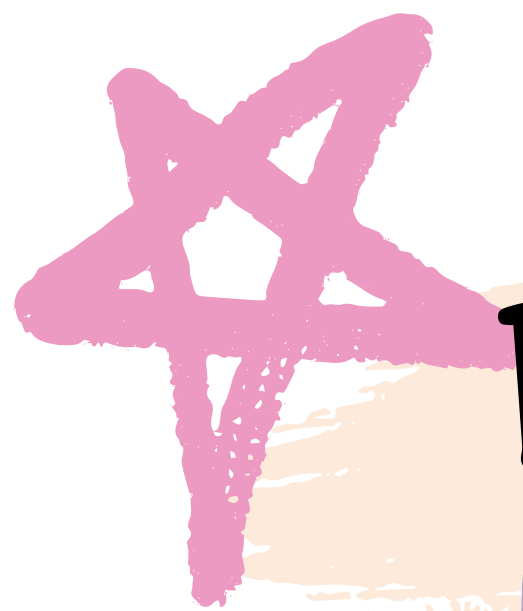
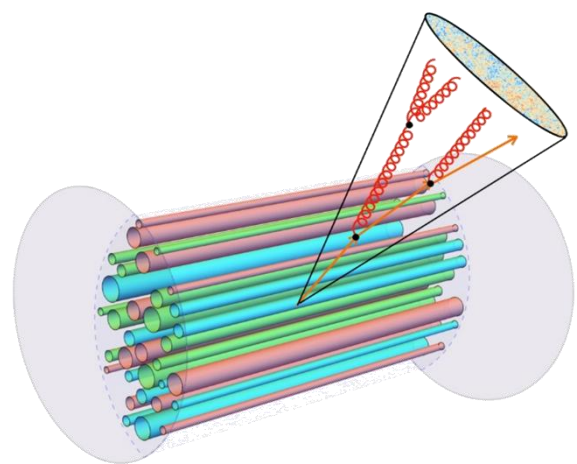
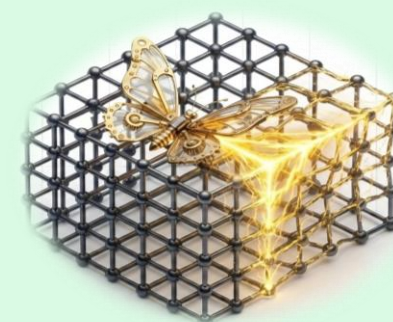




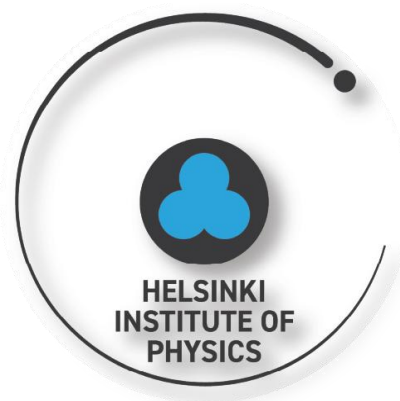
Determination of the leading Lyapunov Exponent for the Boost-Invariant Glasma fields



Leading Lyapunov exponent in the Glasma
Pooja, Dana Avramescu, Tuomas Lappi
Phys. Rev. D 114, 014010 (2026) 



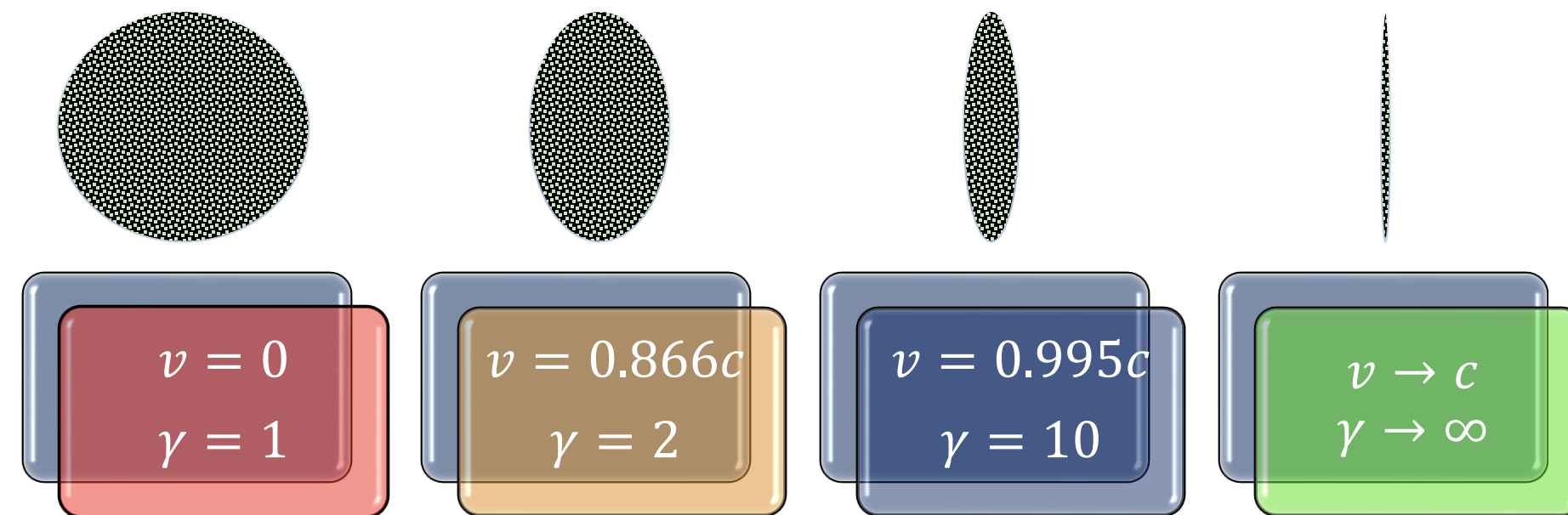
EMMI Workshop:
Probing the early stages with jets



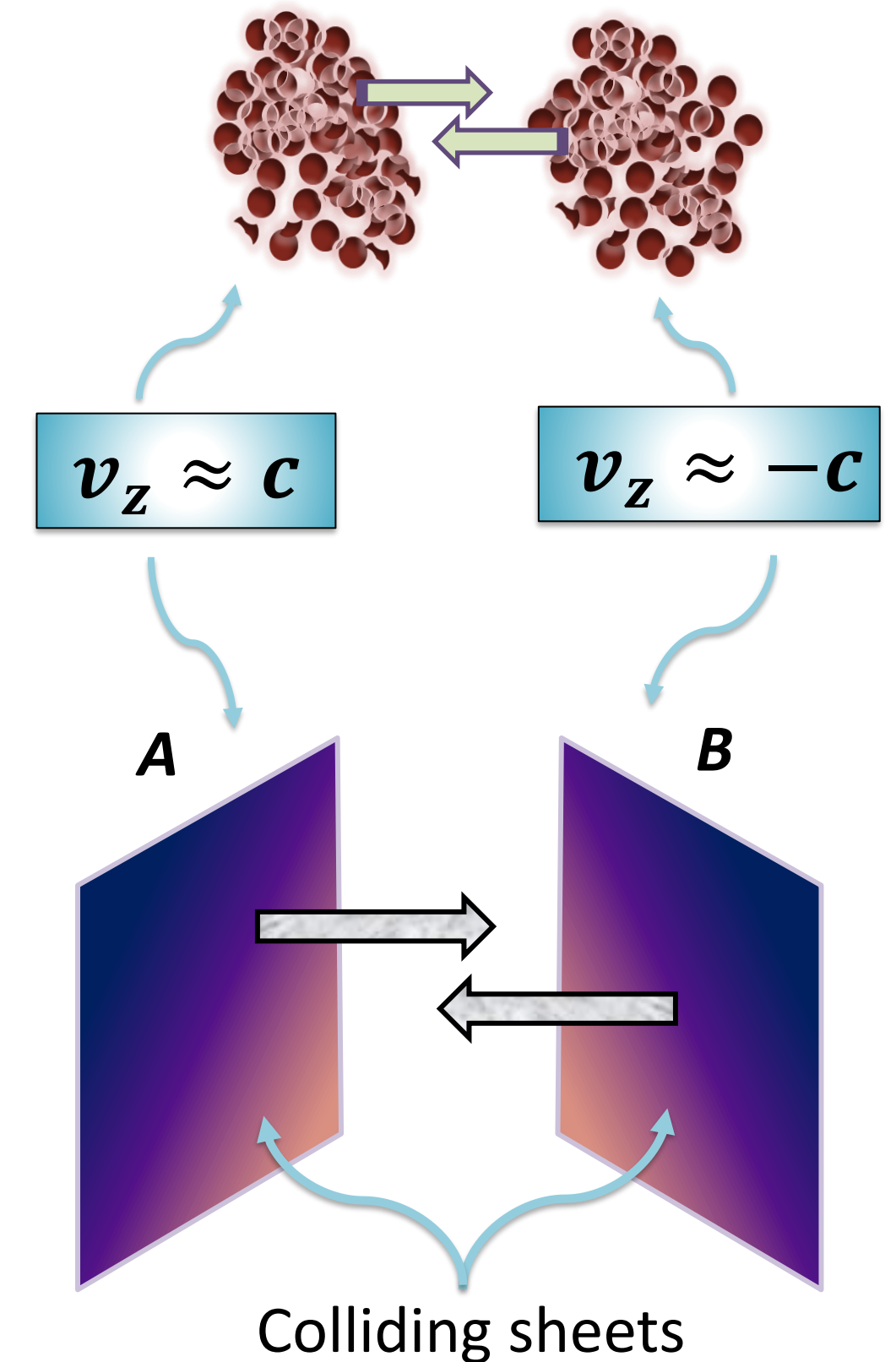
Pooja
University of Jyväskylä
Center of Excellence in Quark Matter

Heavy-Ion Collisions (HICs)

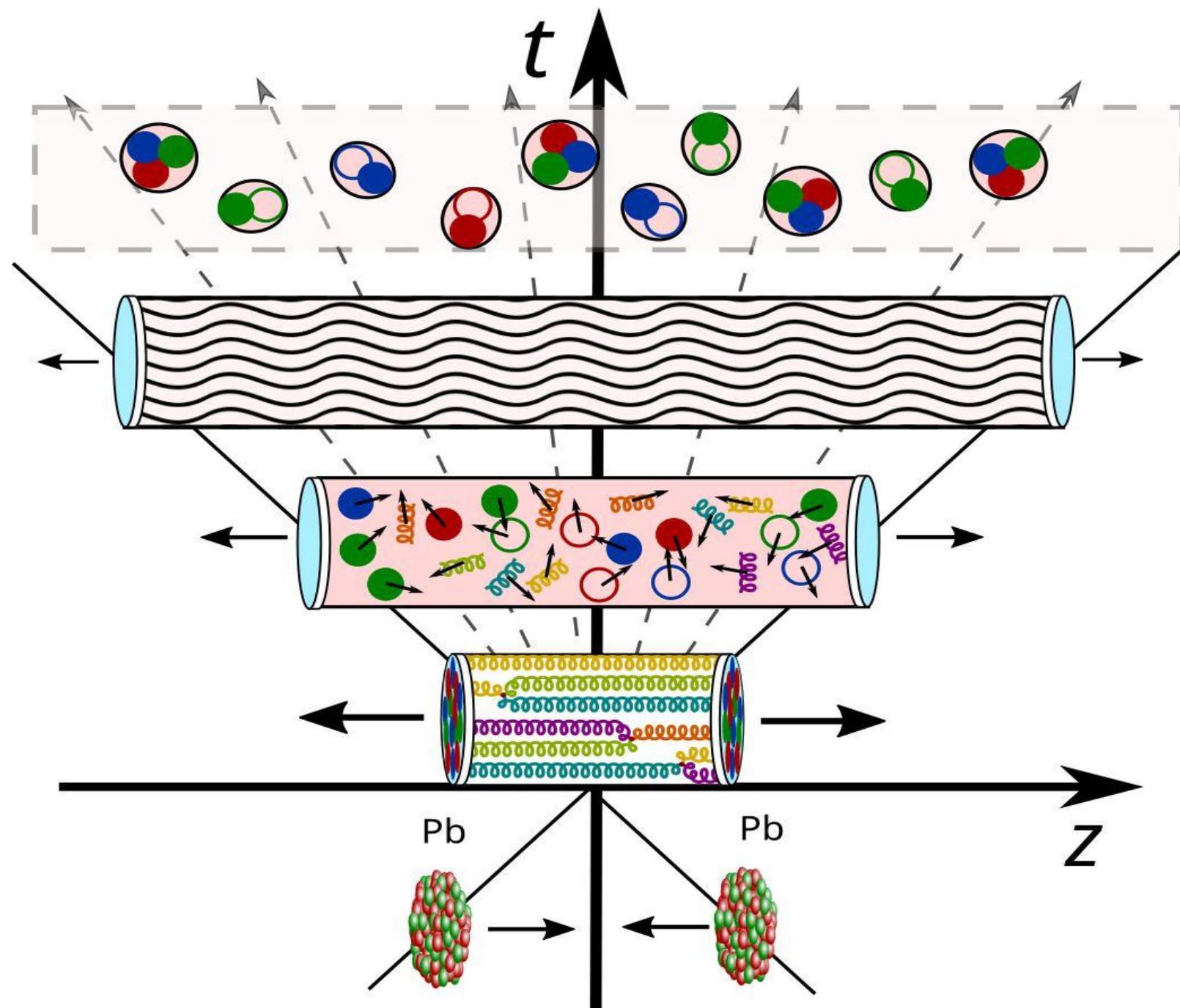
2



*Lorentz contraction along the longitudinal direction :
Two thin colliding sheets*



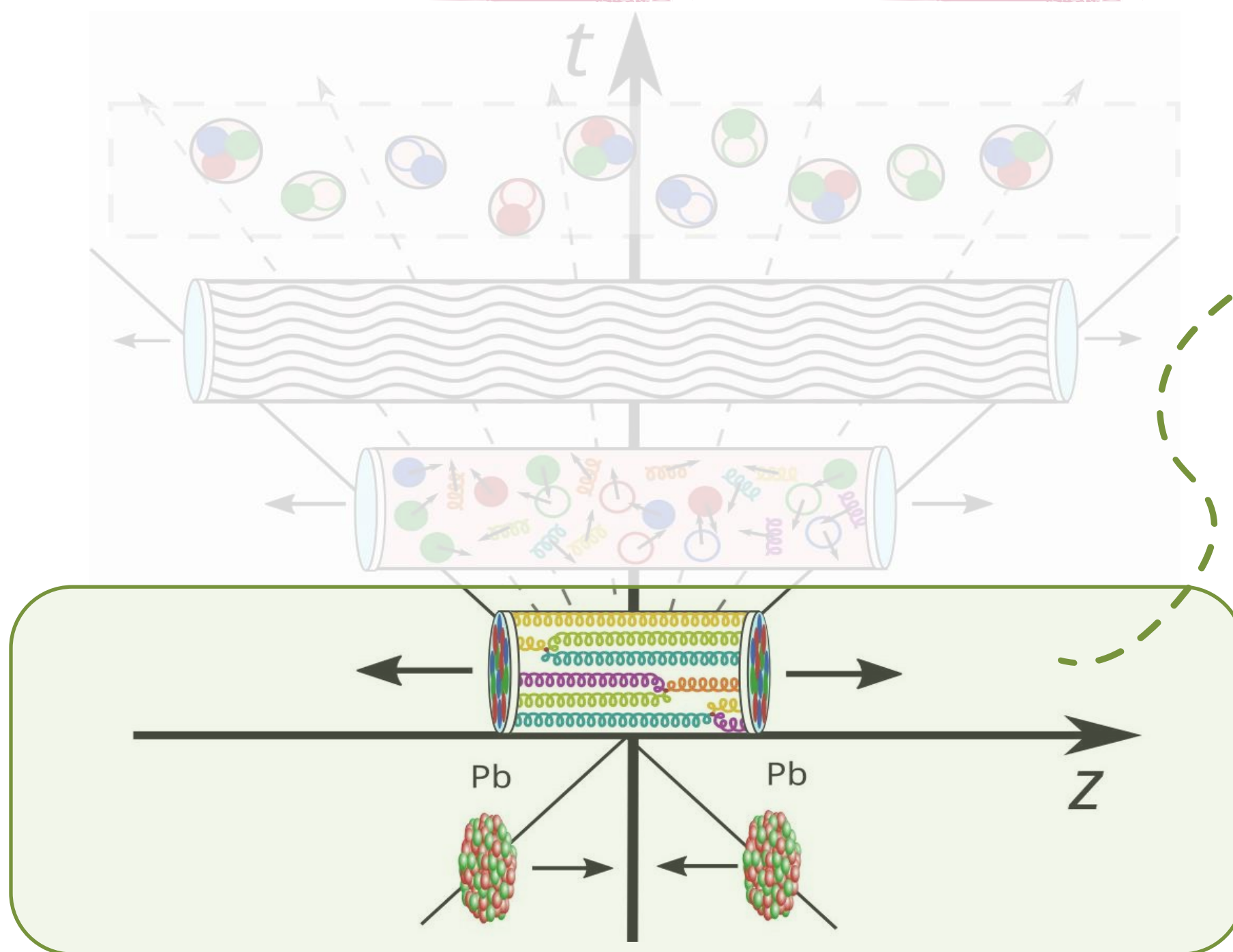
Different stages of HICs



J. Berges, M. P. Heller, A. Mazeliauskas, R. Venugopalan, [Rev. Mod. Phys. 93 \(2021\) 3, 035003](#)

Initial stage of HICs

4



Glasma

- ❖ **Color glass condensate**
Effective field theory for high-energy QCD
- ❖ Gluons as color fields
- ❖ **Classical gauge fields**
High occupation number
- ❖ Real-time classical lattice gauge theory
Numerical solution
- ❖ Longitudinal flux tubes of color fields
- ❖ **Out-of-equilibrium** medium

T. Lappi, Phys. Lett. B 643 (2006) 11-16

F. Gelis, Int. J. Mod. Phys. A 28 (2013) 1330001

Glasma: The initial condition

5

$$x = \frac{\text{Parton momentum}}{\text{Nucleon momentum}}$$

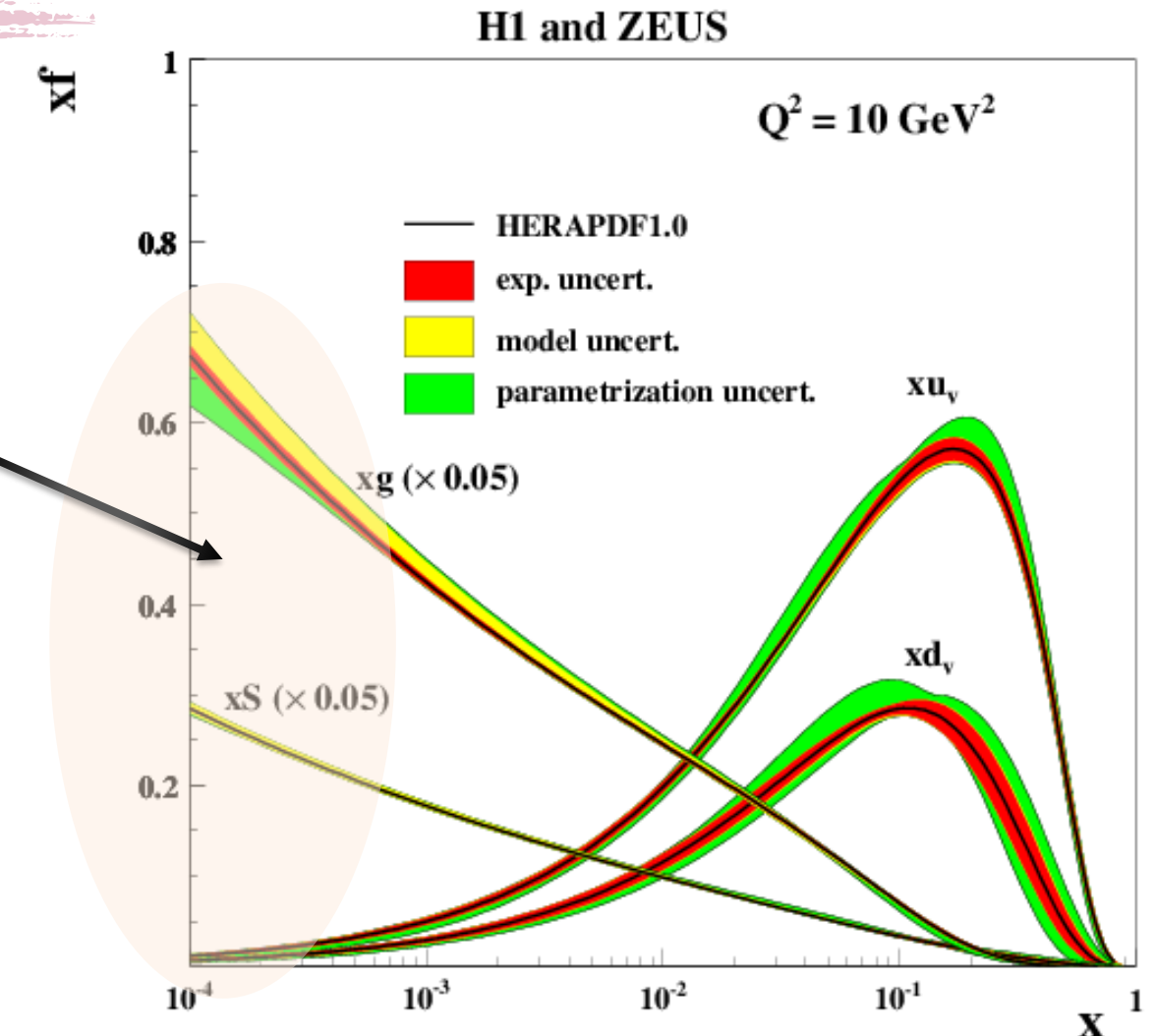
$$x \ll 1$$

Sea quarks + antiquarks
Sea gluons

$$x \approx 1$$

Valence quarks

The small- x
region is
dominated by
the gluons



R. Placakyte (on behalf of the H1 and ZEUS Collaborations),
Proceedings, 31st International Conference on Physics in collisions (PIC 2011)

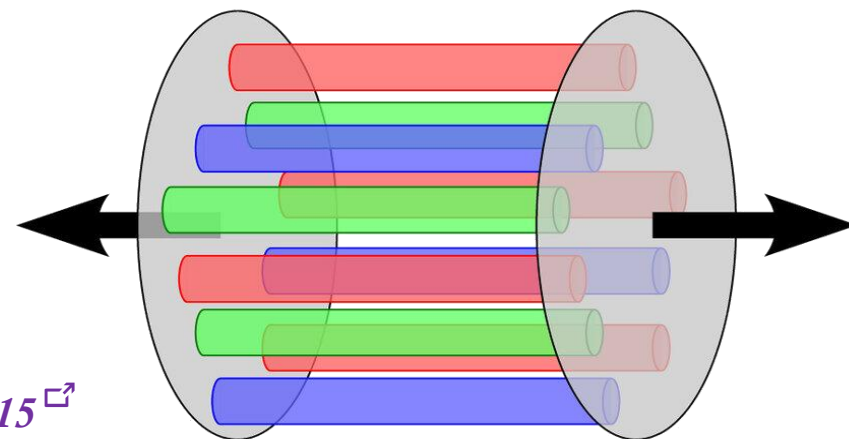
- **Slow partons:** *Gluons* dominate the nucleonic wave function.
- **Fast partons:** *Random sources* of the classical gluon fields.

Color-Glass Condensate (CGC)

6

- ❖ **Color:** Gluons have “colors”.
- ❖ **Glass:** Partons (quarks and gluons) with $x \approx 1$ are very fast ($v \approx c$). So, they appear frozen in lab, like molecules in glasses.
- ❖ **Condensate:** Many small- x gluons: Gluon density is very high and saturated.

The dynamical evolution of the CGC:
Classical Yang-Mills (CYM) equations



K. Fukushima,
Acta Phys. Polon. B 42 (2011) 2697-2715

Pooja
University of Jyväskylä

Condensate
($x \approx 0$)
Gluon fields

$$\mathcal{D}_\mu F^{\mu\nu} = J^\nu$$

Glass
($x \approx 1$)
Source

EMMI Workshop:
Probing the early stages with jets

Wednesday,
July 15, 2026

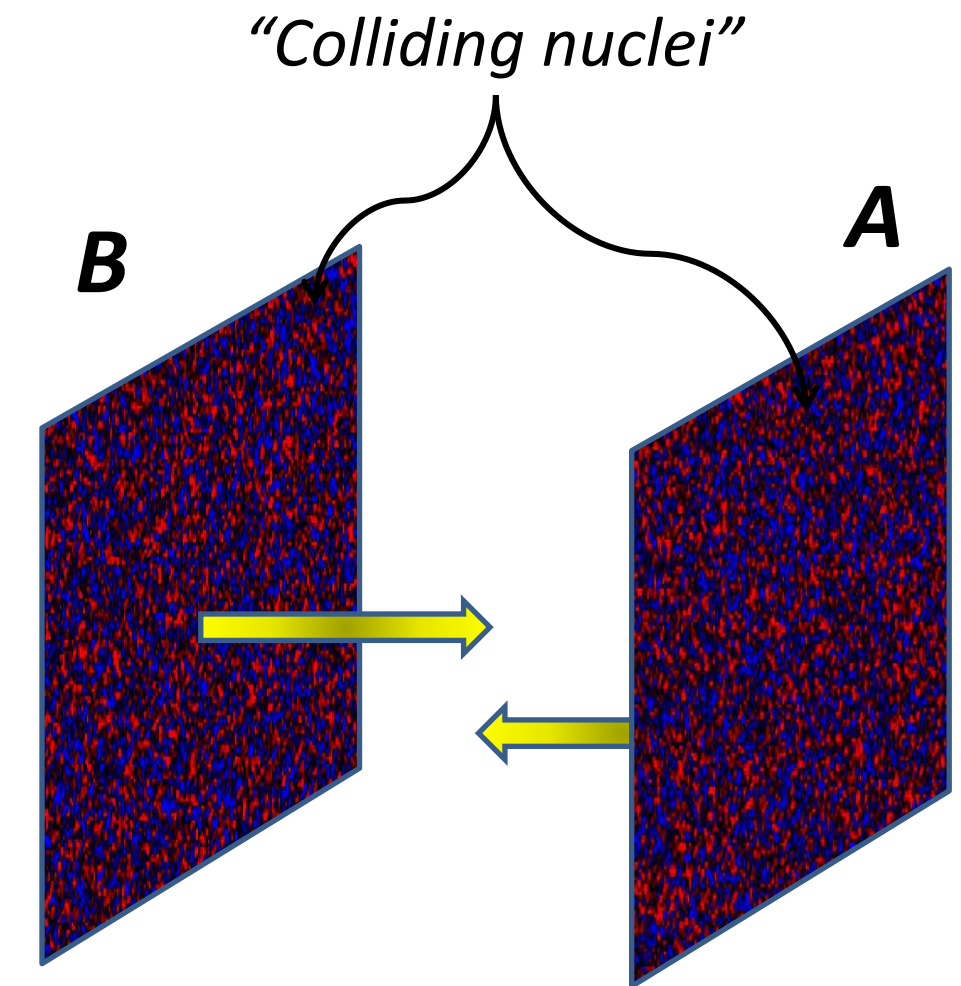
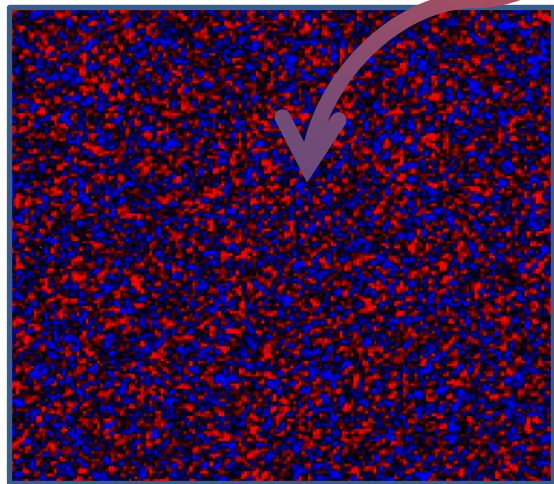
McLerran-Venugopalan (MV) model

7

$$\begin{aligned}\langle \rho^a(x_\perp) \rangle &= 0 \\ \langle \rho^a(x_\perp) \rho^b(y_\perp) \rangle &= (g\mu)^2 \delta^{ab} \delta^{(2)}(x_\perp - y_\perp)\end{aligned}$$

L. McLerran, R. Venugopalan, Phys. Rev. D 49 (1994) 2233-2241

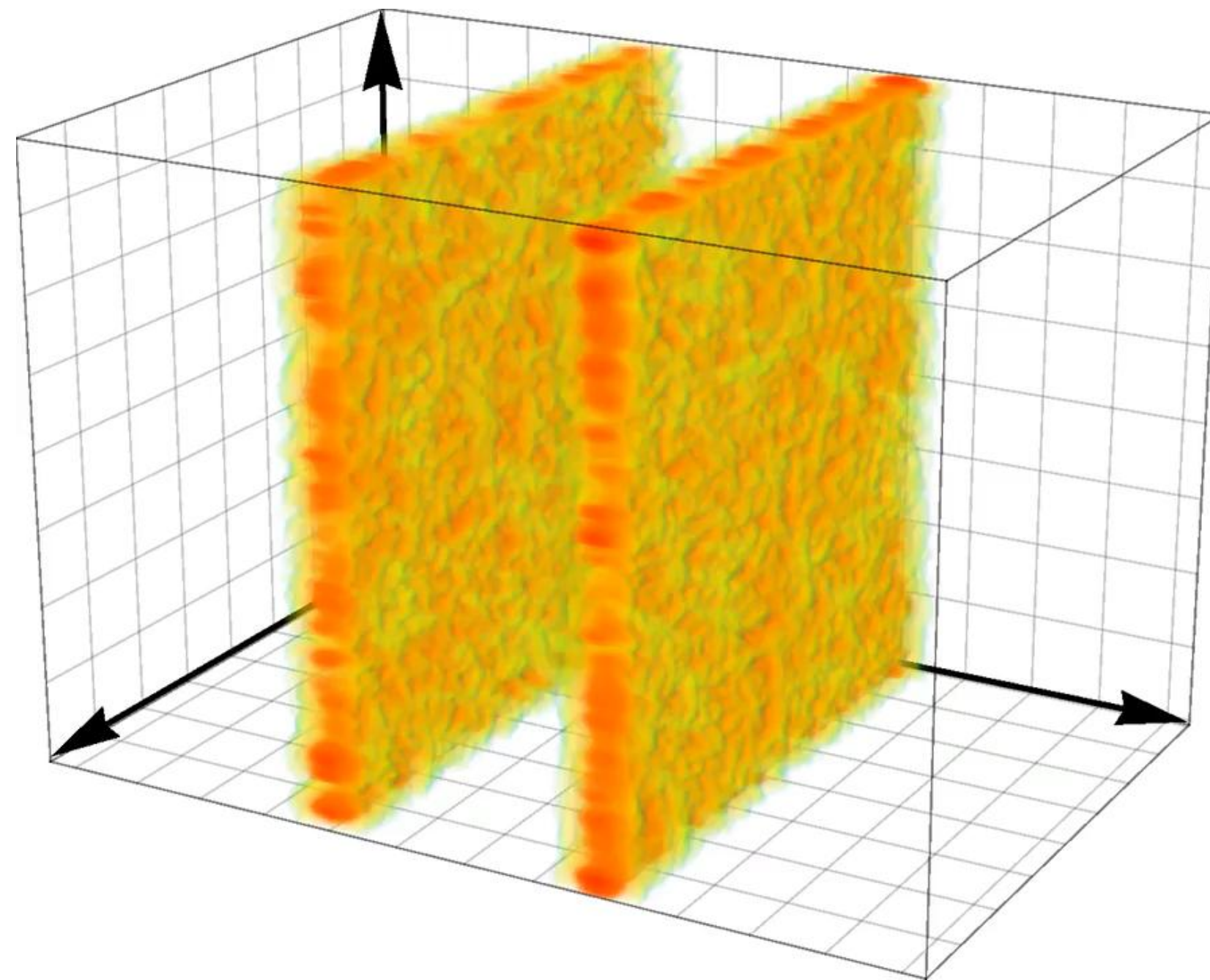
MV model parameter, $g^2\mu \sim Q_s$
 Q_s : Saturation scale of Glasma



Provides initial condition where large- x partons are modeled as random color charges on a $2D$ plane, sourcing soft gluon fields.

