DGLAP resummation in JIMWLK

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A. Kovner, M. Lublinsky, V. V. Skokov and Z. Zhao, arXiv:2308.15545 [hep-ph], in JHEP

A. Kovner, M. Lublinsky, M. Nefedov, V. Skokov arXiv: 2601.02131 [hep-ph], in JHEP?

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High Energy Scattering

Target $(\rho^{t} = \rho^{-},\ Q^{T})$ **Projectile** $(\rho^{p} = \rho^{+},\ Q^{P})$

$$\langle \mathbf{T}| \quad \rightarrow \qquad\qquad \leftarrow \quad |\mathbf{P}\rangle \qquad\qquad \rho^{+} \sim \int \mathbf{dk^{+} a^{\dagger}\, T\, a}$$

Any observable $\hat{\mathcal{O}}(\rho^{t},\ \rho^{p})$

$$\langle \hat{\mathcal{O}} \rangle_{\mathbf{Y}} \; = \; \langle \mathbf{T}\, \langle\, \mathbf{P}| \;\; \hat{\mathcal{O}}(\rho^{t},\ \rho^{p}) \;\; |\mathbf{P}\rangle\, \mathbf{T}\rangle \qquad\qquad \mathbf{Y} \sim \ln(\mathrm{s})$$

How do these averages change with increase in energy of the process?

$$\partial_{\mathbf{Y}} \langle \hat{\mathcal{O}} \rangle_{\mathbf{Y}} \; = -\mathcal{H}\, \langle \hat{\mathcal{O}} \rangle_{\mathbf{Y}} \qquad\qquad \mathcal{H} \;\rightarrow\; \text{the HE effective Hamiltonian}$$

$\mathcal{H}$ **defines the high energy limit of QCD and is universal**

$$\mathcal{H} \; = \; \mathcal{H}_{\mathbf{LO}}(\alpha_{\mathrm{s}}) \, + \, \mathcal{H}_{\mathbf{NLO}}(\alpha_{\mathrm{s}}^{\mathbf{2}}) \, + \ldots ; \qquad\qquad \mathcal{H} \; = \; \mathcal{H}[\rho^{t},\ \delta/\delta\rho^{t}]$$

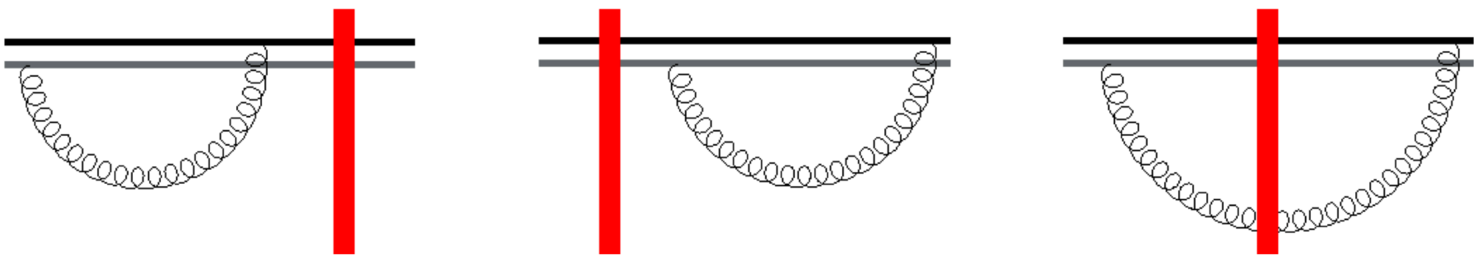
JIMWLK Hamiltonian is a limit of $\mathcal{H}$ for dilute partonic system ($\rho^{p} \rightarrow \mathbf{0}$) which scatters on a dense target. It accounts for linear gluon emission + multiple rescatterings.

