Computation of DIS diffractive structure functions

Mats Kampshoff
Department of Physics & Astronomy
The University of Kansas

Saturation

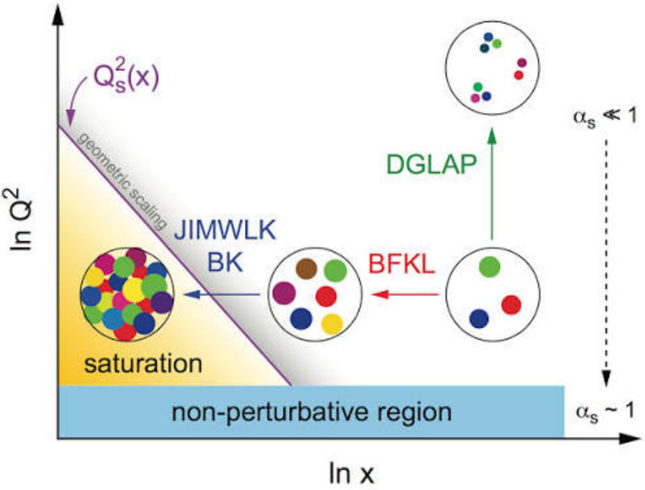


Image from E. Sichtermann, 2015

Following introduction follows arXiv: 2206.13161, 2308.10690, 2401.17251 and 2510.24171.

Inclusive diffractive DIS

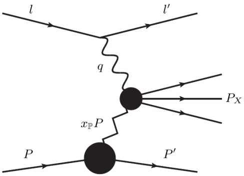


Image from arXiv:2401.17251

Search for Saturation: Want process highly sensitive to gluon distribution.

Inclusive diffractive DIS

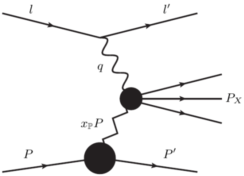


Image from arXiv:2401.17251

Search for Saturation: Want process highly sensitive to gluon distribution.

Diffractive: All digrams $\sim$ squared dipole amplitude, no interference with other processes.

Inclusive diffractive DIS

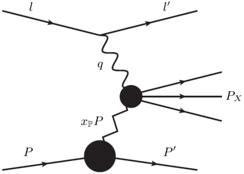


Image from arXiv:2401.17251

Search for Saturation: Want process highly sensitive to gluon distribution.

Diffractive: All digrams $\sim$ squared dipole amplitude, no interference with other processes.

Inclusive: Perturbative treatment possible for lower x, Saturation regime.

Inclusive diffractive DIS

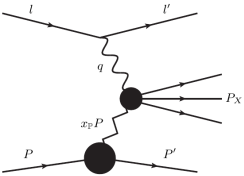


Image from arXiv:2401.17251

Search for Saturation: Want process highly sensitive to gluon distribution.

Diffractive: All digrams $\sim$ squared dipole amplitude, no interference with other processes.

Inclusive: Perturbative treatment possible for lower x, Saturation regime.

DIS: Clean process, no convolution between targets

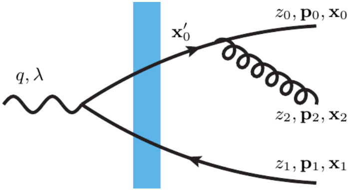


Image from arXiv:2206.13161

Description with LCPT + CGC: Process factorizes, perturbative photon wave function (LCPT) interacts with target colour field (CGC).

Inclusive diffractive DIS

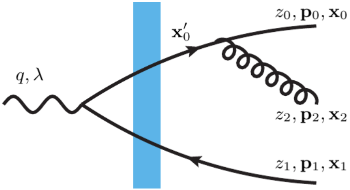


Image from arXiv:2206.13161

Description with LCPT + CGC: Process factorizes, perturbative photon wave function (LCPT) interacts with target colour field (CGC).

Results in terms of diffractive structure function

$$x_P F_{L,T}(x_P, Q^2, M_X^2, t) = \frac{1}{(2\pi)^2 \alpha_{\mathrm{em}}} Q^2 (Q^2 + M_X^2) \frac{\mathrm{d}\sigma^D_{\gamma A}}{\mathrm{d}t \mathrm{d}M_X^2}$$

Inclusive diffractive DIS

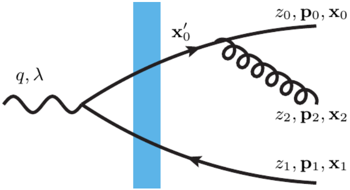


Image from arXiv:2206.13161

Description with LCPT + CGC: Process factorizes, perturbative photon wave function (LCPT) interacts with target colour field (CGC).