How nuclei collide: A visual sequence

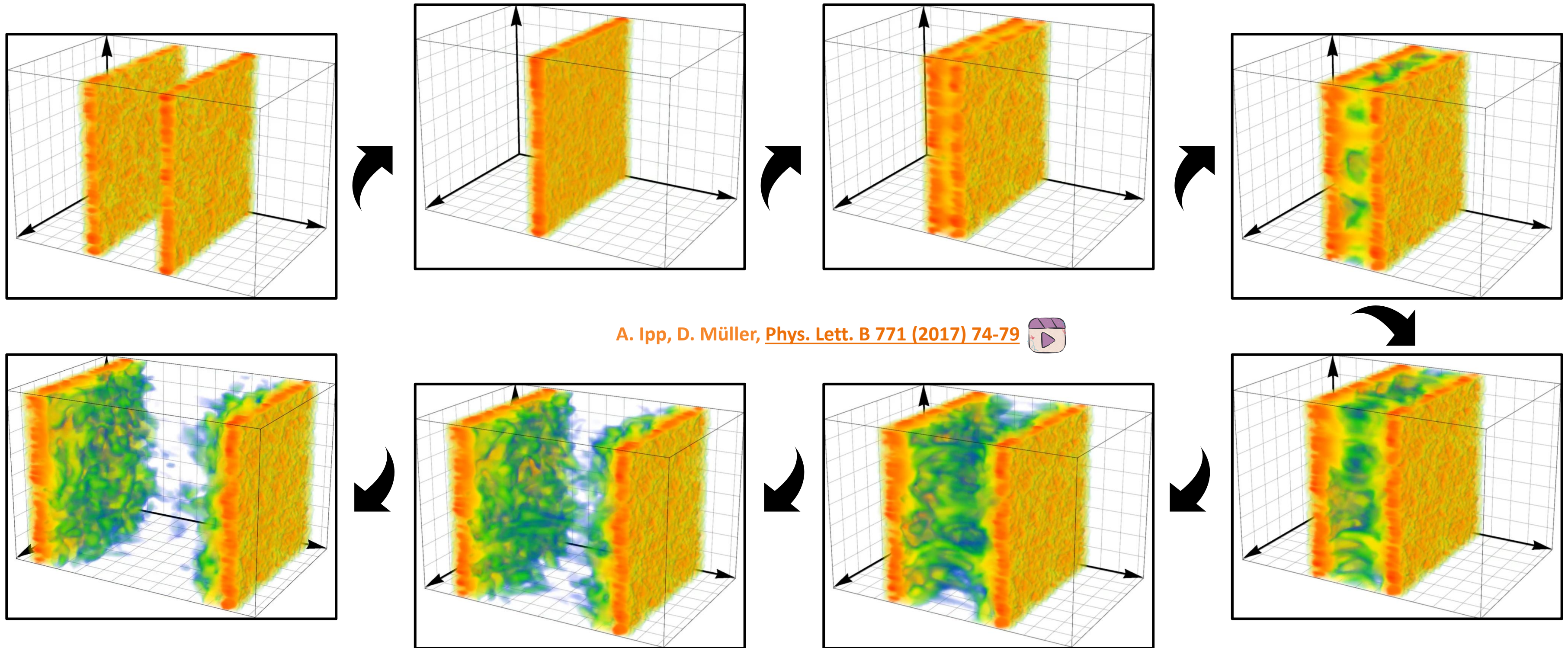
8



A. Ipp, D. Müller, [Phys. Lett. B 771 \(2017\) 74-79](#)[?]

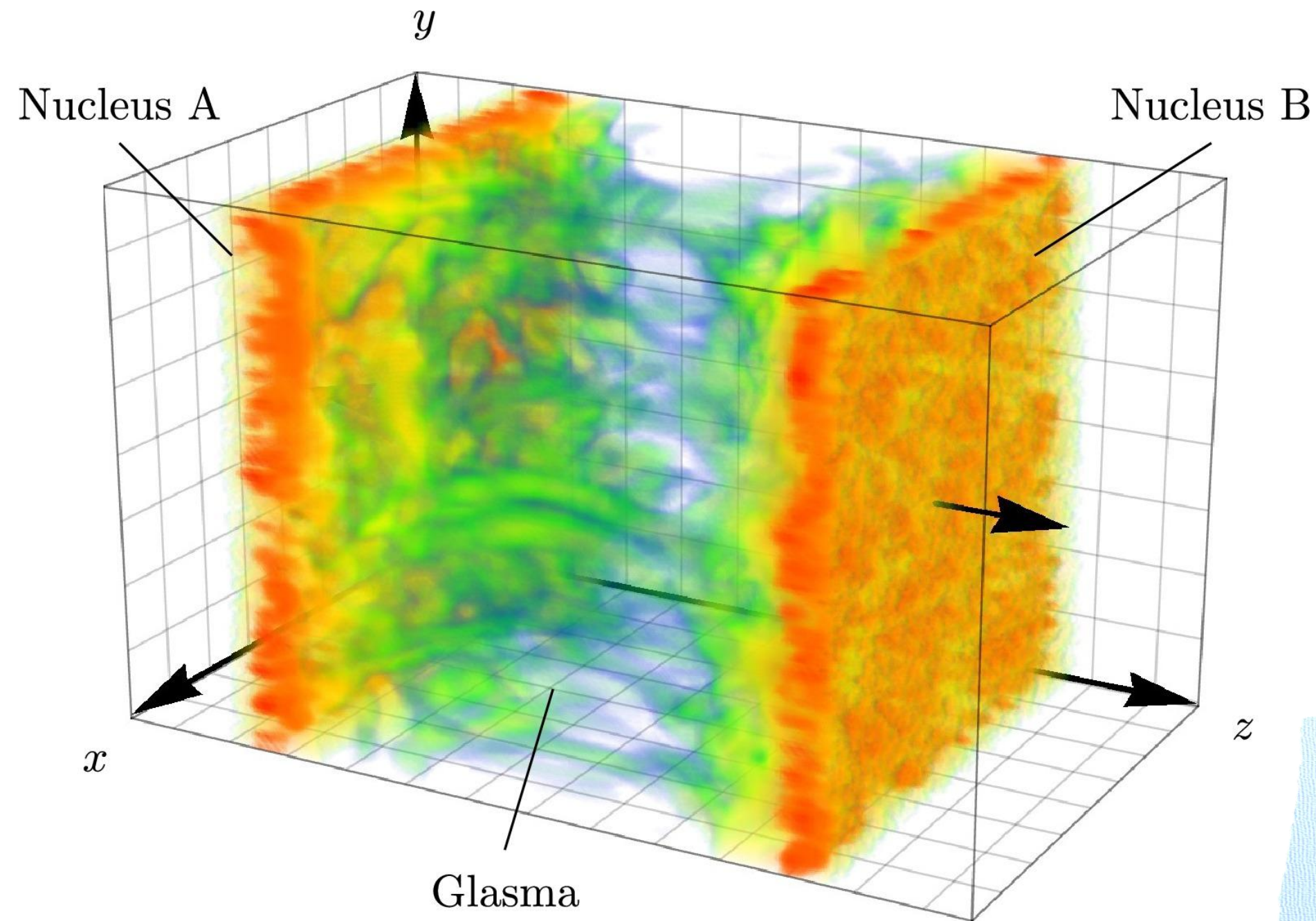
Animatic sketch of collision of two nuclei

9

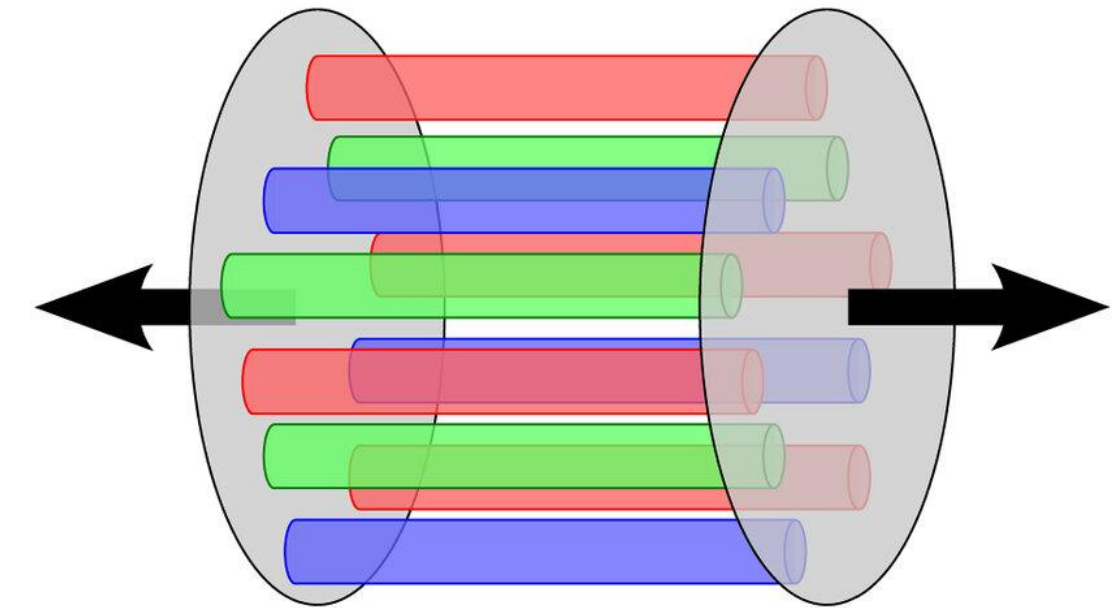


Features of Glasma

10



A. Ipp, D. Müller, *Phys. Lett. B* 771 (2017) 74-79



K. Fukushima, *Acta Phys.Polon.B* 42 (2011) 2697-2715

- Relevant scale: Saturation momentum, Q_s
- Initial longitudinal color flux tubes of $\delta x_T \simeq Q_s^{-1}$
- Fields dilute after $\tau \simeq Q_s^{-1}$

Evolution of expanding Glasma: CYM equations

11

Due to the large density,
the gluon field behaves
like a classical field.



Dynamics is governed
by classical EoMs,
namely the **Classical
Yang-Mills (CYM)**
equations.

**Classical
Yang-Mills
(CYM)
Equations**

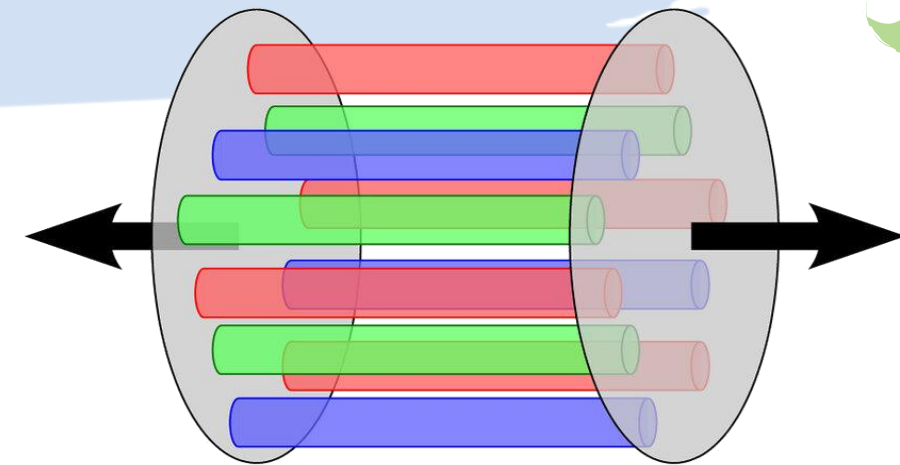
$$\begin{aligned}\dot{A}_i &= \frac{1}{\tau} E^i \\ \dot{A}_\eta &= \tau E_\eta\end{aligned}$$

$$\begin{aligned}\dot{E}^i &= \tau [D_j, F_{ji}] - \frac{ig}{\tau} [A_\eta, [D_i, A_\eta]] \\ \dot{E}_\eta &= \frac{1}{\tau} [D_i, [D_i, A_\eta]]\end{aligned}$$

*Evolution of the system is studied assuming Glasma as the initial condition,
and evolving this condition by virtue of the **CYM** equations.*

Instability of boost-invariant Glasma fields

Pooja, D. Avramescu, T. Lappi
Phys. Rev. D 114, 014010 [↗](#)



B. Muller, A. Trayanov, *Phys. Rev. Lett.* 68 (1992) 3387

U. W. Heinz, C. R. Hu, S. Leupold, S. G. Matinyan, B. Muller, *Phys. Rev. D* 55 (1997) 2464

P. Romatschke, R. Venugopalan, *Phys. Rev. D* 74 (2006) 045011

P. Romatschke, Raju Venugopalan, *Phys. Rev. L* 96 (2006) 062302

T. Kunihiro, B. Müller, A. Ohnishi, A. Schäfer, T. T. Takahashi, A. Yamamoto, *Phys. Rev. D* 82 (2010) 114015

Sylwia Bazak, Stanisław Mrówczyński, *Phys. Rev. C* 109 (2024) 024903

Is Glasma chaotic?

13

Butterfly Effect:

Small changes in initial conditions lead to unpredictable, significant results.

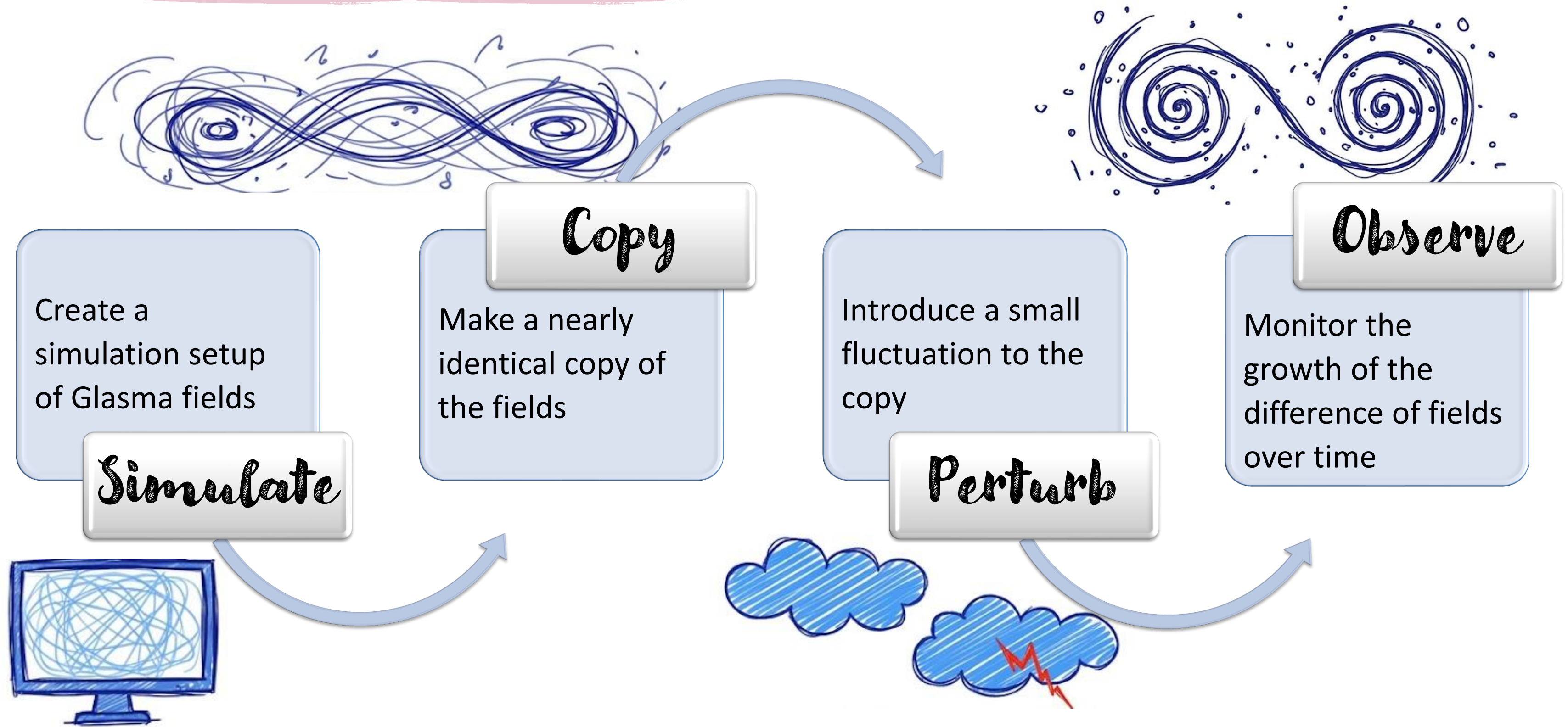


Ingredients for chaos in Glasma:

- ❖ **Non-Abelian Nature:** Gluon fields self-interact. This non-linear interaction is a source of complexity and chaos.
- ❖ **Classical Field Dynamics:** In the early, dense stage, gluons can be treated as classical Yang-Mills fields, highly non-linear.
- ❖ **Sensitivity to Initial Conditions:** The initial color charges in the nuclei are random. Small changes here should propagate.

Seeding the chaos

14



Seeding the chaos in initial classical fields

15

Perturb any one of the initial fields:

Perturbed field	Unperturbed field	Small perturbation
$E'_\eta{}^a(\tau = 0, \mathbf{x}_\perp)$	$E_\eta{}^a(\tau = 0, \mathbf{x}_\perp)$	$+\frac{2\chi^a(\mathbf{x}_\perp)}{a_\perp^2}$
$A_i'^a(\tau = 0, \mathbf{x}_\perp)$	$A_i^a(\tau = 0, \mathbf{x}_\perp)$	$+\frac{2\xi_i^a(\mathbf{x}_\perp)}{a_\perp}$

$$\begin{aligned}\langle \chi^a(\mathbf{x}_\perp) \rangle &= 0 \\ \langle \chi^a(\mathbf{x}_\perp) \chi^b(\mathbf{y}_\perp) \rangle &= \alpha^2 \delta^{ab} \delta_{\mathbf{x}_\perp, \mathbf{y}_\perp}\end{aligned}$$

$$\begin{aligned}\langle \xi_i^a(\mathbf{x}_\perp) \rangle &= 0 \\ \langle \xi_i^a(\mathbf{x}_\perp) \xi_j^b(\mathbf{y}_\perp) \rangle &= \beta^2 \delta^{ab} \delta_{ij} \delta_{\mathbf{x}_\perp, \mathbf{y}_\perp}\end{aligned}$$

leads to perturbed B'_η

$$\begin{aligned}U'_i(\tau = 0, \mathbf{x}_\perp) &= U_i(\tau = 0, \mathbf{x}_\perp) \cdot \exp[ig a_\perp \delta A_i^a(\tau = 0, \mathbf{x}_\perp)] T^a \\ B_\eta(\tau_n, \mathbf{x}_\perp) &= -F_{xy}(\tau_n, \mathbf{x}_\perp) \approx -\frac{1}{g a_\perp^2} [U_{i\hat{j}}(\tau, \mathbf{x}_\perp)]_{ah}\end{aligned}$$

anti-Hermitian

α, β : Perturbation amplitude

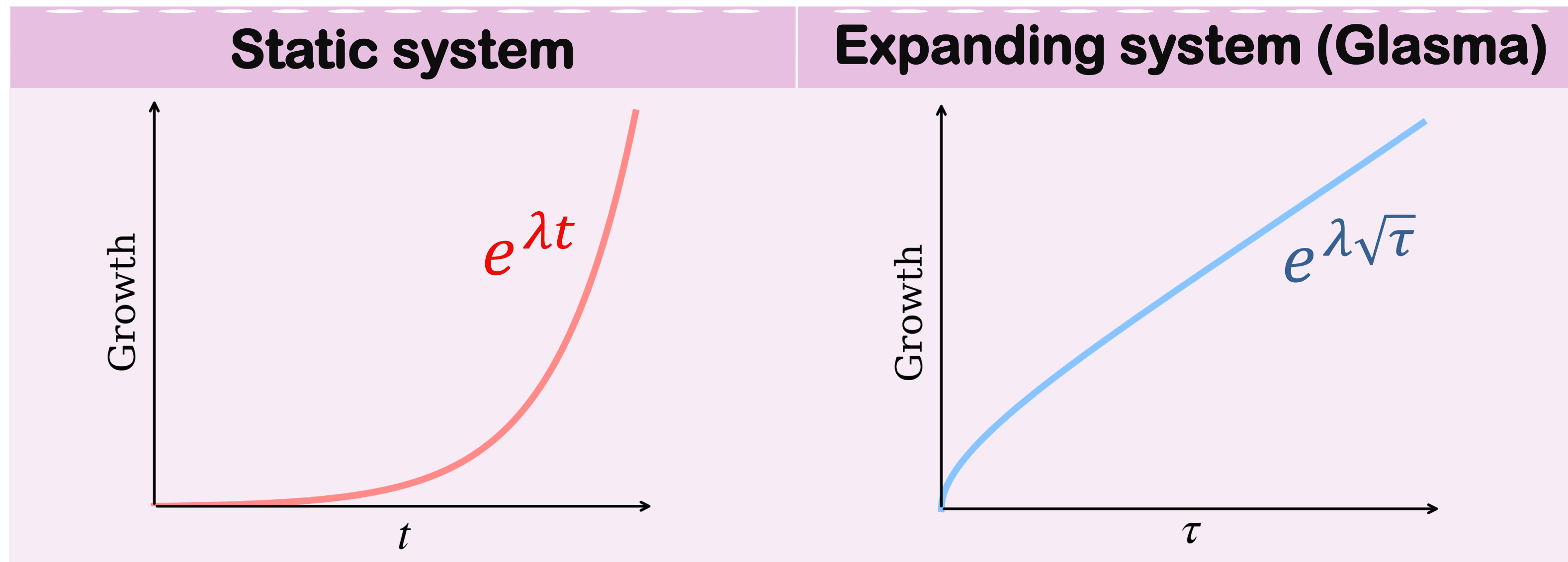
***Solve CYM
equations of
motion
separately for
 E, A , and E', A'***



***Measure
 $\left(\delta E_\eta(\tau)\right)^2$ and
 $\left(\delta B_\eta(\tau)\right)^2$ vs τ***

Impact of longitudinal expansion

17



Growth in the Glasma is not exponential in time, but exponential in the square root of the proper time, τ .

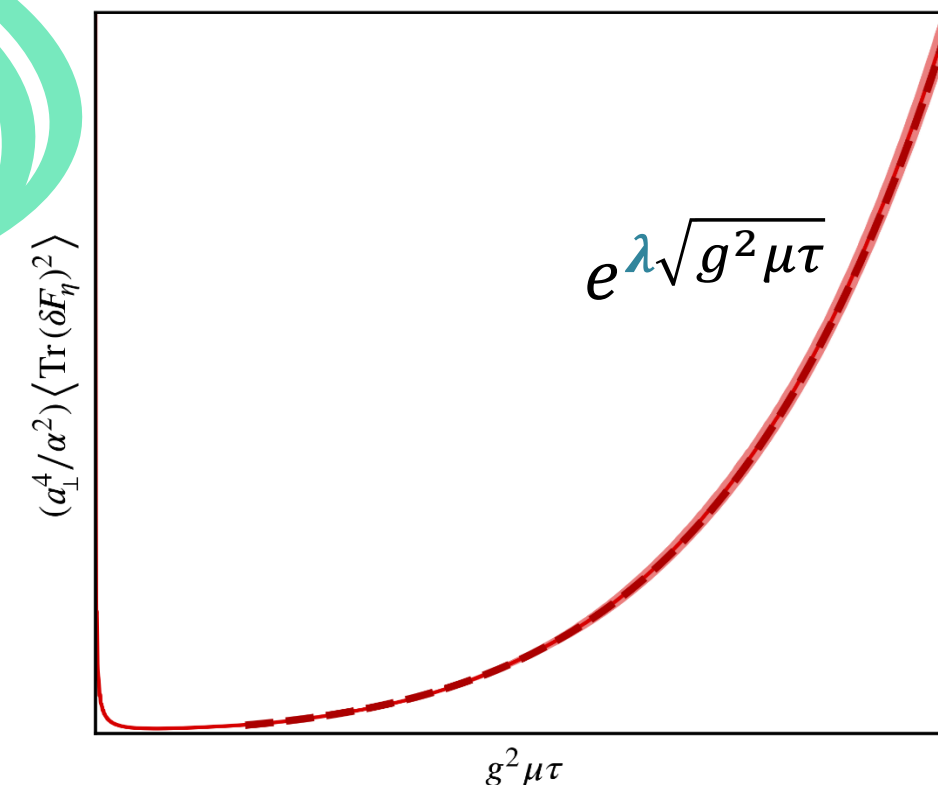
As the field perturbations attempt to grow, the boost invariant longitudinal expansion of the colliding system stretches physically and dilutes the growth.

Impact of longitudinal expansion of Glasma

18

In a static box, chaotic perturbations grow exponentially with linear time ($e^{\lambda t}$). However, in the expanding system, longitudinal expansion stretches the system. This dilutes the energy density, modifying the chaotic growth into a square-root trajectory.

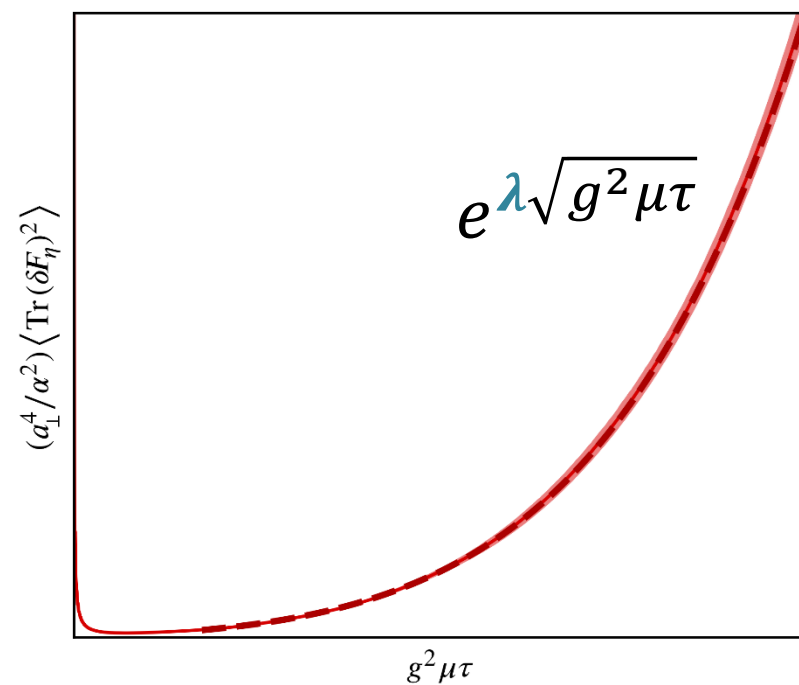
$$\left\langle \text{Tr} \left(\delta F_{\eta}(\tau) \right)^2 \right\rangle \propto e^{\lambda \sqrt{g^2 \mu \tau}}$$



Impact of longitudinal expansion of Glasma

19

Longitudinal expansion dictates a square-root time dependence.



The tracked perturbation in the longitudinal fields, $F = E, B$

The signature of classical chaos: exponential growth

The leading Lyapunov exponent (the intrinsic chaoticity being extracted)

Square-root time dependency: A direct physical consequence of the 2+1 D boost-invariant Glasma

$$\left\langle \text{Tr} \left(\delta F_{\eta}(\tau) \right)^2 \right\rangle \propto e^{\lambda \sqrt{g^2 \mu \tau}}$$

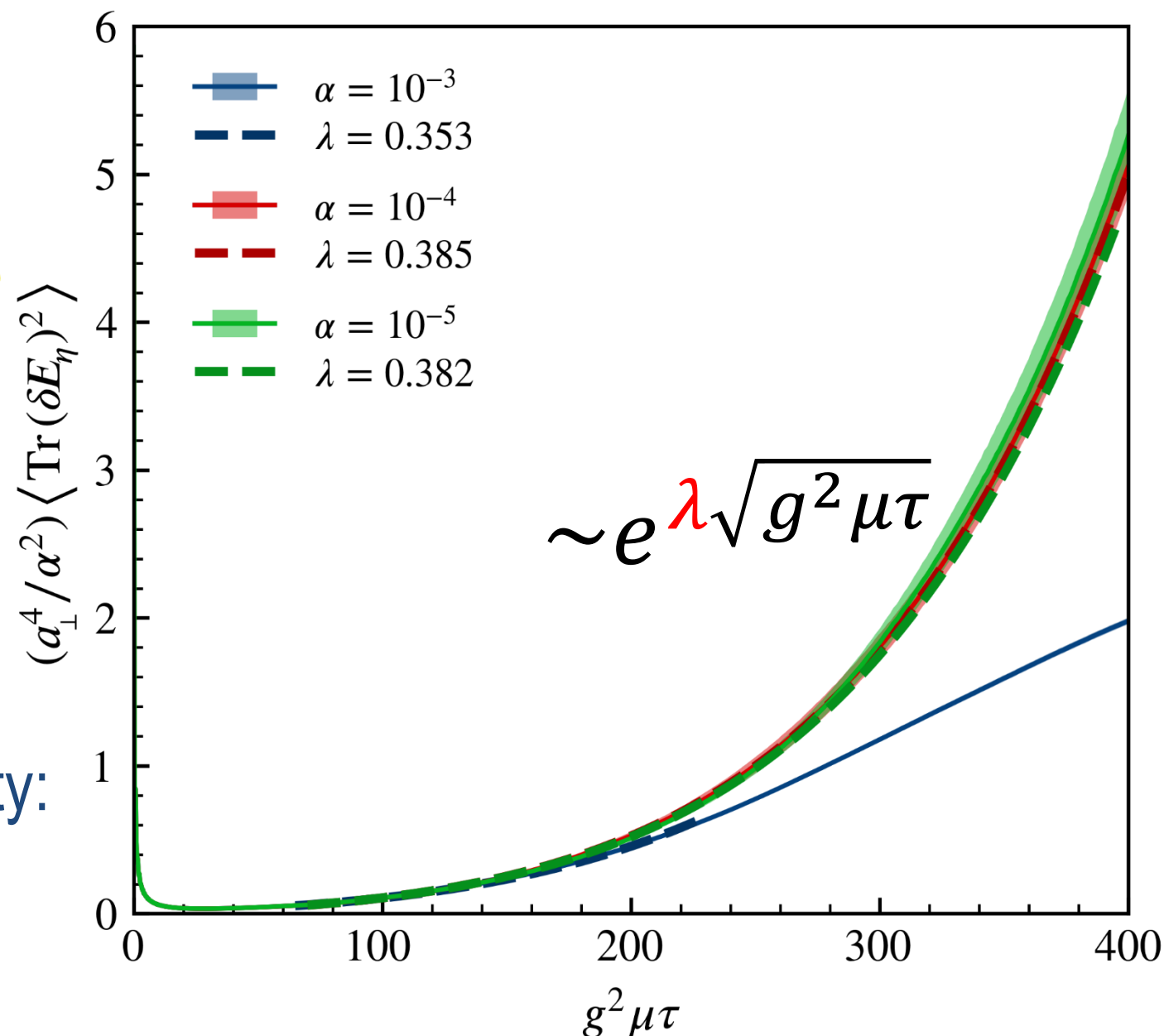
The MV model parameter, $g^2 \mu \sim Q_s$
 Q_s : Saturation scale of Glasma

Step 0: Check linearity

20

Are we in the
linear regime?