LO JIMWLK Hamiltonian

Jalilian Marian, Iancu, McLerran, Weigert, Leonidov, Kovner (1997-2002)

$$\mathcal{H}^{\mathrm{JIMWLK}}_{\mathrm{LO}} = \int_{x,y,z} K_{\mathrm{LO}} \left\{ J^a_L(x) J^a_L(y) + J^a_R(x) J^a_R(y) - 2 J^a_L(x) S^{ab}_A(z) J^b_R(y) \right\}$$

$$K_{\mathrm{LO}}(x, y, z) = \frac{\alpha_s}{2\,\pi^2} \frac{(x-z)_i (y-z)_i}{(x-z)^2 (y-z)^2}$$



$$S_A{}^{cd}(z) = \mathcal{P} \exp\left\{ i \int dx^+\, T^a\, \alpha^a(z, x^+) \right\}^{cd}. \qquad ``\Delta"\alpha = \rho_t \qquad (\mathrm{YM})$$

Here $\rho^p \to J_L$ and $\hat{S}\rho^p \to J_R$ are left and right SU(N) generators:

$$J^a_L(x) S^{ij}_A(z) = (T^a S_A(z))^{ij}\, \delta^2(x-z) \qquad J^a_R(x) S^{ij}_A(z) = (S_A(z) T^a)^{ij}\, \delta^2(x-z)$$

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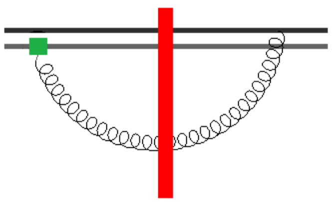
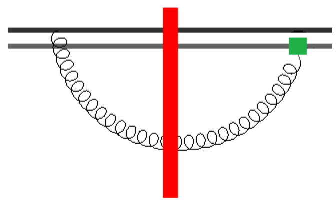
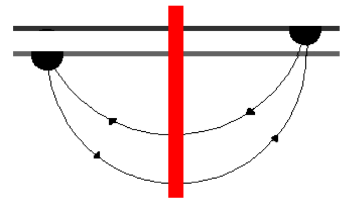
JIMWLK Hamiltonian @ NLO

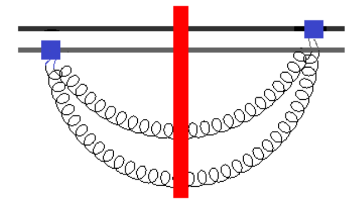
Kovner, ML & Mulian (2013) based on Balitsky & Chirilli (2007), Grabovsky (2013); ML & Mulian (2016)







(a)



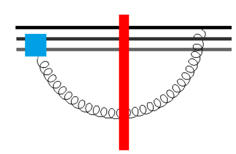
(b)

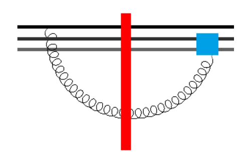
$$\mathcal{H}^{NLO\ JIMWLK} = \int_{x,y,z} K_{JSJ}(x,y;z)\left[J_L^a(x)J_L^a(y) + J_R^a(x)J_R^a(y) - 2J_L^a(x)S_A^{ab}(z)J_R^b(y)\right]$$

$$+ \int_{x\,y\,z\,z'} K_{JSSJ}(x,y;z,z')\left[f^{abc}f^{def}J_L^a(x)S_A^{be}(z)S_A^{cf}(z')J_R^d(y) - N_c J_L^a(x)S_A^{ab}(z)J_R^b(y)\right]$$

$$+ \int_{x,y,z,z'} K_{q\bar{q}}(x,y;z,z')\left[2\,J_L^a(x)\,tr[S_F^\dagger(z)\,t^a\,S_F(z')t^b]\,J_R^b(y) \;-\; J_L^a(x)\,S_A^{ab}(z)\,J_R^b(y)\right]$$

