Photo-production of a heavy-quark pair: interplay between Sudakov and saturation effects

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Introduction

TMDs with unpolarized beams

TMDs are crucial to describe hard processes in polarized collisions

(e.g. Drell-Yan and semi-inclusive DIS)

8 leading-twist TMDs

Sivers function

correlation between transverse spin of the nucleon and transverse momentum of the quark

Boer-Mulders function

correlation between transverse spin and transverse momentum of the quark in unpolarized nucleon

nucleon polarization

quark polarization

	U	L	T
U	f_1 number density q		$f_{1T}^{\perp}$ Sivers
L		g_1 helicity Δq	g_{1T}
T	$h_1^{\perp}$ Boer Mulders	$h_{1L}^{\perp}$	h_1 transversity $h_{1T}^{\perp}$

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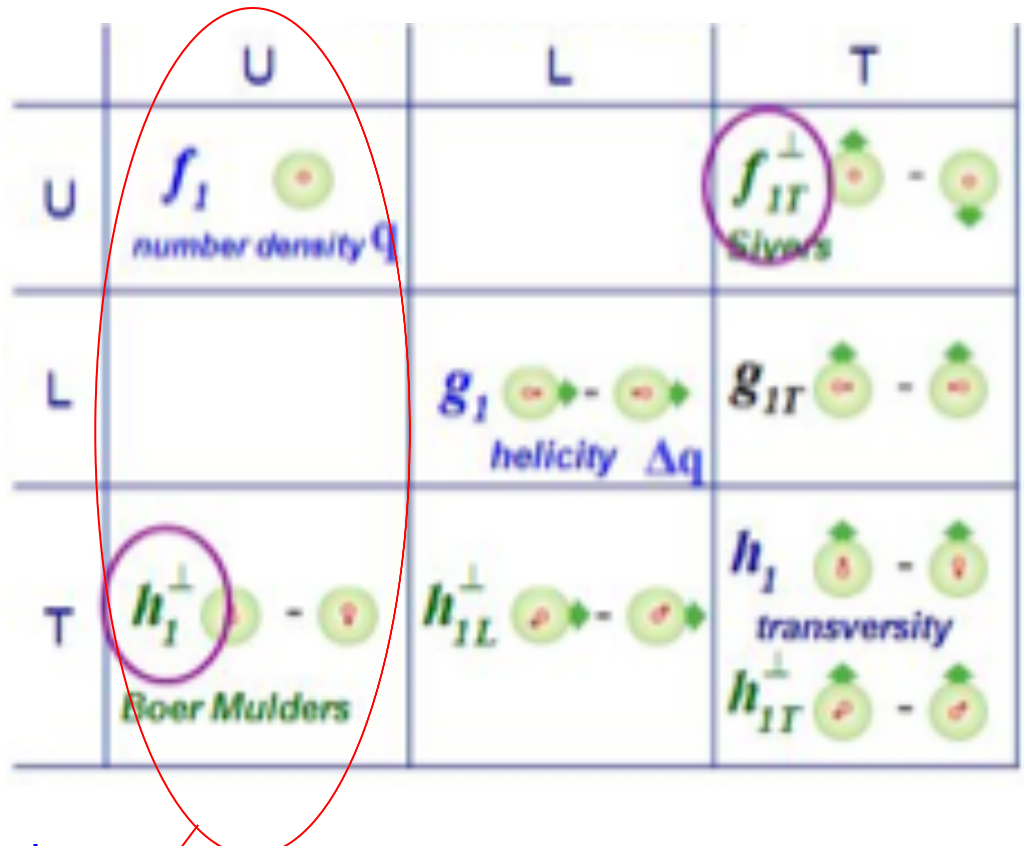

Sivers function

correlation between transverse spin of the nucleon and transverse momentum of the quark

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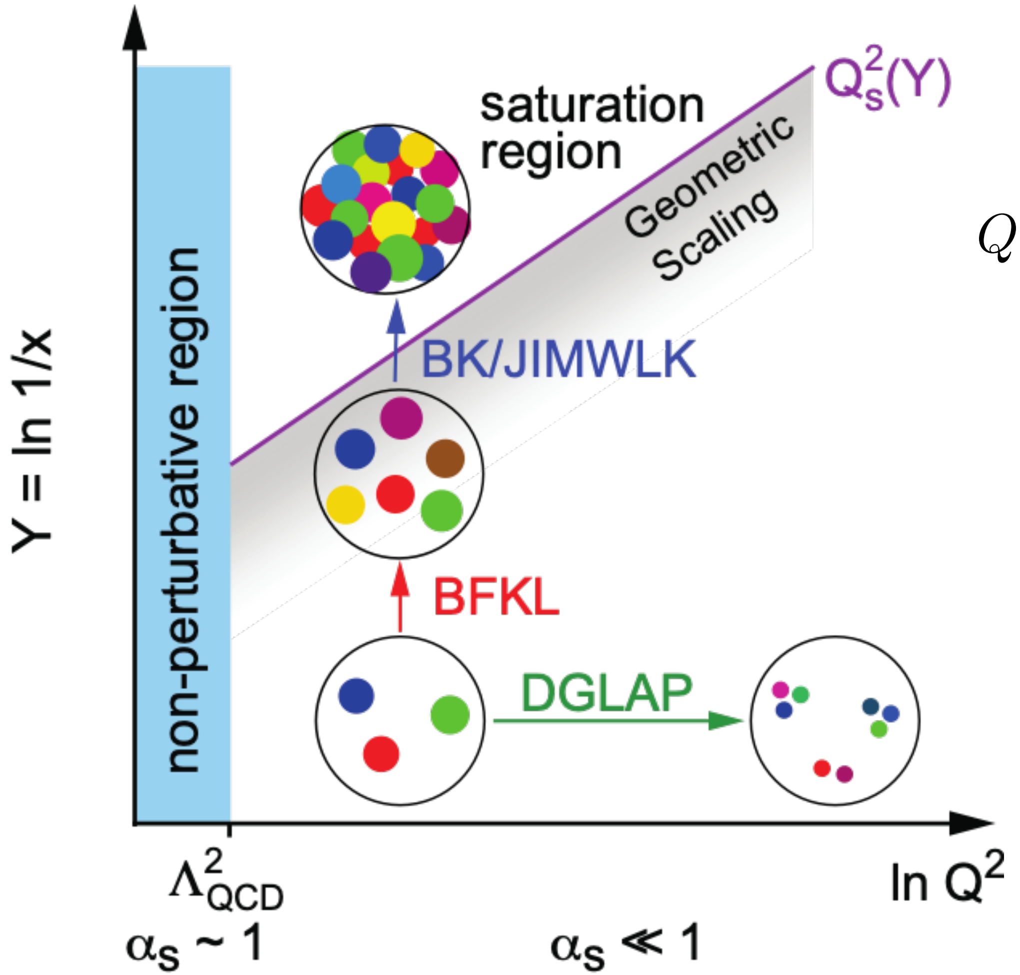
correlation between transverse spin and transverse momentum of the quark in unpolarized nucleon

