Generalized transverse momentum distributions s at small-x

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SB, Hagiwara, Šarić, Vivoda, 2603.06092

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GTMDs

Belitsky, Ji, Yuan (2003)

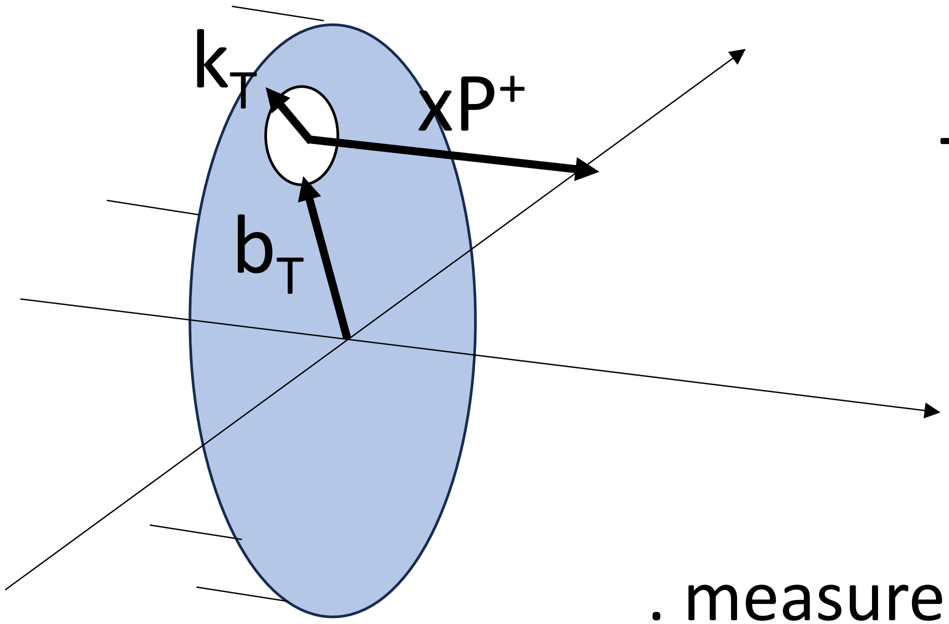
Meissner, Metz, Schlegel (2009)

Lorce, Pasquini (2011)

. partons carry finite k_T in addition to xP^+

. imaging/tomography: the (x,k_T) distributions of partons depend on the location within the proton -> (x,k_T,b_T) or (x,k_T, Δ_T)

$$W(x, \boldsymbol{k}, \boldsymbol{\Delta}) = \int \frac{\mathrm{d}z^- \mathrm{d}^2\boldsymbol{z}}{(2\pi)^3} \mathrm{e}^{\mathrm{i}xP^+z^-} \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{z}} \langle p - \Delta/2\Lambda' | \bar{\psi}(-z/2)\gamma^+\psi(z/2) | p + \Delta/2 \rangle$$



$\boldsymbol{\Delta} = 0$ $\quad\quad$ $\int \mathrm{d}^2\boldsymbol{k}$

TMD $f(x, \boldsymbol{k})$ $\quad\quad$ GPD $f(x, \boldsymbol{\Delta})$

$\int \mathrm{d}^2\boldsymbol{k}$ $\quad\quad$ $\int \mathrm{d}x$

PDF $f(x)$ $\quad\quad$ FF $F(\boldsymbol{\Delta})$

. measurements via e. g. exclusive dijets in ep or UPCs

GTMDs

. attractive to consider quark and gluon GTMDs at small-x

. connect to the conventional high-energy language of Pomerons/Odderons

. practical computations of small-x GTMDs through Pomerons/Odderons and their high-energy evolutions -> apply to processes

. similar analysis exist for TMDs, and for unpolarized GTMDs

Metz, Zhou (2011)

Dominguez, Qiu, Xiao, Yuan (2012)

Boer, Echevarria, Mulders, Zhou (2016)

Boer, Cotogno, van Daal, Mulders, Signori, Zhou (2016)

Kovchegov, Sievert (2015, 2017, 2018)

…

Boer, Echevarria, Mulders, Zhou (2016)

Hatta, Nakagawa, Zhao, Xiao, Hua-Zhong (2017)

Bhattacharya, Boussarie, Hatta (2024)

Bhattacharya, Kang, Padilla, Pentalla (2026)

Kovchegov, Sun, Santiago (2026)

…

. complete analysis including also proton helicity flips has not been done, even in the strict eikonal case

Recall some known results at small-x

. Small-x gluon is maximally linearly polarized (along its k_T)

$$f_1^g(x,\boldsymbol{k}) = \frac{\boldsymbol{k}^2}{2M^2} h_1^{\perp g}(x,\boldsymbol{k})$$

Metz, Zhou (2011)

Dominguez, Qiu, Xiao, Yuan (2012)

unpolarized TMD

Linearly polarized TMD

. Small-x gluon spin and OAM are maximally anti-aligned

$$\boldsymbol{k}^2 f_1^g(x,\boldsymbol{k},\boldsymbol{\Delta}) = -M^2 C^g(x,\boldsymbol{k},\boldsymbol{\Delta})$$

Bhattacharya, Boussarie, Hatta (2024)

