibrium corrections

arXiv:2605.12791 [hep-ph]

Jet momentum broadening out of equilibrium



The broadening process is intrinsically direction dependent.
No symmetry reason for momentum broadening to be a single scalar.

We begin with the general definition of jet momentum broadening tensor:

$$\hat{q}^{ij} = \frac{(2\pi)^4}{4p\,\nu_a} \sum_{bcd} \int d\Gamma_{kp'k'} \delta^{(4)}(P + K - P' - K') \left| \mathscr{M}^{ab}_{cd} \right|^2 q^i q^j \, f_b(X, K) \tilde{f}_c(X, P') \tilde{f}_d(X, K')$$

We incorporate the deviations from equilibrium to jet momentum broadening in a *systematic fashion*

For a generic δf: $f_b = f_{0,b} + \delta f_b = f_{0,b}\left[1 + \tilde{f}_{0,b}\phi_k\right],$ $f_{0,b} \rightarrow$ local equilibrium distribution

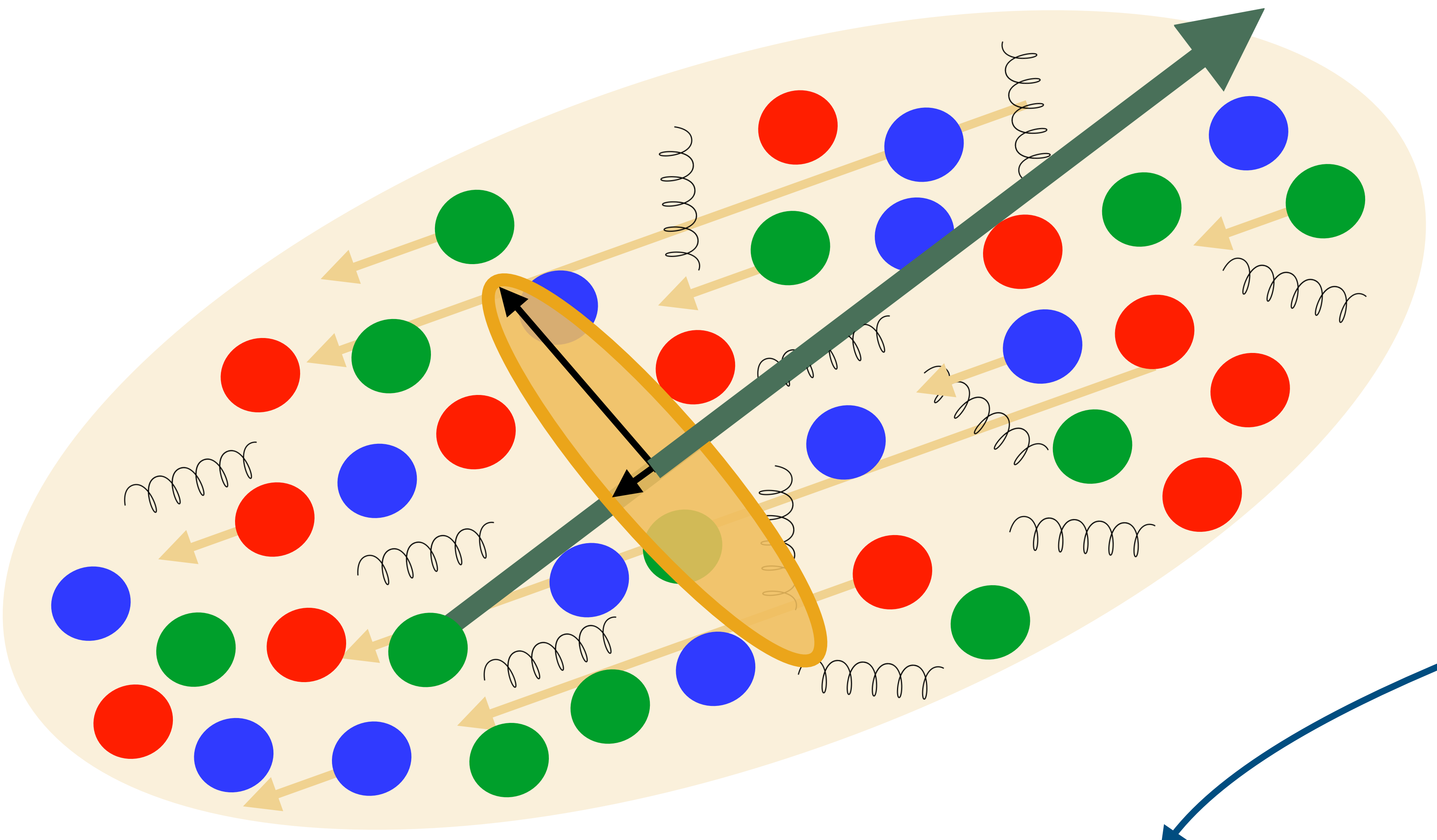
which induces the decomposition (for a linearized collision operator)

$$\hat{q}^{ij}(P, U) = \hat{q}^{ij}_{\rm eq}(P, U) + \delta\hat{q}^{ij}(P, U)$$

U^μ is the medium's 4-velocity

arXiv:2605.12791 [hep-ph]

Jet momentum broadening out of equilibrium



$$\hat{q}^{ij}(P,U) = \boxed{\hat{q}^{ij}_{\rm eq}(P,U)} + \boxed{\delta\hat{q}^{ij}(P,U)}$$

$$\hat{q}^{ij}_{\rm eq} = \frac{1}{4p\,\nu_a}\sum_{bd}\int d\Gamma_{kp'k'}\left|\mathcal{M}^{ab}_{cd}\right|^2 q^i q^j (2\pi)^4\delta^{(4)}(P+K-P'-K')f^b_{0k}\tilde{f}^d_{0k'}$$

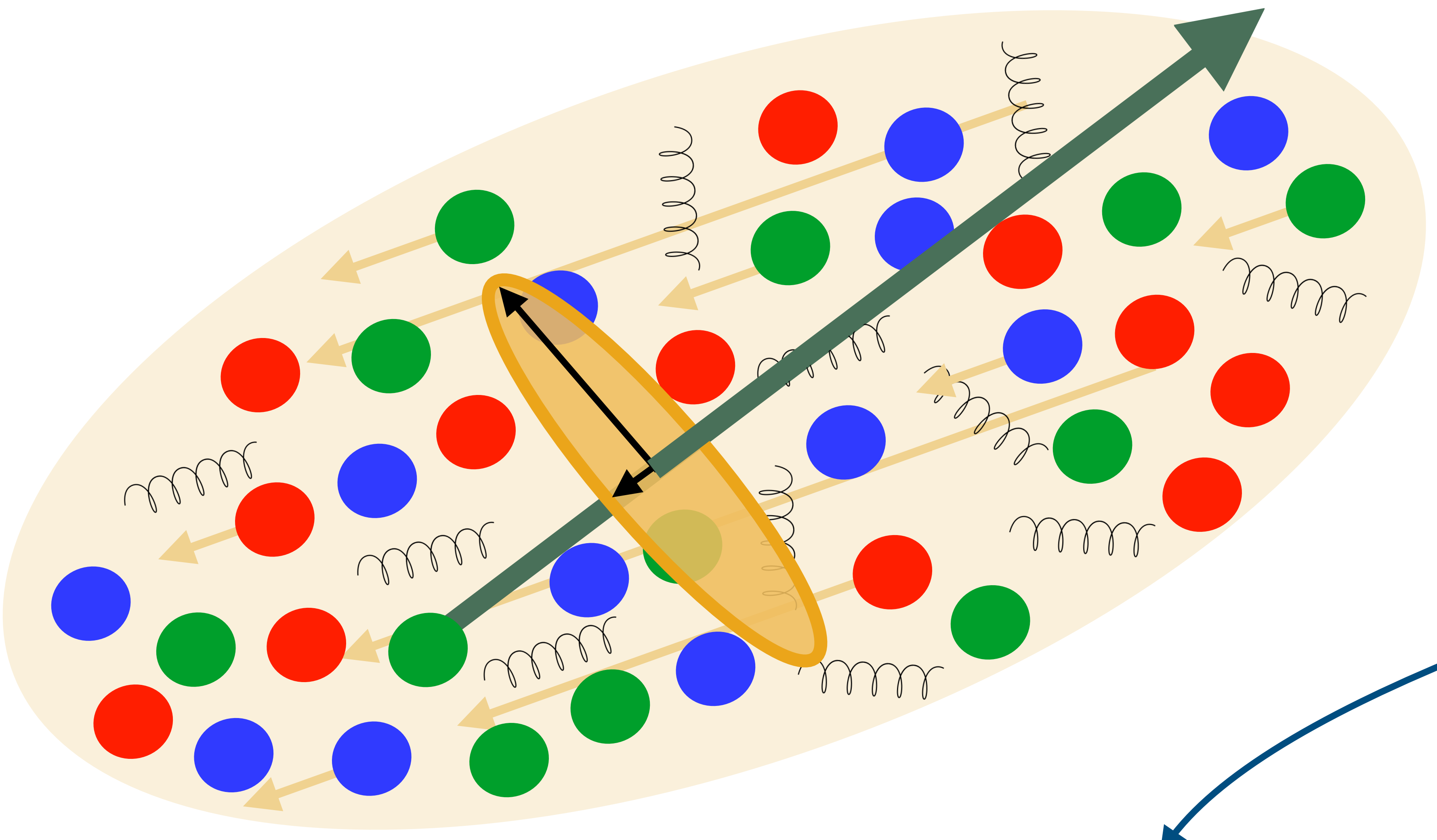
arXiv:2605.12791 [hep-ph]

$$\delta\hat{q}^{ij} = \frac{(2\pi)^4}{4p\,\nu_a}\sum_{bd}\int d\Gamma_{kp'k'}\delta^{(4)}(P+K-P'-K')\,q^i q^j\left|\mathcal{M}^{ab}_{cd}\right|^2\left[f^b_{0k}f^d_{0k'}\tilde{f}^d_{0k'}\phi_{k'} + f^b_{0k}\tilde{f}^b_{0k}\tilde{f}^d_{0k'}\phi_k\right]$$

The scalar function $\phi_{\mathbf{k}}$ encodes all the non-equilibrium contributions coming from the medium.

