

Gluon Radiation From Off-shell Quark in Glasma Fields

Based on: T. Lappi & IS arXiv:2607.XXXX

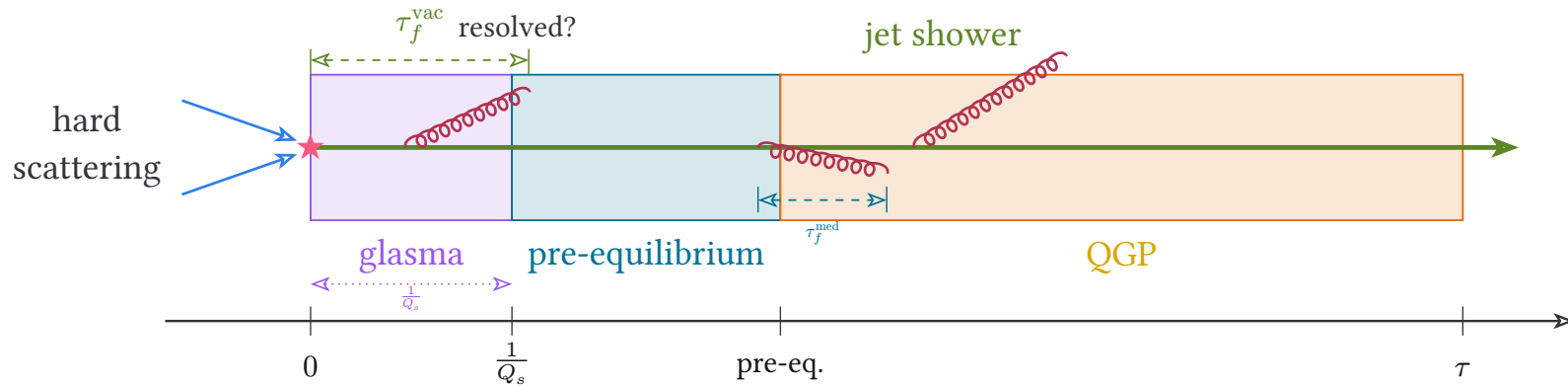
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Jets in heavy ion collisions



Medium

1. Initial Glasma Fields
2. Out-of-equilibrium evolution
2. Hydrodynamical QGP

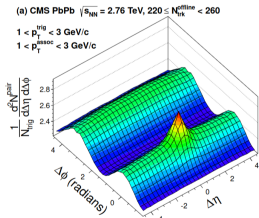
Jet

1. Hard production $\tau_{\text{hard}} \sim 1/p_T$
2. Vacuum-like shower
3. Jet-medium Shower

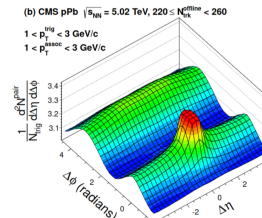
HIC vs Small Systems

Heavy-ion collisions

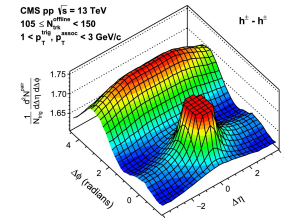
- ▶ Strong signals of collectivity
- ▶ Clear quenching observed
- ▶ QGP energy loss dominates



PbPb



pPb



high-mult. pp

System size

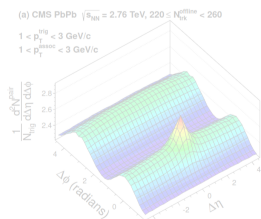
- ▶ Energy loss v.s. system size
- ▶ Early-time v.s. late-time energy loss

[Chatrchyan & others Phys. Lett. B 718, 795–814 2013; Chatrchyan & others Phys. Lett. B 724, 213–240 2013; Khachatryan & others Phys. Lett. B 765, 193–220 2017; Hayrapetyan & others Phys. Rev. Lett. 136, 162301 2026]

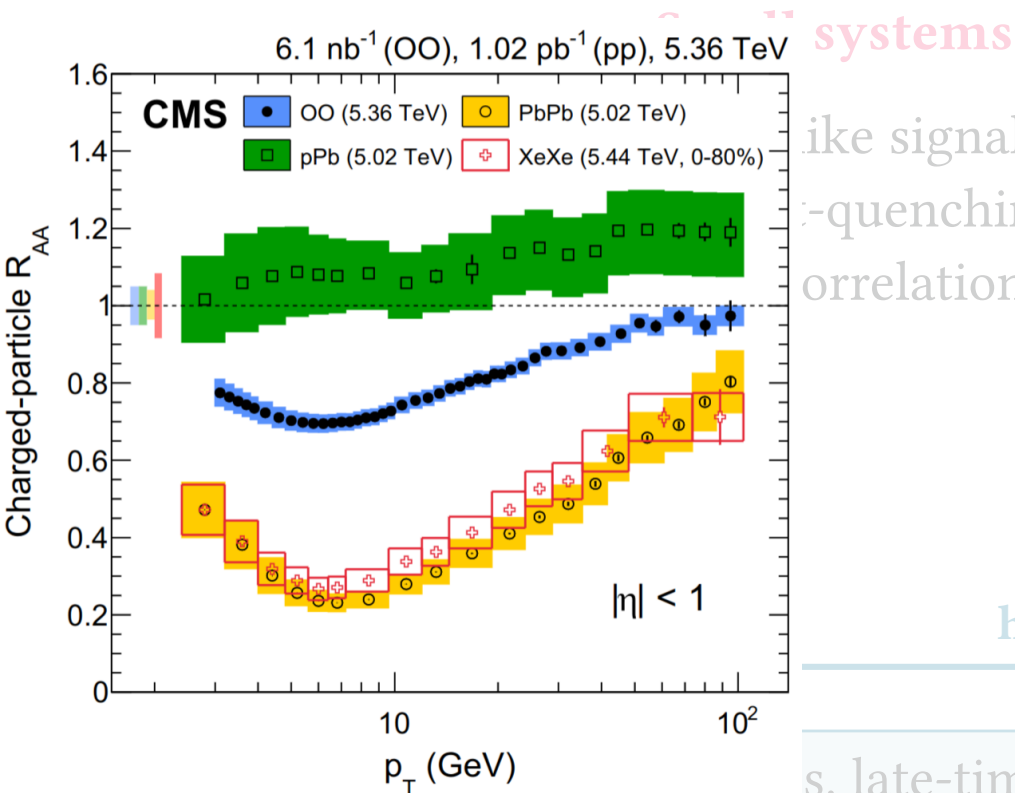
HIC vs Small Systems

Heavy-ion collisions

- ▶ Strong signals of collective flow
- ▶ Clear quenching observed
- ▶ QGP energy loss dominates

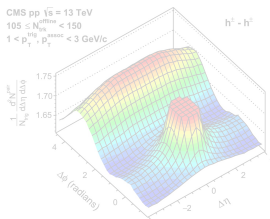


PbPb



Small systems

- ▶ Like signals
- ▶ Quenching signal
- ▶ Correlations



high-mult. pp

s. late-time energy loss

▶ Energy loss v.s. system size

[Chatrchyan & others Phys. Lett. B 718, 795–814 2013; Chatrchyan & others Phys. Lett. B 724, 213–240 2013; Khachatryan & others Phys. Lett. B 765, 193–220 2017; Hayrapetyan & others Phys. Rev. Lett. 136, 162301 2026]