Linear regime stability:
 $\alpha, \beta \ll 1$



$$\left\langle \text{Tr} \left(\delta F_{\eta}(\tau) \right)^2 \right\rangle \propto e^{\lambda \sqrt{g^2 \mu \tau}}$$

$F = E, B$ λ : Lyapunov exponent

- Scaling by α^2 confirms that the dynamics remain strictly in linear regime for $\alpha < 10^{-3}$
- Initial transient phase: Other modes die away
- Later, a very clear **exponential growth**

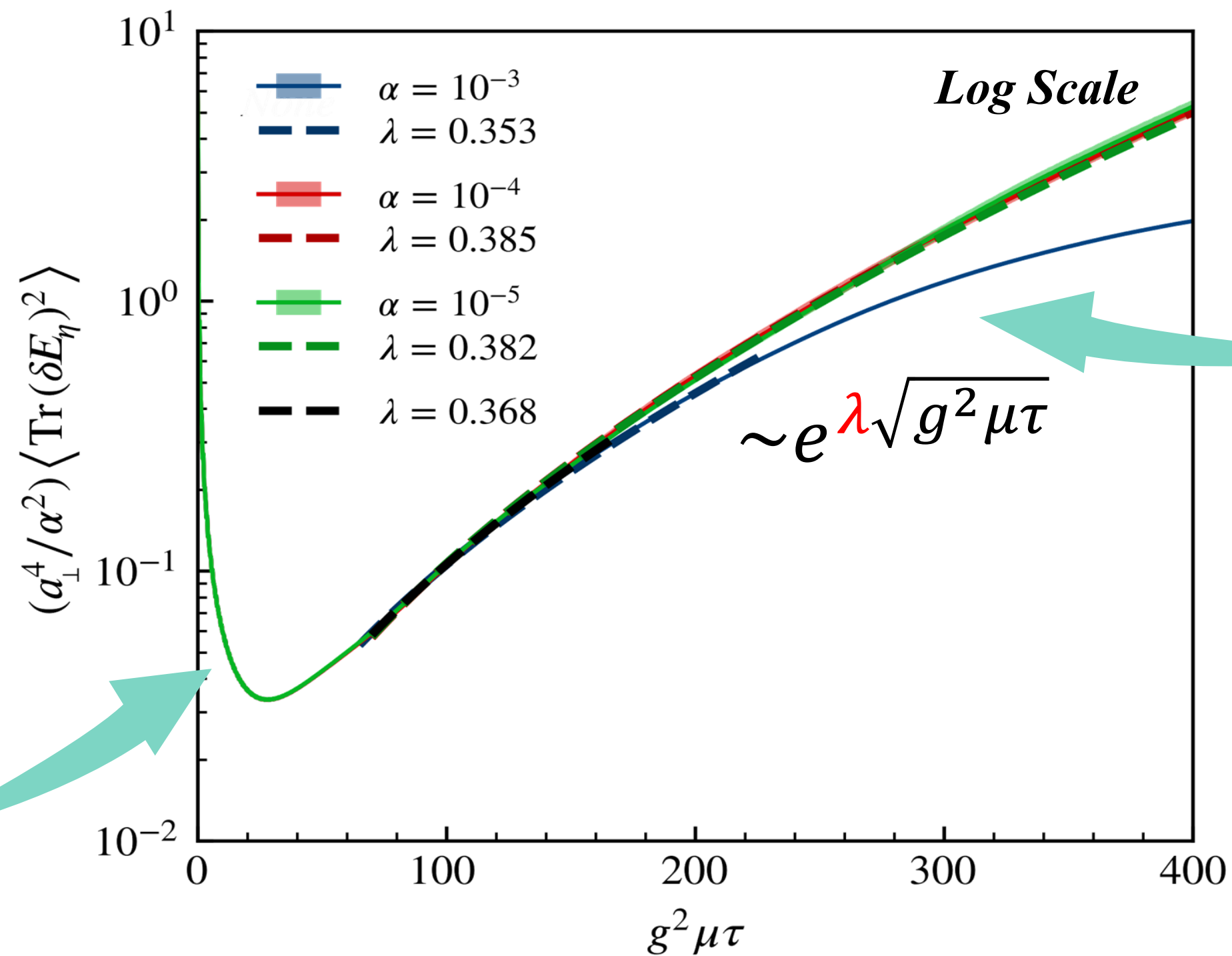


Chaoticity: Perturbations grow exponentially as a **square root of time** due to **boost-invariant expansion** of the Glasma fields.

The awakening of chaos

21

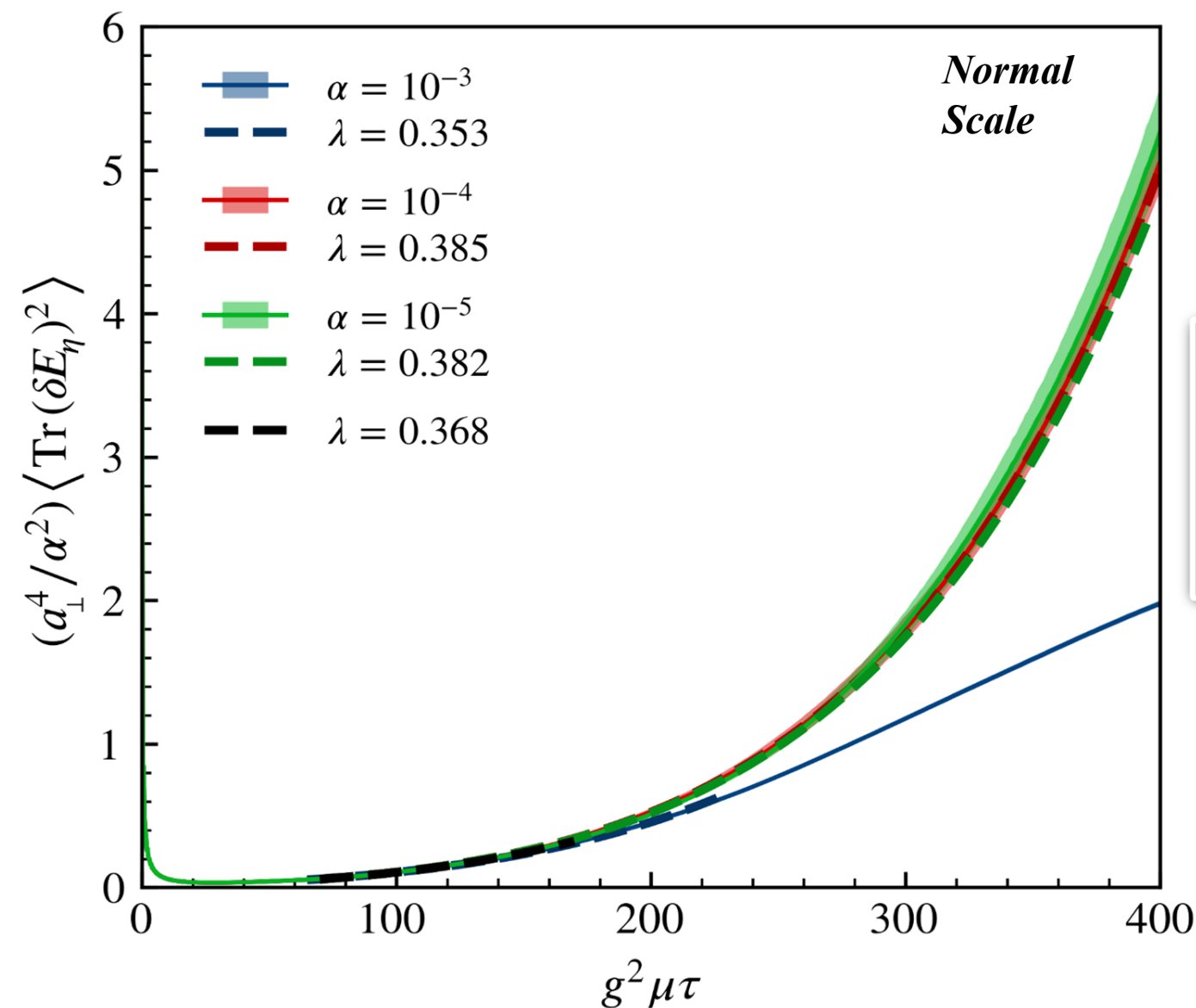
Transient phase:
System takes
time to digest
the perturbation



Exponential phase:
Clear linear
growth on the
logarithm scale

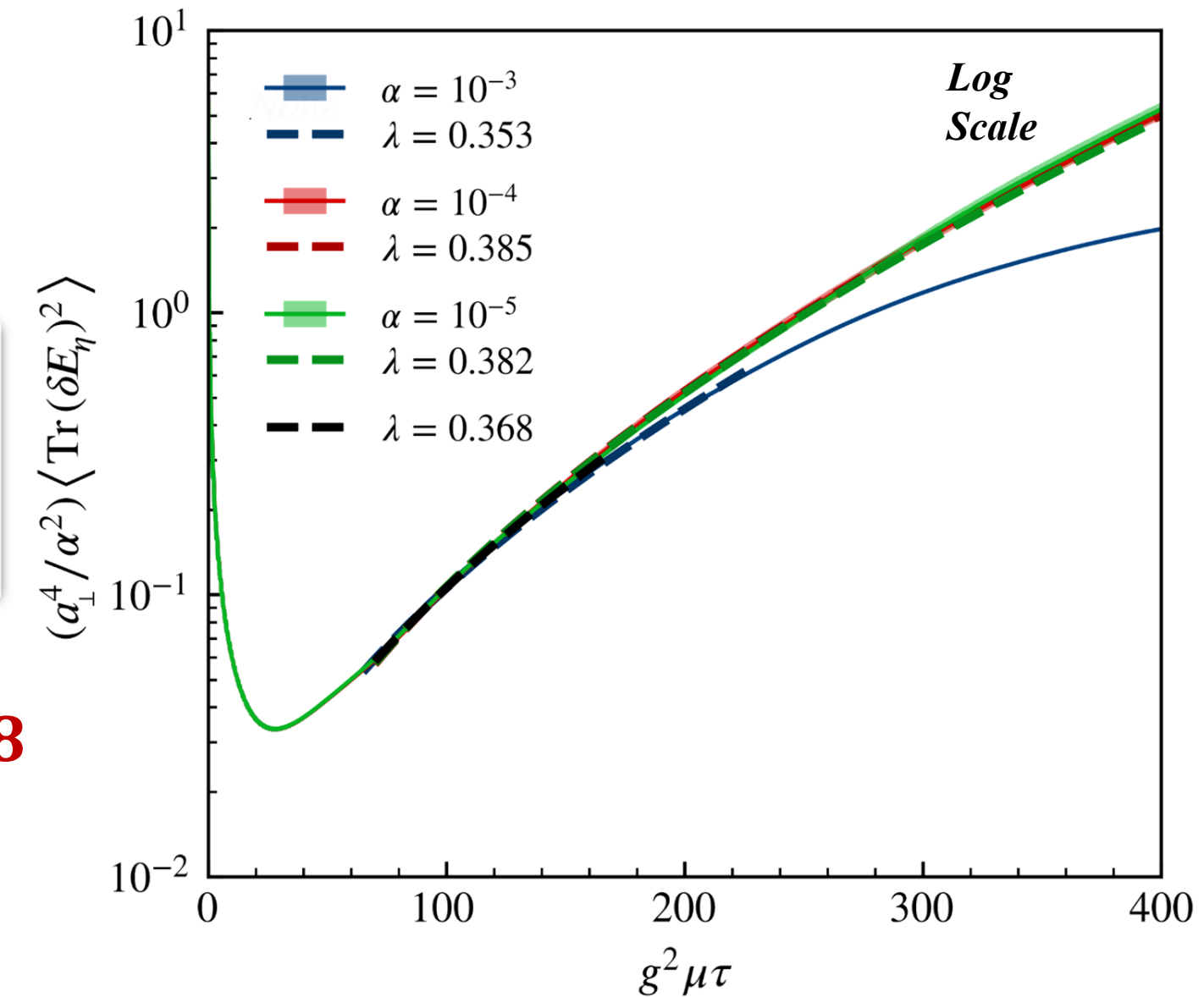
White noise perturbation

22



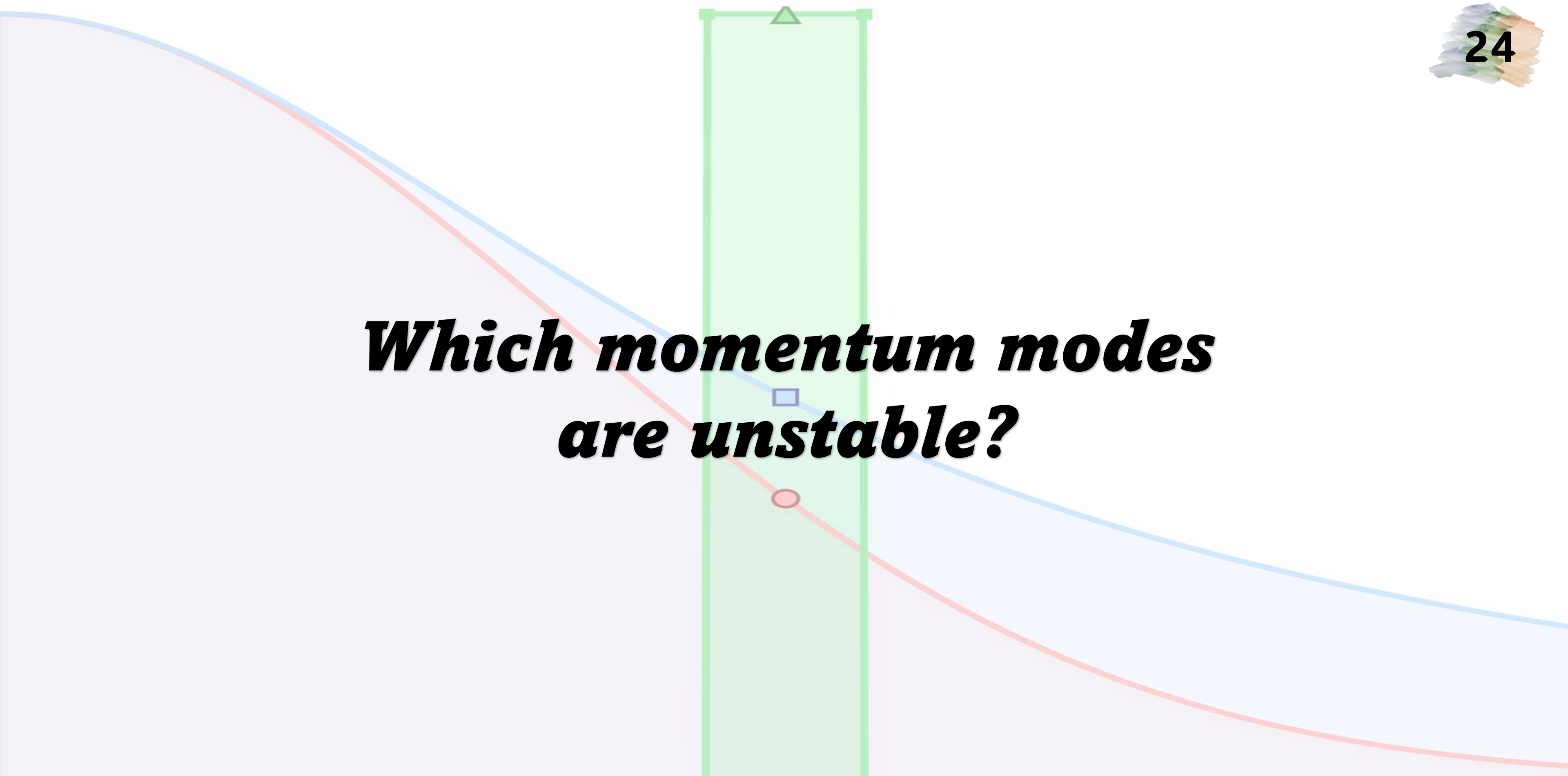
$$\langle \text{Tr}(\delta E_{\eta}(\tau))^2 \rangle \propto e^{\lambda \sqrt{g^2 \mu \tau}}$$

$\lambda = 0.35, \dots, 0.38$



- Perturbations remain in linear regime for $\alpha < 10^{-3}$
- White noise excites all momentum modes equally.
- Clearly visible exponential growth at late times, thus we extract the **Lyapunov exponent, λ**

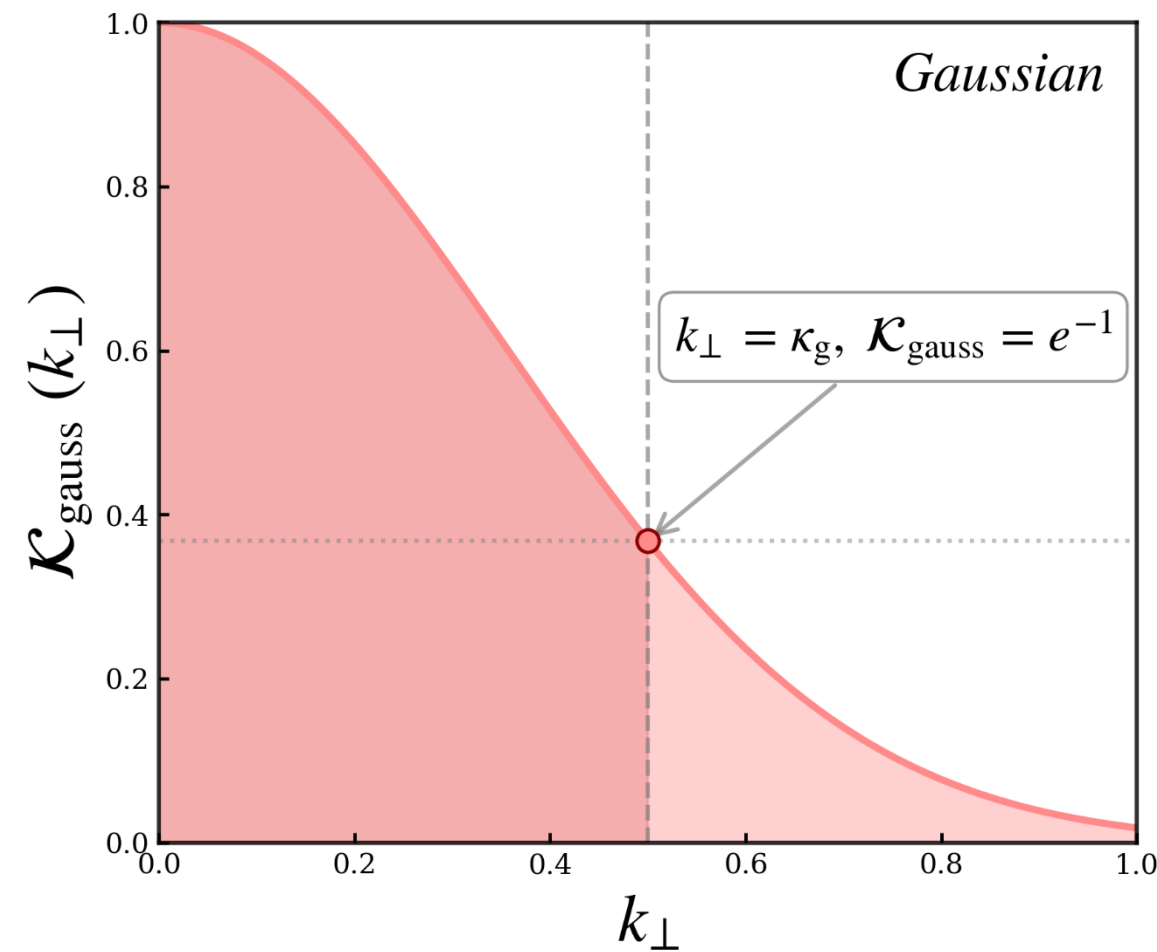
***Does the chaos in the Glasma
originate from specific instabilities at
specific momentum scales?***



***Which momentum modes
are unstable?***

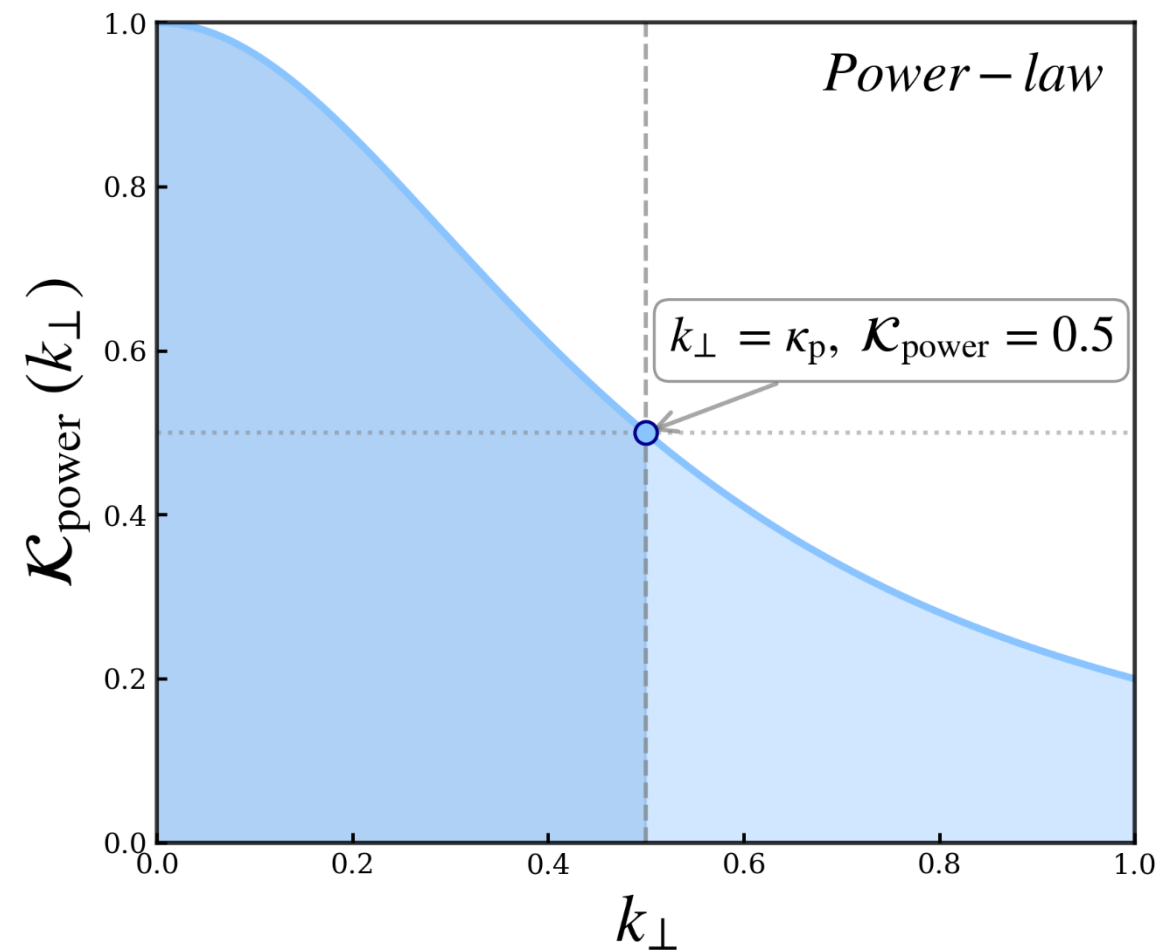
Momentum-space filtering

25



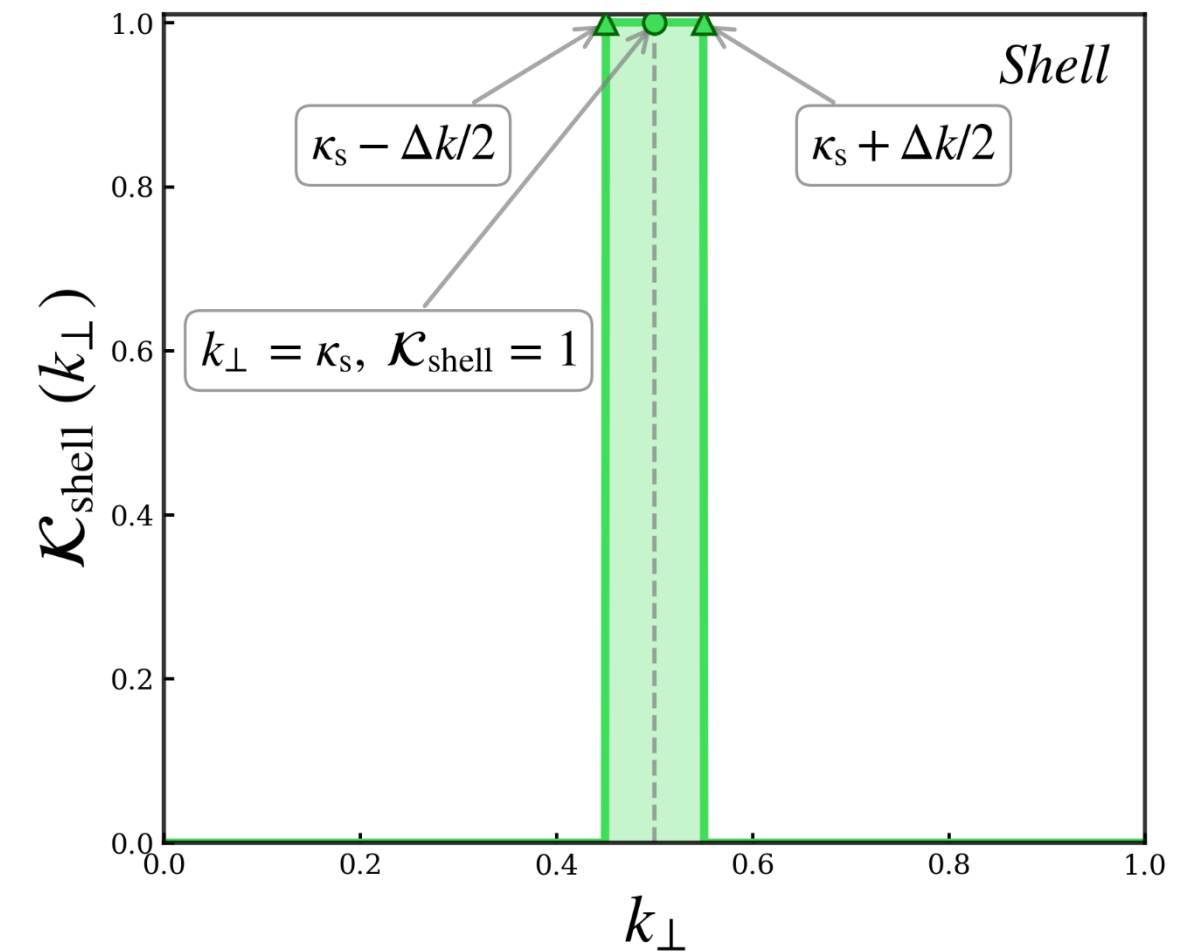
$$\mathcal{K}_{\text{gauss}}(k_{\perp}) = \exp\left(-\frac{\kappa_{\perp}^2}{\kappa_g^2}\right)$$

- Aggressively suppresses high-momentum ultraviolet modes
- Focuses on infrared modes



$$\mathcal{K}_{\text{power}}(k_{\perp}) = \frac{\kappa_p^2}{k_{\perp}^2 + \kappa_p^2}$$

- Milder suppression of high-momentum modes
- Preserves the ultraviolet tail

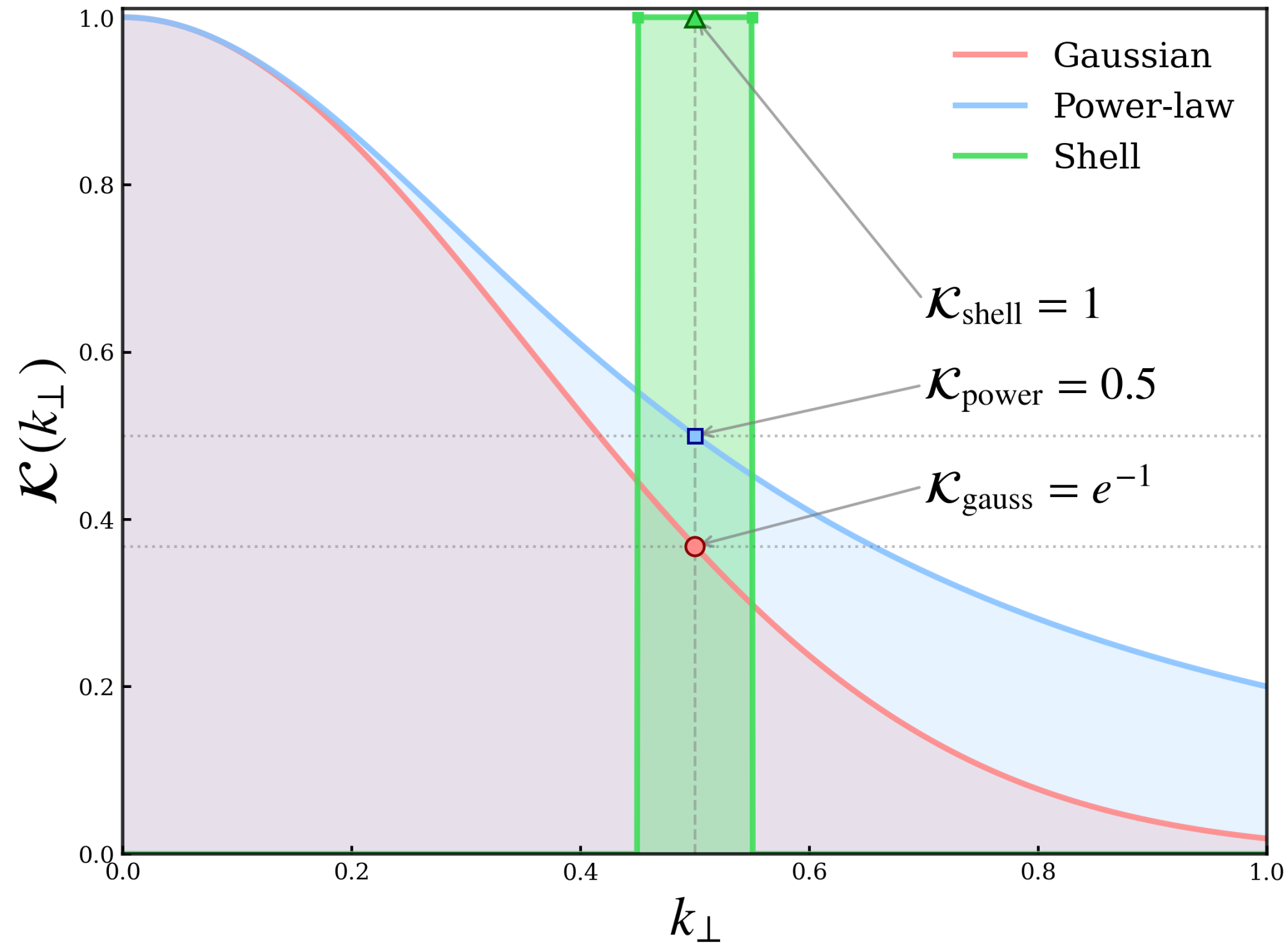


$$\mathcal{K}_{\text{shell}}(k_{\perp}) = \begin{cases} 1, & \kappa_s - \frac{\Delta k}{2} \leq k_{\perp} \leq \kappa_s + \frac{\Delta k}{2} \\ 0, & \text{otherwise} \end{cases}$$

- Isolates a narrow band of momenta
- Tests specific momentum modes independently

