Quantum evolutions for diffractive jet production

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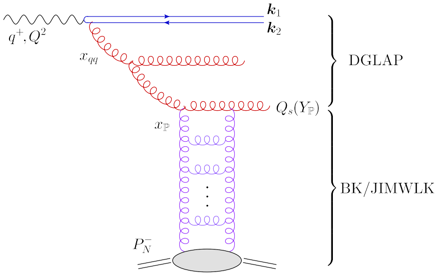
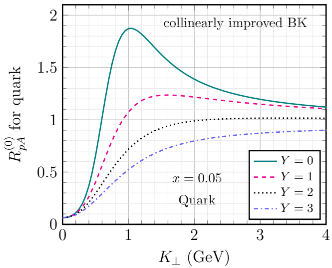
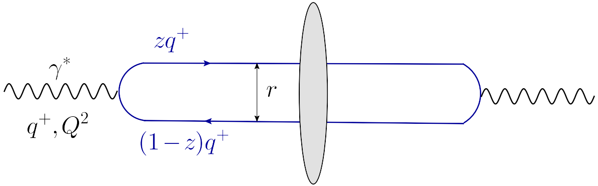




- Cross-section = photon wavefunction ($\gamma^* \to q\bar{q}$) $\otimes$ dipole cross-section



$$\sigma_{\gamma^* A}(Q^2, x) = \int \mathrm{d}^2 r \int_0^1 \mathrm{d}z \left|\Psi_{\gamma^* \to q\bar{q}}(r, z; Q^2)\right|^2 \underbrace{\sigma_{\text{dipole}}(r, A, x)}_{2\pi R_A^2\, T_A(r,x)}$$

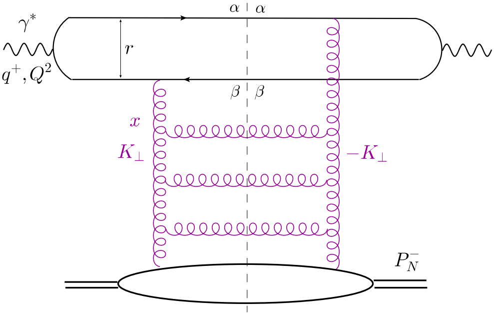
- QCD dynamics encoded in the dipole amplitude $T_A(r, x)$

- Transition from weak to strong scattering at $r \sim 1/Q_s$:

$$T_A(r, x) \simeq \begin{cases} r^2 Q_s^2(A, x) & \text{for } rQ_s \ll 1 \text{ (color transparency)} \\ 1 & \text{for } rQ_s \gtrsim 1 \text{ (black disk/saturation)} \end{cases}$$

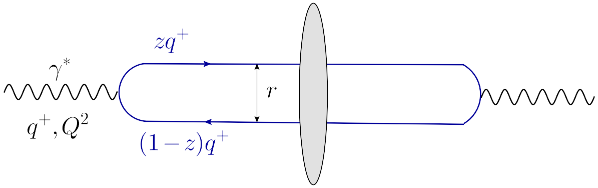
High energy evolution: BK/JIMWLK

- Decrease x starting at $x_0 \sim 10^{-2}$: non-linear generalisation of BFKL



$$T_A(r,x) \simeq \begin{cases} \left(r^2 Q_s^2(A,x)\right)^{\gamma_s} & \text{for } rQ_s \ll 1 \text{ (weak scattering)} \\ 1 & \text{for } rQ_s \gtrsim 1 \text{ (black disk/saturation)} \end{cases}$$

$$Q_s^2(A,x) \simeq Q_s^2(p,x_0) A^{1/3} \left(\frac{x_0}{x}\right)^{\lambda_s}, \quad \lambda_s \simeq 0.25, \quad \gamma_s \simeq 0.63$$



$$\sigma_{\gamma^* A}(Q^2, x) = \int \mathrm{d}^2 r \int_0^1 \mathrm{d}z \left|\Psi_{\gamma^* \to q\bar{q}}(r, z; Q^2)\right|^2 2\pi R_A^2\, T_A(r, x)$$

