Concept for Provision of Flat Beams

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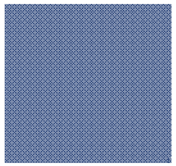
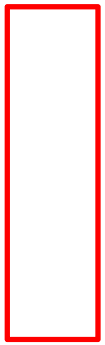
- Motivation
- Beam line to create flat beams (brief)
- rms- and eigen emittances
- Emittance transfer experiment EmTEx
- Emittance transfer beam line for FAIR
- Brilliances from flat beams

Motivation

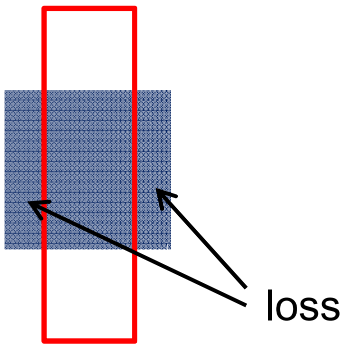
- beam from linac is generally „round“, i.e. $\varepsilon_x = \varepsilon_y$



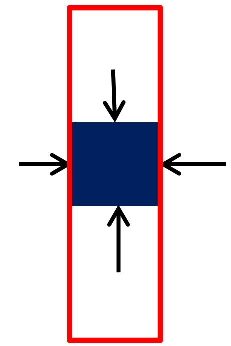
- MTI injection imposes „flat“ ring acceptances, i.e. $A_x < A_y$



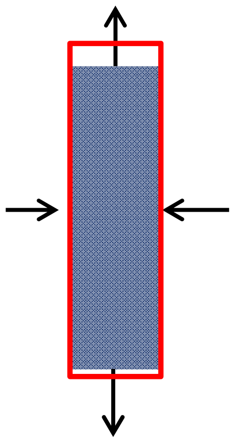
- even if $\varepsilon_x \cdot \varepsilon_y < A_x \cdot A_y$, the MTI-efficiency might be poor



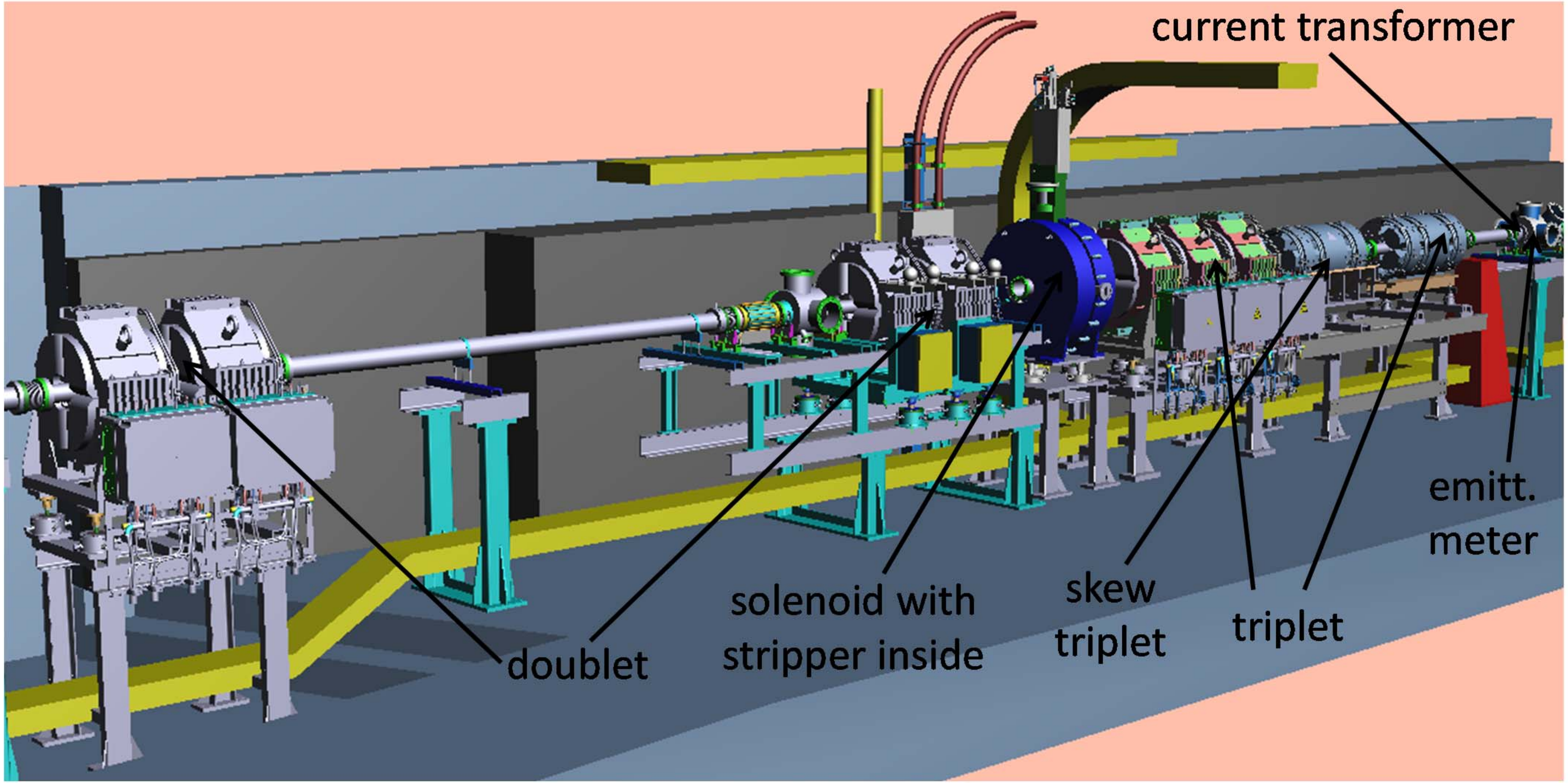
- shrinkage in both planes requires brighter beam from source and/or cooling



- emittance transfer preserves $\varepsilon_x \cdot \varepsilon_y$, hence it does not require brighter source beams nor cooling



Emittance Transfer Beam Line (example)



rms emittances defined through beam's second moments :

- a_i, b_i : two coordinates of particle i
- $<ab>$: mean of product $a_i b_i$
- C is 2nd moments matrix (symmetric)

$$E_x{}^2 = <xx><x'x'> - <xx'>^2$$

$$C_x = \begin{bmatrix} <xx> & <xx'> \\ <x'x> & <x'x'> \end{bmatrix}, \quad E_x^2 = det\, C_x$$

$$C_y = \begin{bmatrix} <yy> & <yy'> \\ <y'y> & <y'y'> \end{bmatrix}, \quad E_y^2 = det\, C_y$$

Transport of Moments

linear transport from 1 → 2 through matrices :

$$\begin{bmatrix} x \\ x' \end{bmatrix}_2 = M_x \begin{bmatrix} x \\ x' \end{bmatrix}_1 \qquad M_x = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}, \ det\, M_x = 1$$

beam moments transport by matrix equation :

$$C_{x2} = M_x\, C_{x1}\, M_x^T$$

analogue in y

Generalization to 4d

$$E_{4d}^2 = det \begin{bmatrix} <xx> & <xx'> & <xy> & <xy'> \\ <x'x> & <x'x'> & <x'y> & <x'y'> \\ <yx> & <yx'> & <yy> & <yy'> \\ <y'x> & <y'x'> & <y'y> & <y'y'> \end{bmatrix}$$

transport of moments from 1 → 2 as usual :

$$\begin{bmatrix} x \\ x' \\ y \\ y' \end{bmatrix}_2 = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix} \begin{bmatrix} x \\ x' \\ y \\ y' \end{bmatrix}_1 \quad , \; det\, M = 1$$

$$C_2 = M\, C_1\, M^T$$

<u>if x & y planes are not coupled</u>

$$E_{4d}^2 = det \begin{bmatrix} <xx> & <xx'> & 0 & 0 \\ <x'x> & <x'x'> & 0 & 0 \\ 0 & 0 & <yy> & <yy'> \\ 0 & 0 & <y'y> & <y'y'> \end{bmatrix} = (E_x \cdot E_y)^2$$

transport of moments from 1 → 2 as usual :

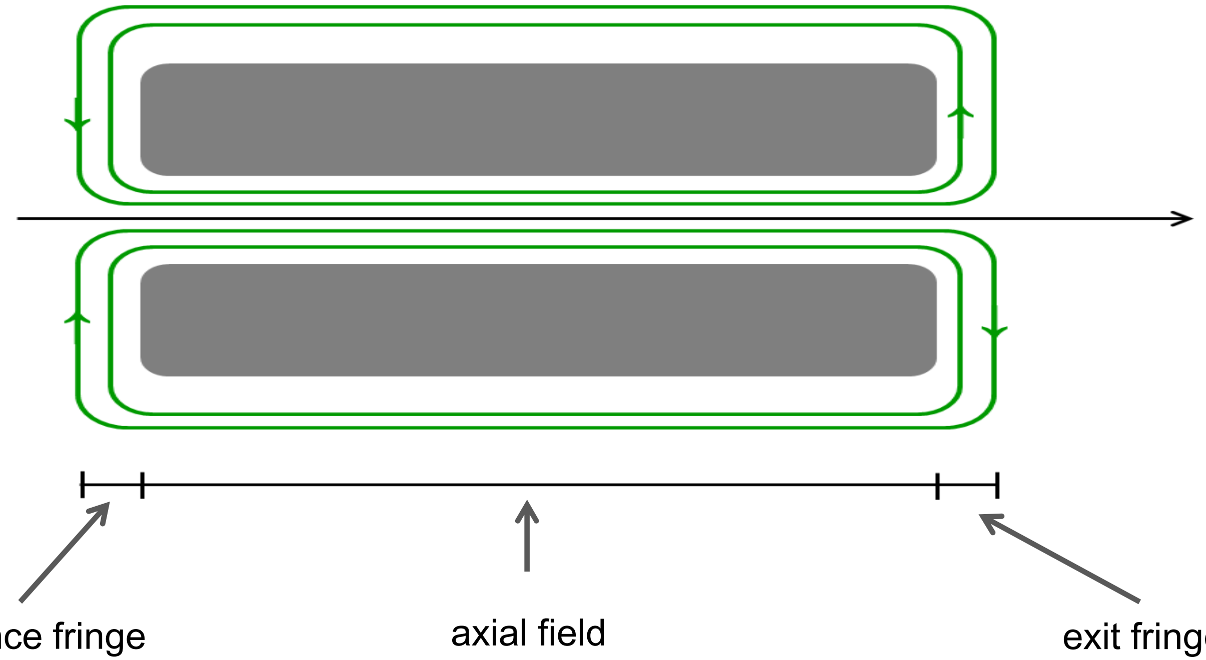
$$M = \begin{bmatrix} m_{11} & m_{12} & 0 & 0 \\ m_{21} & m_{22} & 0 & 0 \\ 0 & 0 & m_{33} & m_{34} \\ 0 & 0 & m_{43} & m_{44} \end{bmatrix} \quad , \; det\, M = det\, M_x \cdot det\, M_y = 1 \cdot 1 = 1$$

