Jet transport coefficients in the Glasma

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Based on work with Liliana Apolinário [b] and José Soares [b]

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EMMI workshop: Probing the early stages with jets
(Darmstadt, Germany)

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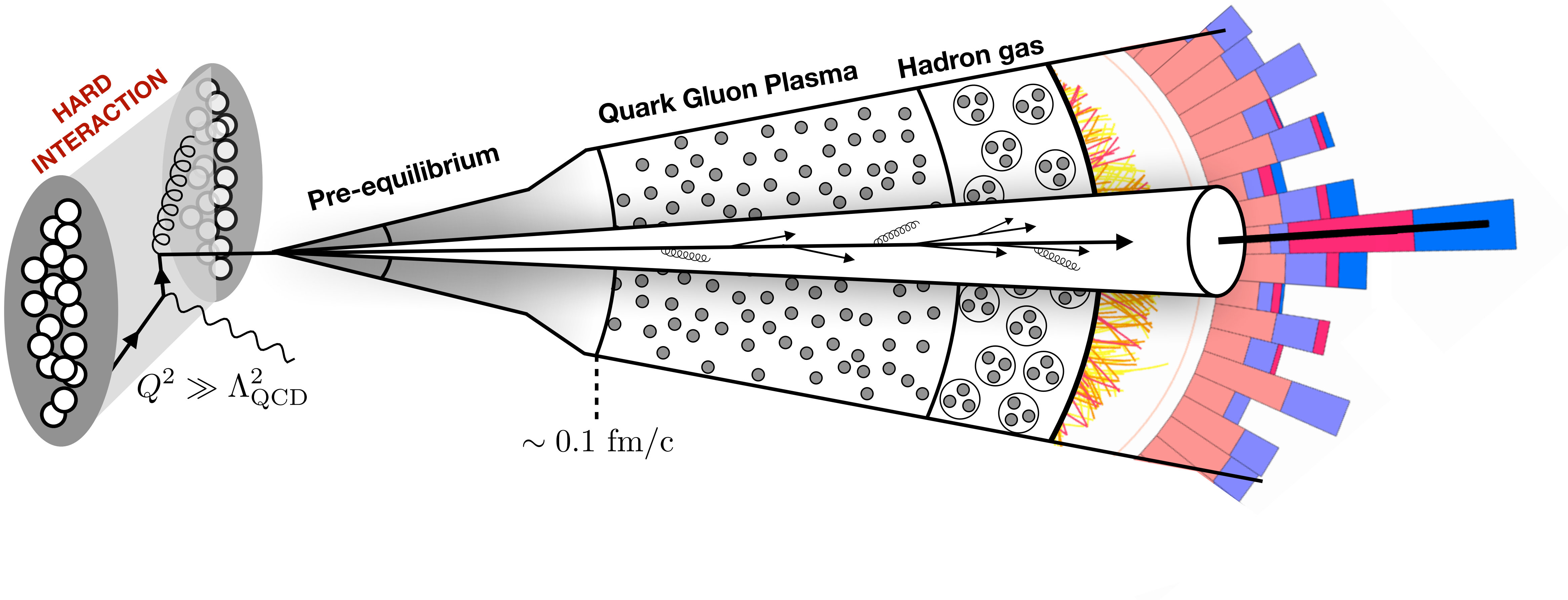
[b] *Laboratório de Instrumentação e Física Experimental de Partículas (LIP)*

Introduction

- ➡ Shortly after a Heavy Ion Collision (HIC) a de-confined, fluid-like state emerges: the **Quark Gluon Plasma (QGP)**
- ➡ Jets in HICs are modified with respect to the p-p reference ⟶ **Jet Quenching** (one of the main QGP signals)



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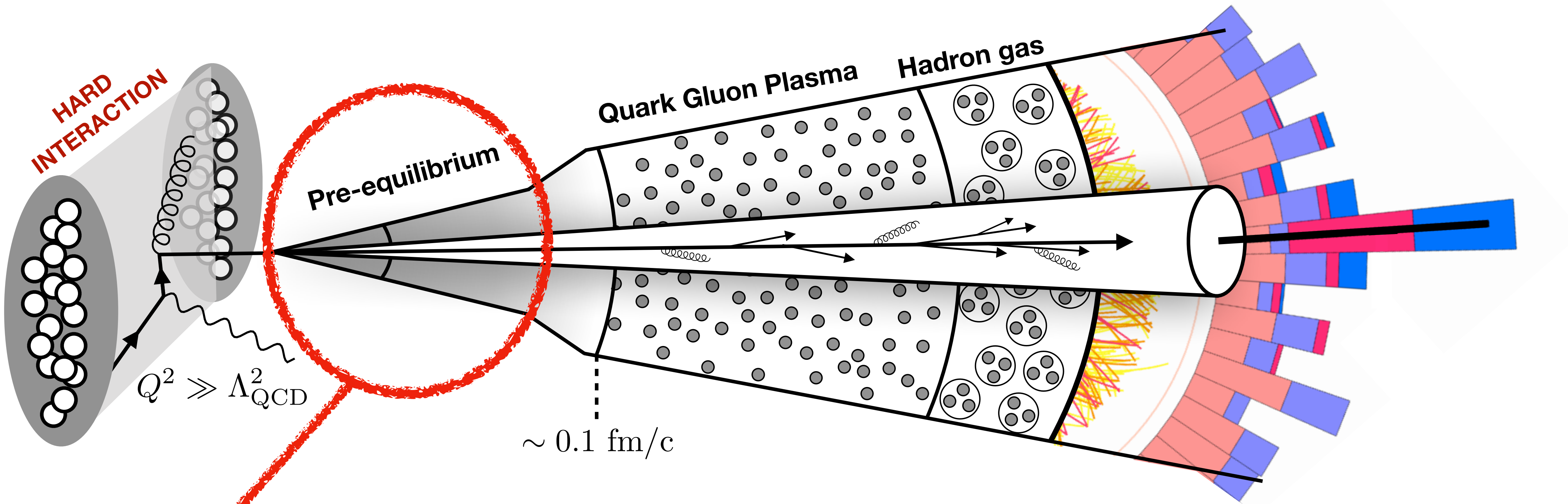

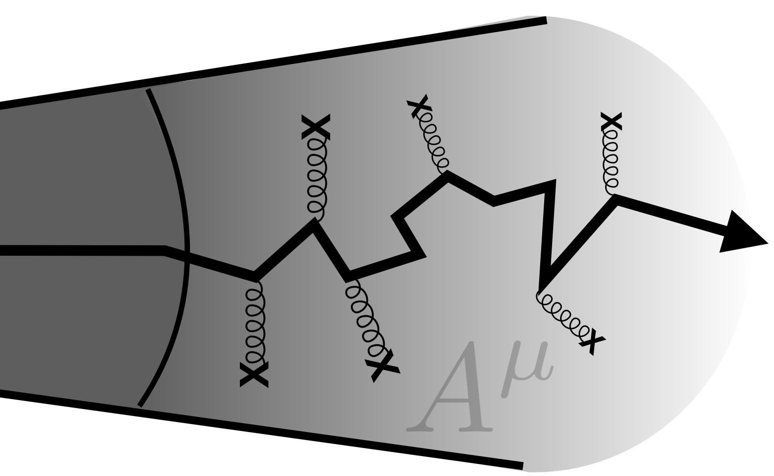

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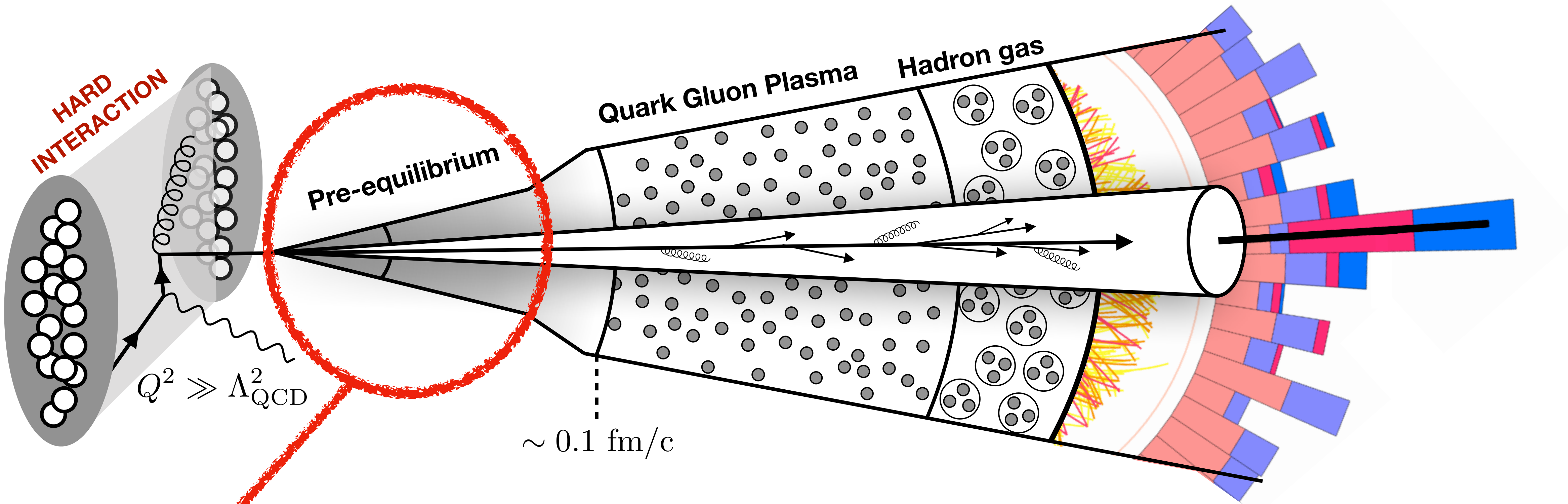

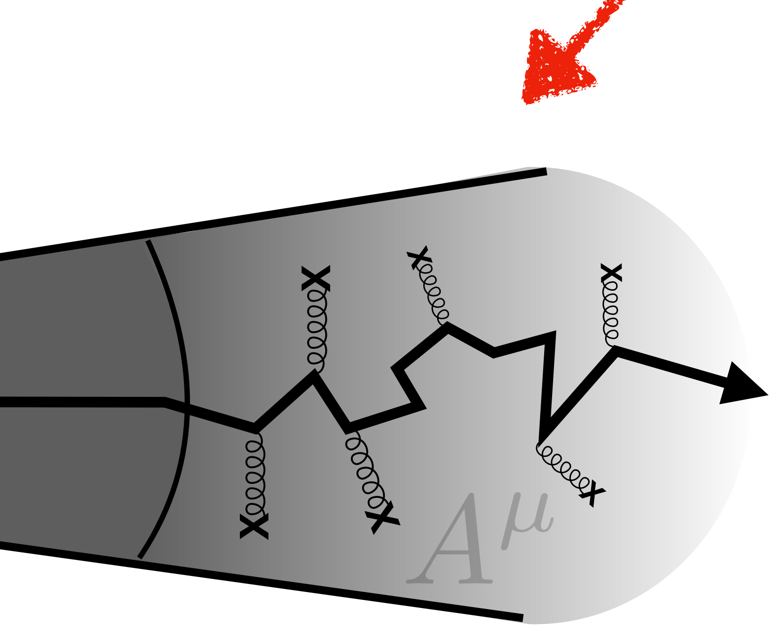

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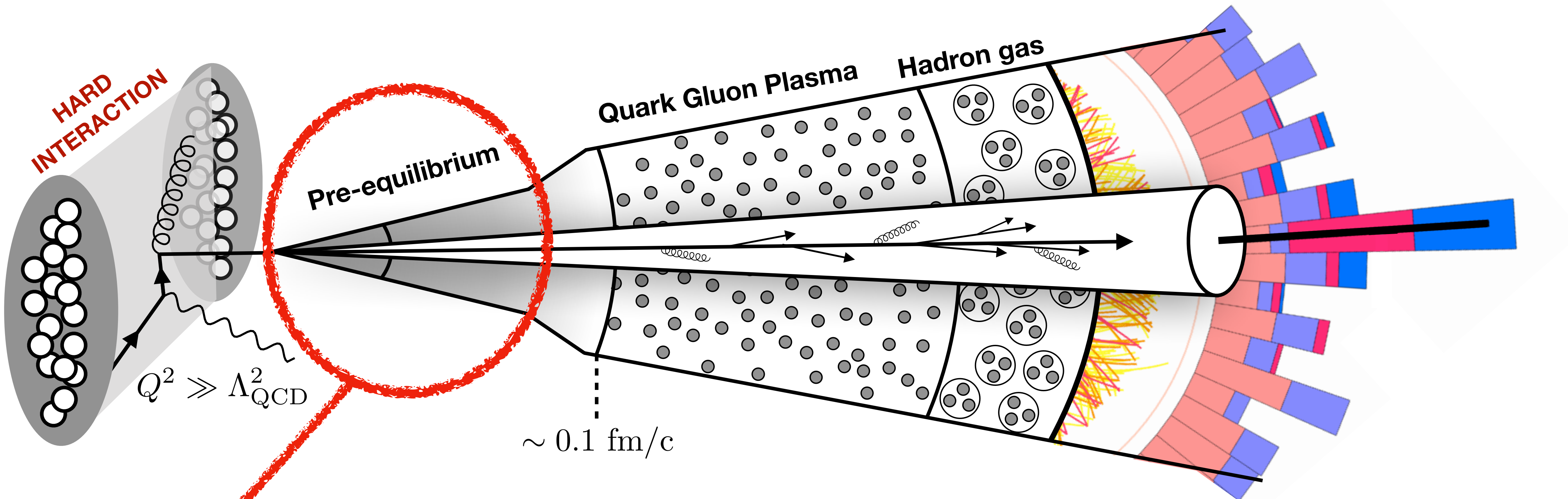

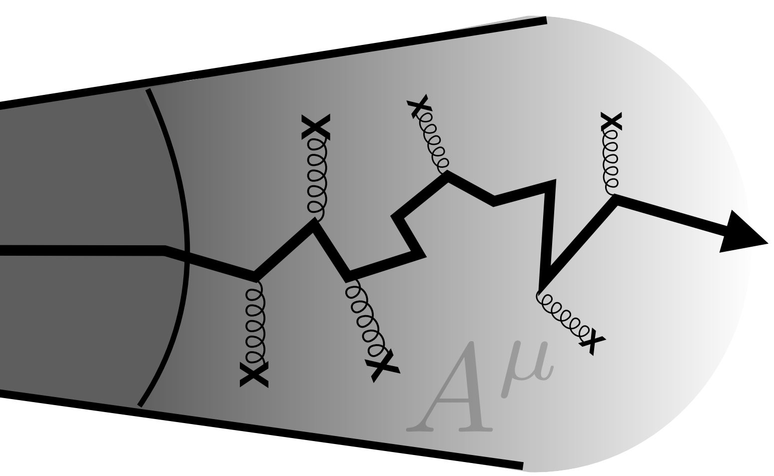

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Some even propose that **impact of pre-eq phase** $\approx$ **impact of QGP!!**

- *Can we make a theoretically robust estimation of the transport coefficients characterizing the pre-eq phase?*

The Color Glass Condensate

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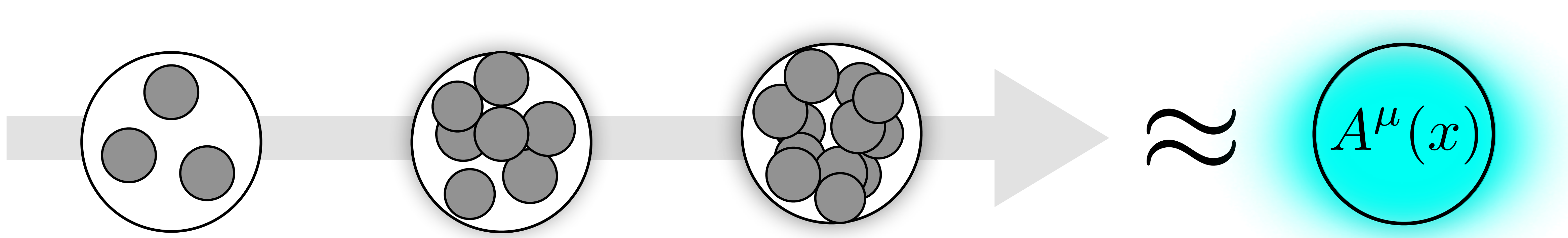
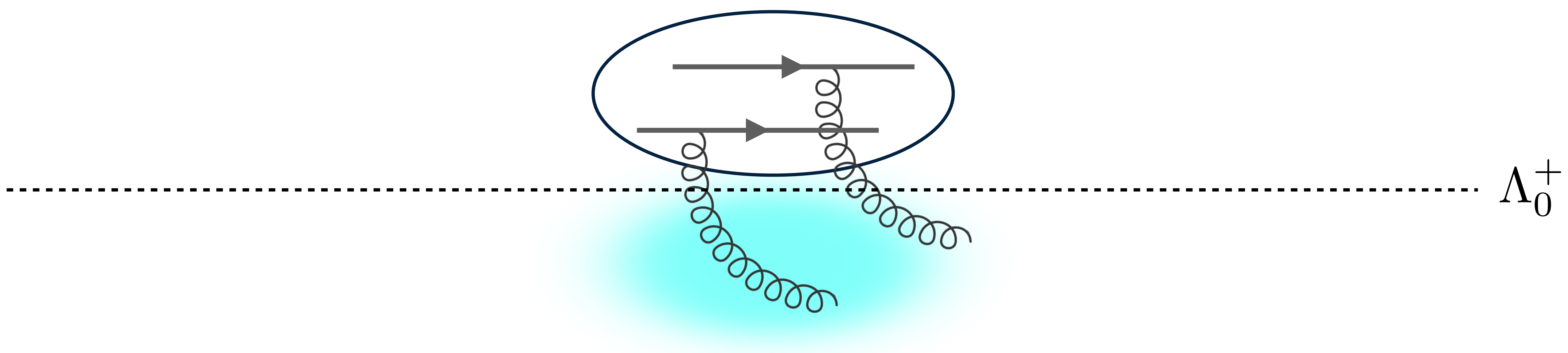


Moderate energy: Dilute hadrons

High energy: Gluon-dense hadrons

CGC: Classical fields generated by a color current



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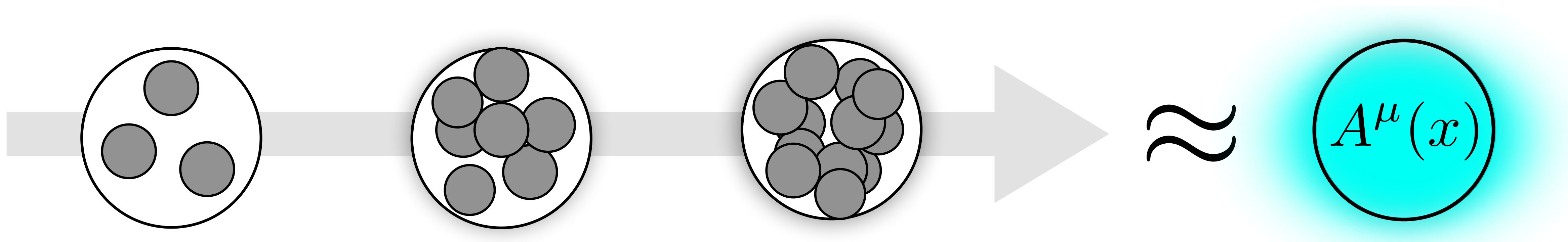

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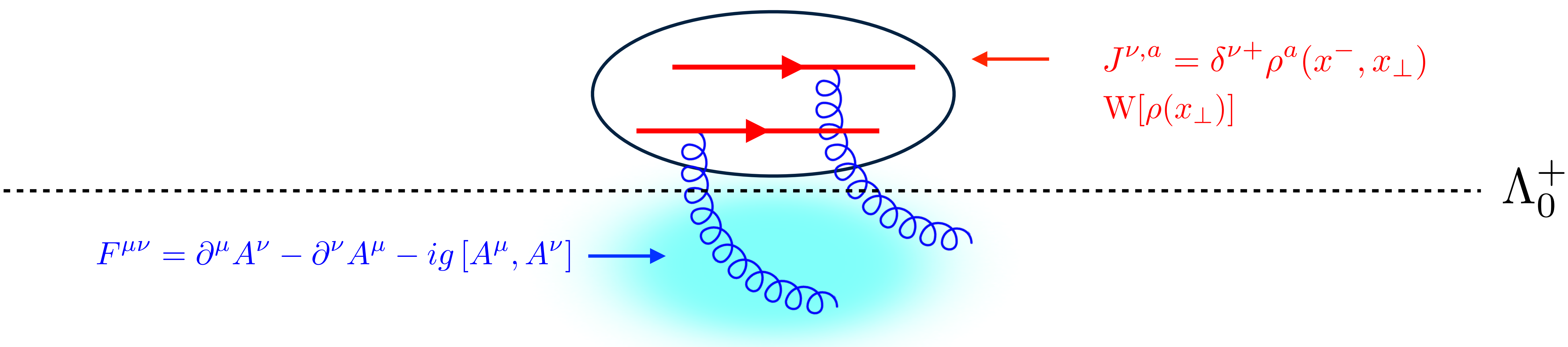

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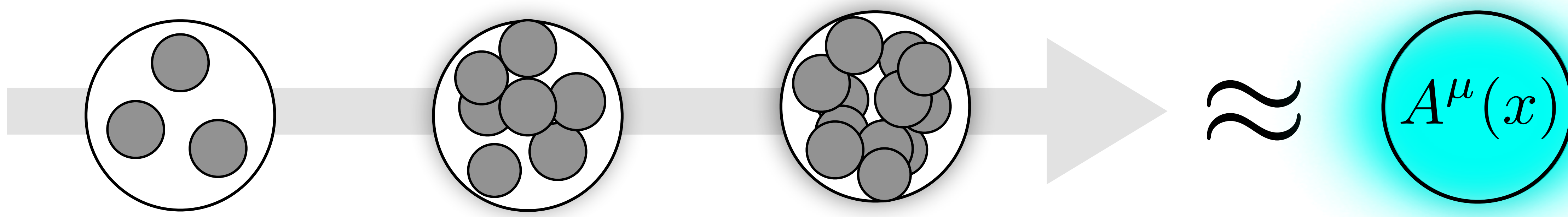