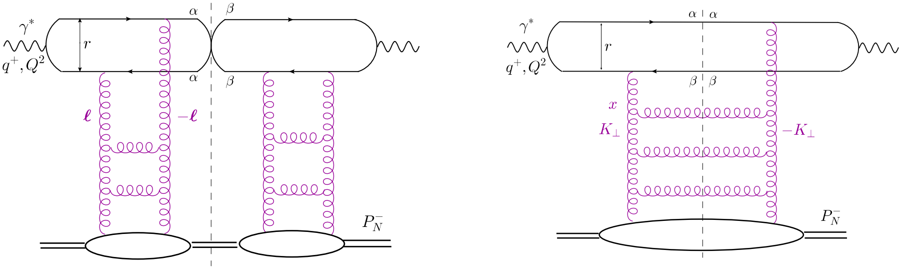
- Competition between scattering and photon virtuality
  - strong scattering requires large dipoles: $r \gtrsim 1/Q_s$
  - virtuality limits the dipole size: $r \lesssim 1/\bar{Q}$ with $\bar{Q}^2 \equiv z(1-z)Q^2$
- Strong scattering/saturation dominates only for relatively low virtuality:

$$Q^2 \lesssim Q_s^2(A, x) \sim 1\ \text{GeV}^2 \quad \text{for } A \sim 200\ \ \&\ \ x \sim 10^{-2}$$

- Possible contamination from non-perturbative physics: $\Lambda_{\rm QCD} \sim 0.2$ GeV

Elastic scattering

- Compare elastic scattering (left) and inelastic (right)



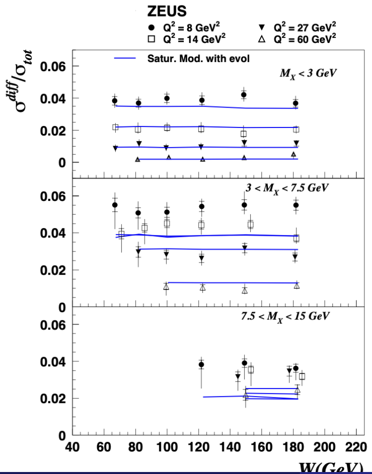
$$\sigma_{\rm el}(Q^2,x) = \int {\rm d}^2 r \int {\rm d}z \big|\Psi(r,z;Q^2)\big|^2\, 2\pi R_A^2\, |T(r,x)|^2$$

- Amplitude squared $\Longrightarrow$ Diffraction abhors weak scattering
- $\sigma_{\rm el}(Q^2,x)$ is controlled by strong scattering even when $Q^2 \gg Q_s^2(A,x)$
  - aligned jets: $r \sim 1/Q_s$ and $z(1-z) \sim Q_s^2/Q^2 \ll 1$

One vs two Pomerons... at HERA

- W/o saturation: σ_{el} would rise with $\frac{1}{x}$ (or with A) like two Pomerons

$$T^2 \simeq \left[r^2 Q_s^2(A,x)\right]^2 \propto \frac{A^{2/3}}{x^{2\lambda_s}} \qquad \text{vs.} \qquad T \simeq r^2 Q_s^2(A,x) \propto \frac{A^{1/3}}{x^{\lambda_s}}$$



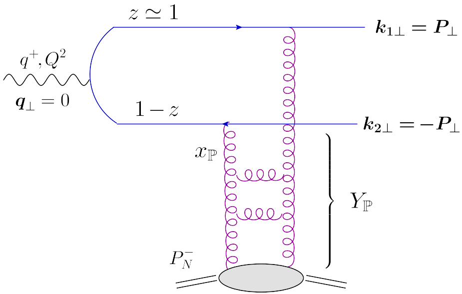
- But it doesn't! Double integral controlled by saturation: $T \sim 1$, or $r \sim 1/Q_s$
- Almost the same scaling as σ_{tot}:

$$\frac{\sigma_{el}}{\sigma_{tot}} \sim \frac{1}{\ln\left(Q^2/Q_s^2\right)}$$

- Weak x-dependence confirmed by HERA *(Bartels, Golec-Biernat, Kowalski, 2002)*
- Would be interesting to also check the A-dependence at the EIC