Results in terms of diffractive structure function

$$x_P F_{L,T}(x_P, Q^2, M_X^2, t) = \frac{1}{(2\pi)^2 \alpha_{\mathrm{em}}} Q^2 (Q^2 + M_X^2) \frac{\mathrm{d}\sigma^D_{\gamma A}}{\mathrm{d}t \mathrm{d}M_X^2}$$

Resulting high-dimensional integrals with Monte Carlo methods

Inclusive diffractive DIS

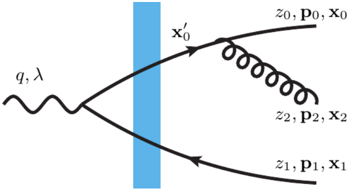


Image from arXiv:2206.13161

Description with LCPT + CGC: Process factorizes, perturbative photon wave function (LCPT) interacts with target colour field (CGC).

Results in terms of diffractive structure function

$$x_P F_{L,T}(x_P, Q^2, M_X^2, t) = \frac{1}{(2\pi)^2 \alpha_{\mathrm{em}}} Q^2 (Q^2 + M_X^2) \frac{\mathrm{d}\sigma^D_{\gamma A}}{\mathrm{d}t \mathrm{d}M_X^2}$$

Resulting high-dimensional integrals with Monte Carlo methods

For now focus on $q\bar{q}$ interaction with target, $q\bar{q} + q\bar{q}g$ final state.

Outline of calculation

$$\frac{\mathrm{d}\sigma^{D}_{\gamma A}}{\mathrm{d}t\,\mathrm{d}M_X^2} \sim \sum_n \int \frac{\mathrm{d}\sigma^{D}_{\gamma A}}{\mathrm{d}p^n} I_{M_X} I_t \, dp^n$$

Outline of calculation

$$\frac{\mathrm{d}\sigma^D_{\gamma A}}{\mathrm{d}t\mathrm{d}M_X^2} \sim \sum_n \int \frac{\mathrm{d}\sigma^D_{\gamma A}}{\mathrm{d}p^n} I_{M_X} I_t dp^n$$

$$\mathcal{M}_{\gamma A \to \{p\}_n A} \sim \sum_m \int \langle \{p\}_n | \{q\}_m \rangle \langle \{q\}_m | \, S - 1 \, | \{k\}_m \rangle \langle \{k\}_m | \gamma \rangle \, dq^m dk^m \tag{1}$$

with $\langle \{k\}_m | \gamma \rangle$, $\langle \{p\}_n | \{q\}_m \rangle$ lightcone wave funcions and $\langle \{q\}_m | \, S - 1 \, | \{k\}_m \rangle$ the dipole amplitude.

Outline of calculation

$$\frac{d\sigma^D_{\gamma A}}{dt dM_X^2} \sim \sum_n \int \frac{\mathrm{d}\sigma^D_{\gamma A}}{\mathrm{d}p^n} I_{M_X} I_t dp^n$$

$$\mathcal{M}_{\gamma A \to \{p\}_n A} \sim \sum_m \int \langle \{p\}_n | \{q\}_m \rangle \langle \{q\}_m | S - 1 | \{k\}_m \rangle \langle \{k\}_m | \gamma \rangle \, dq^m dk^m \tag{1}$$

with $\langle \{k\}_m | \gamma \rangle$, $\langle \{p\}_n | \{q\}_m \rangle$ lightcone wave funcions and $\langle \{q\}_m | S - 1 | \{k\}_m \rangle$ the dipole amplitude.

For the FSR contribution discussed here, $n = 2, 3$ and $m = 2$.

Light-cone wave functions

ψ_a^b : Coefficient of state $|b\rangle$ in the unitary time evolution of state $|a\rangle$, e.g.

Light-cone wave functions

ψ_a^b : Coefficient of state $|b\rangle$ in the unitary time evolution of state $|a\rangle$, e.g.

$$U(t)\,|p_1 p_2\rangle \sim \left(|p_1 p_2\rangle + \sum_{k_1 k_2} \psi_{p_1 p_1}^{k_1 k_2}\,|k_1 k_2\rangle + \sum_{k_1 k_2 l_1} \psi_{p_1 p_1}^{k_1 k_2 l_1}\,|k_1 k_2 l_1\rangle + ... \right)$$

Light-cone wave functions

ψ_a^b : Coefficient of state $|b\rangle$ in the unitary time evolution of state $|a\rangle$, e.g.

$$U(t)\,|p_1 p_2\rangle \sim \left(|p_1 p_2\rangle + \sum_{k_1 k_2} \psi_{p_1 p_1}^{k_1 k_2}\,|k_1 k_2\rangle + \sum_{k_1 k_2 l_1} \psi_{p_1 p_1}^{k_1 k_2 l_1}\,|k_1 k_2 l_1\rangle + \ldots \right)$$

Computed perturbatively. The later algebra ensues almost exclusively from th lcwf.