I will discuss those for gluons

→ I consider only hadronic/nuclear states that are *unpolarized*

Small x and saturation



$$Q_s^2(x) \sim x^{-0.3} A^{1/3}$$

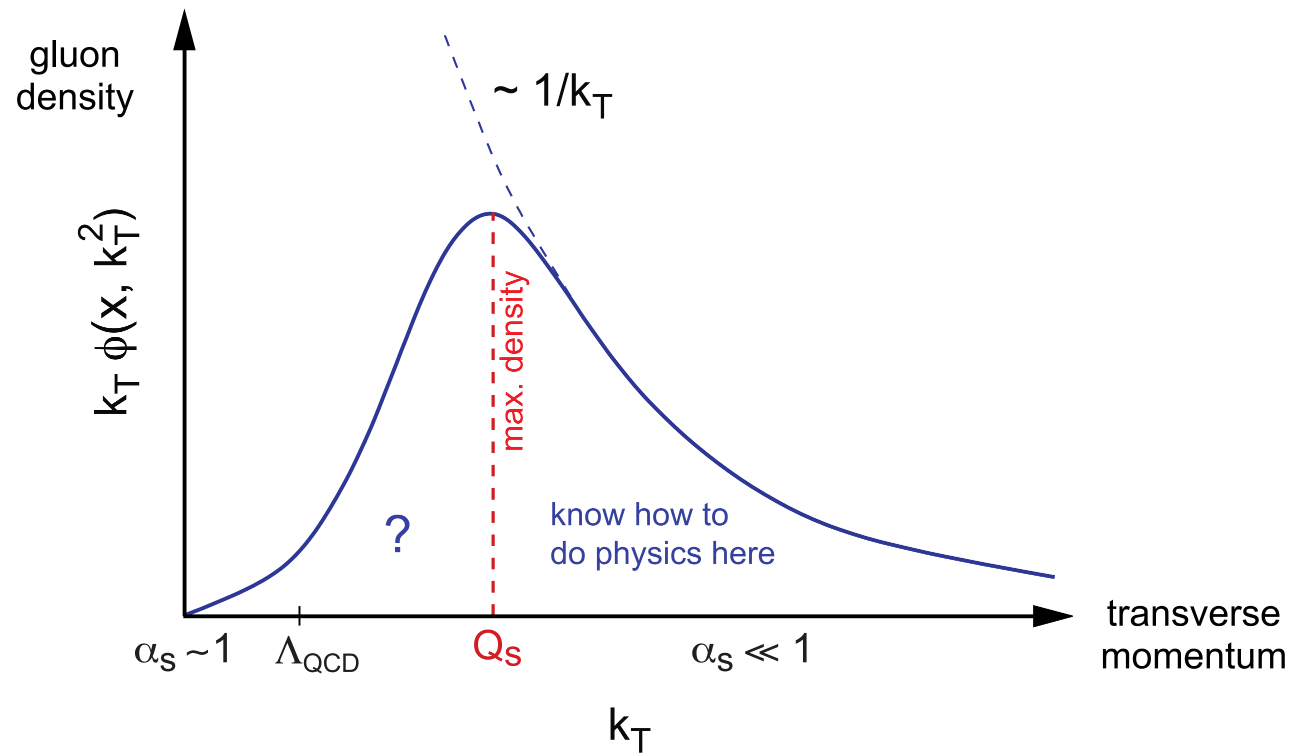
at small-x, the gluon transverse momentum plays an important role

so what does small-x physics have to say about gluon TMDs ?

The saturation scale

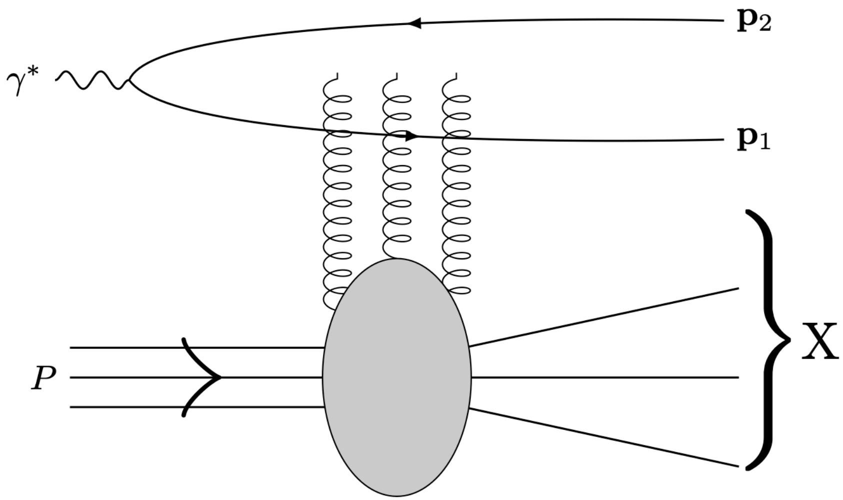
The saturation scale $Q_S(x)$ is the momentum scale which characterizes the transition between the dilute and dense regimes

at small-x, the typical gluon transverse momentum is no more Λ_{QCD} , it is instead $Q_S(x)$



the dynamics is non-linear, but the theory stays weakly coupled $\alpha_s(Q_s) \ll 1$

$Q\bar{Q}$ photo-production at small x



$$\langle k_t \rangle \sim Q_s$$

$$Q_s(x) \gg \Lambda_{QCD}$$

(*) the photon may also be virtual, but a large Q^2 value is not needed

The hard scale is: $\quad |p_{1t}|, \ |p_{2t}| \sim \mathbf{P} \gg Q_s$

The semi-hard scale is:

$$|k_t|^2 = |p_{1t} + p_{2t}|^2 = |p_{1t}|^2 + |p_{2t}|^2 + 2|p_{1t}||p_{2t}|cos\Delta\phi$$

→ the small-x gluon's transverse momentum (di-jet imbalance)

The back-to-back regime: TMD factorization

Generic definitions of gluon TMDs

I consider only hadronic/nuclear states that are *unpolarized*

$$2\int \frac{d\xi^+ d^2\boldsymbol{\xi}_t}{(2\pi)^3 p_A^-} e^{ixp_A^-\xi^+ - ik_t\cdot\boldsymbol{\xi}_t} \left\langle A \right| \mathrm{Tr}\left[F^{i-}\left(\xi^+, \boldsymbol{\xi}_t\right) F^{j-}(0)\right] \left| A\right\rangle$$

$$= \frac{\delta_{ij}}{2}\mathcal{F}(x, k_t) + \left(\frac{k_i k_j}{k_t^2} - \frac{\delta_{ij}}{2}\right)\mathcal{H}(x, k_t)$$

unpolarized gluon TMD

linearly-polarized gluon TMD

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unpolarized gluon TMD

linearly-polarized gluon TMD

- at small x, $\mathcal{F} = \mathcal{H}$ in the linear (a.k.a. BFKL) regime:

$$\mathcal{F}(x,k_t) = UGD(x,k_t) + \mathcal{O}(Q_s^2/k_t^2)$$

$$\mathcal{H}(x,k_t) = UGD(x,k_t) + \mathcal{O}(Q_s^2/k_t^2)$$

Kotko, Kutak, CM, Petreska, Sapeta, van Hameren (2015)

CM, Roiesnel, Taels (2017)

so-called unintegrated gluon distribution

The back-to-back regime at LO

$$|p_{1t}|, |p_{2t}| \gg |k_t|, Q_s$$

- a factorization can be established in the small *x* limit, for nearly back-to-back di-jets