Momentum Broadening in Glasma

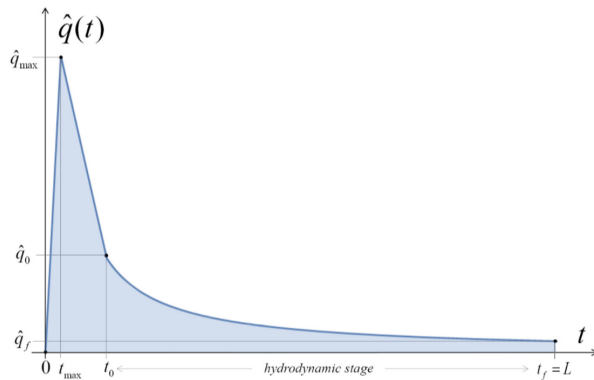
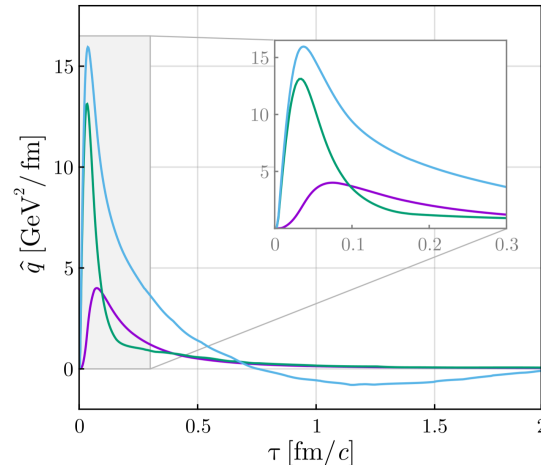
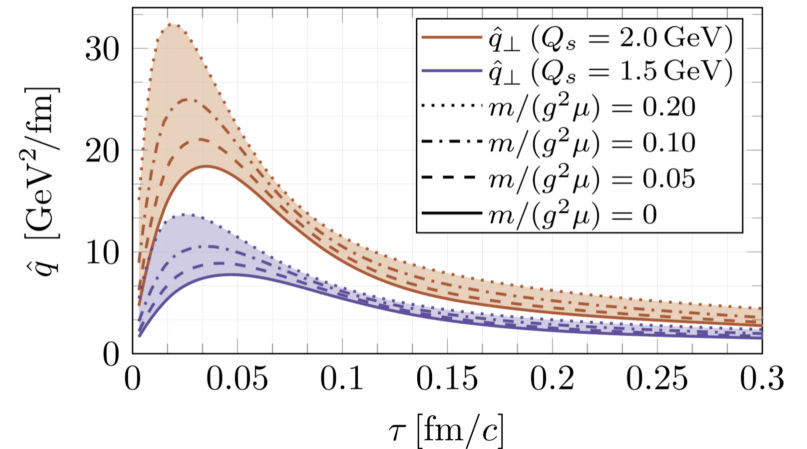


Figure 3: Schematic temporal evolution of $\hat{q}(t)$.

[Carrington et al. Phys. Lett. B 834, 137464 2022]



[Avramescu et al. Phys. Rev. D 107, 114021 2023]



[Ipp et al. Phys. Lett. B 810, 135810 2020]

► Large broadening $\hat{q}_{\text{glasma}} \gg \hat{q}_{\text{QGP}}$

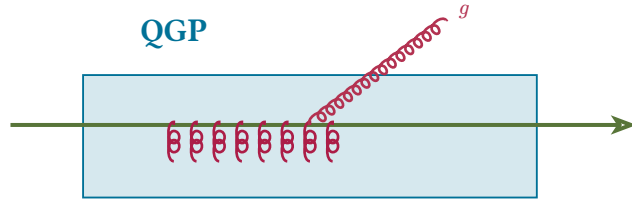
► Short lifetime

► BDMPS-Z Energy loss: $\Delta E \sim \hat{q} L^2$

[Baier et al. Nucl. Phys. B 483, 291–320 1997]

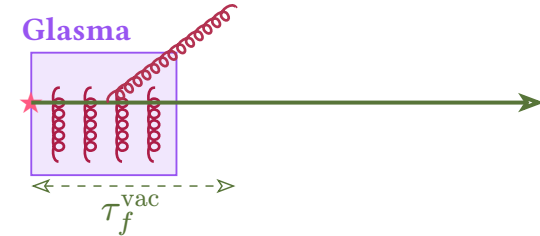
Medium-Induced Radiation

Standard picture



- ▶ Medium length $L \gg 1/Q_s$
- ▶ Multiple scatterings (LPM)
- ▶ Vacuum/medium separation
→ Jet resolved as on-shell

Early-time CGC



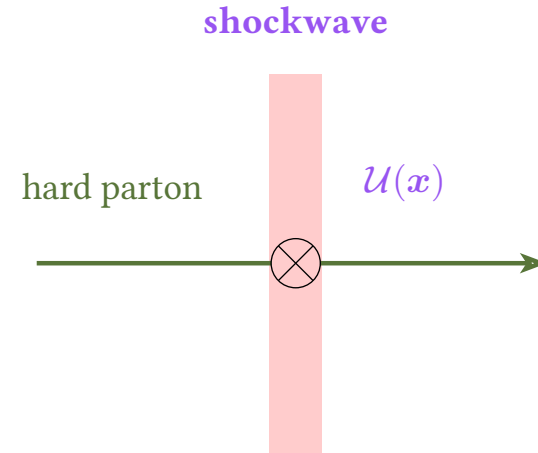
- ▶ Short-lived glasma fields
- ▶ Highly virtual jet
- ▶ Vacuum-like splittings
→ Virtuality vs glasma resolution

First emission: vacuum-like vs glasma modification

$$\tau_f^{\text{vac}} \Leftrightarrow 1/Q_s.$$

Shockwave Framework

- ▶ Dense small- x gluons
 - Classical color field A^μ .
- ▶ Fast target is a thin sheet in x^+ .
 - $A^\mu(x) = \delta^{\mu-} \delta(x^+) \alpha(x)$.
- ▶ Crossing the sheet described by
 - Wilson line $\mathcal{U}(x)$.



- ▶ Frozen background field during the short interaction.

[Balitsky Nucl. Phys. B 463, 99–160 1996; Kovchegov Phys. Rev. D 61, 74018 2000; Iancu & Venugopalan Quark-gluon plasma 4 249–3363 2003; Gelis & Jalilian-Marian Phys. Rev. D 66, 14021 2002]

LCPT

- In light-cone gauge $A^+ = 0$

$$\begin{aligned} S_F(x, y) &= \int dq^- e^{i(x^+ - y^+) \cdot q^-} \frac{\not{q}}{q^2 + i\varepsilon} \\ &= \int dq^- e^{i(x^+ - y^+) \cdot q^-} \frac{1}{2q^+} \frac{\not{q}}{q^- - \frac{\mathbf{q}}{2q^+} + i\varepsilon} + \frac{\gamma^+ \left(q^- - \frac{\mathbf{q}}{2q^+} \right)}{2q^+ \left(q^- - \frac{\mathbf{q}}{2q^+} \right)} \\ &= \theta(x^+ - y^+) \frac{1}{2q^+} \frac{\not{q}}{q^- - \frac{\mathbf{q}}{2q^+} + i\varepsilon} + \frac{\gamma^+}{2q^+} \delta(x^+ - y^+) + \text{Reverse Order} \end{aligned} \tag{1}$$

- Old-fashioned perturbation theory with x^+ time-ordering (c.f. *T. Lappi's* talk)