$$+ \int_{w,x,y,z,z'} K_{JJSSJ}(w;x,y;z,z')f^{acb}\left[J_L^d(x)\,J_L^e(y)\,S_A^{dc}(z)\,S_A^{eb}(z')\,J_R^a(w) - J_L^a(w)\,S_A^{cd}(z)\,S_A^{be}(z')\,J_R^d(x)\,J_R^e(y)\right]$$
$$+ \int_{w,x,y,z} K_{JJSJ}(w;x,y;z)\,f^{bde}\left[J_L^d(x)\,J_L^e(y)\,S_A^{ba}(z)\,J_R^a(w) \;-\; J_L^a(w)\,S_A^{ab}(z)\,J_R^d(x)\,J_R^e(y)\right]$$
$$+ \int_{w,x,y} K_{JJJ}(w;x,y)f^{deb}\left[J_L^d(x)\,J_L^e(y)\,J_L^b(w) \;-\; J_R^d(x)\,J_R^e(y)\,J_R^b(w)\right].$$





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Motivation and Objectives

Precise saturation physics phenomenology at NLO is badly needed.

The JIMWLK Hamiltonian at NLO is known for some years, but there are problems there.

- No known recipe for numerical evaluation
- Large transverse logarithms emerge: $\mathcal{H} \sim \alpha_s(\# + \alpha_s(\# + \mathbf{Log}))$,
  If the Log is large, then $\alpha_s \mathbf{Log} \sim \mathbf{1}$ – not a small correction to LO
  All have to be identified, clearly separated, and independently resummed.

Proper resummation requires understanding of physics beyond NLO!

- Running coupling effects (UV divergent) – rcJIMWLK
- DGLAP logs: Large transverse logs of the $\log(\mathbf{Q^T}/\mathbf{Q^P})$ type

1) Anti-collinear single logs ($Q^T \gg Q^P$) $\rightarrow$ DGLAP of the projectile
2) Collinear single and double logs ($Q^T \ll Q^P$) $\rightarrow$ DGLAP of the target

Large UV Logs in NLO Kernels

$$\mathbf{X} = \mathbf{x} - \mathbf{z}$$
$$Y = y - z$$

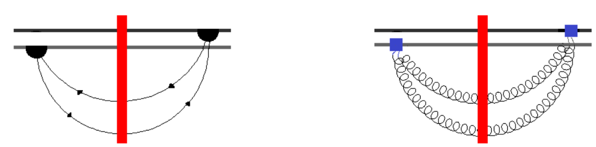
$$\mathbf{K_{JSJ}}(\mathbf{b}\,\mathrm{terms}) = \frac{\alpha_s^2}{16\pi^3}\left\{-\mathbf{b}\frac{(\mathbf{x}-\mathbf{y})^2}{\mathbf{X}^2\mathbf{Y}^2}\ln(\mathbf{x}-\mathbf{y})^2\mu^2 + \frac{\mathbf{b}}{\mathbf{X}^2}\ln\mathbf{Y}^2\mu^2 + \frac{\mathbf{b}}{\mathbf{Y}^2}\ln\mathbf{X}^2\mu^2\right\} + \cdots$$

Here μ is the normalization point, $\mathbf{b} = \frac{11}{3}\mathbf{N_c} - \frac{2}{3}\mathbf{n_f}$, $\quad \mathbf{b}\,\ln\mu_*^2/\mu^2 \to \alpha_s(\mu_*^2)$
Huge ambiguity in identifying μ_*

Resum large Logs into an effective kernel $\quad \mathbf{K_{LO}} = \frac{\alpha_s}{2\pi^2}\frac{\mathbf{XY}}{\mathbf{X^2Y^2}} \to \mathbf{K_{rc}} = \frac{\alpha_s[\mathrm{running}]}{2\pi^2}\frac{\mathbf{XY}}{\mathbf{X^2Y^2}}$

$$\int_{\mathbf{x\,y\,z,z'}}\mathbf{K_{JSSJ}}(\mathbf{x},\mathbf{y};\mathbf{z},\mathbf{z}')\,\mathbf{J_L^a}(\mathbf{x})\mathbf{J_R^b}(\mathbf{y})\,\left[\,\mathbf{D^{ab}}(\mathbf{z},\mathbf{z}')\,\right] \quad \sim \; \mathbf{b}\,\mathrm{imes}\,(\mathrm{UV\ divergent\ Log})$$

$$\mathbf{D^{ab}}(\mathbf{z},\mathbf{z}') \equiv \mathbf{Tr}[\mathbf{T^aS_A}(\mathbf{z})\mathbf{T^bS_A^\dagger}(\mathbf{z}')]$$