Light-cone wave functions

ψ_a^b : Coefficient of state $|b\rangle$ in the unitary time evolution of state $|a\rangle$, e.g.

$$U(t)\,|p_1 p_2\rangle \sim \left(|p_1 p_2\rangle + \sum_{k_1 k_2} \psi_{p_1 p_1}^{k_1 k_2} \, |k_1 k_2\rangle + \sum_{k_1 k_2 l_1} \psi_{p_1 p_1}^{k_1 k_2 l_1} \, |k_1 k_2 l_1\rangle + \ldots \right)$$

Computed perturbatively. The later algebra ensues almost exclusively from th lcwf.

Both for evolution from photon to pre-shockwave dipole and from post-shockwave to final state.

Dipole Amplitude

Target at low x: High gluon density means large field occupation numbers and small α, quantum fluctuations doubly suppressed $\rightarrow$ classical approximation.

Dipole Amplitude

Target at low x: High gluon density means large field occupation numbers and small α, quantum fluctuations doubly suppressed $\rightarrow$ classical approximation.

Obtain target colour field from classical YM equation, model target colour density.

Dipole Amplitude

Target at low x: High gluon density means large field occupation numbers and small α, quantum fluctuations doubly suppressed $\rightarrow$ classical approximation.

Obtain target colour field from classical YM equation, model target colour density.

For slowly varying colour density: DA $N \sim 1 - \exp\left(-\frac{Q_s}{4} r^2\right)$, r dipole size. Q_S saturation scale, depends on x as well as the target colour density and transverse center of mass (impact parameter) b.

Dipole Amplitude

Target at low x: High gluon density means large field occupation numbers and small α, quantum fluctuations doubly suppressed $\rightarrow$ classical approximation.

Obtain target colour field from classical YM equation, model target colour density.

For slowly varying colour density: DA $N \sim 1 - \exp\left(-\frac{Q_s}{4} r^2\right)$, r dipole size. Q_S saturation scale, depends on x as well as the target colour density and transverse center of mass (impact parameter) b.

Instead of BK evolution of DA, use model like GBW fit to data.

Diffractive structure function (FSR contribution, $\int dt$)

$$x_P F_{L,T}^{(3)} = \frac{N_C}{2\pi} Q^2 (Q^2 + M_X^2) \sum_f e_f^2 \int \left(\mathcal{K}_{\mathrm{LO}} + \frac{\alpha C_F}{2\pi} \mathcal{K}_{\mathrm{NLO}} \right)$$

$$\cdot (1 - S_{01})(1 - \bar{S}_{01}^\dagger) \frac{1}{(2\pi)^4} \delta(1 - z_0 - z_1) d\mathbf{x}_0 d\mathbf{x}_1 d\bar{\mathbf{x}}_0 d\bar{\mathbf{x}}_1 dz_0 dz_1$$

Diffractive structure function (FSR contribution, $\int dt$)

$$x_P F_{L,T}^{(3)} = \frac{N_C}{2\pi} Q^2 (Q^2 + M_X^2) \sum_f e_f^2 \int \left(\mathcal{K}_{\mathrm{LO}} + \frac{\alpha C_F}{2\pi} \mathcal{K}_{\mathrm{NLO}} \right)$$
$$\cdot (1 - S_{01})(1 - \bar{S}_{01}^\dagger) \frac{1}{(2\pi)^4} \delta(1 - z_0 - z_1) d\mathbf{x}_0 d\mathbf{x}_1 d\bar{\mathbf{x}}_0 d\bar{\mathbf{x}}_1 dz_0 dz_1$$

$$\mathcal{F}_\lambda(z_0, z_1, z_{\overline{0}}, z_{\overline{1}}) = \begin{cases} 4[z_0 z_1 z_{\overline{0}} z_{\overline{1}}]^{3/2} Q^2 K_0(|\mathbf{x}_{01}| Q \sqrt{z_0 z_1}) K_0(|\mathbf{x}_{\overline{01}}| Q \sqrt{z_{\overline{0}} z_{\overline{1}}}) & \lambda = L \\ z_0 z_1 z_{\overline{0}} z_{\overline{1}} (z_0 z_{\overline{0}} + z_1 z_{\overline{1}}) \frac{\mathbf{x}_{01} \cdot \mathbf{x}_{\overline{01}}}{|\mathbf{x}_{01}||\mathbf{x}_{\overline{01}}|} Q^2 K_1(|\mathbf{x}_{01}| Q \sqrt{z_0 z_1}) K_1(|\mathbf{x}_{\overline{01}}| Q \sqrt{z_{\overline{0}} z_{\overline{1}}}) & \lambda = T \end{cases}$$