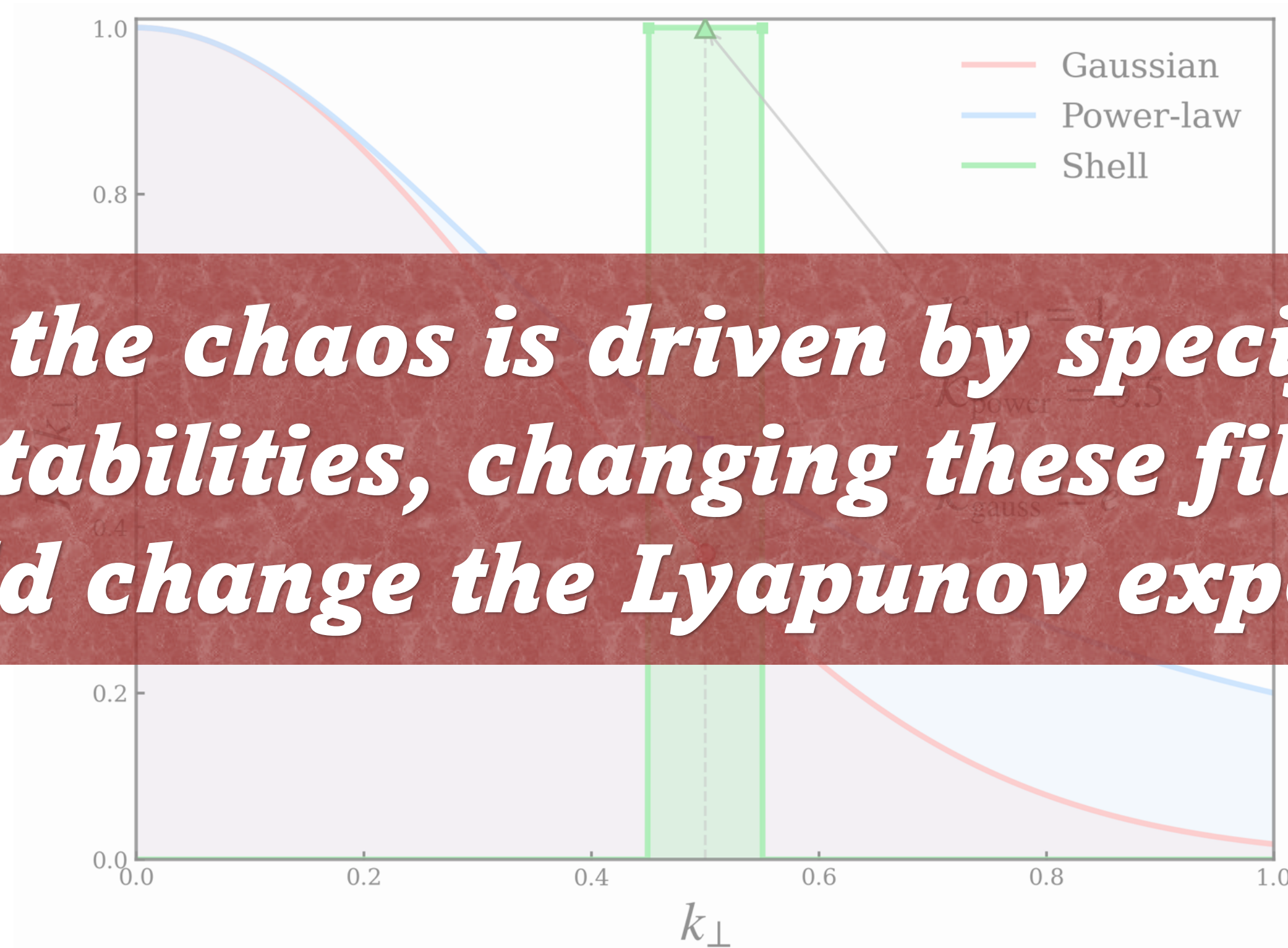
Momentum-space filtering

26



Momentum-space filtering

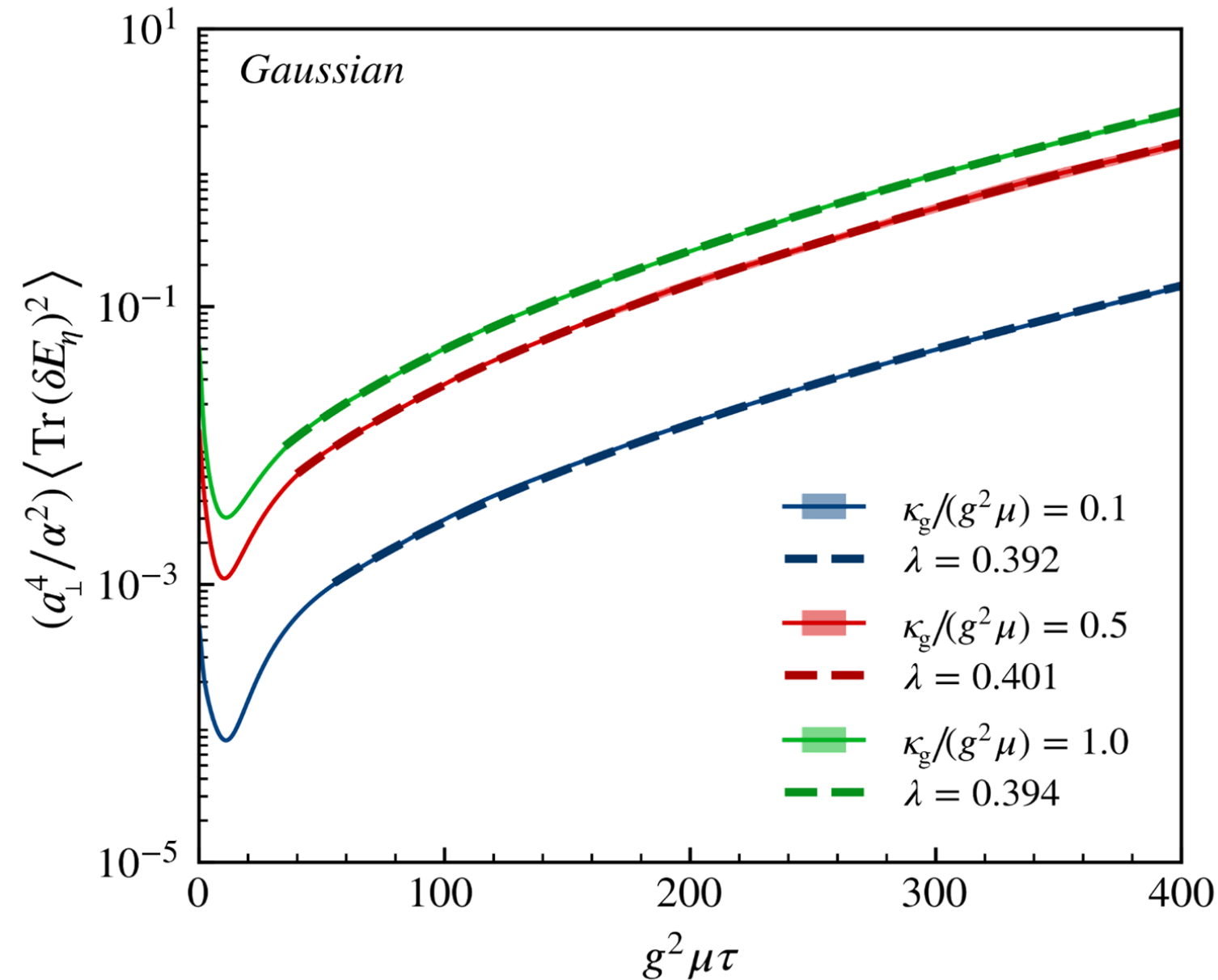
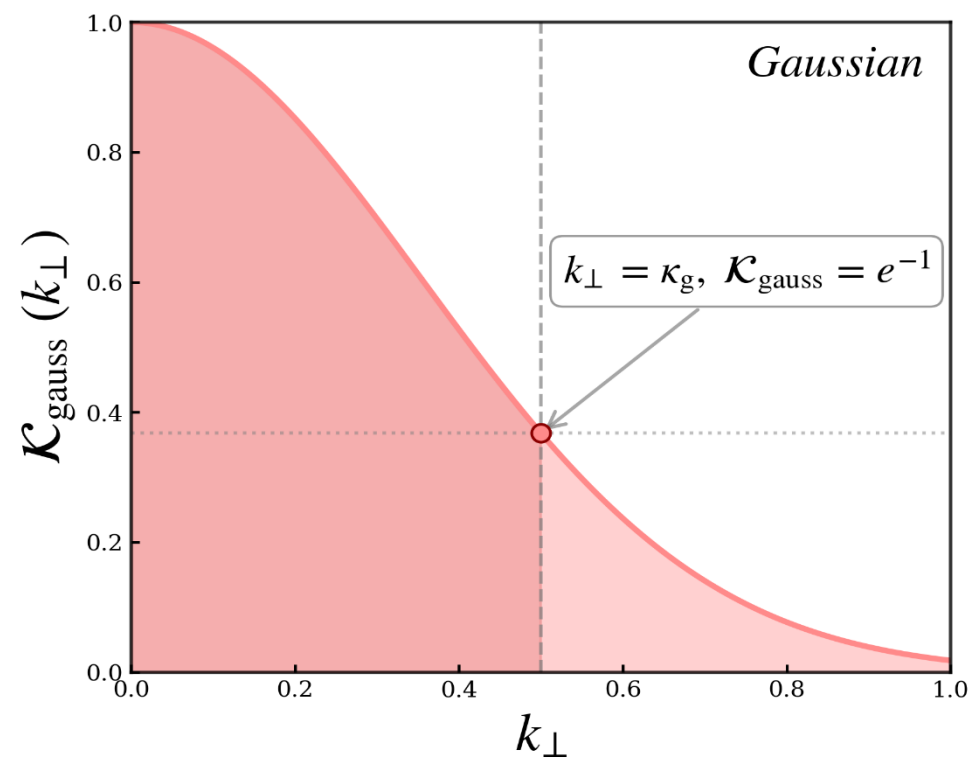
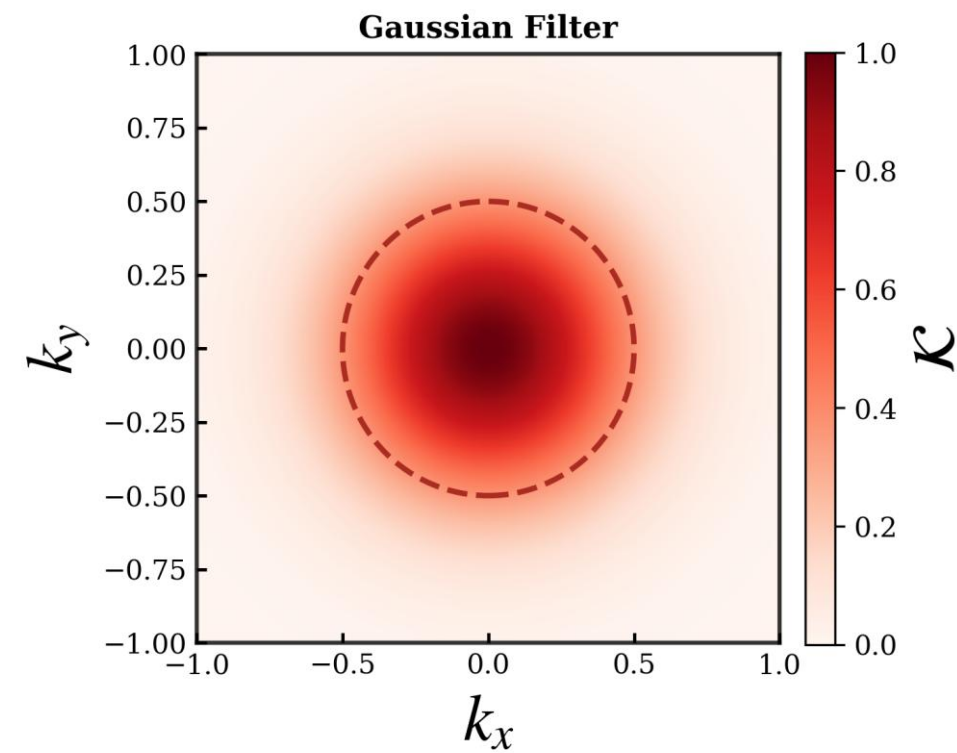
27



If the chaos is driven by specific instabilities, changing these filters should change the Lyapunov exponent.

Gaussian-filtered perturbations

28



$$\mathcal{K}_{\text{gauss}}(k_{\perp}) = \exp\left(-\frac{\kappa_{\perp}^2}{\kappa_g^2}\right)$$

- Aggressively suppresses high-momentum ultraviolet modes
- Focuses on infrared modes

Gaussian-filtered perturbations

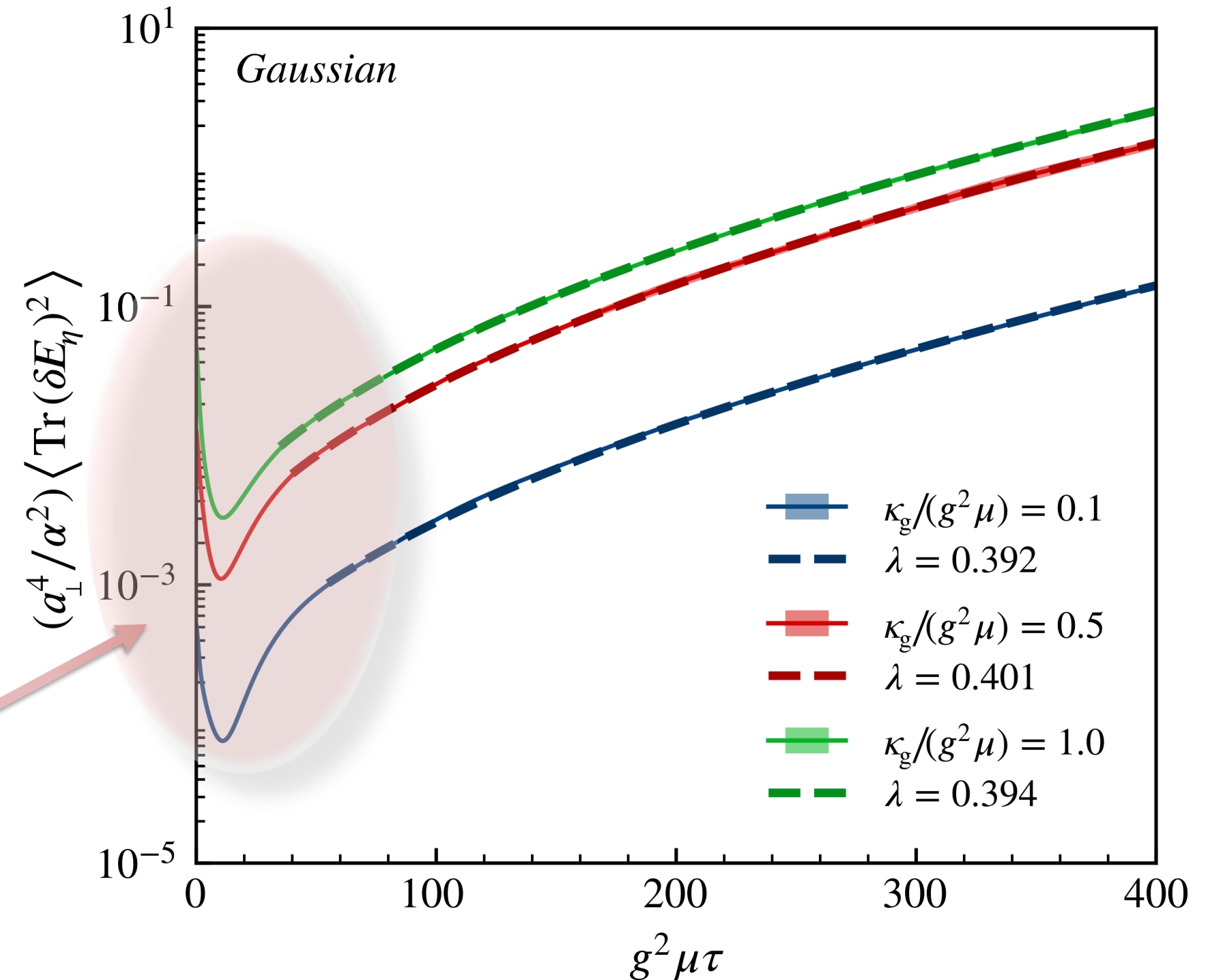
29

$$a_{\perp}^4 \left\langle \text{Tr} \left(\delta E_{\eta}(\tau = 0) \right)^2 \right\rangle \propto a_{\perp}^2 \kappa_g^2$$

Initial amplitude of the perturbations $\propto \kappa_g$

For smaller values of κ_g , the filter suppresses a larger portion of the momentum spectrum, resulting in fewer modes contributing to the initial fluctuations.

Hence, we get a smaller initial intercept on the y-axis.



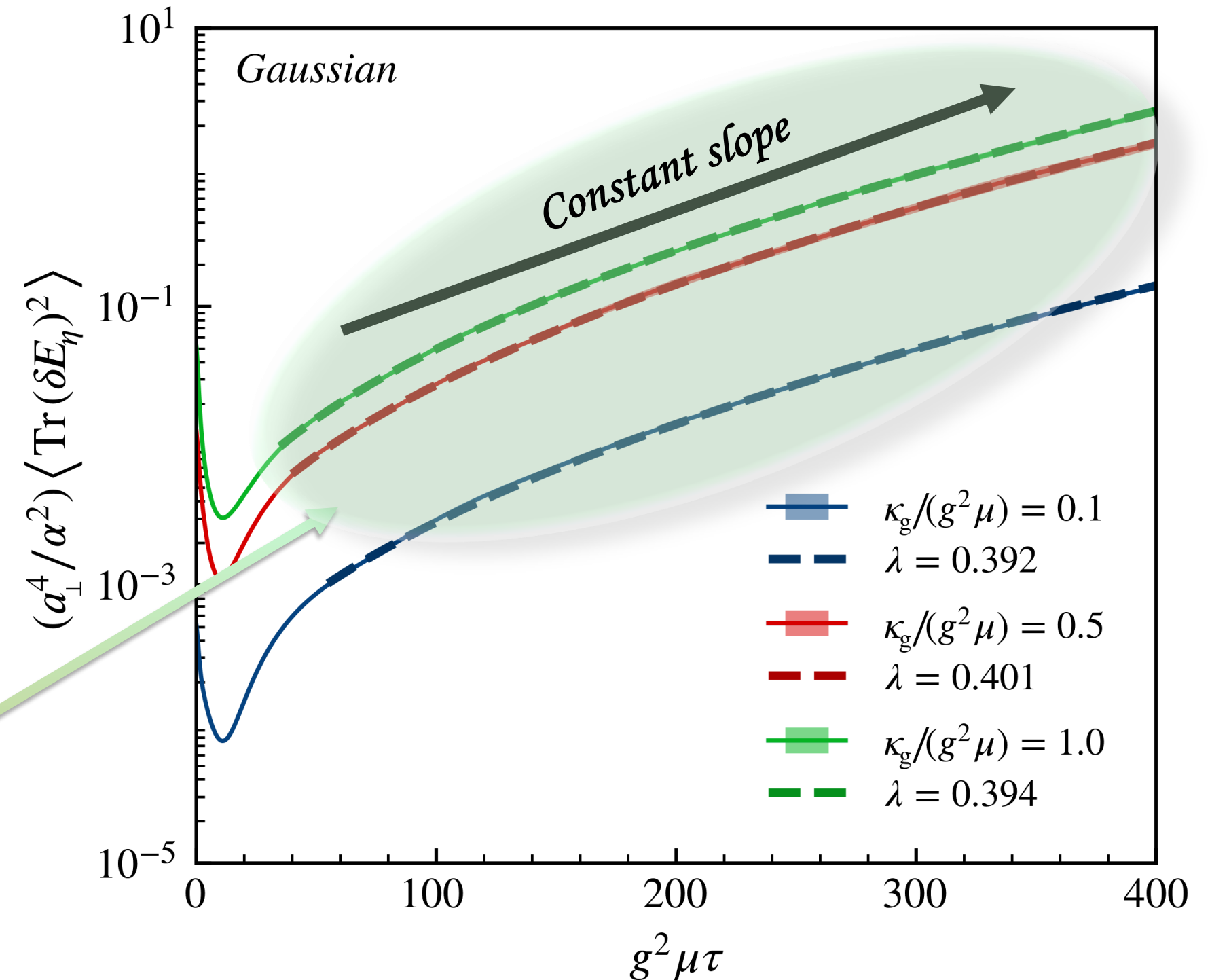
Gaussian-filtered perturbations

30

$$\left\langle \text{Tr} \left(\delta E_\eta(\tau) \right)^2 \right\rangle \propto e^{\lambda \sqrt{g^2 \mu \tau}}$$

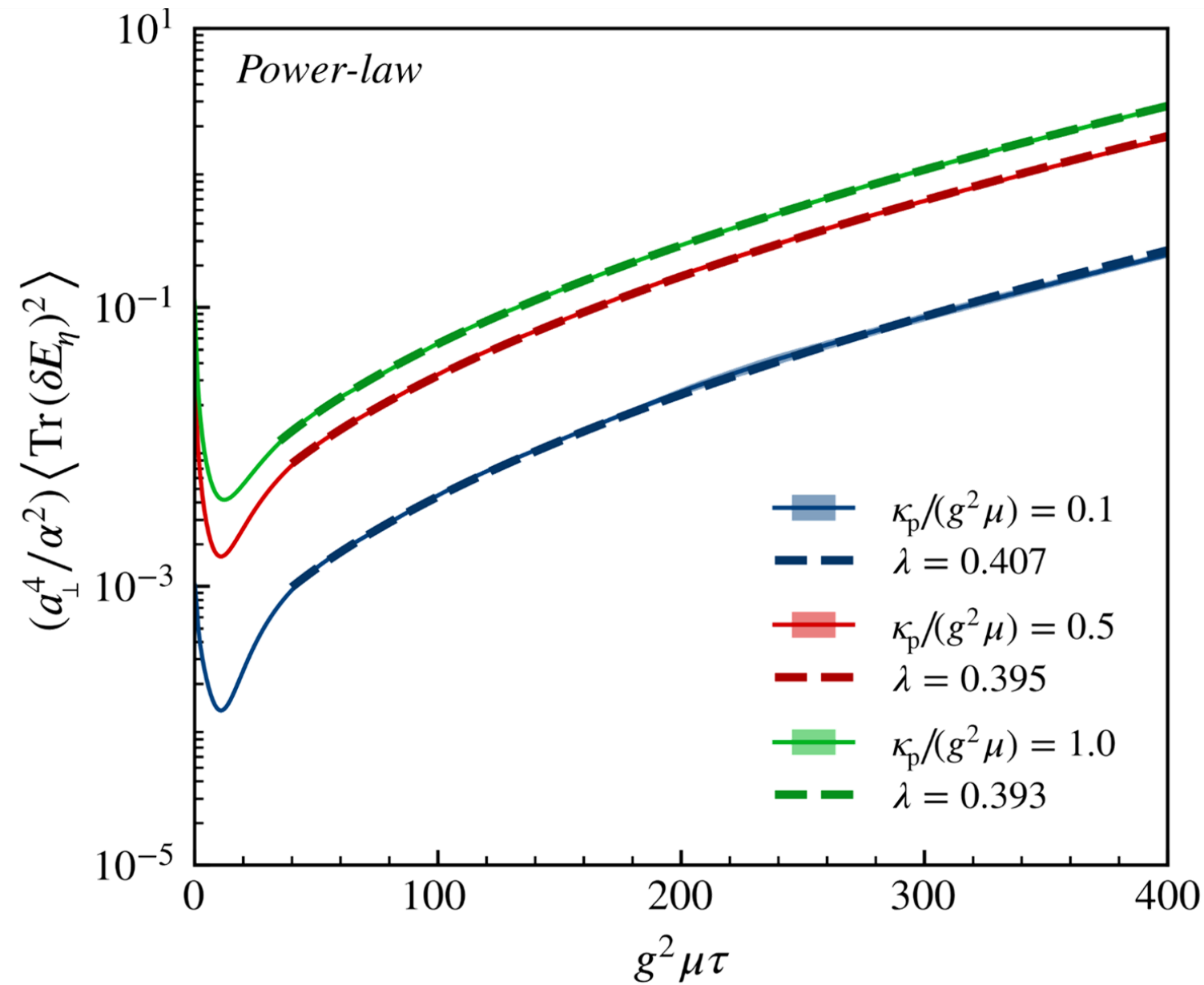
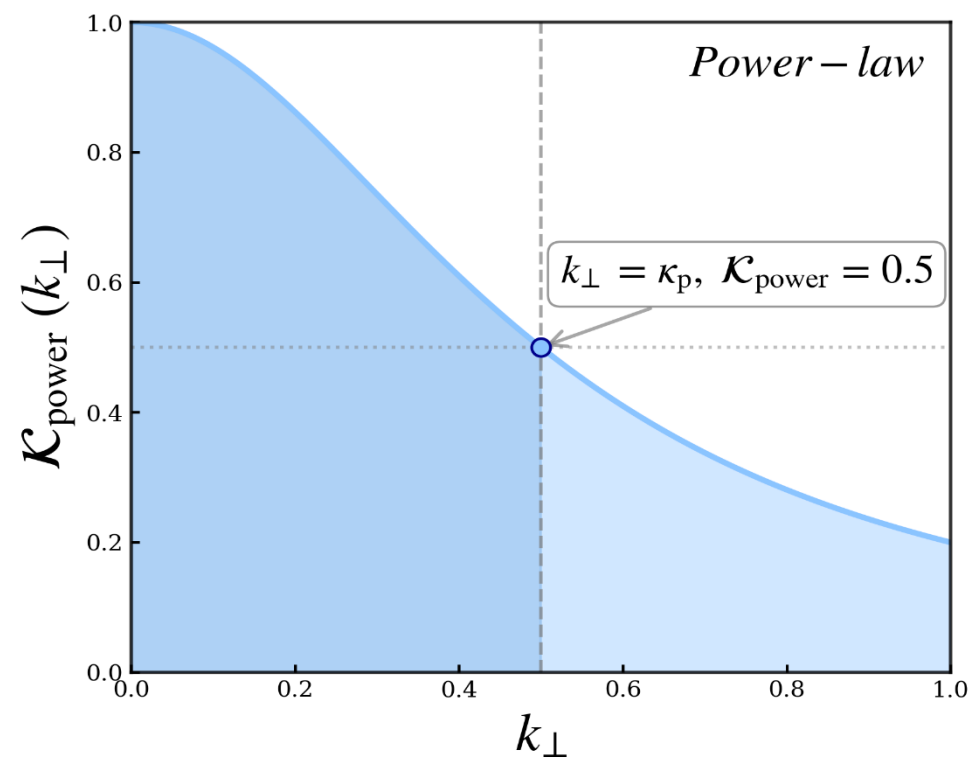
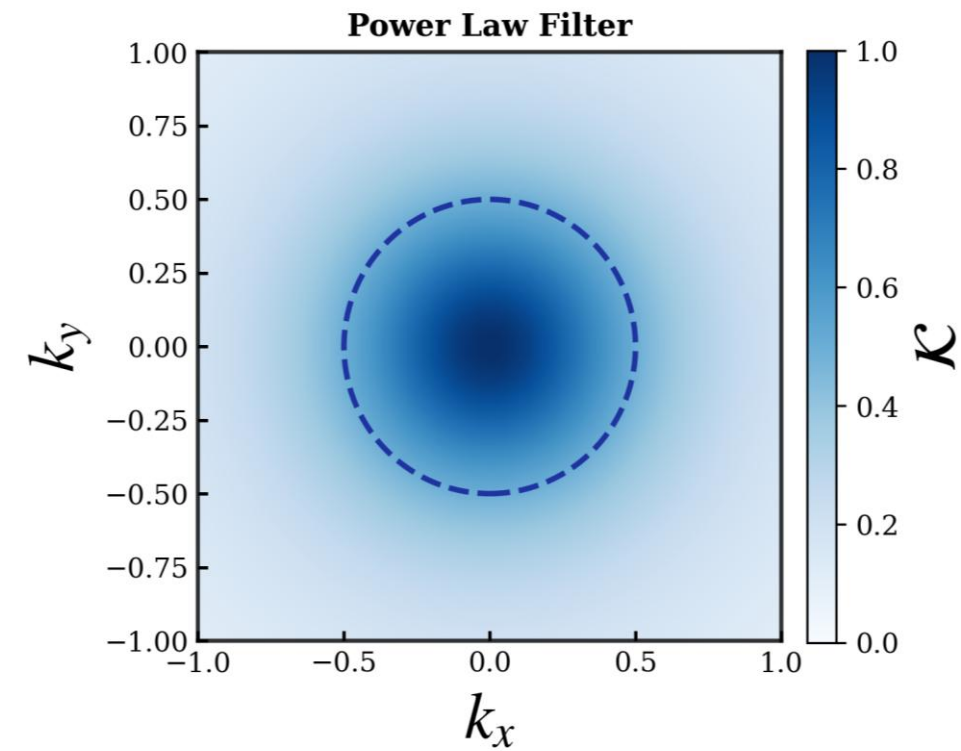
The slope is insensitive to the filter scale, κ_g .

Clean exponential growth:
 $\lambda \approx 0.39 - 0.40$



Power-law filtered perturbations

31



$$\mathcal{K}_{\text{power}}(k_{\perp}) = \frac{\kappa_p^2}{k_{\perp}^2 + \kappa_p^2}$$

- Milder suppression of high-momentum modes (algebraic fall-off)
- Preserves the ultra-violet tail

Power-law filtered perturbations

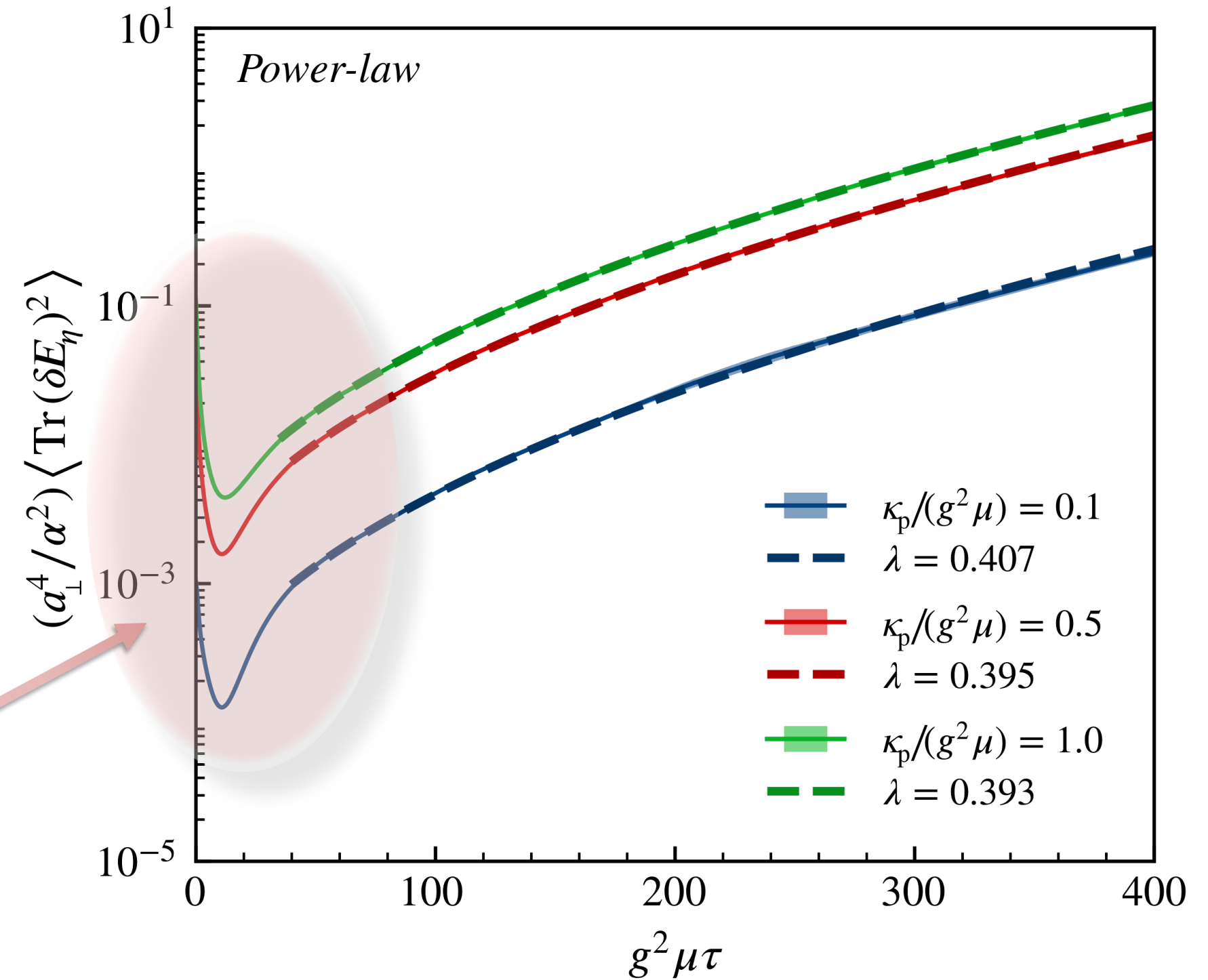
32

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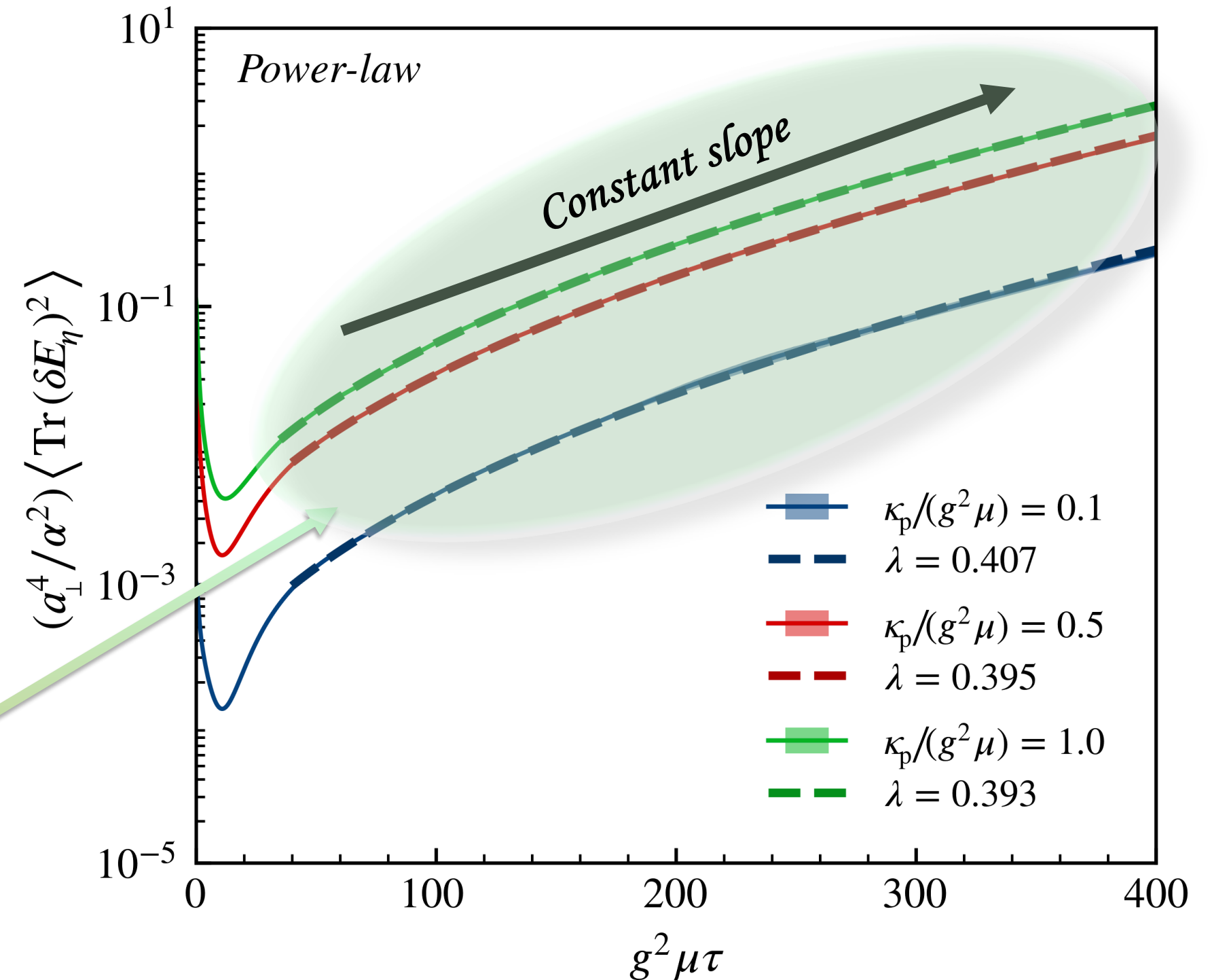
Power-law filtered perturbations