(a) (b)

The UV divergence in $JSSJ$ is trivial: when the two gluons are too close to each other ($z \sim z'$), they cannot be resolved by the target and hence should be counted as a single gluon scattering. We are thus prompted to introduce a "resolution scale" Q

Dressed Wilson line

Within the finite resolution Q, bare gluons $\to$ dressed gluons, bare Wilson lines $\to$ *dressed Wilson lines*, $\mathbf{S} \to \mathbf{S_Q}$

$$\mathbf{S^{ab}_Q(z) = S^{ab}_A(z) + \frac{\alpha_s}{2\pi^2}\int_0^1 d\xi\, \sigma(\xi) \int^{Q^{-1}} \frac{d^2Z}{Z^2}\left(D^{ab}(z+(1-\xi)Z, z-\xi Z) \,-\, N_c\, S^{ab}_A(z)\right)}$$

ξ is the fraction of longitudinal momentum carried by one of the gluons.

$$\sigma(\xi) = \left[\frac{1}{\xi(1-\xi)}\left(\xi^2 + (1-\xi)^2 + \xi^2(1-\xi)^2\right)\right]_+ ; \quad \mathbf{2N_c}\int_0^1 d\xi\sigma(\xi) \,=\, -\frac{\mathbf{11N_c}}{\mathbf{3}} \to -\mathrm{b}$$

This is the P_{gg} splitting function except that we introduce the "+" prescription both for $\xi = 1$ and $\xi = 0$ poles The "+" prescription emerges from the $1/\xi$ subtraction absorbed into (LO)^2 part of the evolution.

The sign is negative – correcting for the over-subtraction in the LO.

We go beyond the usual DGLAP: we allow simultaneous scattering of all gluons.

For $\mathrm{Q} > \mathrm{Q^T}$, $\mathrm{S_Q} \simeq \mathrm{S_A}$ - the target does not resolve gluon splitting at distances smaller than $1/\mathrm{Q^T}$.

Anti-collinear DGLAP resummation

$\mathbf{S_Q \sim S_A\,[1+\alpha_s\,\#\,Log(Q^2/Q^T)]}$. At $Q = Q^P$, $\mathbf{S_Q}$ is very different from $\mathbf{S_A}$. This large Log has to be resummed via inclusion of multiple consecutive DGLAP splittings:

$$\frac{\partial \mathbf{S_Q(z)}}{\partial \ln \mathbf{Q}} = -\frac{\alpha_s(\mathbf{Q^2})}{2\pi^2}\int_\xi \sigma(\xi)\int_{\phi_\mathbf{Q}}[\mathbf{D_Q(z)} - \mathbf{N_c\,S_Q(z)}]$$

$$\mathbf{D_Q(z_1,z_2) \equiv Tr[T^a S_Q(z_1) T^b S^+_Q(z_2)]}$$

Resummed JIMWLK kernel with running coupling

$$\mathcal{H}^{\rm res.}_{\rm JIMWLK} = \int_{\mathbf{x,y,z}} \mathbf{K(x,y;z)}\left[\mathbf{J^a_L(x)\,J^a_L(y) + J^a_R(x)J^a_R(y) - 2J^a_L(x)\,S^{ab}_{Q*}(z)\,J^b_R(y)}\right]$$

$$\mathbf{K(x,y,z)} = \frac{\alpha_s(\mu_*^2)}{2\pi^2}\frac{\mathbf{XY}}{\mathbf{X^2Y^2}}; \qquad \mu_\star^2 = \min(\mathbf{X^{-2},Y^{-2}}); \qquad \mathbf{Q^2_\star} = \max(\mathbf{X^{-2},Y^{-2}})$$