The equilibrium contribution has been extensively studied before, see arXiv:0811.1603 [hep-ph], arXiv:1502.03730 [hep-ph], arXiv:2312.00447 [hep-ph], arXiv:2303.12595 [hep-ph], arXiv:1509.07773 [hep-ph].

Jet momentum broadening out of equilibrium



$$\hat{q}^{ij}(P,U) = \boxed{\hat{q}^{ij}_{\rm eq}(P,U)} + \boxed{\delta\hat{q}^{ij}(P,U)}$$

$$\hat{q}^{ij}_{\rm eq} = \frac{1}{4p\,\nu_a}\sum_{bd}\int d\Gamma_{kp'k'}\left|\mathcal{M}^{ab}_{cd}\right|^2 q^i q^j (2\pi)^4\delta^{(4)}(P+K-P'-K')f^b_{0k}\tilde{f}^d_{0k'}$$

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$$\delta\hat{q}^{ij} = \frac{(2\pi)^4}{4p\,\nu_a}\sum_{bd}\int d\Gamma_{kp'k'}\delta^{(4)}(P+K-P'-K')\,q^i q^j\left|\mathcal{M}^{ab}_{cd}\right|^2\left[f^b_{0k}f^d_{0k'}\tilde{f}^d_{0k'}\phi_{k'} + f^b_{0k}\tilde{f}^b_{0k}\tilde{f}^d_{0k'}\phi_k\right]$$

The scalar function $\phi_{\mathbf{k}}$ encodes all the non-equilibrium contributions coming from the medium.

How do we parametrize it?

The equilibrium contribution has been extensively studied before, see arXiv:0811.1603 [hep-ph], arXiv:1502.03730 [hep-ph], arXiv:2312.00447 [hep-ph], arXiv:2303.12595 [hep-ph], arXiv:1509.07773 [hep-ph].

The Method of Moments: A General Strategy for the Boltzmann Equation

1949

Rarefied gas dynamics

Grad expands f in tensorial Hermite polynomials → the classic 13-moment equations.

Grad, *Comm. Pure Appl. Math.* **2**, 331 (1949) · *Struchtrup & Torrilhon, Phys. Fluids* ***15**, 2668–2680 (2003)*

1965

Plasma physics

Moment and Chapman–Enskog closures give Braginskii's transport equations and modern gyrofluid models.

Braginskii, Reviews of Plasma Physics Vol. 1 (1965) · arXiv:2201.11561

1984

Radiative & neutrino transport

Radiation hydrodynamics used in supernova and merger simulations.

*Levermore, JQSRT **31**, 149 (1984)*

1987

Maximum entropy / extended thermodynamics

Fix the closure by maximizing entropy at fixed moments.

*Dreyer, J. Phys. A **20**, 6505–6517 (1987) · Levermore, J. Stat. Phys. **83**, 1021–1065 (1996)*

1995

Semiconductor & phonon transport

Max-entropy closure → energy-transport / hydrodynamic device models.

*Anile & Muscato, Phys. Rev. B **51**, 16728 (1995)*

2012

Relativistic fluids / heavy-ion

Second-order moment closure → causal, stable dissipative hydrodynamics; the 14-moment truncation is the workhorse for the QGP.

Israel–Stewart (1979) · DNMR (2012, arXiv:1202.4551)

★ *our setting*

→ **2026**: this talk

The Method of Moments

W. Israel and J. M. Stewart, Phys. Lett. 58A, 213 (1976)

This provides a systematic and improvable way to solve kinetic theory

We expand ϕ_k in a ***complete and orthogonal basis*** formed by the irreducible tensors

$$1,\, K^{\langle\mu\rangle},\, K^{\langle\mu}K^{\nu\rangle},\, K^{\langle\mu}K^{\nu}K^{\lambda\rangle},\, \ldots\,.$$

Angle brackets denote symmetric, traceless projections orthogonal to the fluid four-velocity,

$$A^{\langle\mu\rangle} = \Delta^{\mu}{}_{\nu}A^{\nu}, \qquad \Delta_{\mu\nu} = g_{\mu\nu} + U_\mu U_\nu,$$

The expansion then reads

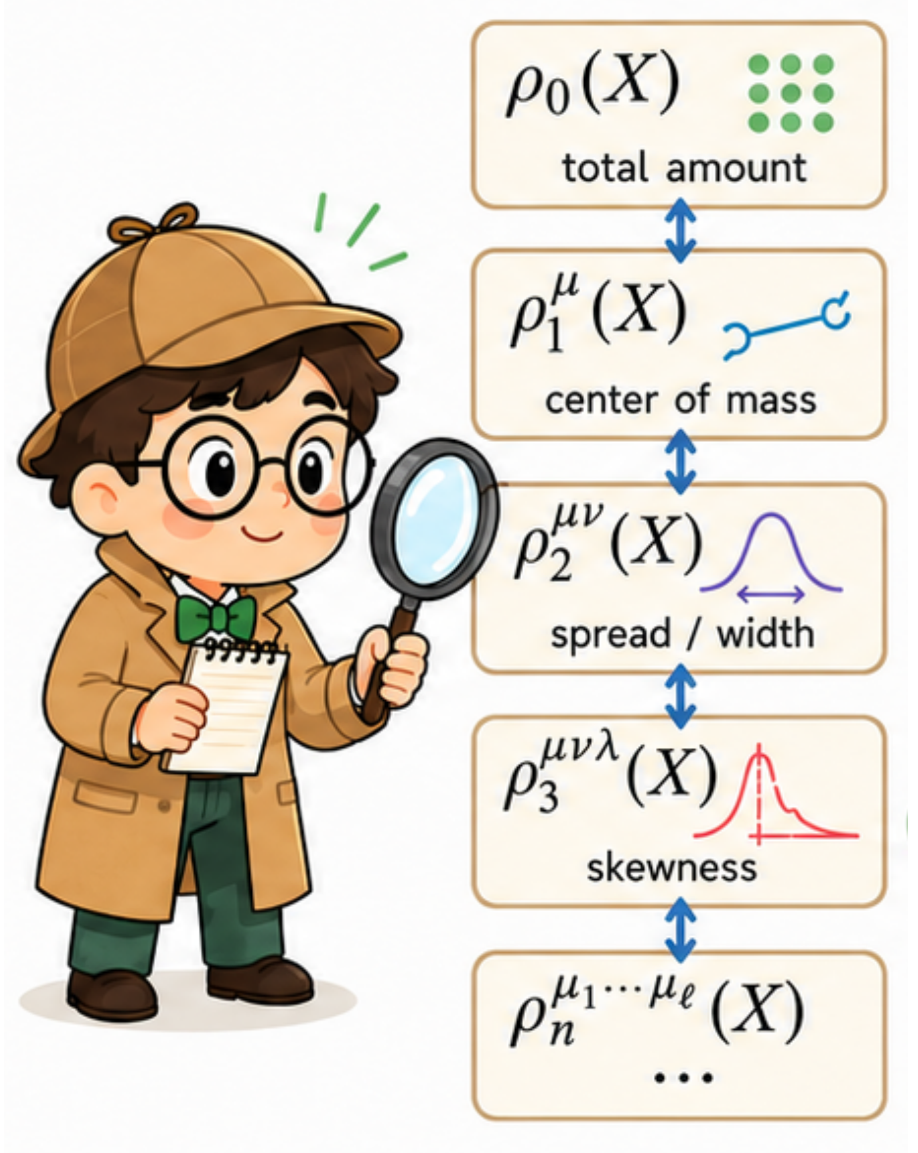
$$\phi_k = \sum_{\ell=0}^{\infty}\sum_{n=0}^{N_\ell} \mathscr{H}^{(\ell)}_{\mathbf{k}n}\rho_n^{\mu_1\cdots\mu_\ell}K_{\langle\mu_1}\cdots K_{\mu_\ell\rangle}$$

$\mathscr{H}^{(\ell)}_{\mathbf{k}n} \rightarrow$ polynomials of $E_{\mathbf{k}}$

$\ell \rightarrow$ rank

where the moments are defined as

$$\rho_n^{\mu_1\cdots\mu_\ell}(X) = \int \frac{d^3\mathbf{k}}{(2\pi)^3k^0}E_{\mathbf{k}}^n\, K^{\langle\mu_1}\cdots K^{\mu_\ell\rangle}\,\delta f(X,K),$$



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Solving this infinite hierarchy of coupled PDEs

↓

solving the Boltzmann equation

↓

used to reconstruct the full distribution function.