unpolarized GTMD

Spin-orbit GTMD : $\langle L^z S^z \rangle \to -1$

. proton is unpolarized

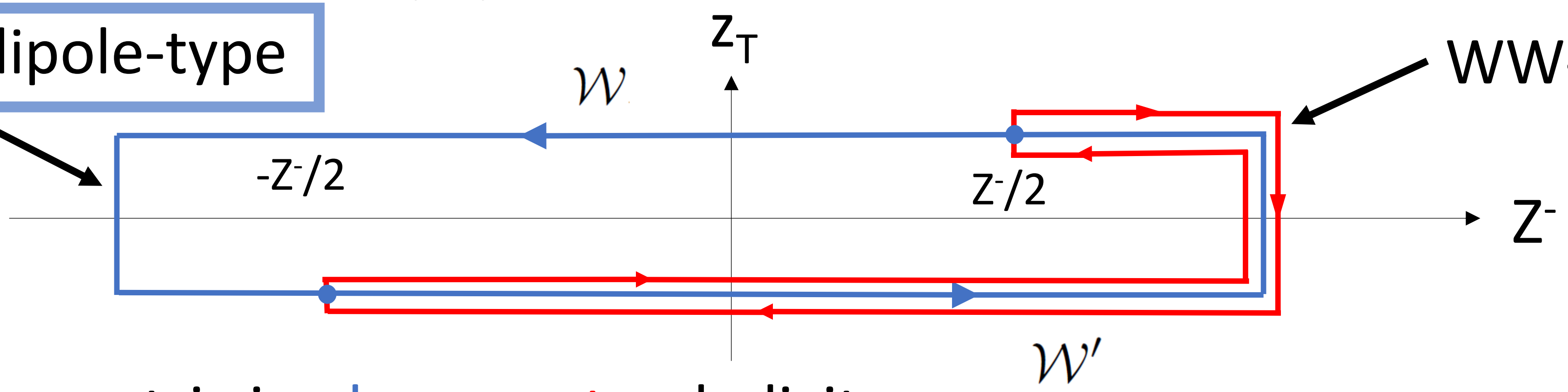
are there more such relations?

Gluon GTMDs

$$W^{ij}_{\Lambda'\Lambda}(x,\xi,\boldsymbol{k},\boldsymbol{\Delta}) = \frac{2}{xP^+}\int \frac{\mathrm{d}z^-\mathrm{d}^2\boldsymbol{z}}{(2\pi)^3}\,\mathrm{e}^{\mathrm{i}xP^+z^- -\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{z}}\langle p'\Lambda'|\mathrm{tr}\left(F^{+i}(-z/2)\boxed{\mathcal{W}}F^{+j}(z/2)\boxed{\mathcal{W}'}\right)|p\Lambda\rangle$$

. This work: dipole-type

WW-type



. spin density matrix in gluon-proton helicity space

$$H^g_{\Lambda'\lambda',\Lambda\lambda} = \begin{pmatrix} \frac{1}{2}(U^g+L^g)_{\boxed{\Lambda'\Lambda}} & \frac{1}{2}T^g_{R,\Lambda'\Lambda} \\ \frac{1}{2}T^g_{L,\Lambda'\Lambda} & \frac{1}{2}(U^g-L^g)_{\Lambda'\Lambda} \end{pmatrix}_{\boxed{\lambda'\lambda}}$$

linearly polarized

$T^g_R \equiv -R^iR^jW^{ij}$

$\boldsymbol{L}=(1,-\mathrm{i})$ $\boldsymbol{R}=(1,\mathrm{i})$

unpolarized

$U^g \equiv \delta^{ij}W^{ij}$

longitudinally polarized

$L^g \equiv -\mathrm{i}\epsilon^{ij}W^{ij}$

Lorce, Pasquini (2013)

Gluon GTMDs

Lorce, Pasquini (2013)

unpolarized GTMD ($F_{1,1}$)

spin-orbit ($F_{1,4}$)

OAM ($G_{1,4}$)

➡ rich decompositions, e. g.

$$H^g_{+\frac{1}{2}+1,+\frac{1}{2}+1} = \frac{1}{2}\left[\left(\boxed{S^{0,+;g}_{1,1a}} + S^{0,-;g}_{1,1a}\right) + \mathrm{i}\frac{\boldsymbol{k}\times\boldsymbol{\Delta}}{M^2}\left(\boxed{S^{0,+;g}_{1,1b}} + \boxed{S^{0,-;g}_{1,1b}}\right)\right]$$

$$H^g_{-\frac{1}{2}+1,-\frac{1}{2}+1} = \frac{1}{2}\left[\left(S^{0,+;g}_{1,1a} - \boxed{S^{0,-;g}_{1,1a}}\right) - \mathrm{i}\frac{\boldsymbol{k}\times\boldsymbol{\Delta}}{M^2}\left(S^{0,+;g}_{1,1b} - S^{0,-;g}_{1,1b}\right)\right]$$

Meissner, Metz, Schlegel (2009)

helicity GTMD ($G_{1,1}$)

. GTMD notations

helicity transfer

$$(\Delta l_z)_t^{\Delta S_z, c_P}(x, \xi, \boldsymbol{k}, \boldsymbol{\Delta})$$

OAM transfer $\Delta l_z = \Lambda - \lambda - (\Lambda' - \lambda')$ = S, P, D, F waves

Gluon GTMDs

no ~ k_R Δ_R - type terms: $2(\boldsymbol{k}\cdot\boldsymbol{\Delta})k_R\Delta_R = \boldsymbol{k}^2\Delta_R^2 + \boldsymbol{\Delta}^2 k_R^2$
(not independent)

. with gluon helicity flips

$\Delta_{R,L} = \Delta^1 \pm \mathrm{i}\Delta^2$

$$H^g_{+\frac{1}{2}\boxed{+1},+\frac{1}{2}\boxed{-1}} = -\frac{1}{2}\left[\frac{k_R^2}{M^2}\left(D^{2,+;g}_{1,1a} + D'^{2,+;g}_{1,1a}\right) + \boxed{\frac{\Delta_R^2}{M^2}}\left(D^{2,+;g}_{1,1b} + D'^{2,+;g}_{1,1b}\right)\right]$$

linearly polarized gluons (forward limit)

related to transversity-type GPDs

same, but with longitudinally polarized
proton (vanishes at small-x)

Boer et al (2016)

. small-x gluons in off-forward proton are linearly polarized as k ± Δ/2, so naturally the different D-wave GTMDs will become related

. typically Δ is ignored in high-energy (s>>t), but in this case it is essential

Gluon GTMDs: proton helicity flips

Im -> gluon Sivers in forward limit

$$H^g_{+\frac{1}{2}+1,-\frac{1}{2}+1} = \frac{1}{2}\left[-\frac{k_L}{M}\left(\boxed{P^{0,+;g}_{1,1a}} - P^{0,-;g}_{1,1a}\right) - \frac{\Delta_L}{M}\left(P^{0,+;g}_{1,1b} - P^{0,-;g}_{1,1b}\right)\right]$$

$$H^g_{-\frac{1}{2}+1,+\frac{1}{2}+1} = \frac{1}{2}\left[\frac{k_R}{M}\left(\boxed{P^{0,+;g}_{1,1a}} + P^{0,-;g}_{1,1a}\right) + \frac{\Delta_R}{M}\left(\boxed{P^{0,+;g}_{1,1b}} + P^{0,-;g}_{1,1b}\right)\right],$$