$$C_2 = M\, C_1\, M^T$$

Solenoid

$$\kappa := \frac{B}{2(B\rho)}$$

$$\alpha(L) = -2\kappa L$$



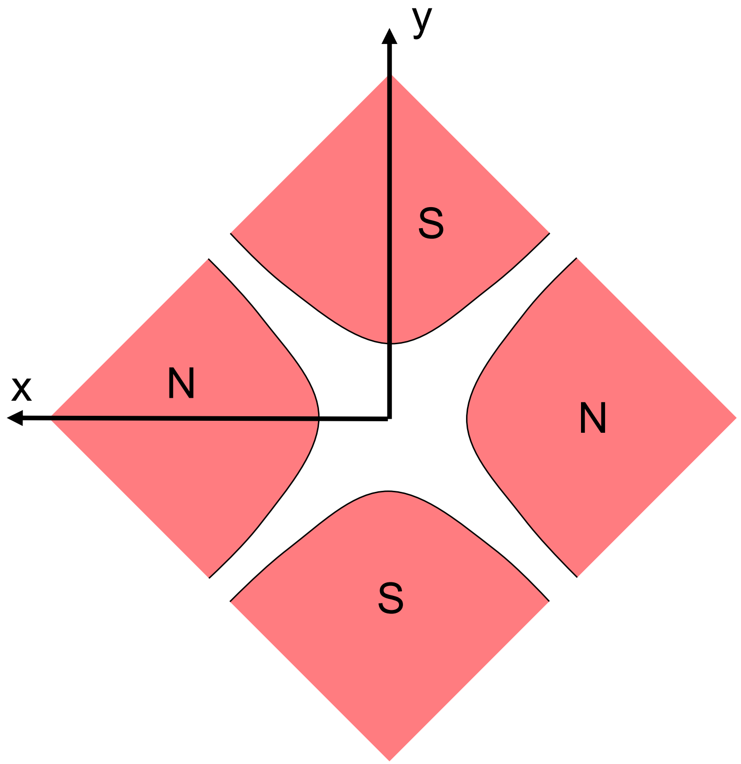
$$M_{fi} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \kappa & 0 \\ 0 & 0 & 1 & 0 \\ -\kappa & 0 & 0 & 1 \end{bmatrix}$$

$$M_{||} = \begin{bmatrix} 1 & -\frac{1}{2\kappa} sin(\alpha) & 0 & \frac{1-cos(\alpha)}{2\kappa} \\ 0 & cos(\alpha) & 0 & -sin(\alpha) \\ 0 & \frac{cos(\alpha)-1}{2\kappa} & 1 & -\frac{sin\alpha)}{2\kappa} \\ 0 & sin(\alpha) & 0 & cos(\alpha) \end{bmatrix}$$

$$M_{fo} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -\kappa & 0 \\ 0 & 0 & 1 & 0 \\ \kappa & 0 & 0 & 1 \end{bmatrix}$$

complete solenoid matrix $M_{sol} = M_{fo} \cdot M_{||} \cdot M_{fi}$

Skew Quadrupole



Transport matrix of a thin hor. foc. quadrupole, focusing length 1/q, rotated clockwise by 45° :

$$M_{SkewQuad} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -q & 0 \\ 0 & 0 & 1 & 0 \\ -q & 0 & 0 & 1 \end{bmatrix}$$

Eigen Emittances

- Linear, Hamiltonian elements preserve:
  - rms emittance $E_{4d}^2 = det \begin{bmatrix} <xx> & <xx'> & <xy> & <xy'> \\ <x'x> & <x'x'> & <x'y> & <x'y'> \\ <yx> & <yx'> & <yy> & <yy'> \\ <y'x> & <y'x'> & <y'y> & <y'y'> \end{bmatrix}$
  - the two eigen emittances

$$\varepsilon_1 = \frac{1}{2}\sqrt{-tr[(CJ)^2] + \sqrt{tr^2[(CJ)^2] - 16det(C)}}$$

$$\varepsilon_2 = \frac{1}{2}\sqrt{-tr[(CJ)^2] - \sqrt{tr^2[(CJ)^2] - 16det(C)}}$$

$E_{4d} = \varepsilon_1 \cdot \varepsilon_2$

$$C = \begin{bmatrix} <xx> & <xx'> & <xy> & <xy'> \\ <x'x> & <x'x'> & <x'y> & <x'y'> \\ <yx> & <yx'> & <yy> & <yy'> \\ <y'x> & <y'x'> & <y'y> & <y'y'> \end{bmatrix} \qquad J := \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

- linear, x↔y coupling elements do NOT preserve hor. & ver. emittances

eigen-Emittances

- if, and only if there is no x↔y coupling, i.e. $C = \begin{bmatrix} <xx> & <xx'> & 0 & 0 \\ <x'x> & <x'x'> & 0 & 0 \\ 0 & 0 & <yy> & <yy'> \\ 0 & 0 & <y'y> & <y'y'> \end{bmatrix}$
  - rms-emittances = eigen emittances
- if there is any coupling
  - rms-emittance ≠ eigen emittance

term „eigen emittance“ is quite unknown, since one generally assumes uncoupled beams

How Elements do Change Emittances

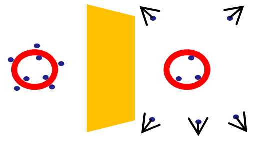
Element	**rms$_{x,y}$**	**4d-rms**	**Eigen$_{1,2}$**
drift	no	no	no
quadrupole	no	no	no
tilted quadrupole	yes	no	no
dipole	no	no	no
tilted dipole	yes	no	no
solenoid	yes	no	no
solenoid fringe	yes	no	yes
solenoid axial field	yes	no	yes

Emittance Transfer from x → y

!! Do not Confuse with Emittance Exchange !!

- if we want emittance transfer, we must change the eigen emittances !!!!

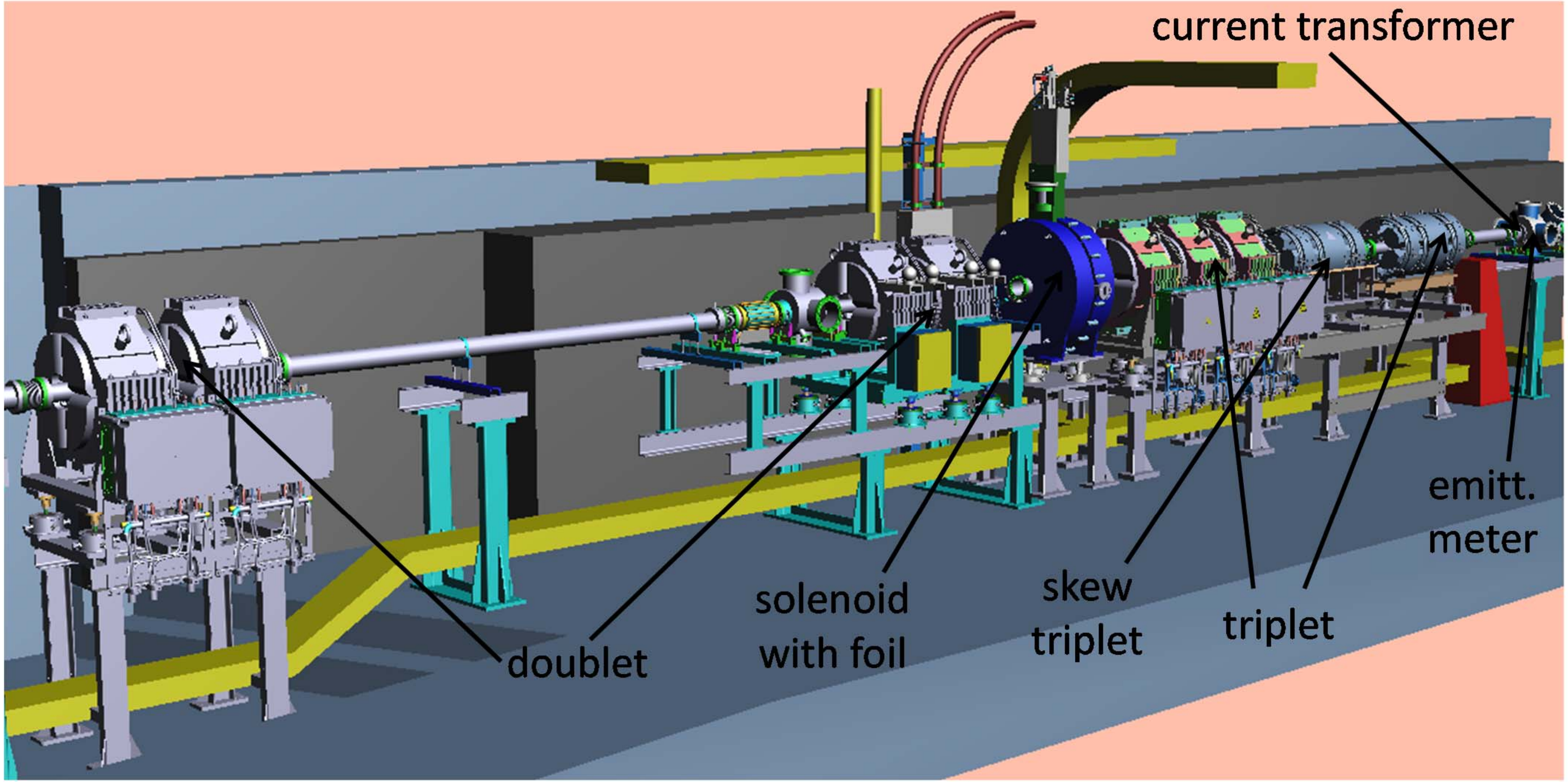
- we must do something non-Hamiltonian to the beam, like:
    - synchrotron radiation: stochastic
    - slits, cups: remove particles (good beam ions) from the system
    - stripping: remove particles (electrons) from the system



- we need a controlled non-Hamiltonian action that should not cause “good particle” loss