Quark Propagator

Eikonal propagation

$$\begin{aligned}
 S_{F(x,y)_{\beta\alpha}} &= \text{Diagram: A horizontal line with two black dots at the ends. The left dot is labeled 'x' and the right dot is labeled 'y'. Above the line, an arrow labeled 'p' points from 'x' to a red vertical rectangle. Inside the rectangle is a red circle with a cross. An arrow labeled 'k' points from the rectangle to 'y'. } \\
 &= \mathbf{1}_{\beta\alpha} \delta^{(2)}(\mathbf{x} - \mathbf{y}) \delta(x^+ - y^+) \text{sign}(x^- - y^-) \frac{\gamma^+}{4} + \int \frac{d^3 p}{(2\pi)^3} \int \frac{d^3 k}{(2\pi)^3} \\
 &\quad \frac{\bar{p} \gamma^+ \bar{k}}{4p^+ k^+} (2\pi) \delta(p^+ - k^+) e^{-ix \cdot \bar{p} + iy \cdot \bar{k}} \int d^2 z e^{-iz \cdot (p - k)} \\
 &\quad \times \left\{ \theta(k^+) \theta(x^+ - y^+) \mathcal{U}_F(x^+, y^+; z)_{\beta\alpha} - \theta(-k^+) \theta(y^+ - x^+) \mathcal{U}_F(y^+, x^+; z)_{\beta\alpha} \right\}.
 \end{aligned} \tag{2}$$

[Balitsky Nucl. Phys. B 463, 99–160 1996; Kovchegov Phys. Rev. D 61, 74018 2000; Iancu & Venugopalan Quark-gluon plasma 4 249–3363 2003; Gelis & Jalilian-Marian Phys. Rev. D 66, 14021 2002; Gelis & Venugopalan Phys. Rev. D 69, 14019 2004; Fujii et al. Nucl. Phys. A 780, 146–174 2006]

Gluon Propagator

Adjoint representation

$$\begin{aligned}
 G_{ab}^{\mu\nu}(x, y) &= \text{Diagram: A wavy line with momentum } p \text{ from point } x \text{ to a red shaded box containing a crossed circle, followed by a wavy line with momentum } k \text{ to point } y. \\
 &= G_{ab, \text{inst}}^{\mu\nu}(x, y) + \int \frac{d^3 p}{(2\pi)^3} \int \frac{d^3 k}{(2\pi)^3} \frac{1}{2k^+} \sum_{\lambda} \varepsilon_{\lambda}^{\mu}(\bar{p}) \varepsilon_{\lambda}^{*\nu}(\bar{k}) \\
 &\quad \times (2\pi) \delta(p^+ - k^+) e^{-ix \cdot \bar{p} + iy \cdot \bar{k}} \int d^2 z e^{-iz \cdot (\mathbf{p} - \mathbf{k})} \\
 &\quad \times \left\{ \theta(k^+) \theta(x^+ - y^+) \mathcal{U}_A(x^+, y^+; \mathbf{z})_{ab} - \theta(-k^+) \theta(y^+ - x^+) \mathcal{U}_A(y^+, x^+; \mathbf{z})_{ab} \right\}.
 \end{aligned} \tag{3}$$

[Balitsky Nucl. Phys. B 463, 99–160 1996; Kovchegov Phys. Rev. D 61, 74018 2000; Iancu & Venugopalan Quark-gluon plasma 4 249–3363 2003; Gelis & Jalilian-Marian Phys. Rev. D 66, 14021 2002; Gelis & Venugopalan Phys. Rev. D 69, 14019 2004; Fujii et al. Nucl. Phys. A 780, 146–174 2006]

Kinematics

- Incoming hard quark:

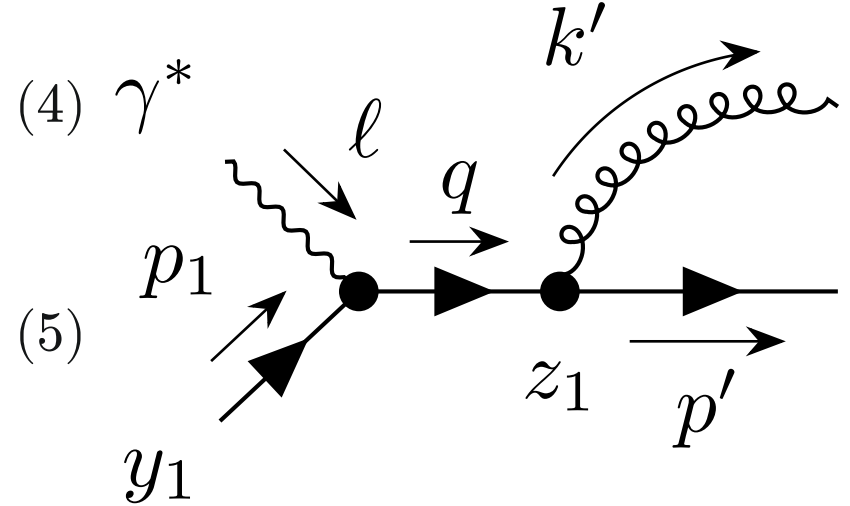
$$p_1 = (0^+, p_1^-, \mathbf{0}), \quad \ell = (\ell^+, \ell^-, \mathbf{0}) \quad (4)$$

- On-shell Outgoing pair:

$$p' = (p'^+, p'^-, \mathbf{p}'), \quad k' = (k'^+, k'^-, \mathbf{k}') \quad (5)$$

- Gluon momentum fraction

$$y = \frac{k'^+}{q'^+} = \frac{k'^+}{p'^+ + k'^+} \quad (6)$$

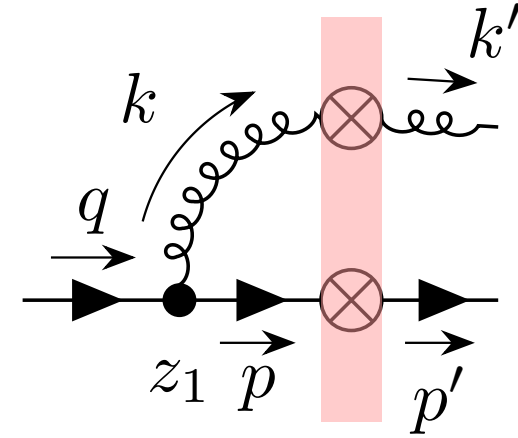
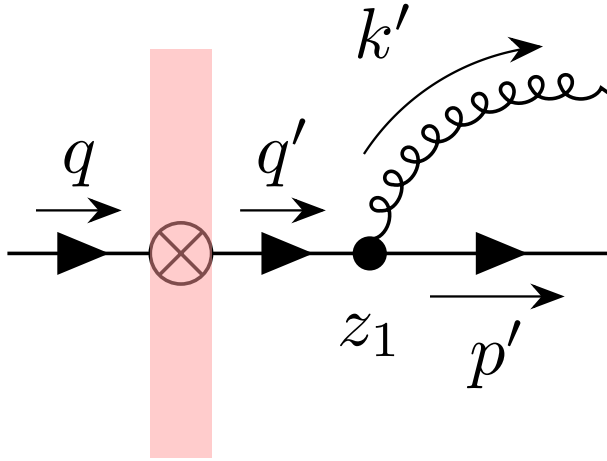


b : quark position

r : gluon position

$u = r - b$: qg size

On-Shell Case



► **A**: Emission **after** the shockwave

► **B**: Emission **before** the shockwave

► On-shell parent $\rightarrow$ two time orderings across the shockwave.