14-Moment Approximation

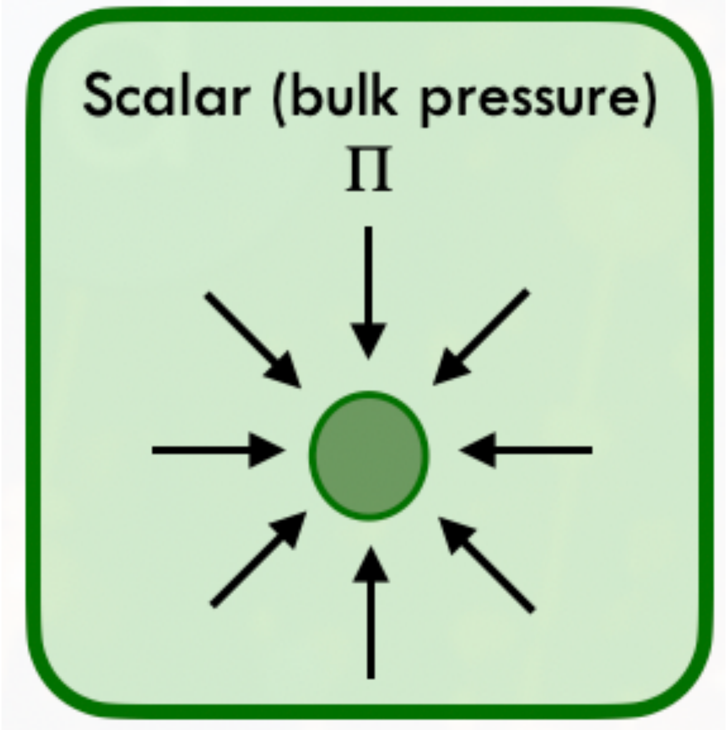
We adopt the lowest-order truncation compatible with ***causal and stable*** hydrodynamic evolution, namely the 14-moment approximation, in which the series is truncated at rank two so that moments with $\ell \geq 3$ are neglected,

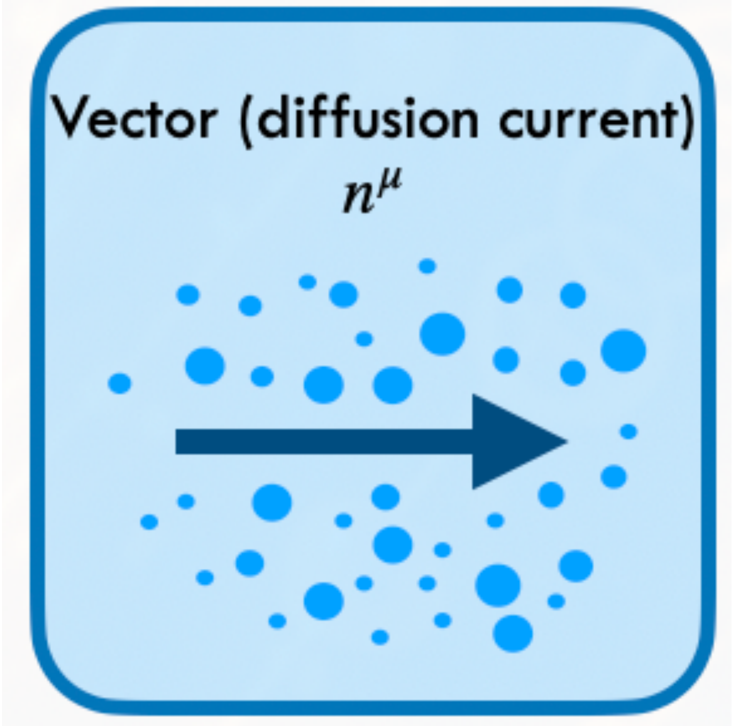
arXiv:2311.15063 [nucl-th]

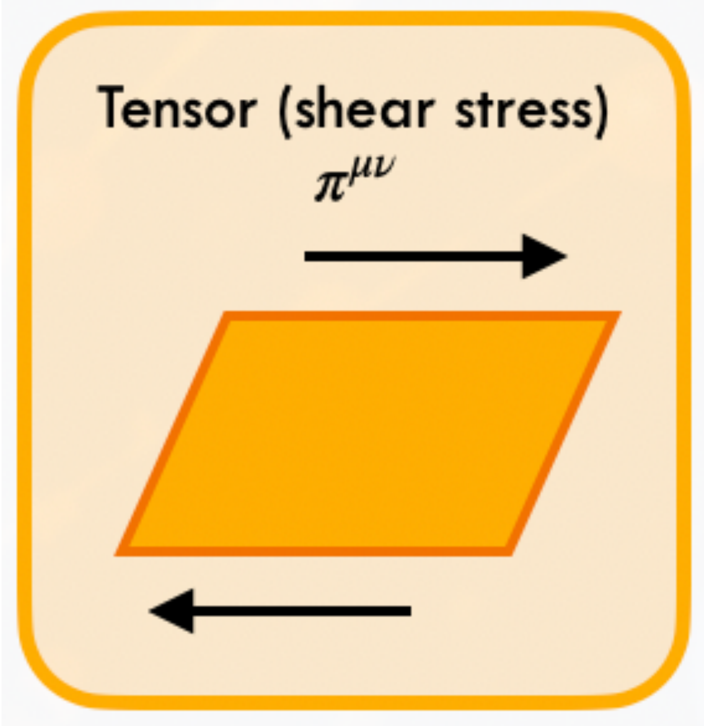
$$\rho_r^{\mu_1 \cdots \mu_\ell} = 0, \qquad \ell \geq 3.$$

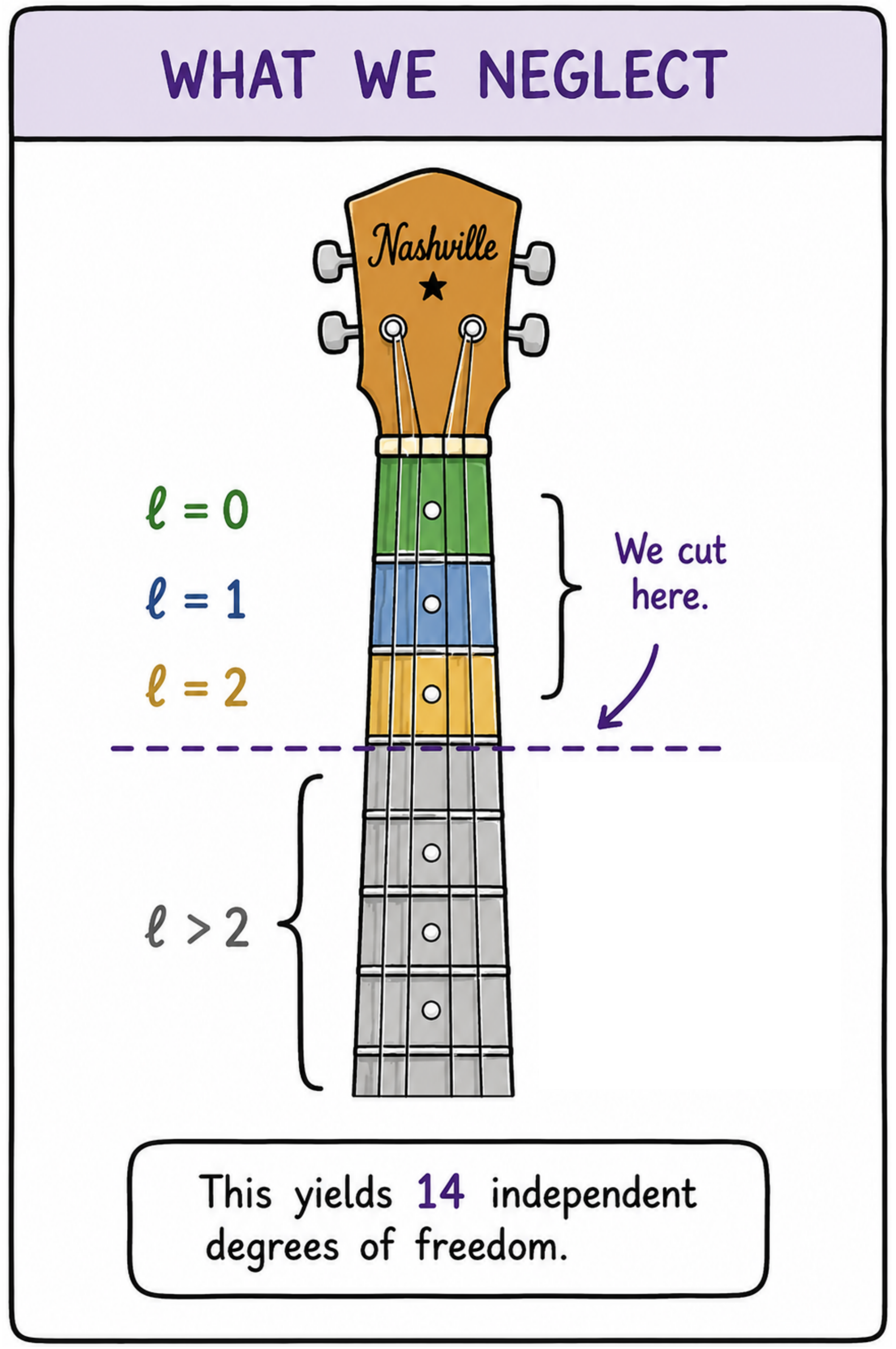
This retains only the scalar ($\ell = 0$), vector ($\ell = 1$), and tensor ($\ell = 2$) moments,