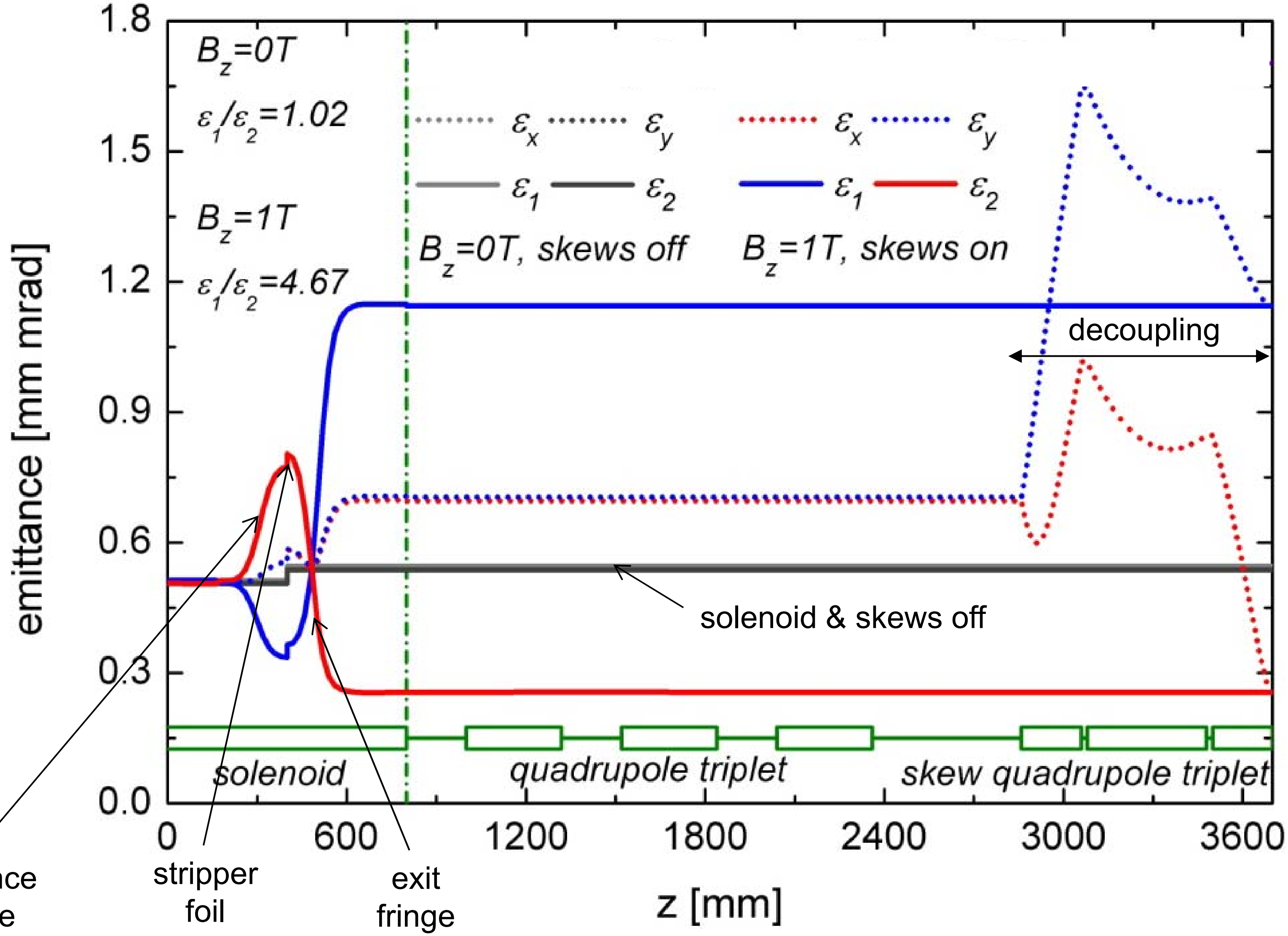
- a solution: fake a stand-alone solenoid fringe field
    - e-beam: by placing source inside solenoid,   D. Edwards, XXth LINAC-Conf. (2000),
    - ion beam: by placing stripper inside solenoid,   PRST-AB **14** 064201 (2011)

Experimental Set-up for Proof of Principle:
EmTEx: Emittance Transfer Experiment



total length: 13 m, to be integrated into the existing
transfer line from the UNILAC to the SIS-18 synchrotron

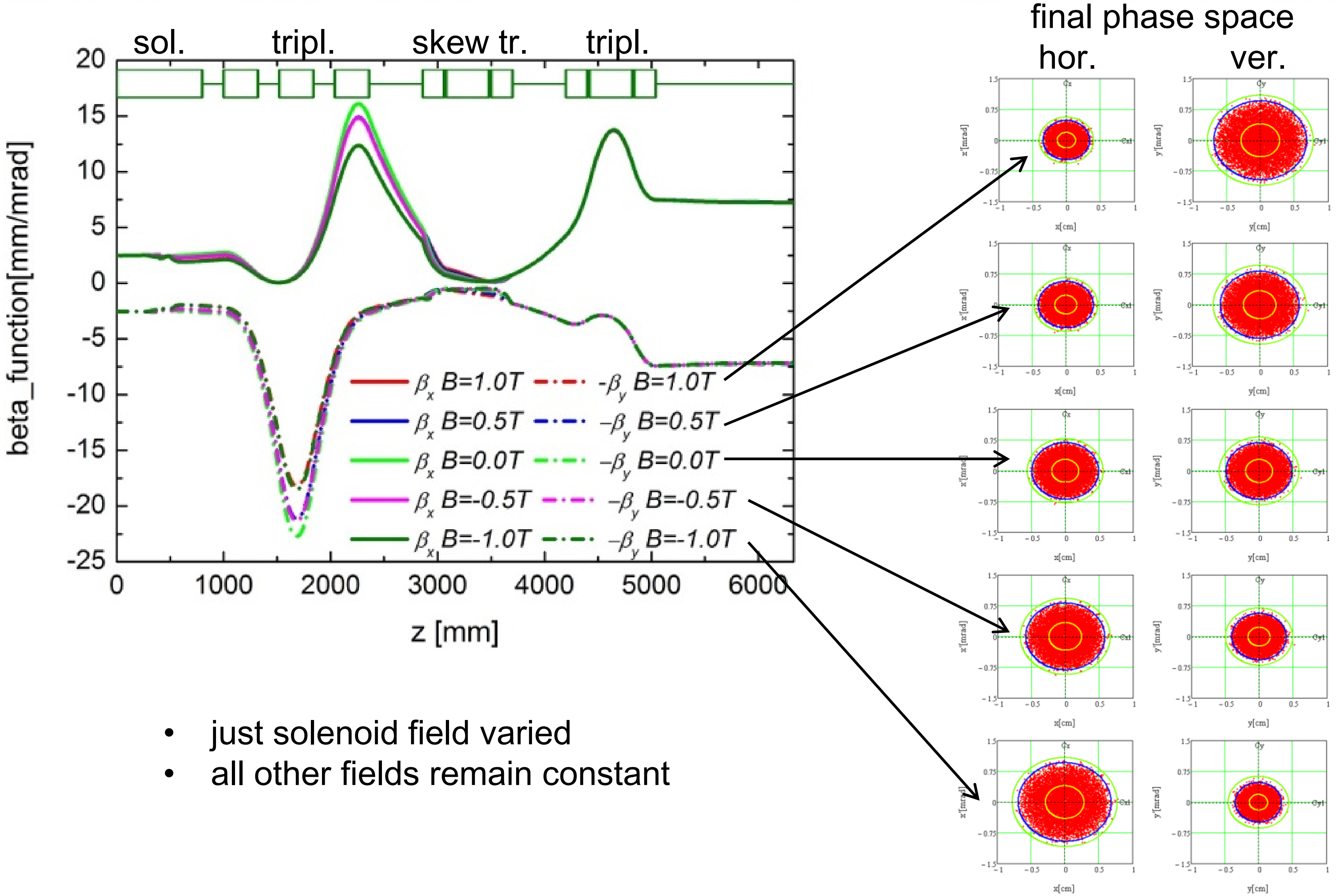
Evolution of Emittances at EmTEX



Decoupling after Solenoid

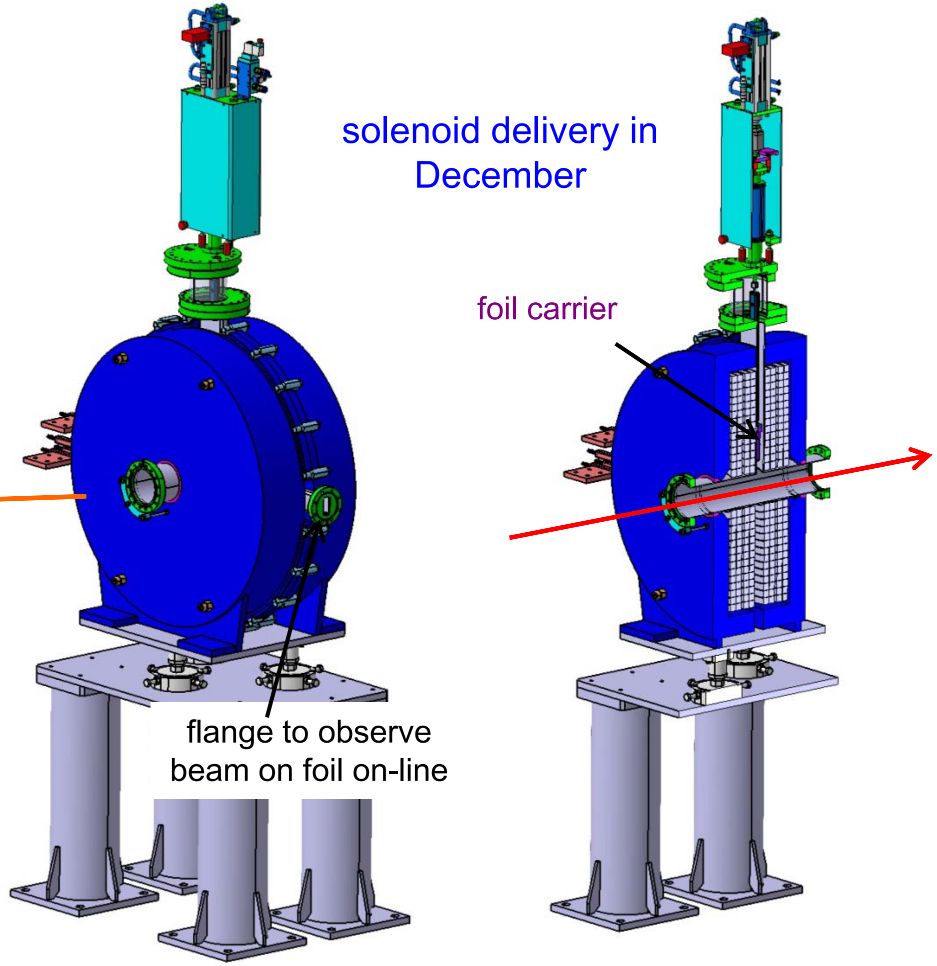
- the gradients of the decoupling skews do practically not depend on the solenoid field
- i.e. for a wide range of coupling solenoid field strengths, a constant set of skew-gradients provides very good decoupling
- accordingly, the solenoid field is a single-knob tool to partition transverse emittances
- the decoupled Twiss parameters (behind skews) do not depend on the solenoid field
- a constant set of skew-gradients will always result in same Twiss params behind skews, just the transverse emittances are different
- we still do not understand why it works so well
- feature was confirmed by two independend calculations done at MSU and Univ. Liverpool
- this very pleasant feature holds also very well if bends, space charge, and multi-charge operation, straggeling & scattering is included
- details in Phys. Rev. ST Accel. & Beams **14** 064201 and **16** 044201