The Wave Function

- ▶ The $q \rightarrow qg$ wave function

$$\Psi^{q \rightarrow qg}(p', k') = \begin{array}{c} \text{Diagram: A horizontal line with a black dot at position } z_1. \text{ An incoming quark line from the left is labeled } q' \text{ with an arrow pointing right. An outgoing quark line to the right is labeled } p' \text{ with an arrow pointing right. A wavy gluon line goes from the dot } z_1 \text{ upwards and to the right, labeled } k' \text{ with an arrow pointing right.} \end{array} = \frac{\bar{u}(q' - k') \not{\epsilon}_\lambda^*(k') u(q')}{(q' - k')^- - q'^- - k'^-} \quad (7)$$

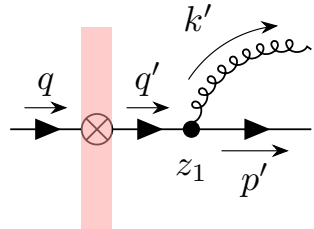
- ▶ On-shell parent $\rightarrow$ Energy denominator simply: $(k - yq')^{-2} \neq \infty$.
- ▶ Fourier Transform $q' \rightarrow r - b$:

$$\begin{aligned} \phi_0^\lambda(u) &= \int d^2 q' e^{i u \cdot q'} \Psi^{q \rightarrow qg}(p', k') \\ &= 2i \sqrt{1-y} \frac{u \cdot \epsilon_\lambda^*}{2\pi u^2} [\delta_{\lambda,s} + (1-y)\delta_{\lambda,-s}] . \end{aligned} \quad (8)$$

[Marquet Nucl. Phys. A 796, 41–60 2007]

Amplitude

After

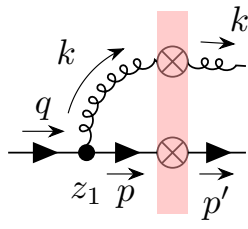


$$\mathcal{A}_{\text{after}} = \int d^2b d^2r e^{-i\mathbf{b} \cdot \mathbf{p}'} e^{-i\mathbf{r} \cdot \mathbf{k}'} \phi_0^\lambda(\mathbf{r} - \mathbf{b}) t^a \mathcal{U}_F(\mathbf{v}). \quad (9)$$

► Transverse center of mass

$$\rightarrow \mathbf{v} = \mathbf{b} + y(\mathbf{r} - \mathbf{b}).$$

Before



$$\mathcal{A}_{\text{before}} = \int d^2b d^2r e^{-i\mathbf{b} \cdot \mathbf{p}'} e^{-i\mathbf{r} \cdot \mathbf{k}'} \phi_0^\lambda(\mathbf{r} - \mathbf{b}) t^b \mathcal{U}_F(\mathbf{b}) \mathcal{U}_A^{ba}(\mathbf{r}). \quad (10)$$

► Transverse positions:

→ $\mathbf{b}$: quark

→ $\mathbf{r}$: gluon

Total Amplitude

$$\mathcal{A}_{\text{on-shell}} = \mathcal{N}_0 \int d^2b d^2r e^{-i\delta\mathbf{r} \cdot (\mathbf{p}' + \mathbf{k}')} e^{-i\mathbf{R} \cdot \frac{\mathbf{k}' - \mathbf{p}'}{2}} \phi_0^\lambda(\delta\mathbf{r})$$

$$\left[\underbrace{t^b \mathcal{U}_F(\mathbf{b}) \mathcal{U}_A(\mathbf{r})^{ba}}_{\text{After}} - \underbrace{t^a \mathcal{U}_F(\mathbf{v})}_{\text{Before}} \right]. \quad (11)$$

- ▶ Amplitude $\rightarrow$ difference between **Before** and **After**.
- ▶ In vacuum: $\rightarrow \mathcal{U} \rightarrow 1 \Rightarrow \mathcal{A} \rightarrow 0.$

Kinematics

- Incoming hard quark:

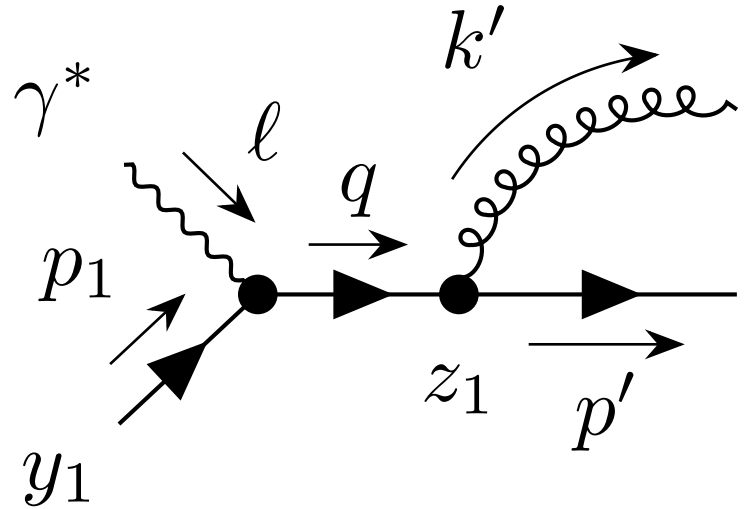
$$p_1 = (0^+, p_1^-, \mathbf{0}), \quad \ell = (\ell^+, \ell^-, \mathbf{0}) \quad (12)$$

- On-shell Outgoing pair:

$$p' = (p'^+, p'^-, \mathbf{p}'), \quad k' = (k'^+, k'^-, \mathbf{k}') \quad (13)$$

- Gluon momentum fraction

$$y = \frac{k'^+}{q'^+} = \frac{k'^+}{p'^+ + k'^+} \quad (14)$$



b : quark position

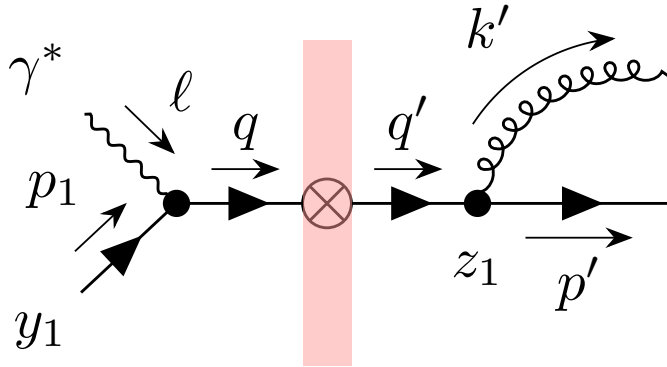
r : gluon position

$u = r - b$: qg size

Off-shell Amplitude

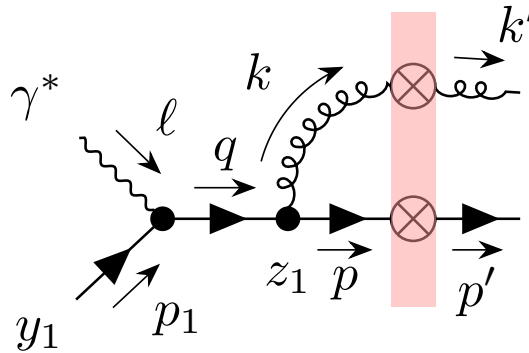
- **A:** emission **after** the shockwave