Dominguez, CM, Xiao and Yuan (2011)

$$\mathrm{d}\sigma \propto H^{ij}(\mathbf{P})\left[\frac{1}{2}\delta^{ij}\mathcal{F}(x,k_t) + \left(\frac{k^i k^j}{k_t^2} - \frac{1}{2}\delta^{ij}\right)\mathcal{H}(x,k_t)\right]$$

hard factors

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hard factors

- gauge links are missing in the previous definition, their structure for this process implies that the gluon TMDs are of the Weizsäcker Williams type, which at small-x gives

$$\mathcal{F}_{WW}(x,k_t) = -\frac{4}{g^2}\int \frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3}\, e^{-ik_t\cdot(\mathbf{x}-\mathbf{y})}\left\langle \mathrm{Tr}\left[(\partial_i U_{\mathbf{x}})U^\dagger_{\mathbf{y}}(\partial_i U_{\mathbf{y}})U^\dagger_{\mathbf{x}}\right]\right\rangle_x$$

similarly for H_{WW} with projection onto the other 2d Lorentz structure

ITMD factorization

provides matching with BFKL at k_t ~ **P**

$$\mathrm{d}\sigma \propto H^{ij}(\mathbf{P},k_t)\left[\frac{1}{2}\delta^{ij}\mathcal{F}(x,k_t)+\left(\frac{k^ik^j}{k_t^2}-\frac{1}{2}\delta^{ij}\right)\mathcal{H}(x,k_t)\right]$$

hard factors

Kotko, Kutak, CM, Petreska, Sapeta, van Hameren (2015 - 2016)
Altinoluk, Boussarie, Kotko (2019)

ITMD factorization

provides matching with BFKL at $k_t \sim \mathbf{P}$

$$\mathrm{d}\sigma \propto H^{ij}(\mathbf{P},k_t)\left[\frac{1}{2}\delta^{ij}\mathcal{F}(x,k_t)+\left(\frac{k^i k^j}{k_t^2}-\frac{1}{2}\delta^{ij}\right)\mathcal{H}(x,k_t)\right]$$

hard factors

Kotko, Kutak, CM, Petreska, Sapeta, van Hameren (2015 - 2016)
Altinoluk, Boussarie, Kotko (2019)

- TMD factorization involves $H^{ij}(\mathbf{P},k_t=0)$

  needs $\mathbf{P} \gg k_t,\ Q_s$

- improved TMD (ITMD) factorization involves

$$H^{ij}(\mathbf{P},k_t)=H^{ij}(\mathbf{P},k_t=0)+\sum_n c_n(k_t/\mathbf{P})^n$$

all-order resummation of higher “kinematic” twists

also valid away from $\Delta\Phi = \pi$, when $k_t \sim \mathbf{P}$

Processes sensitive to $\mathcal{H}$

- factorization may be rewritten

$$d\sigma \propto H^{ns}(\mathbf{P},k_t)\mathcal{F}(x,k_t) + H^{h}(\mathbf{P},k_t)\Big(\underbrace{\mathcal{H}(x,k_t) - \mathcal{F}(x,k_t)}_{=\,0\text{ in BFKL regime}}\Big)$$

projections onto "non-sense" polarization

$$H^{ns} = H^{ij}k^ik^j/k_t^2$$

projections onto linear polarization

$$H^{h} = H^{ij}(k^ik^j/k_t^2 - \delta^{ij}/2)$$

emergence, due to non-linear effects, of
small-x gluons which are not fully linearly polarized

Processes sensitive to $\mathcal{H}$

- factorization may be rewritten

$$d\sigma \propto H^{ns}(\mathbf{P},k_t)\mathcal{F}(x,k_t) + H^h(\mathbf{P},k_t)\Big(\underbrace{\mathcal{H}(x,k_t) - \mathcal{F}(x,k_t)}_{=\,0\text{ in BFKL regime}}\Big)$$

projections onto
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projections onto linear polarization

$$H^h = H^{ij}(k^ik^j/k_t^2 - \delta^{ij}/2)$$

- processes for which the H^h hard factors are non-zero:

  - dijets in deep inelastic scattering (e+p or e+A)
  - heavy-quark pair production (in photo-production or p+A collisions)
  - trijets or more

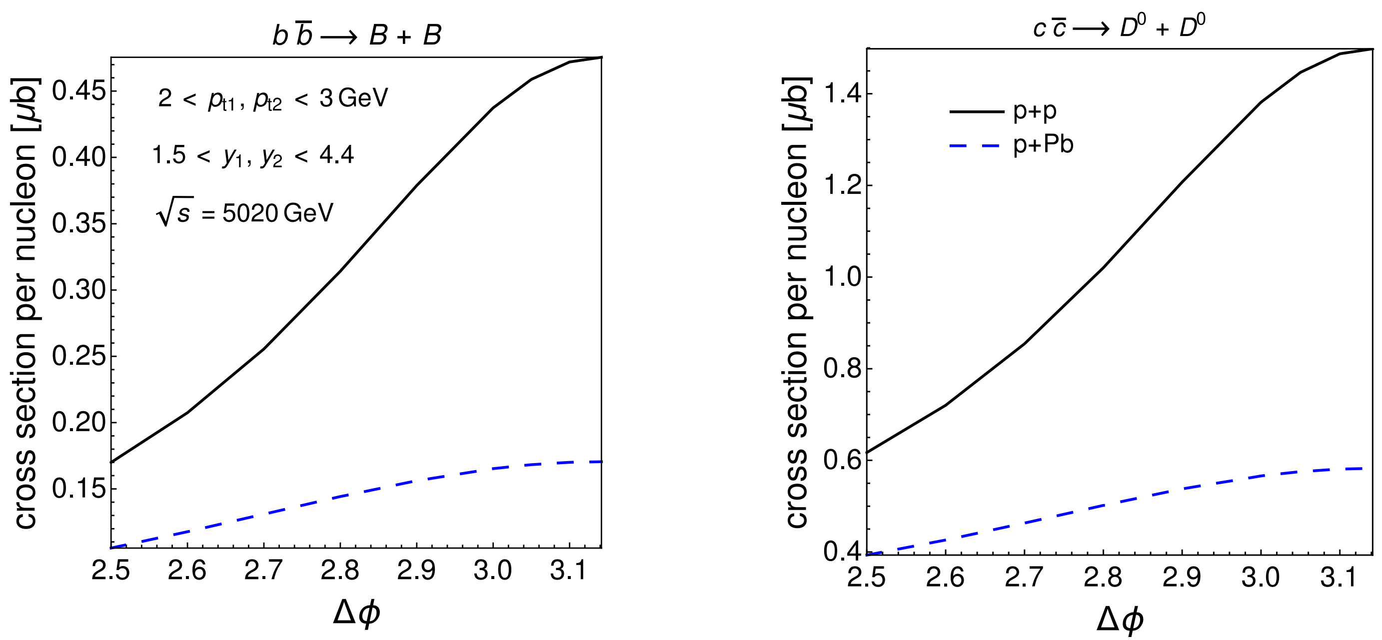
Altinoluk, Boussarie, CM, Taels (2019 - 2020)
Altinoluk, CM, Taels (2021)

the polarized gluons come with a cos(2ϕ) modulation
(at small k_t / **P**) where ϕ is the angle between k_t and **P**

Forward $Q\bar{Q}$ pair in p+A collisions

- preliminary study performed for HL-LHC yellow report

CM, Giacalone (2018)