Diffractive particle production

- Leading order in α_s: a quark-antiquark pair in a colour singlet state
- Rapidity gap $Y_{\mathbb{P}} = \ln(1/x_{\mathbb{P}})$ between hadronic target and diffractive system



$$x_{\mathbb{P}} = \frac{Q^2 + M_{q\bar{q}}^2}{2P\cdot q} \ll 1$$

$$M_{q\bar{q}}^2 \simeq \frac{P_\perp^2}{z(1-z)} \sim Q^2$$

$$z(1-z) \sim \frac{Q_s^2(Y_{\mathbb{P}})}{Q^2} \ll 1$$

- Typical events: the produced particles are semi-hard: $P_\perp \sim 1/r \sim Q_s(Y_{\mathbb{P}})$
- "Aligned jet": the quark $(z \simeq 1)$ is aligned with the photon (forward)
- The antiquark is very soft: $1 - z \ll 1$ (midrapidities)

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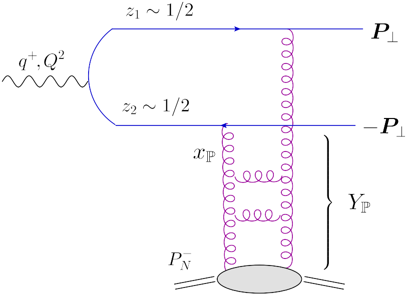


$$z_1 \sim z_2 \sim \frac{1}{2}\,, \qquad P_\perp \sim \frac{1}{r} \sim Q$$

$$\frac{\mathrm{d}\sigma_{\mathrm{el}}^{\gamma^* A \to q\bar{q}A}}{\mathrm{d}z_1 \mathrm{d}z_2 \mathrm{d}^2\boldsymbol{P}} \propto \underbrace{\alpha_{\mathrm{em}} r^2}_{\gamma^* \to q\bar{q}} \; \underbrace{Q_s^4 r^4}_{T_{q\bar{q}}^2}$$

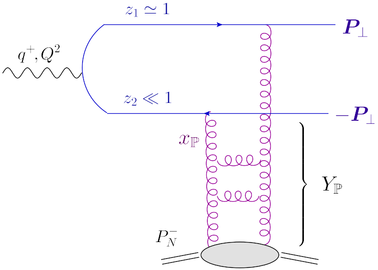
$$P_\perp \sim \frac{1}{r} \implies \frac{1}{P_\perp^6}$$

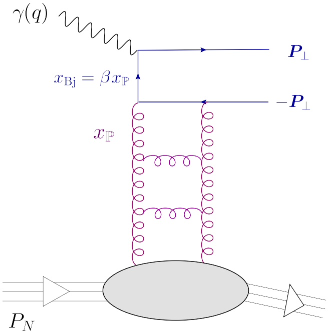
- Hard dijets ($P_\perp \sim Q$) are possible too, but strongly suppressed: higher twist
- Most naturally produced via additional radiation: 2+1 jets

  (cf. talk by Patricia G.-E.)

Diffractive SIDIS: TMD factorisation

- Mesure only the quark: integrate over z_2 at fixed $x_{\mathbb{P}}$ and for $Q^2 \gg P_\perp^2$
  - leading twist from "aligned jet configurations": $z_2 \sim \frac{P_\perp^2}{Q^2} \ll 1$
- Soft antiquark can be viewed as a constituent of the target (low $k_2^+ = z_2 q^+$)





- Target picture: the Pomeron splits into a quark-antiquark pair $\pm \boldsymbol{P}$
  - t-channel quark absorbs the photon and becomes the aligned jet

Diffractive SIDIS: TMD factorisation

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  - leading twist from "aligned jet configurations": $z_2 \sim \frac{P_\perp^2}{Q^2} \ll 1$
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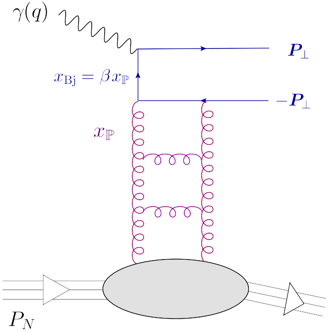
- customary to use the β variable
- longitudinal fraction w.r.t. Pomeron

$$\beta \equiv \frac{x_{\rm Bj}}{x_{\mathbb{P}}} = \frac{Q^2}{Q^2 + M_{q\bar{q}}^2}$$

$$\frac{\mathrm{d}\sigma^{\gamma_T^* A \to qAX}}{\mathrm{d}\ln(1/\beta)\,\mathrm{d}^2\boldsymbol{P}} = \frac{8\pi^2 \alpha_{em} e_f^2}{Q^2}\, \mathcal{Q}_{\mathbb{P}}(\beta, Y_{\mathbb{P}}, P_\perp^2)$$