$$y_1^+ < 0 < z_1^+$$



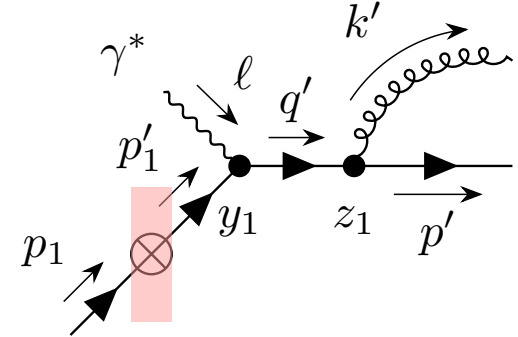
- **B:** emission **before** the shockwave

$$y_1^+ < z_1^+ < 0$$



- **C:** shockwave **before** the photon vertex

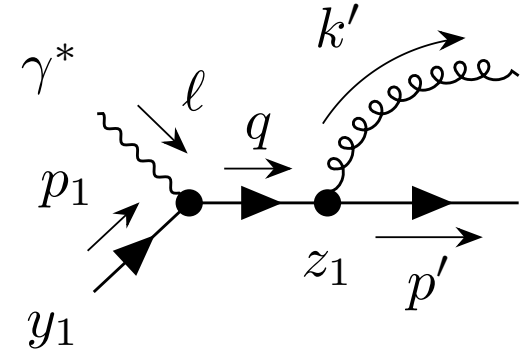
$$0 < y_1^+ < z_1^+$$



Vacuum radiation

► Minus energy

• $z_1^+ : P^- = \bar{p}^- + \bar{k}^-$ • $y_1^+ : Q^- = p_1^- + \ell^-$



► **A:** $y_1^+ < 0 < z_1^+$

$$\int_{-\infty}^0 dy_1^+ \int_0^{\infty} dz_1^+ \mathcal{V}$$

$$= - \frac{1}{P^- - \bar{q}^- + i\varepsilon} \frac{1}{Q^- - \bar{q}^- + i\varepsilon}$$

► **B:** $y_1^+ < z_1^+ < 0$

$$\int_{-\infty}^0 dy_1^+ \int_{y_1^+}^0 dz_1^+ \mathcal{V}$$

$$= \frac{1}{P^- - Q^- - i\varepsilon} \frac{1}{Q^- - \bar{q}^- + i\varepsilon}$$

► **C:** $0 < y_1^+ < z_1^+$

$$\int_0^{\infty} dy_1^+ \int_{y_1^+}^{\infty} dz_1^+ \mathcal{V}$$

$$= \frac{1}{P^- - \bar{q}^- + i\varepsilon} \frac{1}{Q^- - P^- - i\varepsilon}$$

Energy conservation

- The sum of the three time-orderings gives a delta function in minus energy

$$\begin{aligned}
 & \frac{1}{P^- - \bar{q}^- + i\varepsilon} \frac{1}{Q^- - P^- - i\varepsilon} - \frac{1}{P^- - \bar{q}^- + i\varepsilon} \frac{1}{Q^- - \bar{q}^- + i\varepsilon} \\
 & + \frac{1}{P^- - Q^- - i\varepsilon} \frac{1}{Q^- - \bar{q}^- + i\varepsilon} \\
 & = \frac{1}{Q^- - \bar{q}^- + i\varepsilon} \left[\frac{1}{Q^- - P^- - i\varepsilon} - \frac{1}{Q^- - P^- + i\varepsilon} \right] \propto \frac{1}{Q^- - \bar{q}^- + i\varepsilon} \delta(Q^- - P^-)
 \end{aligned} \tag{15}$$

Emission **After** Shockwave

- ▶ Photon vertex:

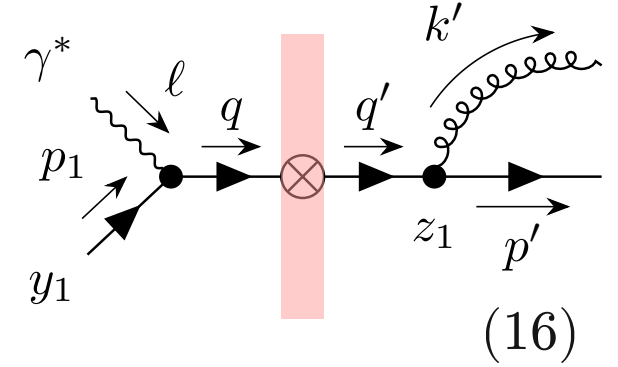
$$\bar{u}(\bar{q}) \not{\epsilon}_\lambda(\ell) u(\bar{p}) = 2\sqrt{p_1^-} \ell^+$$

- ▶ Energy denominator ($q = 0$):

$$p_1^- + \ell^- - \bar{p}^- - \bar{k}^- + i\varepsilon = -\frac{k^2}{2\ell^+ y(1-y)} \quad (17)$$

- ▶ Weizsäcker-Williams wave function:

$$\phi_0^\lambda(\mathbf{u}) = 2i\sqrt{1-y} \frac{\mathbf{u} \cdot \boldsymbol{\varepsilon}_\lambda^*}{2\pi u^2} [\delta_{\lambda,s} + (1-y)\delta_{\lambda,-s}] \quad (18)$$



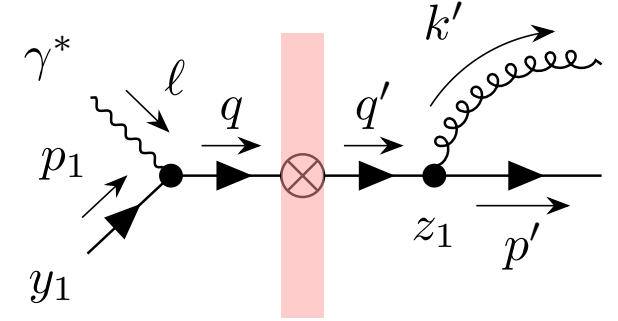
Emission **After** Shockwave

- Weizsäcker-Williams wave function:

$$\phi_0^\lambda(\mathbf{u}) = 2i\sqrt{1-y} \frac{\mathbf{u} \cdot \boldsymbol{\varepsilon}_\lambda^*}{2\pi\mathbf{u}^2} [\delta_{\lambda,s} + (1-y)\delta_{\lambda,-s}] \quad (19)$$

- Amplitude for ordering A:

$$\mathcal{A}_A = -2eg \frac{\sqrt{p_1^- \ell^+}}{p_1^- + \ell^-} \int d^2b d^2r e^{-i\mathbf{b} \cdot \mathbf{p}'} e^{-i\mathbf{r} \cdot \mathbf{k}'} \phi_0^\lambda(\mathbf{u}) t^a \mathcal{U}_F(\mathbf{v})_{\beta\alpha} . \quad (20)$$



Emission **Before** Shockwave

- Off-shell virtuality:

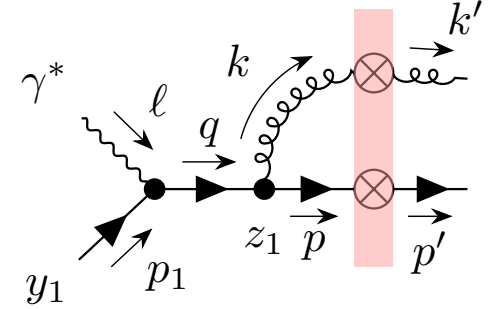
$$M^2 = y(1 - y)(2\ell^+ p_1^- - Q^2)$$

- Energy denominator ($q = 0$):

$$p_1^- + \ell^- - \bar{p}^- - \bar{k}^- + i\varepsilon = -\frac{k^2 - M^2 - i\varepsilon}{2\ell^+ y(1 - y)} \quad (21)$$