$$\phi_k = \quad [E_0 + B_0 m^2 - D_0 E_k - 4B_0 E_k^2]\Pi \qquad + \lambda_n n_\alpha K^\alpha \quad + B_2 \pi_{\alpha\beta} K^\alpha K^\beta$$









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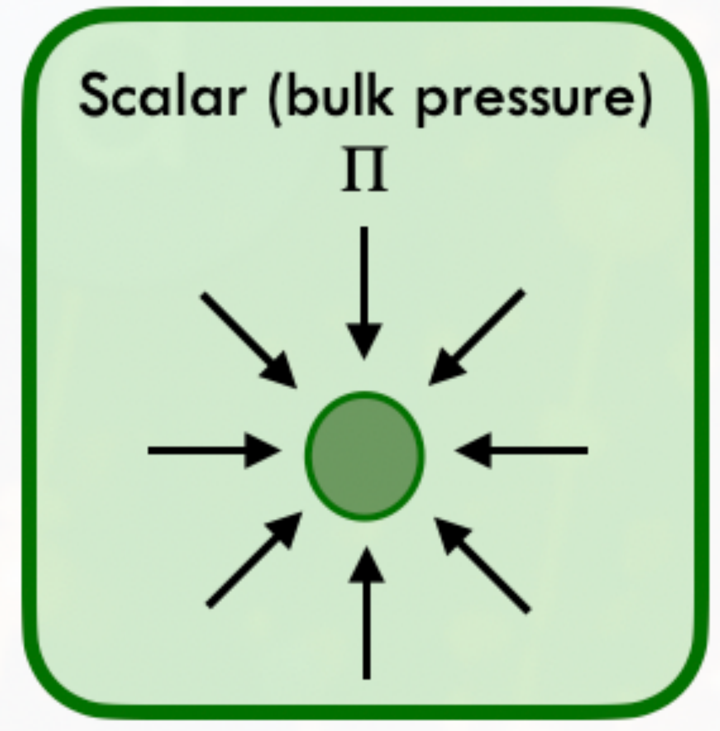
arXiv:1202.4551 [nucl-th]

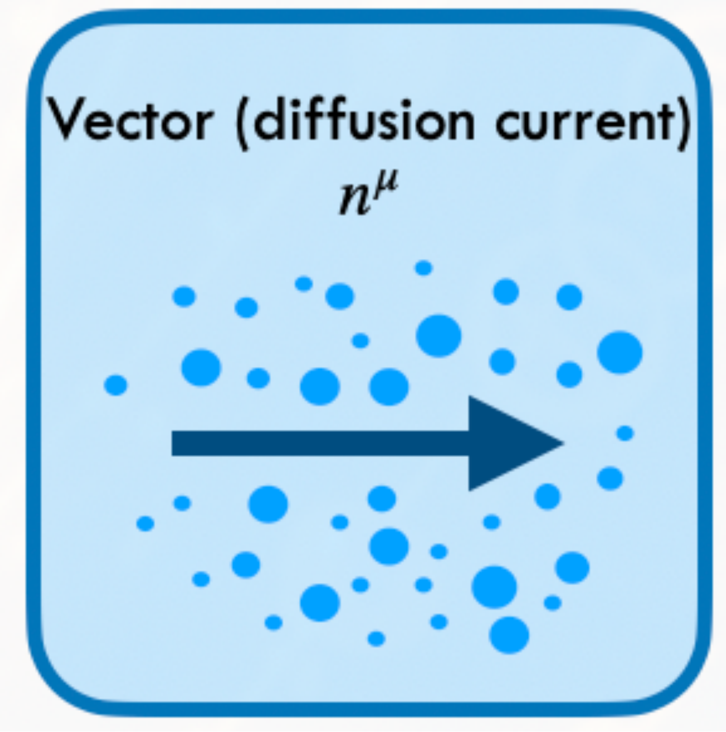
The expression can be systematically improved by including higher-rank moments!

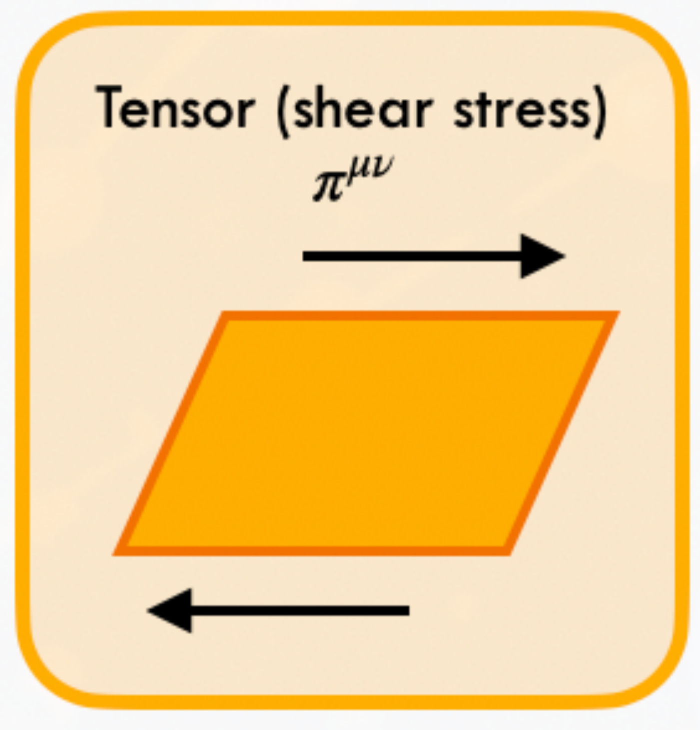
Connection with Hydrodynamics

$$\phi_k = \quad [E_0 + B_0 m^2 - D_0 E_k - 4B_0 E_k^2]\Pi \qquad + \lambda_n n_\alpha K^\alpha \quad + B_2 \pi_{\alpha\beta} K^\alpha K^\beta$$

arXiv:2311.15063 [nucl-th]







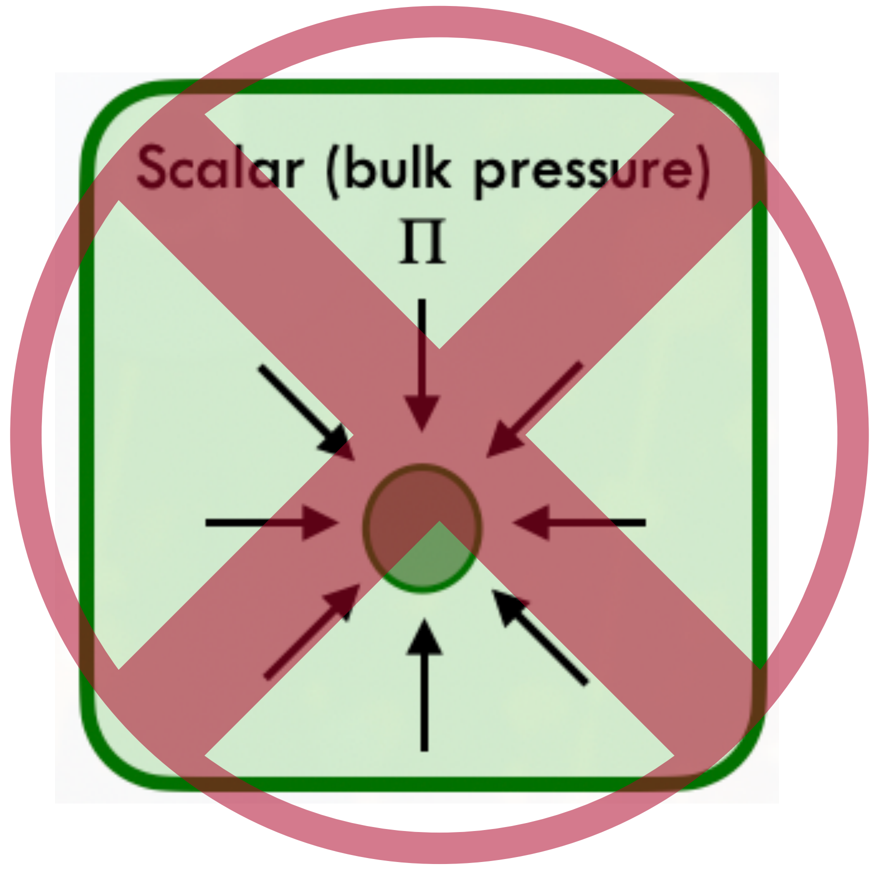
We consider massless particles at zero baryon chemical potential.