Decoupling & Matching Feature

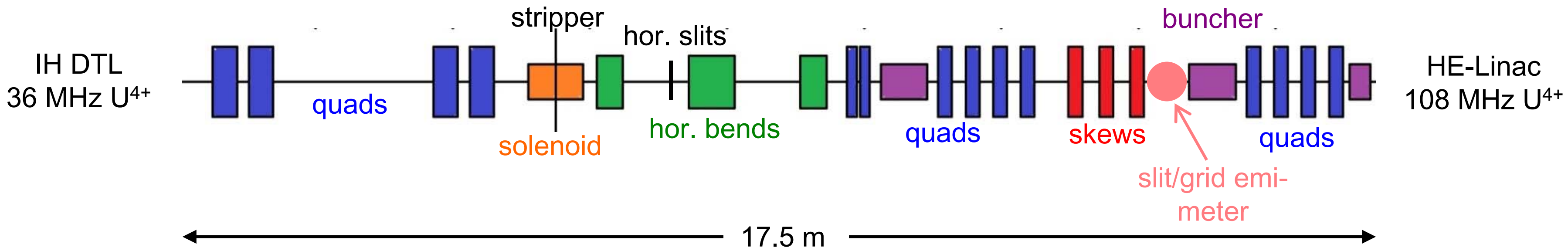


- just solenoid field varied
- all other fields remain constant

experiment planned for summer 2014



Emittance Transfer for Uranium @ FAIR



- matching to stripper being placed inside solenoid (small beam spot, double waist)
- stripping, x↔y coupling, change of eigen emittances
- dispersive charge-state separation/selection
- re-bunching & -focusing
- telescopic transformation of <xy>, <xy'>, <x'y>, <x'y'> to <xx'>, <yy'>
- measurement of 4d second beam moments
- decoupling
- long. & transv. matching to DTL

Decoupling & Matching Features

- to minimize brilliance loss along DTL, matching incl. space charge is required
- matching must be re-done for all emittance partion scenarios
- fortunately, DTL`s periodic solution depends very weekly on the emittance ratio:

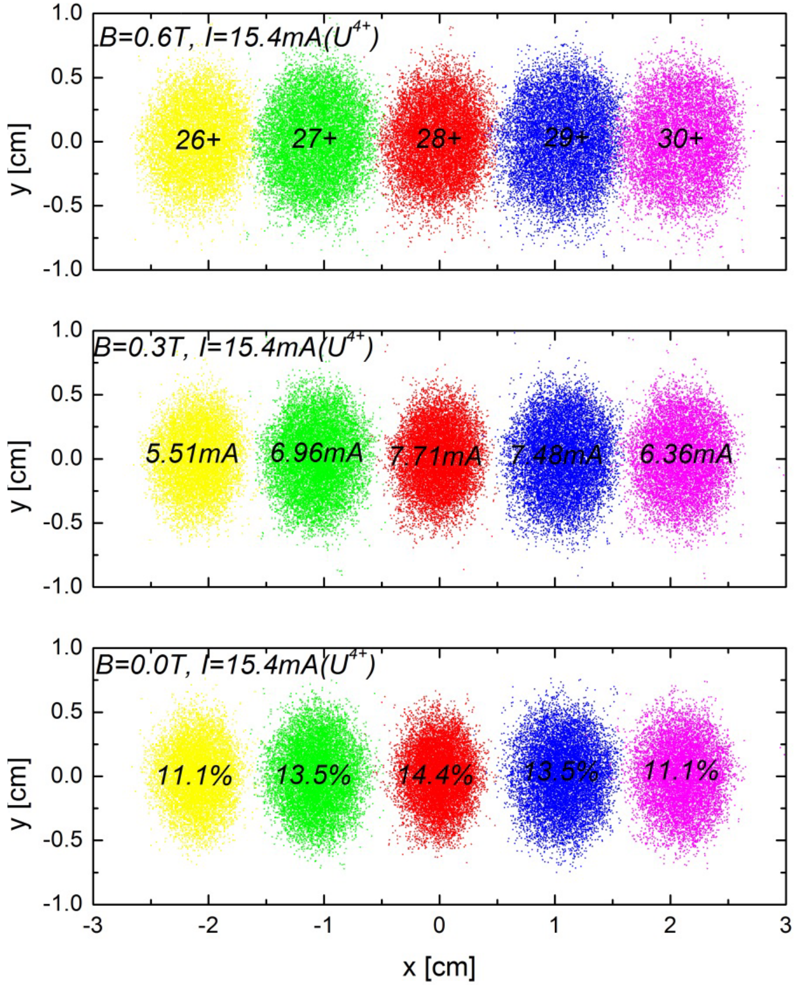
(example Alvarez DTL, $\sigma_{\perp 0}$ = 70°)

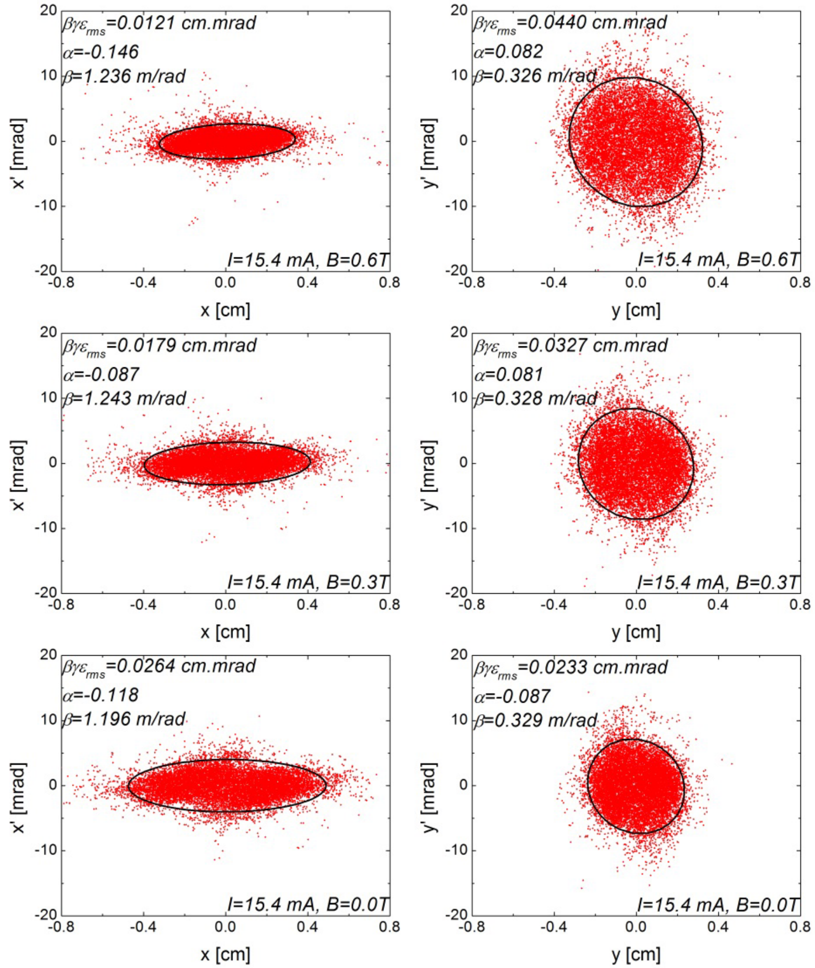
$\varepsilon_y / \varepsilon_x$	σ_x [deg]	σ_y [deg]	σ_l [deg]	β_x [m]	α_x	β_y [m]	α_y	β_l [deg/mrad]	α_l
1	50,0	50,0	28,0	1,3	0,1	0,4	0,0	3,0	-0,1
2	46,5	53,4	27,7	1,3	0,1	0,4	0,0	3,0	-0,1
3	44,4	55,3	27,8	1,4	0,1	0,4	0,0	3,0	-0,1
5	41,9	57,5	27,8	1,5	0,1	0,4	0,0	3,0	-0,1
8	39,7	59,3	27,9	1,6	0,1	0,4	0,0	3,0	-0,1

- this weak dependence together with the decoupling features facilitates the operation significantly
- i.e. emittance partitioning can be set with one knob (solenoid) w/o re-matching

Decoupling & Matching Features

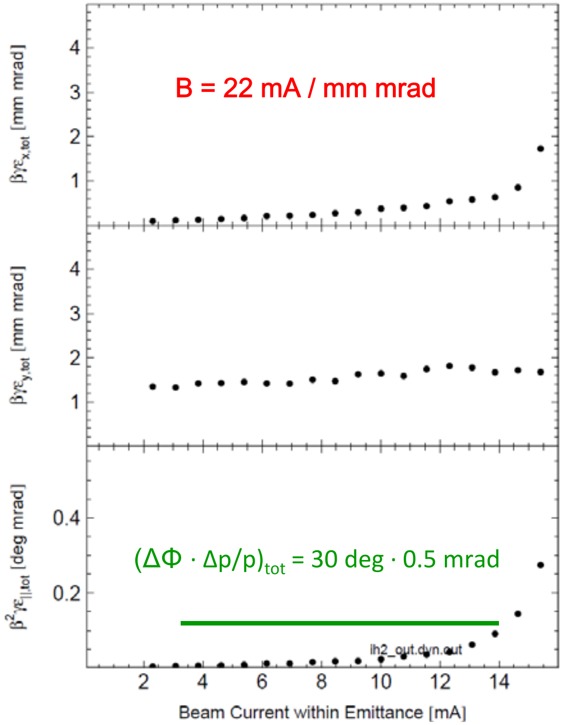
charge separation & final distributions vs solenoid field strength
(all other field strengths remain constant)