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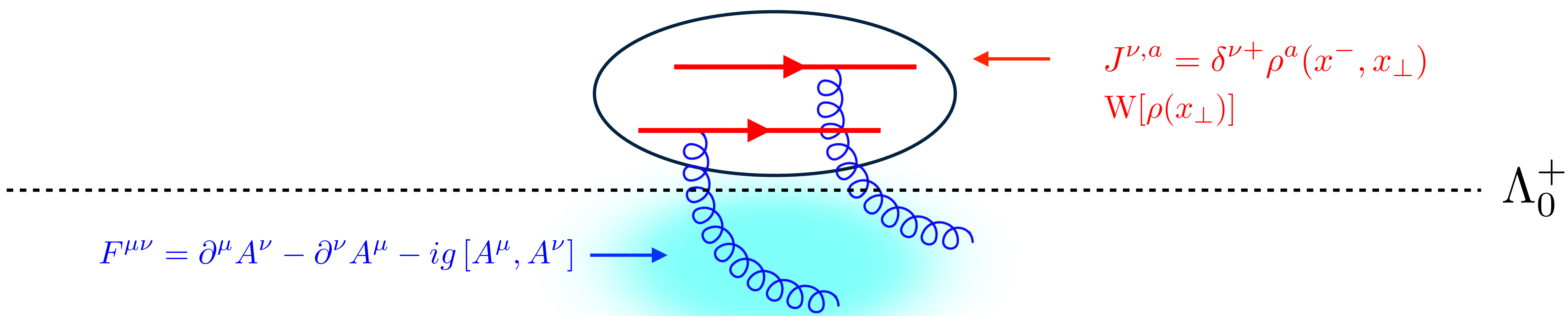

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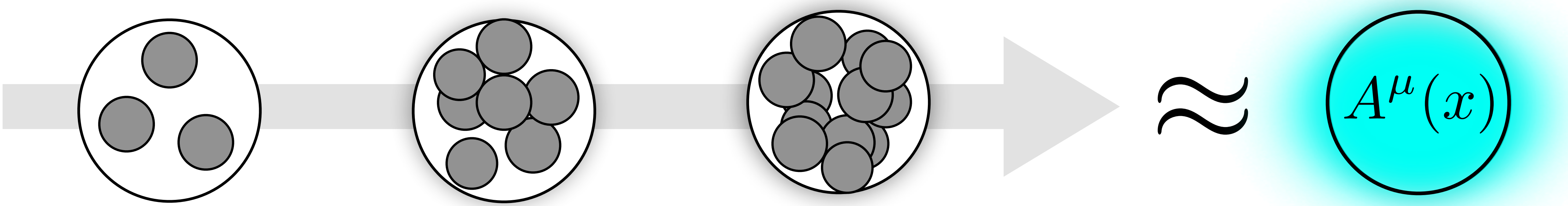

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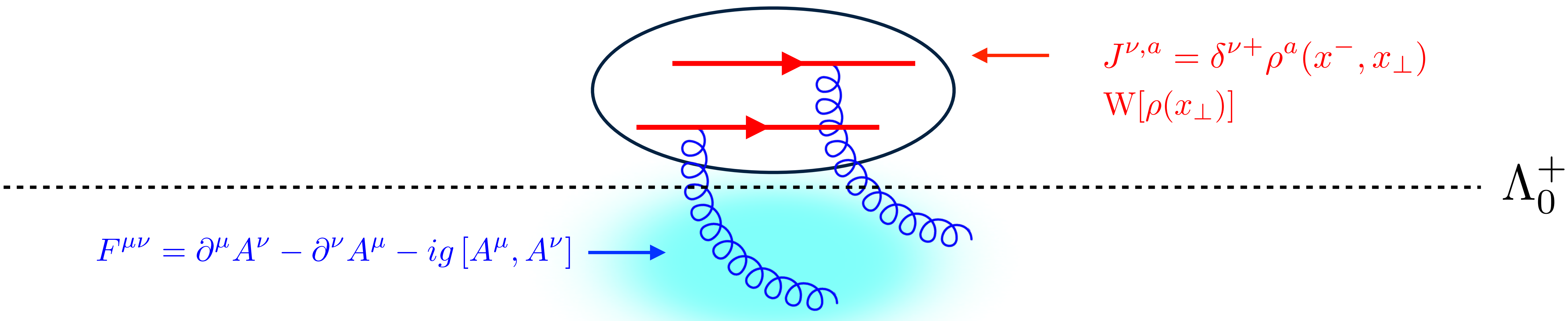

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➡ Color charge distributions for large nuclei: **McLerran-Venugopalan (MV) model** *(Gaussian statistics)*

$$\langle \rho^a(x_\perp) \rangle = 0 \quad , \quad \langle \rho^a(x_\perp)\rho^b(y_\perp) \rangle = g^2\mu^2\delta^{ab}\delta^{(2)}(x_\perp - y_\perp)$$

The Glasma Fields

Kovner, McLerran and Weigert, Phys. Rev. D 52 (1995)

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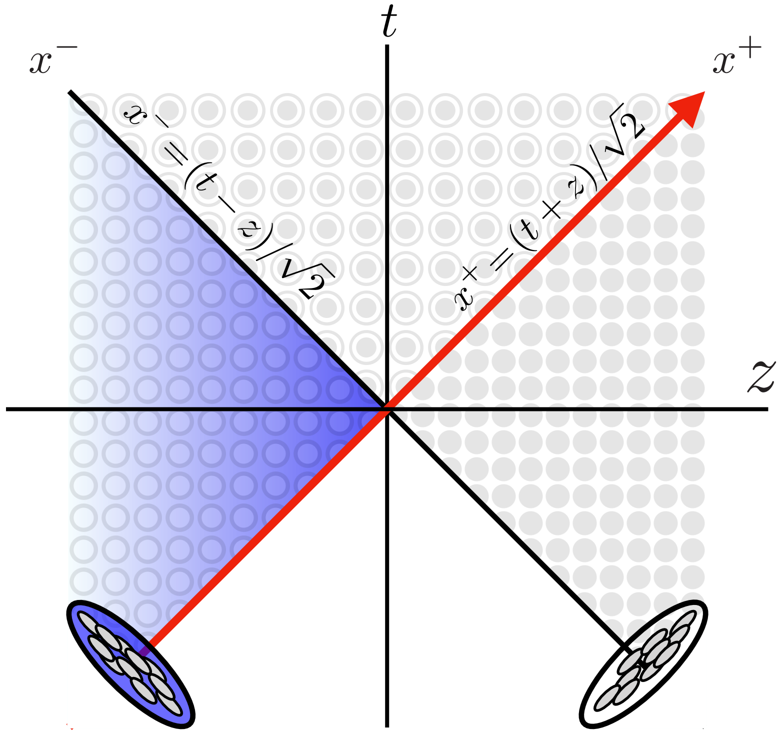

$$[D_\mu, F^{\mu\nu}] = J_1^\nu$$

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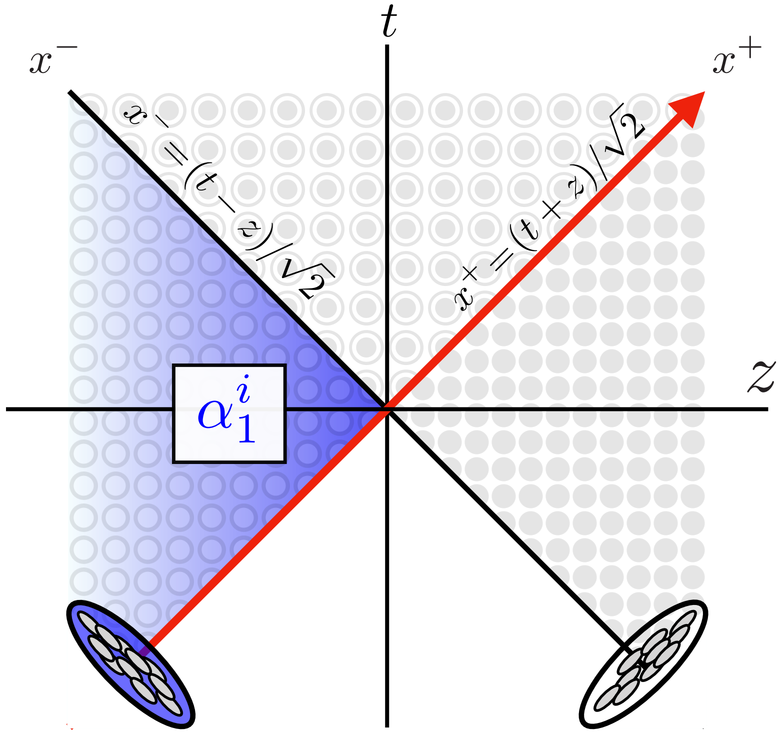

$$[D_\mu, F^{\mu\nu}] = J_1^\nu \xrightarrow[\text{(Solving in LC gauge } A^-=0)]{} \boxed{\alpha_1^i(x_\perp) = -\frac{1}{ig}U_1(x_\perp)\partial^i U_1^\dagger(x_\perp)}$$

with the Wilson line defined as: $U_1(x_\perp) = P^- \exp\left\{-ig\int_{-\infty}^{\infty} dz^- \frac{1}{\nabla^2}\rho_1(z^-, x_\perp)\right\}$

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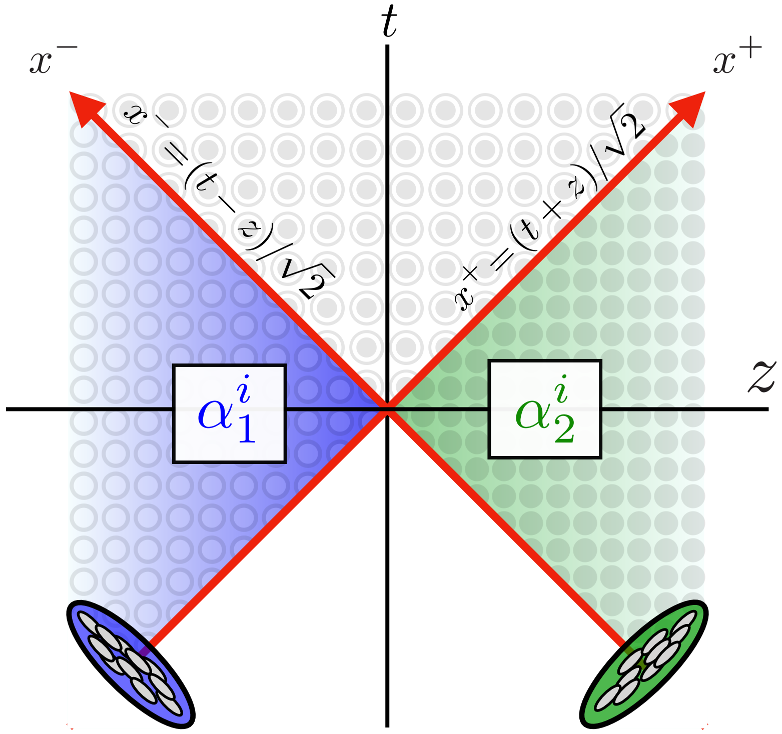

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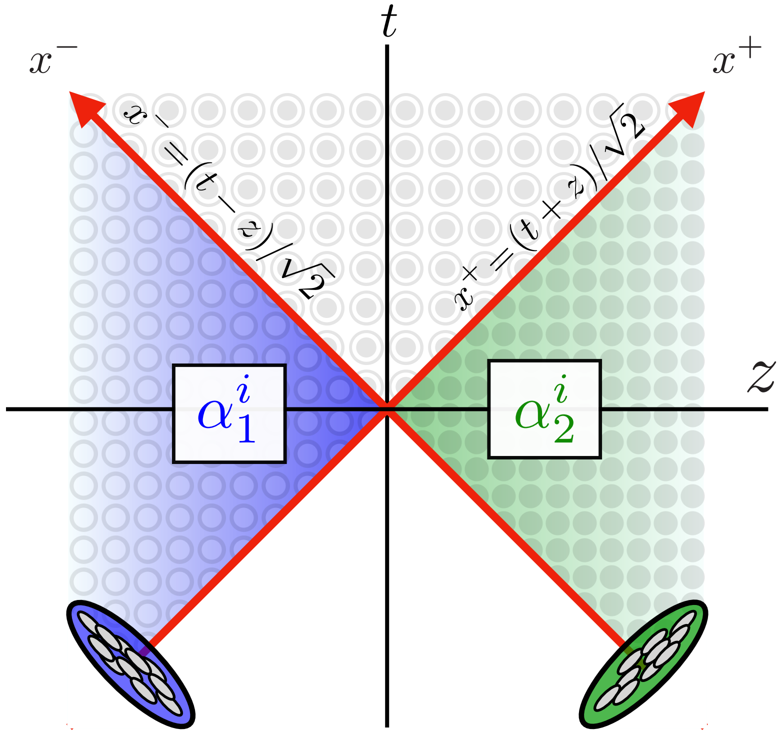

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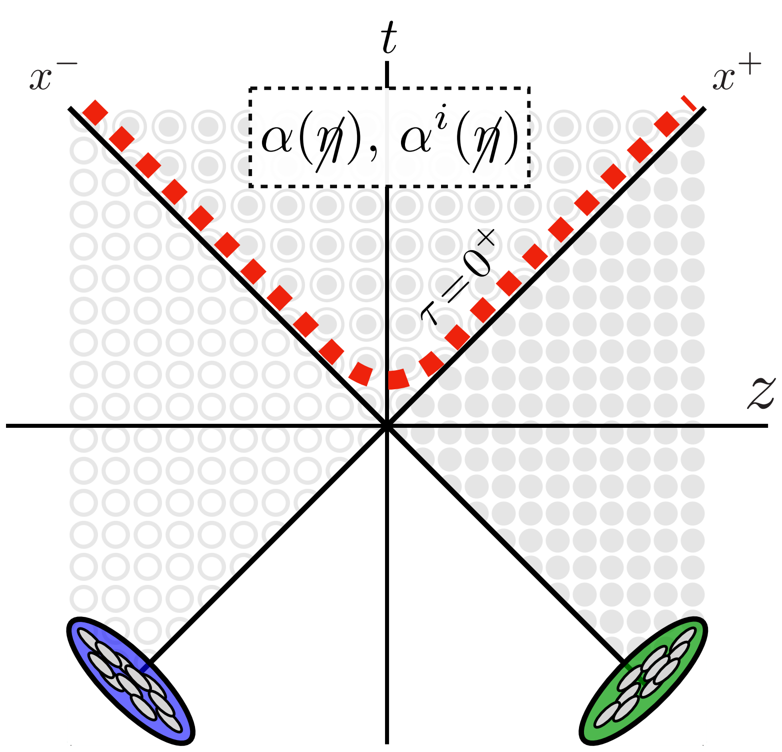

$$[D_\mu, F^{\mu\nu}] = J_1^\nu + J_2^\nu$$

(In Fock-Schwinger gauge gauge $A^+x^- + A^-x^+ = 0$)

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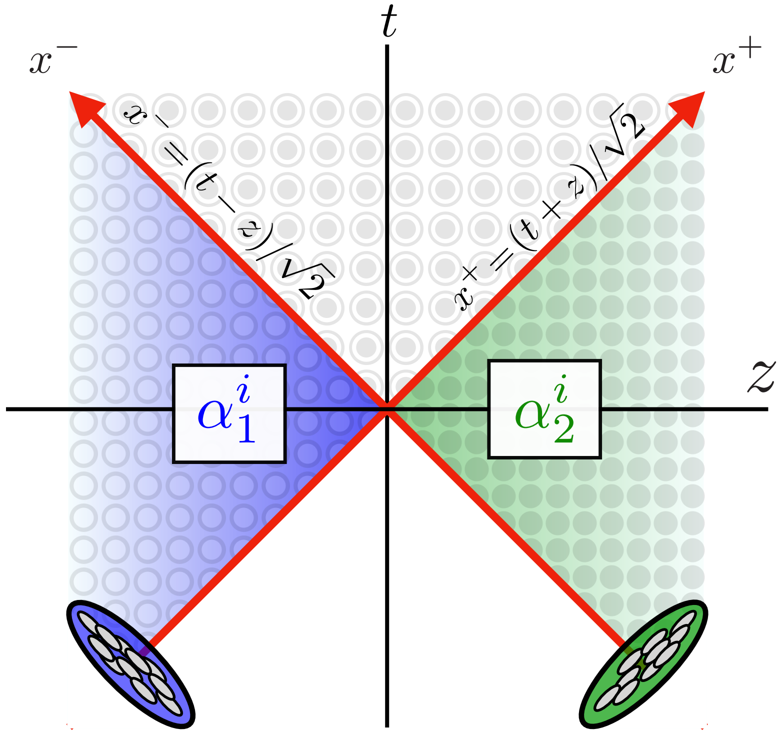

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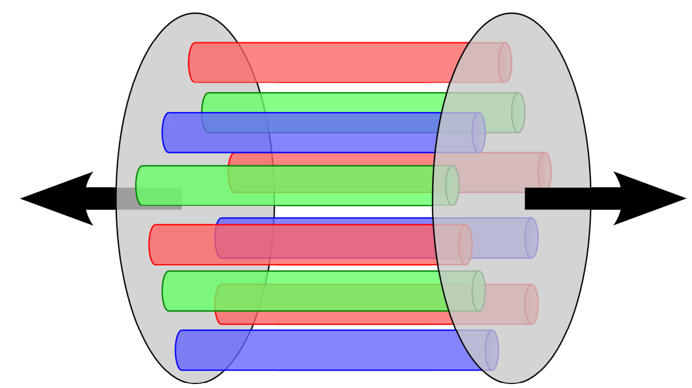

Credit: Fukushima; Acta Phys.Polon. 2011

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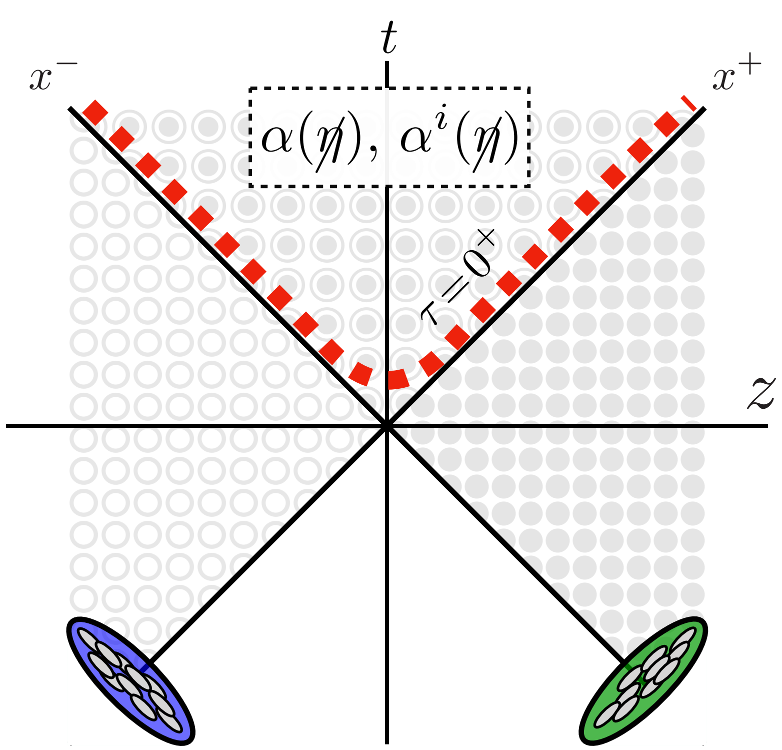

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➡ Longitudinal chromo-electric and chromo-magnetic fields (**flux tubes**):

$$\boxed{\begin{aligned} E^\eta(\tau = 0^+, x_\perp) &= -ig\delta^{ij}[\alpha_1^i(x_\perp), \alpha_2^j(x_\perp)] \\ B^\eta(\tau = 0^+, x_\perp) &= -ig\varepsilon^{ij}[\alpha_1^i(x_\perp), \alpha_2^j(x_\perp)] \end{aligned}}$$

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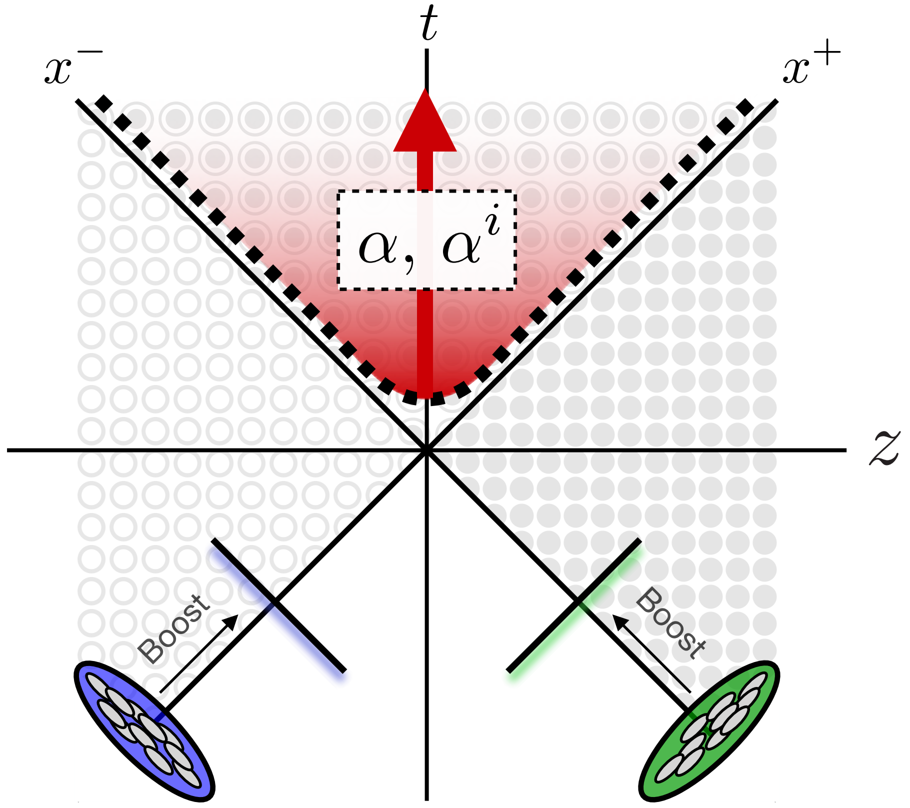