33

$$\left\langle \text{Tr} \left(\delta E_\eta(\tau) \right)^2 \right\rangle \propto e^{\lambda \sqrt{g^2 \mu \tau}}$$

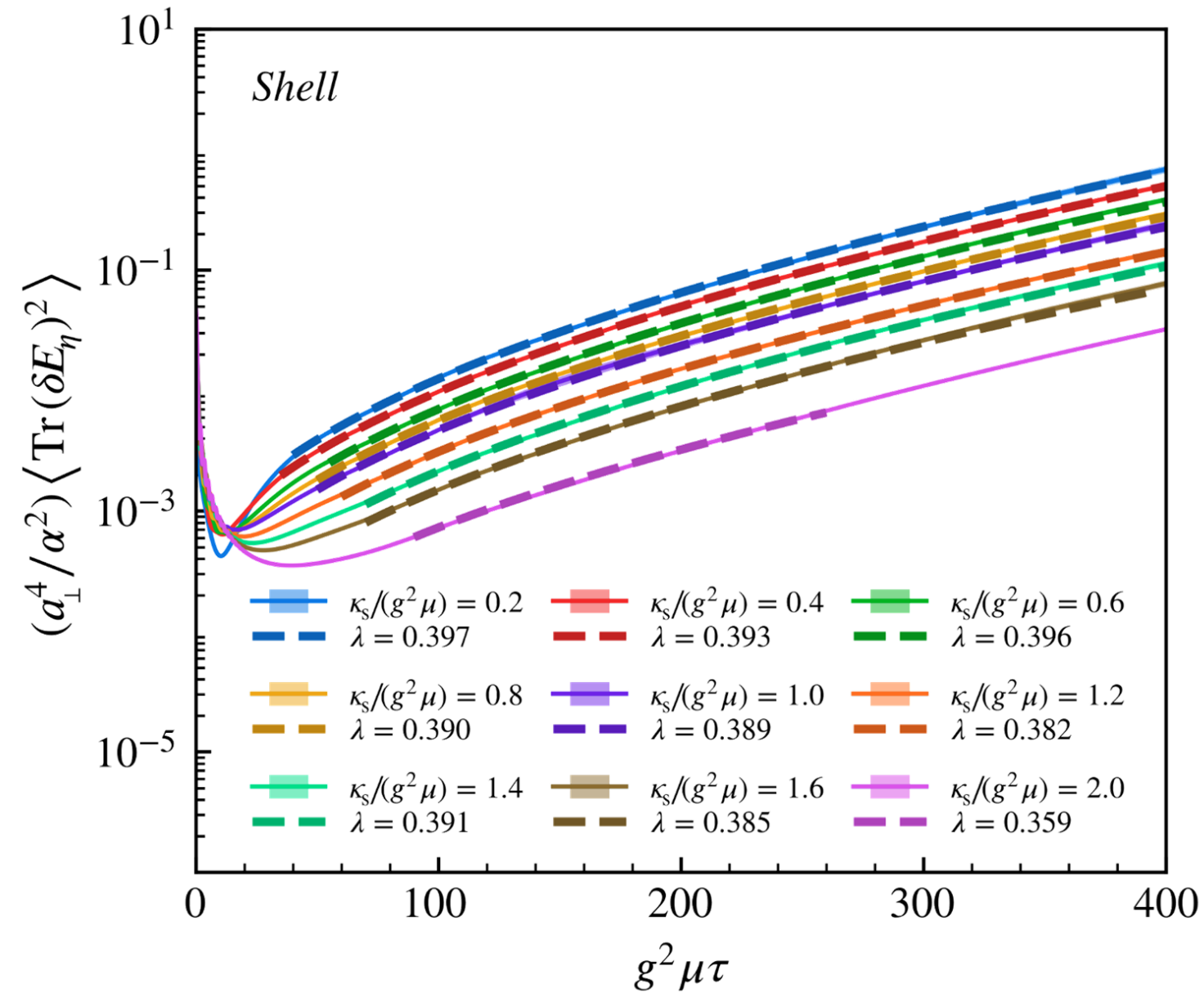
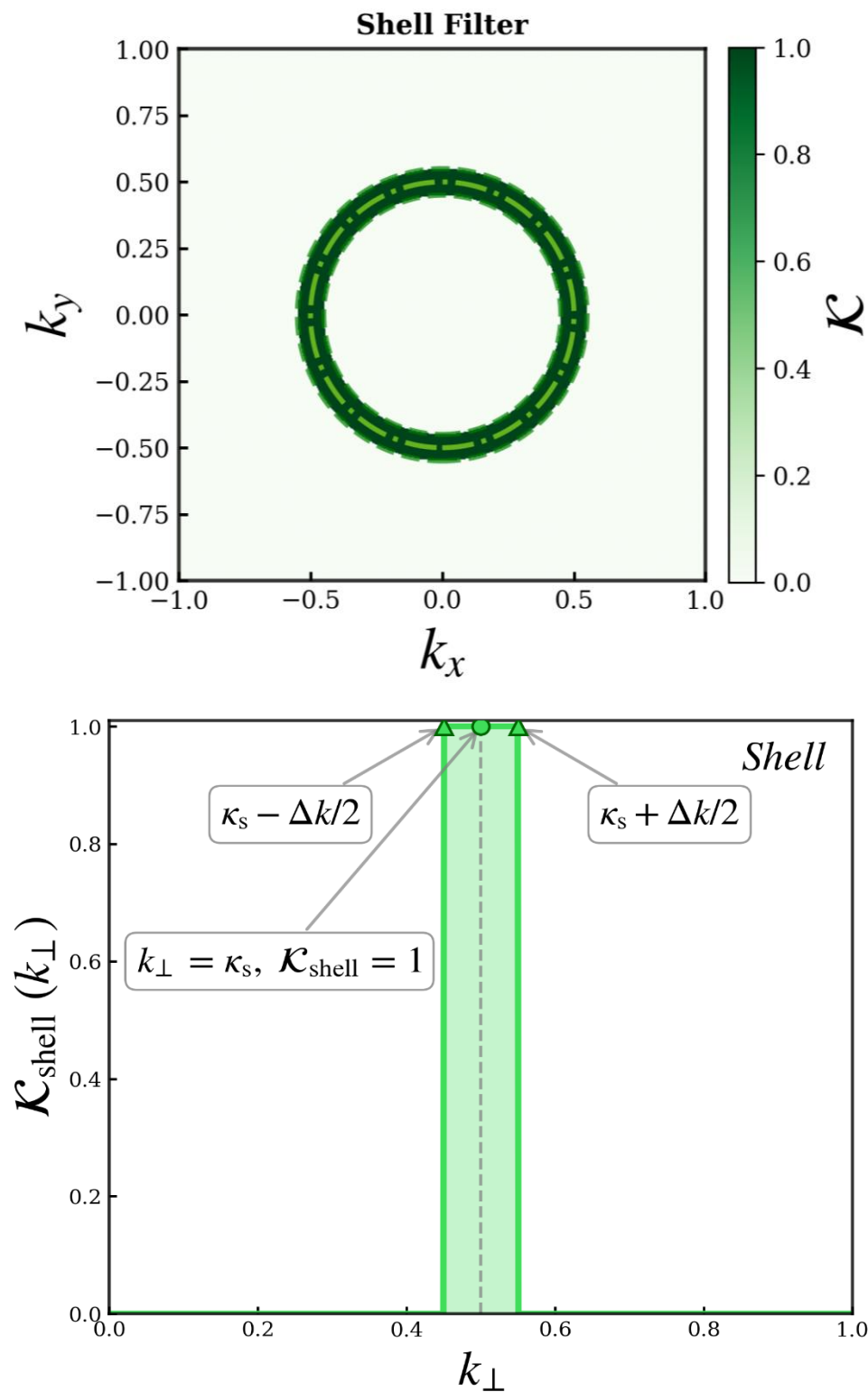
The slope is insensitive to the filter scale, κ_p .

Clean exponential growth:
 $\lambda \approx 0.39 - 0.40$



Shell-filtered perturbations

34



$$\mathcal{K}_{\text{shell}}(k_{\perp}) = \begin{cases} 1, & \kappa_s - \frac{\Delta k}{2} \leq k_{\perp} \leq \kappa_s + \frac{\Delta k}{2} \\ 0, & \text{otherwise} \end{cases}$$

- Isolates a narrow band of momenta
- Tests specific momentum modes independently

Shell-filtered perturbations

35

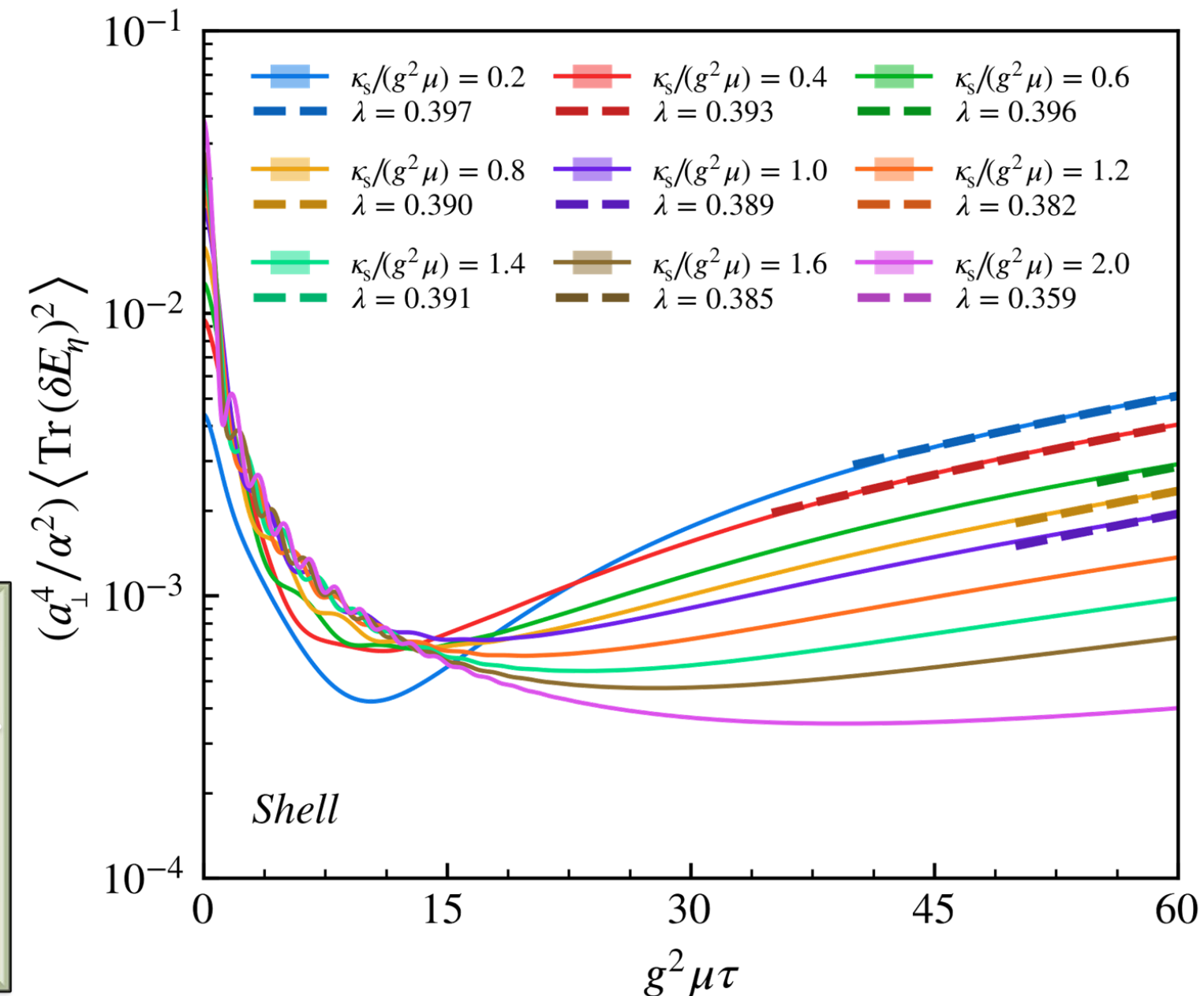
Clear vertical hierarchy
in the growth curve

Early time

Higher momentum shells (larger κ_s) exhibit larger initial amplitudes due to the larger phase-space area $A \sim 2\pi\kappa_s\Delta k$.

Late time

Higher- κ_s modes have smaller overlap with the unstable modes, leading to lower late time amplitudes.



Shell-filtered perturbations

36

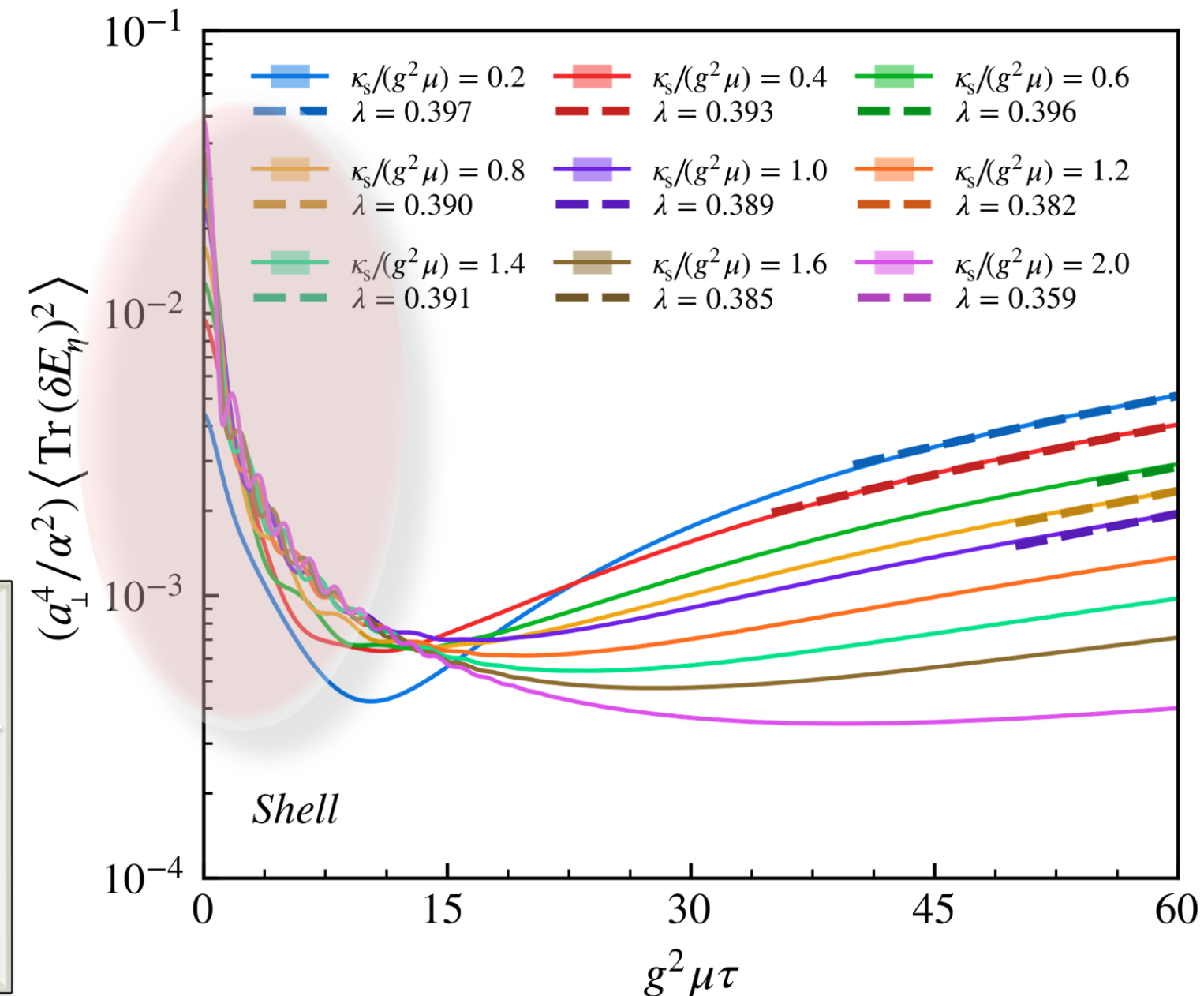
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Shell-filtered perturbations

37

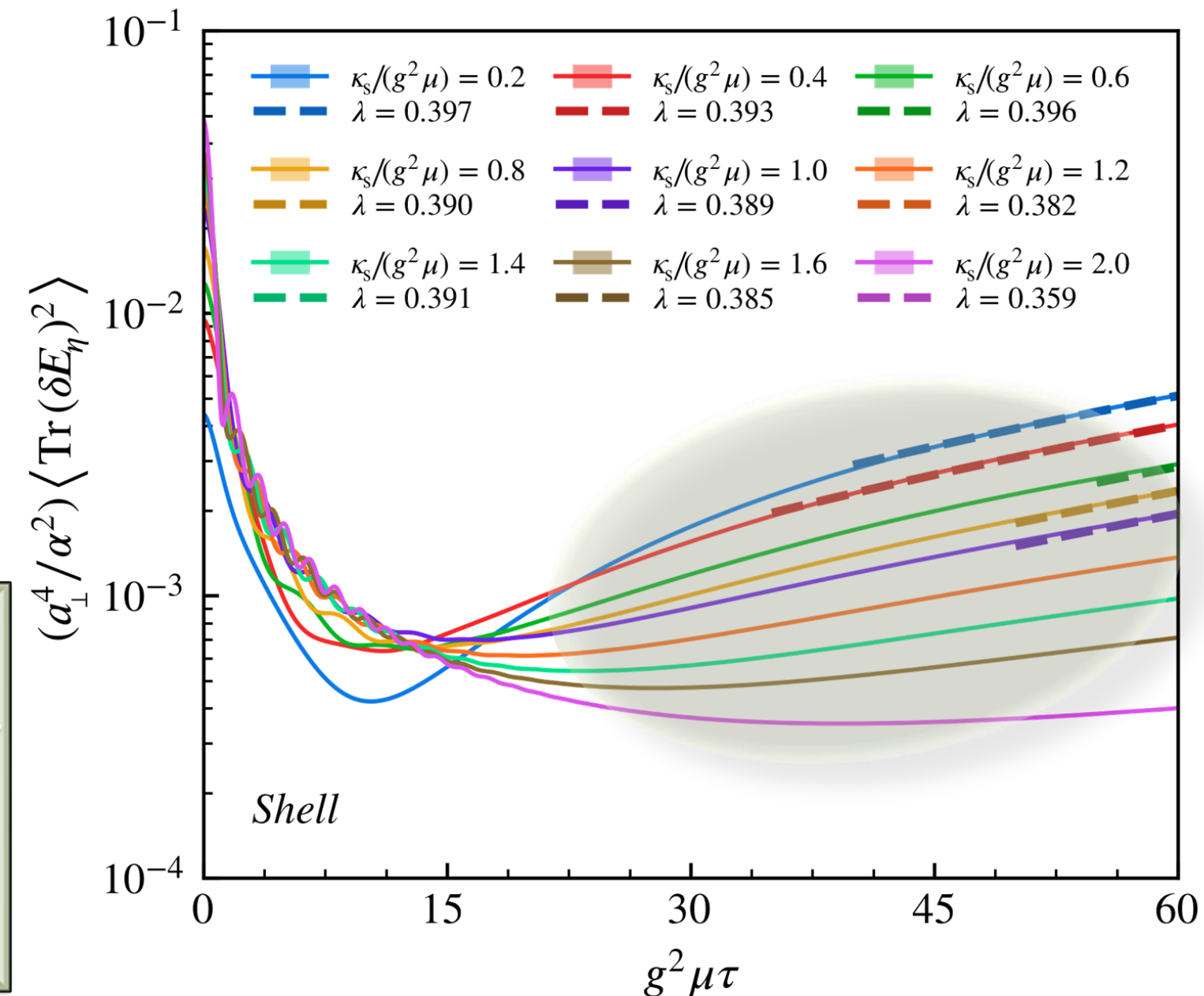
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Late time

Higher- κ_s modes have smaller overlap with the unstable modes, leading to lower late time amplitudes.



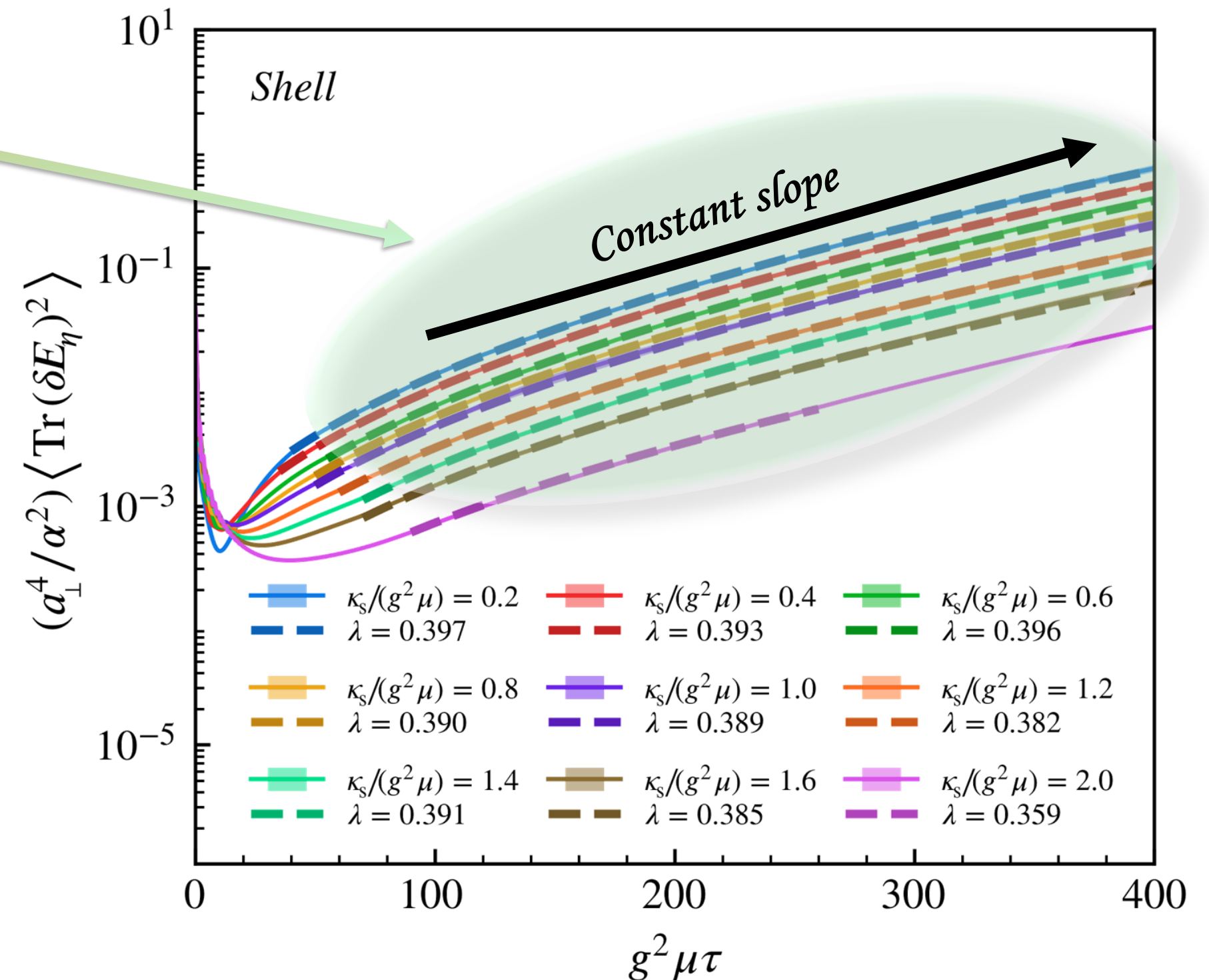
Shell-filtered perturbations

38

Despite the oscillatory start and shifting amplitudes, every single shell eventually locks into the same slope:

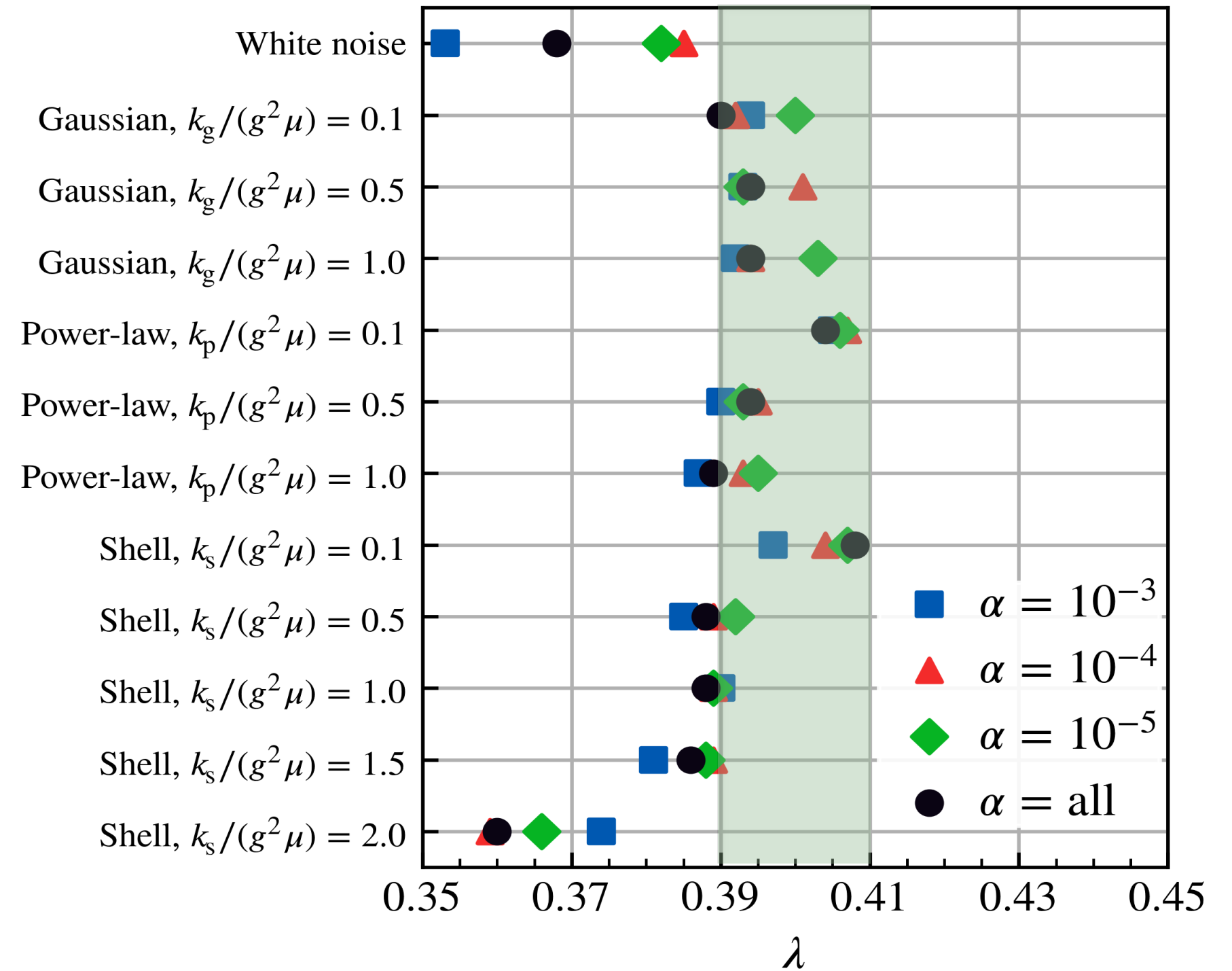
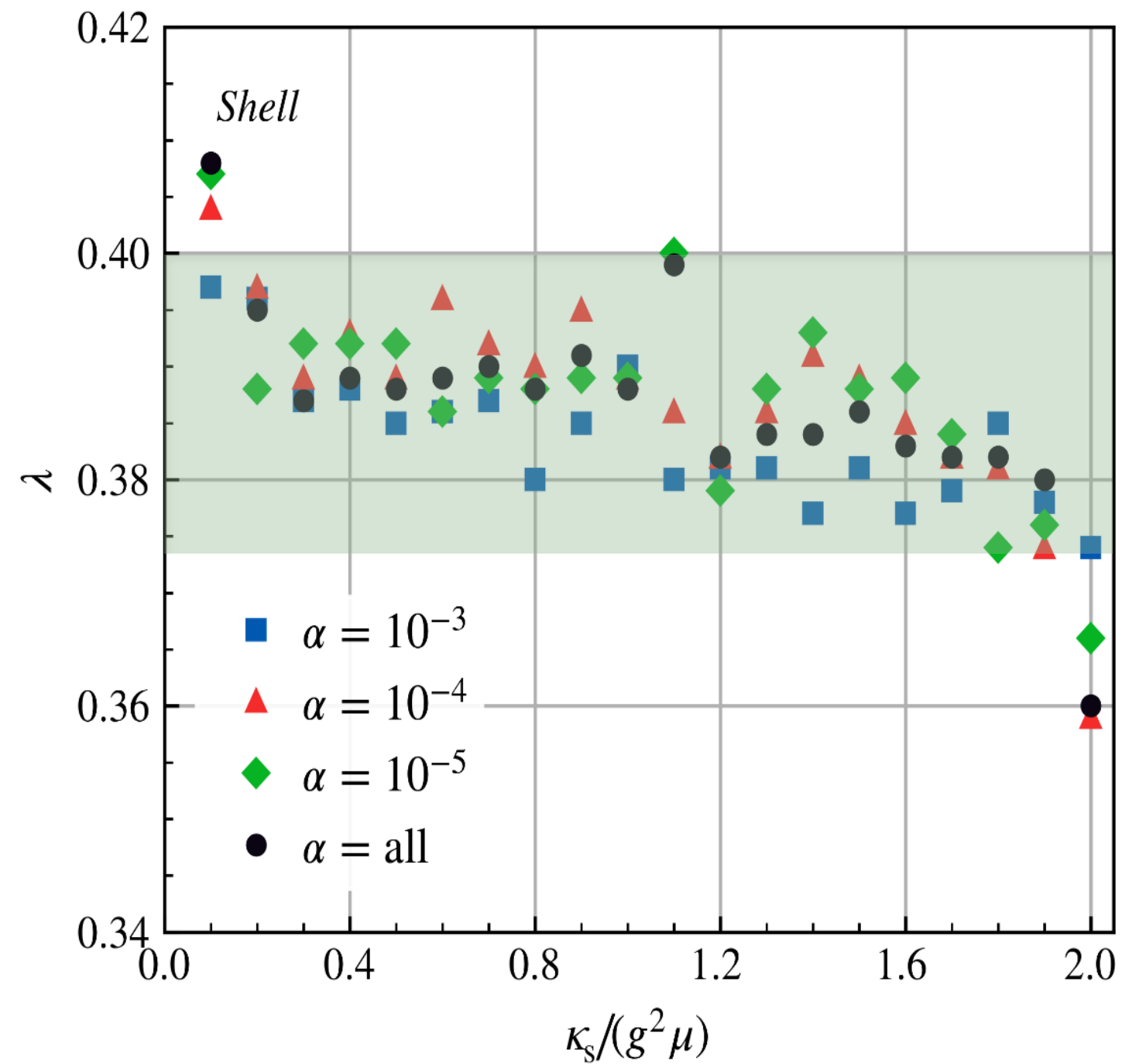
$$\lambda \approx 0.38 - 0.40$$

- The unstable mode has a nontrivial momentum-space structure, rather than being associated with a single momentum scale.
- Need a Fourier-space analysis of the perturbation.