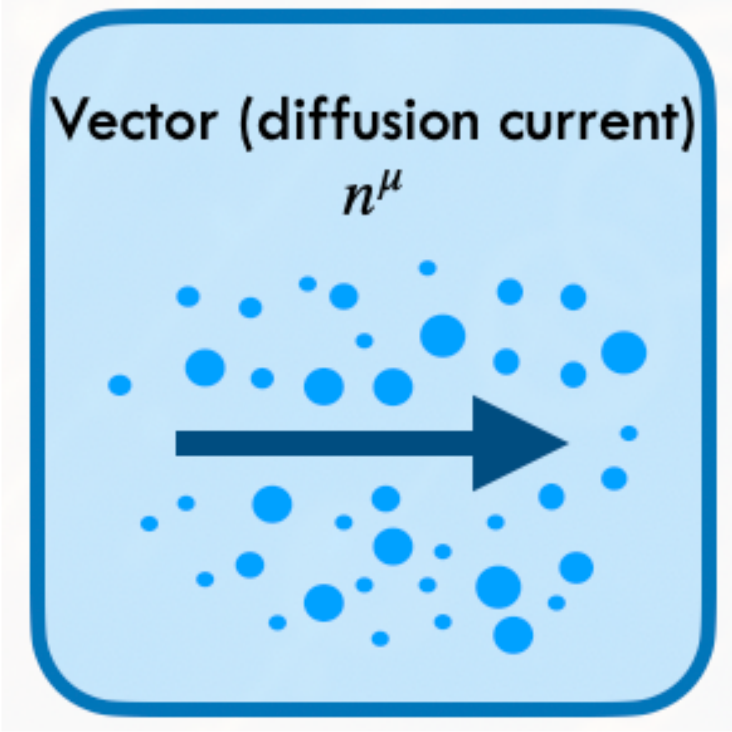
arXiv:2605.12791 [hep-ph]

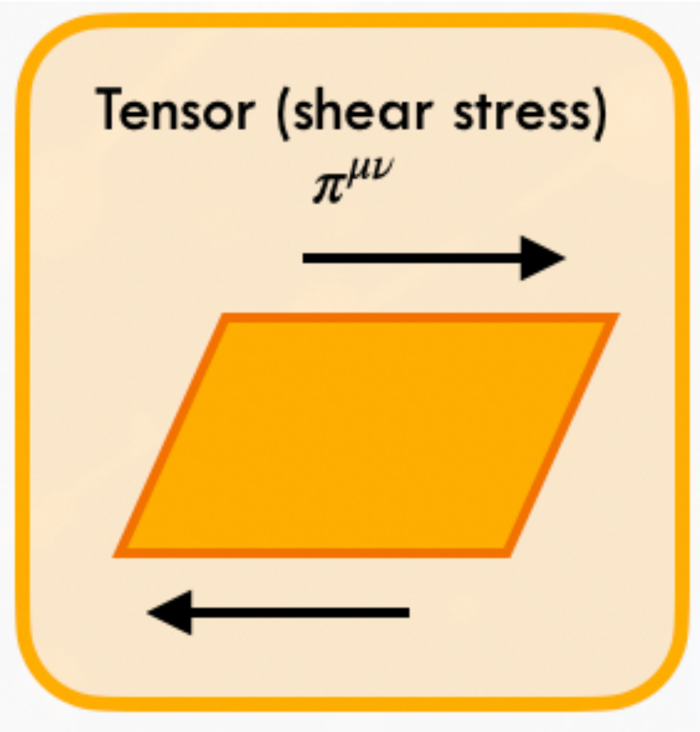
Connection with Hydrodynamics

$$\phi_k = \quad [E_0 + B_0 m^2 - D_0 E_k - 4B_0 E_k^2]\Pi \qquad + \lambda_n n_\alpha K^\alpha \qquad + B_2 \pi_{\alpha\beta} K^\alpha K^\beta$$

arXiv:2311.15063 [nucl-th]







We consider massless particles at zero baryon chemical potential.

For our massless system the bulk viscous scalar $\Pi = 0$.

(we neglect running-coupling corrections that generate a nonzero bulk viscosity coefficient)

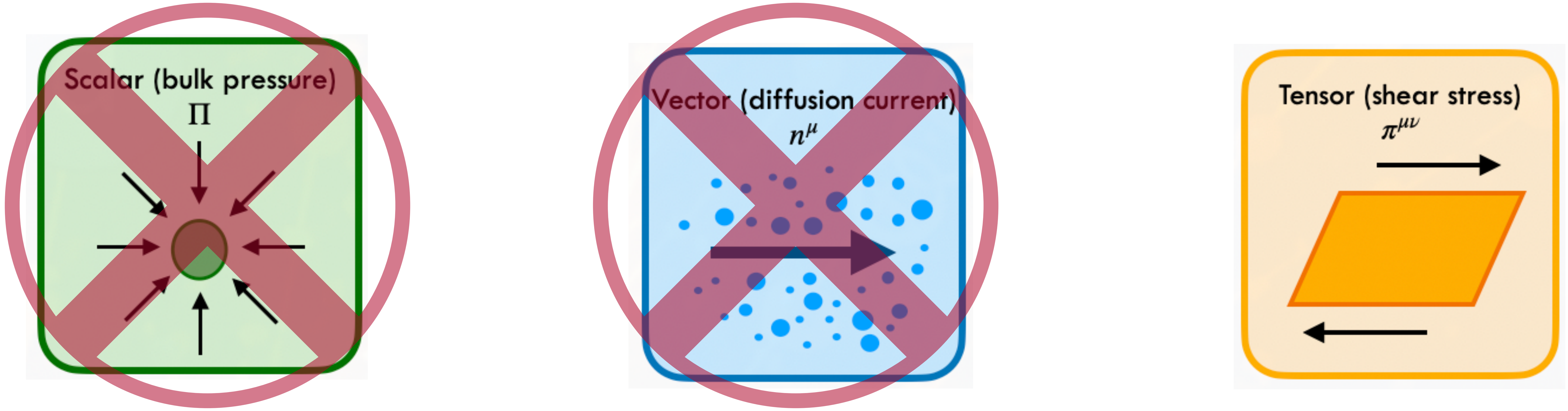
arXiv:hep-ph/0608012.

arXiv:2605.12791 [hep-ph]

Connection with Hydrodynamics

$$\phi_k = \quad [E_0 + B_0 m^2 - D_0 E_k - 4B_0 E_k^2]\Pi \qquad +\lambda_n n_\alpha K^\alpha \quad +B_2 \pi_{\alpha\beta} K^\alpha K^\beta$$

arXiv:2311.15063 [nucl-th]



We consider massless particles at zero baryon chemical potential.

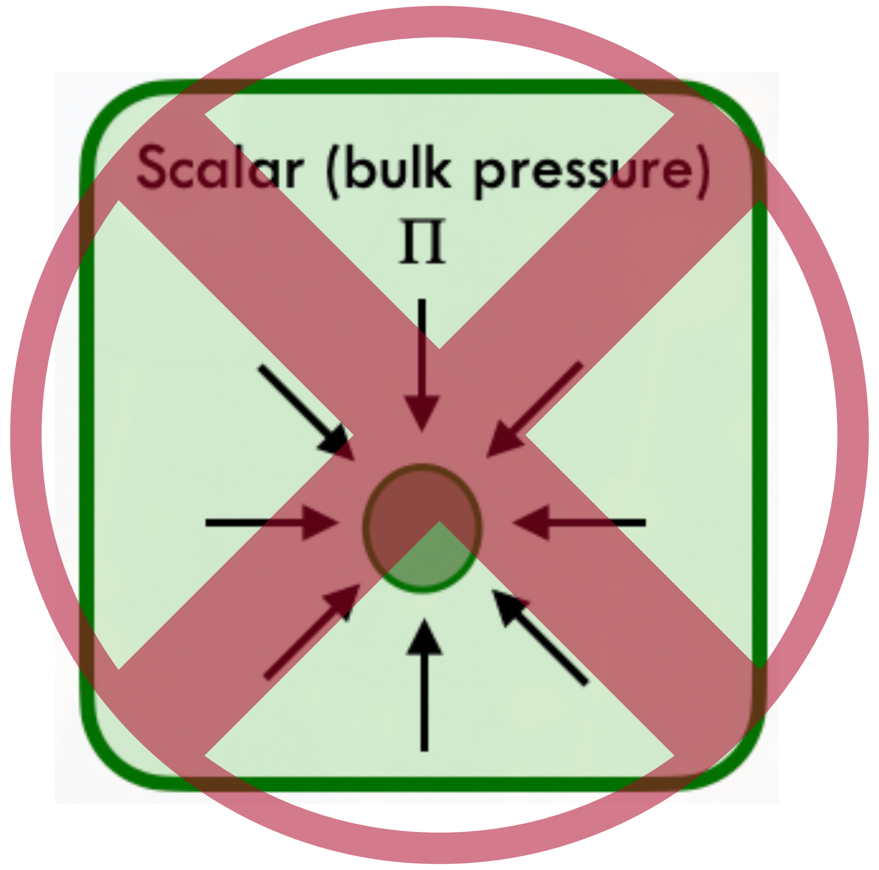
Landau frame → energy-diffusion current vanishes.
At zero μ_B the charge-diffusion current $\mathcal{J}^\mu$ vanishes.