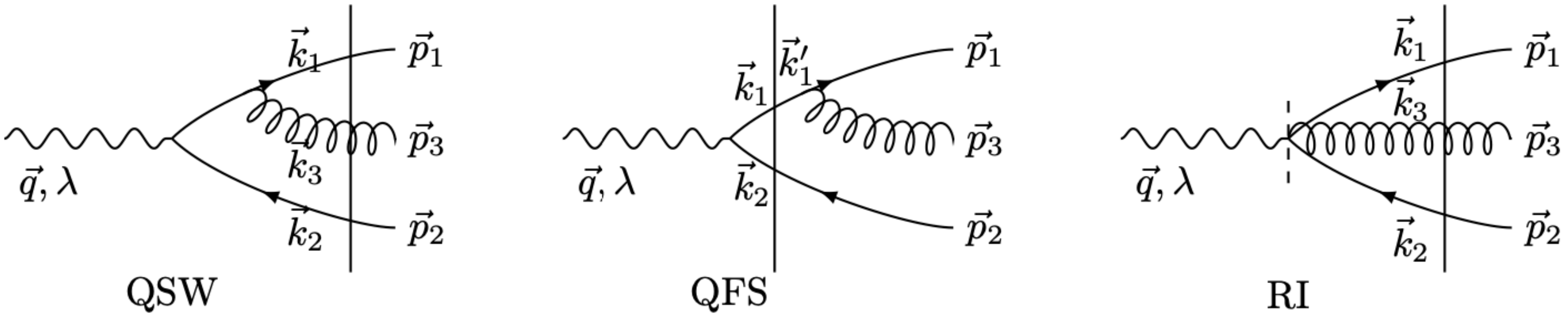
soft-gluon resummation needs to be implemented as well

important near $\Delta\Phi = \pi$, when $\log(\mathbf{P}/k_t)$ becomes large

NLO corrections and QCD evolution: di-jet case

Real emission diagrams

Altinoluk, Boussarie, CM and Taels (2020)



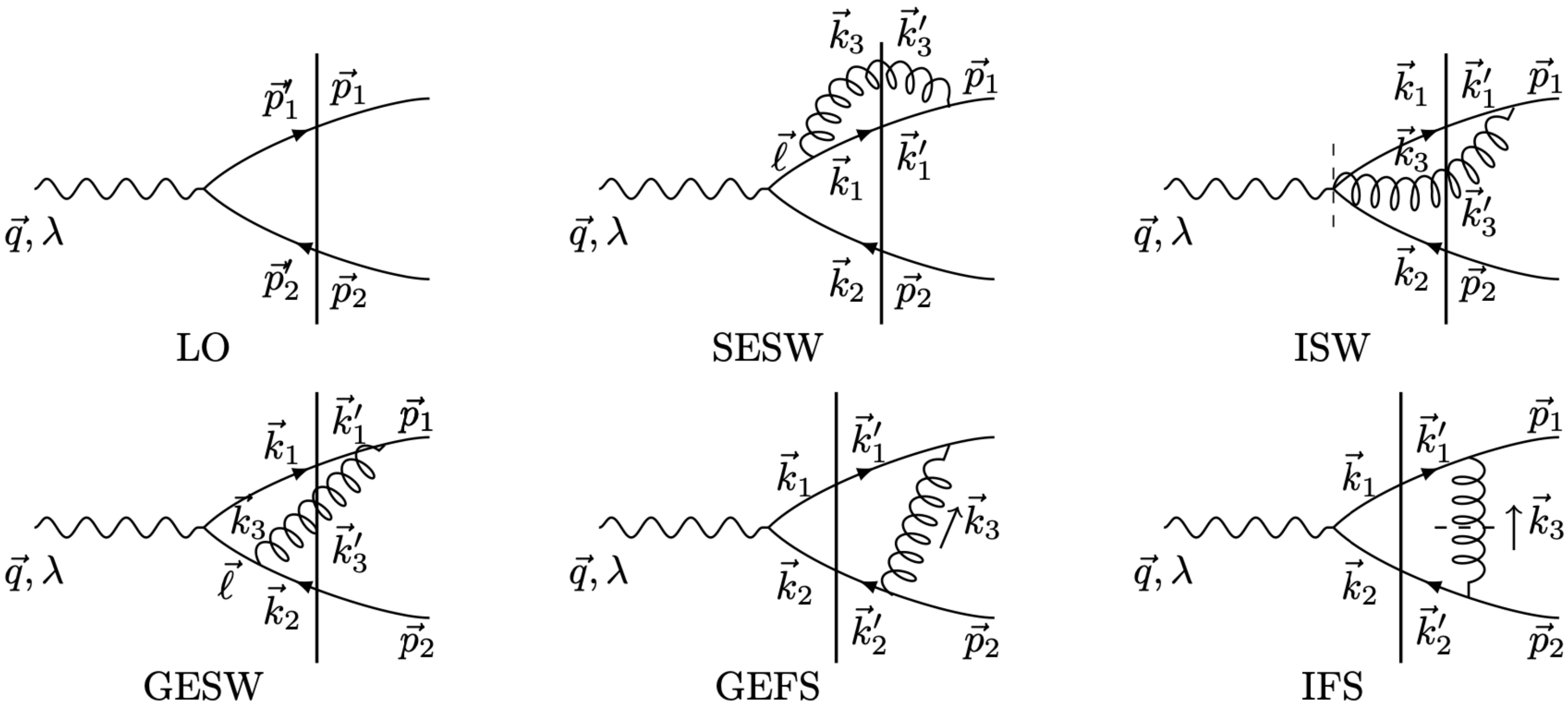
linearly-polarized gluon TMD involved at NLO, even for photo-production

see also

Caucal, Salazar and Venugopalan (2021)
Bergabo and Jalilian-Marian (2022)
Iancu and Mulian (2023)

Virtual diagrams

Caucal, Salazar and Venugopalan (2021)