Re -> GPD E

$$H^g_{+\frac{1}{2}+1,-\frac{1}{2}-1} = -\frac{1}{2}\left[\frac{k_R}{M}\boxed{P^{2,+;g}_{1,1a}} + \frac{\Delta_R}{M}P^{2,+;g}_{1,1b}\right]$$

$$H^g_{-\frac{1}{2}+1,+\frac{1}{2}-1} = -\frac{1}{2}\left[\frac{k_R^3}{M^3}\boxed{F^{2,+;g}_{1,1a}} + \frac{\Delta_R^3}{M^3}F^{2,+;g}_{1,1b}\right]$$

Linearly polarized gluon in transversely polarized proton

total of 16 complex GTMDs at leading twist

Relation to gluon dipole at small-x

Dominguez, Marquet, Xiao, Yuan (2011)

Hatta, Xiao, Yuan (2016)

Boer, Cotogno, Van Daal, Mulders, Signori, Zhou (2016)

Proton and gluon spin indices **factorize**

(does not hold for WW type)

$$\lim_{x\to 0} xW^{ij}_{\Lambda'\Lambda}(x, \boxed{\xi = 0}, \boldsymbol{k}, \boldsymbol{\Delta}) = \frac{2N_c}{\alpha_S}\left(k^i + \frac{1}{2}\Delta^i\right)\left(k^j - \frac{1}{2}\Delta^j\right) S_{\Lambda'\Lambda}(\boldsymbol{k}, \boldsymbol{\Delta})$$

For skewness corrections, see

Bhattacharya, He, Kang, Padilla, Penttala (2026)

Kovchegov, Santiago, Sun (2026)

. all GTMDs in principle described by Wilson line dipole

$$S_{\Lambda'\Lambda}(\boldsymbol{k}, \boldsymbol{\Delta}) = \int \frac{\mathrm{d}^2\boldsymbol{x}}{(2\pi)^2} \int \frac{\mathrm{d}^2\boldsymbol{y}}{(2\pi)^2} \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y})+\mathrm{i}\boldsymbol{\Delta}\cdot\frac{\boldsymbol{x}+\boldsymbol{y}}{2}} \frac{1}{N_c} \frac{\langle p'\Lambda'|\mathrm{tr}\left[V^\dagger(\boldsymbol{x})V(\boldsymbol{y})\right]|p\Lambda\rangle}{\langle P\Lambda|P\Lambda\rangle}$$

$$V(\boldsymbol{x}) = P\exp\left[-ig\int_{-\infty}^{\infty} \mathrm{d}x^- A^+(x^-, \boldsymbol{x})\right]$$

Dipole decomposition

Boussarie, Hatta, Szymanowski, Wallon (2020)

Hagiwara, Hatta, Pasechnik, Zhou (2020)

proton helicity flips!

(ΔxS)-type

$$\mathcal{S}_{\Lambda'\Lambda}(\boldsymbol{k},\boldsymbol{\Delta}) = \delta_{\Lambda\Lambda'}\mathcal{S}(\boldsymbol{k},\boldsymbol{\Delta}) + \boxed{\delta_{\Lambda,-\Lambda'}}\frac{2\Lambda k^1 + \mathrm{i}k^2}{M}\mathcal{S}^{\perp}_{1T}(\boldsymbol{k},\boldsymbol{\Delta}) + \boxed{\delta_{\Lambda,-\Lambda'}}\frac{2\Lambda\Delta^1 + \mathrm{i}\Delta^2}{M}\mathcal{S}_T(\boldsymbol{k},\boldsymbol{\Delta})$$

(kxS)-type

$$\mathcal{P} + \mathrm{i}\frac{(\boldsymbol{k}\cdot\boldsymbol{\Delta})}{M^2}\mathcal{O}$$

spin-independent

Pomeron and Odderon

$$\frac{(\boldsymbol{k}\cdot\boldsymbol{\Delta})}{M^2}\mathcal{P}^{\perp}_{1T} + \mathrm{i}\mathcal{O}^{\perp}_{1T}$$

$$\mathcal{P}_T + \mathrm{i}\frac{(\boldsymbol{k}\cdot\boldsymbol{\Delta})}{M^2}\mathcal{O}_T$$

transverse spin vs helicity basis

$$|\boldsymbol{S}_\perp\rangle = \frac{1}{\sqrt{2}}\left(|\Lambda=+\rangle + \mathrm{e}^{\mathrm{i}\phi_S}|\Lambda=-\rangle\right)$$

spin-dependent Pomerons and Odderons

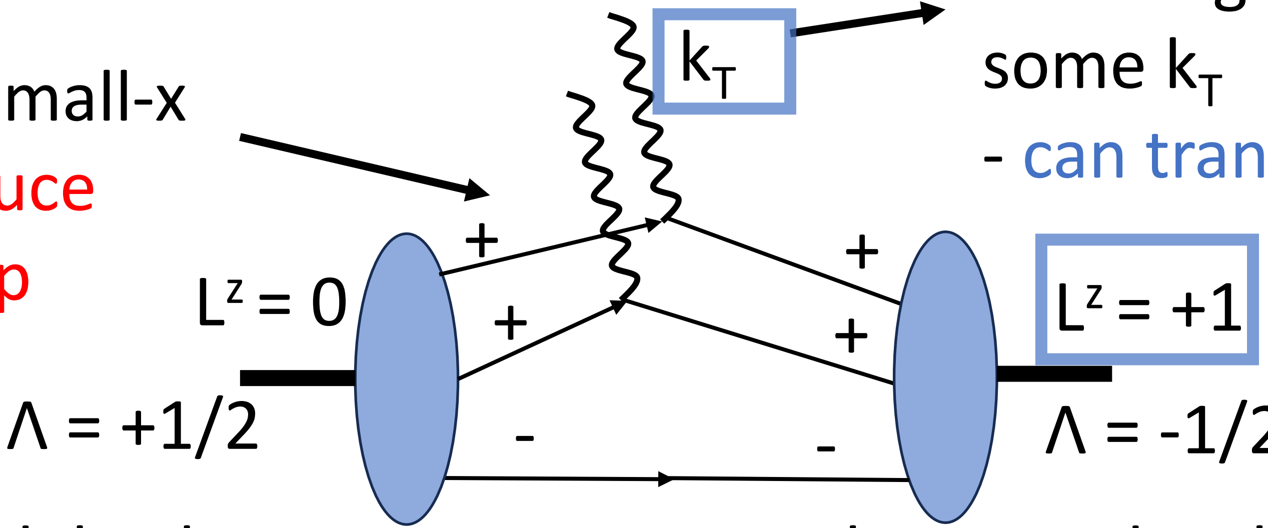
➡ 3 independent complex functions vs total of 16 GTMD