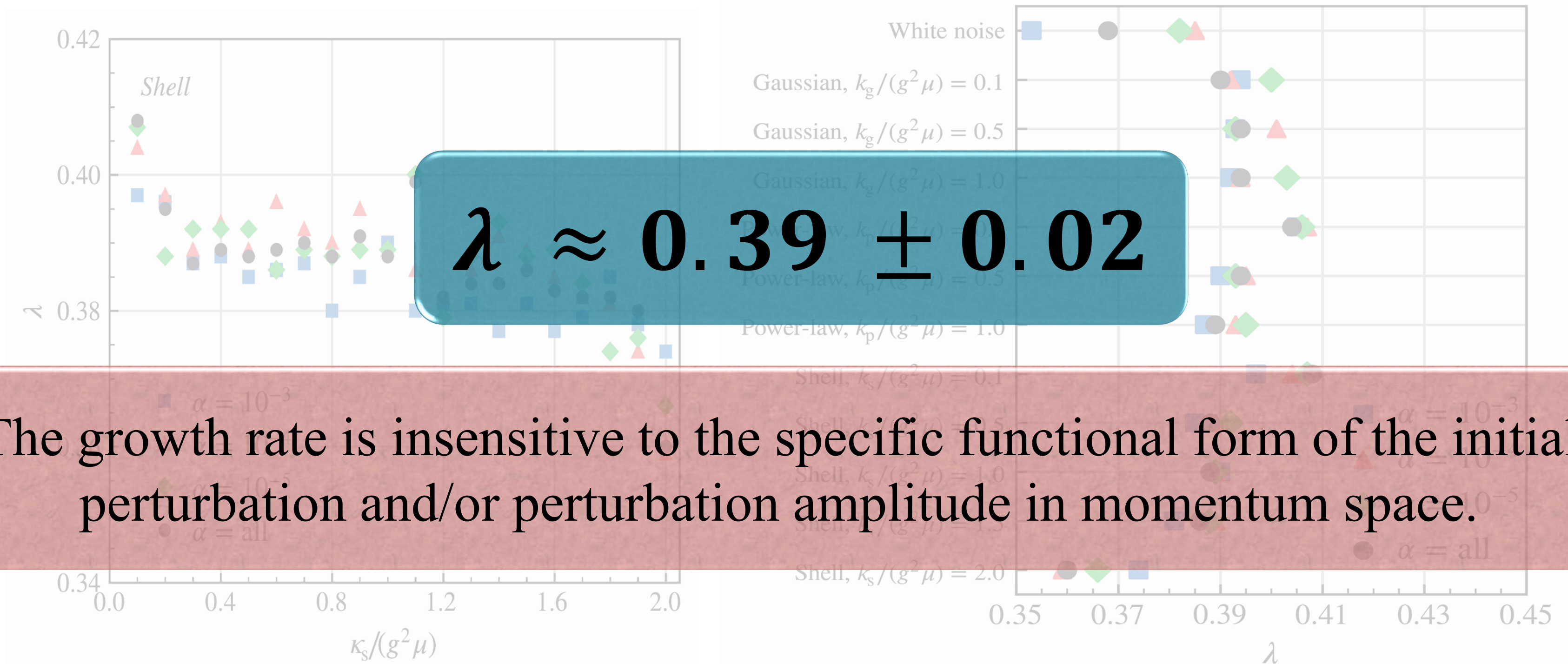
Robustness of the Lyapunov exponent

39



Robustness of the Lyapunov exponent

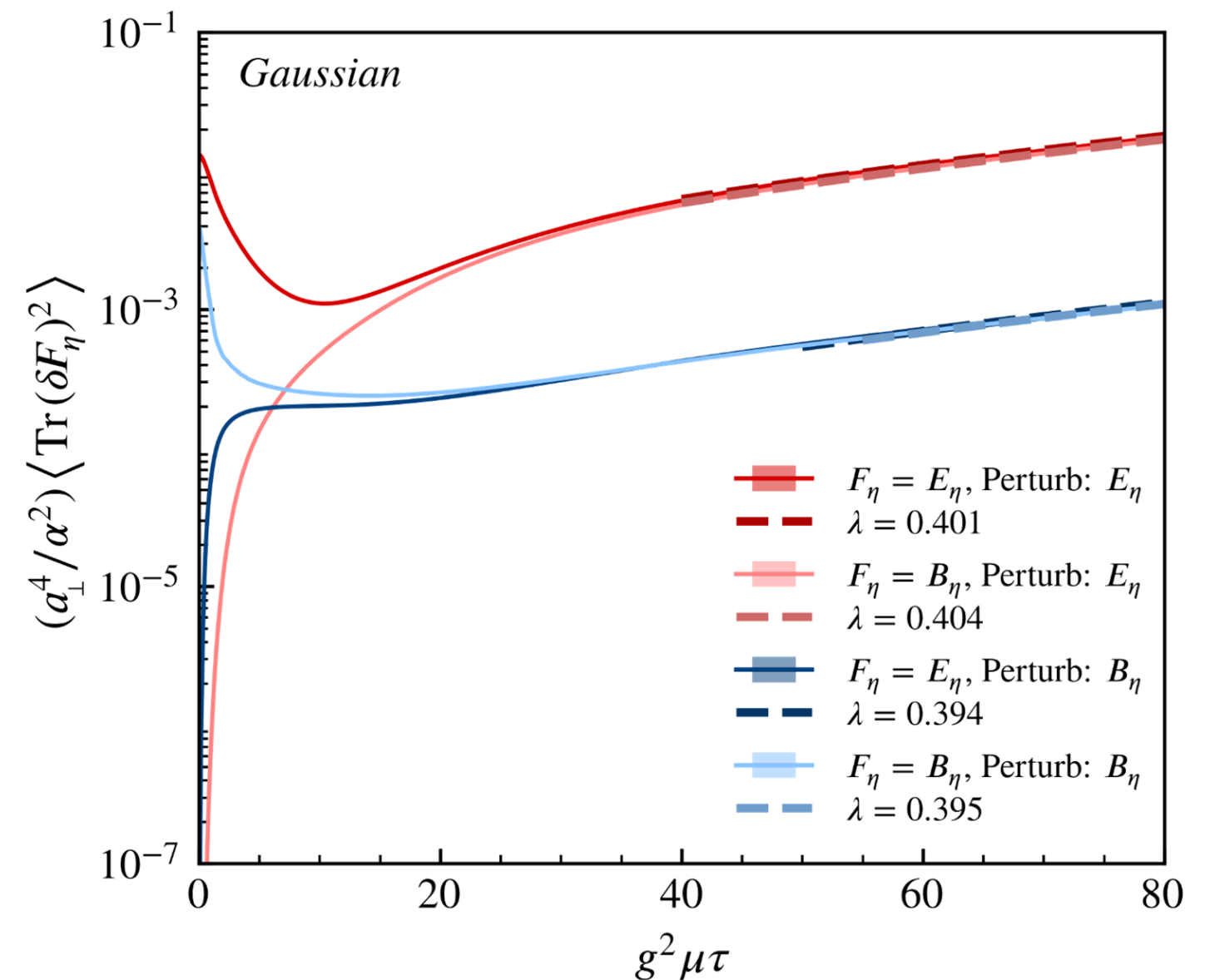
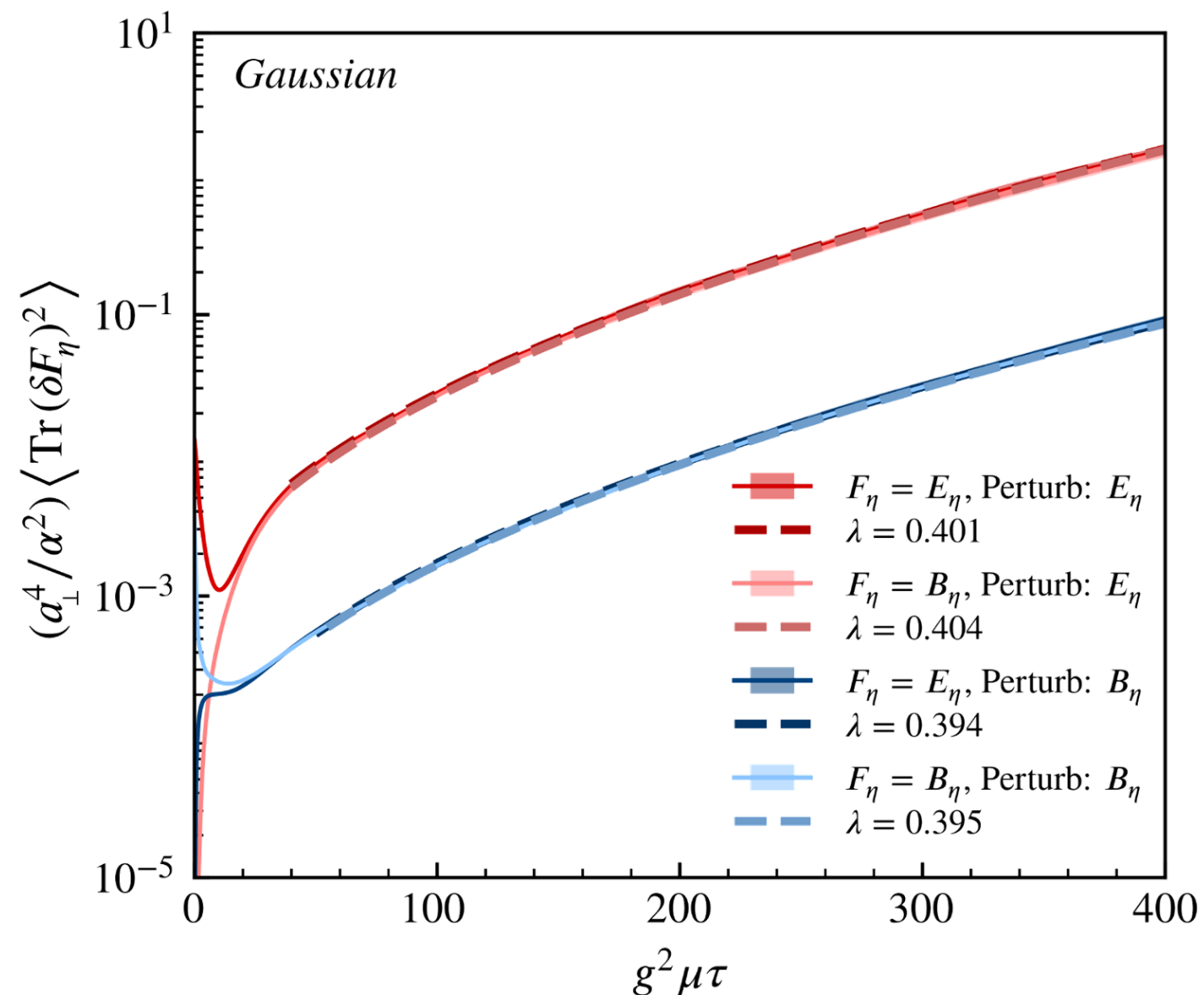
40



The growth rate is insensitive to the specific functional form of the initial perturbation and/or perturbation amplitude in momentum space.

Electric vs magnetic field perturbations

41



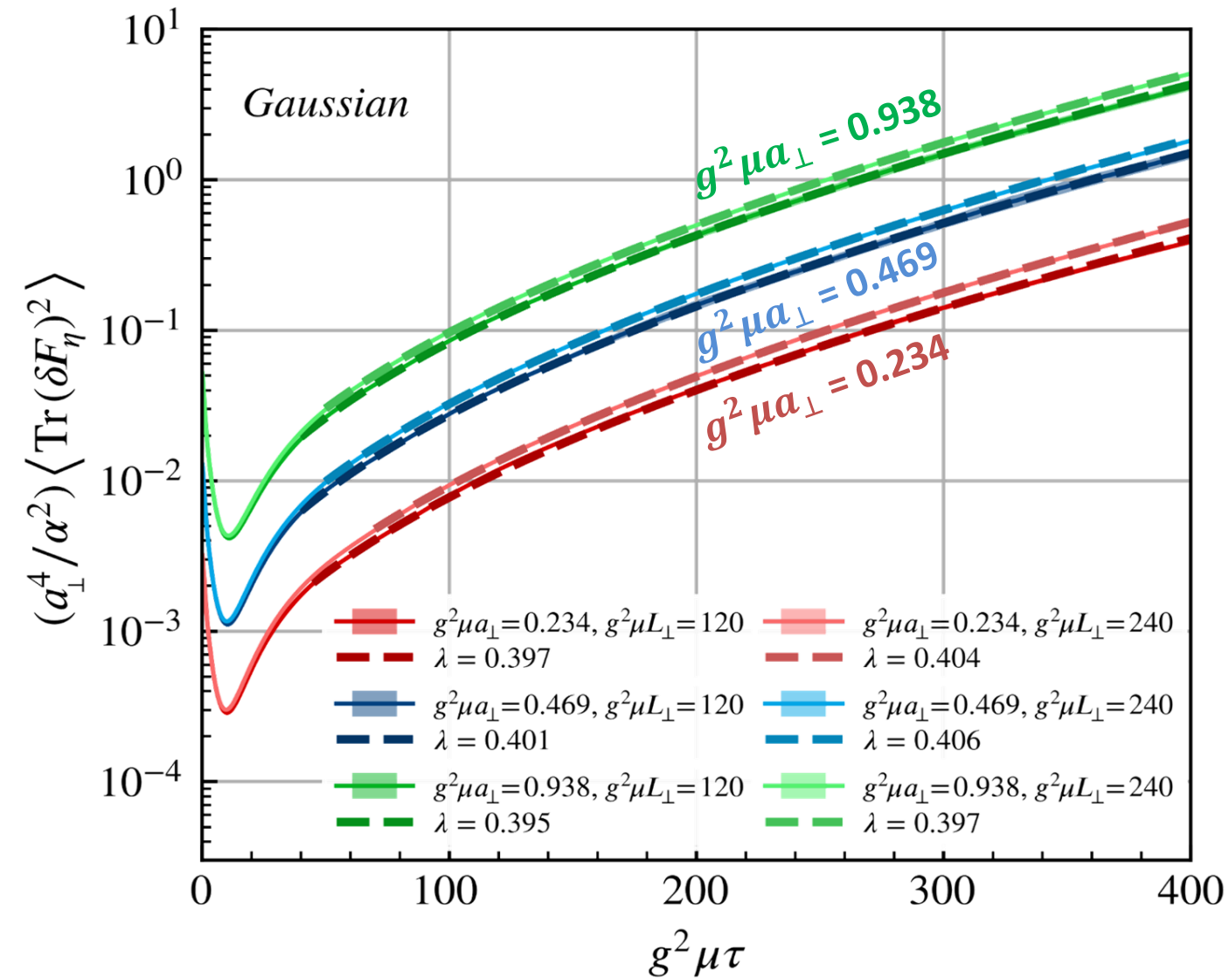
- ✓ Initialize one, measure the other
- ✓ Both couple to same unstable mode

Lyapunov exponent is insensitive to:

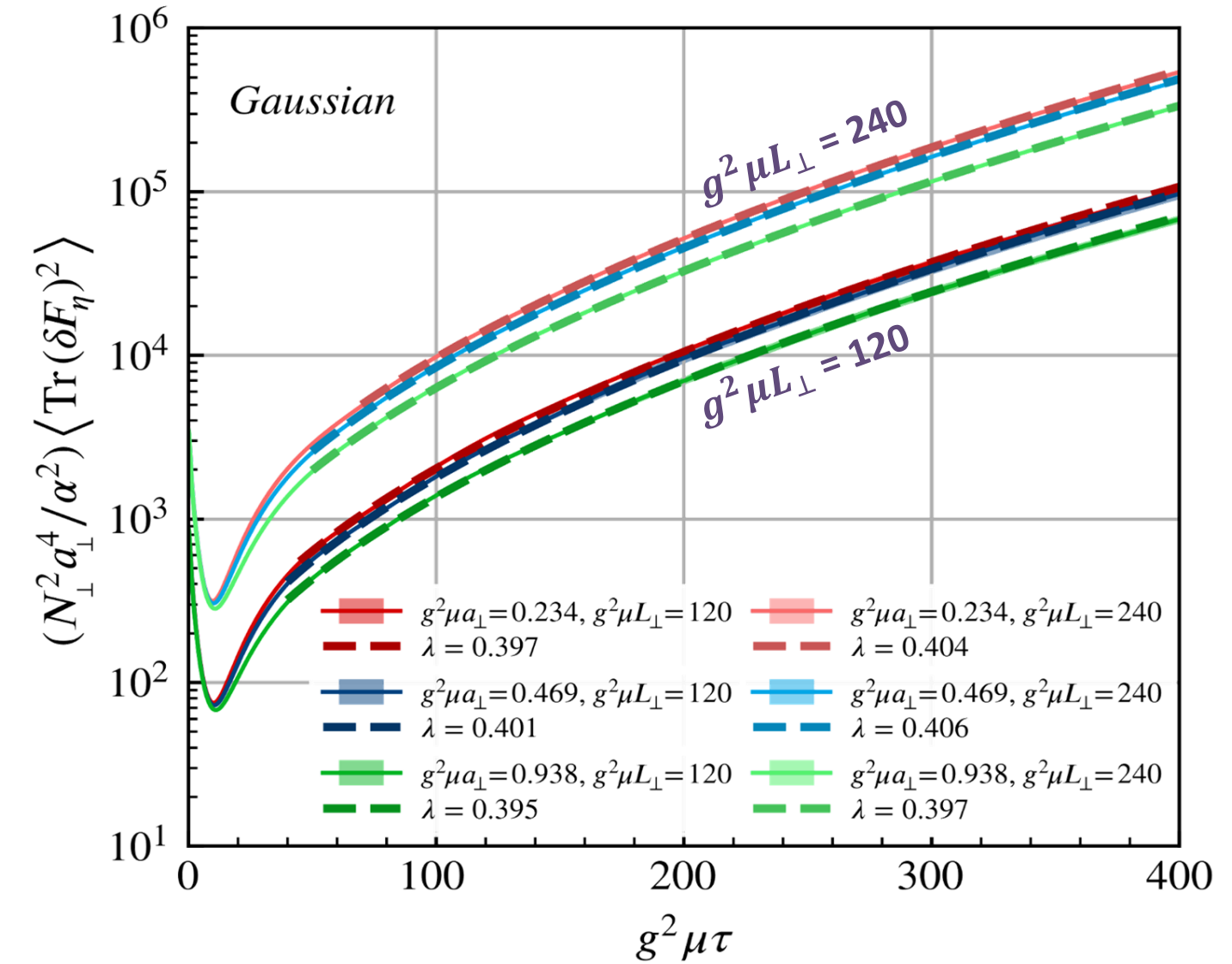
- Component of field perturbed
- Component of field measured

Lattice parameters dependence ??

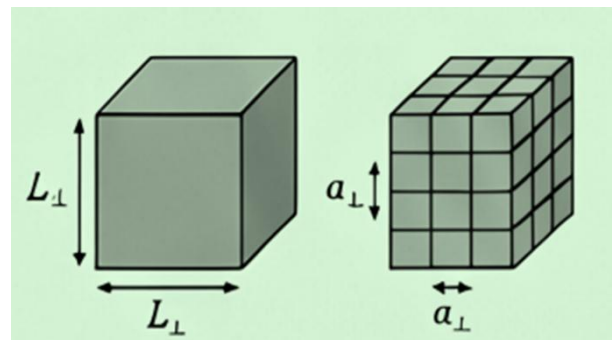
42



Dependence on lattice volume



Dependence on lattice spacing



The Lyapunov exponent ($\lambda \approx 0.38 - 0.40$) of the Glasma is independent of the numerical discretization details.

Conclusions

43

$$\left(\delta F_\eta(\tau)\right)^2 \sim \exp\left[(0.39 \pm 0.02)\sqrt{g^2\mu\tau}\right]$$

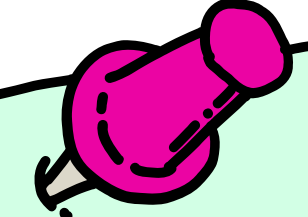
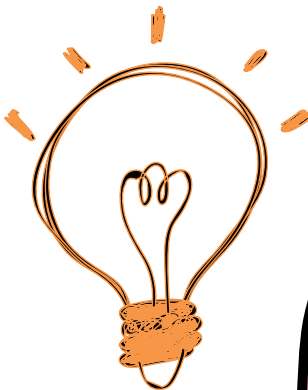
leads to
growth rate, $\Upsilon(\tau)$

$$\frac{d}{d\tau}\delta F_\eta(\tau) = \Upsilon(\tau)F_\eta(\tau)$$

where

$$\Upsilon(\tau) = (0.1 \pm 0.005)\sqrt{\frac{g^2\mu}{\tau}}$$

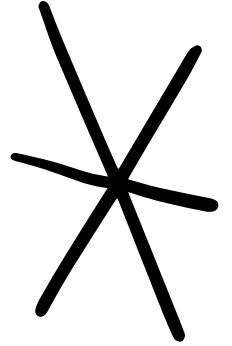
- ❑ We discovered an **unstable leading mode** in the non-linear Glasma fields.
- ❑ Chaos in the Glasma is a scale-invariant, collective phenomenon.
- ❑ The leading mode couples to all momenta scales in the background fields efficiently.
- ❑ The Lyapunov exponent sets the fundamental timescale for entropy production and transition from the classical fields to a thermalized plasma.



Summary

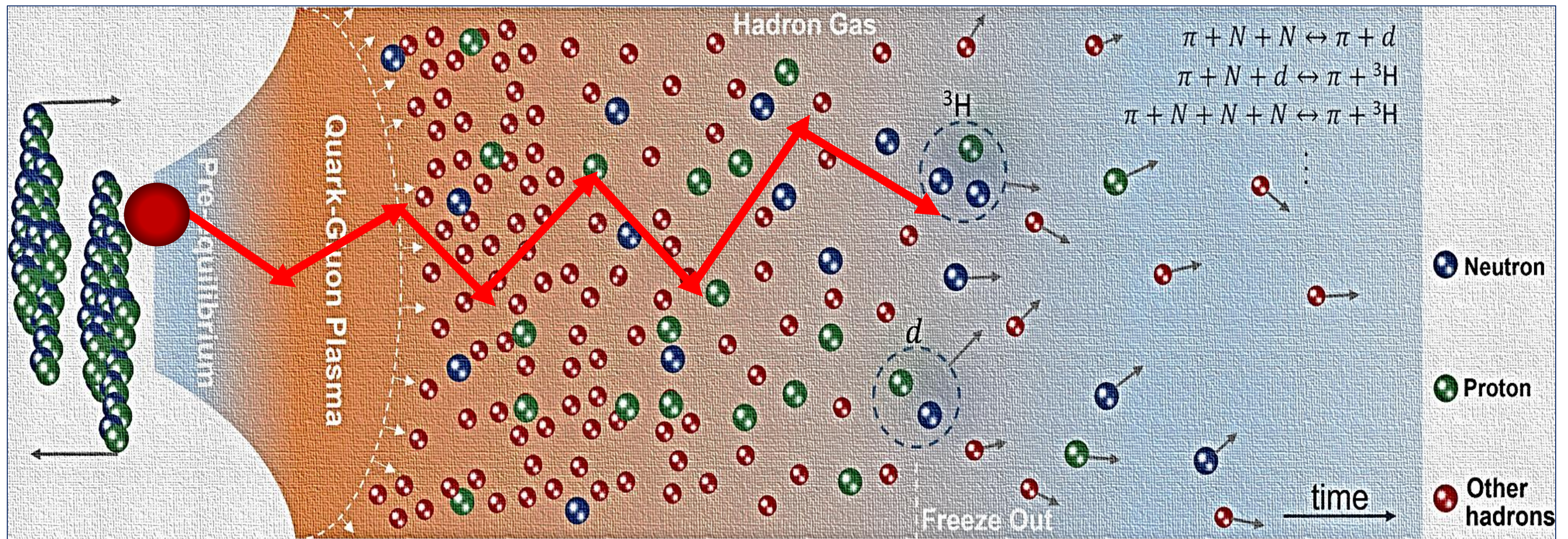
$$\left\langle \text{Tr} \left(\delta F_\eta(\tau) \right)^2 \right\rangle \propto e^{\lambda \sqrt{g^2 \mu \tau}}$$

In linear regime of the perturbation, the Lyapunov exponent for the Glasma is $\lambda \sim 0.4$.



Work in progress: Heavy quarks' trajectories

45



K.J. Sun, R. Wang, C.M. Ko, Y.G. Ma, C. Shen, *Nature Communications*, 15 (2024) 1, 1074 [↗](#)

HQs can probe the entire evolution of the medium, from the **early stage** up to hadronization.

Work in progress: Heavy quarks' trajectories

Equations of motion of heavy quarks (Wong Equations)

$$\frac{dx^i}{dt} = \frac{p^i}{E}, \quad E = \sqrt{\mathbf{p}^2 + m^2}$$

$$\frac{dp^i}{dt} = gQ_a F_a^{i\mu} \frac{p_\mu}{E}$$

$$\frac{dQ_a}{dt} = g f_{abc} A_b^\mu \frac{p_\mu}{E} Q_c$$

- Moving beyond field fluctuations to track actual particles (heavy quarks)
- Solving Wong equations for heavy quarks moving in the background CYM glasma system
- Goal: Measure chaotic signatures directly from the diffusion of heavy quarks

Seeding the chaos in HQs

47

$$x_0^{i'} = x_0^i + \chi^i a_\perp$$

$$\begin{aligned}\langle \chi^i(\mathbf{x}_\perp) \rangle &= 0 \\ \langle \chi^i(\mathbf{x}_\perp) \chi^j(\mathbf{y}_\perp) \rangle &= \alpha_x^2 \delta^{ij} \delta_{\mathbf{x}_\perp, \mathbf{y}_\perp}\end{aligned}$$

$$p_0^{i'} = p_0^i + \frac{\xi^i}{a_\perp}$$

$$\begin{aligned}\langle \xi^i(\mathbf{x}_\perp) \rangle &= 0 \\ \langle \xi^i(\mathbf{x}_\perp) \xi^j(\mathbf{y}_\perp) \rangle &= \alpha_p^2 \delta^{ij} \delta_{\mathbf{x}_\perp, \mathbf{y}_\perp}\end{aligned}$$

$$Q_0^{a'} = (Q_0^a + \zeta^a) \frac{|Q_0^a|}{|(Q_0^a + \zeta^a)|}$$

$$\begin{aligned}\langle \zeta^a(\mathbf{x}_\perp) \rangle &= 0 \\ \langle \zeta^a(\mathbf{x}_\perp) \zeta^b(\mathbf{y}_\perp) \rangle &= \alpha_Q^2 \delta^{ab} \delta_{\mathbf{x}_\perp, \mathbf{y}_\perp}\end{aligned}$$

$\alpha_x, \alpha_p, \alpha_Q$:

Perturbation amplitudes

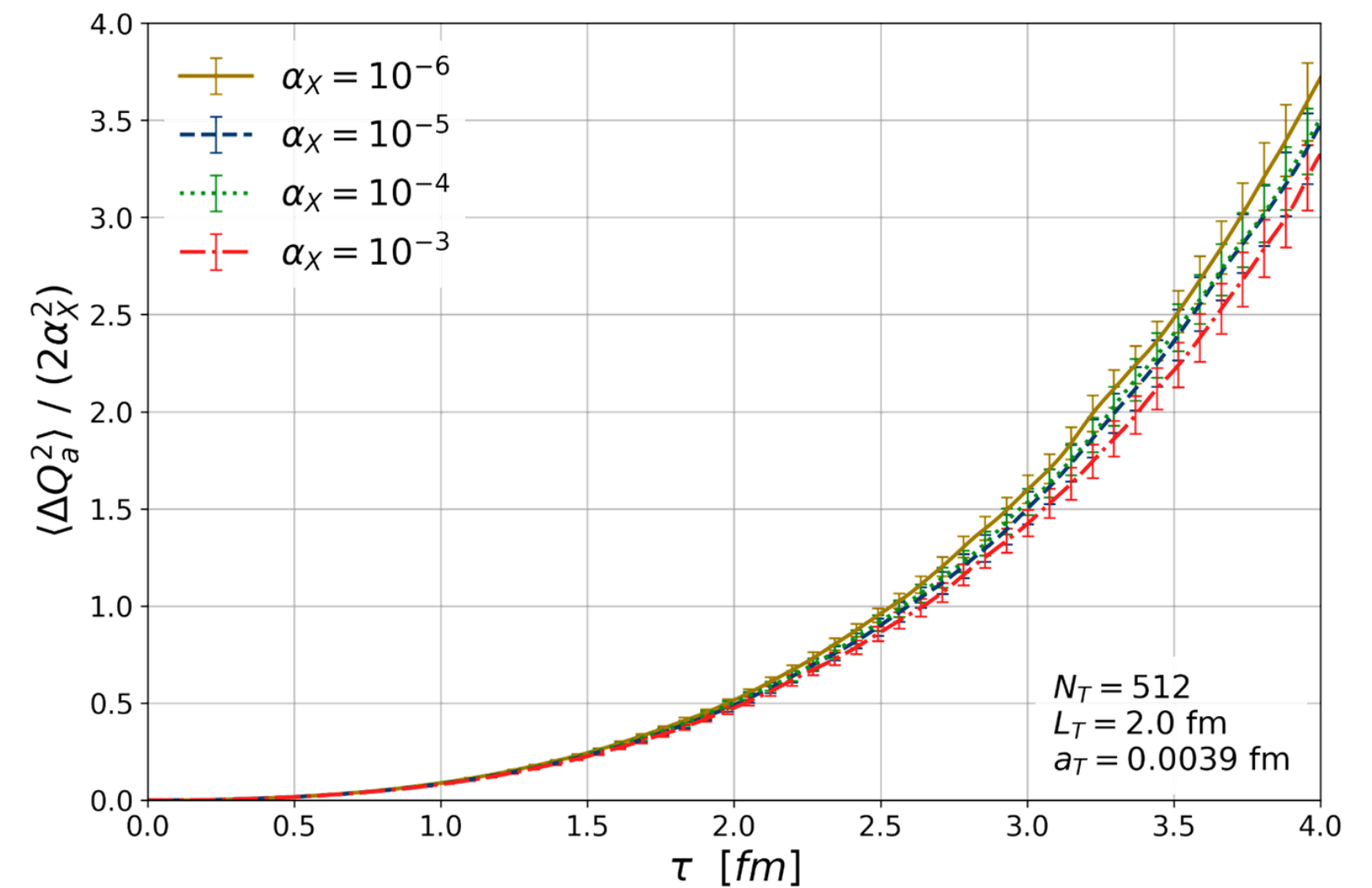
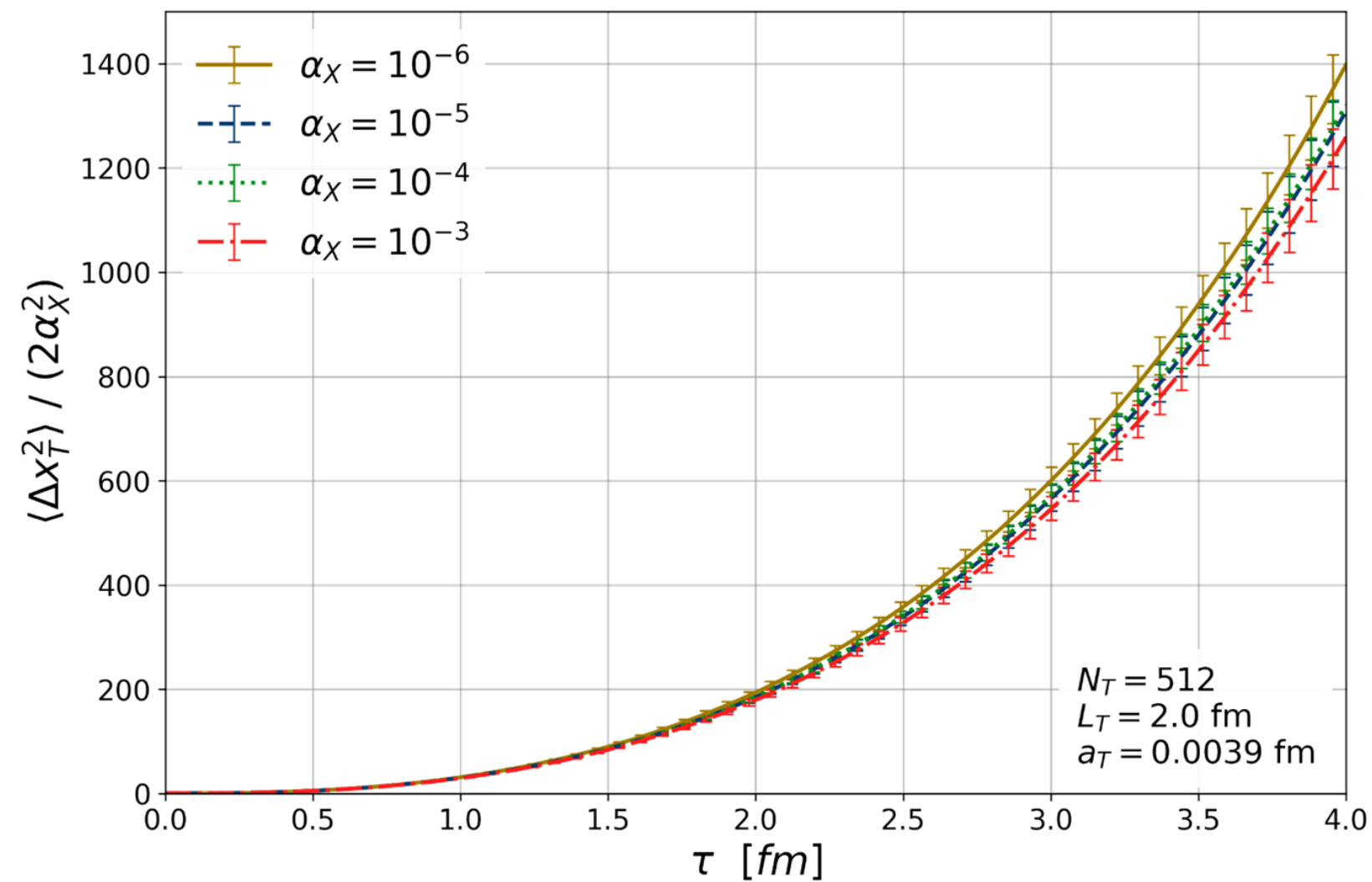
Perturbation in position