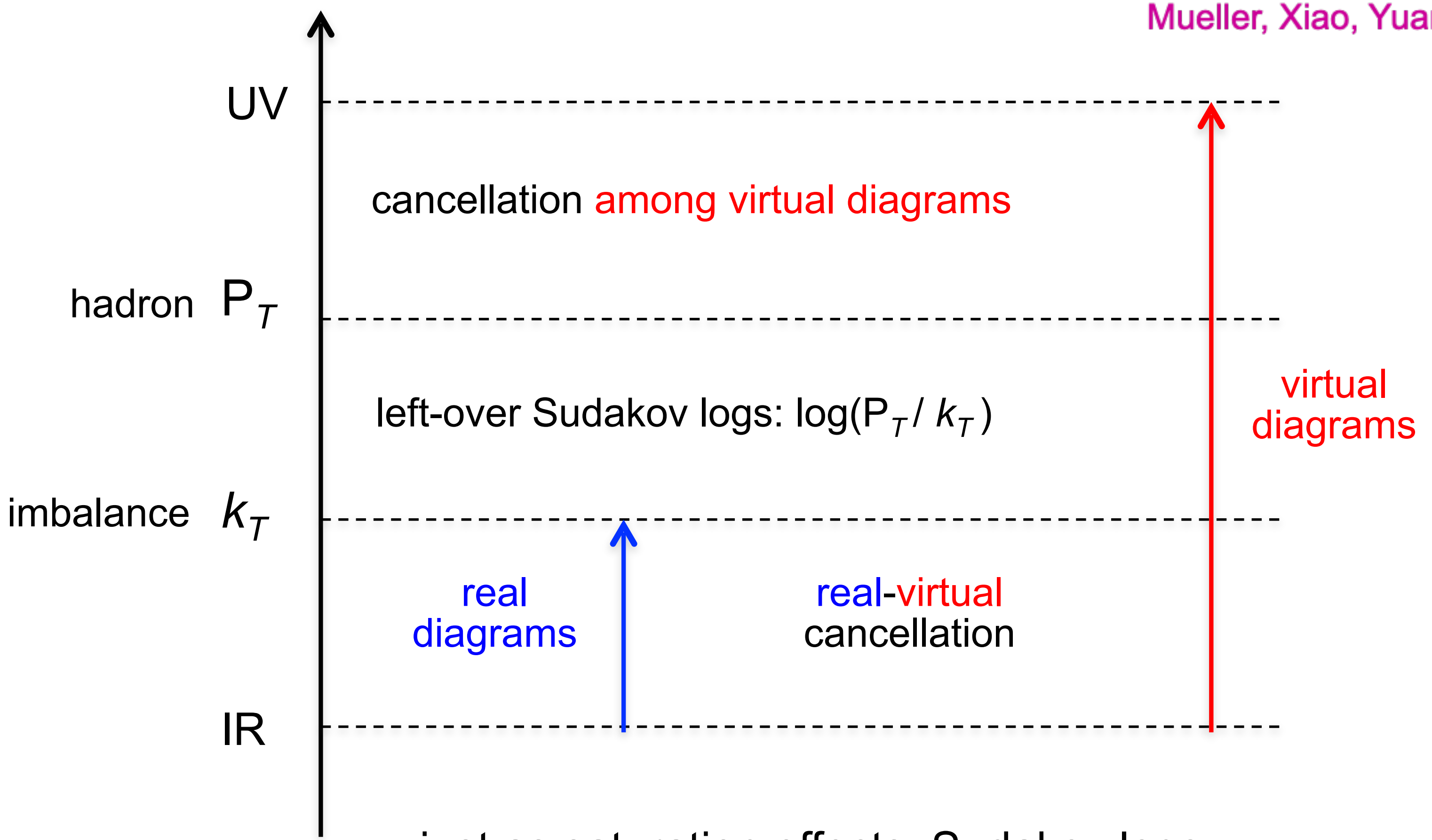
full NLO CGC is UV and soft finite
collinear divergences give DGLAP evolution of the fragmentation function
rapidity divergences give small-x evolution (BK/JIMWLK)

see also Taels, Altinoluk, Beuf and CM (2022)
Bergabo and Jalilian-Marian (2022)

Sudakov logarithms

- they arise from "incomplete" cancellation of IR divergences

Mueller, Xiao, Yuan (2013)

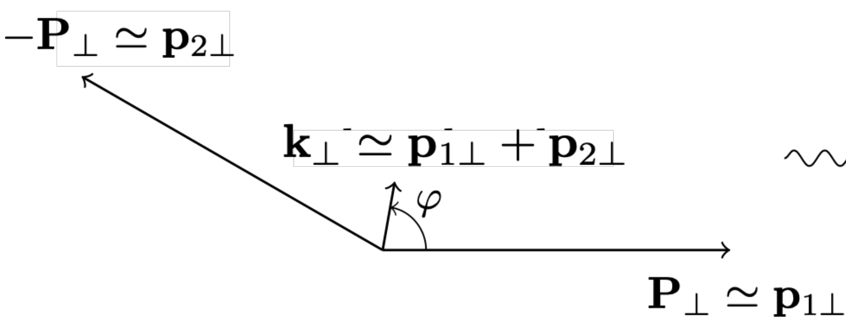


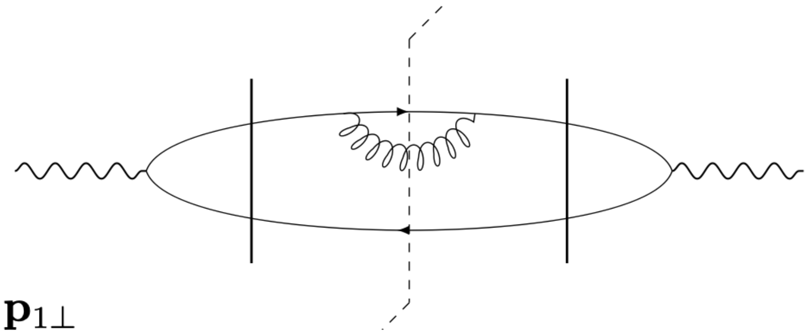
just as saturation effects, Sudakov logs
are important in the back-to-back regime $k_T \ll P_T$

The back-to-back regime at NLO

full NLO + TMD limit

Taels, Altinoluk, Beuf and CM (2022)





Remnants of soft-collinear generate Sudakov double log with wrong sign!

$$\mathrm{d}\sigma^{\mathrm{TMD}}_{\mathrm{NLO}} = \mathrm{d}\sigma^{\mathrm{TMD}}_{\mathrm{LO}} \times \frac{\alpha_s N_c}{4\pi} \ln\left(\frac{\mathbf{P}_\perp^2 (\mathbf{b}-\mathbf{b}')^2}{c_0^2}\right)^2$$

$$\mathbf{P}_\perp^2 \sim \mu^2$$
$$(\mathbf{b}-\mathbf{b}')^2 \sim 1/\mathbf{k}_\perp^2$$

this is due to an over-subtraction of the small-x rapidity logarithms

Sudakov and small-x logs aren't completely separated in phase space!

Kinematically-constrained evolution

Taels, Altinoluk, Beuf and CM (2022)

To obtain $$d\sigma^{\text{NLO}}_{\text{TMD}} \text{ ''} = \text{''} \; d\sigma^{\text{LO}}_{\text{TMD}} \times \left(-\frac{\alpha_s N_c}{4\pi}\right) \ln^2(\mathbf{P}^2|\mathbf{x}-\mathbf{y}|^2)$$

and then write

$$\mathcal{F}_{WW}(x,k_t;P) = -\frac{4}{g^2}\int \frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3}\; e^{-ik_t\cdot(\mathbf{x}-\mathbf{y})}e^{-S_{sud}(\mathbf{P},\mathbf{x}-\mathbf{y})}\left\langle \mathrm{Tr}\left[(\partial_i U_{\mathbf{x}})U^\dagger_{\mathbf{y}}(\partial_i U_{\mathbf{y}})U^\dagger_{\mathbf{x}}\right]\right\rangle_x$$

, re-summing the small-x logs and Sudakov logs separately, the rapidity subtraction must be altered

this leads to a kinematically-constrained small-x evolution

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this leads to a kinematically-constrained small-x evolution

→ in the small-x evolved LO contribution, the kernel of the JIMWLK equation now contains an extra theta term $\theta\left[(k_g^+/k_f^+)\mathbf{P}^2 - \mathbf{k}_g^2\right]$

confirmed beyond large Nc and double logs in

Caucal, Salazar, Schenke, Venugopalan (2022)