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(Extremely Lorentz-contracted nuclei) $\approx \rho_1(x_\perp)\delta(x^-)\delta^{\nu+} + \rho_2(x_\perp)\delta(x^+)\delta^{\nu-}$

$$\tau = \sqrt{2x^+x^-} > 0 \longrightarrow [D_\mu, F^{\mu\nu}] = 0$$

$$\nu = \tau \longrightarrow \ ig\tau\,[\alpha, \partial_\tau\alpha] - \frac{1}{\tau}\left[D_i, \partial_\tau\alpha^i\right] = 0$$

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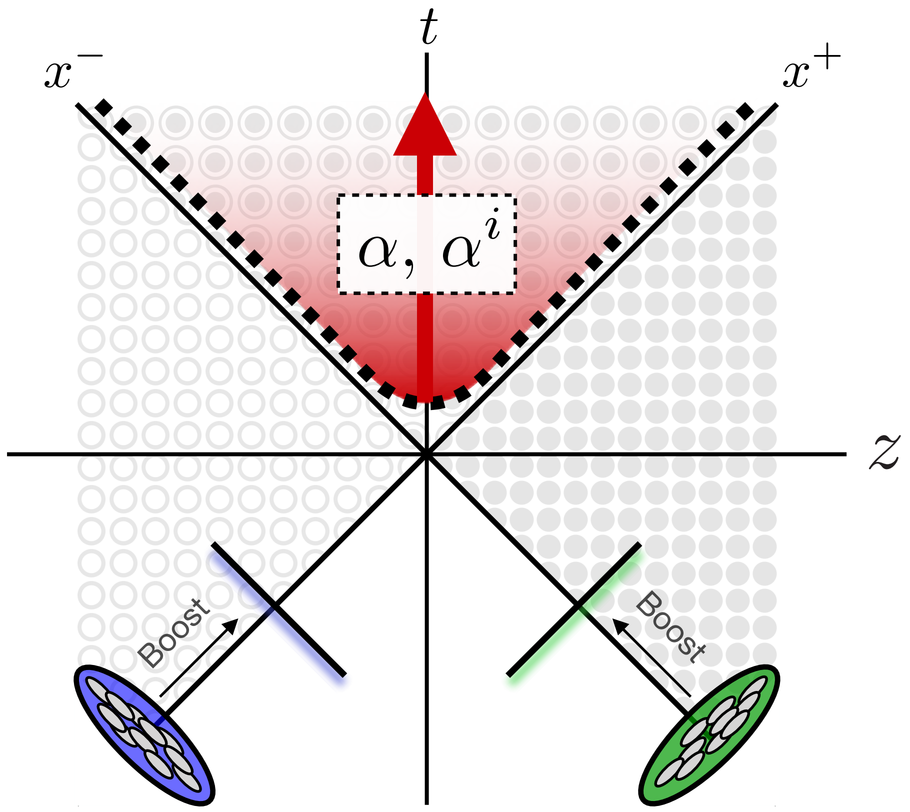

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➡ Typically solved through numerical methods. **Analytical approximations** include:

- Weak field limit (i.e. expanding in orders of one of the sources), e.g. PGR and S. Demirci; PRD, 2023
- Expanding the fields in powers of the proper time, e.g. M. E. Carrington, A. Czajka, S. Mrowczynski; PRC, 2022
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The Glasma Fields (at $\tau > 0^+$)

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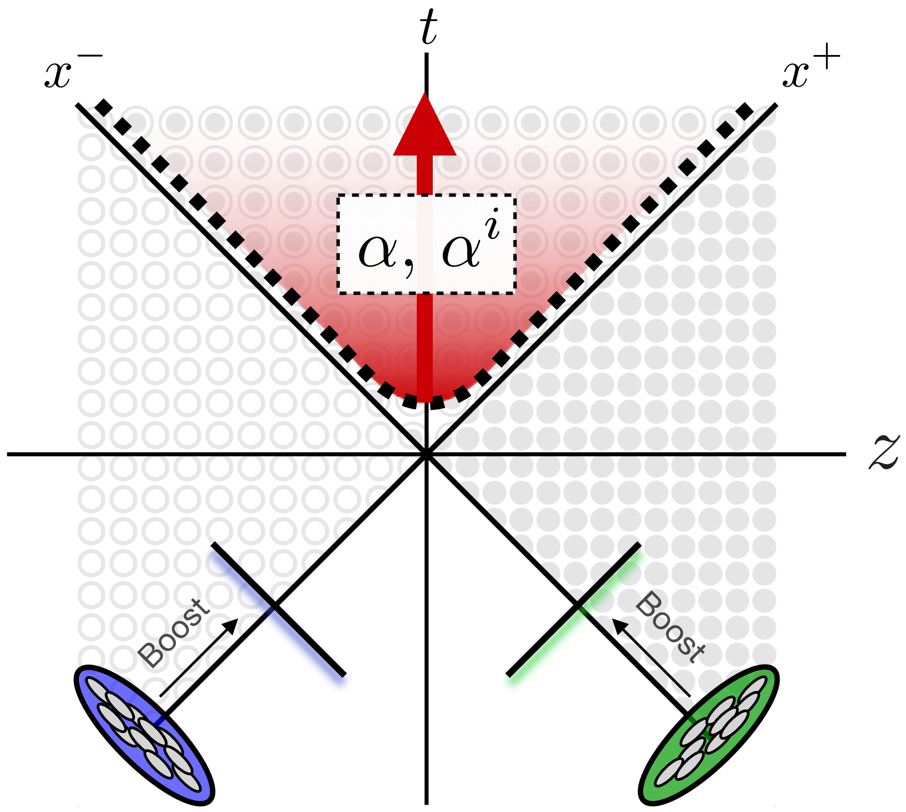

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➡ We employ the **linearized Yang-Mills equations** (as in PGR and T. Lappi; PRD, 2021):

$$\partial_\tau\frac{1}{\tau}\partial_\tau(\tau^2\beta) = \tau\partial_i\partial^i\beta$$
$$\partial_\tau(\tau\partial_\tau\beta^i) = \tau\partial^k\partial_k\beta^i$$

Where we impose $\partial_i\beta^i = 0$ **(Coulomb gauge condition)** ⟶ **Amplitude of transverse component minimized**

➡ **Free plane wave** solution of linearized Yang-Mills equations in momentum space:

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$$\longrightarrow \quad \beta(\tau, k_\perp) = \beta_0(k_\perp)\frac{2J_1(k\tau)}{k\tau} \quad , \quad \beta^i(\tau, k_\perp) = \beta_0^i(k_\perp)J_0(k\tau)$$

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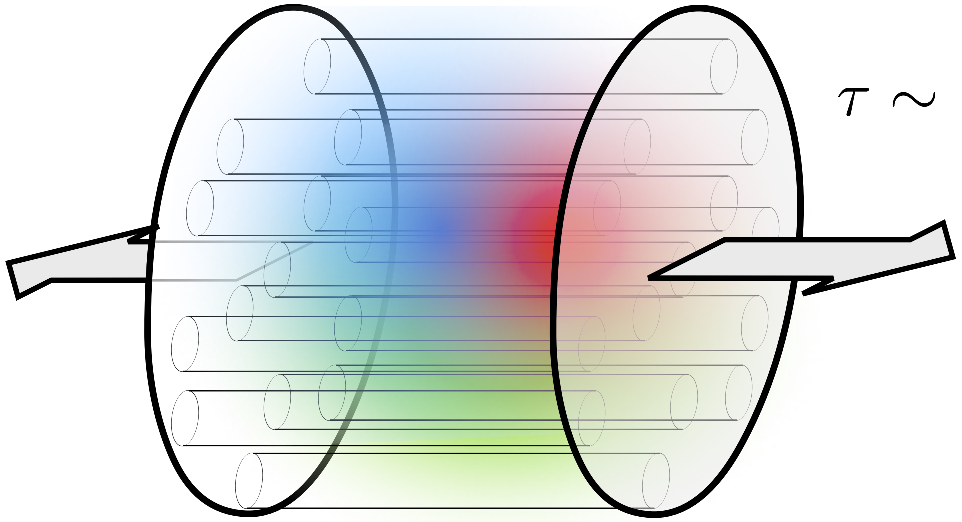
➡ Boundary conditions obtained by **matching to** $\tau = 0^+$ **solution (** $E^\eta(\tau = 0^+, x_\perp),\ B^\eta(\tau = 0^+, x_\perp)$ **)**:

$$\beta(\tau, k_\perp) = \frac{\tau}{k}E_0^\eta(k_\perp)J_1(k\tau) \, , \quad \beta^i(\tau, k_\perp) = -i\frac{\epsilon^{ij}k^j}{k^2}B_0^\eta(k_\perp)J_0(k\tau)$$

(Where $E^\eta(\tau = 0^+, x_\perp) = -ig\delta^{ij}[\alpha_1^i(x_\perp), \alpha_2^j(x_\perp)]$, $B^\eta(\tau = 0^+, x_\perp) = -ig\varepsilon^{ij}[\alpha_1^i(x_\perp), \alpha_2^j(x_\perp)]$)

➡ Chromo-electric and chromo-magnetic fields at $\tau > 0$:

$$\boxed{\begin{aligned} E^\eta(\tau, k_\perp) &= E_0^\eta(k_\perp)J_0(k\tau) \\ E^i(\tau, k_\perp) &= i\epsilon^{ij}\frac{k^j}{k}B_0^\eta(k_\perp)J_1(k\tau) \\ B^\eta(\tau, k_\perp) &= B_0^\eta(k_\perp)J_0(k\tau) \\ B^i(\tau, k_\perp) &= -i\epsilon^{ij}\frac{k^j}{k}E_0^\eta(k_\perp)J_1(k\tau) \end{aligned}}$$



➡ **Free plane wave** solution of linearized Yang-Mills equations in momentum space:

$$\partial_\tau \frac{1}{\tau}\partial_\tau(\tau^2\beta) = \tau\partial_i\partial^i\beta \qquad \partial_\tau(\tau\partial_\tau\beta^i) = \tau\partial^k\partial_k\beta^i \longrightarrow \boxed{\beta(\tau,k_\perp) = \beta_0(k_\perp)\frac{2J_1(k\tau)}{k\tau}} \;,\; \boxed{\beta^i(\tau,k_\perp) = \beta_0^i(k_\perp)J_0(k\tau)}$$

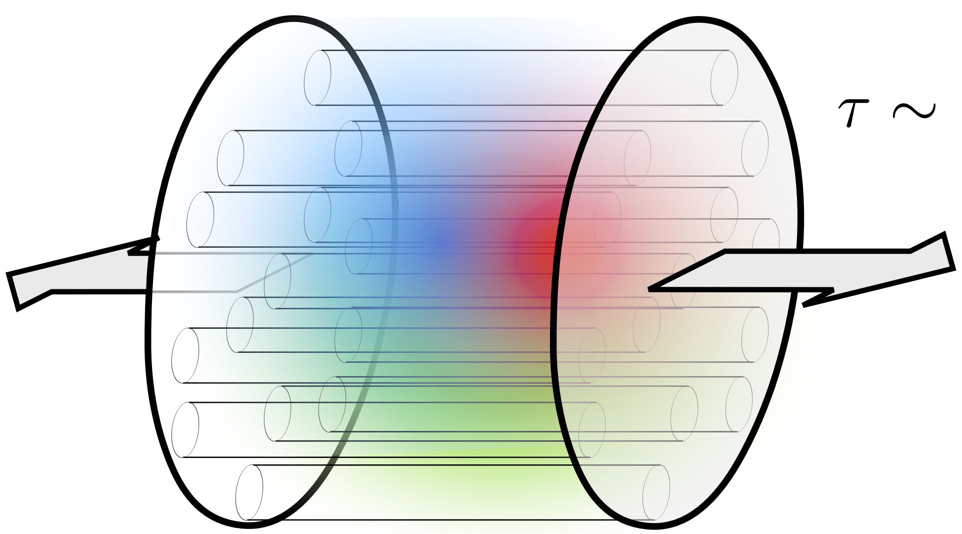
➡ Boundary conditions obtained by **matching to** $\tau = 0^+$ **solution (** $E^\eta(\tau=0^+,x_\perp),\ B^\eta(\tau=0^+,x_\perp)$**)**:

$$\boxed{\beta(\tau,k_\perp) = \frac{\tau}{k}E_0^\eta(k_\perp)J_1(k\tau)} \;,\; \boxed{\beta^i(\tau,k_\perp) = -i\frac{\epsilon^{ij}k^j}{k^2}B_0^\eta(k_\perp)J_0(k\tau)}$$

(Where $E^\eta(\tau=0^+,x_\perp) = -ig\delta^{ij}[\alpha_1^i(x_\perp),\alpha_2^j(x_\perp)]$, $B^\eta(\tau=0^+,x_\perp) = -ig\varepsilon^{ij}[\alpha_1^i(x_\perp),\alpha_2^j(x_\perp)]$)

➡ Chromo-electric and chromo-magnetic fields at $\tau > 0$:

$$\boxed{\begin{aligned} E^\eta(\tau,k_\perp) &= E_0^\eta(k_\perp)J_0(k\tau) \\ E^i(\tau,k_\perp) &= i\epsilon^{ij}\frac{k^j}{k}B_0^\eta(k_\perp)J_1(k\tau) \\ B^\eta(\tau,k_\perp) &= B_0^\eta(k_\perp)J_0(k\tau) \\ B^i(\tau,k_\perp) &= -i\epsilon^{ij}\frac{k^j}{k}E_0^\eta(k_\perp)J_1(k\tau) \end{aligned}}$$



➡ This solution effectively resums the UV divergencies present in the power series in τ (see H. Fujii, K. Fukushima, Y. Hidaka; PRC (2008))

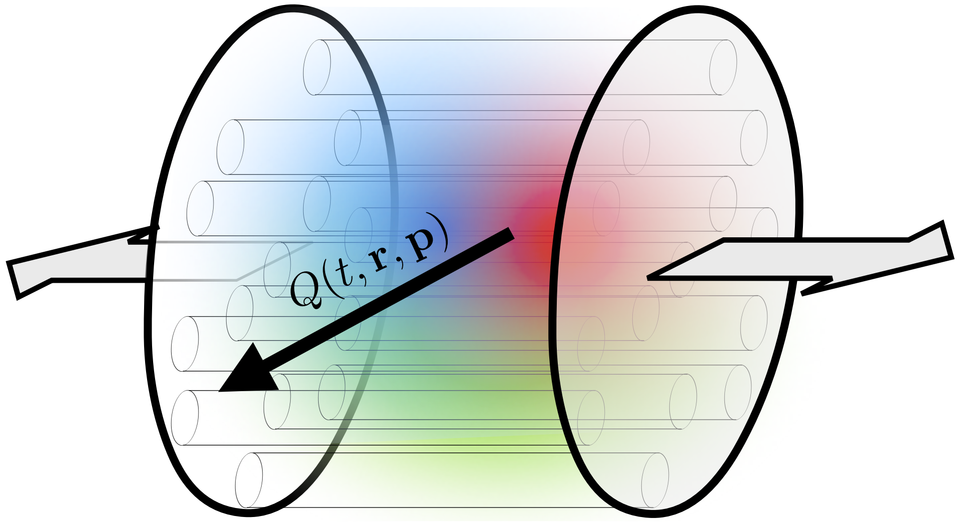
Fokker-Planck Equation

M. E. Carrington, A. Czajka, S. Mrowczynski; PRC, 2022
Stanislaw Mrowczynski; Eur. Phys. Journal A, 2018

- *How do quarks move through the Glasma?* Let $Q(t, \mathbf{r}, \mathbf{p})$ be the **quark distribution function**

➡ We start from the following **Vlasov-type equation**: $\underbrace{\Big(\partial_t + \mathbf{v}\cdot\nabla\Big)}_{\equiv\,\mathcal{D}\ \text{“Material derivative”}} Q(t, \mathbf{r}, \mathbf{p}) - \frac{1}{2}\left\{\mathbf{F}(t, \mathbf{r}), \nabla_p Q(t, \mathbf{r}, \mathbf{p})\right\} = 0$

Based on the following approximations:
1. **Collision-less** (low concentration of hard probes in the Glasma)
2. **Mean field** ($\mathbf{F}$ dominated by ensemble-averaged field)



M. E. Carrington, A. Czajka, S. Mrowczynski; PRC, 2022
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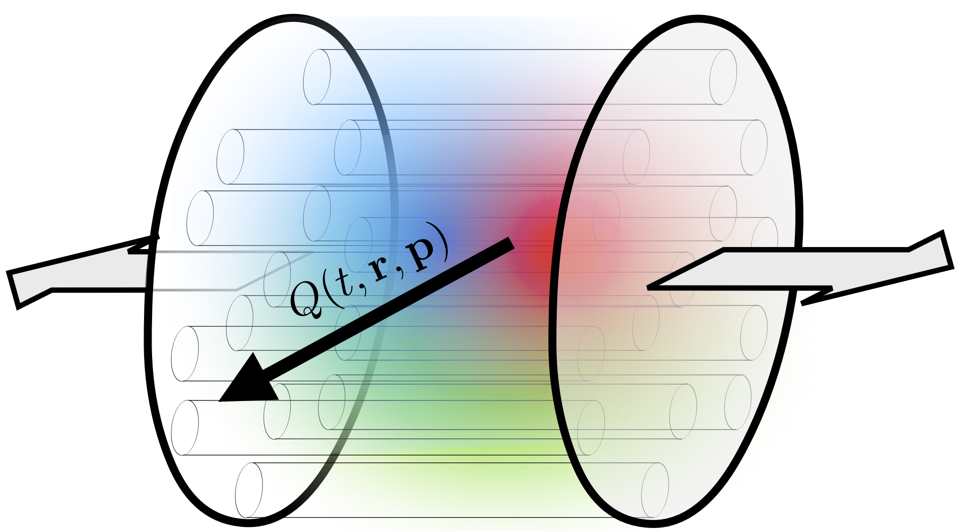
“Regular” and “Turbulent” components



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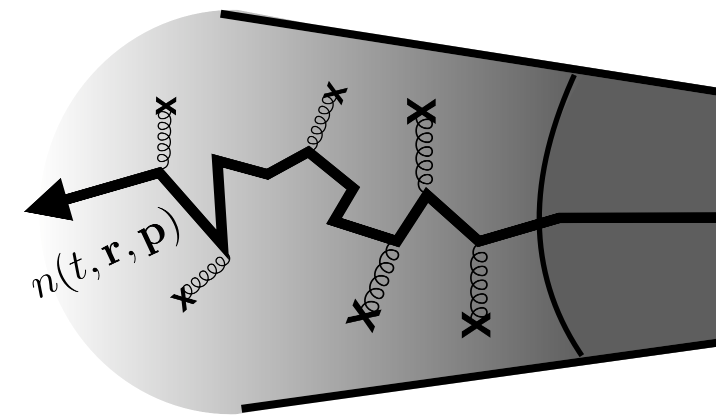
➡ By substituting, taking the ensemble average and tracing over color indices one arrives at a **Fokker-Planck transport equation**:

$$\Big(\mathcal{D} - \nabla_p^\alpha X^{\alpha\beta}(\mathbf{v})\nabla_p^\beta - \nabla^\alpha Y^\alpha(\mathbf{v})\Big) n(t, \mathbf{x}, \mathbf{p}) = 0$$



Where the collision terms are defined as:

$$X^{\alpha\beta}(\boldsymbol{v}) \equiv \frac{1}{2N_c}\int_0^t \mathrm{d}t' \mathrm{Tr}\left[\langle \mathcal{F}^\alpha(t, \mathbf{x})\mathcal{F}^\beta(t', \mathbf{y})\rangle\right] \quad , \quad Y^\alpha(\mathbf{v}) \sim X^{\alpha\beta}\frac{v^\beta}{T}$$

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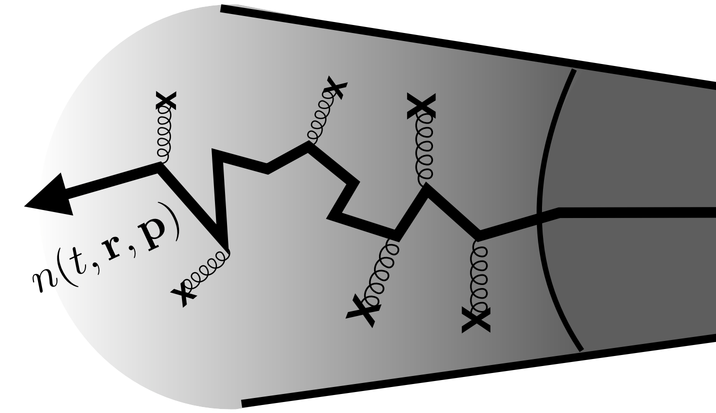

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M. E. Carrington, A. Czajka, S. Mrowczynski; PRC, 2022
Stanislaw Mrowczynski; Eur. Phys. Journal A, 2018

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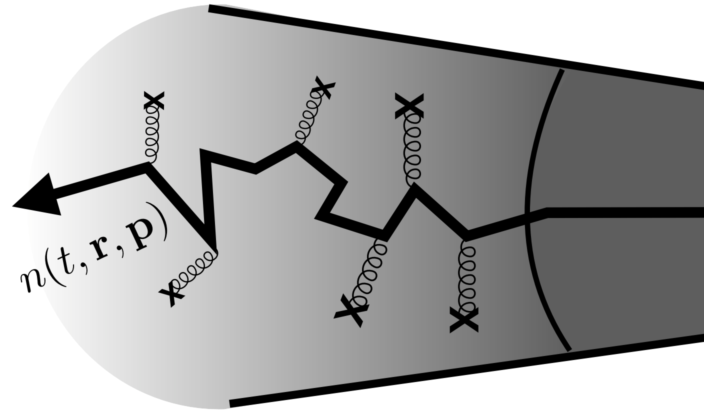