Diffractive structure function (FSR contribution, $\int dt$)

$$x_P F_{L,T}^{(3)} = \frac{N_C}{2\pi} Q^2 (Q^2 + M_X^2) \sum_f e_f^2 \int \left(\mathcal{K}_{\mathrm{LO}} + \frac{\alpha C_F}{2\pi} \mathcal{K}_{\mathrm{NLO}} \right)$$

$$\cdot (1 - S_{01})(1 - \bar{S}_{01}^\dagger) \frac{1}{(2\pi)^4} \delta(1 - z_0 - z_1) d\mathbf{x}_0 d\mathbf{x}_1 d\bar{\mathbf{x}}_0 d\bar{\mathbf{x}}_1 dz_0 dz_1$$

$$\mathcal{F}_\lambda(z_0, z_1, z_{\overline{0}}, z_{\overline{1}}) = \begin{cases} 4[z_0 z_1 z_{\overline{0}} z_{\overline{1}}]^{3/2} Q^2 K_0(|\mathbf{x}_{01}| Q\sqrt{z_0 z_1}) K_0(|\mathbf{x}_{\overline{01}}| Q\sqrt{z_{\overline{0}} z_{\overline{1}}}) & \lambda = L \\ z_0 z_1 z_{\overline{0}} z_{\overline{1}} (z_0 z_{\overline{0}} + z_1 z_{\overline{1}}) \frac{\mathbf{x}_{01} \cdot \mathbf{x}_{\overline{01}}}{|\mathbf{x}_{01}||\mathbf{x}_{\overline{01}}|} Q^2 K_1(|\mathbf{x}_{01}| Q\sqrt{z_0 z_1}) K_1(|\mathbf{x}_{\overline{01}}| Q\sqrt{z_{\overline{0}} z_{\overline{1}}}) & \lambda = T \end{cases}$$

$$\mathcal{K}_{\mathrm{LO}} = \mathcal{F}(z_0, z_1) J_0(\sqrt{z_0 z_1} |\mathbf{x}_{01} - \bar{\mathbf{x}}_{01}| M_X)$$

Diffractive structure function (FSR contribution, $\int dt$)

$$\mathcal{K}_{\mathsf{NLO}} = \mathcal{F}(z_0, z_1) f_{\mathsf{UV}} + \int \delta(1 - \bar{z}_0 - \bar{z}_1) \mathcal{F}(z_0, z_1, \bar{z}_0 \bar{z}_1)(F_{\mathsf{sub}} + F_{\mathsf{nsoft}} + F_{02} + F_{012A} + F_{012B}) d\bar{z}_0 d\bar{z}_1$$

Diffractive structure function (FSR contribution, $\int dt$)

$$\mathcal{K}_{\mathsf{NLO}} = \mathcal{F}(z_0, z_1) f_{\mathsf{UV}} + \int \delta(1 - \bar{z}_0 - \bar{z}_1) \mathcal{F}(z_0, z_1, \bar{z}_0 \bar{z}_1)(F_{\mathsf{sub}} + F_{\mathsf{nsoft}} + F_{02} + F_{012A} + F_{012B}) d\bar{z}_0 d\bar{z}_1$$

$$\begin{aligned} f^{\mathrm{UV}} = & (2\log(z_0 z_1) - 3) \times \left[\log\left(\frac{M_X e^{\gamma_E} |\mathbf{x}_{01}||\mathbf{x}_{\overline{01}}|}{2|\mathbf{x}_{01} - \mathbf{x}_{\overline{01}}|} \right) J_0\left(\overline{M}_X |\mathbf{x}_{01} - \mathbf{x}_{\overline{01}}| \right) + \frac{\pi}{2} Y_0\left(\overline{M}_X |\mathbf{x}_{01} - \mathbf{x}_{\overline{01}}| \right) \right] \\ & + \left[11 - \pi^2 + \log^2 z_0 + \log^2 z_1 - 4 \log z_0 \log z_1 \right] J_0\left(\overline{M}_X |\mathbf{x}_{01} - \mathbf{x}_{\overline{01}}| \right). \end{aligned}$$