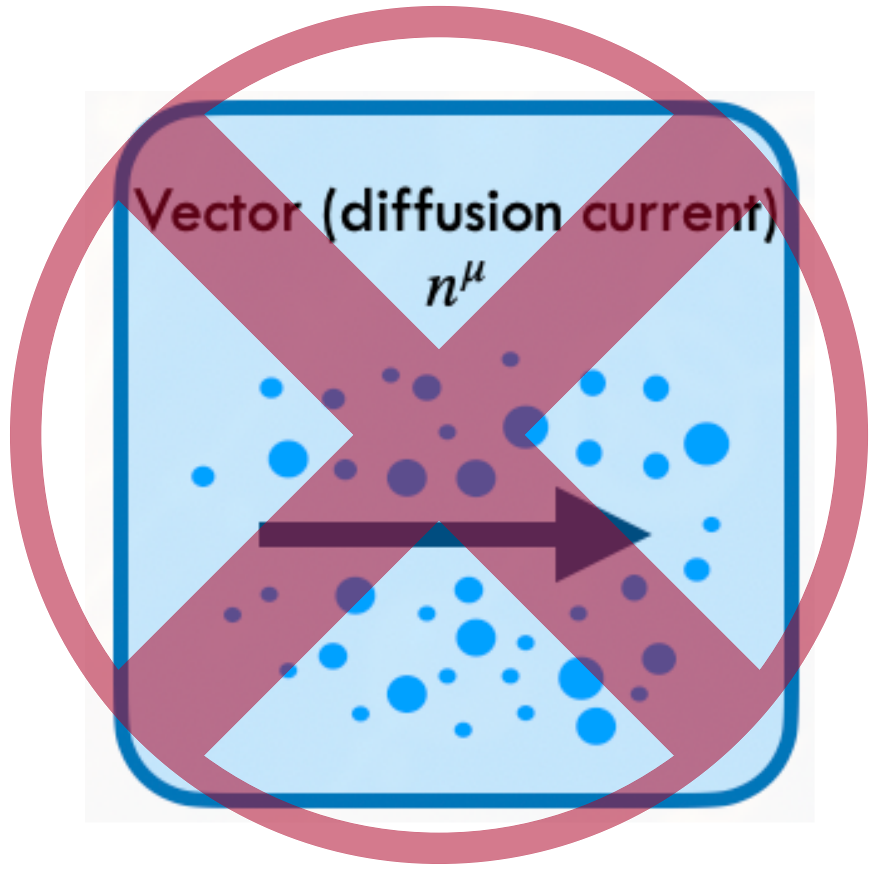
arXiv:2605.12791 [hep-ph]

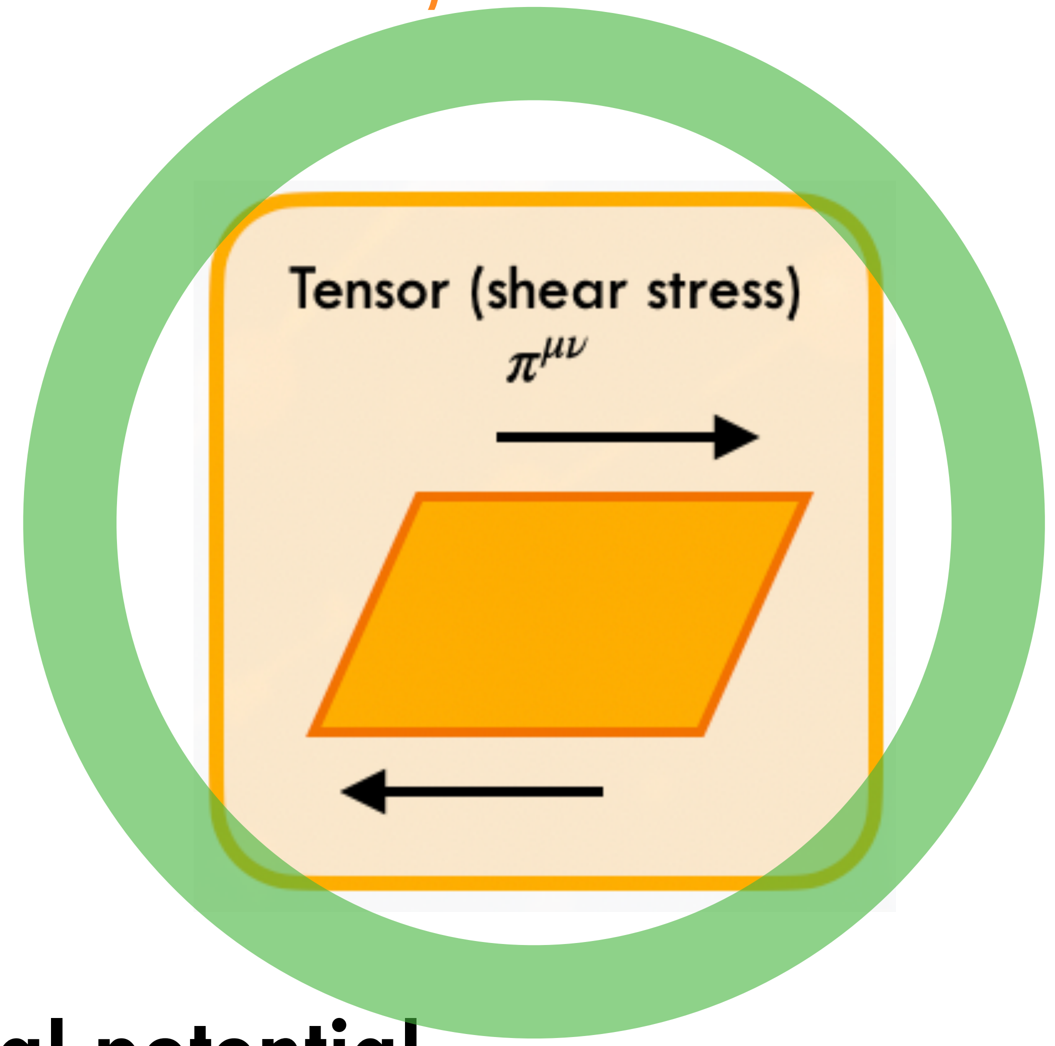
Connection with Hydrodynamics

$$\phi_k = \quad [E_0 + B_0 m^2 - D_0 E_k - 4B_0 E_k^2]\Pi \qquad + \lambda_n n_\alpha K^\alpha \qquad + B_2 \pi_{\alpha\beta} K^\alpha K^\beta$$

arXiv:2311.15063 [nucl-th]







We consider massless particles at zero baryon chemical potential.

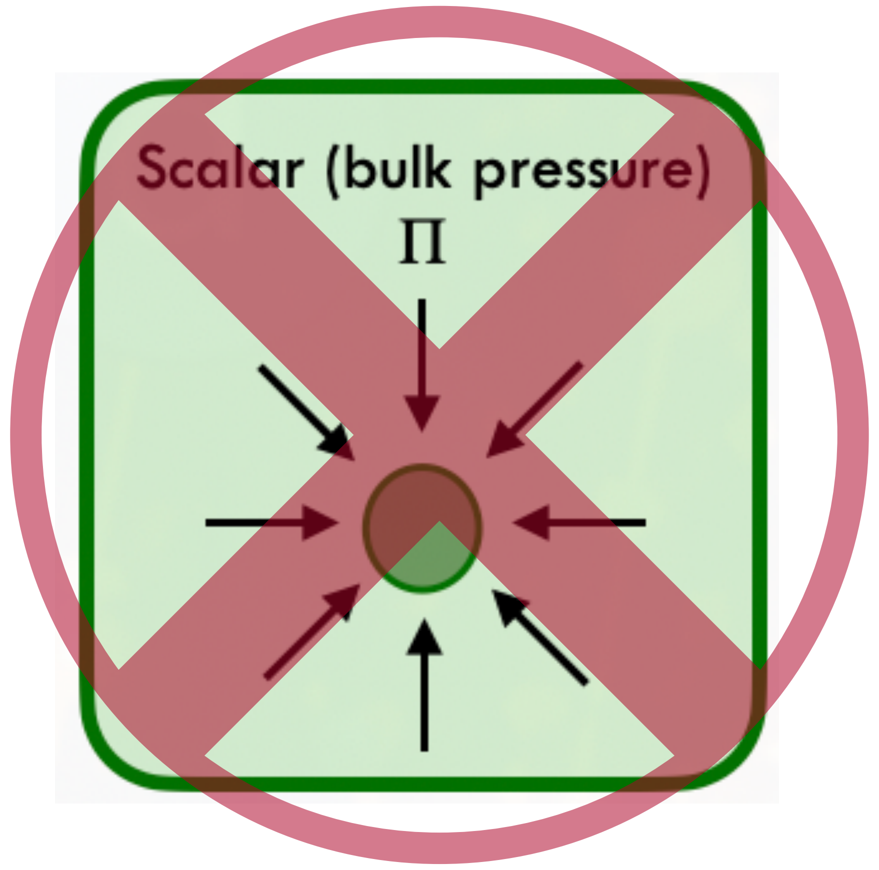
$$\boxed{\phi_k = \frac{1}{2J_{42}} \pi_{\mu\nu} K^\mu K^\nu}$$