Asymmetry suppressed by evolution

- without Sudakov resummation, the CGC predicts sizable <cos(2ϕ)>

$$\mathrm{d}\sigma \propto H_F(\mathbf{P})\mathcal{F}(x,k_t) + \cos(2\phi) H_H(\mathbf{P})\mathcal{H}(x,k_t)$$

$$< \cos(2\phi) > \propto \frac{\mathcal{H}(x,k_t)}{\mathcal{F}(x,k_t)}$$

with $\mathcal{F}$ & $\mathcal{H}$ of similar magnitude

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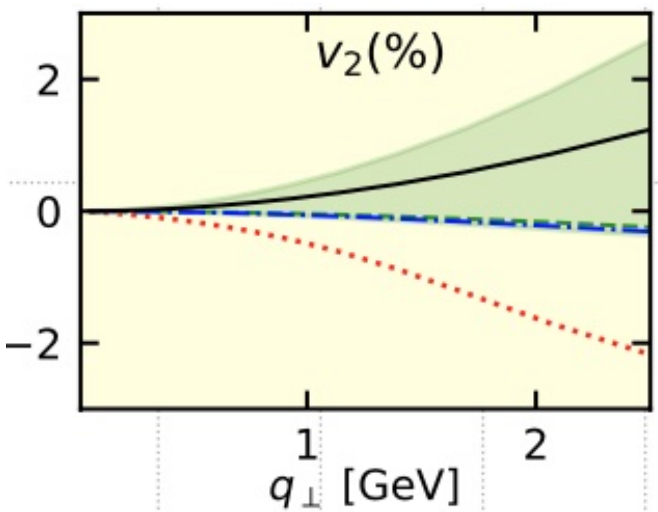
$$< \cos(2\phi) > \propto \frac{\mathcal{H}(x,k_t)}{\mathcal{F}(x,k_t)}$$ with $\mathcal{F}$ & $\mathcal{H}$ of similar magnitude

- however TMD evolution suppresses the asymmetry

Boer, Mulders, Zhou and Zhou (2017)



full NLO

LO + Sudakov & BK

LO

Caucal, Salazar, Schenke, Stebel and Venugopalan (2024)

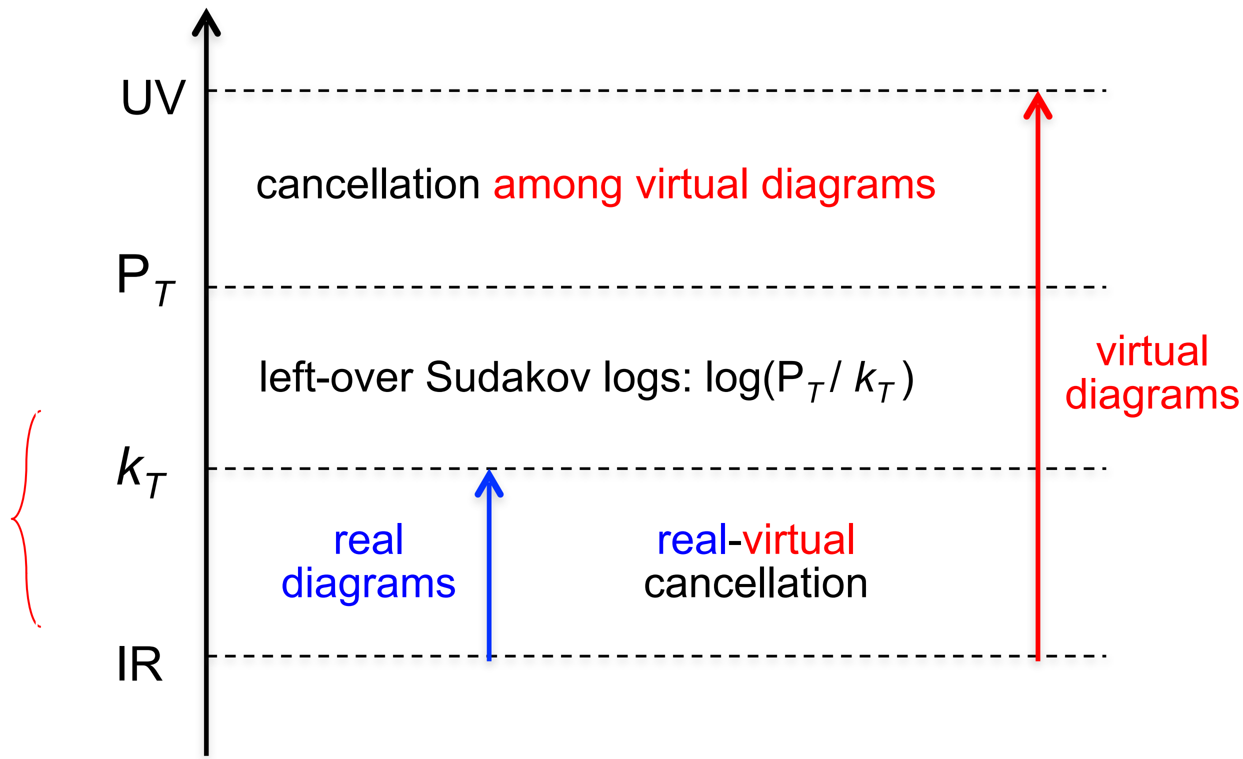
Heavy quark-antiquark pair

Using heavy quarks

- we computed the Sudakov factor for heavy quark production

following the method in Hatta, Xiao, Yuan, Zhou (2021)

we considered this regime for the heavy quark mass, unifying the massive calculation and the massless limit



CM, Y. Shi and B.-W. Xiao, arXiv:2510.18949

Asymetry with heavy $Q\bar{Q}$ pair

CM, Y. Shi and B.-W. Xiao, arXiv:2510.18949

- we computed the Sudakov factor for heavy quark production

following the method in Hatta, Xiao, Yuan, Zhou (2021), we obtain:

$$\mathcal{F}_{WW}(x, k_t; P) \longrightarrow -\frac{4}{g^2}\int \frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3}\, e^{-ik_t\cdot(\mathbf{x}-\mathbf{y})}e^{-S_{sud}(\mathbf{P},\mathbf{x}-\mathbf{y})}\left\langle \mathrm{Tr}\left[(\partial_i U_{\mathbf{x}})U^\dagger_{\mathbf{y}}(\partial_i U_{\mathbf{y}})U^\dagger_{\mathbf{x}}\right]\right\rangle_x$$