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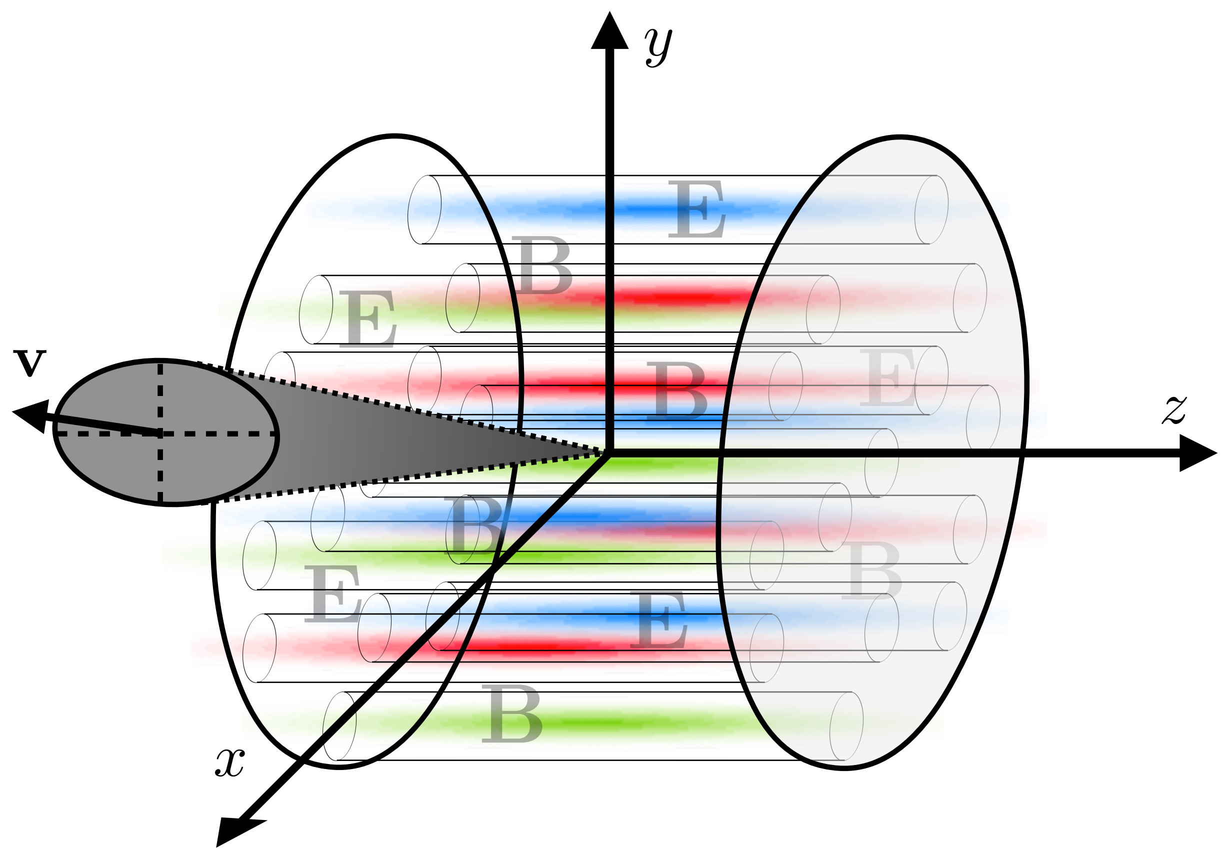
➡ Substituting the chromodynamic fields, one obtains (**M. E. Carrington, A. Czajka, S. Mrowczynski; PRC, 2022**):

$$X^{\alpha\beta}(\boldsymbol{v}) = \frac{g^2}{2N_c}\int_0^t \mathrm{d}t'\Big[\langle E_a^\alpha(t,\mathbf{x})E_a^\beta(t',\mathbf{y})\rangle + \epsilon^{\beta\gamma\gamma'}v^\gamma\langle E_a^\alpha(t,\mathbf{x})B_a^{\gamma'}(t',\mathbf{y})\rangle$$
$$+\,\epsilon^{\alpha\gamma\gamma'}v^\gamma\langle B_a^{\gamma'}(t,\mathbf{x})E_a^\beta(t',\mathbf{y})\rangle + \epsilon^{\alpha\gamma\gamma'}\epsilon^{\beta\delta\delta'}v^\gamma v^\delta\langle B_a^{\gamma'}(t,\mathbf{x})B_a^{\delta'}(t',\mathbf{y})\rangle\Big]$$

With e.g. $E^z(\tau,\mathbf{x}_\perp) = -ig\delta^{ij}\int\frac{d^2\mathbf{k}_\perp}{(2\pi)^2}\int d^2\mathbf{u}_\perp[\alpha_1^i(\mathbf{u}_\perp),\alpha_2^j(\mathbf{u}_\perp)]J_0(k_\perp\tau)e^{i\mathbf{k}_\perp(\mathbf{x}-\mathbf{u})_\perp}$ and $\mathbf{y} = \mathbf{x} - \boldsymbol{v}(t-t')$



Transport Coefficients in the Glasma

➡ We have: $\frac{\mathrm{d}E}{\mathrm{d}x} = -\frac{v}{T}\frac{v^\alpha v^\beta}{v^2}X^{\alpha\beta}(\boldsymbol{v})$, $\hat{q} = \frac{2}{v}\left(\delta^{\alpha\beta} - \frac{v^\alpha v^\beta}{v^2}\right)X^{\alpha\beta}(\boldsymbol{v})$ with $\boldsymbol{v} = \mathbf{p}/E_\mathbf{p}$

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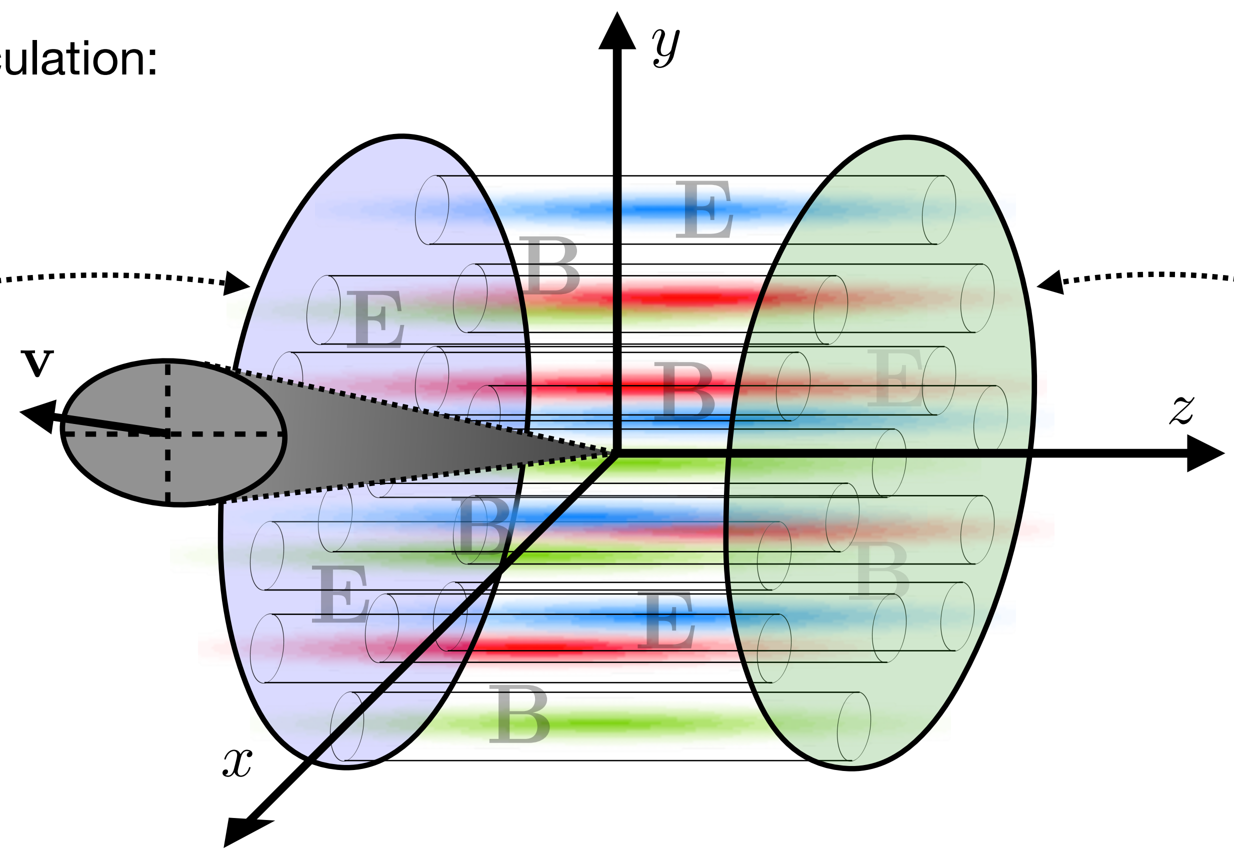
$$\boxed{\begin{aligned} X^{\alpha\beta}(\boldsymbol{v}) = \frac{g^2}{2N_c}\int_0^t \mathrm{d}t' \Big[&\langle E_a^\alpha(t,\mathbf{x})E_a^\beta(t',\mathbf{y})\rangle + \epsilon^{\beta\gamma\gamma'}v^\gamma\langle E_a^\alpha(t,\mathbf{x})B_a^{\gamma'}(t',\mathbf{y})\rangle \\ &+ \epsilon^{\alpha\gamma\gamma'}v^\gamma\langle B_a^{\gamma'}(t,\mathbf{x})E_a^\beta(t',\mathbf{y})\rangle + \epsilon^{\alpha\gamma\gamma'}\epsilon^{\beta\delta\delta'}v^\gamma v^\delta\langle B_a^{\gamma'}(t,\mathbf{x})B_a^{\delta'}(t',\mathbf{y})\rangle\Big] \end{aligned}}$$

With e.g. $E^z(\tau,\mathbf{x}_\perp) = -ig\delta^{ij}\int\frac{d^2\mathbf{k}_\perp}{(2\pi)^2}\int d^2\mathbf{u}_\perp[\alpha_1^i(\mathbf{u}_\perp),\alpha_2^j(\mathbf{u}_\perp)]J_0(k_\perp\tau)e^{i\mathbf{k}_\perp(\mathbf{x}-\mathbf{u})_\perp}$ and $\mathbf{y} = \mathbf{x} - \boldsymbol{v}(t-t')$

➡ Building blocks of the calculation:

2-point correlators

$$\langle\alpha_1^{a,i}\alpha_1^{b,j}\rangle \equiv W_1^{ij}\delta^{ab}$$



$$\langle\alpha_2^{c,k}\alpha_2^{d,l}\rangle \equiv W_2^{kl}\delta^{cd}$$

➡ In terms of the **Weiszäcker-Williams distributions**:

- $$\frac{dE}{dx} = \frac{v_\perp}{T}\frac{g^4(N_c^2-1)}{2}\int_0^t dt' \int_{k_\perp q_\perp}\int_{u_\perp v_\perp} W_1^{ik}(u_\perp, v_\perp) W_2^{jl}(u_\perp, v_\perp)\, \epsilon^{ij}\epsilon^{kl}\left(\frac{v_\perp \times k_\perp}{v_\perp k_\perp}\right)\left(\frac{v_\perp \times q_\perp}{v_\perp q_\perp}\right) J_1(k_\perp t) J_1(q_\perp t') e^{ik_\perp (x-u)_\perp} e^{iq_\perp (y-v)_\perp}$$

- $$\begin{aligned}\hat{q} = \frac{g^4}{v_\perp}(N_c^2-1)\int_0^t dt' \int_{k_\perp q_\perp}\int_{u_\perp v_\perp} & W_1^{ik}(u_\perp, v_\perp) W_2^{jl}(u_\perp, v_\perp) \\ \times \Bigg[& \epsilon^{ij}\epsilon^{kl}\left(v_\perp J_0(k_\perp t) + i\left(\frac{v_\perp \cdot k_\perp}{v_\perp k_\perp}\right) J_1(k_\perp t)\right)\left(v_\perp J_0(q_\perp t') + i\left(\frac{v_\perp \cdot q_\perp}{v_\perp q_\perp}\right) J_1(q_\perp t')\right) \\ & + \delta^{ij}\delta^{kl}\left(J_0(k_\perp t) + i\left(\frac{v_\perp \cdot k_\perp}{k_\perp}\right) J_1(k_\perp t)\right)\left(J_0(q_\perp t') + i\left(\frac{v_\perp \cdot q_\perp}{q_\perp}\right) J_1(q_\perp t')\right)\Bigg] e^{ik_\perp (x-u)_\perp} e^{iq_\perp (y-v)_\perp}\end{aligned}$$

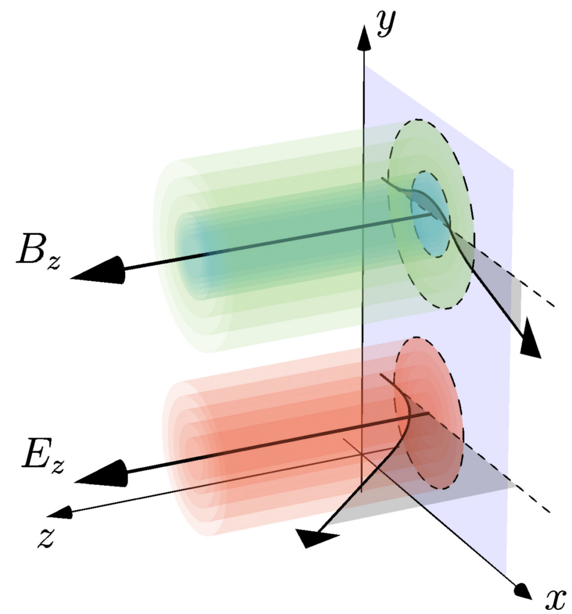
Transport coefficients at midrapidity ($\eta = 0$, $v_{\parallel} = 0$)

➡ In terms of the **Weiszäcker-Williams distributions** and the **initial fields**:

- $$\frac{dE}{dx} = \frac{v_\perp}{T}\frac{g^4(N_c^2-1)}{2}\int_0^t dt' \int_{k_\perp q_\perp}\int_{u_\perp v_\perp} W_1^{ik}(u_\perp, v_\perp) W_2^{jl}(u_\perp, v_\perp)\, \epsilon^{ij}\epsilon^{kl}\left(\frac{v_\perp \times k_\perp}{v_\perp k_\perp}\right)\left(\frac{v_\perp \times q_\perp}{v_\perp q_\perp}\right) J_1(k_\perp t) J_1(q_\perp t') e^{ik_\perp (x-u)_\perp} e^{iq_\perp (y-v)_\perp}$$
$$= \frac{v_\perp}{T}\frac{g^2}{2N_c}\int_0^t dt' \int_{k_\perp q_\perp} \langle B_0^z(k_\perp) B_0^z(q_\perp)\rangle \left(\frac{v_\perp \times k_\perp}{v_\perp k_\perp}\right)\left(\frac{v_\perp \times q_\perp}{v_\perp q_\perp}\right) J_1(k_\perp t) J_1(q_\perp t') \quad \Big\} \longrightarrow \sim \frac{\hat{q}_\parallel}{T}$$

- $$\hat{q} = \frac{g^4}{v_\perp}(N_c^2-1)\int_0^t dt' \int_{k_\perp q_\perp}\int_{u_\perp v_\perp} W_1^{ik}(u_\perp, v_\perp) W_2^{jl}(u_\perp, v_\perp)$$
$$\times \Bigg[\epsilon^{ij}\epsilon^{kl}\left(v_\perp J_0(k_\perp t) + i\left(\frac{v_\perp \cdot k_\perp}{v_\perp k_\perp}\right) J_1(k_\perp t)\right)\left(v_\perp J_0(q_\perp t') + i\left(\frac{v_\perp \cdot q_\perp}{v_\perp q_\perp}\right) J_1(q_\perp t')\right)$$
$$+ \delta^{ij}\delta^{kl}\left(J_0(k_\perp t) + i\left(\frac{v_\perp \cdot k_\perp}{k_\perp}\right) J_1(k_\perp t)\right)\left(J_0(q_\perp t') + i\left(\frac{v_\perp \cdot q_\perp}{q_\perp}\right) J_1(q_\perp t')\right)\Bigg] e^{ik_\perp (x-u)_\perp} e^{iq_\perp (y-v)_\perp}$$
$$= \frac{g^4}{v_\perp}(N_c^2-1)\int_0^t dt' \int_{k_\perp q_\perp}\Bigg[\langle B_0^z(k_\perp) B_0^z(q_\perp)\rangle \left(v_\perp J_0(k_\perp t) + i\left(\frac{v_\perp \cdot k_\perp}{v_\perp k_\perp}\right) J_1(k_\perp t)\right)\left(v_\perp J_0(q_\perp t') + i\left(\frac{v_\perp \cdot q_\perp}{v_\perp q_\perp}\right) J_1(q_\perp t')\right) \quad \Big\} \longrightarrow \hat{q}_\perp$$
$$+ \langle E_0^z(k_\perp) E_0^z(q_\perp)\rangle \left(J_0(k_\perp t) + i\left(\frac{v_\perp \cdot k_\perp}{k_\perp}\right) J_1(k_\perp t)\right)\left(J_0(q_\perp t') + i\left(\frac{v_\perp \cdot q_\perp}{q_\perp}\right) J_1(q_\perp t')\right)\Bigg] \quad \Big\} \longrightarrow \hat{q}_z$$

The momentum broadening coefficient has **two components**: one emerging from B_0^z (in the plane transverse to the beam), and another from E_0^z (in the beam direction)



On the other hand, the drag coefficient depends exclusively on the initial chromo-magnetic fields

(Borrowed from
A. Ipp, D. I. Müller, D. Schuh; PLB, 2020)

Transport coefficients at midrapidity ($\eta = 0$, $v_{\parallel} = 0$)

➡ We decompose the Weiszäcker-Williams distributions into two components: the **unpolarized** and **linearly polarized** gluon distributions

$$W^{ij}(r_\perp) = \frac{1}{2}\left(\delta^{ij}G(r_\perp) + \left(\delta^{ij} - 2\frac{r^i r^j}{r^2}\right)h(r_\perp)\right)$$

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➡ The transport coefficients then become:

- $$\frac{\mathrm{d}E}{\mathrm{d}x} = -\frac{g^4}{4}\frac{v_\perp}{T}(N_c^2-1)\int_0^t \mathrm{d}t' \int_0^{2\pi} \frac{\mathrm{d}\theta_{t_\perp}}{2\pi}\frac{\mathrm{d}\theta_{s_\perp}}{2\pi} \sin(\theta_{t_\perp}) \sin(\theta_{s_\perp}) \left(G_1(r_\perp)G_2(r_\perp) - h_1(r_\perp)h_2(r_\perp)\right)$$

- $$\hat{q} = \frac{g^4}{2}(N_c^2-1)\int_0^t \mathrm{d}t' \int_0^{2\pi} \frac{\mathrm{d}\theta_t}{2\pi}\frac{\mathrm{d}\theta_s}{2\pi}\frac{1}{v_\perp}\Big[(v_\perp - \cos\theta_t)(v_\perp - \cos\theta_s)\left(G_1(r_\perp)G_2(r_\perp) - h_1(r_\perp)h_2(r_\perp)\right) \\ + (1 - v_\perp\cos\theta_t)(1 - v_\perp\cos\theta_s)\left(G_1(r_\perp)G_2(r_\perp) + h_1(r_\perp)h_2(r_\perp)\right)\Big]$$

where: $r_\perp^2 = (v_\perp \Delta t + t'\cos\theta_{t_\perp} - t\cos\theta_{s_\perp})^2 + (t'\sin\theta_{t_\perp} - t\sin\theta_{s_\perp})^2$.

Transport coefficients at midrapidity ($\eta = 0$, $v_{\parallel} = 0$)

➡ We decompose the Weiszäcker-Williams distributions into two components: the **unpolarized** and **linearly polarized** gluon distributions

$$W^{ij}(r_\perp) = \frac{1}{2}\left(\delta^{ij} G(r_\perp) + \left(\delta^{ij} - 2\frac{r^i r^j}{r^2}\right) h(r_\perp)\right)$$

➡ The transport coefficients then become:

- $$\frac{\mathrm{d}E}{\mathrm{d}x} = -\frac{g^4}{4}\frac{v_\perp}{T}(N_c^2-1)\int_0^t \mathrm{d}t' \int_0^{2\pi} \frac{\mathrm{d}\theta_{t_\perp}}{2\pi}\frac{\mathrm{d}\theta_{s_\perp}}{2\pi} \sin(\theta_{t_\perp})\sin(\theta_{s_\perp})\left(G_1(r_\perp)G_2(r_\perp) - h_1(r_\perp)h_2(r_\perp)\right)$$
- $$\hat{q} = \frac{g^4}{2}(N_c^2-1)\int_0^t \mathrm{d}t' \int_0^{2\pi} \frac{\mathrm{d}\theta_t}{2\pi}\frac{\mathrm{d}\theta_s}{2\pi}\frac{1}{v_\perp}\Big[(v_\perp - \cos\theta_t)(v_\perp - \cos\theta_s)\left(G_1(r_\perp)G_2(r_\perp) - h_1(r_\perp)h_2(r_\perp)\right) + (1 - v_\perp\cos\theta_t)(1 - v_\perp\cos\theta_s)\left(G_1(r_\perp)G_2(r_\perp) + h_1(r_\perp)h_2(r_\perp)\right)\Big]$$

where: $r_\perp^2 = (v_\perp \Delta t + t'\cos\theta_{t_\perp} - t\cos\theta_{s_\perp})^2 + (t'\sin\theta_{t_\perp} - t\sin\theta_{s_\perp})^2$.