Weak target field approximation – linearization

$$\mathbf{S^{ab}_Q(x)} = \delta^{\mathbf{ab}} + \mathbf{igT^{c_1}_{ab}}\int_{\mathbf{z_1}} \mathbf{R^{(1)}_Q(x-z_1)}\alpha^{\mathbf{c_1}}(\mathbf{z_1})$$
$$+\frac{\mathbf{(ig)^2}}{\mathbf{2}}\mathbf{T^{ab}_{c_1c_2}}\int_{\mathbf{z_1,z_2}} \mathbf{R^{(2)}_Q(x-z_1,x-z_2)}\alpha^{\mathbf{c_1}}(\mathbf{z_1})\alpha^{\mathbf{c_2}}(\mathbf{z_2}) + \mathbf{O(g^3)},$$

DGLAP-like equation

$$\frac{\partial}{\partial \ln Q^2}\mathcal{R}^{(2)}_Q(p) = -\alpha_s(Q^2)\left[\Pi_2\mathcal{R}^{(2)}_Q(p) + source\right],$$

$$\mathcal{R}^{(2)}_{\mathbf{Q}}(\mathbf{p}) \equiv \begin{pmatrix} R^{(2)}_Q(p) \\ r^{(2)}_Q(p) \end{pmatrix} \qquad \mathbf{\Pi_2} = \begin{pmatrix} \beta_0 - 2N_c\frac{11}{3} & \frac{4C_F n_F}{3N_c} \\ -3N_c & 0 \end{pmatrix}$$

Solution at LLA

$$\mathbf{R^{(2),LLA}_Q(p)} \sim \left(\frac{\alpha_s(\max[\mathbf{Q^2},\mathbf{p^2}])}{\alpha_s(\mathbf{Q^2})}\right)^{-\lambda_-/\beta_0} + (\lambda_+ \leftrightarrow \lambda_-),$$

$$\lambda_\pm = \frac{\beta_0}{2} - \frac{11}{3}N_c \pm \sqrt{\left(\frac{\beta_0}{2} - \frac{11}{3}N_c\right)^2 - 4C_F n_F}$$

Linearized JIMWLK Hamiltonian with resummation

$$\hat{\mathbf{H}}^{(\mathrm{res.lin.})}_{\mathbf{JIMWLK}} = \int_{\mathbf{x},\mathbf{y}} \Bigg\{ \frac{\mathbf{f}^{[\mathbf{c_1 a b_1}}\mathbf{f}^{\mathbf{c_2}]\mathbf{a b_2}}}{\mathbf{N_c}} \Bigg[- \int_{\mathbf{z}} \bar{\mathbf{K}}(\mathbf{x},\mathbf{y},\mathbf{z}) \alpha^{\mathbf{c_1}}(\mathbf{x}) \alpha^{\mathbf{c_2}}(\mathbf{y}) - \int_{\mathbf{z_1},\mathbf{z_2}} \bar{\mathbf{K}}^{(2)}(\mathbf{x},\mathbf{y},\mathbf{z_1},\mathbf{z_2}) \alpha^{\mathbf{c_1}}(\mathbf{z_1}) \alpha^{\mathbf{c_2}}(\mathbf{z_2}) + \int_{\mathbf{z}} \bar{\mathbf{K}}^{(1)}(\mathbf{x},\mathbf{y},\mathbf{z}) \Big(\alpha^{\mathbf{c_1}}(\mathbf{z}) \alpha^{\mathbf{c_2}}(\mathbf{x}) + \alpha^{\mathbf{c_1}}(\mathbf{z}) \alpha^{\mathbf{c_2}}(\mathbf{y}) \Big) \Bigg] \frac{\delta^2}{\delta\alpha^{\mathbf{b_1}}(\mathbf{x}) \delta\alpha^{\mathbf{b_2}}(\mathbf{y})} \Bigg\}$$