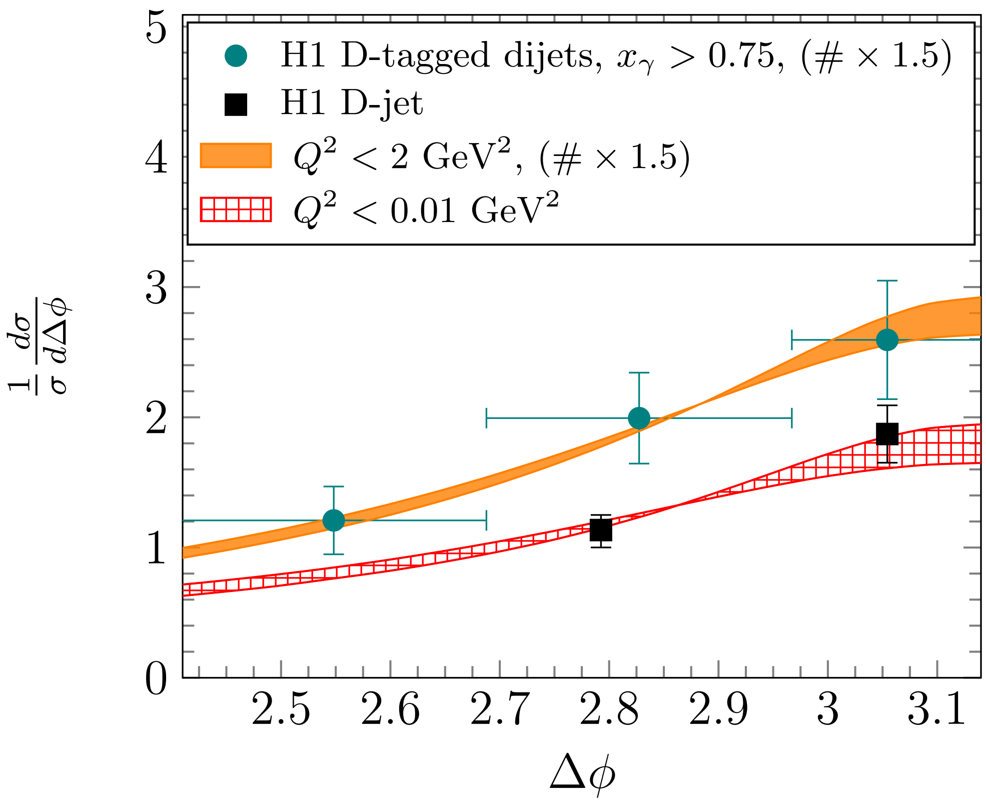
$$\times\left[1-\alpha_s\sum_{n>0} c_{2n}\cos(2n\phi_{\mathbf{P},\mathbf{x}-\mathbf{y}})\right]$$

additional cos(2nϕ) factor implies non-zero asymmetry for the $\mathcal{F}$ term

Comparison with HERA e+p data

CM, Y. Shi and B.-W. Xiao, arXiv:2510.18949

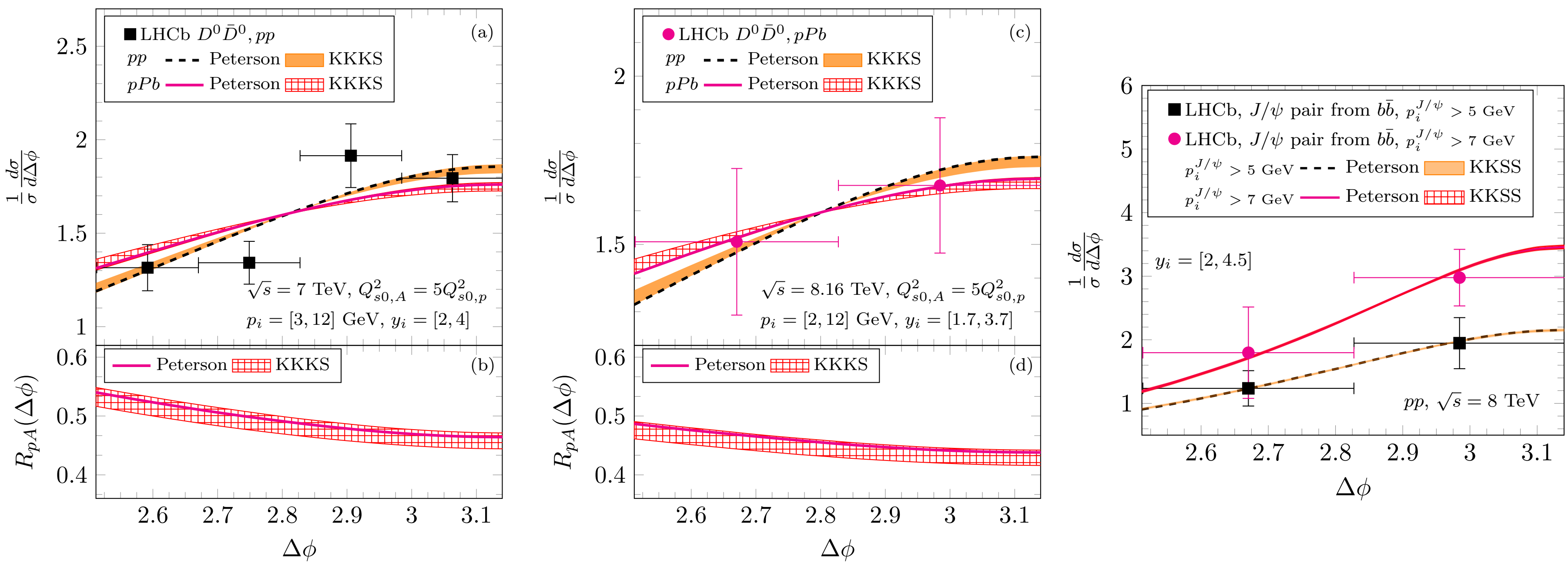
first description of HERA data



Comparison with LHC p+A data

Z. Gao, CM, Y. Shi and B.-W. Xiao, arXiv:2605.01527

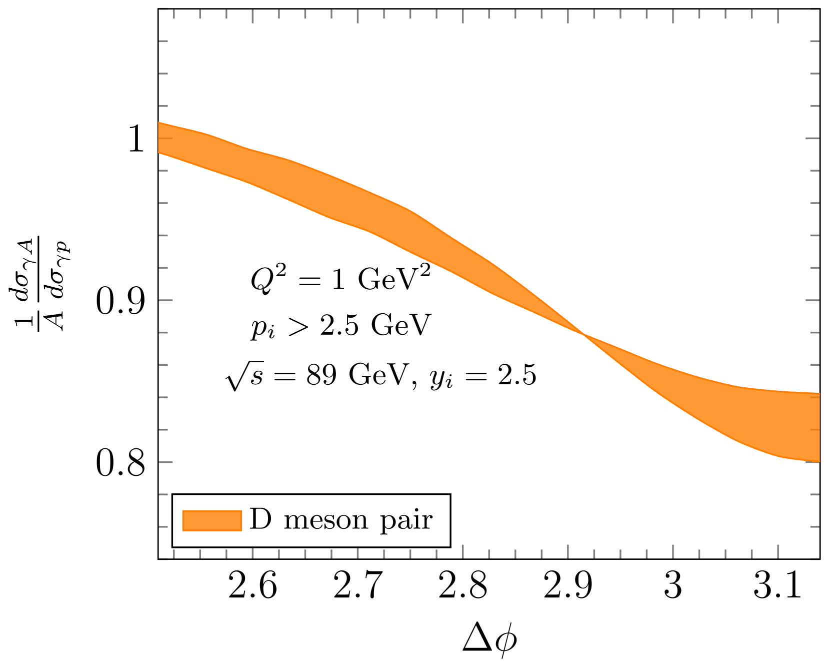
we also compare to p+Pb data in order to further validate our massive Sudakov resummation