- Cross-section = Hard factor (γ^* absorption) $\times$ Quark DTMD

The quark diffractive TMD

- Unintegrated quark distribution of the Pomeron (relabel $\beta \to x$) :

$$\mathcal{Q}_{\mathbb{P}}(x, Y_{\mathbb{P}}, P_\perp^2) = \frac{S_\perp N_c}{4\pi^3} \underbrace{\Psi_{\mathbb{P}}(x, Y_{\mathbb{P}}, P_\perp^2)}_{\text{occupation number}}$$

- Bessel-Fourier transform of the dipole amplitude $T(r, Y_{\mathbb{P}})$

$$\Psi_{\mathbb{P}}(x, Y_{\mathbb{P}}, P_\perp^2) \simeq x \begin{cases} 1, & P_\perp \lesssim \tilde{Q}_s(x, Y_{\mathbb{P}}) \\ \dfrac{\tilde{Q}_s^4(x, Y_{\mathbb{P}})}{P_\perp^4}, & P_\perp \gg \tilde{Q}_s(x, Y_{\mathbb{P}}) \end{cases}$$

- Rapid decay $\sim 1/P_\perp^4$ at $P_\perp^2 \gg \tilde{Q}_s^2(x, Y_{\mathbb{P}}) \equiv (1-x) Q_s^2(Y_{\mathbb{P}})$

- Diffractive PDF controlled by saturation: $P_\perp \lesssim \tilde{Q}_s$

$$x q_{\mathbb{P}}(x, x_{\mathbb{P}}, Q^2) \equiv \int^{Q^2} \mathrm{d}P_\perp^2 \, \mathcal{Q}_{\mathbb{P}}(x, Y_{\mathbb{P}}, P_\perp^2) \propto x(1-x)\, Q_s^2(Y_{\mathbb{P}}) \propto A^{1/3}$$

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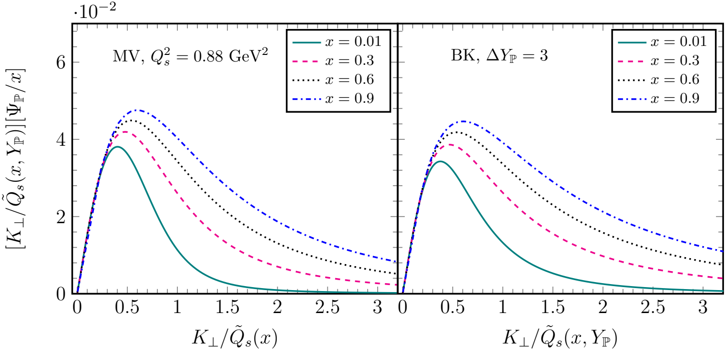
- F_2^D grows like the gluon PDF (one Pomeron), and not like its squared

$$x q_{\mathbb{P}}(x, x_{\mathbb{P}}, Q^2) \equiv \int^{Q^2} \mathrm{d}P_\perp^2 \, \mathcal{Q}_{\mathbb{P}}(x, Y_{\mathbb{P}}, P_\perp^2) \propto x(1-x)\, Q_s^2(Y_{\mathbb{P}}) \propto A^{1/3}$$

Numerical results *(2402.14748)*

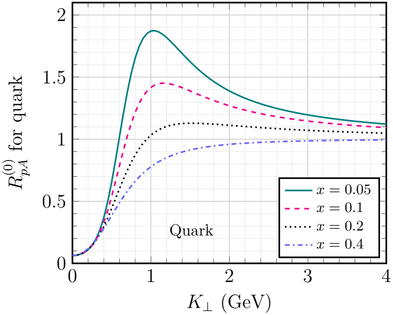
- Left: McLerran-Venugopalan model. Right: BK evolution with $Y_{\mathbb{P}}$
- $\Psi_{\mathbb{P}}(x, Y_{\mathbb{P}}, K_\perp^2)$ multiplied by $K_\perp$: Pronounced maximum at $K_\perp \simeq \tilde{Q}_s$



- BK evolution: increasing $Q_s(Y_{\mathbb{P}})$, approximate geometric scaling
  - (roughly) a function of the ratio $K_\perp/\tilde{Q}_s(x, Y_{\mathbb{P}})$
  - small anomalous dimension $1/K^{4\gamma_s}$ with $\gamma_s \sim 0.63$