. all GTMDs can be calculated in terms of Pomerons and Odderons

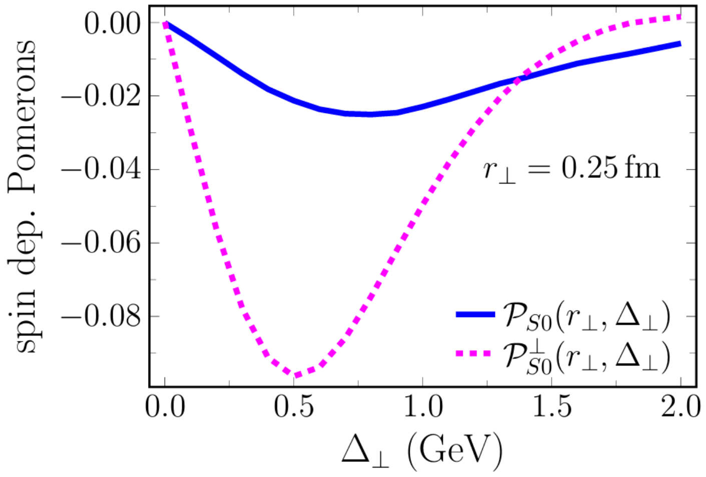
Proton vs parton helicity flip

eikonal vertex – small-x gluon cannot induce parton helicity flip

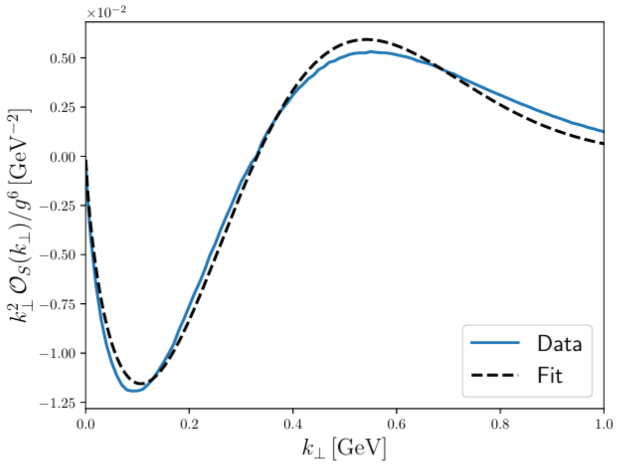
small-x gluons carry some k_T
- can transfer OAM



. In light-cone models, the $L^z \neq 0$ components become developed by the Melosh rotation Pasquini, Yuan (2010)



SB, Dumitru (2025)



SB, Dumitru, Hechenberger, Stebel (2026)

spin dep Pomerons and Odderons in LCwf model

Full set of relations between gluon GTMDs

. proton is unpolarized

$$\boldsymbol{k}^2 x S^{0,-;g}_{1,1b} \approx M^2 x S^{0,+;g}_{1,1a}$$

unpolarized ↔ spin-orbit connection

$$\boldsymbol{k}^2 x D^{2,+;g}_{1,1a} = -4\boldsymbol{k}^2 x D^{2,+;g}_{1,1b} = M^2 x S^{0,+;g}_{1,1a}$$

unpolarized ↔ linearly-polarized

polarized along k ≈ polarized along Δ (new relation)

. proton is transversely polarized

$$\boldsymbol{k}^2 x \boxed{P^{0-;g}_{1,1a}} \approx -(\boldsymbol{k}\cdot\boldsymbol{\Delta}) x P^{0+;g}_{1,1a} \,,$$

$$\boldsymbol{k}^2 x \boxed{P^{0-;g}_{1,1b}} \approx \boldsymbol{k}^2 x P^{0+;g}_{1,1a} + (\boldsymbol{k}\cdot\boldsymbol{\Delta}) x P^{0+;g}_{1,1b}$$

Related to the GPD $\tilde{E}^g$ but the GPD itself vanishes due to k_T -> -k_T symmetry (GTMD eikonal but GPD is not)

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Full set of relations between gluon GTMDs

. proton is transversely polarized

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$$xP_{1,1a}^{2,+;g} \approx xP_{1,1a}^{0,+;g} - 2xP_{1,1b}^{0,-;g}$$

$$\boxed{xP_{1,1b}^{2,+;g} \approx xP_{1,1b}^{0,+;g} - \frac{x}{2}P_{1,1a}^{0,-;g}}$$

$$\boldsymbol{k}^2 xF_{1,1a}^{2,+;g} \approx M^2 xP_{1,1a}^{0,+;g}$$

unpolarized ↔ linearly-polarized gluon in transversely proton

. in the forward limit collapses to the known relations

$$f_{1T}^{\perp g} = h_1^g = -\frac{\boldsymbol{k}^2}{2M^2}h_{1T}^{\perp g}$$

$\mathrm{Im}(P_{1,1a}^{2,+;g})_{\boldsymbol{\Delta}=0}$

$\mathrm{Im}(P_{1,1a}^{0,+;g})_{\boldsymbol{\Delta}=0}$

$\mathrm{Im}(F_{1,1a}^{2,+;g})_{\boldsymbol{\Delta}=0}$

Boer, Echevarria, Mulders, Zhou (2016)