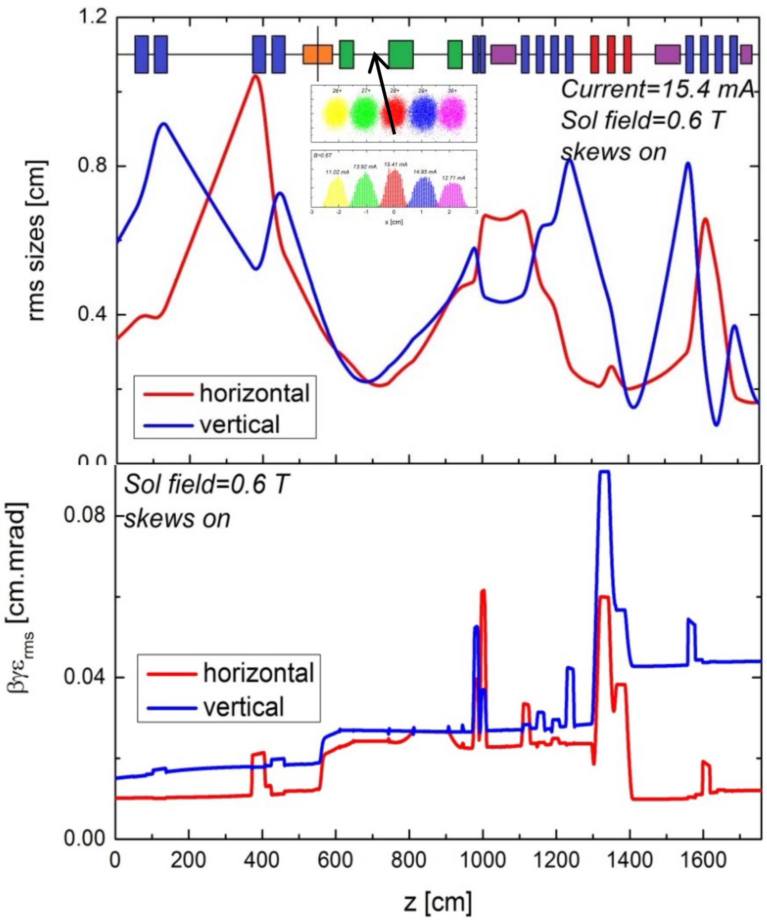




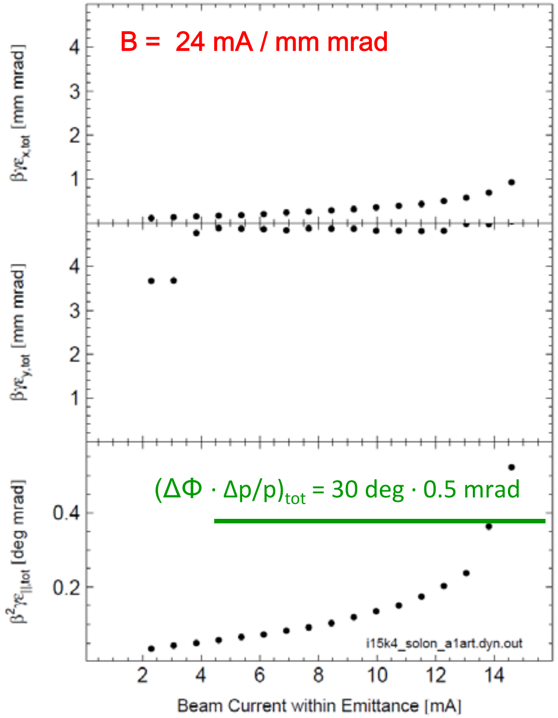
Full Beam Current: Solenoid Field of 0.6 T

entrance:





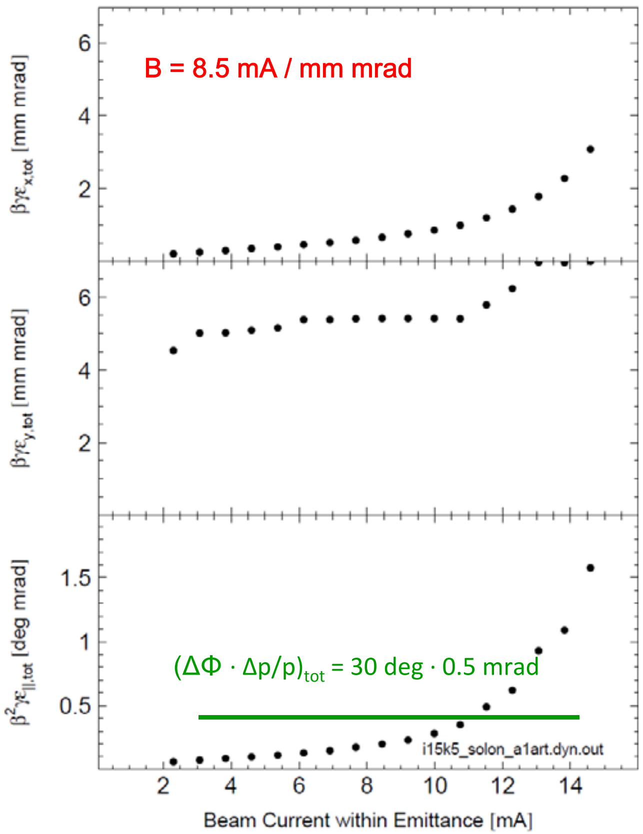
exit:

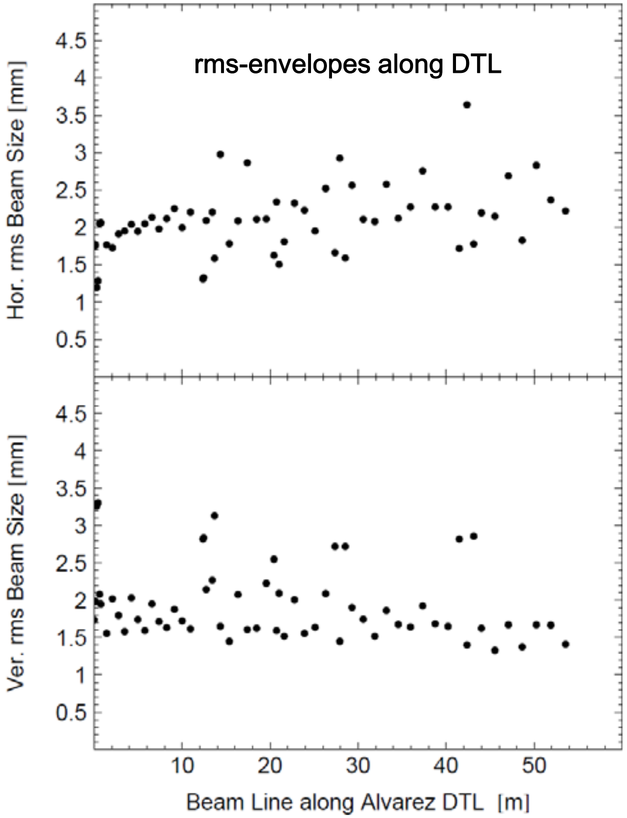


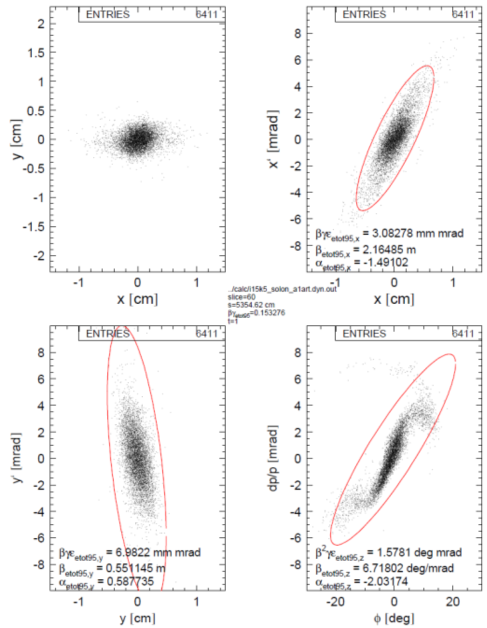
brilliance change = +11%

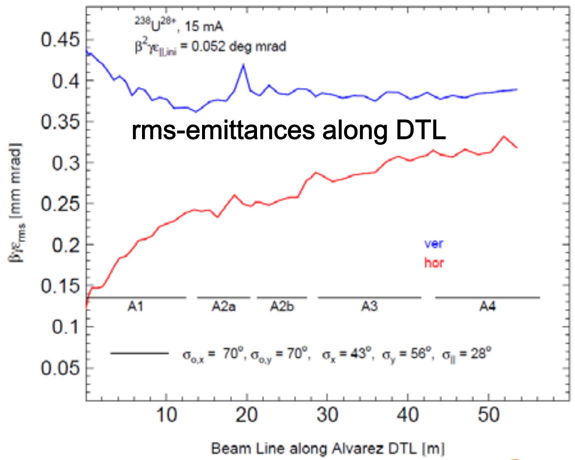
After Acceleration of Flat Beam along DTL

- transmission 100%
- brilliance change = -65%



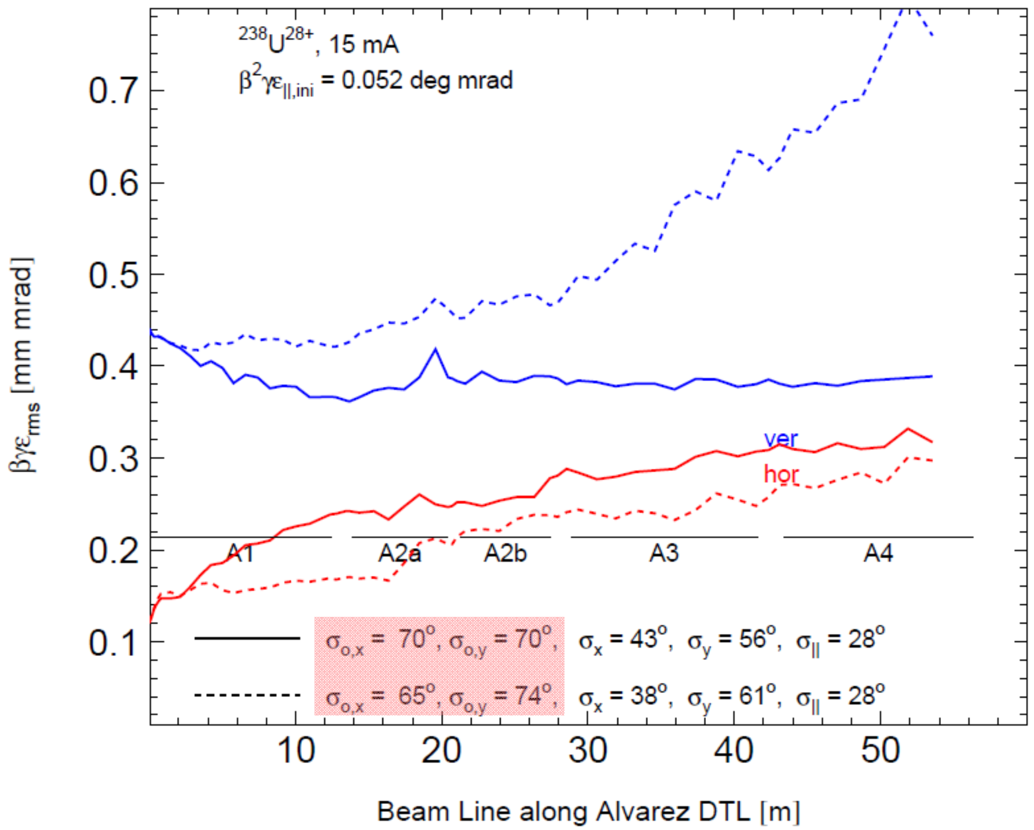




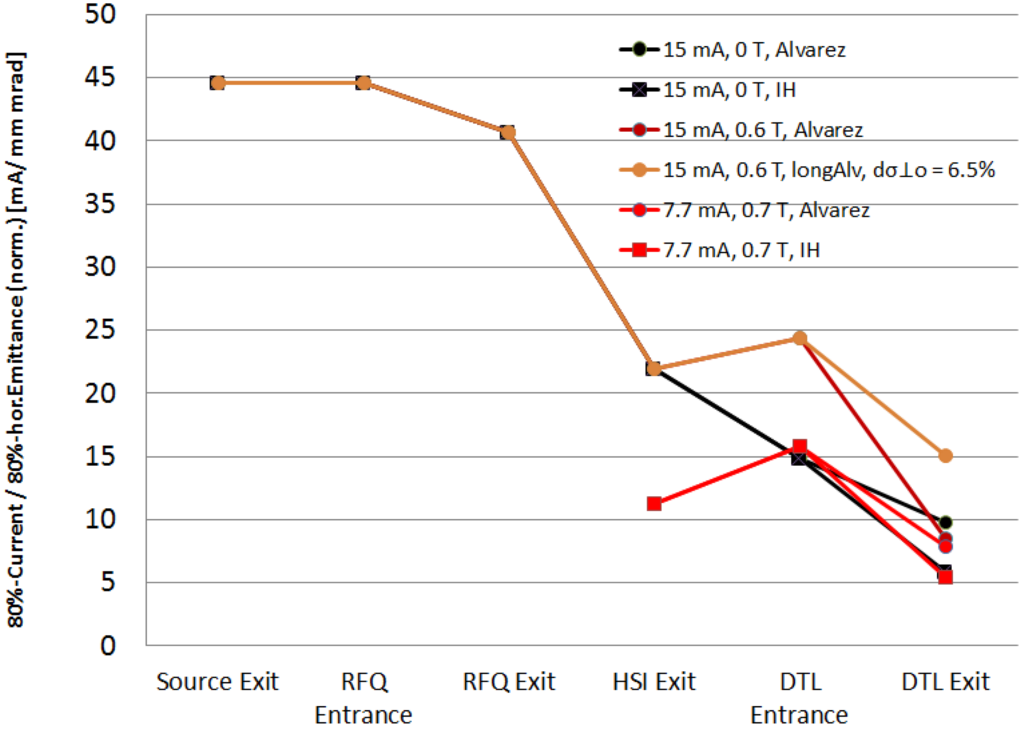


Acceleration of Flat Beam along DTL

- for time being, brilliance gain in matching section is not preserved along the DTL
- possible solution: transverse tune separation in combination with careful inter-tank matching