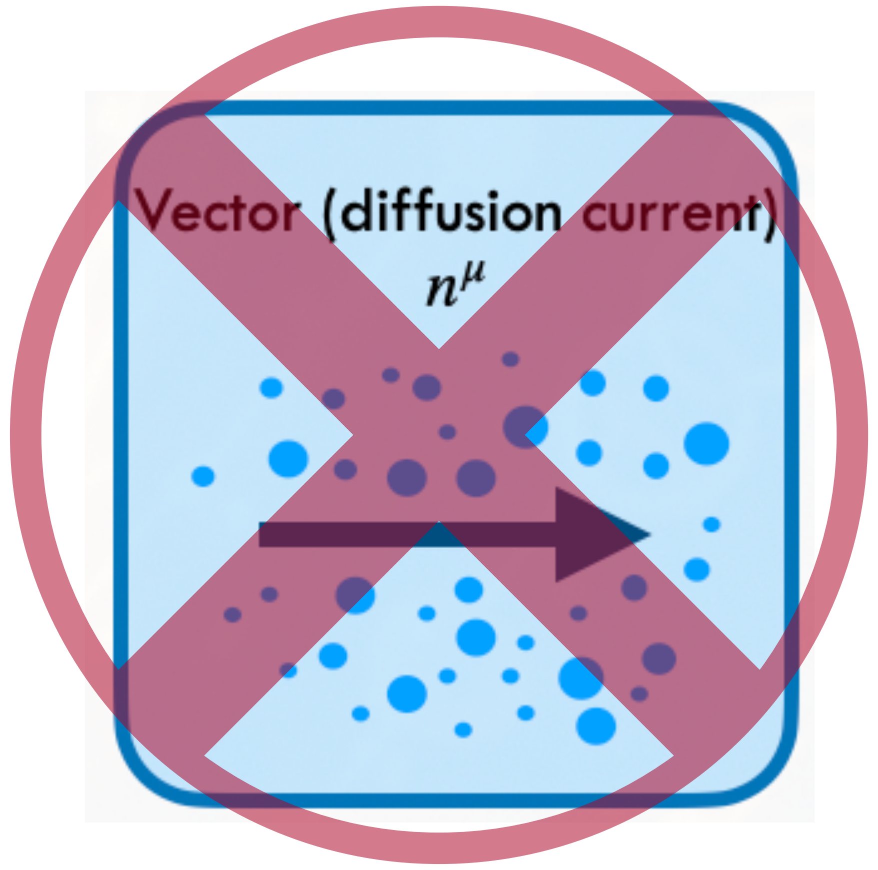
arXiv:2605.12791 [hep-ph]

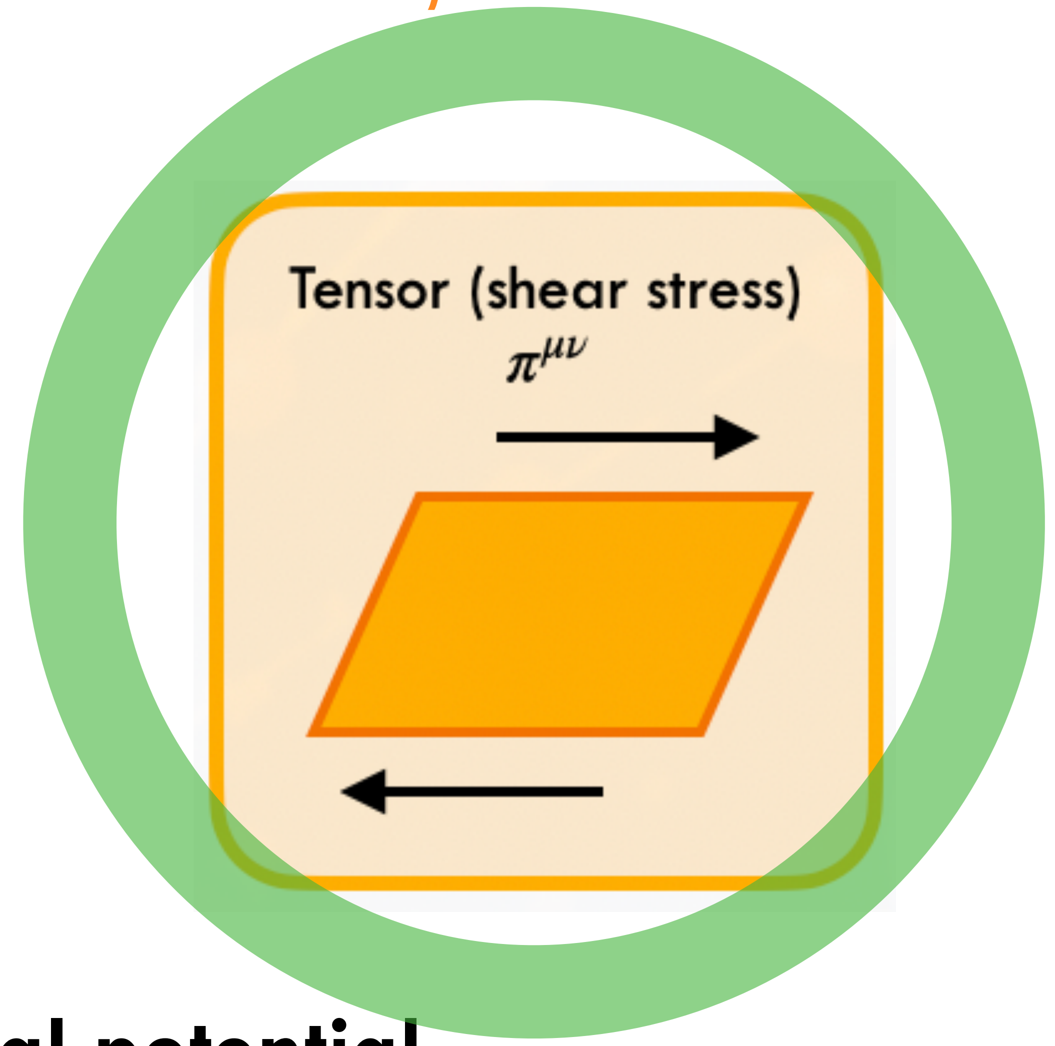
Connection with Hydrodynamics

$$\phi_k = \quad [E_0 + B_0 m^2 - D_0 E_k - 4B_0 E_k^2]\Pi \qquad + \lambda_n n_\alpha K^\alpha \qquad + B_2 \pi_{\alpha\beta} K^\alpha K^\beta$$

arXiv:2311.15063 [nucl-th]







We consider massless particles at zero baryon chemical potential.

$$\boxed{\phi_k = \frac{1}{2J_{42}} \pi_{\mu\nu} K^\mu K^\nu}$$

Working in linear response around local equilibrium

$$\delta \hat{q}^{ij} = \boxed{\mathcal{K}^{ijab}}\, \pi_{ab}$$

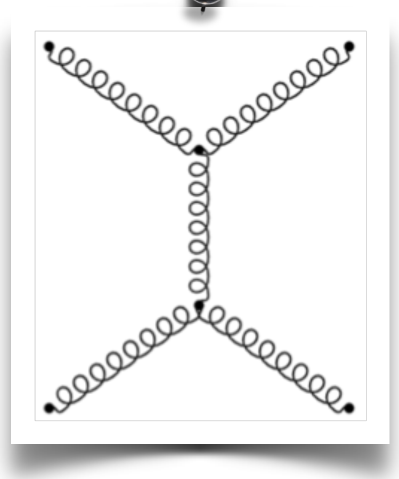
Fixed by the collision kernel!