Boer, Cotogno, van Daal, Mulders, Signori, Zhou (2016)

GPD limit

$$\mathcal{P}(\boldsymbol{k},\boldsymbol{\Delta}) = \mathcal{P}_0(|\boldsymbol{k}|,|\boldsymbol{\Delta}|) + \left[\frac{2(\boldsymbol{k}\cdot\boldsymbol{\Delta})^2}{\boldsymbol{k}^2\boldsymbol{\Delta}^2} - 1\right] 2\mathcal{P}_\epsilon(|\boldsymbol{k}|,|\boldsymbol{\Delta}|) + \ldots$$

at leading twist only Pomerons contribute..

$$xH^g = \frac{2N_c}{\alpha_S}\int \mathrm{d}^2\boldsymbol{k}\boldsymbol{k}^2\mathcal{P}_0\,,$$ Hatta, Xiao, Yuan (2017)

$$xE^g = \frac{4N_c}{\alpha_S}\int \mathrm{d}^2\boldsymbol{k}\boldsymbol{k}^2\left[\frac{\boldsymbol{k}^2}{2M^2}(\mathcal{P}_{1T,0}^\perp + \mathcal{P}_{1T,\epsilon}^\perp) + \mathcal{P}_{T,0}\right]$$

GPD E is eikonal and governed by spin-dep Pomerons

Hatta, Zhou (2022)

. **Observable for the GPD E?**

-> direct way: tag the polarization of outgoing proton -> is it possible??

-> indirect way: single spin asymmetry in exclusive J/psi in e + p$^\uparrow$ Koempel, Kroll, Metz, Zhou (2011)

Lansberg, Massacrier, Szymanowski, Wagner (2022)

$$\text{SSA} = \frac{\frac{1}{2m_N}(1+\xi)|\Delta_T|\sin(\phi_{\vec{\Delta}})\boxed{\Im(\mathcal{H}^g\mathcal{E}^{g\star})}}{(1-\xi^2)|\mathcal{H}^g|^2 + \frac{\xi^4}{1-\xi^2}|\mathcal{E}^g|^2 - 2\xi^2\Re(\mathcal{H}^g\mathcal{E}^{g\star})}$$

Vanishes in the eikonal limit (CFFs are purely imaginary)

. instead, look at double spin asymmetry $\vec{e}$ + p$^\uparrow$

SB, Dumitru (2025)

$$\boxed{\text{DSA} \sim \frac{\Re(\mathcal{H}^g\mathcal{E}^{g*})}{|\mathcal{H}^g|^2}}$$

GPD limit

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. gluon transversity GPDs

new

$$xH_T^g = \frac{2N_c}{\alpha_S}\int \mathrm{d}^2\boldsymbol{k}\boldsymbol{k}^2\frac{\boldsymbol{k}^2}{2M^2}\mathcal{P}_{1T,0}^{\perp},$$

$$xE_T^g = -\frac{8N_c}{\alpha_S}\frac{M^2}{\boldsymbol{\Delta}^2}\int \mathrm{d}^2\boldsymbol{k}\boldsymbol{k}^2\left[\mathcal{P}_\epsilon + \boxed{\frac{\boldsymbol{k}^2}{M^2}\mathcal{P}_{1T,\epsilon}^{\perp} + 2\mathcal{P}_{T,\epsilon}}\right]$$

$$x\tilde{H}_T^g = \frac{4N_c}{\alpha_S}\frac{M^2}{\boldsymbol{\Delta}^2}\int \mathrm{d}^2\boldsymbol{k}\boldsymbol{k}^2\boxed{\left[\frac{\boldsymbol{k}^2}{M^2}\mathcal{P}_{1T,\epsilon}^{\perp} + 2\mathcal{P}_{T\epsilon}\right]}.$$

new

spin-dep elliptic pieces are new

not considered in Hatta, Xiao, Yuan (2017)

$$xE_T^g + \boxed{2x\tilde{H}_T^g} = -\frac{8N_c}{\alpha_S}\frac{M^2}{\boldsymbol{\Delta}^2}\int \mathrm{d}^2\boldsymbol{k}\boldsymbol{k}^2\mathcal{P}_\epsilon$$

. transversity GPD could be probed even in unpolarized DVCS or exclusive J/psi

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\mathrm{d}Q^2\mathrm{d}^2\boldsymbol{\Delta}} \sim A_0 A_2\cos(2\phi_{\Delta l})$$

Hatta, Xiao, Yuan (2017)

Mantysaari, Roy, Salazar, Schenke (2021)

γ(+1) -> γ(+1)

γ(+1) -> γ(-1) ~ $-\frac{e_q^2\alpha_s\Delta_\perp^2}{4Q^2M^2}E_{Tg}(x,\Delta_\perp)$

Seaquark GTMDs at small-x

Lorce, Pasquini (2013)

$$W^{[\Gamma]}_{\Lambda'\Lambda} = \frac{1}{2}\int \frac{\mathrm{d}z^- \mathrm{d}^2\boldsymbol{z}}{(2\pi)^3} \mathrm{e}^{\mathrm{i}xP^+z^-} \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{z}} \langle p'\Lambda'|\bar{\psi}(-z/2)\boxed{\mathcal{W}}\Gamma\psi(z/2)|p\Lambda\rangle$$

. at the leading twist $\boxed{U^q \equiv W^{[\gamma^+]},\ L^q \equiv W^{[\gamma^+\gamma_5]}},\ T^q_R \equiv W^{[\mathrm{i}\sigma^{R+}\gamma_5]}$

. similar to gluon case we have decomposition into S, P, D-wave GTMDs

. quarks emerge through the interaction with the small-x gluons, also taking care of shockwave-gauge link „interaction”

$$\begin{aligned}\psi(z)\bar{\psi}(w) \to\ S(z,w) &= \theta(z^-)\theta(w^-)S_0(z,w) + \theta(z^-)\theta(-w^-)\int \mathrm{d}^4y\delta(y^-)S_0(z,y)\gamma^- S_0(y,w)V(\boldsymbol{y}) \\ &- \theta(-z^-)\theta(w^-)\int \mathrm{d}^4y\delta(y^-)S_0(z,y)\gamma^- S_0(y,w)V^\dagger(\boldsymbol{y}) + \theta(-z^-)\theta(-w^-)S_0(z,w)\end{aligned}$$