The resummed kernels

$$\bar{\mathbf{K}}(\mathbf{x},\mathbf{y},\mathbf{z}) = \frac{\alpha_s(\mu^2_\star(\mathbf{x},\mathbf{y},\mathbf{z}))}{2\pi^2} \frac{\mathbf{X}\mathbf{Y}}{\mathbf{X}^2\mathbf{Y}^2}$$

$$\bar{\mathbf{K}}^{(1)}(\mathbf{x},\mathbf{y},\mathbf{z}) = \int_{\bar{\mathbf{z}}} \frac{\alpha_s(\mu^2_\star(\mathbf{x},\mathbf{y},\bar{\mathbf{z}}))}{2\pi^2} \frac{\mathbf{X}\mathbf{Y}}{\mathbf{X}^2\mathbf{Y}^2} \mathbf{R}^{(1)}_{\mathbf{Q}_\star(\mathbf{x},\mathbf{y},\mathbf{z})}(\bar{\mathbf{z}} - \mathbf{z})$$

$$\bar{\mathbf{K}}^{(2)}(\mathbf{x},\mathbf{y},\mathbf{z_1},\mathbf{z_2}) = \int_{\mathbf{z}} \frac{\alpha_s(\mu^2_\star(\mathbf{x},\mathbf{y},\mathbf{z}))}{2\pi^2} \frac{\mathbf{X}\mathbf{Y}}{\mathbf{X}^2\mathbf{Y}^2} \mathbf{R}^{(2)}_{\mathbf{Q}_\star(\mathbf{x},\mathbf{y},\mathbf{z})}(\mathbf{z} - \mathbf{z_1}, \mathbf{z} - \mathbf{z_2})$$

BFKL equation with DGLAP resummation

$$\langle \hat{\mathbf{H}}^{(\mathrm{res.lin.})}_{\mathbf{JIMWLK}}\,(\alpha^{\mathbf{a}}(\mathbf{x})-\alpha^{\mathbf{a}}(\mathbf{y}))^{\mathbf{2}}\rangle_{\mathbf{Y}} = \int_{\mathbf{z_1},\mathbf{z_2}} \mathbf{K}^{(\mathrm{res.})}_{\mathbf{D}}(\mathbf{x},\mathbf{y},\mathbf{z_1},\mathbf{z_2})\langle(\alpha^{\mathbf{a}}(\mathbf{z_1})-\alpha^{\mathbf{a}}(\mathbf{z_2}))^{\mathbf{2}}\rangle_{\mathbf{Y}}$$

$\mathbf{K}^{(\mathrm{res.})}_{\mathbf{D}}$ – **includes anti-collinear resummation and running of the coupling.**

Relating the field correlators to charge correlators

$$\mathbf{k}^{\mathbf{4}}\langle\alpha^{\mathbf{a}}(\mathbf{k})\alpha^{\mathbf{a}}(-\mathbf{k})\rangle_{\mathbf{Y}} = \mathbf{g}^{\mathbf{2}}(\mathbf{k}^{\mathbf{2}})\langle\rho^{\mathbf{a}}_{\mathbf{T}}(\mathbf{k})\rho^{\mathbf{a}}_{\mathbf{T}}(-\mathbf{k})\rangle_{\mathbf{Y}}.$$

momentum-space BFKL equation for the $\langle\rho^a_T(k)\rho^a_T(-k)\rangle_Y$-correlator:

$$\frac{\partial}{\partial\mathbf{Y}}\langle\rho^{\mathbf{a}}_{\mathbf{T}}(\mathbf{k})\rho^{\mathbf{a}}_{\mathbf{T}}(-\mathbf{k})\rangle_{\mathbf{Y}} = -\int_{\mathbf{q}} \mathbf{K}^{(\mathrm{res.})}_{\mathbf{BFKL}}(\mathbf{k},\mathbf{q})\langle\rho^{\mathbf{a}}_{\mathbf{T}}(\mathbf{q})\rho^{\mathbf{a}}_{\mathbf{T}}(-\mathbf{q})\rangle_{\mathbf{Y}},$$