The R_{pA} ratio for diffractive SIDIS *(2606.04169)*

- The ratio the D-SIDIS cross-sections in eA and respectively ep
  - TMD factorisation $\Rightarrow$ the ratio of the respective TMDs
  - normalised to unity at large $K_\perp \gg Q_s(A, Y=0) = 1$ GeV



$$R_{pA}(x, Y_{\mathbb{P}}, K_\perp) \equiv \frac{\Psi_{\mathbb{P}}^{(A)}(x, x_{\mathbb{P}}, K_\perp^2)}{A^{2/3}\Psi_{\mathbb{P}}^{(p)}(x, x_{\mathbb{P}}, K_\perp^2)}$$

$$Q_s^4(A) = A^{2/3}\, Q_s^4(p)$$

Peak height $R_{\max} \sim \ln^2 \dfrac{Q_s^2(A)}{\Lambda^2}$

- The peak disappears with increasing x: two competing mechanisms
  - hard scattering at small x, hard splitting at moderate x

The R_{pA} ratio for diffractive SIDIS (2606.04169)

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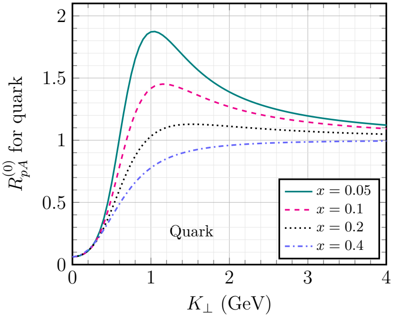
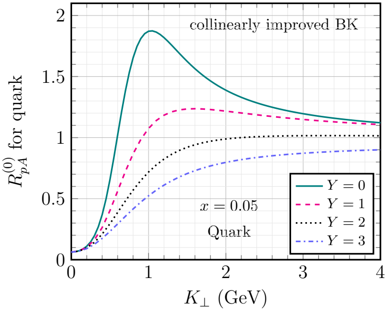
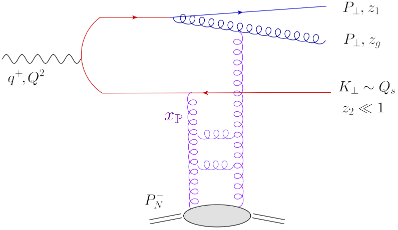


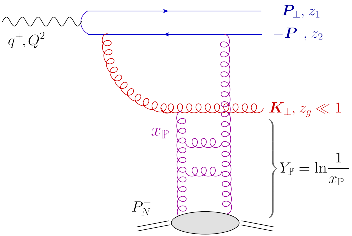


- The peak disappears with increasing x: two competing mechanisms
- It also disappears with increasing $Y_{\mathbb{P}}$ for a given x: proton evolves faster

2+1 diffractive jets (2112.06353, 2207.06268)

- "Hard jets": cross-section with leading-twist power-tail at large $P_\perp \gg Q_s$
  - $1/P_\perp^2$ for single jets, $1/P_\perp^4$ for dijets ...
- Naturally produced via a hard splitting: e.g. $q \to q + g$ or $\gamma \to q\bar{q}$





- A system of 3 partons ($q\bar{q}g$) which must suffer strong elastic scattering
  - Left: hard quark-gluon pair + semi-hard antiquark ($K_\perp \sim Q_s$)
  - Right: hard quark-antiquark pair + semi-hard gluon
- Large effective dipole ($q\bar{q}$, or gg), with $R \sim 1/Q_s$, hence $T(R) \sim 1$