➡ We employ a variant of the **MV model** regularized through a **running coupling** prescription:

$$G_{\mathrm{MV}}(r_\perp) = \frac{g^4\bar{\mu}_0^2}{4\pi g^2 N_c}\left(mrK_1(mr) - 2K_0(mr)\right)\frac{1 - \exp\left\{\frac{g^4\bar{\mu}_0^2 N_c}{4\pi m^2}\left(mrK_1(mr) - 1\right)\right\}}{\frac{g^4\bar{\mu}_0^2}{4\pi m^2}\left(mrK_1(mr) - 1\right)}$$

$$h_{\mathrm{MV}}(r_\perp) = -\frac{g^4\bar{\mu}_0^2}{4\pi g^2 N_c} mrK_1(mr)\frac{1 - \exp\left\{\frac{g^4\bar{\mu}_0^2 N_c}{4\pi m^2}\left(mrK_1(mr) - 1\right)\right\}}{\frac{g^4\bar{\mu}_0^2}{4\pi m^2}\left(mrK_1(mr) - 1\right)}.$$

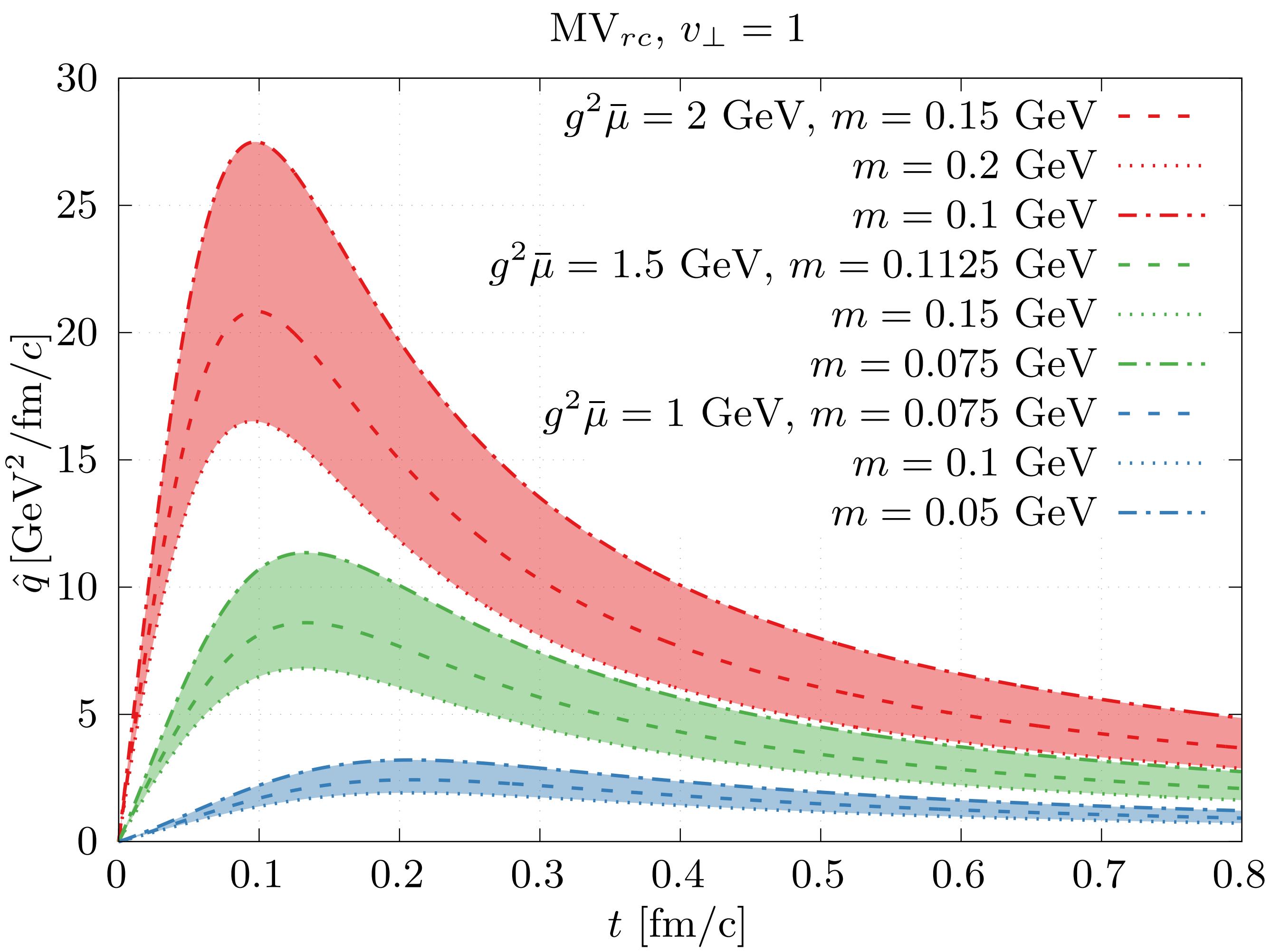
With $g^4\bar{\mu}_0^2 \to g^2(\bar{\mu}^2)g^2(r^2)\bar{\mu}^2$, where:

$$g^2(r^2) = g^2(\bar{\mu}^2)\frac{\log\left(\frac{\bar{\mu}^2}{\Lambda^2}\right)}{\log\left(\frac{4e^{-2\gamma_e}}{\Lambda^2 r^2} + e\right)}.$$

As previously done in T. Lappi and S. Schlichting; PRD, 2018 , PGR and T. Lappi; PRD, 2021

Results ($\eta = 0$, $v_{\parallel} = 0$)

➡ For the momentum broadening coefficient: $\hat{q} = \frac{1}{v_\perp N_c} \int_0^t \mathrm{d}t' \left(\delta^{\alpha\beta} - \frac{v_\perp^\alpha v_\perp^\beta}{v_\perp^2} \right) \mathrm{Tr}\left[\langle \mathcal{F}^\alpha(t,\mathbf{x}) \mathcal{F}^\beta(t',\mathbf{y}) \rangle \right]$

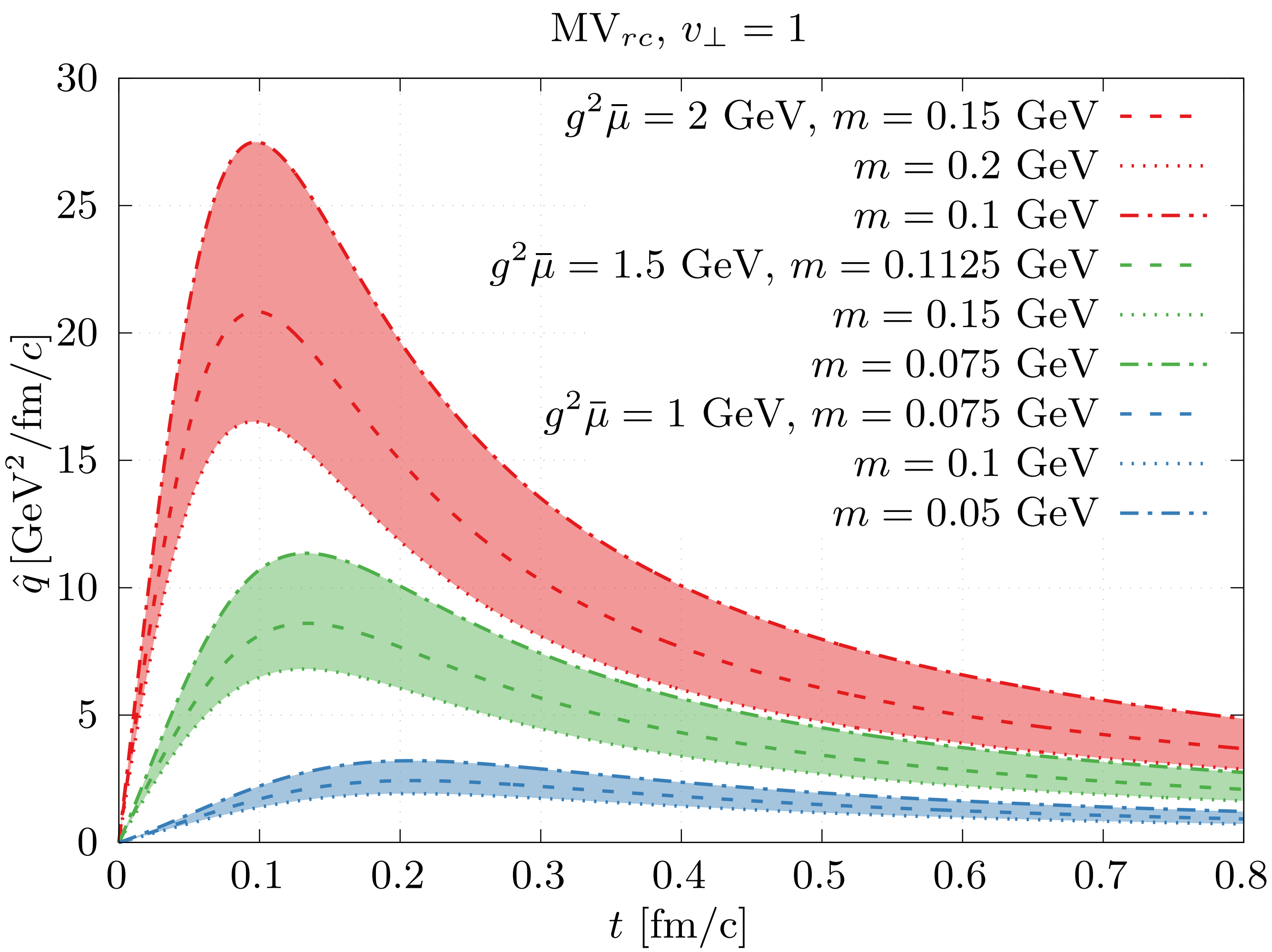


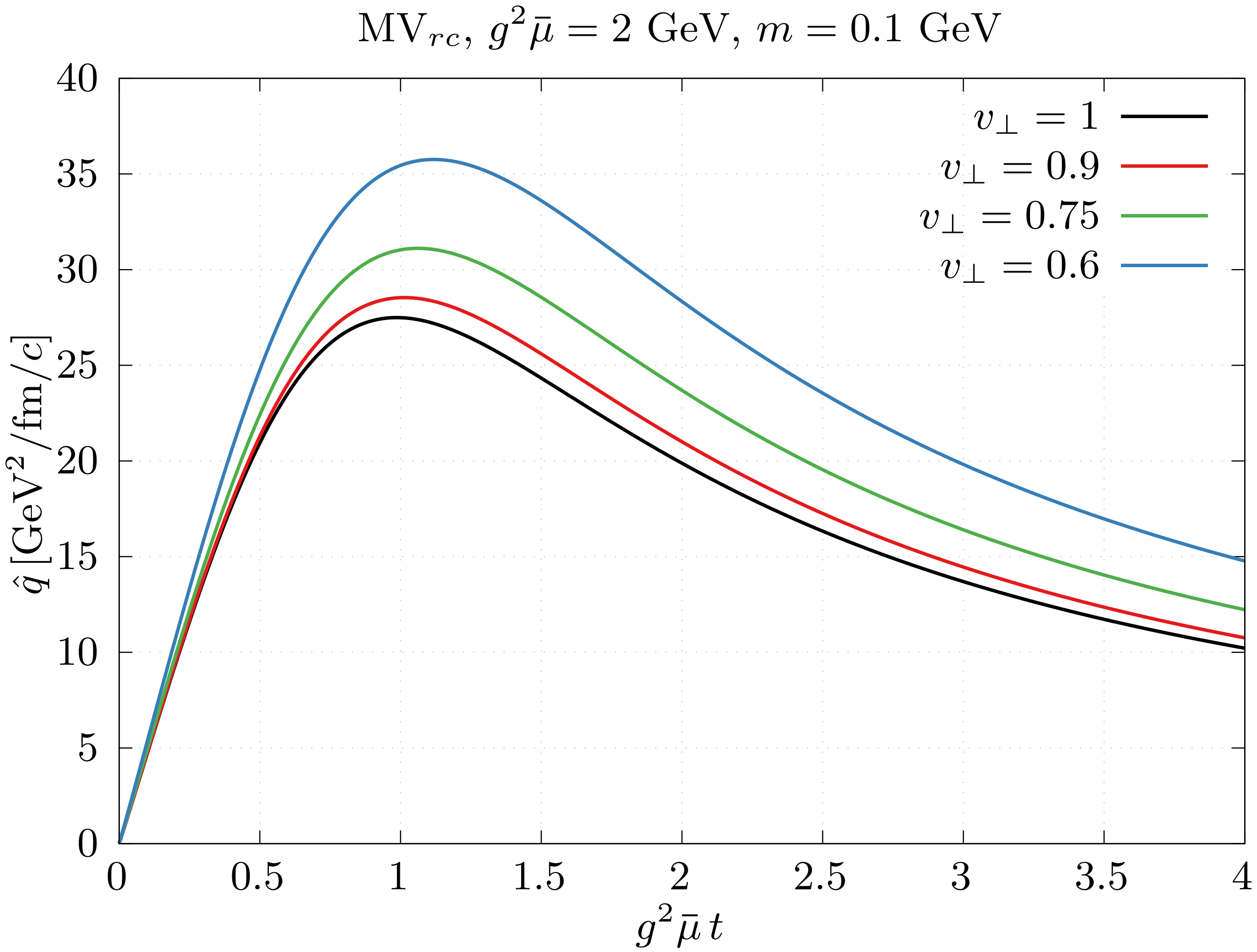
- Very strong growth early in the evolution
- Lower IR regulator $\longrightarrow$ more momentum broadening (softer modes participate in the interaction)

Results ($\eta = 0$, $v_{\parallel} = 0$)

➡ For the momentum broadening coefficient:

$$\hat{q} = \frac{1}{v_\perp N_c} \int_0^t \mathrm{d}t' \left(\delta^{\alpha\beta} - \frac{v_\perp^\alpha v_\perp^\beta}{v_\perp^2} \right) \mathrm{Tr}\left[\langle \mathcal{F}^\alpha(t, \mathbf{x}) \mathcal{F}^\beta(t', \mathbf{y}) \rangle \right]$$





- Very strong growth early in the evolution
- Lower IR regulator $\longrightarrow$ more momentum broadening (softer modes participate in the interaction)
- Lower probe velocity $\longrightarrow$ more momentum broadening (the probe spends more time within the correlation length of the Glasma fields)

Results ($\eta = 0$, $v_\parallel = 0$)

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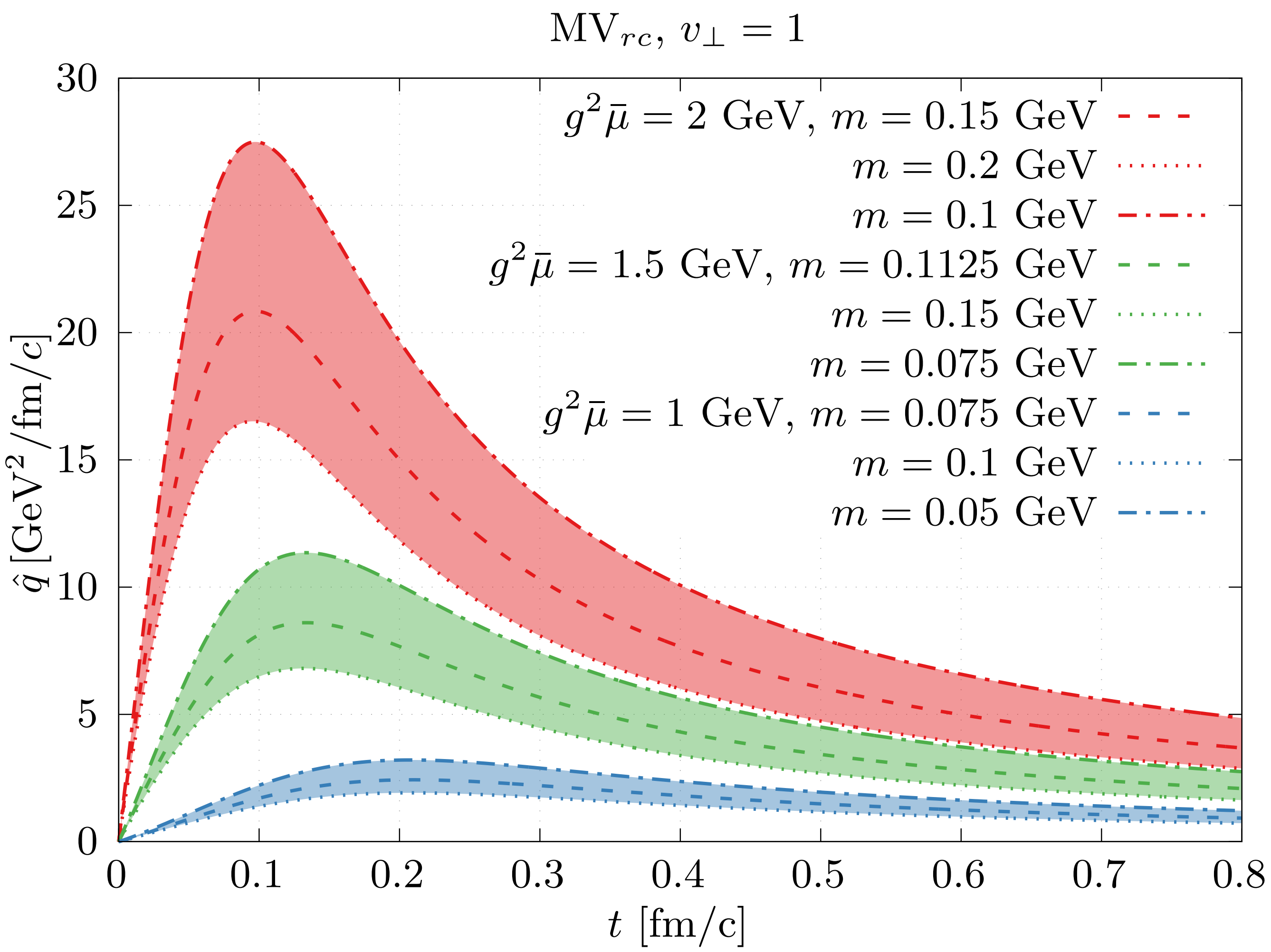
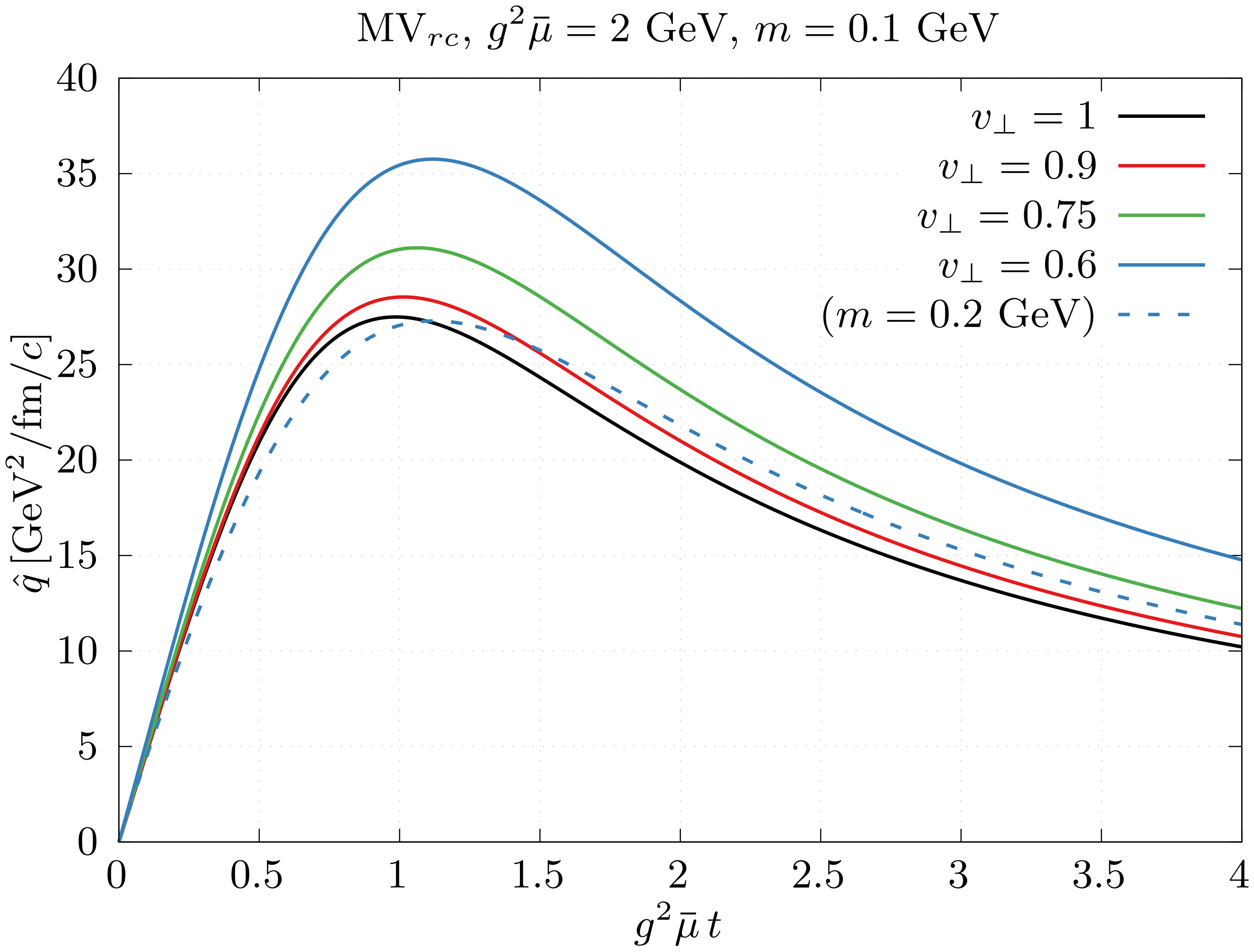
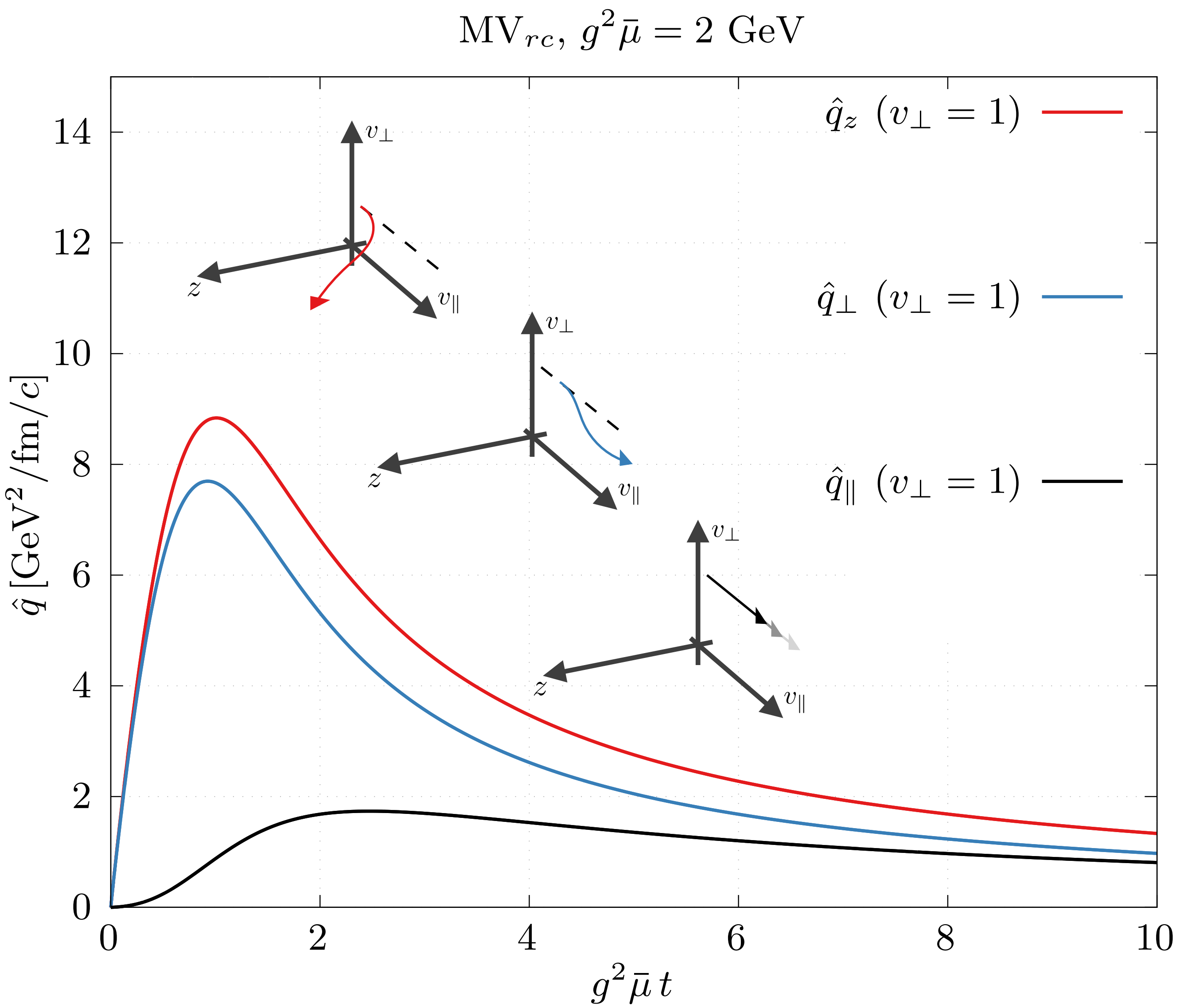