Microscopics of the system!

arXiv:2605.12791 [hep-ph]

General tensorial structure of $\delta\hat{q}^{ij}$

Computed with screened t-channel gluon exchange!



arXiv:1401.3751 [hep-ph]
arXiv:2312.00447 [hep-ph]

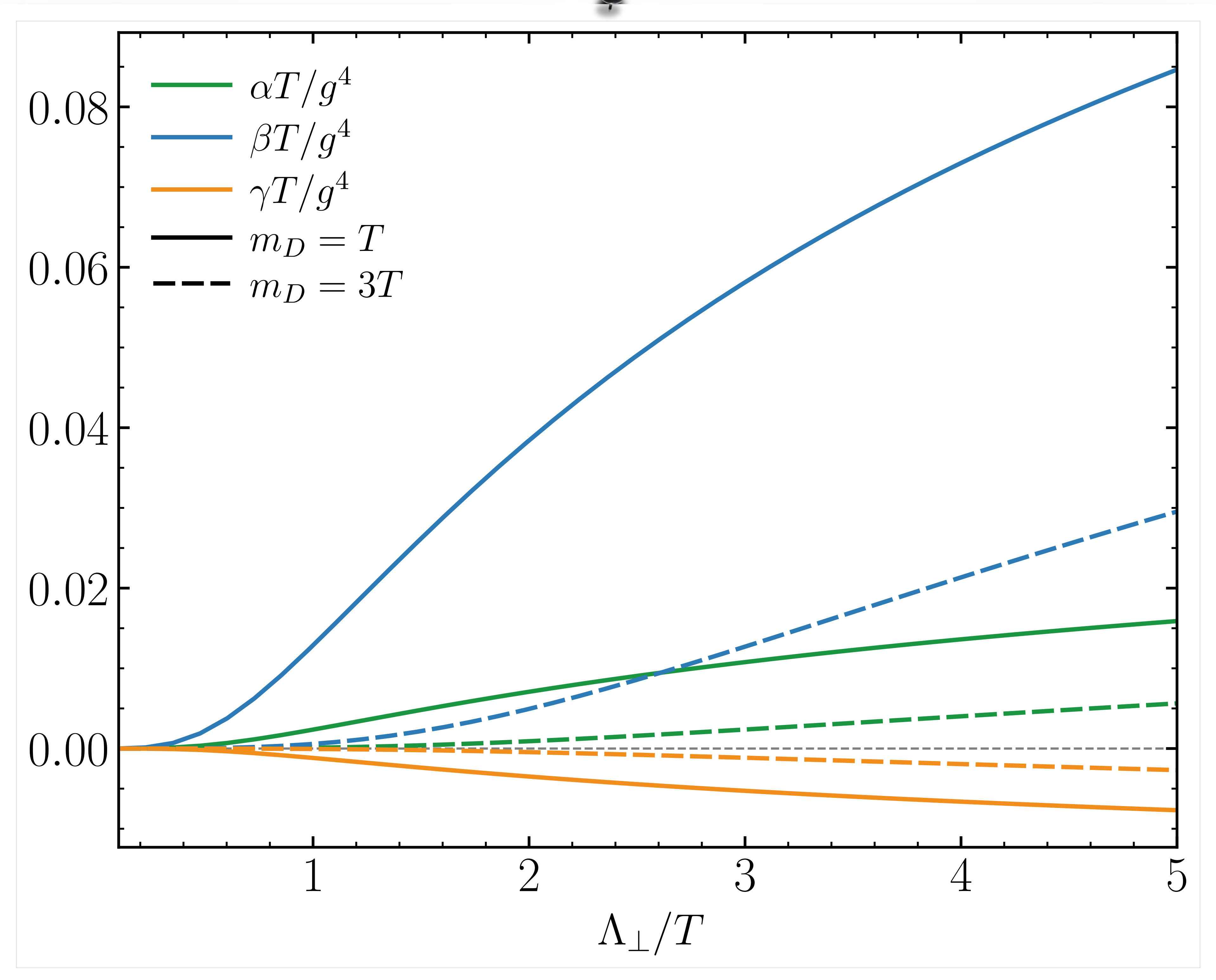
$\delta\hat{q}$ is then given by,

$$\delta\hat{q}^{ij} = \frac{(2\pi)^4}{4\nu_a p}\sum_{bd}\int d\Gamma_{kp'k'}\, q^i q^j \delta(p+k-p'-k')\,|\,\mathscr{M}^{ab}_{cd}(P,K;P',K')\,|^2$$
$$\times f_{0k}\tilde{f}_{0k'}\left[\frac{f_{0k'}}{2J_{42}}\pi_{\mu\nu}K'^{\mu}K'^{\nu} + \frac{\tilde{f}_{0k}}{2J_{42}}\pi_{\mu\nu}K^{\mu}K^{\nu}\right]$$

After performing all angular integrations, we recover exactly the same tensor structure as before

$$\delta\hat{q}^{ij} = \begin{pmatrix} \alpha\pi^{xx}+\beta\pi^{zz} & \alpha\pi^{xy} & (\alpha+\gamma)\pi^{xz} \\ \alpha\pi^{xy} & \alpha\pi^{yy}+\beta\pi^{zz} & (\alpha+\gamma)\pi^{yz} \\ (\alpha+\gamma)\pi^{xz} & (\alpha+\gamma)\pi^{yz} & (\alpha+\beta+3\gamma)\pi^{zz} \end{pmatrix}$$

Coefficients $\alpha T, \beta T$, and γT defined as functions of the transverse momentum cutoff $\Lambda_\perp/T$ for a purely gluonic medium



arXiv:2605.12791 [hep-ph]

Summary

✓ Out-of-equilibrium jet momentum broadening in QCD effective kinetic theory through a moment expansion of the medium distribution function.

✓ Within the 14-moment approximation, we demonstrate that viscous corrections to jet momentum broadening are governed by the hydrodynamic shear-stress tensor.

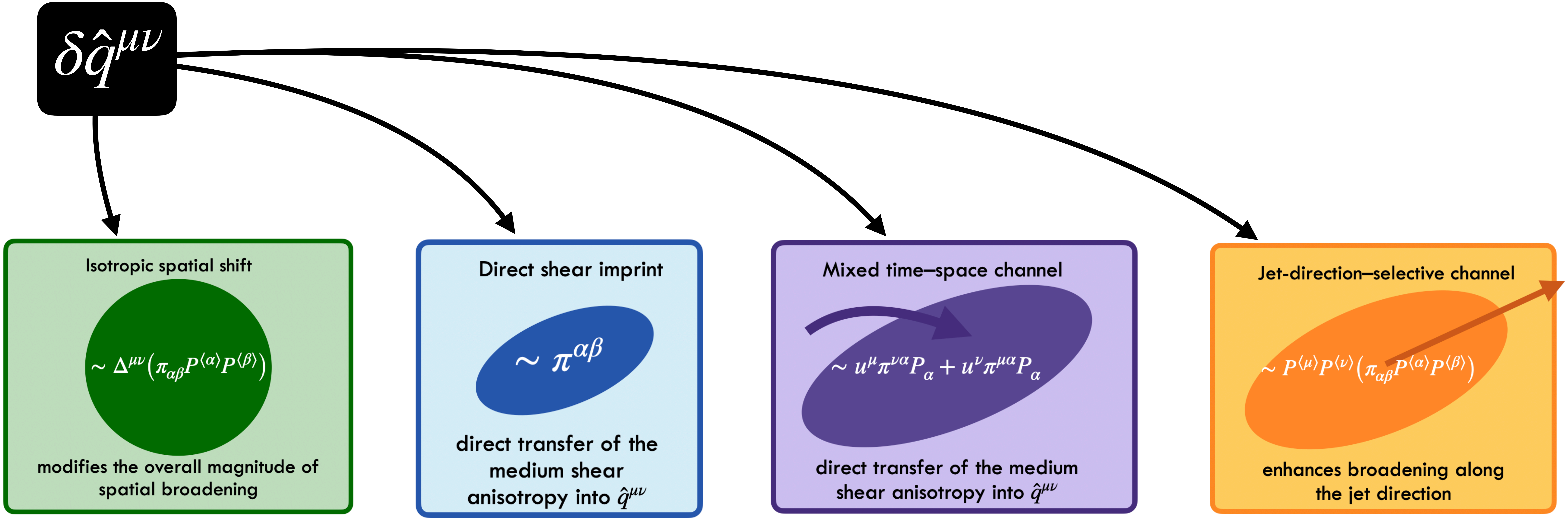
✓ Our work provides a **systematic** route to incorporate realistic medium evolution into jet transport calculations, offering an alternative to simplified homogeneous (“brick”) backgrounds commonly employed in studies of jet-medium interactions.

This gives a ready-to-use formula for the out-of-equilibrium viscous corrections to $\hat{q}$!

arXiv:2605.12791 [hep-ph]

Covariant QCD jet momentum broadening

The momentum-broadening coefficient is promoted to a rank-two tensor $\delta\hat{q}^{\mu\nu}$ depending on the jet momentum, the local fluid 4-velocity, and the dissipative currents. The tensor has additional information and can be described as,



isotropic / magnitude → jet-aligned / directional