2+1 diffractive jets *(2112.06353, 2207.06268)*

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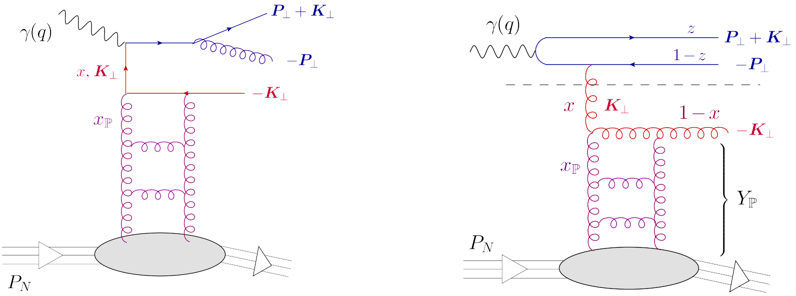
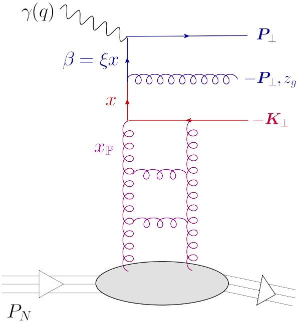
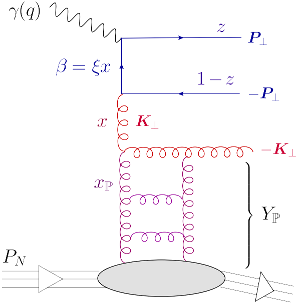


- TMD factorisation: the semi-hard parton can be transferred to the target
  - the same quark DTMD $\mathcal{Q}_{\mathbb{P}}(x, x_{\mathbb{P}}, K_\perp^2)$ as in SIDIS: universality
  - gluon DTMD $\mathcal{G}_{\mathbb{P}}(x, x_{\mathbb{P}}, K_\perp^2)$: UGD of the Pomeron

NLO corrections to diffractive SIDIS *(2402.14748)*

- Measure only the hard quark: NLO contribution, but leading-twist tail $1/P_\perp^2$
- DGLAP splitting functions: $P_{qq}(\xi)$ (left) $P_{qg}(\xi)$ (right)
  - target evolution recovered from the evolution of the photon projectile





- Both DGLAP and CSS evolutions for diffractive PDFs/TMDs

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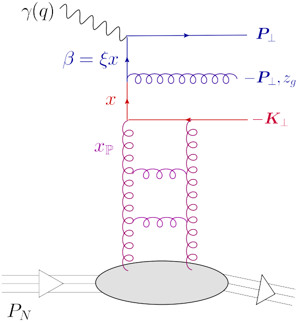

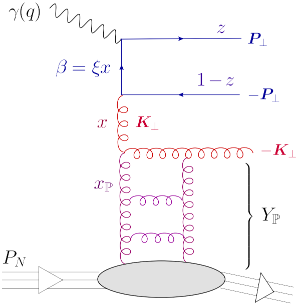

$$\Delta Q_{\mathbb{P}}(\beta, x_{\mathbb{P}}, P_\perp^2) = \frac{\alpha_s}{2\pi^2}\frac{1}{P_\perp^2}\int_\beta^1 \mathrm{d}\xi\, \frac{\xi^2 + (1-\xi)^2}{2}\, \frac{\beta}{\xi} G_{\mathbb{P}}\left(\frac{\beta}{\xi}, x_{\mathbb{P}}, P_\perp^2\right)$$

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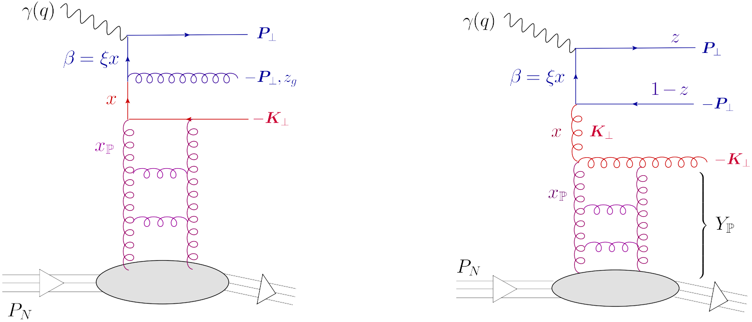
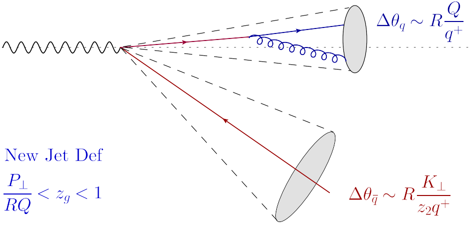


$$\Delta \mathcal{Q}_{\mathbb{P}}(\beta, x_{\mathbb{P}}, P_\perp^2) = \frac{\alpha_s C_F}{2\pi^2}\,\frac{1}{P_\perp^2}\int_\beta^{1-\xi_0} \mathrm{d}\xi\, \frac{1+\xi^2}{1-\xi}\,\frac{\beta}{\xi}\, q_{\mathbb{P}}\left(\frac{\beta}{\xi}, x_{\mathbb{P}}, P_\perp^2\right)$$