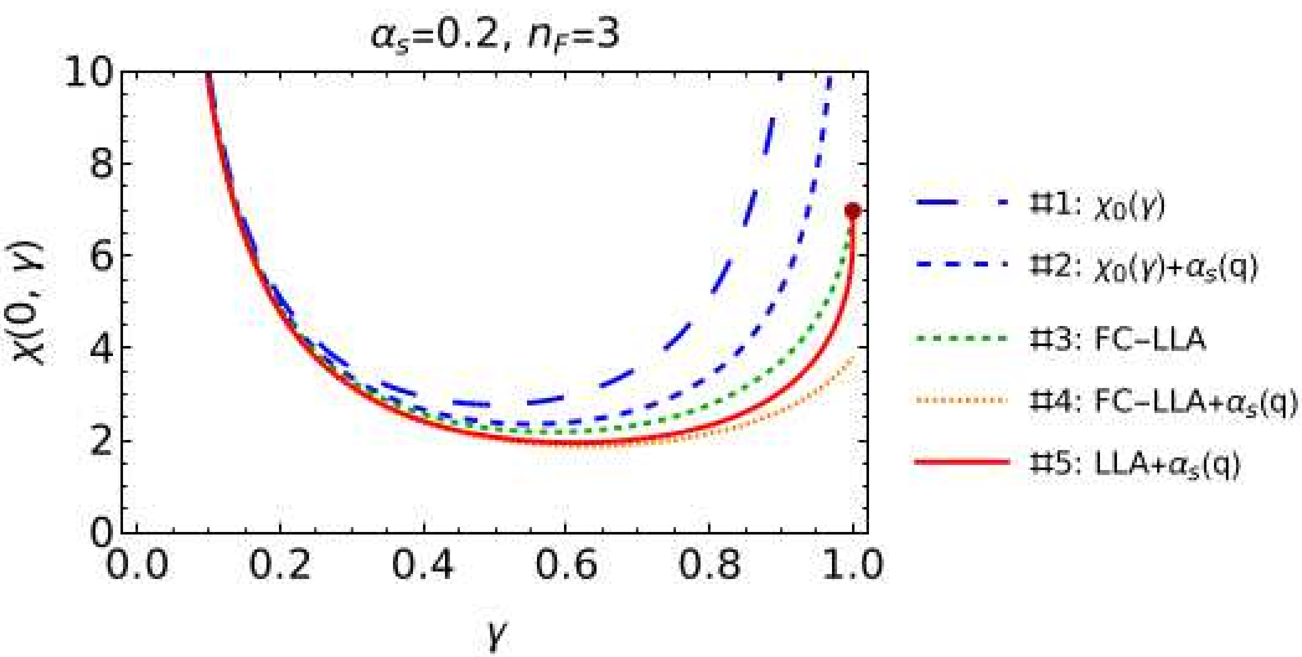
Generalized Characteristic Function

$$\int_{\mathbf{q}} \mathbf{K_{BFKL}(k,q)(q^2)^{\gamma} e^{in\phi_q}} = -\frac{\alpha_s(\mathbf{k^2})\mathbf{N_c}}{\pi}(\mathbf{k^2})^{\gamma}\mathbf{e^{in\phi_k}}\chi(\mathbf{n},\gamma,\alpha_s(\mathbf{k^2}))\,.$$

$$\chi(\mathbf{n},\gamma,\alpha_s) = \chi_0(\mathbf{n},\gamma) + \sum_{\mathbf{m=1}}^{\infty}(\alpha_s \mathbf{N_c})^{\mathbf{m}}\chi_{\mathbf{m}}(\mathbf{n},\gamma).$$