Diffractive structure function (FSR contribution, $\int dt$)

$$\mathcal{K}_{\text{NLO}} = \mathcal{F}(z_0, z_1) f_{\text{UV}} + \int \delta(1 - \bar{z}_0 - \bar{z}_1)\mathcal{F}(z_0, z_1, \bar{z}_0\bar{z}_1)(F_{\text{sub}} + F_{\text{nsoft}} + F_{02} + F_{012A} + F_{012B}) d\bar{z}_0 d\bar{z}_1$$

$$\begin{aligned} f^{\mathrm{UV}} = {} & (2\log(z_0 z_1) - 3) \times \left[\log\left(\frac{M_X e^{\gamma_E}|\mathbf{x}_{01}||\mathbf{x}_{\overline{01}}|}{2|\mathbf{x}_{01} - \mathbf{x}_{\overline{01}}|}\right) J_0\left(\overline{M}_X|\mathbf{x}_{01} - \mathbf{x}_{\overline{01}}|\right) + \frac{\pi}{2} Y_0\left(\overline{M}_X|\mathbf{x}_{01} - \mathbf{x}_{\overline{01}}|\right)\right] \\ & + \left[11 - \pi^2 + \log^2 z_0 + \log^2 z_1 - 4\log z_0 \log z_1\right] J_0\left(\overline{M}_X|\mathbf{x}_{01} - \mathbf{x}_{\overline{01}}|\right). \end{aligned}$$

$$F^{02} = \frac{\pi}{2}\frac{z_0 z_{\overline{0}} + z_1 z_{\overline{1}}}{\sqrt{z_0 z_1 z_{\overline{0}} z_{\overline{1}}}} \times \left[J_0\left(M_X|\mathbf{x}_{01}|\sqrt{z_0 z_1}\right) Y_0\left(M_X|\mathbf{x}_{\overline{01}}|\sqrt{z_{\overline{0}} z_{\overline{1}}}\right) + Y_0\left(M_X|\mathbf{x}_{01}|\sqrt{z_0 z_1}\right) J_0\left(M_X|\mathbf{x}_{\overline{01}}|\sqrt{z_{\overline{0}} z_{\overline{1}}}\right)\right]$$

Diffractive structure function (FSR contribution, $\int dt$)

$$
\begin{aligned}
F_{\mathrm{sub}} = & \int_0^1 \frac{\mathrm{d}z_2'}{z_2'} \frac{1}{2} \frac{z_0 z_1 + z_{\overline{0}} z_{\overline{1}}}{\sqrt{z_0 z_1 z_{\overline{0}} z_{\overline{1}}}} \\
& \times \Bigg\{ 4\delta(z_0 - z_{\overline{0}}) f_{\mathrm{sub}} \left(M_X \sqrt{z_0 z_1} |\mathbf{x}_{\overline{01}} - \mathbf{x}_{01}|, M_X |\mathbf{x}_{\overline{01}}| \sqrt{z_2'} \right) \\
& \quad - \delta(z_2' - (z_0 - z_{\overline{0}})) f_{\mathrm{sub}} \left(M_X \sqrt{\frac{z_1}{z_0}} |z_{\overline{0}} \mathbf{x}_{\overline{01}} - z_0 \mathbf{x}_{01}|, M_X |\mathbf{x}_{\overline{01}}| \sqrt{z_2' \frac{z_{\overline{0}}}{z_0}} \right) \\
& \quad - \delta(z_2' - (z_0 - z_{\overline{0}})) f_{\mathrm{sub}} \left(M_X \sqrt{\frac{z_{\overline{0}}}{z_{\overline{1}}}} |z_{\overline{1}} \mathbf{x}_{\overline{01}} - z_1 \mathbf{x}_{01}|, M_X |\mathbf{x}_{01}| \sqrt{z_2' \frac{z_1}{z_{\overline{1}}}} \right) \\
& \quad - \delta(z_2' - (z_{\overline{0}} - z_0)) f_{\mathrm{sub}} \left(M_X \sqrt{\frac{z_0}{z_1}} |z_{\overline{1}} \mathbf{x}_{\overline{01}} - z_1 \mathbf{x}_{01}|, M_X |\mathbf{x}_{\overline{01}}| \sqrt{z_2' \frac{z_{\overline{1}}}{z_1}} \right) \\
& \quad - \delta(z_2' - (z_{\overline{0}} - z_0)) f_{\mathrm{sub}} \left(M_X \sqrt{\frac{z_{\overline{1}}}{z_{\overline{0}}}} |z_{\overline{0}} \mathbf{x}_{\overline{01}} - z_0 \mathbf{x}_{01}|, M_X |\mathbf{x}_{01}| \sqrt{z_2' \frac{z_0}{z_{\overline{0}}}} \right) \Bigg\}
\end{aligned}
$$

$$
f_{\mathrm{sub}}(a, b) = J_0(a) \log\left(\frac{b^2 e^{\gamma}}{2a} \right) + \frac{\pi}{2} Y_0(a)
$$