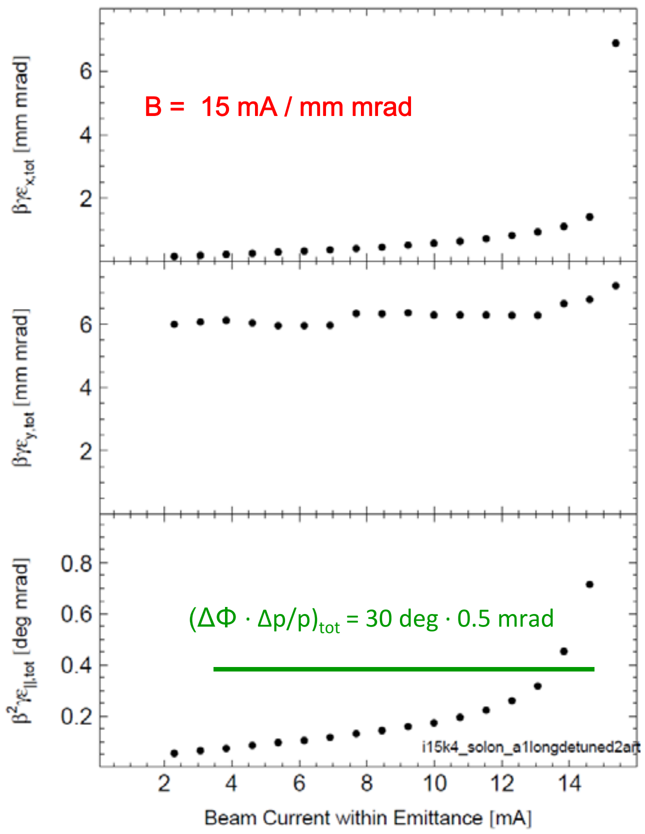
Brilliance along Machine: different Scenarios

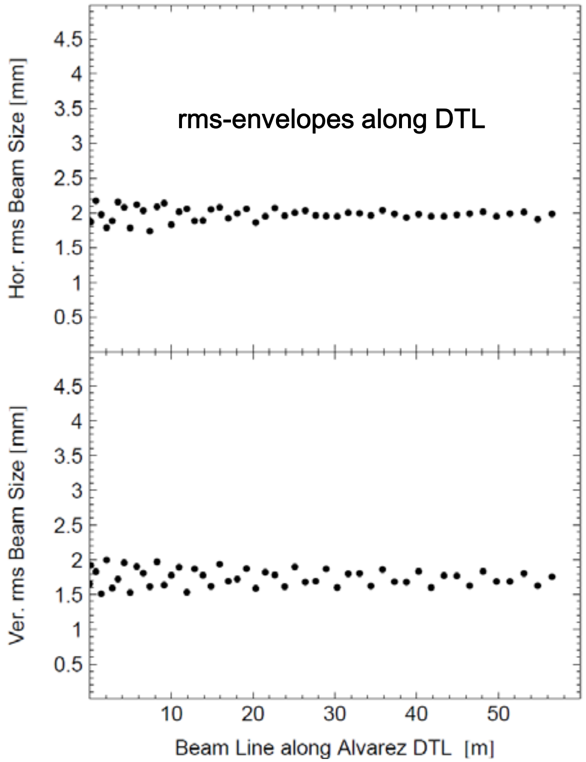


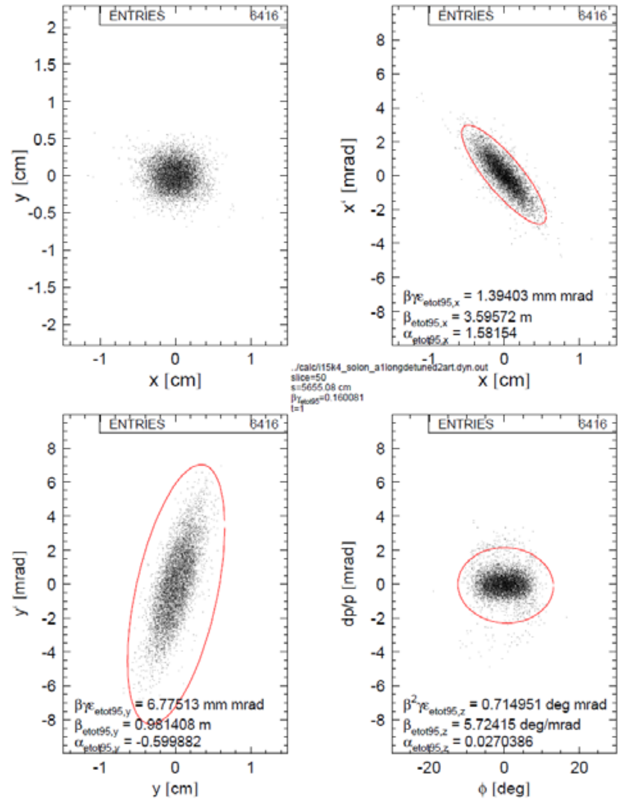
Summary:

- flat beams may provide significant enhancement of hor. brilliance at DTL entrance
- might open a path towards higher hor. brilliance wrt to round beam scheme
- flat beam acceleration is quite prone to brilliance delution from mismatch
- DTL must provide proper periodicity incl. inter-tank transitions

Concept for Provision of Flat Beams: Appendix

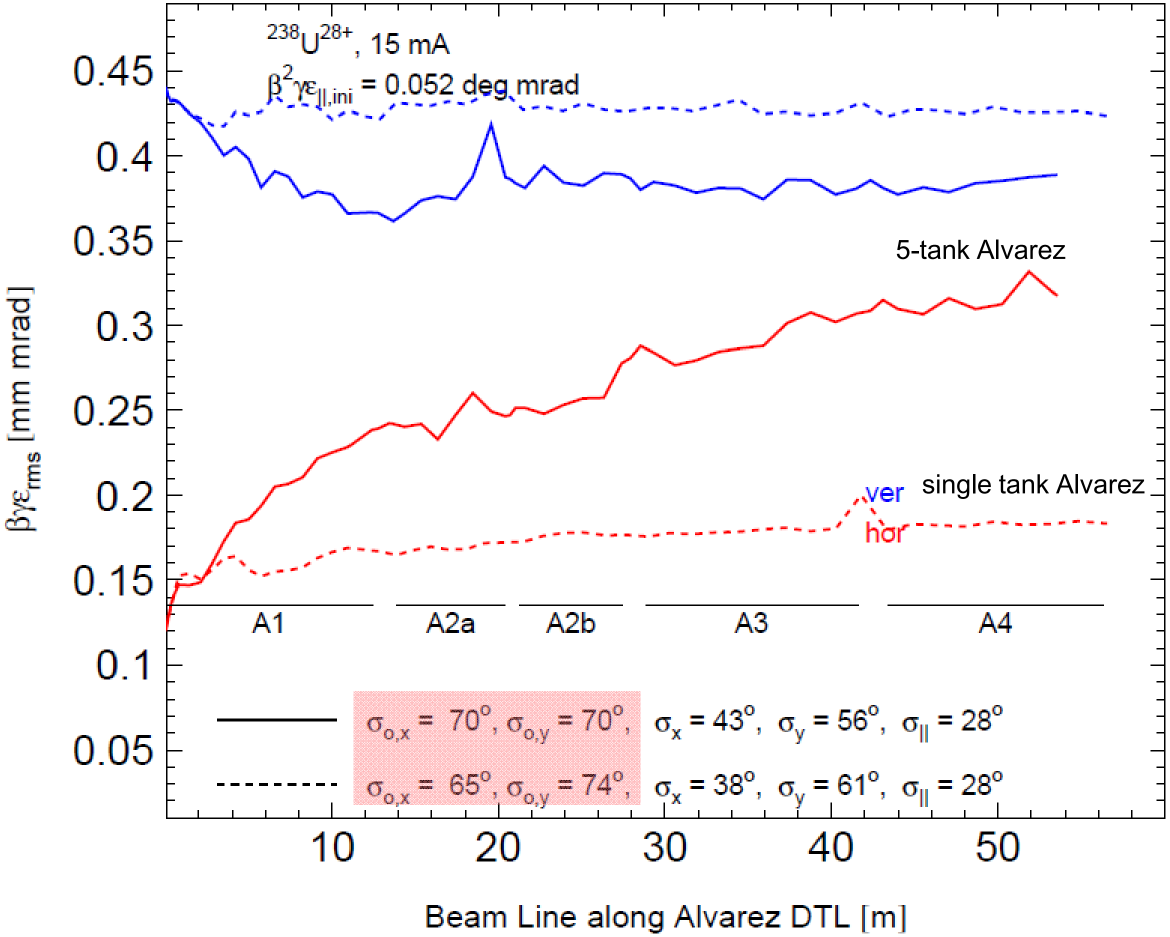






Assumption: Alvarez w/o inter-Tank Section, one single Tank

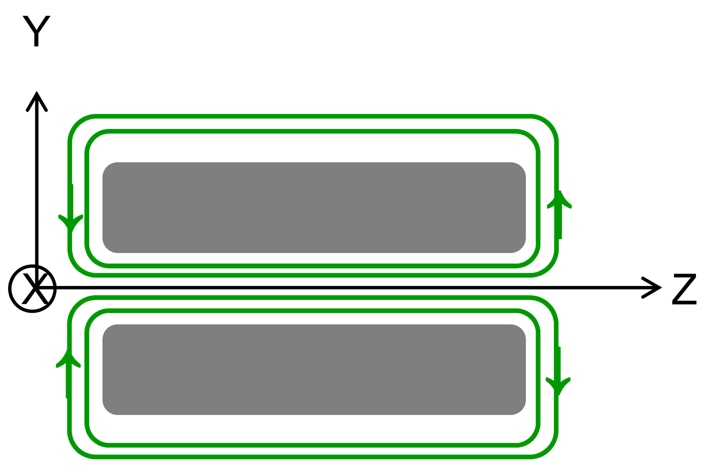
to study the influence of inter-tank mismatch, an (unrealistic) single-tank Alvarez DTL was assumed