Xiao, Yuan, Zhou (2017)

McLerran, Venugopalan (1994)

Bhattacharya, He, Kang, Padilla, Pentalla (2026)

Unpolarized seaquark GTMDs

SB, Hagiwara, Šarić, Vivoda, 2603.06092

. unpolarized

Xiao, Yuan, Zhou (2017)

$$S^{0,+;q}_{1,1a} = X^{S0,+;q}_{1,1a} + \mathrm{i}\frac{\boldsymbol{k}\cdot\boldsymbol{\Delta}}{M^2} Y^{S0,+;q}_{1,1a}$$

$$A(\boldsymbol{q},\boldsymbol{k}) = 2\left[1 - \frac{\boldsymbol{q}\cdot\boldsymbol{k}}{\boldsymbol{k}^2-\boldsymbol{q}^2}\log\left(\frac{\boldsymbol{k}^2}{\boldsymbol{q}^2}\right)\right]$$

$\boldsymbol{q} \equiv \boldsymbol{k} - \boldsymbol{k}_g$

$$xX^{S0,+;q}_{1,1a} = \frac{N_c}{2\pi^2}\int \mathrm{d}^2\boldsymbol{k}_g A(\boldsymbol{q},\boldsymbol{k})\mathcal{P}_0(|\boldsymbol{k}_g|,|\boldsymbol{\Delta}|) + \frac{N_c}{\pi^2}\left[\frac{2(\boldsymbol{k}\cdot\boldsymbol{\Delta})^2}{\boldsymbol{k}^2\boldsymbol{\Delta}^2}-1\right]\int \mathrm{d}^2\boldsymbol{k}_g\left[\frac{2(\boldsymbol{k}\cdot\boldsymbol{k}_g)^2}{\boldsymbol{k}^2\boldsymbol{k}_g^2}-1\right]A(\boldsymbol{q},\boldsymbol{k})\mathcal{P}_\epsilon(|\boldsymbol{k}_g|,|\boldsymbol{\Delta}|)$$

$$xY^{S0,+;q}_{1,1a} = \frac{N_c}{2\pi^2}\int \mathrm{d}^2\boldsymbol{k}_g \frac{\boldsymbol{k}\cdot\boldsymbol{k}_g}{\boldsymbol{k}^2}A(\boldsymbol{q},\boldsymbol{k})\mathcal{O}_0(|\boldsymbol{k}_g|,|\boldsymbol{\Delta}|)\,.$$

Im part governed by Odderon

. spin-orbit

Kovchegov, Santiago, Sun (2025)

$$xX^{S0,-;q}_{1,1b} = \frac{N_c}{2\pi^2}M^2\int \mathrm{d}^2\boldsymbol{k}_g(\boldsymbol{q}\cdot\boldsymbol{k})B_2(\boldsymbol{q},\boldsymbol{k})\mathcal{P}(\boldsymbol{k}_g,\boldsymbol{\Delta})$$

$$xY^{S0,-;q}_{1,1b} = 0\,.$$

$$B_2(\boldsymbol{q},\boldsymbol{k}) = \frac{\log\left(\boldsymbol{k}^2/\boldsymbol{q}^2\right)}{\boldsymbol{k}^2-\boldsymbol{q}^2}$$

$$\boldsymbol{k}^2 xX^{S0,-;q}_{1,1b} = \frac{1}{2}M^2 xX^{S0,+;q}_{1,1a}$$

unpolarized ↔ spin-orbit connection

Bhattacharya, Boussarie, Hatta (2024)

Seaquark GTMDs with proton helicity flips

. quark helicity flip (chiral odd) impossible by the eikonal (chiral even) interaction vertex

. proton helicity flips

quark Sivers in forward limit

$$H^q_{+\frac{1}{2}+\frac{1}{2},-\frac{1}{2}+\frac{1}{2}} = \frac{1}{2}\left[-\frac{k_L}{M}\left(\boxed{P^{0,+;q}_{1,1a}} - P^{0,-;q}_{1,1a}\right) - \frac{\Delta_L}{M}\left(P^{0,+;q}_{1,1b} - P^{0,-;q}_{1,1b}\right)\right]$$

$$H^q_{-\frac{1}{2}+\frac{1}{2},+\frac{1}{2}+\frac{1}{2}} = \frac{1}{2}\left[\frac{k_R}{M}\left(\boxed{P^{0,+;q}_{1,1a}} + P^{0,-;q}_{1,1a}\right) + \frac{\Delta_R}{M}\left(\boxed{P^{0,+;q}_{1,1b}} + P^{0,-;q}_{1,1b}\right)\right],$$

relates to the quark GPD E

$$P^{0,+;q}_{1,1a} = \frac{\boldsymbol{k}\cdot\boldsymbol{\Delta}}{M^2} X^{P0,+;q}_{1,1a} + \mathrm{i}Y^{P0,+;q}_{1,1a}$$

$$P^{0,-;q}_{1,1a} = X^{P0,-;q}_{1,1a} + \mathrm{i}\frac{\boldsymbol{k}\cdot\boldsymbol{\Delta}}{M^2} Y^{P0,-;q}_{1,1a}$$

$$P^{0,+;q}_{1,1b} = X^{P0,+;q}_{1,1b} + \mathrm{i}\frac{\boldsymbol{k}\cdot\boldsymbol{\Delta}}{M^2} Y^{P0,+;q}_{1,1b}$$

$$P^{0,-;q}_{1,1b} = \frac{\boldsymbol{k}\cdot\boldsymbol{\Delta}}{M^2} X^{P0,-;q}_{1,1b} + \mathrm{i}Y^{P0,-;q}_{1,1b}$$