Preliminary results

48

$$x_0^{i'} = x_0^i + \chi^i a_\perp$$

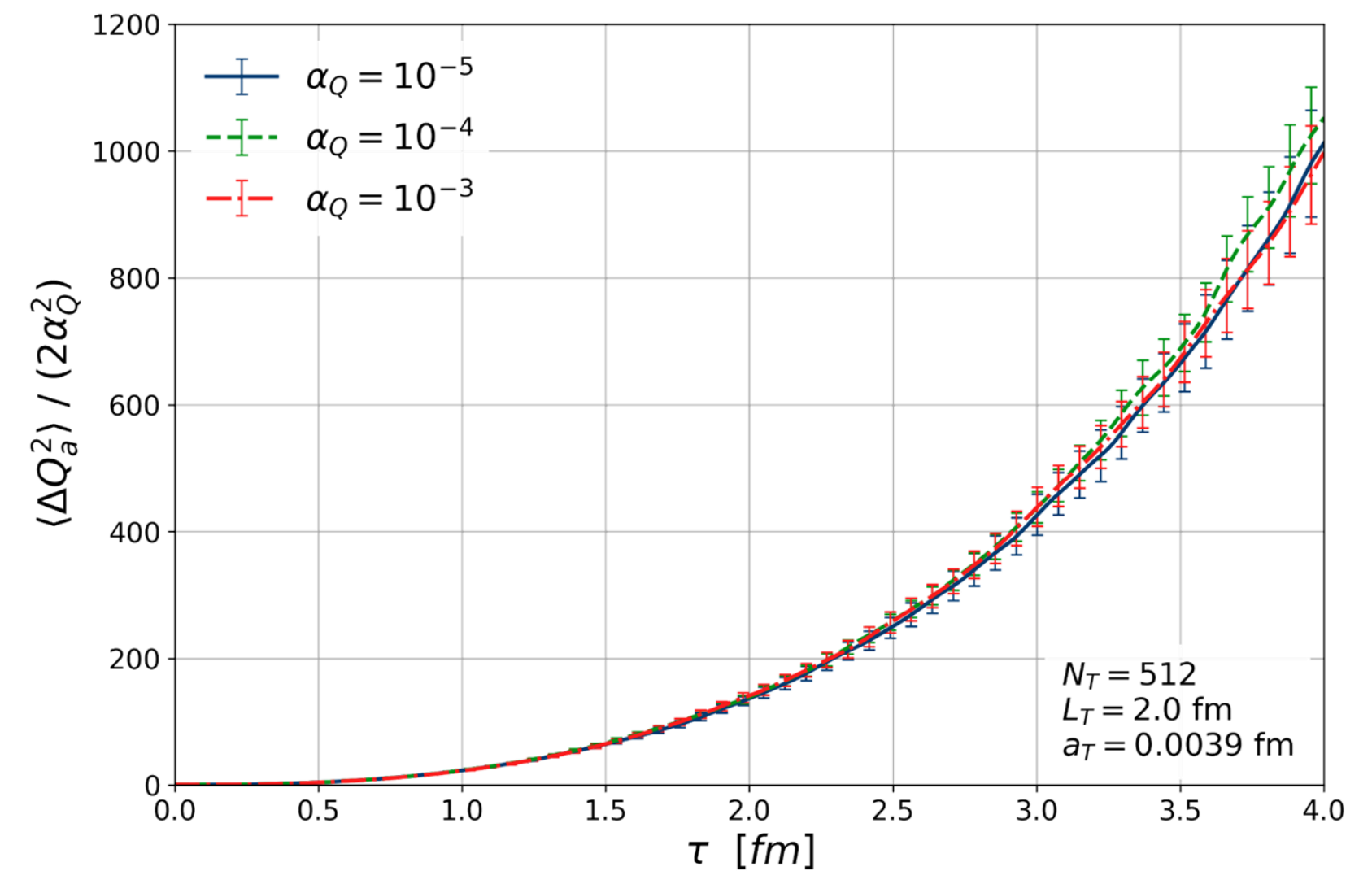
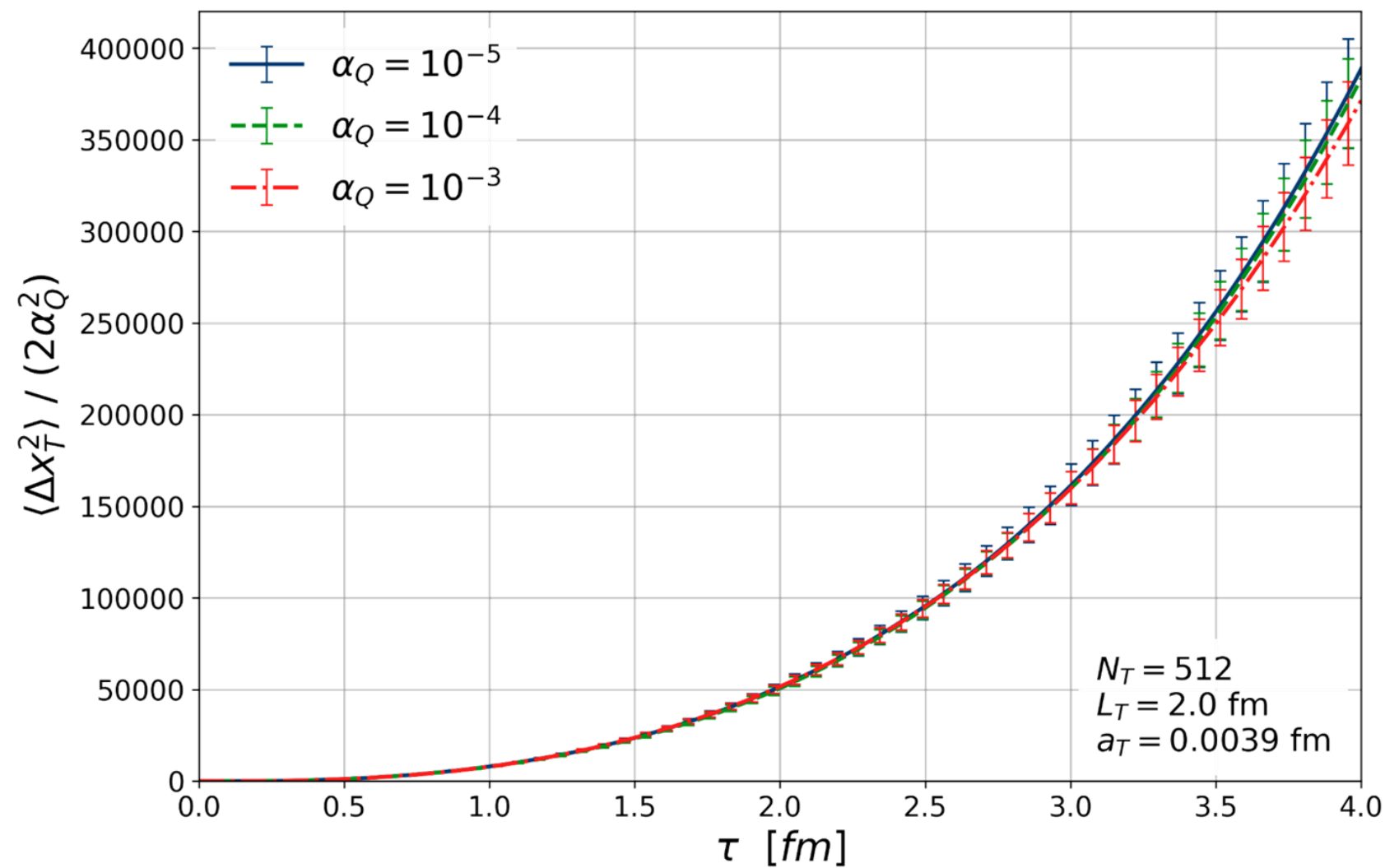
$$\begin{aligned}\langle \chi^i(x_\perp) \rangle &= 0 \\ \langle \chi^i(x_\perp) \chi^j(y_\perp) \rangle &= \alpha_x^2 \delta^{ij} \delta_{x_\perp, y_\perp}\end{aligned}$$



Perturbation in color charge Preliminary results 49

$$Q_0^{a'} = (Q_0^a + \zeta^a) \frac{|Q_0^a|}{|(Q_0^a + \zeta^a)|}$$

$$\begin{aligned}\langle \zeta^a(\mathbf{x}_\perp) \rangle &= 0 \\ \langle \zeta^a(\mathbf{x}_\perp) \zeta^b(\mathbf{y}_\perp) \rangle &= \alpha_Q^2 \delta^{ab} \delta_{\mathbf{x}_\perp, \mathbf{y}_\perp}\end{aligned}$$



The road ahead for Glasma chaos

50

Analyzing leading mode

Extract momentum dependence of the leading mode in Coulomb gauge.

Full spectrum of modes

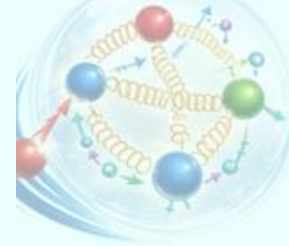
Extract the complete matrix of Lyapunov exponents, rather than just the leading mode.

Broken boost-invariance

Study of exponentially growing modes related to the Weibel instability.



Thank You.



Physics Opportunities

at an Electron-Ion Collider 2027

15-19 February, Bambolim Beach Resort, Goa, India



POETIC 2027



POETIC XII, the twelfth international conference on “Physics Opportunities at an Electron-Ion Collider”, will be held in Goa, India, continuing the established POETIC conference series. The meeting takes place at an important stage in the development of the future EIC program, marked by significant advances in detector technologies, software and computing, machine learning applications, and growing international participation.

Key topics include the structure of nucleons and nuclei in Quantum Chromodynamics, spin physics, jet physics, small-x dynamics, detector technologies, and AI-driven data analysis.

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Appendix

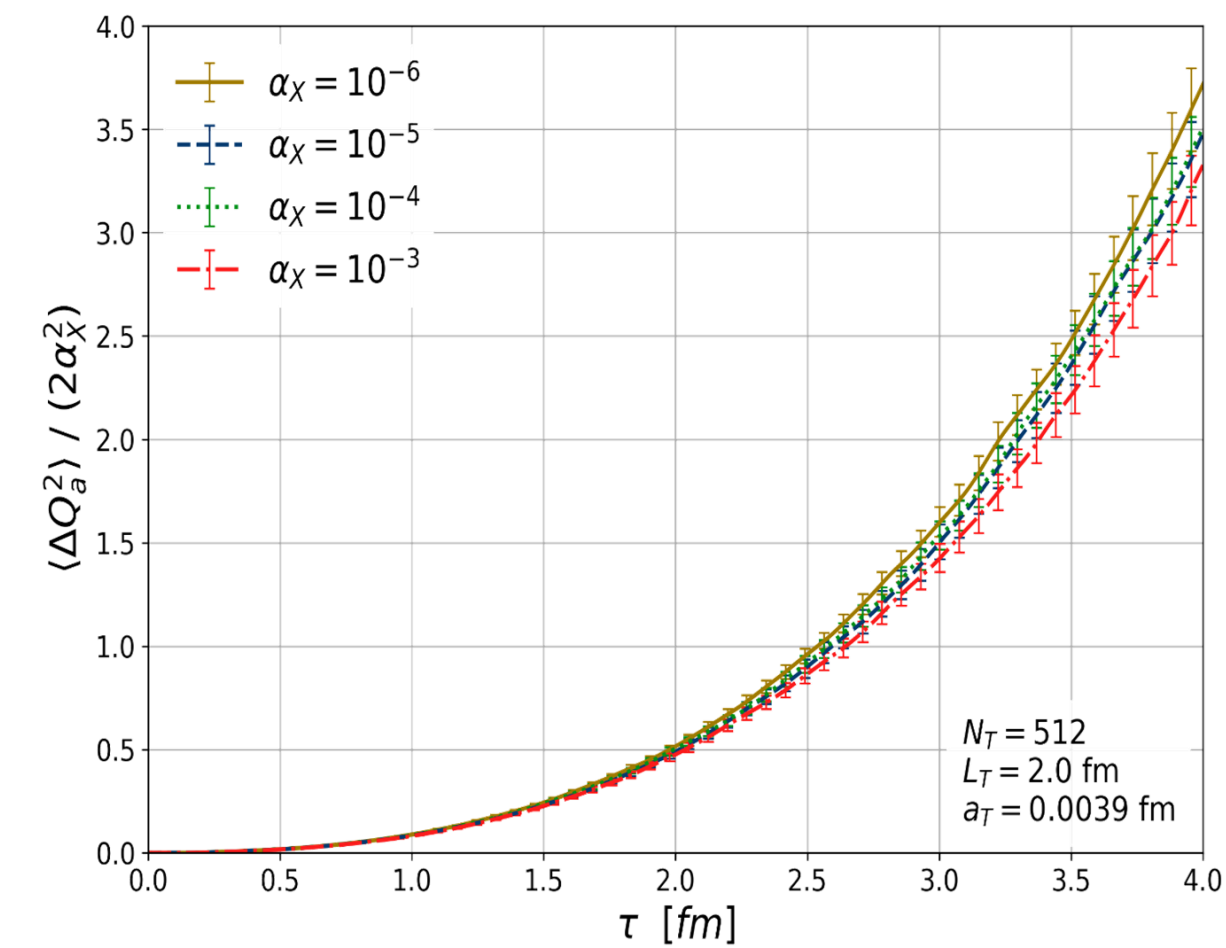
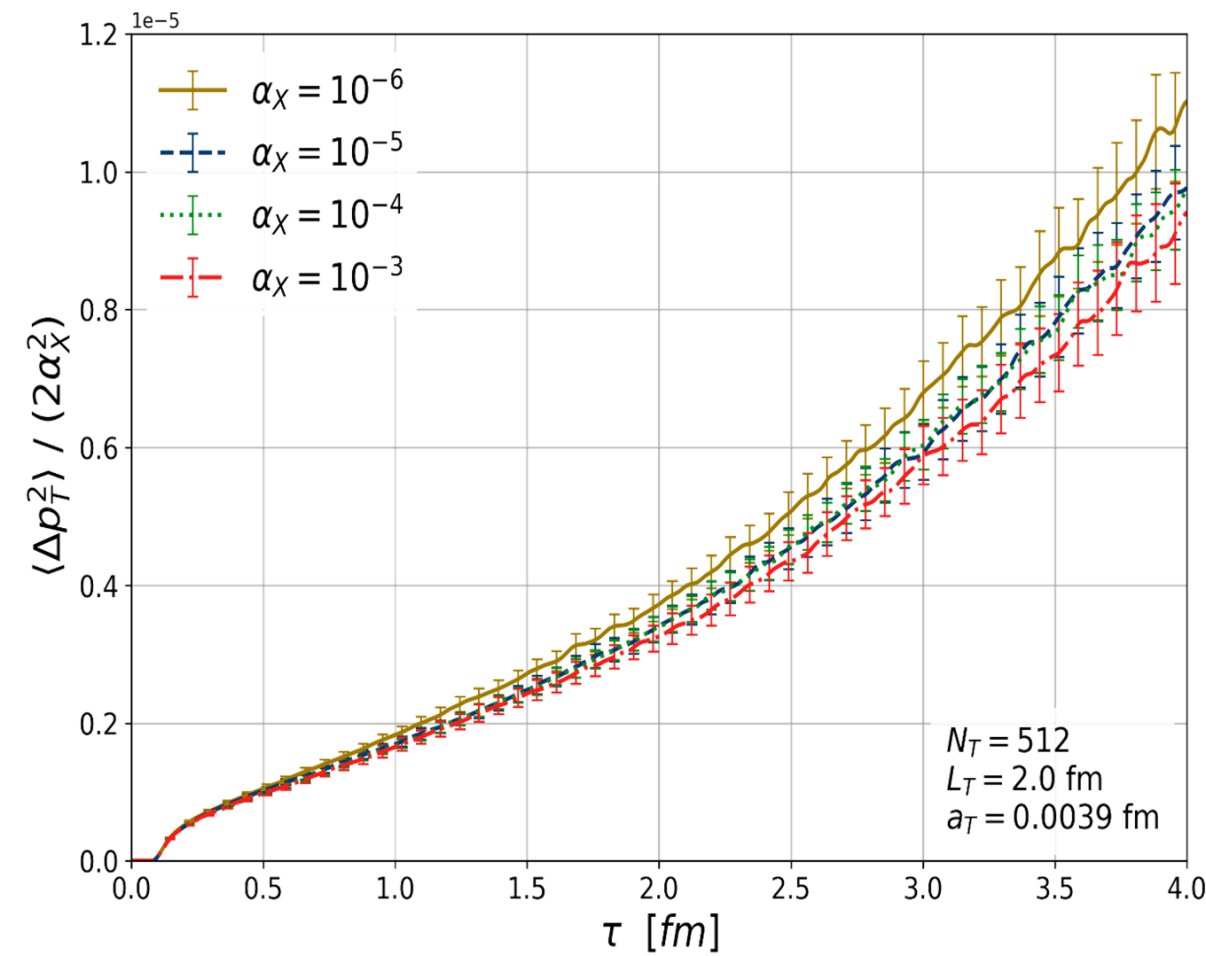
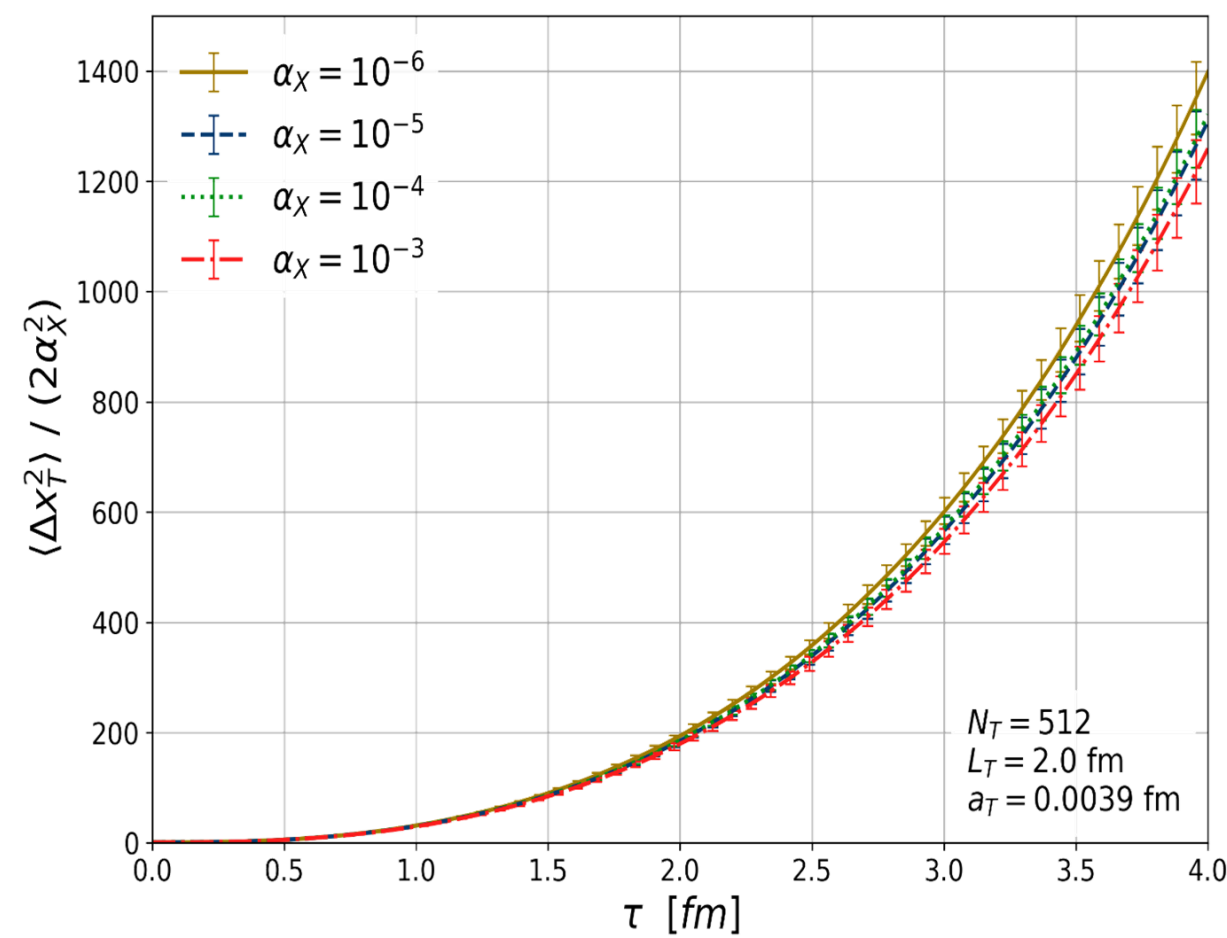


Perturbation in position

54

$$x_0^{i'} = x_0^i + \chi^i a_\perp$$

$$\begin{aligned} \langle \chi^i(x_\perp) \rangle &= 0 \\ \langle \chi^i(x_\perp) \chi^j(y_\perp) \rangle &= \alpha_x^2 \delta^{ij} \delta_{x_\perp, y_\perp} \end{aligned}$$

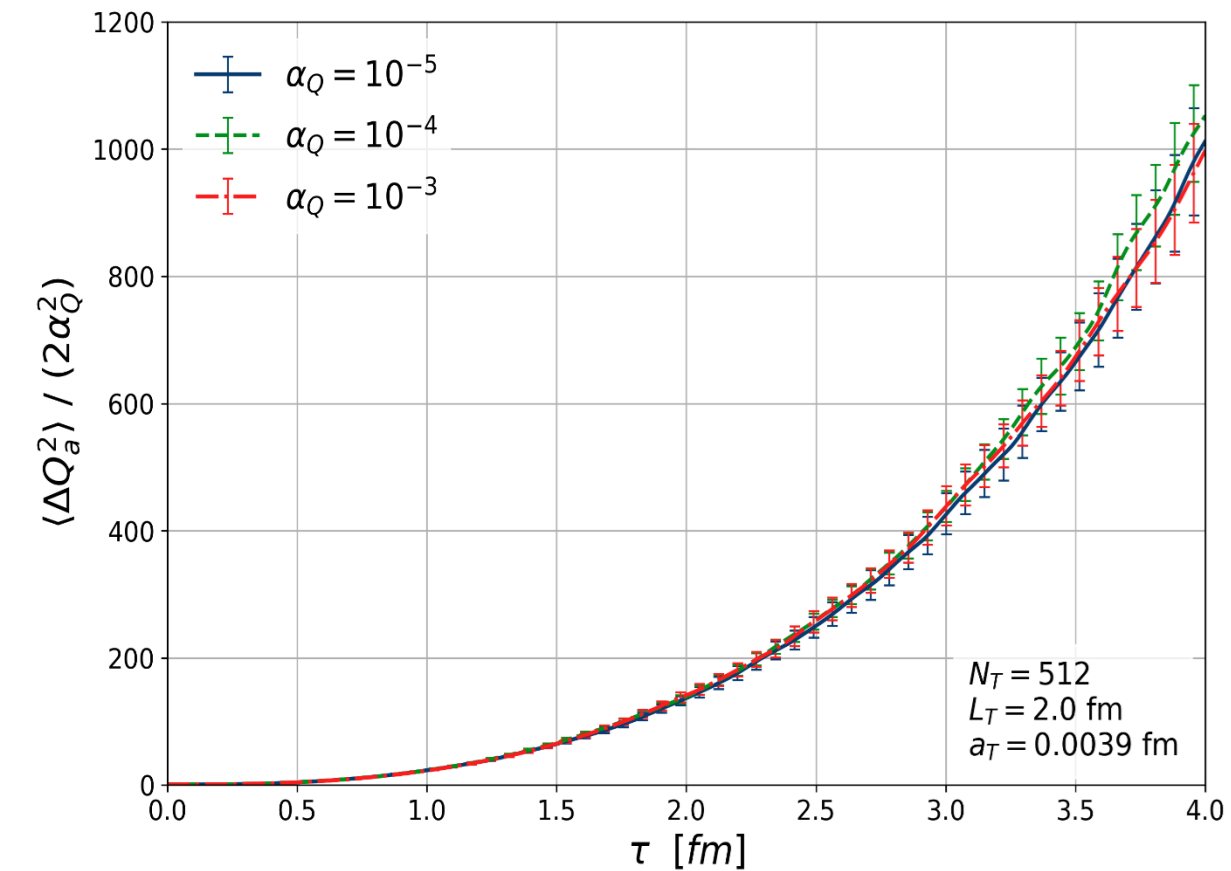
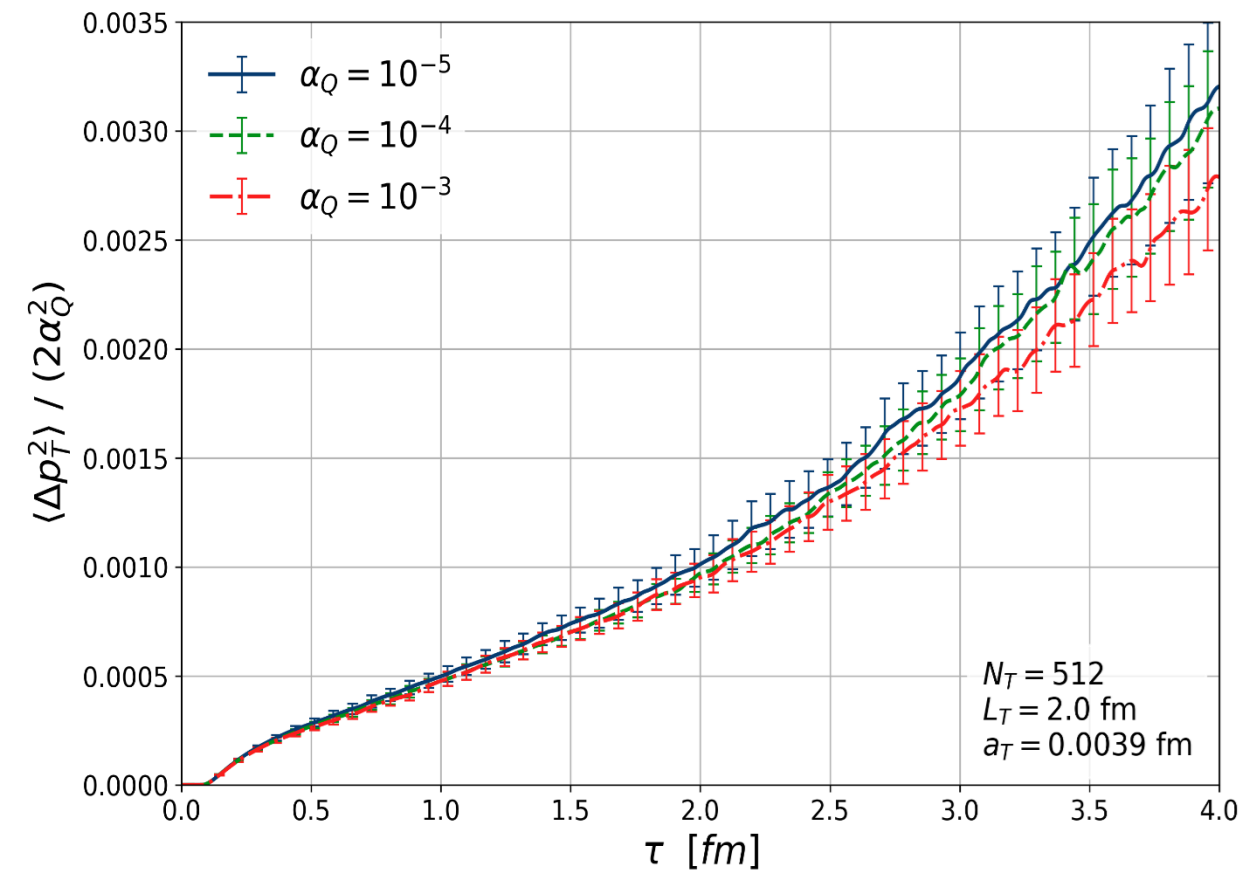
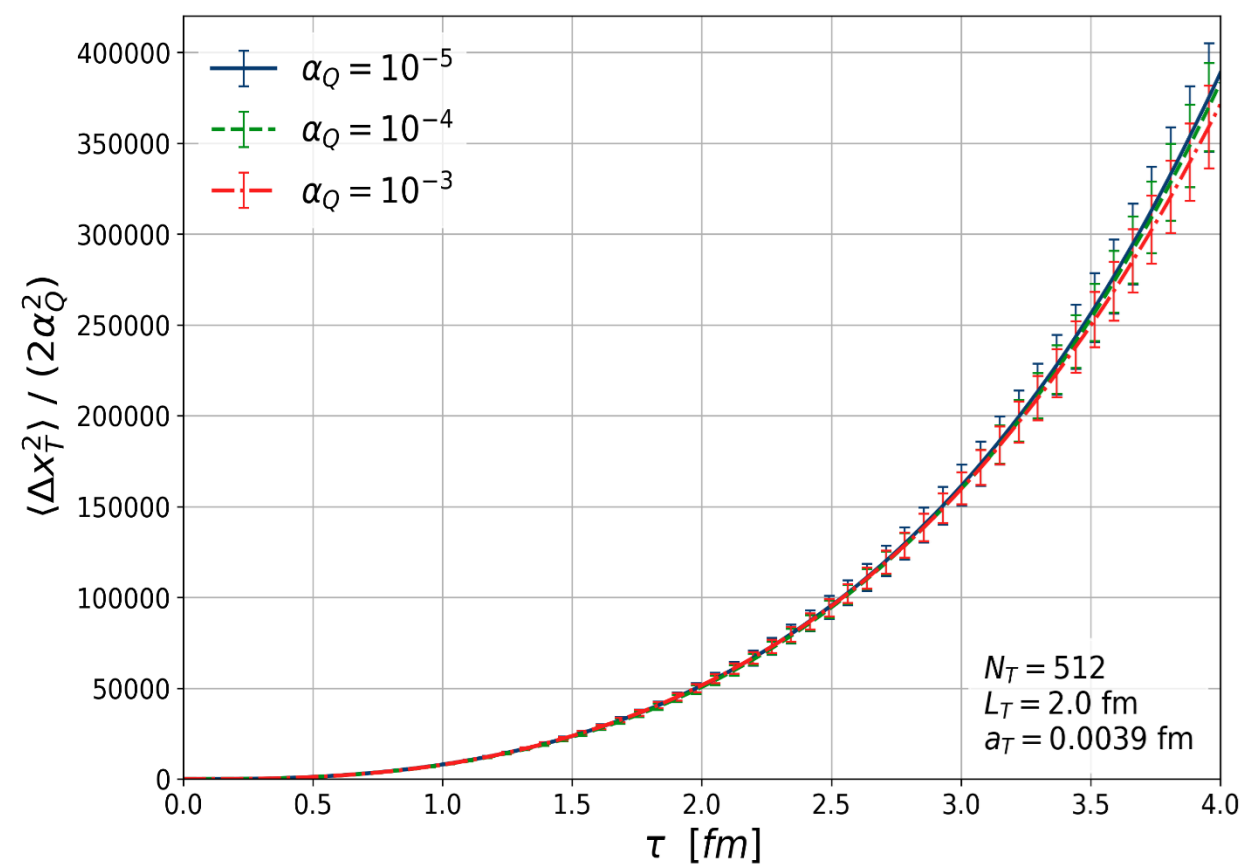