- Measure a quark jet $\Rightarrow$ only large-angle emissions matter



$$\Delta\theta_{qg} \simeq \frac{P_\perp}{z_g q^+} > \frac{Q}{q^+}$$

$$z_g \leq \frac{P_\perp}{Q} \ll 1$$

$$\Longrightarrow \; 1-\xi \geq \xi_0 \equiv \frac{P_\perp}{Q}$$

- Logarithmic dependence upon ξ_0: expand for small ξ_0

$$\Delta\mathcal{Q}_{\mathbb{P}}(\beta, x_{\mathbb{P}}, P_\perp^2, Q^2) = \frac{\alpha_s C_F}{2\pi^2} \ln\frac{Q^2}{P_\perp^2} \frac{\beta q_{\mathbb{P}}\left(\beta, x_{\mathbb{P}}, P_\perp^2\right)}{P_\perp^2}$$

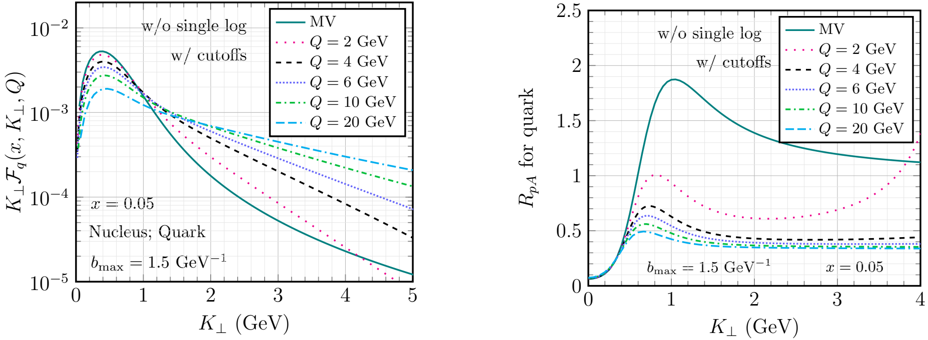
$$+ \frac{\alpha_s C_F}{2\pi^2} \frac{1}{P_\perp^2} \int_\beta^1 \mathrm{d}\xi \, \frac{1+\xi^2}{(1-\xi)_+} \, \frac{\beta}{\xi} q_{\mathbb{P}}\left(\frac{\beta}{\xi}, x_{\mathbb{P}}, P_\perp^2\right) + \mathcal{O}(\xi_0)$$

- Add the virtual corrections: the Sudakov double log *(cf. talk by Cyille M.)*

(with D.N. Triantafyllopoulos and S.-Y. Wei, to appear)

- The CSS/DGLAP evolution changes the tail of the D-TMDs at large $P_\perp$:

$$\frac{Q_s^4(A, Y_{\mathbb{P}})}{P_\perp^4} \sim \frac{A^{2/3}}{P_\perp^4} \longrightarrow \frac{\alpha_s C_F}{2\pi^2} \ln \frac{Q^2}{P_\perp^2} \frac{\beta q_{\mathbb{P}}\left(\beta, x_{\mathbb{P}}, P_\perp^2, A\right)}{P_\perp^2} \sim \frac{A^{1/3}}{P_\perp^2}$$



- CSS evolution with increasing Q^2. Left: Quark DTMD. Right: $R_{pA}(K_\perp)$

CSS evolution for diffractive R_{pA}

(with D.N. Triantafyllopoulos and S.-Y. Wei, to appear)