decomposition into real and imaginary parts

Seaquark GTMDs with proton helicity flips

imaginary parts governed by spin dep Odderon

$$xX^{P0,+;q}_{1,1a} = \frac{N_c}{2\pi^2}\int \mathrm{d}^2\boldsymbol{k}_g A(\boldsymbol{q},\boldsymbol{k})\frac{\boldsymbol{k}_g^2}{\boldsymbol{k}^2}\left[\frac{2(\boldsymbol{k}\cdot\boldsymbol{k}_g)^2}{\boldsymbol{k}^2\boldsymbol{k}_g^2}-1\right]\mathcal{P}^\perp_{1T,0}(|\boldsymbol{k}_g|,|\boldsymbol{\Delta}|) + \left[\frac{2(\boldsymbol{k}\cdot\boldsymbol{\Delta})^2}{\boldsymbol{k}^2\boldsymbol{\Delta}^2}-1\right]\frac{N_c}{\pi^2}\int \mathrm{d}^2\boldsymbol{k}_g A(\boldsymbol{q},\boldsymbol{k})\frac{\boldsymbol{k}_g^2}{\boldsymbol{k}^2}\left[\frac{8(\boldsymbol{k}\cdot\boldsymbol{k}_g)^4}{\boldsymbol{k}^4\boldsymbol{k}_g^4}-\frac{8(\boldsymbol{k}\cdot\boldsymbol{k}_g)^2}{\boldsymbol{k}^2\boldsymbol{k}_g^2}+1\right]\mathcal{P}^\perp_{1T,\epsilon}(|\boldsymbol{k}_g|,|\boldsymbol{\Delta}|),$$

$$\boxed{xY^{P0,+;q}_{1,1a}} = \frac{N_c}{2\pi^2}\int \mathrm{d}^2\boldsymbol{k}_g A(\boldsymbol{q},\boldsymbol{k})\frac{\boldsymbol{k}\cdot\boldsymbol{k}_g}{\boldsymbol{k}^2}\boxed{\mathcal{O}^\perp_{1T,0}(|\boldsymbol{k}_g|,|\boldsymbol{\Delta}|)},$$

$$xY^{P0,-;q}_{1,1a} = \frac{N_c}{2\pi^2}\frac{M^2}{\boldsymbol{k}^2}\int \mathrm{d}^2\boldsymbol{k}_g\left[\left(\frac{(\boldsymbol{k}\cdot\boldsymbol{k}_g)^2}{\boldsymbol{k}^2\boldsymbol{k}_g^2}-1\right)2B_1(\boldsymbol{q},\boldsymbol{k}) + \left(\frac{2(\boldsymbol{k}\cdot\boldsymbol{k}_g)^2}{\boldsymbol{k}^2\boldsymbol{k}_g^2}-\frac{\boldsymbol{k}\cdot\boldsymbol{k}_g}{\boldsymbol{k}_g^2}-1\right)B_2(\boldsymbol{q},\boldsymbol{k})\right]\boldsymbol{k}_g^2\boxed{\mathcal{O}^\perp_{1T,0}(|\boldsymbol{k}_g|,|\boldsymbol{\Delta}|)},$$

$$xX^{P0,+;q}_{1,1b} = \frac{N_c}{2\pi^2}\int \mathrm{d}^2\boldsymbol{k}_g A(\boldsymbol{q},\boldsymbol{k})\left\{\frac{\boldsymbol{k}_g^2}{M^2}\left[1-\frac{(\boldsymbol{k}\cdot\boldsymbol{k}_g)^2}{\boldsymbol{k}^2\boldsymbol{k}_g^2}\right]\left[\mathcal{P}^\perp_{1T,0}(|\boldsymbol{k}_g|,|\boldsymbol{\Delta}|)+\frac{2(\boldsymbol{k}\cdot\boldsymbol{k}_g)^2}{\boldsymbol{k}^2\boldsymbol{k}_g^2}2\mathcal{P}^\perp_{1T,\epsilon}(|\boldsymbol{k}_g|,|\boldsymbol{\Delta}|)\right]+\mathcal{P}_{T,0}(|\boldsymbol{k}_g|,|\boldsymbol{\Delta}|)\right\} + \left[\frac{2(\boldsymbol{k}\cdot\boldsymbol{\Delta})^2}{\boldsymbol{k}^2\boldsymbol{\Delta}^2}-1\right]\frac{N_c}{\pi^2}\int \mathrm{d}^2\boldsymbol{k}_g A\left\{\frac{\boldsymbol{k}_g^2}{M^2}\left[-\frac{4(\boldsymbol{k}\cdot\boldsymbol{k}_g)^4}{\boldsymbol{k}^4\boldsymbol{k}_g^4}+\frac{5(\boldsymbol{k}\cdot\boldsymbol{k}_g)^2}{\boldsymbol{k}^2\boldsymbol{k}_g^2}-1\right]\mathcal{P}^\perp_{1T,\epsilon}+\left[\frac{2(\boldsymbol{k}\cdot\boldsymbol{k}_g)^2}{\boldsymbol{k}^2\boldsymbol{k}_g^2}-1\right]\mathcal{P}_{T,\epsilon}\right\},$$

$$xY^{P0,-;q}_{1,1b} = \frac{N_c}{2\pi^2}\int \mathrm{d}^2\boldsymbol{k}_g B_2\left[-\frac{(\boldsymbol{k}\cdot\boldsymbol{k}_g)^2}{\boldsymbol{k}^2\boldsymbol{k}_g^2}+\frac{\boldsymbol{k}\cdot\boldsymbol{k}_g}{\boldsymbol{k}_g^2}\right]\boldsymbol{k}_g^2\boxed{\mathcal{O}^\perp_{1T,0}},$$

SB, Hagiwara, Šarić, Vivoda, 2603.06092

$$\boxed{f^{\perp,q}_{1T} = -Y^{P0,+}_{1,1a}|_{\boldsymbol{\Delta}=0}}$$

Quark Sivers – result agrees with Zhou et al

Dong, Zheng, Zhou (2017)