- $K_1(\sqrt{-M^2 - i\varepsilon}) \rightarrow i\varepsilon$ selects the side of **Branch cut**

$$K_1(ix) = \begin{cases} H^{(1)}(x) = -\frac{\pi}{2}[J_1(x) + iY_1(x)], & -\pi < \arg(x) < -\frac{\pi}{2} \\ H^{(2)}(x) = -\frac{\pi}{2}[J_1(x) - iY_1(x)], & -\frac{\pi}{2} < \arg(x) < \pi \end{cases} \quad (22)$$



Emission **Before** Shockwave

- $K_1\left(\sqrt{-M^2 - i\varepsilon}\right) \rightarrow i\varepsilon$ selects the side of **Branch cut**

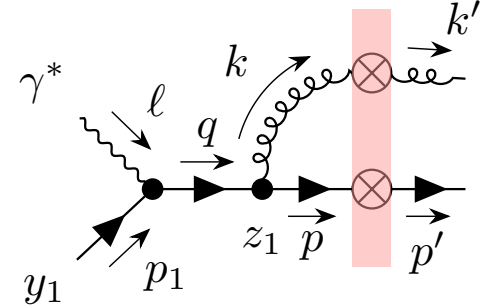
$$K_1(ix) = \begin{cases} H^{(1)}(x) = -\frac{\pi}{2}[J_1(x) + iY_1(x)], & -\pi < \arg(x) < -\frac{\pi}{2} \\ H^{(2)}(x) = -\frac{\pi}{2}[J_1(x) - iY_1(x)], & -\frac{\pi}{2} < \arg(x) < \pi \end{cases} \quad (23)$$

- wave function:

$$\phi_-^\lambda(\mathbf{u}) = i \frac{(-iM)H^{(2)}(-iM|\mathbf{u}|)}{2\pi} \frac{\mathbf{u} \cdot \boldsymbol{\varepsilon}_\lambda^*}{|\mathbf{u}|} 2\sqrt{1-y} [\delta_{\lambda,s} + (1-y)\delta_{\lambda,-s}] \quad (24)$$

- Amplitude:

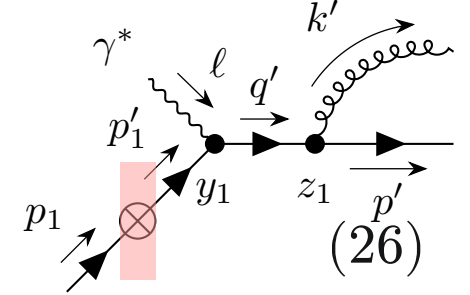
$$\mathcal{A}_B = 2eg \frac{\sqrt{p_1^- \ell^+}}{p_1^- + \ell^-} \int d^2b d^2r e^{-i\mathbf{b} \cdot \mathbf{p}'} e^{-i\mathbf{r} \cdot \mathbf{k}'} \phi_-^\lambda(\mathbf{u}) t^b \mathcal{U}_F(\mathbf{b})_{\beta\alpha} \mathcal{U}_A^{ba}(\mathbf{r}). \quad (25)$$



Shockwave **Before** Photon Vertex

- ▶ Off-shell virtuality:

$$\sqrt{-M^2 + i\varepsilon} = iM$$



- ▶ Energy denominator prescription:

$$p_1^- + \ell^- - p'^- - k'^- - i\varepsilon = -\frac{k'^2 - M^2 + i\varepsilon}{2\ell^+ y(1-y)} \quad (27)$$

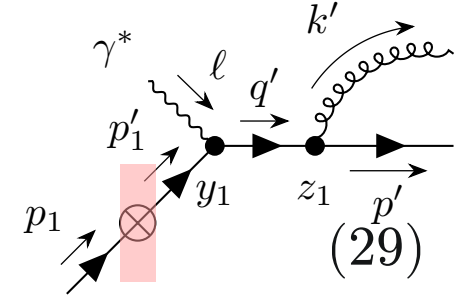
- ▶ Amplitude contains both ϕ_+ and vacuum ϕ_0 :

$$\frac{1}{p_1^- + \ell^- - p'^- - k'^- - i\varepsilon} \frac{\mathcal{N}}{2\ell^+} = - \int d^2\mathbf{r} e^{-i\mathbf{r} \cdot \mathbf{k}'} [\phi_+^\lambda(\mathbf{u}) - \phi_0^\lambda(\mathbf{u})] \quad (28)$$

Shockwave **Before** Photon Vertex

- ▶ Off-shell virtuality:

$$\sqrt{-M^2 + i\varepsilon} = iM$$



- ▶ Energy denominator prescription:

$$p_1^- + \ell^- - p'^- - k'^- - i\varepsilon = -\frac{k'^2 - M^2 + i\varepsilon}{2\ell^+ y(1 - y)} \quad (30)$$

- ▶ Amplitude for ordering C:

$$\mathcal{A}_C = 2eg \frac{\sqrt{p_1^- \ell^+}}{p_1^- + \ell^-} \int d^2 b d^2 r e^{-i\mathbf{b} \cdot \mathbf{p}'} e^{-i\mathbf{r} \cdot \mathbf{k}'} [\phi_0^\lambda(\mathbf{u}) - \phi_+^\lambda(\mathbf{u})] t^a \mathbf{1}_{\beta\alpha}. \quad (31)$$

Total Amplitude

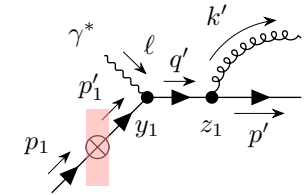
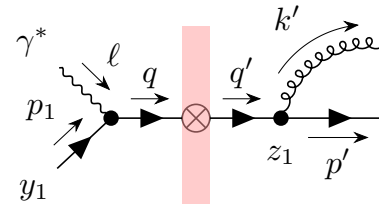
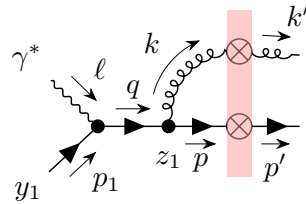
- Use $\mathbf{u} = \mathbf{r} - \mathbf{b}$ and $\mathbf{v} = \mathbf{b} + y(\mathbf{r} - \mathbf{b})$:

$$\mathbf{b} = \mathbf{v} - y\mathbf{u}, \quad \mathbf{r} = \mathbf{v} + (1 - y)\mathbf{u}, \quad (32)$$

- Total amplitude:

$$\mathcal{A}_{\text{total}} = 2eg \frac{\sqrt{p_1^- \ell^+}}{p_1^- + \ell^-} \int d^2\mathbf{u} d^2\mathbf{v} e^{-i\mathbf{v} \cdot (\mathbf{p}' + \mathbf{k}')} e^{-i\mathbf{u} \cdot ((1-y)\mathbf{k}' - y\mathbf{p}')} \quad (33)$$

$$\times \left[\underbrace{\phi_-^\lambda(\mathbf{u}) t^b \mathcal{U}_F(\mathbf{b})_{\beta\alpha} \mathcal{U}_A(\mathbf{r})^{ba}}_{\mathbf{B}} - \underbrace{\phi_0^\lambda(\mathbf{u}) t^a \mathcal{U}_F(\mathbf{v})_{\beta\alpha}}_{\mathbf{A}} + \underbrace{[\phi_0^\lambda(\mathbf{u}) - \phi_+^\lambda(\mathbf{u})] t^a \mathbf{1}_{\beta\alpha}}_{\mathbf{C}} \right]$$



Vacuum Limit

- ▶ Taking $\mathcal{U}_{F/A} \rightarrow 1$:

$$\mathcal{A}_{\text{total}} = 2eg \frac{\sqrt{p_1^- \ell^+}}{p_1^- + \ell^-} \int d^2 \mathbf{u} d^2 \mathbf{v} e^{-i\mathbf{v} \cdot (\mathbf{p}' + \mathbf{k}')} e^{-i\mathbf{u} \cdot ((1-y)\mathbf{k}' - y\mathbf{p}')} t^a [\phi_-^\lambda(\mathbf{u}) - \phi_+^\lambda(\mathbf{u})]. \quad (34)$$

→ On-shell contribution $\neq \phi_0 \rightarrow 0$.