- Very strong growth early in the evolution
- Lower IR regulator $\longrightarrow$ more momentum broadening (softer modes participate in the interaction)
- Lower probe velocity $\longrightarrow$ more momentum broadening (the probe spends more time within the correlation length of the Glasma fields)
- This effect can be partly compensated by increasing the value of m (the screening length of the field decreases)

➡ Separate contributions to momentum broadening:



$$\hat{q}_z + \hat{q}_\perp = \hat{q}$$

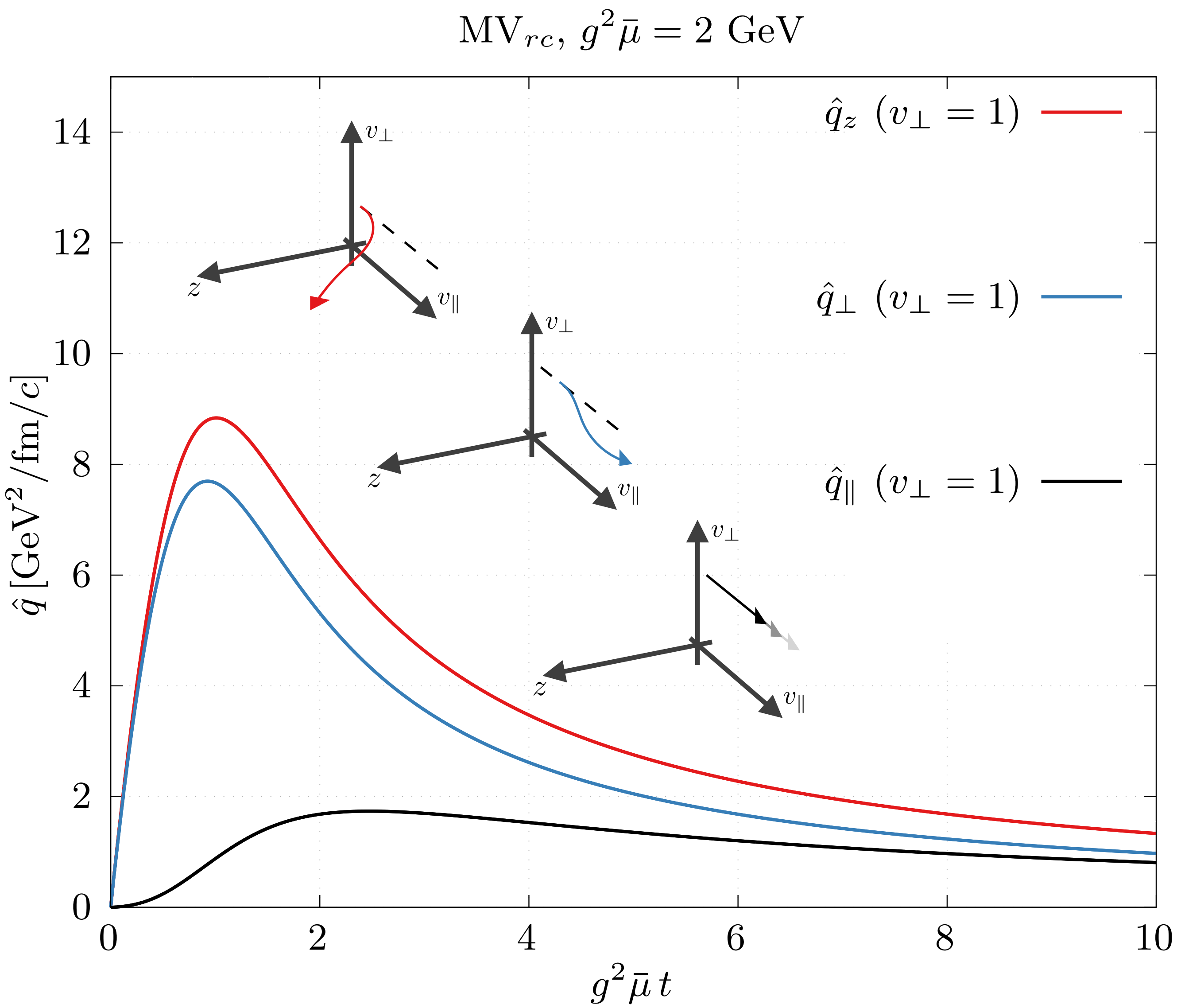
- Larger broadening in the beam direction
- Comparatively small effect in the probe direction
- Difference between $\hat{q}_z$ and $\hat{q}_\perp$ due to linearly polarized gluons:

$$\hat{q}_\perp \sim \int_0^t dt'(\ldots)\left(G_1(r_\perp)G_2(r_\perp) - h_1(r_\perp)h_2(r_\perp)\right)$$

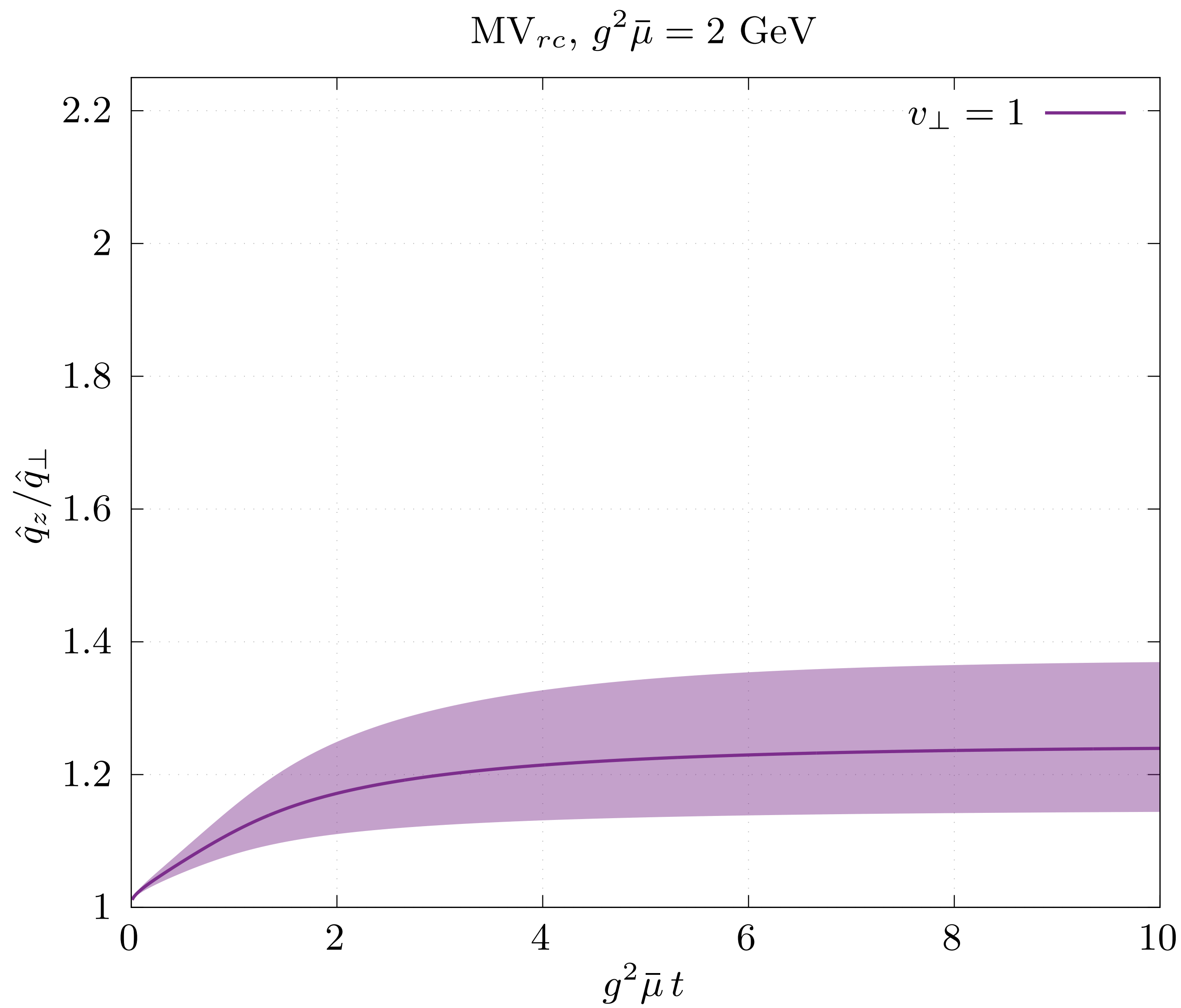
$$\hat{q}_z \sim \int_0^t dt'(\ldots)\left(G_1(r_\perp)G_2(r_\perp) + h_1(r_\perp)h_2(r_\perp)\right)$$

Results ($\eta = 0$, $v_\parallel = 0$)

➡ Separate contributions to momentum broadening:



➡ Anisotropy:



- Larger broadening in the beam direction
- Comparatively small effect in the probe direction
- Difference between $\hat{q}_z$ and $\hat{q}_\perp$ due to linearly polarized gluons:
- Anisotropy grows fast, becomes constant after $t \sim 2/g^2\bar{\mu}$

$$\hat{q}_\perp \sim \int_0^t dt' (\ldots) \left(G_1(r_\perp)G_2(r_\perp) - h_1(r_\perp)h_2(r_\perp)\right)$$

$$\hat{q}_z \sim \int_0^t dt' (\ldots) \left(G_1(r_\perp)G_2(r_\perp) + h_1(r_\perp)h_2(r_\perp)\right)$$

Results ($\eta = 0$, $v_\parallel = 0$)

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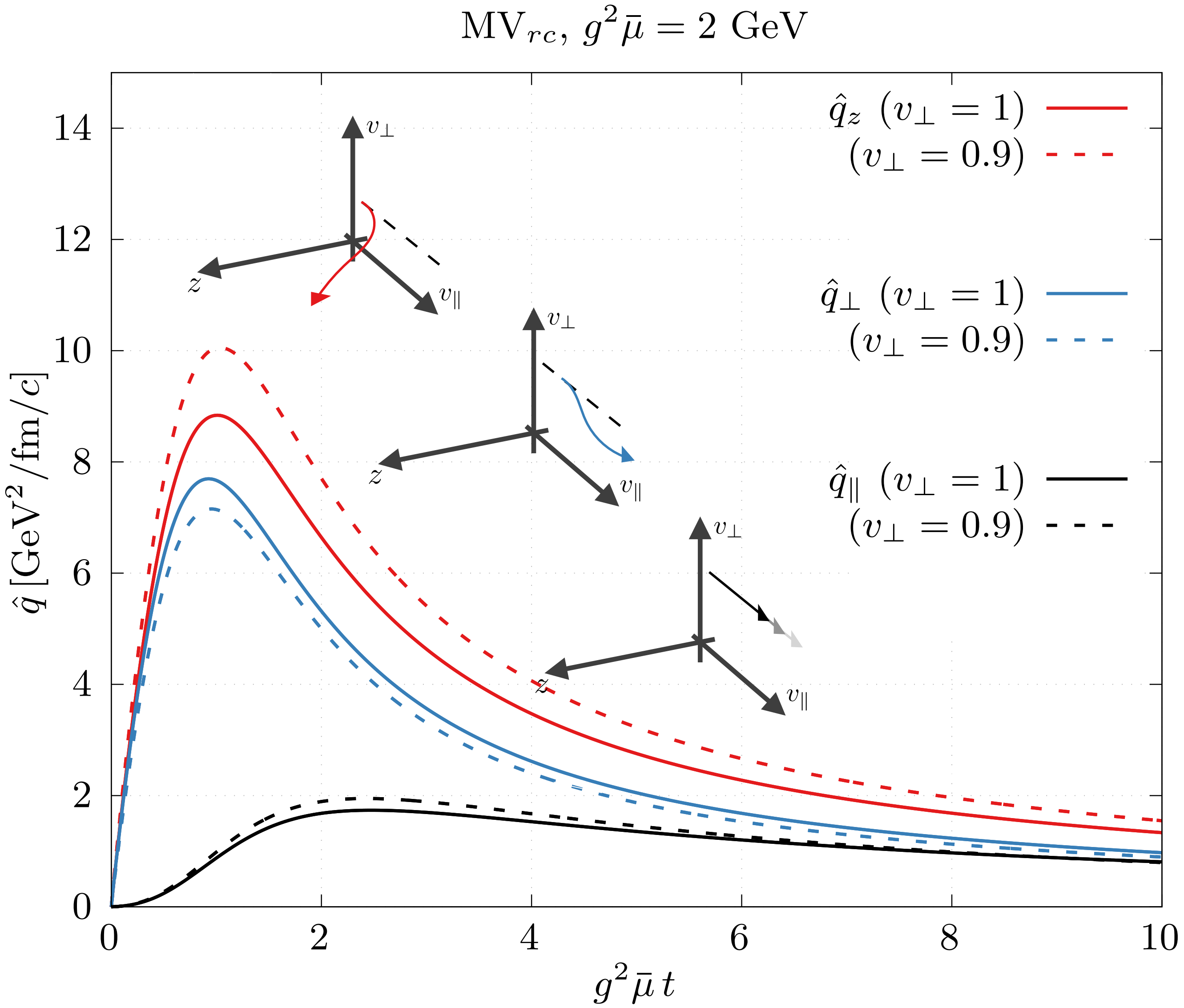

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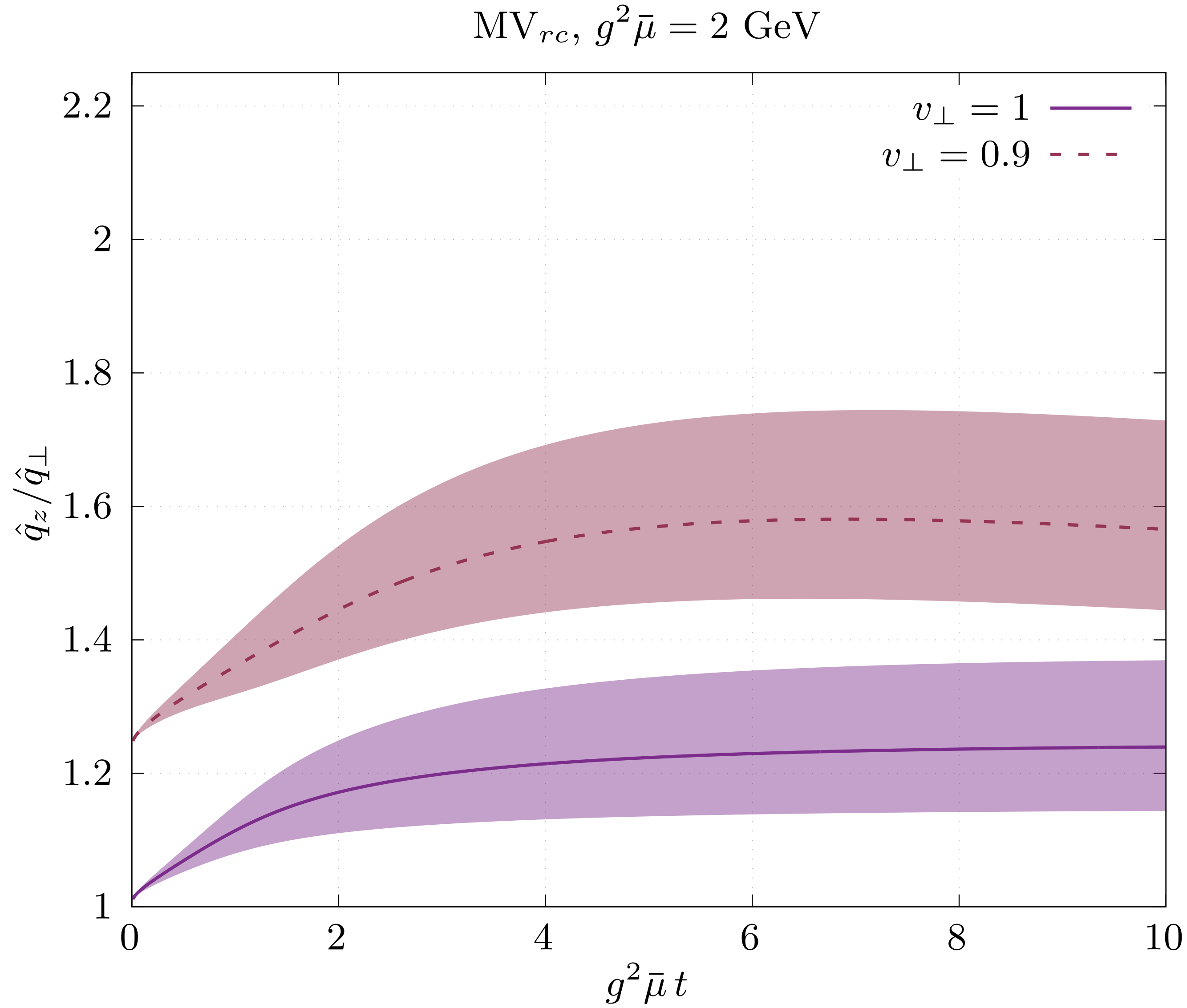