High k_T limit

SB, Hagiwara, Šarić, Vivoda, 2603.06092

. important to verify that in the high-k_T limit all the quark GTMDs become sourced by the gluon GPDs

$$X^{S0,+;q}_{1,1a}(x,\boldsymbol{k},\boldsymbol{\Delta}) = \frac{\alpha_S}{6\pi^2}\frac{\boxed{H^g(x,\boldsymbol{\Delta})}}{\boldsymbol{k}^2} + \frac{2(\boldsymbol{k}\cdot\boldsymbol{\Delta})^2 - \boldsymbol{k}^2\boldsymbol{\Delta}^2}{\boldsymbol{k}^2 M^2}\frac{\alpha_S}{48\pi^2}\frac{\boxed{E^g_T(x,\boldsymbol{\Delta}) + 2\tilde{H}^g_T(x,\boldsymbol{\Delta})}}{\boldsymbol{k}^2}$$

Isotropic sourced by unpolarized GPD

Elliptic sourced by transversity GPDs
(this combination does not flip proton helicity)

$$Y^{S0,+,q}_{1,1a}(x,\boldsymbol{k},\boldsymbol{\Delta}) = -\frac{\alpha_S}{6\pi}\frac{M^2}{\boldsymbol{k}^2}\frac{1}{x}\frac{\boxed{O_1(x,\boldsymbol{\Delta})}}{\boldsymbol{k}^2}$$

twist-3 gluon GPD (d-type)

$$\frac{g}{P^+}\int_{-\infty}^{\infty}\frac{dz^-}{2\pi}\frac{dw^-}{2\pi}e^{i(x_1+x_2)P^+z^-}e^{i(x_2-x_1)P^+w^-}\boxed{d_{abc}}\langle p'\Lambda'|F_a^{+i}(-z/2)F_b^{+j}(w)F_c^{+k}(z/2)|p\Lambda\rangle$$
$$= i\delta_{\Lambda'\Lambda}\Delta^l\left[\delta^{ik}\delta^{jl}O_1(x_1,x_2,\boldsymbol{\Delta}) - \delta^{ij}\delta^{kl}O_1(x_2,x_2-x_1,\boldsymbol{\Delta}) - \delta^{jk}\delta^{il}O_1(x_1,x_1-x_2,\boldsymbol{\Delta})\right]$$
$$+ 2i\Lambda\delta_{\Lambda',-\Lambda}\left(\delta_{\Lambda,-1}L^l + \delta_{\Lambda 1}R^l\right)M$$
$$\times\left[\delta^{ik}\delta^{jl}O_2(x_1,x_2,\boldsymbol{\Delta}) + \delta^{ij}\delta^{kl}O_2(x_2,x_2-x_1,|\boldsymbol{\Delta}|) + \delta^{jk}\delta^{il}O_2(x_1,x_1-x_2,\boldsymbol{\Delta})\right] + \dots,$$

High k_T limit

SB, Hagiwara, Šarić, Vivoda, 2603.06092

$$X^{P0,+;q}_{1,1a}(x,\boldsymbol{k},\boldsymbol{\Delta}) = -\frac{\alpha_S}{12\pi^2}\frac{M^2}{\boldsymbol{k}^2}\frac{\boxed{H^g_T(x,\boldsymbol{\Delta})}}{\boldsymbol{k}^2}$$

$$Y^{P0,+;q}_{1,1a}(x,\boldsymbol{k},\boldsymbol{\Delta}) = -\frac{\alpha_S}{6\pi}\frac{M^2}{\boldsymbol{k}^2}\frac{1}{x}\frac{O_2(x,\boldsymbol{\Delta})}{\boldsymbol{k}^2}$$

Quark Sivers

$$f^{\perp,q}_{1T} = -Y^{P0,+}_{1,1a}|_{\boldsymbol{\Delta}=0}$$

Dong, Zheng, Zhou (2017)

Proton helicity flip GTMDs sourced by gluon transversity GPDs and the gluon GPD E

$$Y^{P0,-;q}_{1,1a}(x,\boldsymbol{k},\boldsymbol{\Delta}) = \frac{\alpha_S}{12\pi}\frac{M^4}{\boldsymbol{k}^4}\frac{1}{x}\frac{O_2(x,\boldsymbol{\Delta})}{\boldsymbol{k}^2}$$

$$X^{P0,+;q}_{1,1b}(x,\boldsymbol{k},\boldsymbol{\Delta}) = \frac{\alpha_S}{24\pi^2}\frac{\boxed{H^g_T(x,\boldsymbol{\Delta})+2E^g(x,\boldsymbol{\Delta})}}{\boldsymbol{k}^2} - \frac{2(\boldsymbol{k}\cdot\boldsymbol{\Delta})^2-\boldsymbol{k}^2\boldsymbol{\Delta}^2}{\boldsymbol{k}^2M^2}\frac{\alpha_S}{48\pi^2}\frac{\boxed{\tilde{H}^g_T(x,\boldsymbol{\Delta})}}{\boldsymbol{k}^2}$$

$$Y^{P0,-;q}_{1,1b}(x,\boldsymbol{k},\boldsymbol{\Delta}) = \frac{\alpha_S}{12\pi}\frac{M^2}{\boldsymbol{k}^2}\frac{1}{x}\frac{O_2(x,\boldsymbol{\Delta})}{\boldsymbol{k}^2}$$

. matches onto the results by Bertone et al

$$S^{0;-;q}_{1,1b} = \frac{(\boldsymbol{b}\cdot\boldsymbol{\Delta}_T)}{2M|\boldsymbol{b}|}C^{L/T}_{q/g}\otimes\left[E^g_T-\xi\widetilde{E}^g_T+2\widetilde{H}^g_T\right]$$

except for spin-orbit case

Bertone, Echevarria, del Rio, Rodini (2025)

Conclusions

. completed the inventory of eikonal gluon and seaquark GTMDs at the leading twist

. computations possible based on models for Pomerons/Odderons

. **gluons**

-> TMD case: out of 8, 5 are eikonal, 2 independent

-> GTMD case: out of 16 (complex), 12 are eikonal, 3 independent

-> GPD case: out of 8, 5 are eikonal and independent

. **quarks**

-> TMD case: out of 8, 2 are eikonal, and independent

-> GTMD case: out of 16 (complex), 6 are eikonal and independent

-> GPD case: out of 8, 2 are eikonal and independent