- ▶ Integral $\int d\mathbf{v} \rightarrow (2\pi)^2 \delta^{(2)}(\mathbf{p}' + \mathbf{k}')$, transverse momentum conservation.
- ▶ Difference $\phi_-^\lambda - \phi_+^\lambda \rightarrow$ minus-momentum conservation

$$\phi_-^\lambda - \phi_+^\lambda = -\phi_J^\lambda(\mathbf{u})$$

$$\text{Momentum space} \Rightarrow \frac{1}{\mathbf{k}^2 - M^2 - i\varepsilon} - \frac{1}{\mathbf{k}^2 - M^2 + i\varepsilon} = 2\pi i \delta(\mathbf{k}^2 - M^2) \quad (35)$$

Vacuum Subtraction

- ▶ The vacuum square contains the usual square of an S -matrix delta function:

$$\int d^2u d^2\bar{u} e^{-il \cdot u} e^{il \cdot \bar{u}} \phi_J^\lambda(u) \phi_J^{\lambda*}(\bar{u}) \propto [\delta(l^2 - M^2)]^2 \quad (36)$$

- ▶ Organize the finite medium-dependent object as

$$\langle |\mathcal{A}|^2 \rangle - |\mathcal{A}_{\text{vac}}|^2 = \langle |\mathcal{A}_{\text{med}}|^2 \rangle + 2 \text{Re} \langle \mathcal{A}_{\text{vac}} \mathcal{A}_{\text{med}}^\dagger \rangle, \quad \mathcal{A}_{\text{med}} = \mathcal{A} - \mathcal{A}_{\text{vac}}. \quad (37)$$

Medium Average

- Dipole and multipole correlators:

$$\begin{aligned} S_{q\bar{q}}^2(\mathbf{v}, \bar{\mathbf{v}}) &= \frac{1}{N_c} \langle \text{Tr} [\mathcal{U}_{F(\mathbf{v})} \mathcal{U}_F^{\dagger(\bar{\mathbf{v}})}] \rangle \\ S_{qg\bar{q}}^3(\mathbf{b}, \mathbf{r}, \bar{\mathbf{v}}) &= \frac{1}{C_F N_c} \langle \text{Tr} [t^b \mathcal{U}_{F(\mathbf{b})} t^a \mathcal{U}_F^{\dagger(\bar{\mathbf{v}})}] \mathcal{U}_{A(\mathbf{r})}^{ba} \rangle \\ S_{qg\bar{q}g}^4(\mathbf{b}, \bar{\mathbf{b}}, \mathbf{r}, \bar{\mathbf{r}}) &= \frac{1}{C_F N_c} \langle \text{Tr} [\mathcal{U}_{F(\mathbf{b})} \mathcal{U}_F^{\dagger(\bar{\mathbf{b}})} t^a t^b] \mathcal{U}_{A(\mathbf{r})}^{ac} \mathcal{U}_A^{\dagger(\bar{\mathbf{r}})cb} \rangle \end{aligned} \tag{38}$$

Square of the Amplitude

$$\begin{aligned} \langle |\mathcal{A}_{\text{med}}|^2 \rangle &= 4e^2 g^2 C_F N_c \frac{p_1^- \ell^+}{(p_1^- + \ell^-)^2} \int d^2 \boldsymbol{v} d^2 \bar{\boldsymbol{v}} d^2 \boldsymbol{u} d^2 \bar{\boldsymbol{u}} e^{-i(\boldsymbol{v} - \bar{\boldsymbol{v}}) \cdot (\boldsymbol{p}' + \boldsymbol{k}')} e^{-i(\boldsymbol{u} - \bar{\boldsymbol{u}}) \cdot ((1-y)\boldsymbol{k}' - y\boldsymbol{p}')} \\ &\times \left\{ \phi_0^\lambda(\boldsymbol{u}) \phi_0^{\lambda*}(\bar{\boldsymbol{u}}) [S^4(\boldsymbol{b}, \bar{\boldsymbol{b}}, \boldsymbol{r}, \bar{\boldsymbol{r}}) - S^3(\boldsymbol{b}, \boldsymbol{r}, \bar{\boldsymbol{v}}) - S^3(\bar{\boldsymbol{b}}, \bar{\boldsymbol{r}}, \boldsymbol{v}) + S^2(\boldsymbol{v}, \bar{\boldsymbol{v}})] \right\} \end{aligned} \quad (39)$$

Square of the Amplitude

$$\begin{aligned}
 \langle |\mathcal{A}_{\text{med}}|^2 \rangle &= 4e^2 g^2 C_F N_c \frac{p_1^- \ell^+}{(p_1^- + \ell^-)^2} \int d^2 v d^2 \bar{v} d^2 u d^2 \bar{u} e^{-i(v-\bar{v}) \cdot (p' + k')} e^{-i(u-\bar{u}) \cdot ((1-y)k' - y p')} \\
 &\times \left\{ \phi_0^\lambda(u) \phi_0^{\lambda*}(\bar{u}) [S^4(b, \bar{b}, r, \bar{r}) - S^3(b, r, \bar{v}) - S^3(\bar{b}, \bar{r}, v) + S^2(v, \bar{v})] \right. \\
 &\quad + [\phi_-^\lambda(u) - \phi_0^\lambda(u)] \phi_0^{\lambda*}(\bar{u}) [S^4(b, \bar{b}, r, \bar{r}) - S^3(b, r, \bar{v})] \\
 &\quad + \phi_0^\lambda(u) [\phi_-^{\lambda*}(\bar{u}) - \phi_0^{\lambda*}(\bar{u})] [S^4(b, \bar{b}, r, \bar{r}) - S^3(\bar{b}, \bar{r}, v)] \\
 &\quad \left. + [\phi_-^\lambda(u) - \phi_0^\lambda(u)] [\phi_-^{\lambda*}(\bar{u}) - \phi_0^{\lambda*}(\bar{u})] [S^4(b, \bar{b}, r, \bar{r}) + 1] \right\}.
 \end{aligned} \tag{40}$$