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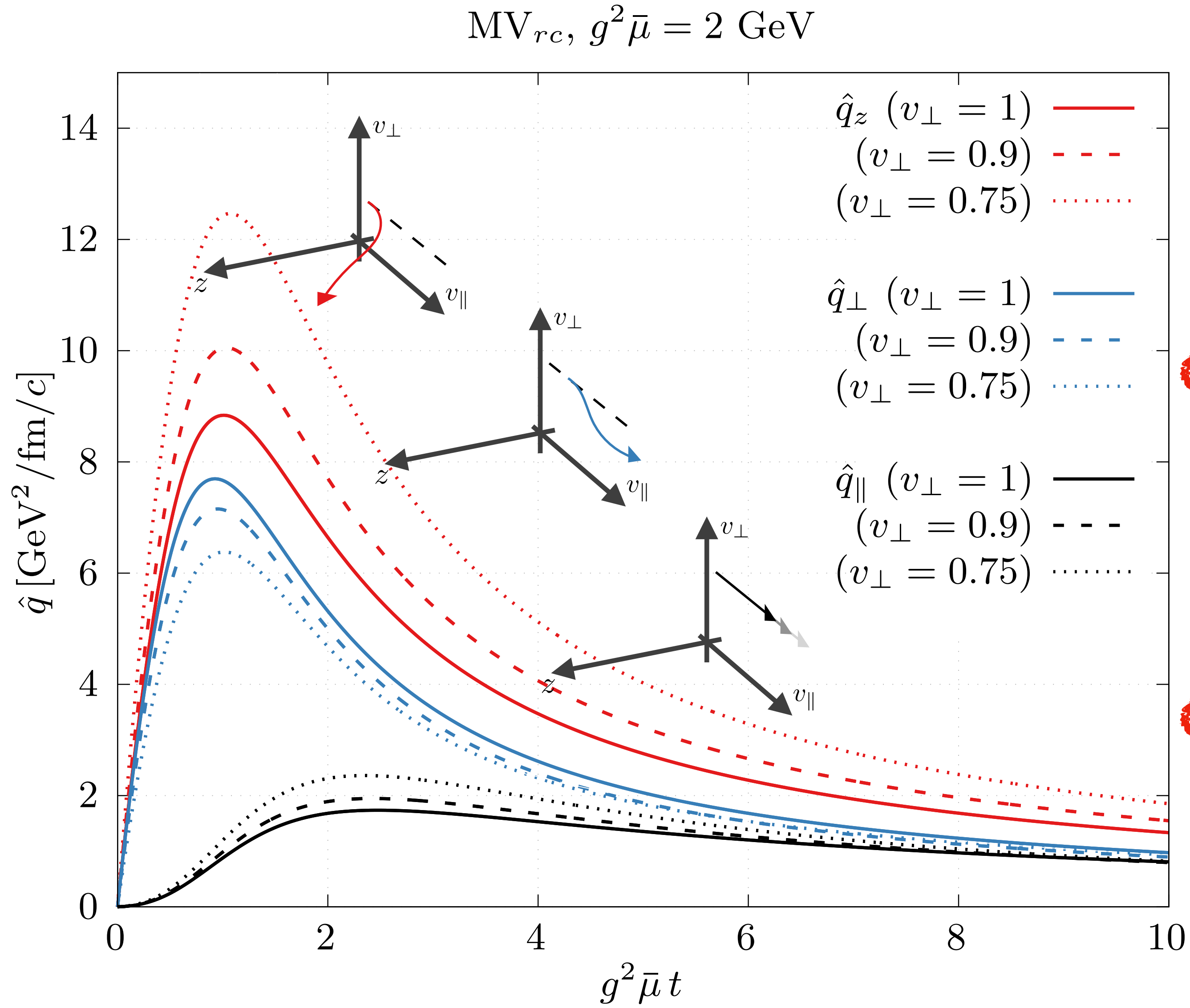
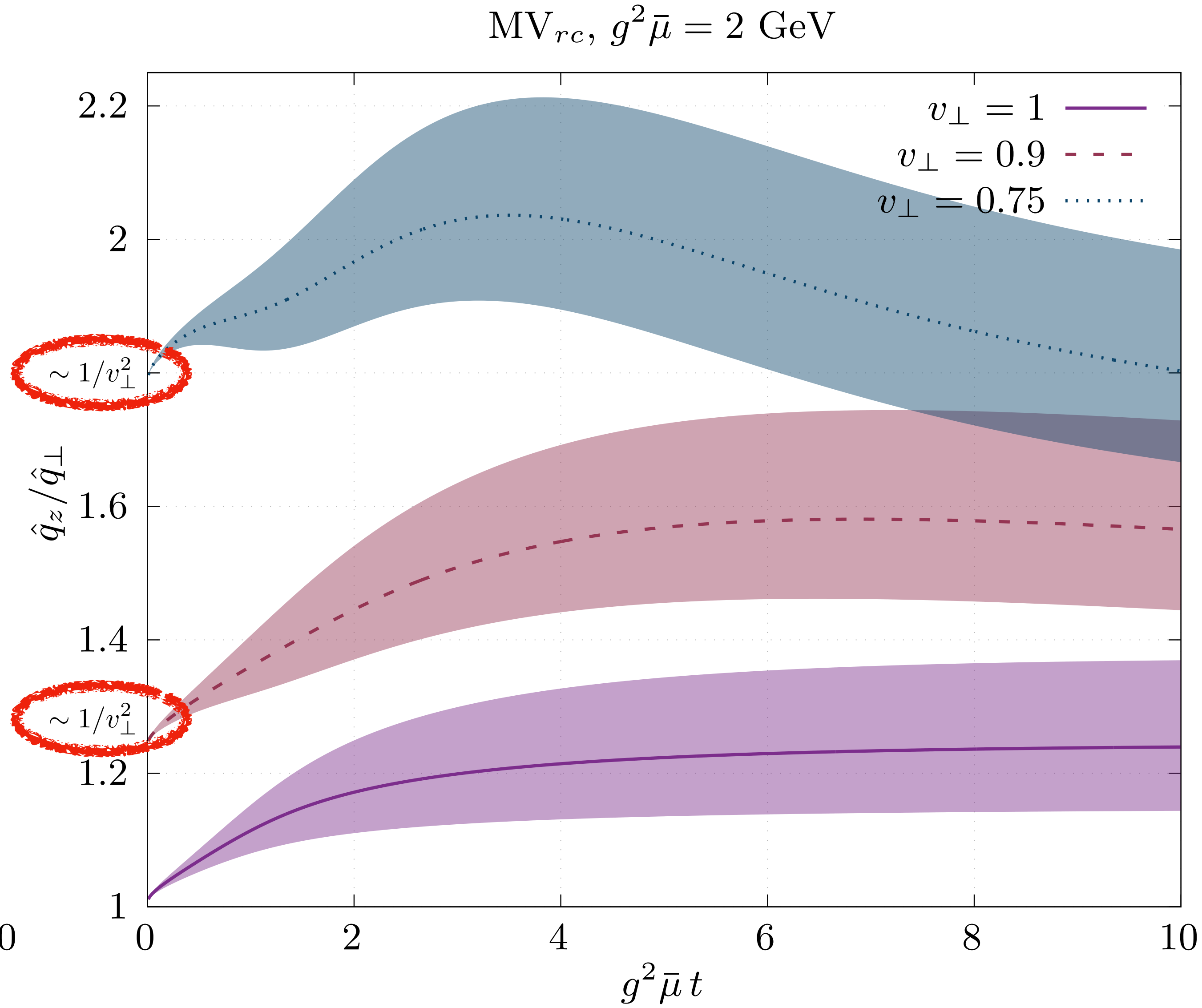
Results ($\eta = 0$, $v_{\parallel} = 0$)

➡ Separate contributions to momentum broadening:



➡ Anisotropy:



- Larger broadening in the beam direction
- Comparatively small effect in the probe direction
- Difference between $\hat{q}_z$ and $\hat{q}_\perp$ due to linearly polarized gluons:
- Anisotropy grows fast, becomes constant after $t \sim 2/g^2\bar{\mu}$
- Initial anisotropy depends on the velocity of the probe as: $\lim_{t\to 0} \hat{q}_z/\hat{q}_\perp = 1/v_\perp^2$

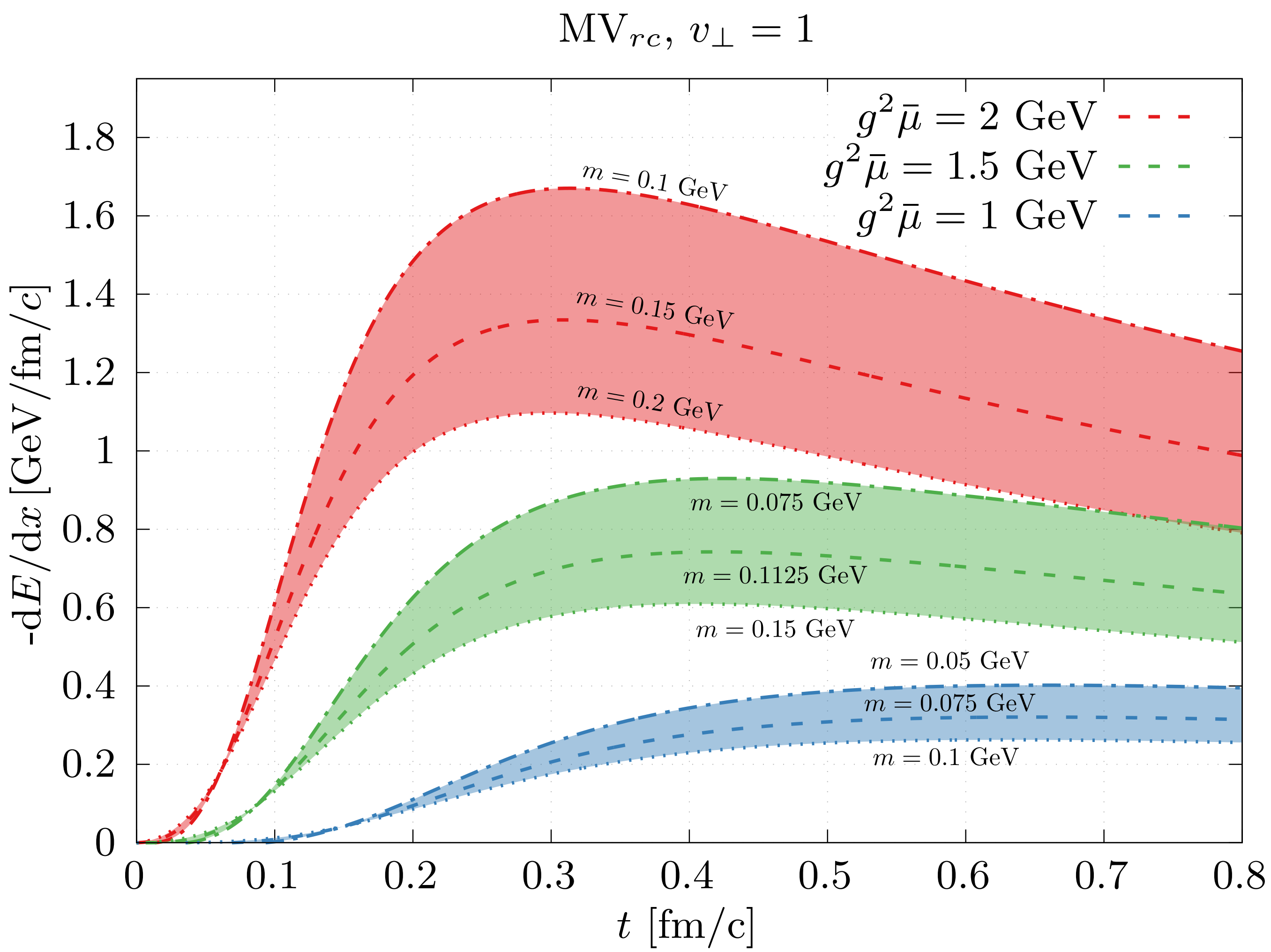
$$\hat{q}_\perp \sim \int_0^t dt' (\ldots) \left(G_1(r_\perp)G_2(r_\perp) - h_1(r_\perp)h_2(r_\perp)\right)$$

$$\hat{q}_z \sim \int_0^t dt' (\ldots) \left(G_1(r_\perp)G_2(r_\perp) + h_1(r_\perp)h_2(r_\perp)\right)$$

Results ($\eta = 0$, $v_{\parallel} = 0$)

➡ For the drag coefficient:

$$\frac{\mathrm{d}E}{\mathrm{d}x} = -\frac{1}{2TN_c}\int_0^t \mathrm{d}t' \frac{v_\perp^\alpha v_\perp^\beta}{v_\perp} \mathrm{Tr}\left[\langle \mathcal{F}^\alpha(t,\mathbf{x})\mathcal{F}^\beta(t',\mathbf{y})\rangle\right]$$



- Slower growth, reaches approximate time independence at $t \sim 2/g^2\bar{\mu}$

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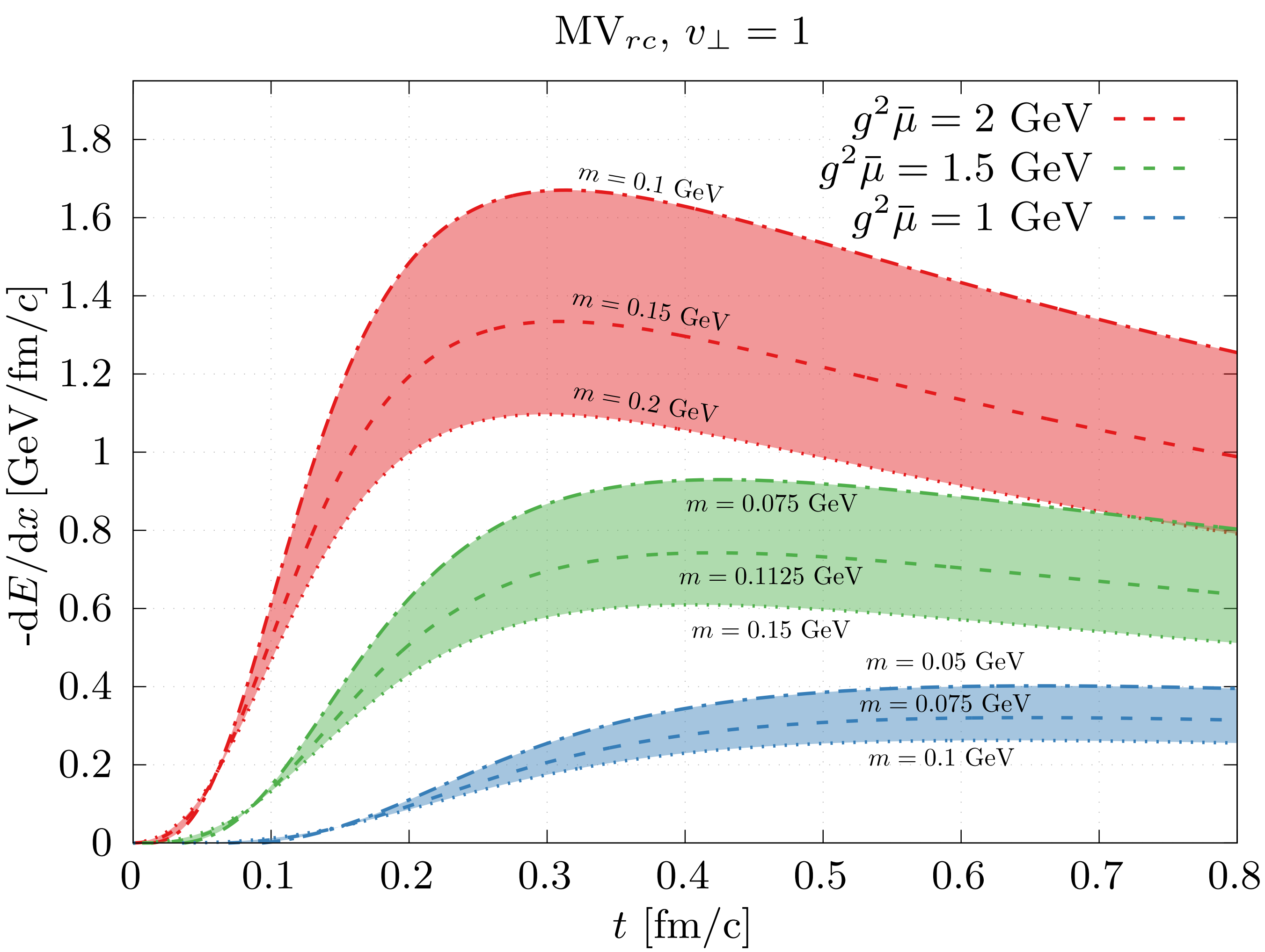
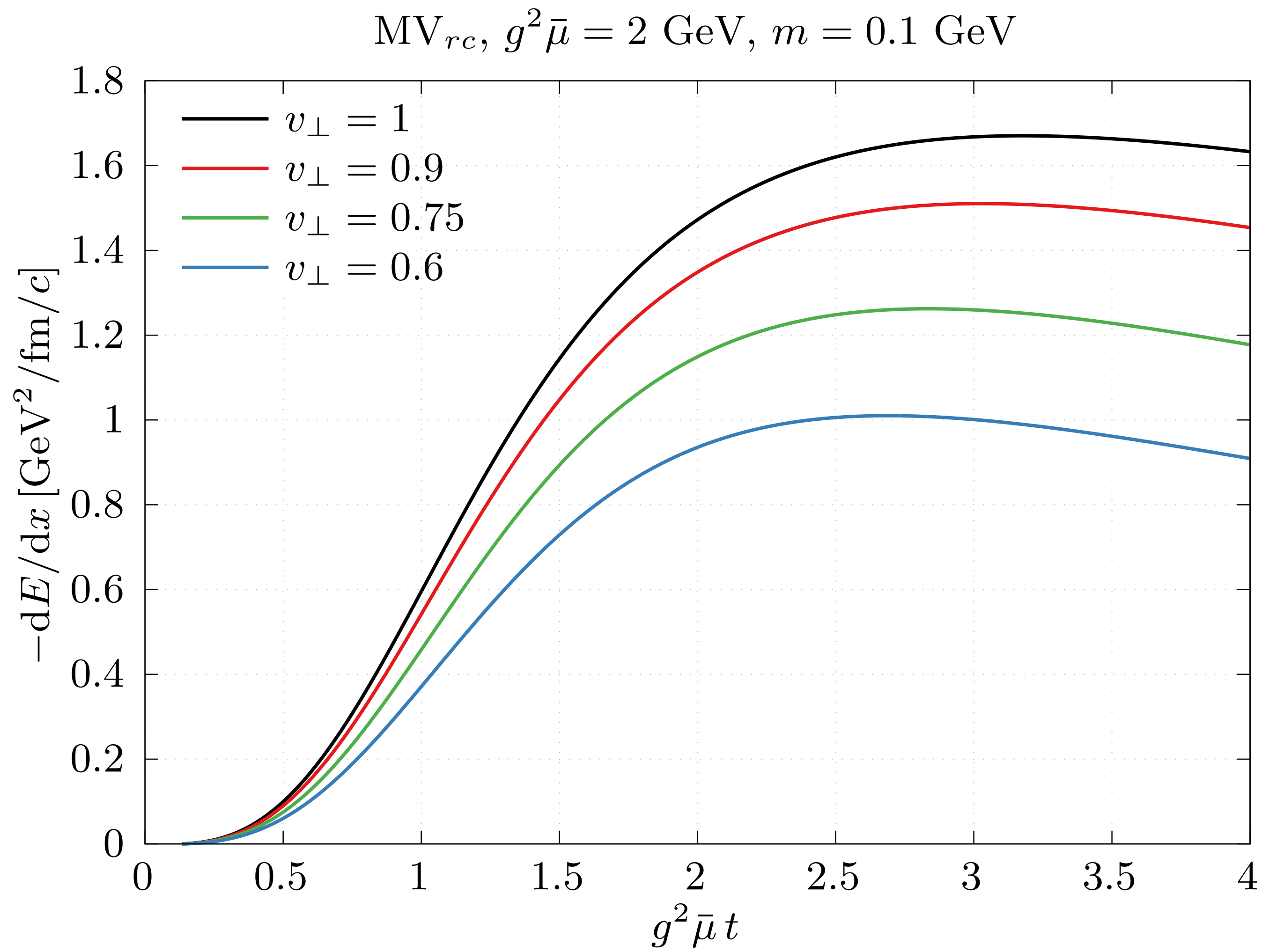
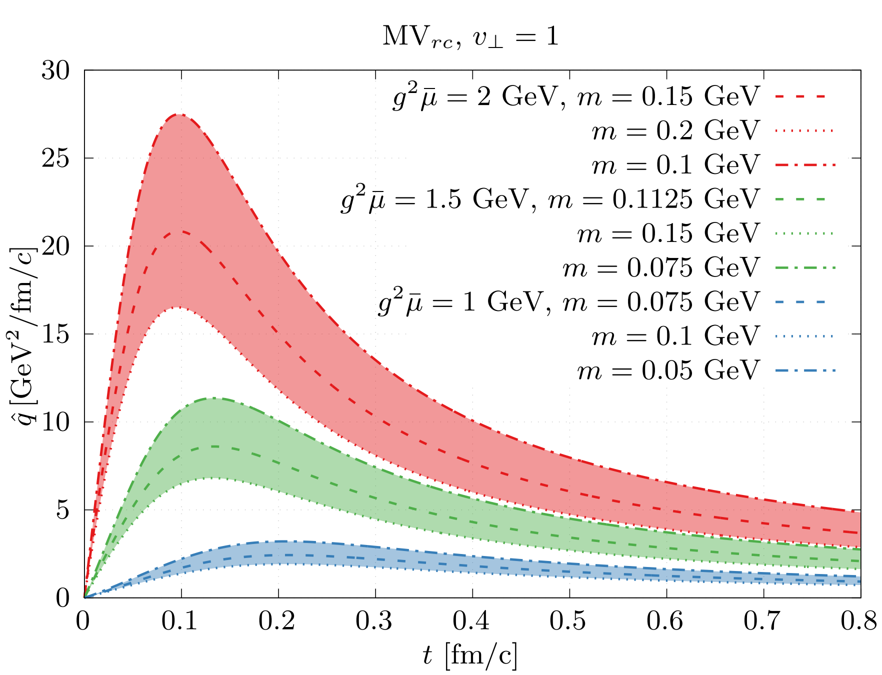


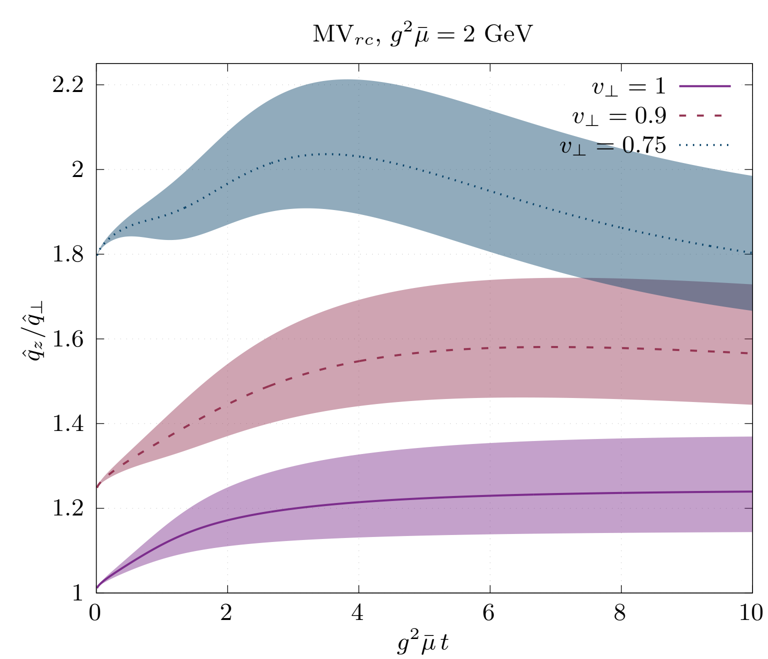


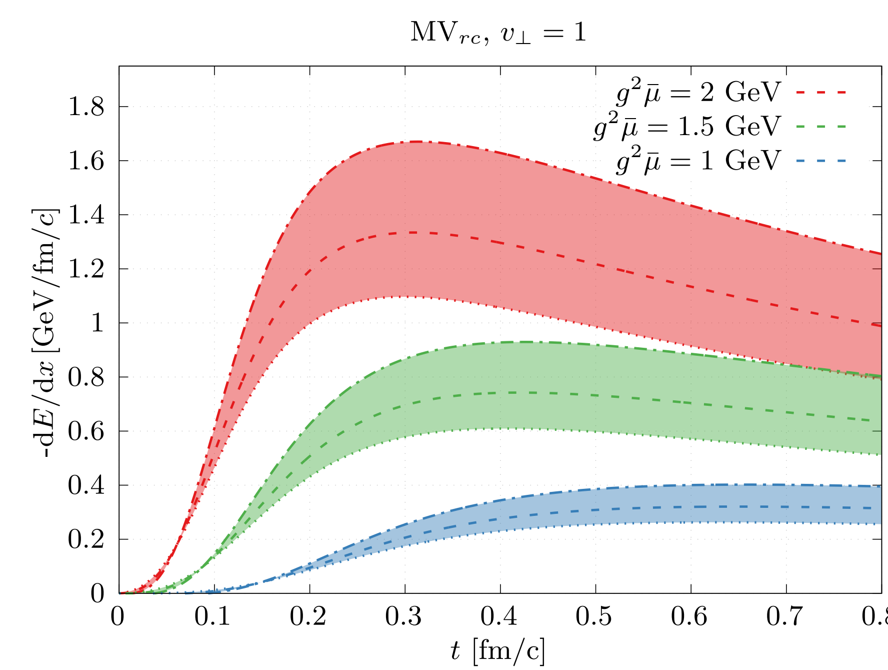
- Slower growth, reaches approximate time independence at $t \sim 2/g^2\bar{\mu}$
- Opposite dependence on velocity as $\hat{q}$ (dE/dx proportional to $v_\perp$)