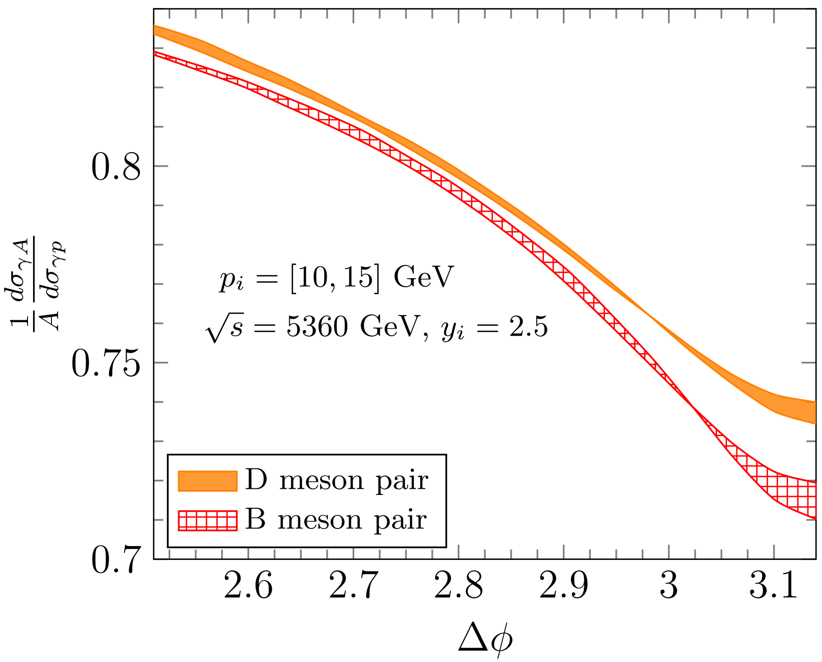
$\Delta\Phi$ distribution

the γ+A / γ+p suppression survives after the Sudakov factor is included

EIC γ+A / γ+p



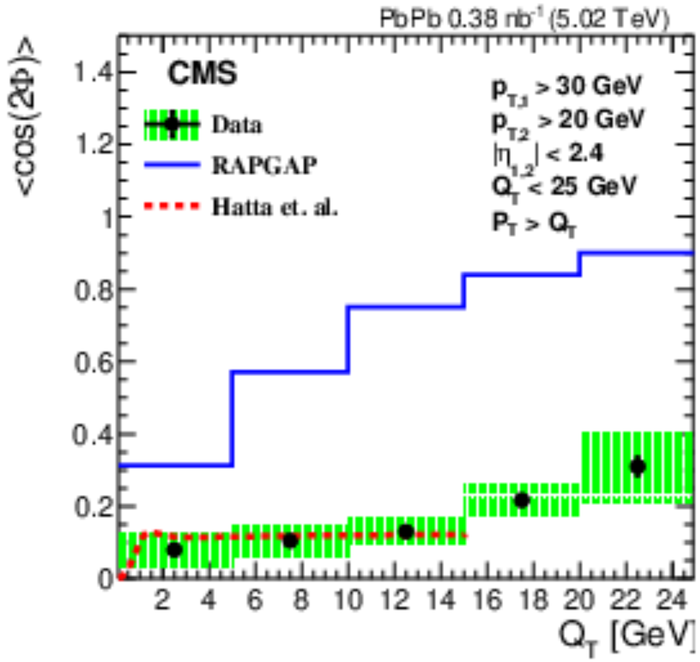
UPC γ+A / γ+p



inverse mass ordering due to suppression of soft gluon emission

The "asymmetry" observables $\langle \cos(n\phi) \rangle$

- measurement previously performed in exclusive UPCs

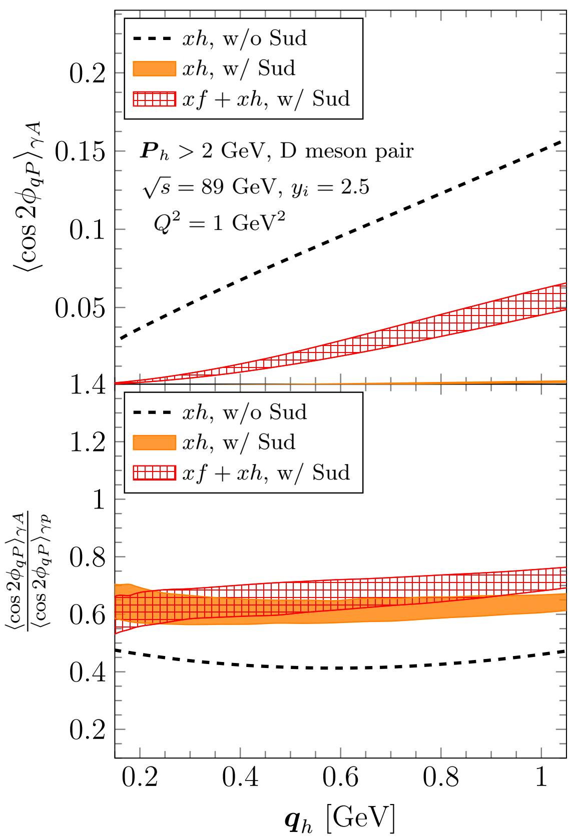


CMS collaboration (2023)

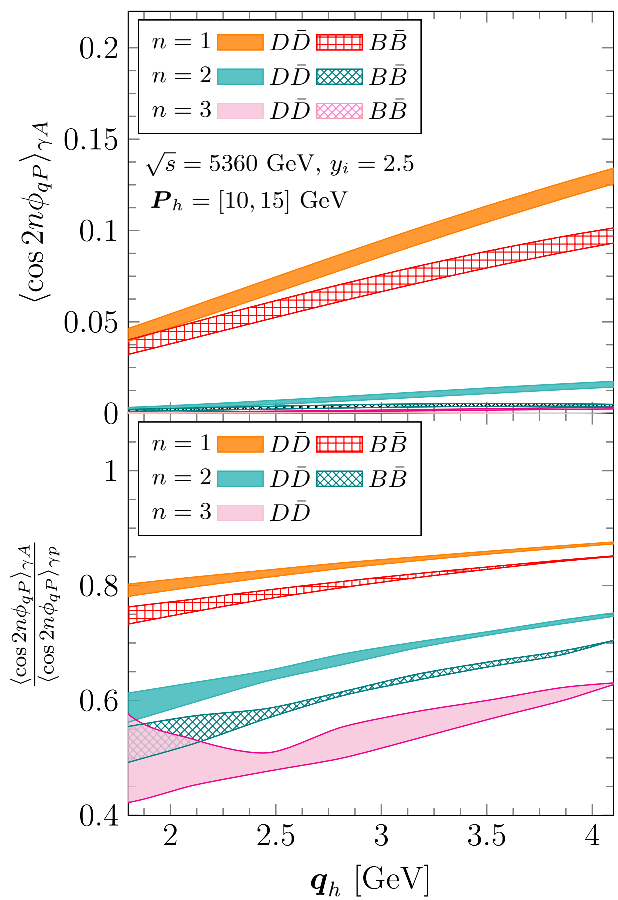
we advocate doing the same with inclusive UPCs

Asymmetry with heavy $Q\bar{Q}$ pair

EIC γ+A & γ+p



UPC γ+A & γ+p



higher harmonics are even more sensitive to non-linear effects

Conclusions

•to match collinear physics and small-x physics in the linear BFKL regime, the necessity of a kinematical constraint in the small-x evolution was recognized a long time ago (led to CCFM equation)

Ciafaloni ('88); Andersson, Gustafson, Samuelsson ('96);
Kwiecinski, Martin, Sutton ('96); Salam ('98)

•more recently, that necessity also emerged in CGC calculations, often in connection with the issue of negative NLO cross sections

Beuf (2014); Hatta, Iancu (2016);
Iancu, Madrigal, Mueller, Soyez, Triantafyllopoulos (2019)

•now it also appears in the context of two-scale processes and TMD physics

•heavy-quark photo-production provides a good testing ground for these theoretical developments, UPC measurements will be attempted at the LHC, and then we'll have the EIC