LO BFKL

LLA: $\ln \mathbf{q^2/k^2} \gg \mathbf{0}$



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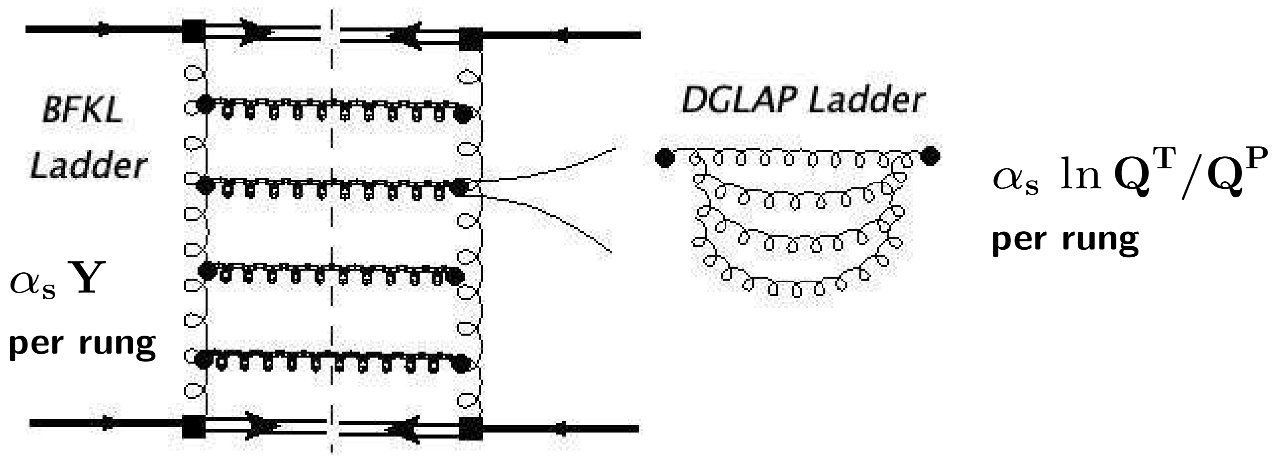
Summary/Outlook

- **Anti-collinear DGLAP resummation inside the JIMWLK Hamiltonian has been performed, including the effects of the running coupling both in the JIMWLK and DGLAP evolutions.**



- **Compared to LO BFKL, the characteristic function is strongly modified close to $\gamma = 1$. The pole at $\gamma = 1$ present in LO BFKL, disappears. The value of the characteristic function for pure glue theory is $\chi(n = 0, \gamma = 1) = \frac{12}{11}\frac{\pi}{4\alpha_s N_c}$, while $\chi(n = 0, \gamma = 1) = \frac{4}{3}\frac{\pi}{4\alpha_s N_c}$ for QCD with 3 massless quarks.**

  The resummed value of $\chi(n = 0, \gamma = 1)$ we obtained differs by a factor $12/11$ (in the pure glue case) from the value conjectured in G. Altarelli, R.D. Ball and S. Forte, Nucl. Phys. B575 (2000) 313

- **We also considered $\gamma^*(Q_P) + \gamma^*(Q_T)$ elastic scattering with $Q_T^2 \gg Q_P^2$ and recovered the LO DGLAP anomalous dimension corresponding to the PDF of the photon with smaller virtuality, Q_P^2. Similarly, our resummation is also valid for DIS of a projectile hadron on a target photon (the target Bjorken limit).**

- **We have found an approximate expression for the BFKL Green's function which includes the anti-collinear resummation with running coupling, and have used the saddle-point approximation to calculate the Pomeron intercept, in gluodynamics.**

- **rcJIMWLK emerges with the scale choice for the running coupling: $\alpha_s(\min(\mathbf{X}^{-2}, \mathbf{Y}^{-2}))$. Consistent with Balitsky and G.A. Chirilli, Phys. Rev. D77 (2008) 014019.**

- **For $n_F = 0$ our resummation formalism reproduces the $\omega = 0$ (Mellin's variable conjugate to Y) results of M. Ciafaloni, D. Colferai, G.P. Salam and A.M. Stasto, Phys. Rev. D68 (2003) 114003. For $n_F \neq 0$, we believe that our formalism is under better control.**

- **Our formalism is on track to generalize previous approaches marrying the BFKL and DGLAP physics. Moreover, it goes beyond the linear regime and is formulated for the full nonlinear theory.**
- **Work on the collinear DGLAP resummation is in progress.**