Assumption: Alvarez w/o inter-Tank Section, one single Tank

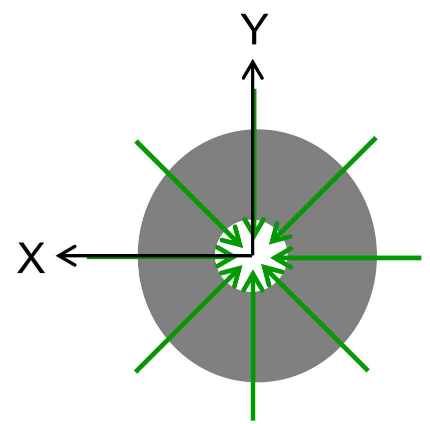
Solenoid

Solenoid side view :



B - Field

Solenoid entrance seen by beam moving towards positive z :



Transport matrix of a solenoid, length L, and field strength B :

$$M_{Solenoid} = \begin{bmatrix} C^2 & \frac{SC}{K} & SC & \frac{S^2}{K} \\ -KSC & C^2 & -KS^2 & CS \\ -SC & -\frac{S^2}{K} & C^2 & \frac{SC}{K} \\ KS^2 & -SC & -KSC & C^2 \end{bmatrix}$$

$$K = \frac{B}{2(B\rho)}$$

beam rigidity

$$C = cos(KL)$$

$$S = sin(KL)$$

How Elements do Change Emittances

- solenoid axial field is not Hamiltonian
- solenoid fringe field is not Hamiltonian
- they are not Hamiltonian, because the do not exist „stand-alone“
- $\vec{\nabla}\vec{B} = \frac{\delta B_x}{\delta x} + \frac{\delta B_y}{\delta y} + \frac{\delta B_z}{\delta z} = 0$ forbids existence of such stand-alone fields !!
- infact: complete solenoids exist, are Hamiltonian, and preserve eigen-emittances

Emittance Transfer from x → y

!! Do not Confuse with Emittance Exchange !!

- we have: uncoupled beam with equal rms emittances in x & y
- we want to:
  1. shrink x-emittance and enlarge y-emittance
  2. finish with an uncoupled beam
- uncoupled beam → rms emittance = eigen-emittances
- we create a furious (but linear) tilted beam line and couple strongly x & y
- such beam line elements will change rms emittances but always preserve eigen-emittances
- such beam line elements will not help, since finally an uncoupled beam is wanted
- decoupling re-equalizes rms emittances to eigen-emittances, but the latter did not change

Stripping inside Solenoid

$$C_0 = \begin{bmatrix} \varepsilon\beta & 0 & 0 & 0 \\ 0 & \frac{\varepsilon}{\beta} & 0 & 0 \\ 0 & 0 & \varepsilon\beta & 0 \\ 0 & 0 & 0 & \frac{\varepsilon}{\beta} \end{bmatrix}$$

round beam, no x-y correlation!

$$R_{in} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & k_{in} & 0 \\ 0 & 0 & 1 & 0 \\ -k_{in} & 0 & 0 & 1 \end{bmatrix}, \quad R_{out} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -k_{out} & 0 \\ 0 & 0 & 1 & 0 \\ k_{out} & 0 & 0 & 1 \end{bmatrix}$$

entrance fringe

exit fringe

$k = \dfrac{B}{2(B\rho)}$ focusing strength of the solenoid

$k = k_{in} = k_{out}$

symplectic transformation, the eigen-emittances are not changed, coupling

$k_{in} \neq k_{out}$

non-symplectic transformation, the eigen-emittances are changed, coupling

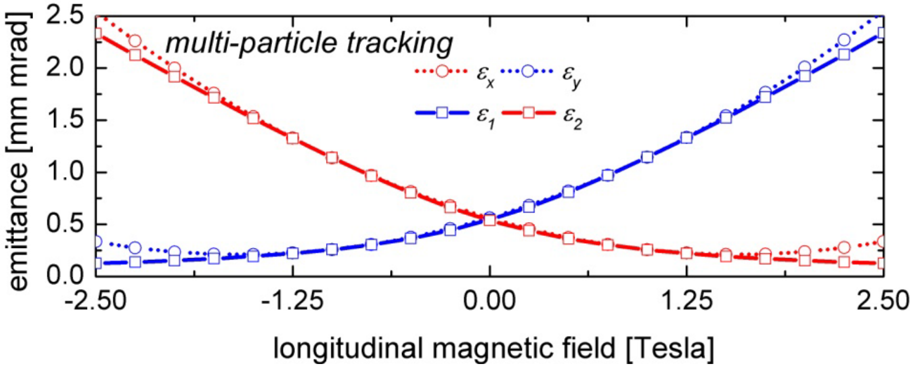
method: couple non-symplectically, decouple symplectically (skew quads)

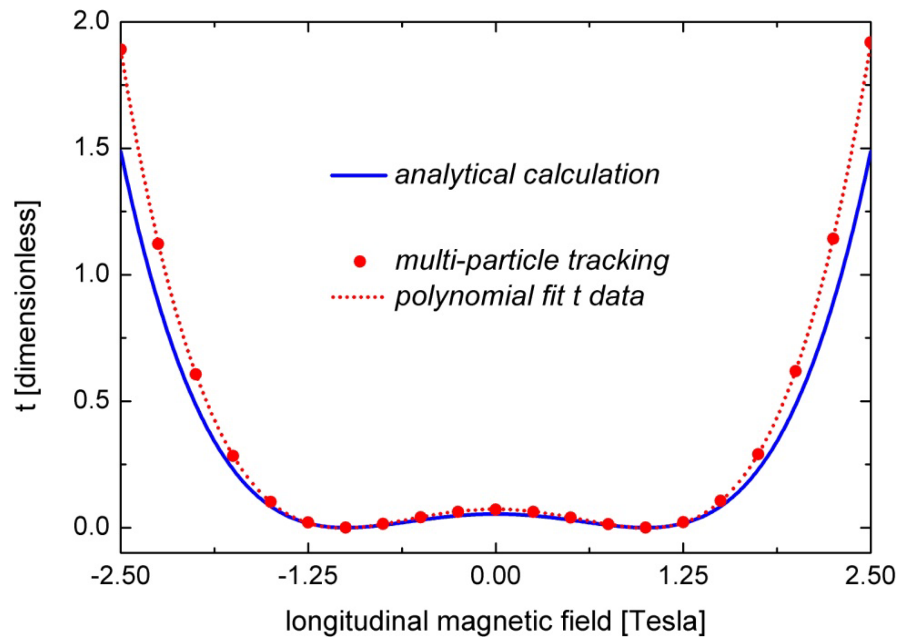
Decoupling Feature of EMTEX

We found unexpected (but very pleasant) feature :

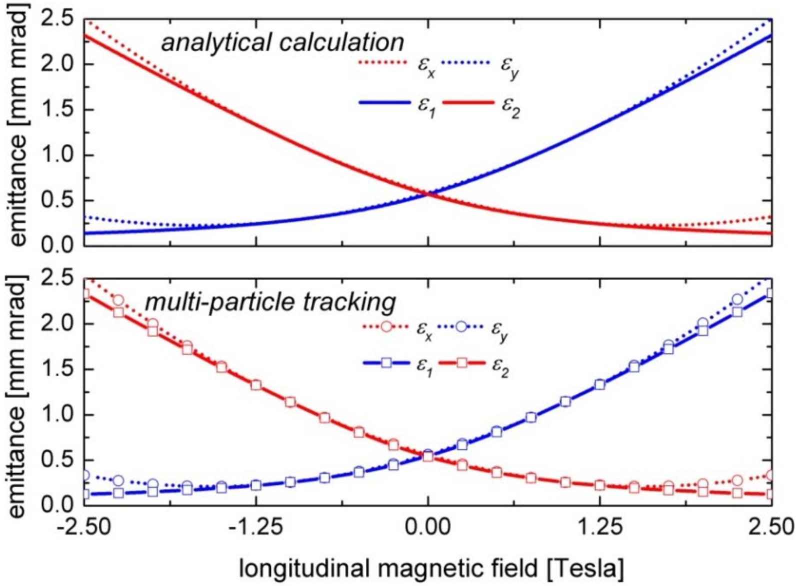
- if decoupling gradients are fixed for one solenoid field, they work fine also for other solenoid field values