Square of the Amplitude

$$\begin{aligned}
 \langle |\mathcal{A}_{\text{med}}|^2 \rangle &= 4e^2 g^2 C_F N_c \frac{p_1^- \ell^+}{(p_1^- + \ell^-)^2} \int d^2 v d^2 \bar{v} d^2 u d^2 \bar{u} e^{-i(v-\bar{v}) \cdot (p' + k')} e^{-i(u-\bar{u}) \cdot ((1-y)k' - y p')} \\
 &\times \left\{ \phi_0^\lambda(u) \phi_0^{\lambda*}(\bar{u}) [S^4(b, \bar{b}, r, \bar{r}) - S^3(b, r, \bar{v}) - S^3(\bar{b}, \bar{r}, v) + S^2(v, \bar{v})] \right. \\
 &\quad + [\phi_-^\lambda(u) - \phi_0^\lambda(u)] \phi_0^{\lambda*}(\bar{u}) [S^4(b, \bar{b}, r, \bar{r}) - S^3(b, r, \bar{v})] \\
 &\quad + \phi_0^\lambda(u) [\phi_-^{\lambda*}(\bar{u}) - \phi_0^{\lambda*}(\bar{u})] [S^4(b, \bar{b}, r, \bar{r}) - S^3(\bar{b}, \bar{r}, v)] \\
 &\quad \left. + [\phi_-^\lambda(u) - \phi_0^\lambda(u)] [\phi_-^{\lambda*}(\bar{u}) - \phi_0^{\lambda*}(\bar{u})] [S^4(b, \bar{b}, r, \bar{r}) + 1] \right\}. \tag{41}
 \end{aligned}$$

► Setting $M \rightarrow 0$ gives $[\phi_-^\lambda(u) - \phi_0^\lambda(u)] \rightarrow 0$

→ recovering [Marquet Nucl. Phys. A 796, 41–60 2007]

Relevant Scales

- ▶ The wave function

$$\phi_-^\lambda(\boldsymbol{u}) \propto M[J_1(Mu) - iY_1(Mu)] \frac{\boldsymbol{u} \cdot \boldsymbol{\varepsilon}_\lambda^*}{u}, \quad (42)$$

→ Selects transverse size $u \approx 1/M$.

- ▶ Dipole correlator

$$S(u) \approx \exp\left(-Q_s^2 \frac{u^2}{4}\right). \quad (43)$$

→ Resolves transverse sizes $u \approx 1/Q_s$.

Small Virtuality $M^2 \ll Q_s^2$

- ▶ Dipole correlators sets transverse size

$$u \approx 1/Q_s \quad \Rightarrow \quad Mu \ll 1. \quad (44)$$

- ▶ Small- Mu expansion:

$$J_1(M|u|) \approx 0, \quad Y_1(M|u|) \approx -\frac{2}{\pi M|u|}, \quad \Rightarrow \quad \phi_-^\lambda(\mathbf{u}) = \phi_0^\lambda(\mathbf{u}). \quad (45)$$

The square matrix element reduces to the on-shell case.

$$\phi_0^\lambda(u) \phi_0^{\lambda*}(\bar{u}) [S^4(b, \bar{b}, r, \bar{r}) - S^3(b, r, \bar{v}) - S^3(\bar{b}, \bar{r}, v) + S^2(v, \bar{v})] \quad (46)$$

High Virtuality $M^2 \gg Q_s^2$

- ▶ Large- Mu Hankel phase:

$$H_1^{(2)}(M|u|) \sim \frac{e^{-iM|u|}}{\sqrt{M|u|}}, \quad M \gg Q_s \Rightarrow u \approx 1/M \ll 1/Q_s. \quad (47)$$

- ▶ Small dipole:

$$S(u) = \exp\left(-Q_s^2 \frac{u^2}{4}\right) \approx 1 - Q_s^2 \frac{u^2}{4}, \quad 1 - S(u) \approx Q_s^2/(4M^2). \quad (48)$$

- ▶ Medium correction:

$$\frac{|\mathcal{A}_{\text{med}}|^2}{|\mathcal{A}_{\text{vac}}|^2} \approx \mathcal{O}\left(\frac{Q_s^2}{M^2}\right). \quad (49)$$

- ▶ Vacuum-like radiation dominates;
→ Suppressed medium corrections.

Intermediate Virtuality $M^2 \approx Q_s^2$

- ▶ Two transverse scales

$$u \approx 1/Q_s \approx 1/M, \quad Mu \approx 1. \quad (50)$$

- ▶ Important contributions from all terms

$$\begin{aligned} & \phi_0^\lambda(u) \phi_0^{\lambda*}(\bar{u}) [S^4(b, \bar{b}, r, \bar{r}) - S^3(b, r, \bar{v}) - S^3(\bar{b}, \bar{r}, v) + S^2(v, \bar{v})] \\ & + [\phi_-^\lambda(u) - \phi_0^\lambda(u)] \phi_0^{\lambda*}(\bar{u}) [S^4(b, \bar{b}, r, \bar{r}) - S^3(b, r, \bar{v})] \\ & + \phi_0^\lambda(u) [\phi_-^{\lambda*}(\bar{u}) - \phi_0^{\lambda*}(\bar{u})] [S^4(b, \bar{b}, r, \bar{r}) - S^3(\bar{b}, \bar{r}, v)] \\ & + [\phi_-^\lambda(u) - \phi_0^\lambda(u)] [\phi_-^{\lambda*}(\bar{u}) - \phi_0^{\lambda*}(\bar{u})] [S^4(b, \bar{b}, r, \bar{r}) + 1]. \end{aligned} \quad (51)$$

- ▶ Interplay between Glasma scales and virtuality.

Conclusions

Takeaways

- ▶ Off-shell radiation introduces a new hard scale M^2 in the shockwave calculation.
- ▶ The glasma resolves transverse sizes $u \approx 1/Q_s$:

$$M^2 \ll Q_s^2 \Rightarrow \text{on-shell limit}, \quad M^2 \gg Q_s^2 \Rightarrow \text{color transparency.} \quad (52)$$

- ▶ The glasma selects part of early shower with virtuality of order $\sim Q_s^2$.

Outlook

- ▶ Compute the medium-induced spectrum $\frac{dN}{dy d^2\mathbf{k}'}$ and compare to on-shell results.
- ▶ Understand interference term $2 \operatorname{Re} \langle \mathcal{A}_{\text{vac}} \mathcal{A}_{\text{med}}^\dagger \rangle$.

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Backup

Wave Functions

- Momentum-space relation to the vacuum wave function:

$$\begin{aligned}\phi_0^\lambda(\mathbf{k}) &= 2\sqrt{1-y}[\delta_{\lambda,s} + (1-y)\delta_{\lambda,-s}] \frac{\mathbf{k} \cdot \boldsymbol{\varepsilon}_\lambda^*}{k^2} \\ \phi_-^\lambda(\mathbf{k}) &= \frac{k^2}{k^2 - M^2 - i\varepsilon} \phi_0^\lambda(\mathbf{k}) \\ \phi_+^\lambda(\mathbf{k}) &= \frac{k^2}{k^2 - M^2 + i\varepsilon} \phi_0^\lambda(\mathbf{k})\end{aligned}\tag{53}$$

- Coordinate-space

$$\phi_-^\lambda(\mathbf{u}) = \phi_Y^\lambda(\mathbf{u}) - \frac{1}{2}\phi_J^\lambda(\mathbf{u}), \phi_+^\lambda(\mathbf{u}) = \phi_Y^\lambda(\mathbf{u}) + \frac{1}{2}\phi_J^\lambda(\mathbf{u})\tag{54}$$

Wave Functions

$\phi_Y^\lambda \rightarrow \phi_0^\lambda$ as $M^2 \rightarrow 0$, while ϕ_J^λ is the on-shell discontinuity.