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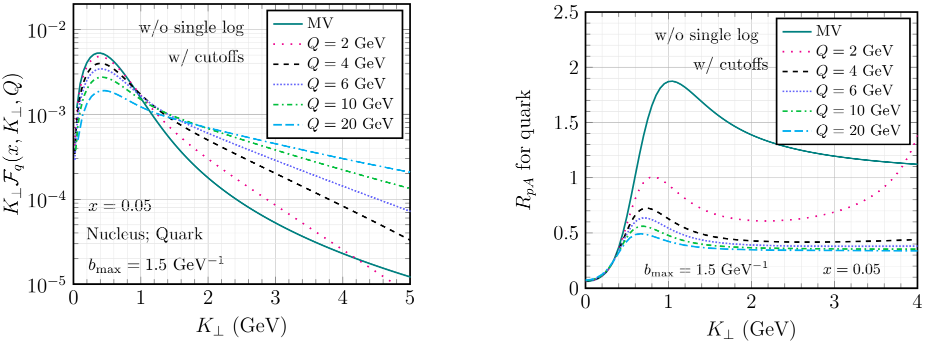
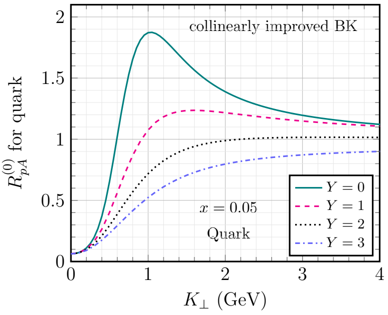
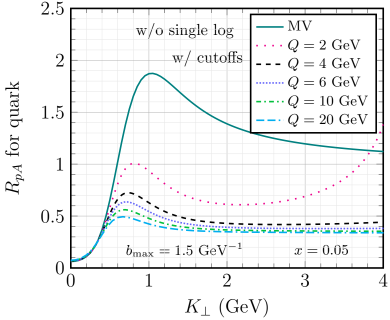


- The peak is strongly suppressed, already for $Q = 2Q_s(A) = 2$ GeV

Instead of conclusions

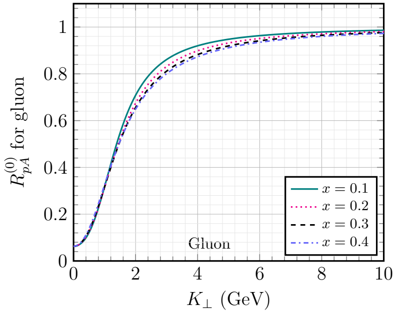
- Diffraction is arguably the high-energy process most sensitive to saturation
- CGC $\Rightarrow$ TMD factorisation for the diffractive production of hard jets
- At tree-level (MV model), the R_{pA} ratio exhibits strong saturation signals
  - suppression at $K_\perp < Q_s(A)$, Cronin peak in some cases

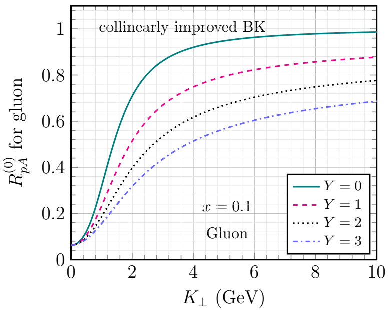




- Suppression at small $K_\perp$ is preserved by all evolutions
- Cronin peak is washed out by all evolutions

- Many similarities with diffractive quarks, but some important differences too:
  - dominate at small x (w.r.t. the Pomeron), no Cronin peak





- Left: tree-level (MV), various x. Right: BK evolution, various $Y \equiv Y_{\mathbb{P}}$

- Many similarities with diffractive quarks, but some important differences too:
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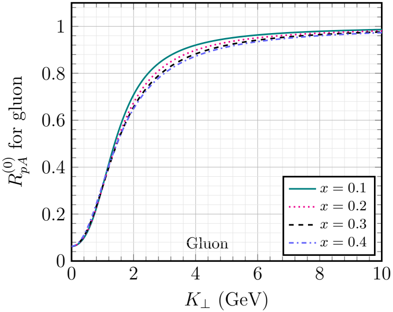

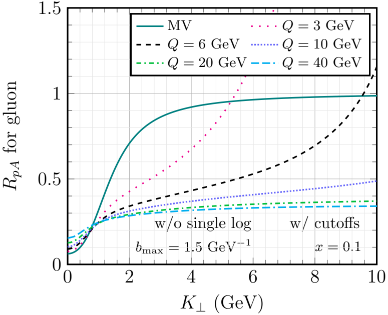

- Left: tree-level (MV), various x. Right: CSS evolution, increasing Q^2
- At large $K_\perp \gg Q_s(A)$, the ratio approaches $R_{pA} = 1/A^{1/3}$
- N.B. CSS results should be trust only for $K_\perp \ll Q$