Summary, conclusions

- We have studied the impact of the Glasma phase over Jet Quenching transport coefficients
- The transport of hard probes is described by means of a Fokker-Planck equation (**S. Mrowczynski; Eur. Phys. Journal A, 2018**, **M. E. Carrington, A. Czajka, S. Mrowczynski; PRC, 2022**)
- The evolution of the medium is described using linearized Yang-Mills equations in combination with non-linear initial conditions (**J.-P. Blaizot, T. Lappi, Y. Mehtar-Tani; Nucl. Phys. A, 2010**, **PGR and T. Lappi; PRD, 2021**)
- These equations effectively resum the UV-divergences of a power series in τ (**H. Fujii, K. Fukushima, Y. Hidaka; PRC, 2008**)
- We observe very strong transverse broadening early in the evolution: maximum before $t \sim 2/g^2\bar{\mu}$
  - More broadening for less relativistic probes
- Anisotropy: different values of broadening along different transverse directions. The ratio $\hat{q}_z/\hat{q}_\perp$ becomes independent of t after $t \sim 2/g^2\bar{\mu}$ (except for very low values of $v_\perp$), tends to $1/v_\perp^2$ at $t \to 0$
  - More anisotropy for less relativistic probes
- Drag coefficient: also reaches a maximum about $t \sim 2/g^2\bar{\mu}$, becomes approximately flat afterwords
  - Unlike $\hat{q}$, it decreases with the velocity of the probe







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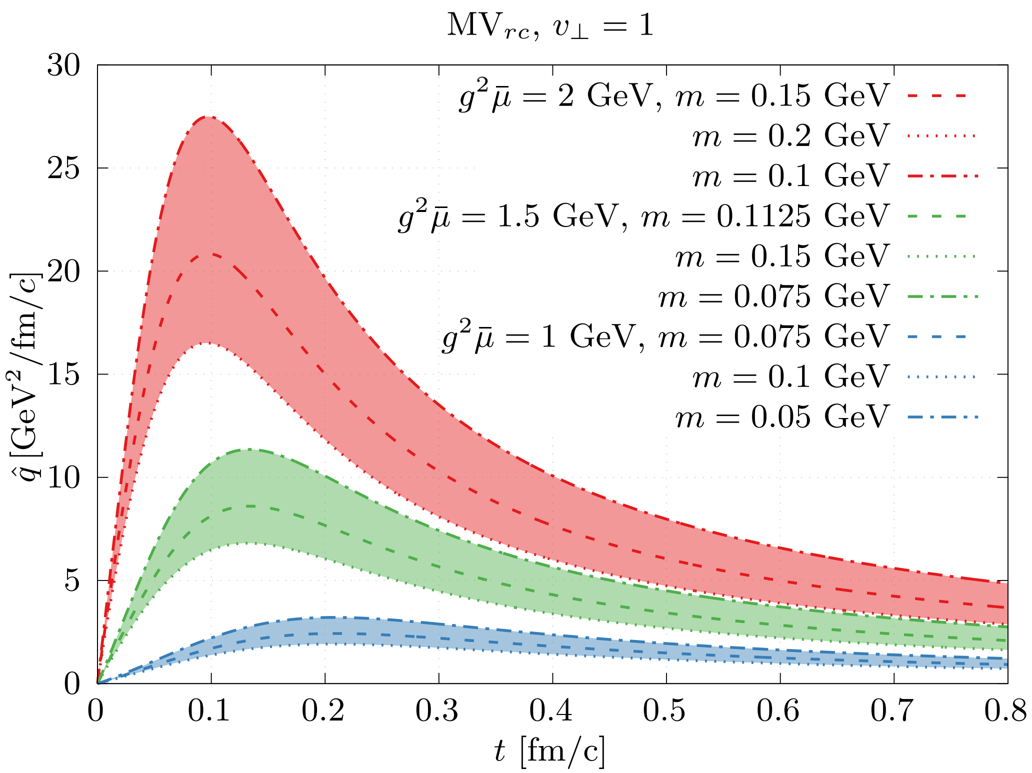
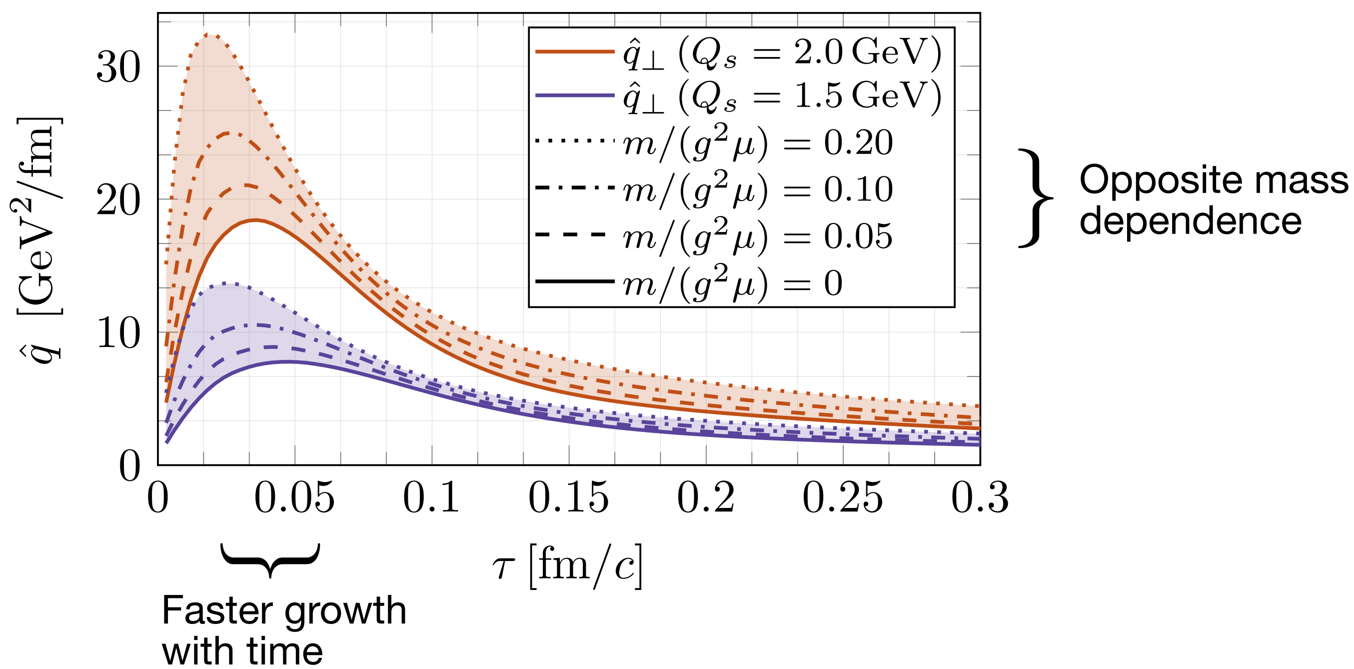
Thank you for your attention

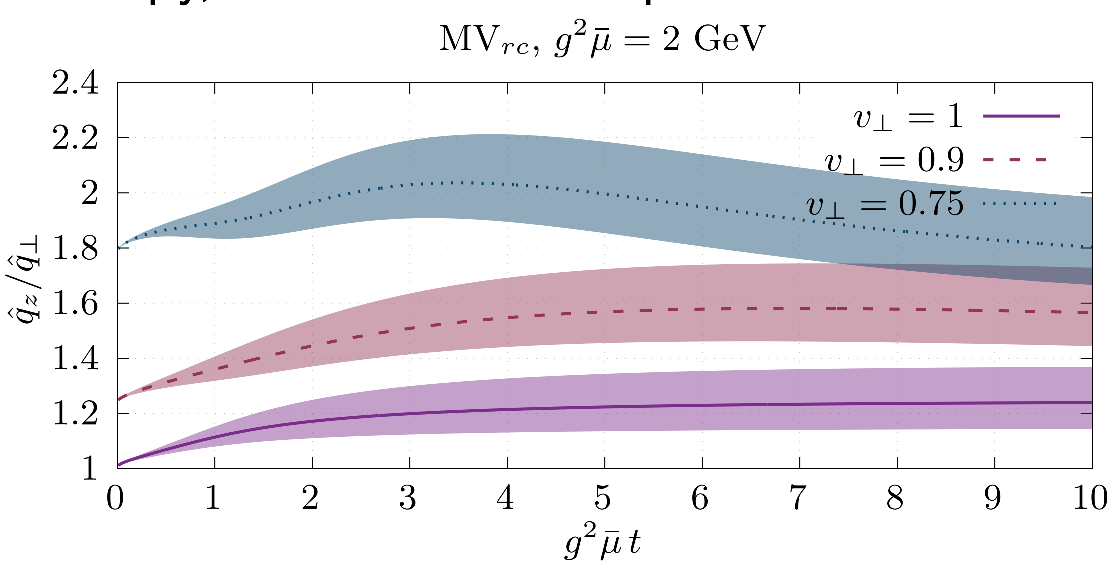
BACK-UP: Comparison to previous results (I)

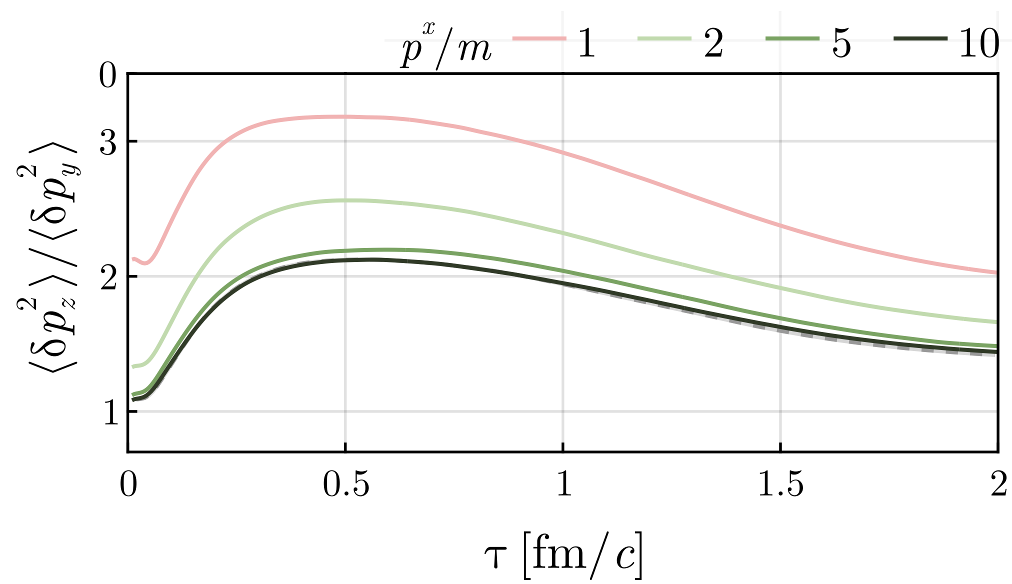
➡ Upon comparison with the results obtained in **A. Ipp, D. I. Müller, D. Schuh; PLB, 2020**, we observe a significantly slower growth, and, remarkably, the opposite IR regulator dependence:





➡ In the results obtained in **D. Avramescu, V. Băran, V. Greco, A. Ipp, D. I. Müller, M. Ruggieri; PRD (2023)** for the anisotropy, we observe a compatible behavior in the $t \to 0$ limit:

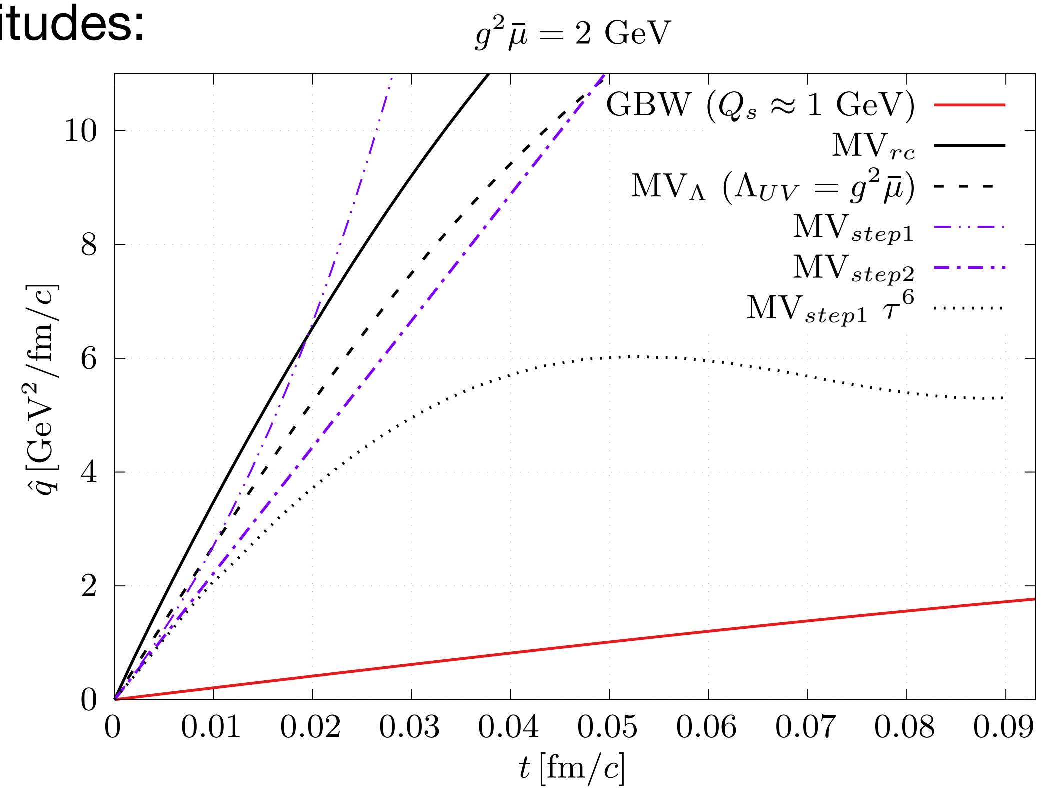


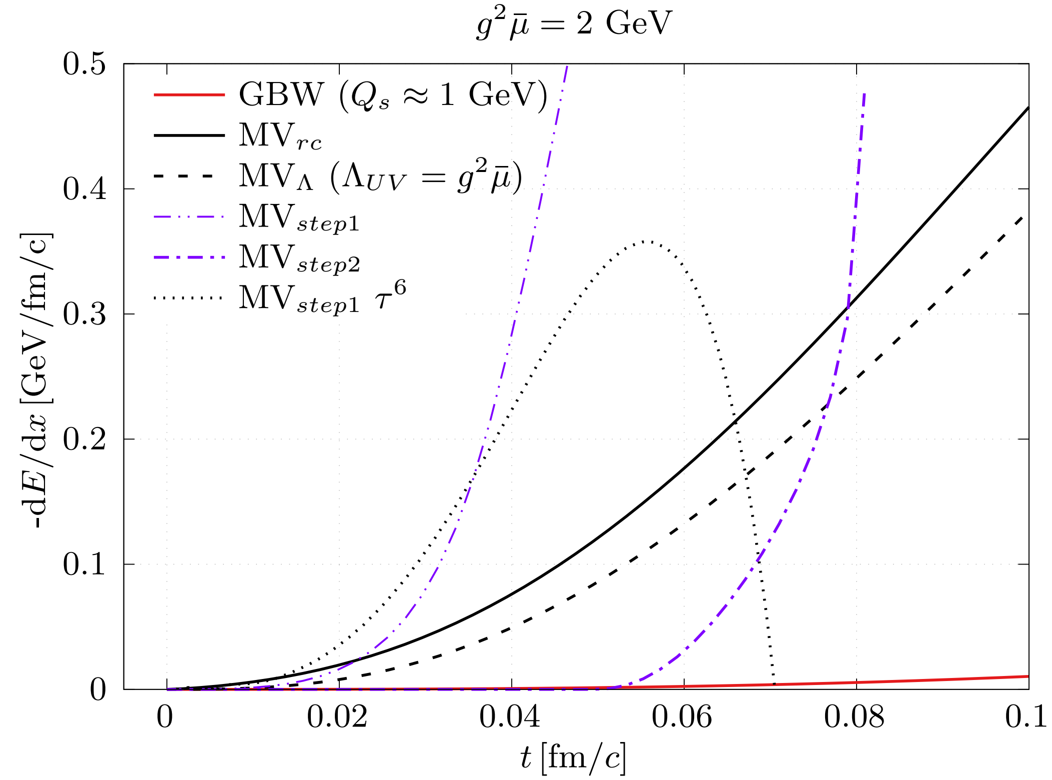


➡ Non-linear corrections and the inclusion of gauge links could improve agreement

BACK-UP: Comparison to previous results (II)

➡ Our results and those obtained in **M. E. Carrington, A. Czajka, S. Mrowczynski; PRC, 2022** show completely different magnitudes:

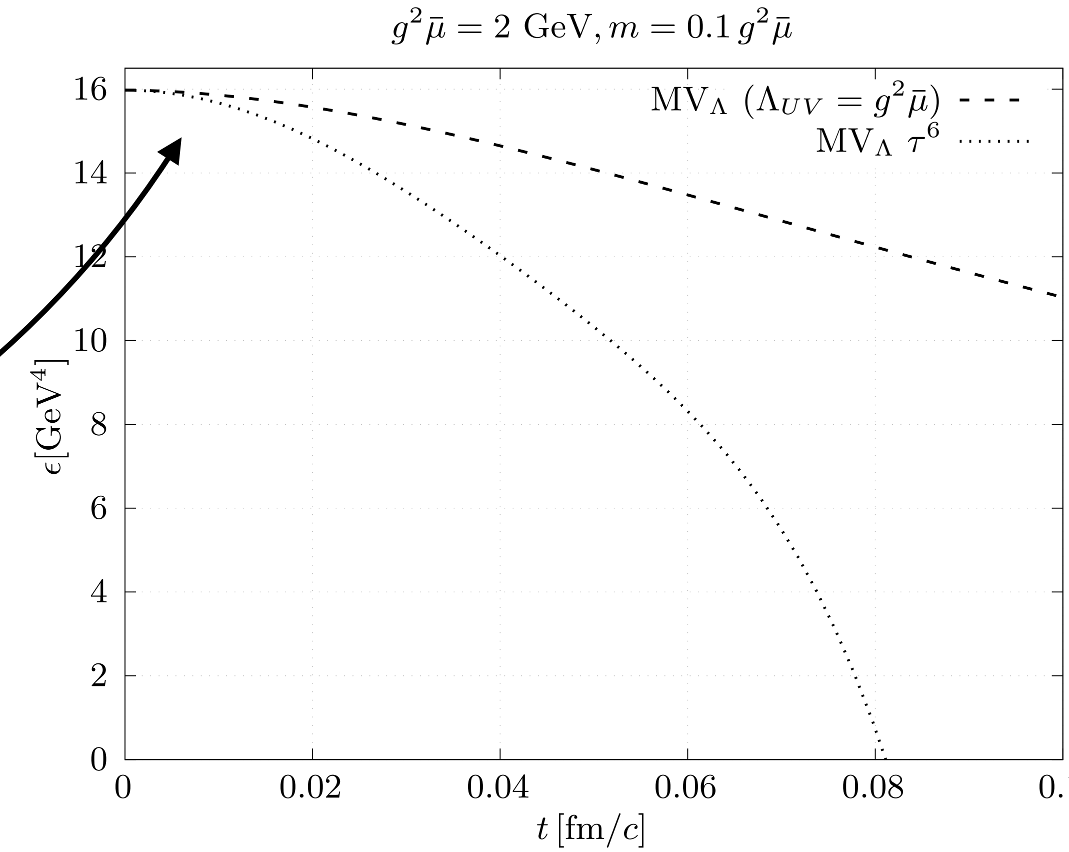




➡ We suggest that the origin of the discrepancy might be the different approaches to the time evolution of the Glasma fields (linearized Yang-Mills vs. τ-expansion to order τ^6).

➡ For instance, in the case of the energy density:

Agreement only around $t \lesssim 0.01$ fm/c



BACK-UP: Dependence on running coupling prescription

➡ Some alternative prescriptions are based on making *only* the coupling in the pre-factor run:

$$G_{\mathrm{MV}}(r_\perp) = \frac{g^2\bar{\mu}^2}{4\pi N_c}\left(mrK_1(mr) - 2K_0(mr)\right)\frac{1-\exp\left\{\frac{g^4\bar{\mu}^2 N_c}{4\pi m^2}\left(mrK_1(mr)-1\right)\right\}}{\frac{g^4\bar{\mu}^2}{4\pi m^2}\left(mrK_1(mr)-1\right)}$$

$$h_{\mathrm{MV}}(r_\perp) = -\frac{g^2\bar{\mu}^2}{4\pi N_c}mrK_1(mr)\frac{1-\exp\left\{\frac{g^4\bar{\mu}^2 N_c}{4\pi m^2}\left(mrK_1(mr)-1\right)\right\}}{\frac{g^4\bar{\mu}^2}{4\pi m^2}\left(mrK_1(mr)-1\right)}$$

➡ There is also some liberty in the exact form of the correspondence between $g^2(Q^2)$ and $g^2(1/r^2)$. For instance, one can take: $g^2(r^2) = \dfrac{g^2(\bar{\mu}^2)}{\ln\left(\frac{4e^{-2\gamma_e-1}}{m^2r^2}+e\right)}$