Diffractive structure function (FSR contribution, $\int dt$)

$$
\begin{aligned}
F_{\text{non-soft}} = \int_0^1 \frac{\mathrm{d}t}{t} \frac{1}{|z_0 - z_{\overline{0}}|} \frac{z_0 z_1 + z_{\overline{0}} z_{\overline{1}}}{\sqrt{z_0 z_1 z_{\overline{0}} z_{\overline{1}}}} \\
\times \Bigg\{ \theta(z_0 - z_{\overline{0}}) f_{\text{non-soft}} \left(t, M_X \sqrt{\frac{z_1}{z_0}} |z_{\overline{0}} \mathbf{x}_{\overline{01}} - z_0 \mathbf{x}_{01}|, M_X |\mathbf{x}_{\overline{01}}| \sqrt{(z_0 - z_{\overline{0}}) \frac{z_{\overline{0}}}{z_0}} \right) \\
+ \theta(z_0 - z_{\overline{0}}) f_{\text{non-soft}} \left(t, M_X \sqrt{\frac{z_{\overline{0}}}{z_{\overline{1}}}} |z_{\overline{1}} \mathbf{x}_{\overline{01}} - z_1 \mathbf{x}_{01}|, M_X |\mathbf{x}_{01}| \sqrt{(z_0 - z_{\overline{0}}) \frac{z_1}{z_{\overline{1}}}} \right) \\
+ \theta(z_{\overline{0}} - z_0) f_{\text{non-soft}} \left(t, M_X \sqrt{\frac{z_0}{z_1}} |z_{\overline{1}} \mathbf{x}_{\overline{01}} - z_1 \mathbf{x}_{01}|, M_X |\mathbf{x}_{\overline{01}}| \sqrt{(z_{\overline{0}} - z_0) \frac{z_{\overline{1}}}{z_1}} \right) \\
+ \theta(z_{\overline{0}} - z_0) f_{\text{non-soft}} \left(t, M_X \sqrt{\frac{z_{\overline{1}}}{z_{\overline{0}}}} |z_{\overline{0}} \mathbf{x}_{\overline{01}} - z_0 \mathbf{x}_{01}|, M_X |\mathbf{x}_{01}| \sqrt{(z_{\overline{0}} - z_0) \frac{z_0}{z_{\overline{0}}}} \right) \Bigg\}
\end{aligned}
$$

$$f_{\text{nsoft}}(t, a, b) = (1 - J_0(tb)) J_0(\sqrt{1 - t^2} a)$$

Change of variables

Change from transverse coordinates $\mathbf{x}_0,\ \mathbf{x}_1,\ \bar{\mathbf{x}}_0,\ \bar{\mathbf{x}}_1$ to $\mathbf{x}_{01} = \mathbf{x}_0 - \mathbf{x}_1,\ \ldots,\ \mathbf{b} = z_o\mathbf{x}_0 + z_1\mathbf{x}_1,\ \ldots,$ Jacobian $\frac{1}{z_0+z_1} = 1$.

Change of variables

Change from transverse coordinates $\mathbf{x}_0,\ \mathbf{x}_1,\ \bar{\mathbf{x}}_0,\ \bar{\mathbf{x}}_1$ to $\mathbf{x}_{01} = \mathbf{x}_0 - \mathbf{x}_1,\ \ldots,\ \mathbf{b} = z_o\mathbf{x}_0 + z_1\mathbf{x}_1,\ \ldots,$ Jacobian $\frac{1}{z_0+z_1} = 1$.

Center of mass only in DA, factorizes out with $1 - S_{01} = T(\mathbf{b})\left(1 - \exp\left(-\frac{Q_S^2}{4}r^2\right)\right)$.

Change of variables

Change from transverse coordinates $\mathbf{x}_0,\ \mathbf{x}_1,\ \bar{\mathbf{x}}_0,\ \bar{\mathbf{x}}_1$ to $\mathbf{x}_{01} = \mathbf{x}_0 - \mathbf{x}_1,\ \ldots,\ \mathbf{b} = z_o \mathbf{x}_0 + z_1 \mathbf{x}_1,\ \ldots,$ Jacobian $\frac{1}{z_0+z_1} = 1$.

Center of mass only in DA, factorizes out with $1 - S_{01} = T(\mathbf{b}) \left(1 - \exp\left(-\frac{Q_S^2}{4} r^2\right)\right)$.

Polar coordinates for $\mathbf{x}_{01} \to r_{01}, \phi_{01}$, only angle differences enter: $\phi = \phi_{01} - \bar{\phi}_{01}$, other angle integrated out.