Perturbation in color charge

55

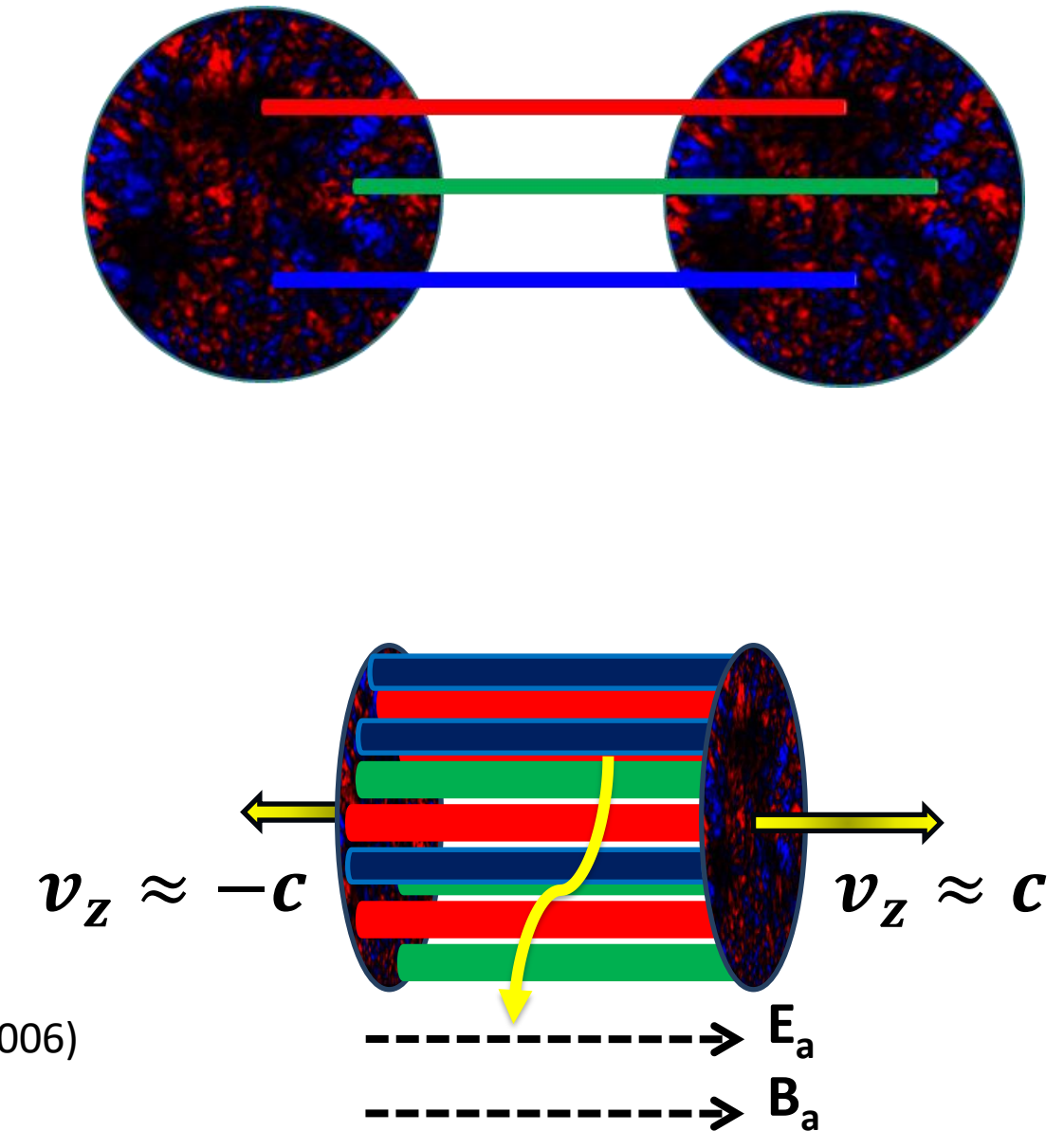
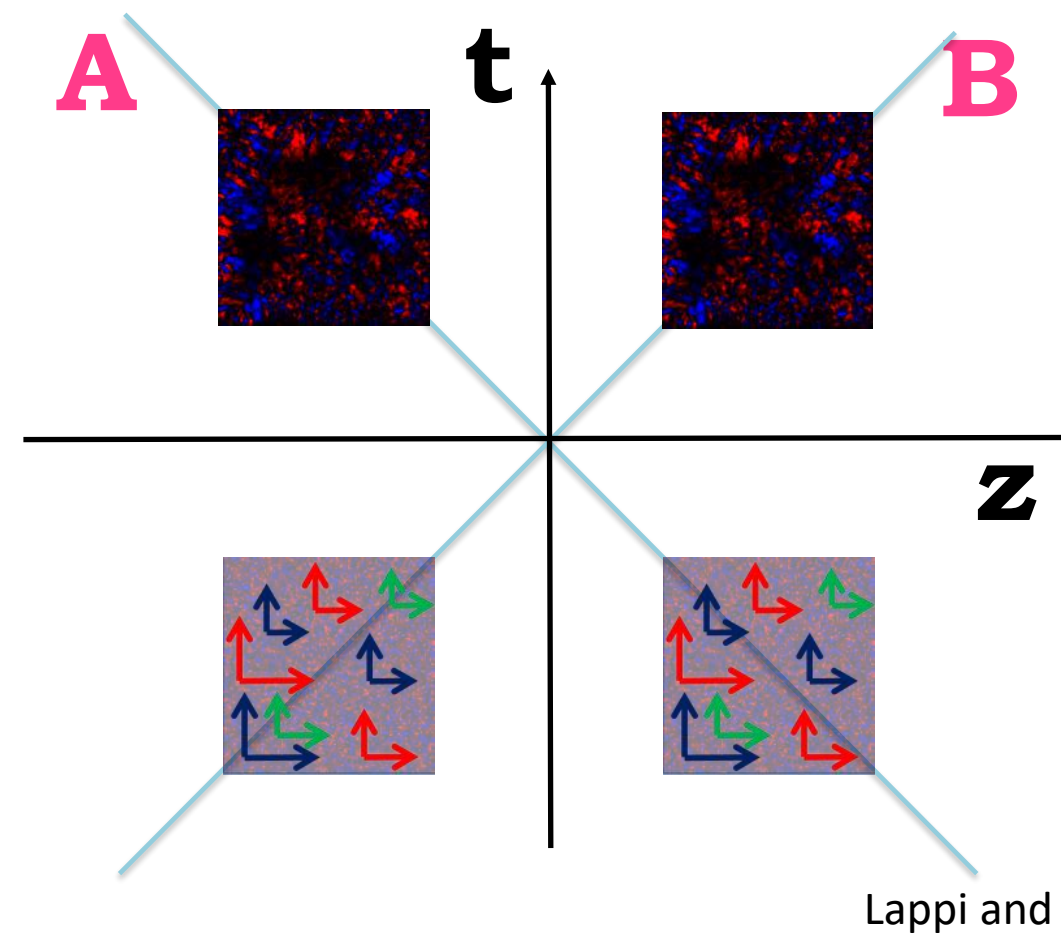
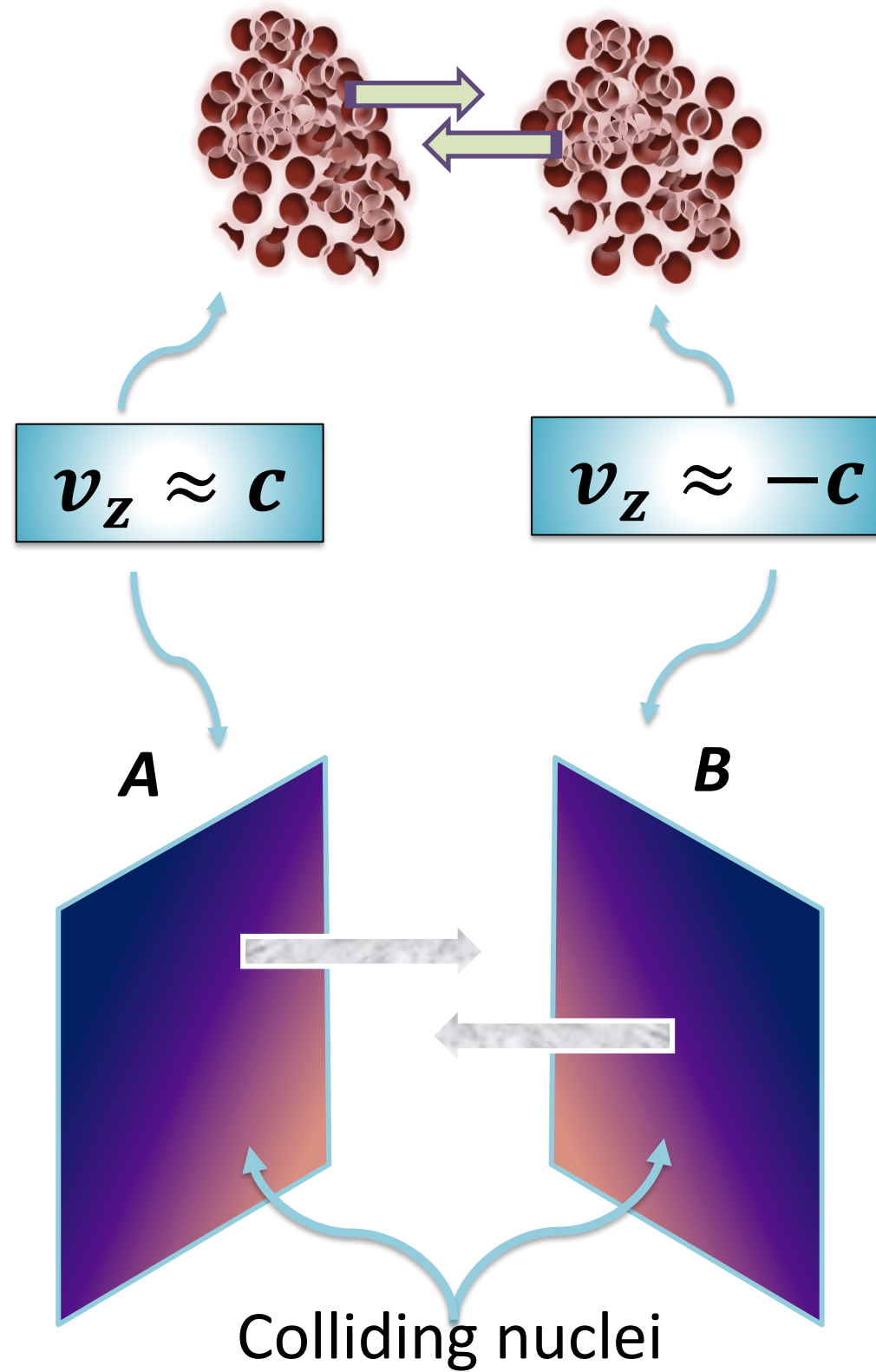
$$Q_0^{a'} = (Q_0^a + \zeta^a) \frac{|Q_0^a|}{|(Q_0^a + \zeta^a)|}$$

$$\begin{aligned} \langle \zeta^a(\mathbf{x}_\perp) \rangle &= 0 \\ \langle \zeta^a(\mathbf{x}_\perp) \zeta^b(\mathbf{y}_\perp) \rangle &= \alpha_Q^2 \delta^{ab} \delta_{\mathbf{x}_\perp, \mathbf{y}_\perp} \end{aligned}$$



Formation of Glasma

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The MV model of color sources

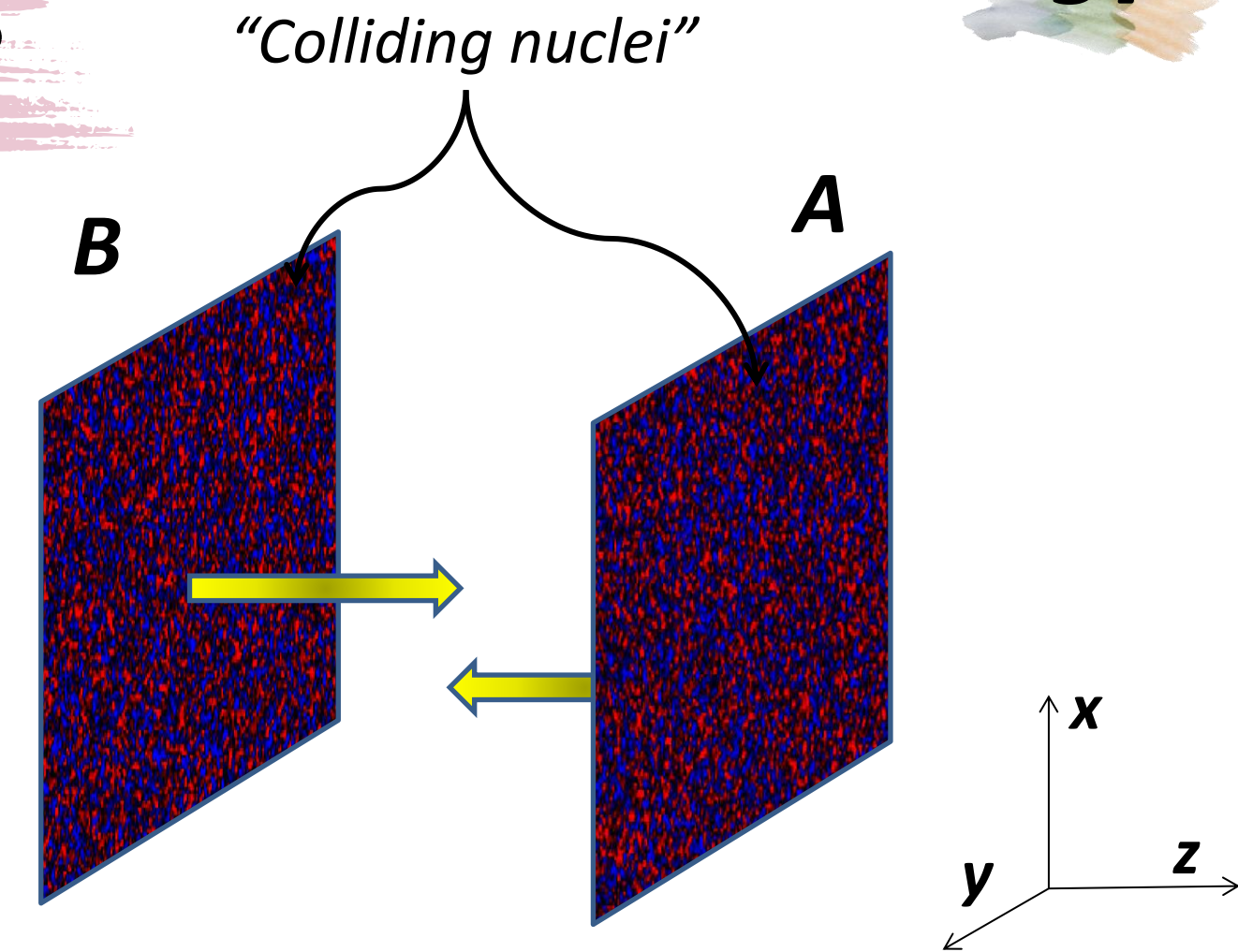
57

- Fast (large momentum) partons

Their dynamics in the Lab frame is slowed down due to time dilation: **static sources of color fields**.

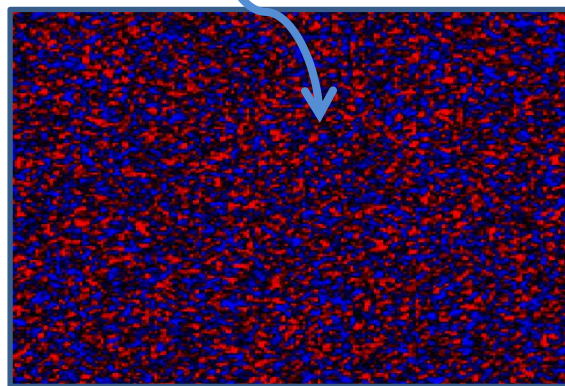
Model of static sources (MV model)

Uncorrelated color density fluctuations on the two nuclei.



$$\langle \rho^a(x_T) \rangle = 0,$$

$$\langle \rho^a(x_T) \rho^b(y_T) \rangle = (g^2 \mu)^2 \delta^{ab} \delta^{(2)}(x_T - y_T)$$



$$g^2 \mu \approx Q_s : \textit{saturation scale}$$

McLerran and Venugopalan (1996)
Kovchegov (1996)

From the correlators of Wilson lines:

$$g^2 \mu = O(Q_s) \quad \text{Lappi (2008)}$$

Evolution of static Background Glasma: CYM Equations

58

Due to the large density,
the gluon field behaves
like a classical field.



Dynamics is governed
by classical EoMs,
namely the **classical**
Yang-Mills (CYM)
equations.

CYM Equations

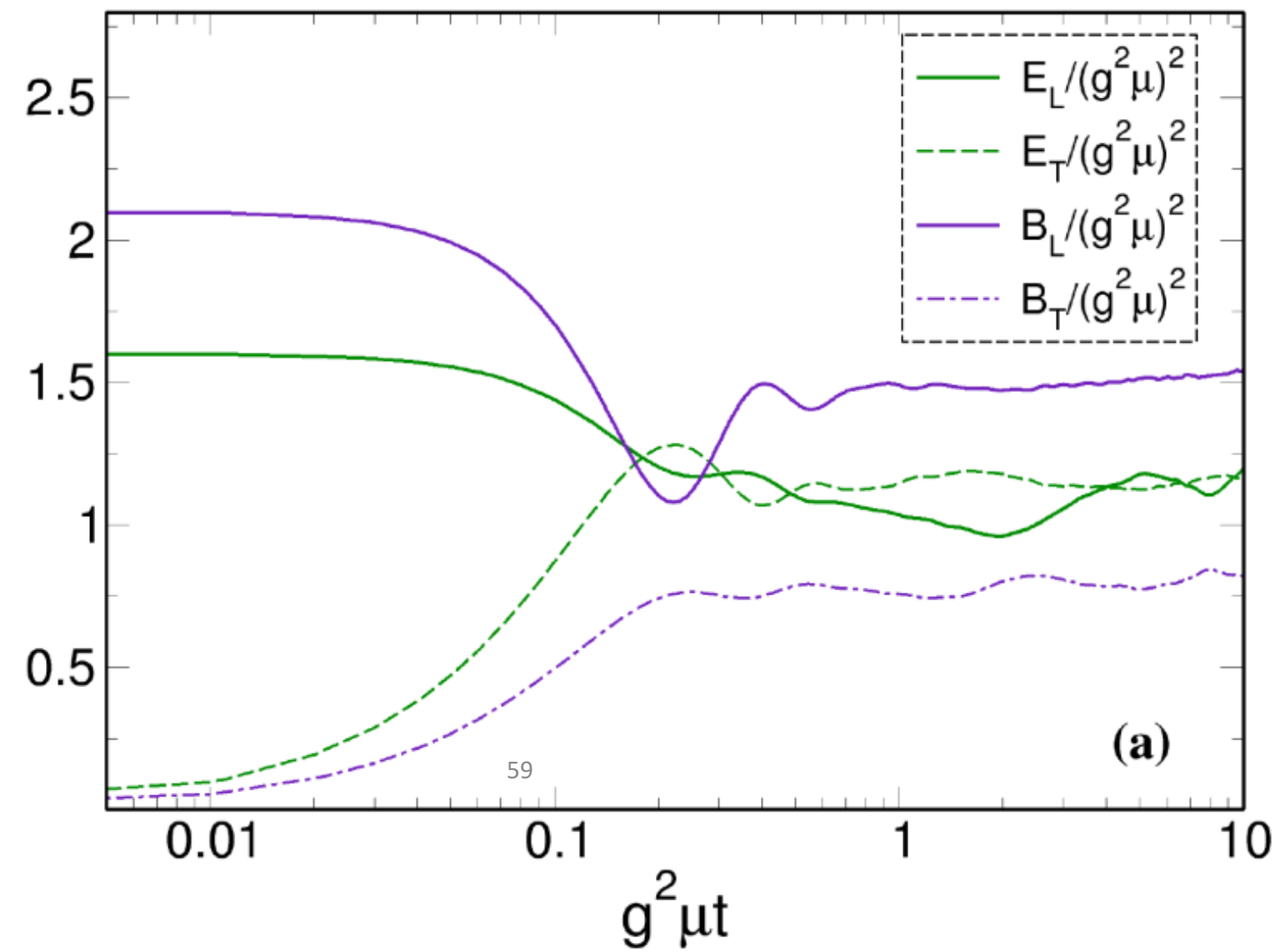
$$\frac{dA_i^a(x)}{dt} = E_i^a(x)$$

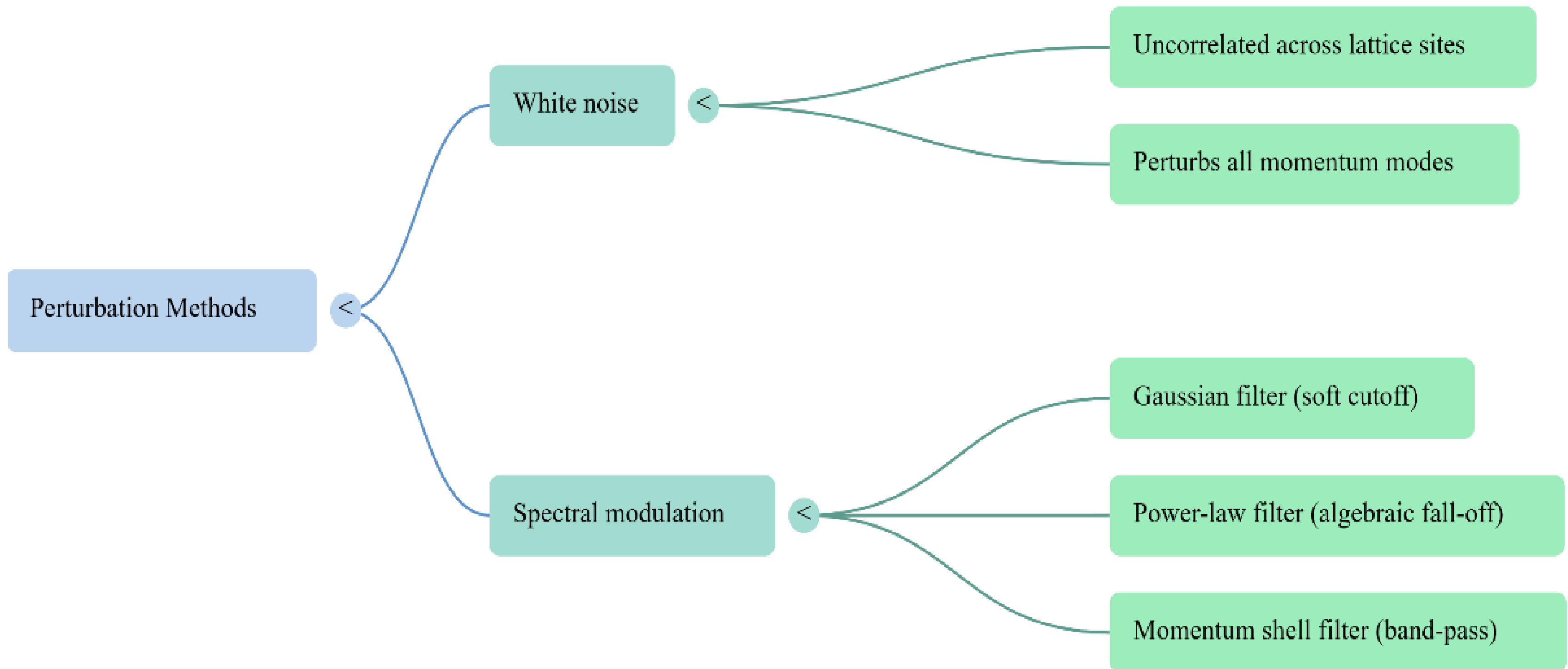
$$\frac{dE_i^a(x)}{dt} = \partial_j F_{ji}^a(x) + g f^{abc} A_j^b(x) F_{ji}^c(x)$$

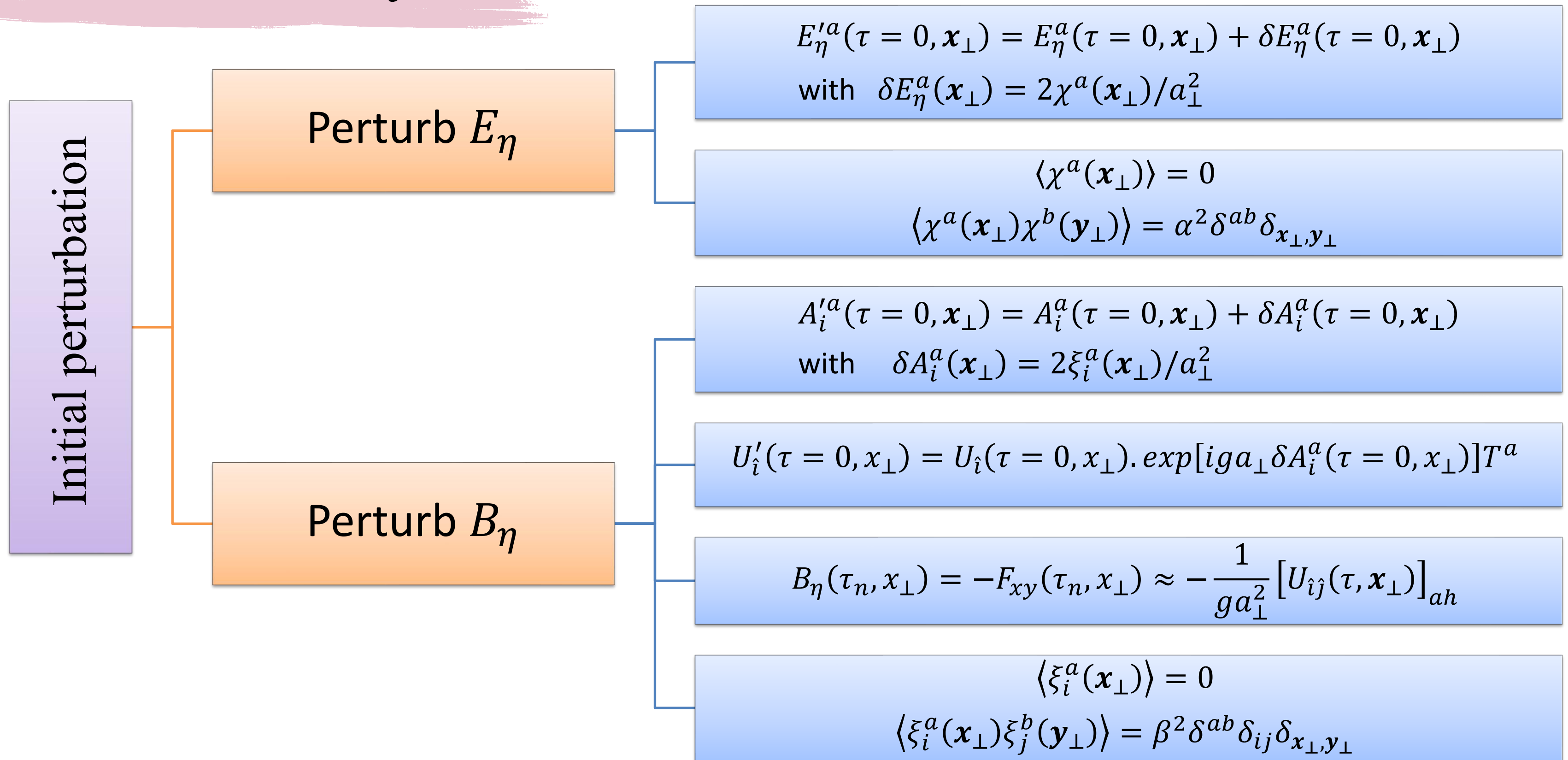
Gluon Field
Strength Tensor

$$\bullet F_{ij}^a(x) = \partial_i A_j^a(x) - \partial_j A_i^a(x) + g f^{abc} A_i^b(x) A_j^c(x)$$

*Evolution of the system is studied assuming Glasma as the initial condition,
and evolving this condition by virtue of the **CYM** equations.*





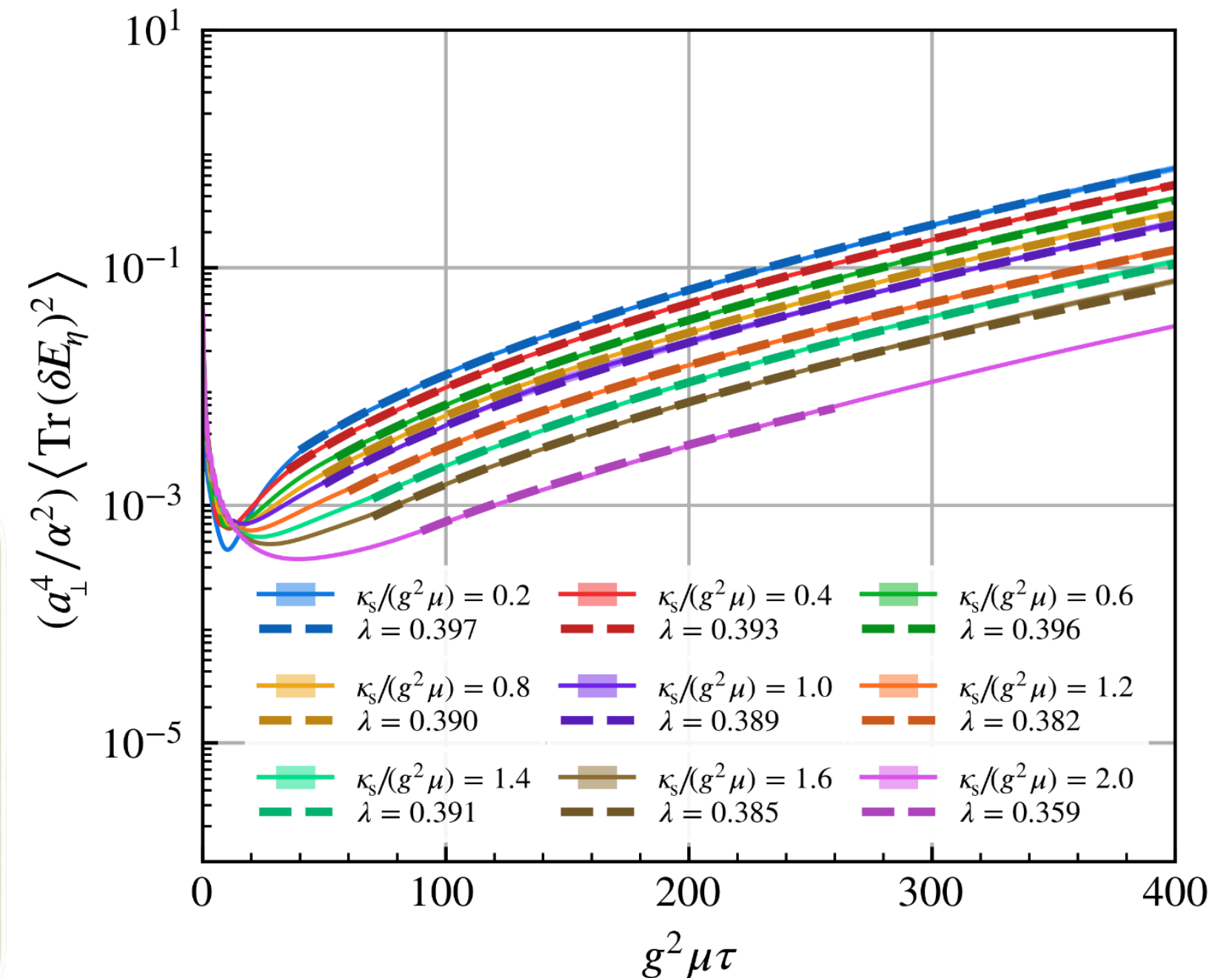


Shell-Filtered Perturbations

62

$$\mathcal{K}_{shell}(k_{\perp}) = \begin{cases} 1, & \kappa_s - \frac{\Delta k}{2} \leq k_{\perp} \leq \kappa_s + \frac{\Delta k}{2} \\ 0, & \text{otherwise} \end{cases}$$

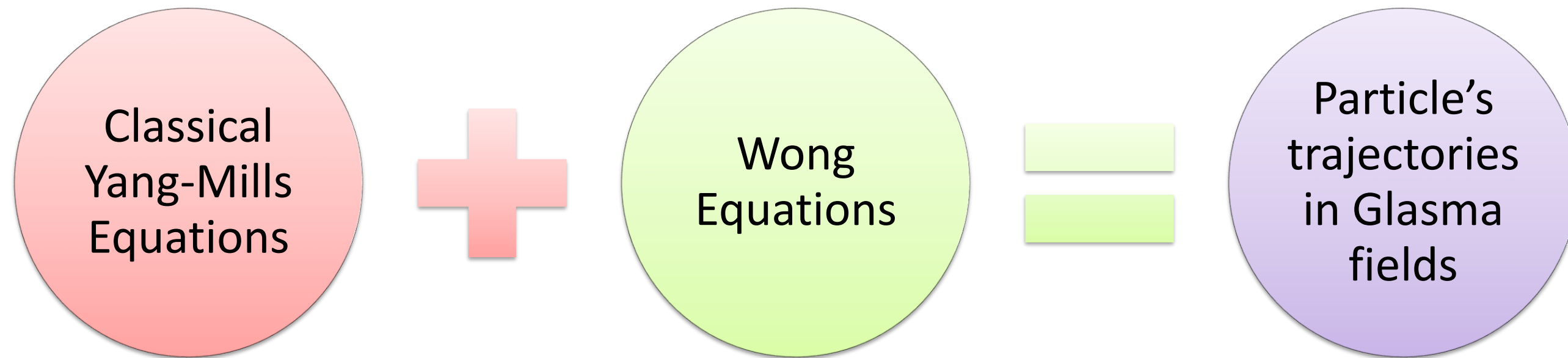
- **Clean exponential growth: $\lambda \approx 0.38 - 0.40$**
- Late-time amplitude of the perturbations $\propto \frac{1}{\kappa_s}$
- Non-trivial momentum space structure: **Need a detailed study in Fourier-space analysis of the perturbation**



Chaotic Behavior of HQs

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Aim: To study the chaotic behavior of the HQs interacting with the dense Glasma fields



Starting Point: Study the chaotic behavior of the Glasma