Change of variables

Change from transverse coordinates $\mathbf{x}_0,\ \mathbf{x}_1,\ \bar{\mathbf{x}}_0,\ \bar{\mathbf{x}}_1$ to $\mathbf{x}_{01} = \mathbf{x}_0 - \mathbf{x}_1,\ ...,\ \mathbf{b} = z_o\mathbf{x}_0 + z_1\mathbf{x}_1,\ ...,$ Jacobian $\frac{1}{z_0+z_1} = 1$.

Center of mass only in DA, factorizes out with $1 - S_{01} = T(\mathbf{b})\left(1 - \exp\left(-\frac{Q_S^2}{4}r^2\right)\right)$.

Polar coordinates for $\mathbf{x}_{01} \to r_{01}, \phi_{01}$, only angle differences enter: $\phi = \phi_{01} - \bar{\phi}_{01}$, other angle integrated out.

Substitution $r \to \sqrt{z_0 z_1}r = R$ so oscillations only in R.

Change of variables

Change from transverse coordinates $\mathbf{x}_0,\ \mathbf{x}_1,\ \bar{\mathbf{x}}_0,\ \bar{\mathbf{x}}_1$ to $\mathbf{x}_{01} = \mathbf{x}_0 - \mathbf{x}_1,\ \ldots,\ \mathbf{b} = z_o\mathbf{x}_0 + z_1\mathbf{x}_1,\ \ldots,$ Jacobian $\frac{1}{z_0+z_1} = 1$.

Center of mass only in DA, factorizes out with $1 - S_{01} = T(\mathbf{b})\left(1 - \exp\left(-\frac{Q_S^2}{4}r^2\right)\right)$.

Polar coordinates for $\mathbf{x}_{01} \to r_{01}, \phi_{01}$, only angle differences enter: $\phi = \phi_{01} - \bar{\phi}_{01}$, other angle integrated out.

Substitution $r \to \sqrt{z_0 z_1} r = R$ so oscillations only in R.

For $\theta(z_0, z_1)$ sampling efficiency either Baker's transform: If $z_0 + z_1 > 1$ $(z_0, z_1) \to (1 - z_1, 1 - z_0)$, or $(z_0, z_1) \to (\max(z_0, z_1), \min(z_0, z_1))$.

Change of variables

Change from transverse coordinates $\mathbf{x}_0$, $\mathbf{x}_1$, $\bar{\mathbf{x}}_0$, $\bar{\mathbf{x}}_1$ to $\mathbf{x}_{01} = \mathbf{x}_0 - \mathbf{x}_1$, ..., $\mathbf{b} = z_o \mathbf{x}_0 + z_1 \mathbf{x}_1$, ..., Jacobian $\frac{1}{z_0 + z_1} = 1$.

Center of mass only in DA, factorizes out with $1 - S_{01} = T(\mathbf{b}) \left(1 - \exp\left(-\frac{Q_S^2}{4} r^2\right)\right)$.

Polar coordinates for $\mathbf{x}_{01} \to r_{01}, \phi_{01}$, only angle differences enter: $\phi = \phi_{01} - \bar{\phi}_{01}$, other angle integrated out.

Substitution $r \to \sqrt{z_0 z_1} r = R$ so oscillations only in R.

For $\theta(z_0, z_1)$ sampling efficiency either Baker's transform: If $z_0 + z_1 > 1$ $(z_0, z_1) \to (1 - z_1, 1 - z_0)$, or $(z_0, z_1) \to (\max(z_0, z_1), \min(z_0, z_1))$.

Most MC sample from unit interval, transform to $\mathbb{R}_+$ with $r \to \frac{r}{1-r}$ or to $\mathbb{R}$ with $x \to \frac{x - \frac{1}{2}}{x(1-x)}$.

MC algorithms: VEGAS vs SUAVE

Vegas: Divide axis into bins, evaluate integral, then change bin sizes such that $\left(\frac{\Delta x_{\text{new}}}{\Delta x_{\text{old}}} \int_{\text{bin}} f(x)\right) = \text{const}$, reiterate.

MC algorithms: VEGAS vs SUAVE

Vegas: Divide axis into bins, evaluate integral, then change bin sizes such that $\left(\frac{\Delta x_{\mathrm{new}}}{\Delta x_{\mathrm{old}}} \int_{\mathrm{bin}} f(x)\right) = \mathrm{const}$, reiterate.

Since binning happens in 1d, best suited for approximately factorizable integrands. Correlations are possible, but numerical effort quickly becomes untenable.

MC algorithms: VEGAS vs SUAVE

Vegas: Divide axis into bins, evaluate integral, then change bin sizes such that $\left(\frac{\Delta x_{\text{new}}}{\Delta x_{\text{old}}} \int_{\text{bin}} f(x)\right) = \text{const}$, reiterate.

Since binning happens in 1d, best suited for approximately factorizable integrands. Correlations are possible, but numerical effort quickly becomes untenable.

Suave: Start with a Vegas grid, find the axis and region with the largest integral fluctuation $\sim \frac{(f(x_i)-I)^2}{I \cdot \sigma}$, bisect this region, expand and refine sub-grids, repeat.

MC algorithms: VEGAS vs SUAVE

Vegas: Divide axis into bins, evaluate integral, then change bin sizes such that $\left(\frac{\Delta x_{\text{new}}}{\Delta x_{\text{old}}} \int_{\text{bin}} f(x)\right) = \text{const}$, reiterate.

Since binning happens in 1d, best suited for approximately factorizable integrands. Correlations are possible, but numerical effort quickly becomes untenable.

Suave: Start with a Vegas grid, find the axis and region with the largest integral fluctuation $\sim \frac{(f(x_i)-I)^2}{I \cdot \sigma}$, bisect this region, expand and refine sub-grids, repeat.

Same factorization preference as Vegas, but expands the grid where necessary (Vegas would have to start with an unwieldy large grid).

MC algorithms: VEGAS vs SUAVE

Vegas: Divide axis into bins, evaluate integral, then change bin sizes such that $\left(\frac{\Delta x_{\text{new}}}{\Delta x_{\text{old}}} \int_{\text{bin}} f(x)\right) = \text{const}$, reiterate.

Since binning happens in 1d, best suited for approximately factorizable integrands. Correlations are possible, but numerical effort quickly becomes untenable.

Suave: Start with a Vegas grid, find the axis and region with the largest integral fluctuation $\sim \frac{(f(x_i)-I)^2}{I\cdot\sigma}$, bisect this region, expand and refine sub-grids, repeat.

Same factorization preference as Vegas, but expands the grid where necessary (Vegas would have to start with an unwieldy large grid).

Per iteration, Vegas reevaluates entire integral, Suave only bisected region. Smaller iterations lead to less impact of initial estimates, suffers less from combining iterations than Vegas.

Convergence analysis

Numerical challenge largely from oscillatory cancellations, shown by $|\cdot|$ converging much faster.
$\sigma \sim \sqrt{n}$ shows proper sampling.

Convergence analysis

Numerical challenge largely from oscillatory cancellations, shown by $|\cdot|$ converging much faster.
$\sigma \sim \sqrt{n}$ shows proper sampling.

Problematic for low β, large Q^2, both enter as frequencies in Bessel functions.

Convergence analysis

Numerical challenge largely from oscillatory cancellations, shown by $|\cdot|$ converging much faster. $\sigma \sim \sqrt{n}$ shows proper sampling.

Problematic for low β, large Q^2, both enter as frequencies in Bessel functions.

SUAVE consistently outperforms VEGAS by a factor up to 15 in numerical effort and about equal time, preferred.

Convergence analysis

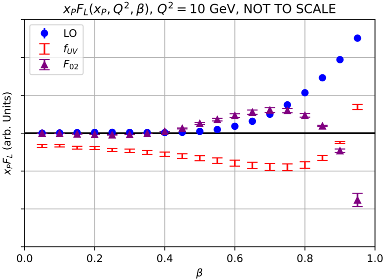
Numerical challenge largely from oscillatory cancellations, shown by $|\cdot|$ converging much faster. $\sigma \sim \sqrt{n}$ shows proper sampling.

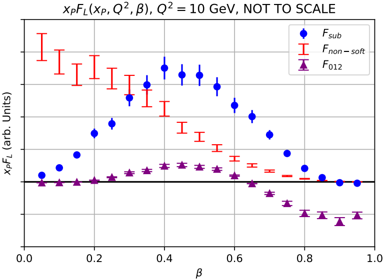
Problematic for low β, large Q^2, both enter as frequencies in Bessel functions.

SUAVE consistently outperforms VEGAS by a factor up to 15 in numerical effort and about equal time, preferred.

Note: Required numerical effort almost exhausts integer range, might need Cuba long int for extreme parameters.

Preliminary Results for F_L (FSR contribution)





Outlook

Need cross checking with U Jyväskylä

Outlook

Need cross checking with U Jyväskylä

Nowhere near optimal numerical efficiency

Outlook

Need cross checking with U Jyväskylä

Nowhere near optimal numerical efficiency

Try also Quasi-Monte-Carlo methods, possible faster convergence rate, better suited for oscillatory integrands.