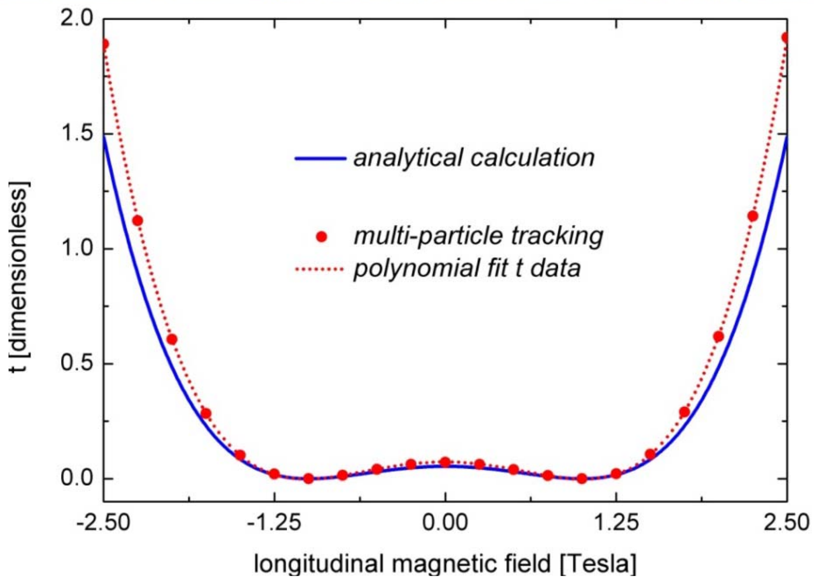
- this feature is not fully understood





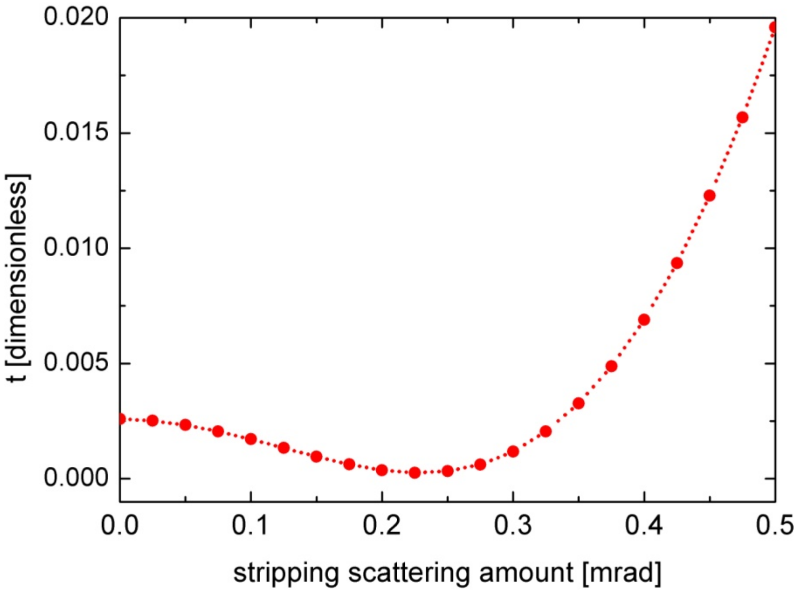
Decoupling Feature of EMTEX

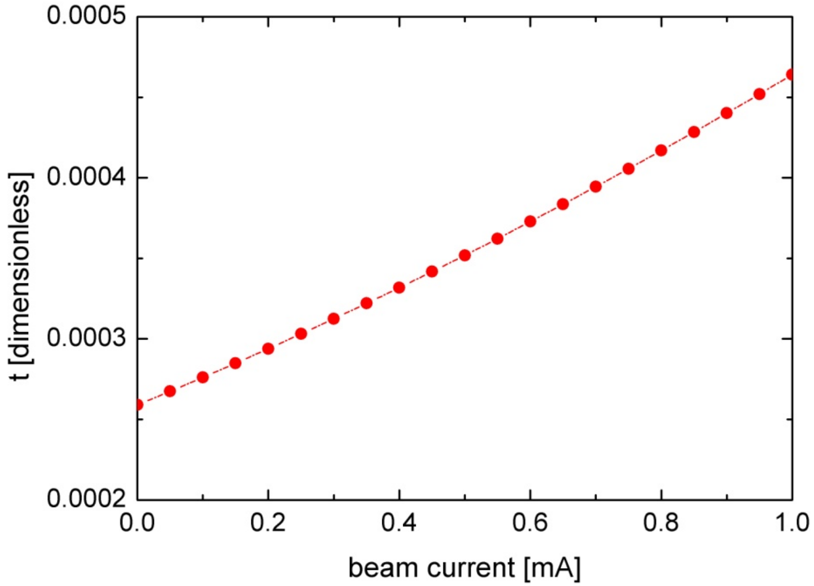




- The set-up provides a single-knob-tool to partition the horizontal and vertical beam rms emittances
- beam well decoupled for a wide range of solenoid fields
- feature understood if decoupling matrix is constructed as $M^{analyt}_{decoupl} = R(\frac{\pi}{4}) \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -u \\ 0 & 0 & u^{-1} & 0 \end{bmatrix} \cdot R(-\frac{\pi}{4})$
- but the EMTEX decoupling matrix is more complex
- however, the feature holds in general, although we do not yet understand why

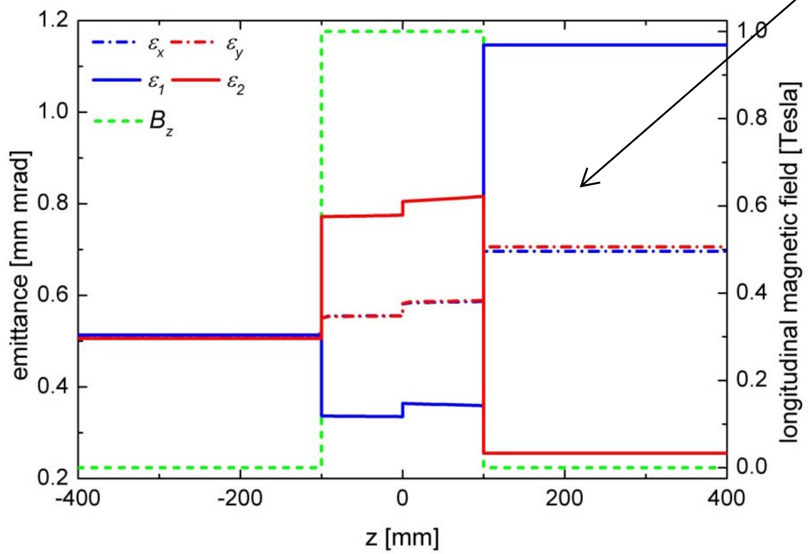
Decoupling Features of EMTEX

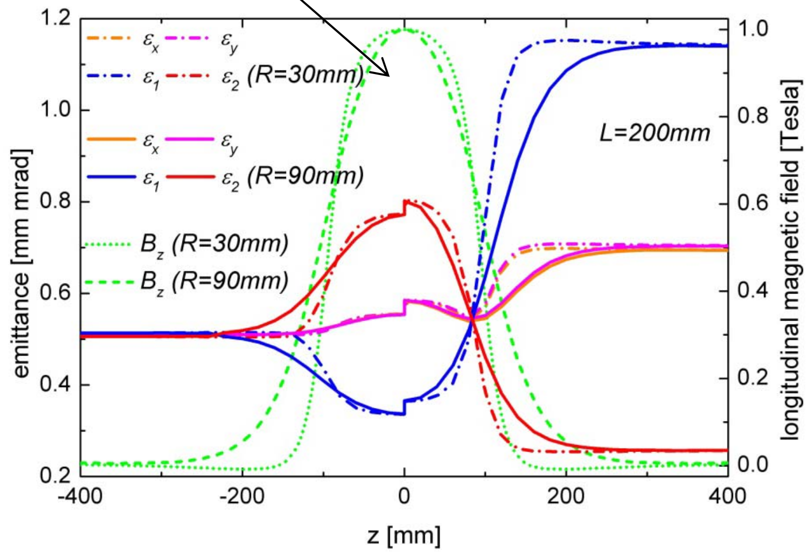


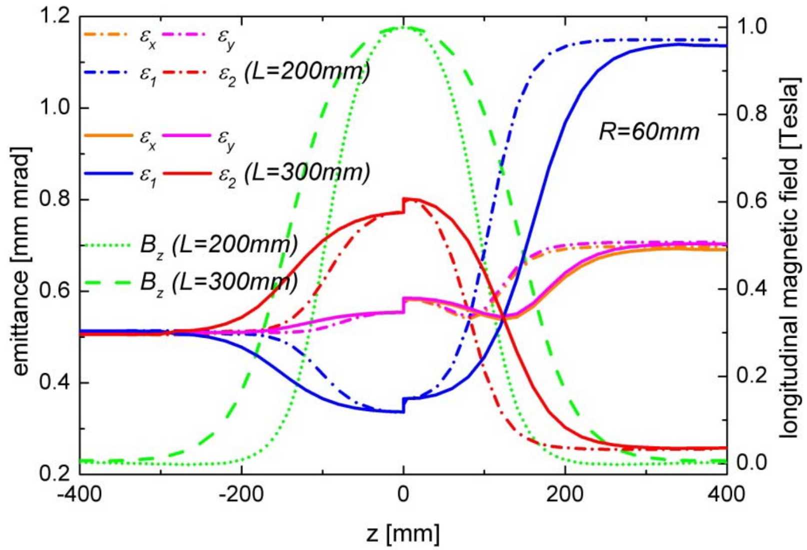


decoupling also quite independent from amount of scattering and beam current

Tracking through 3d-Solenoid Field Map
Comparison: Matrix vs. Tracking

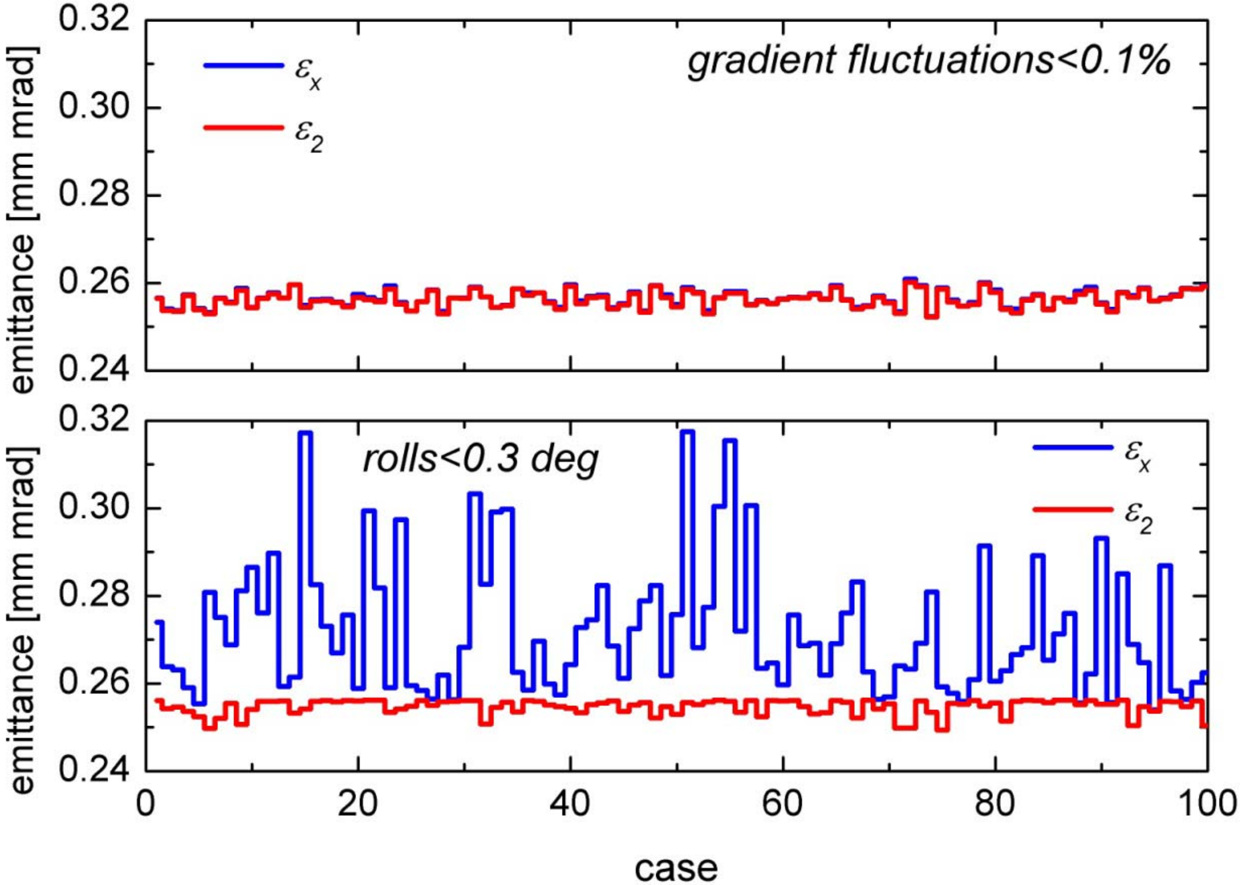






final rms emittances and eigen-emittances do not depend on the exact shape of the fringe field as long as it is reasonably short like solenoids that are commonly in use

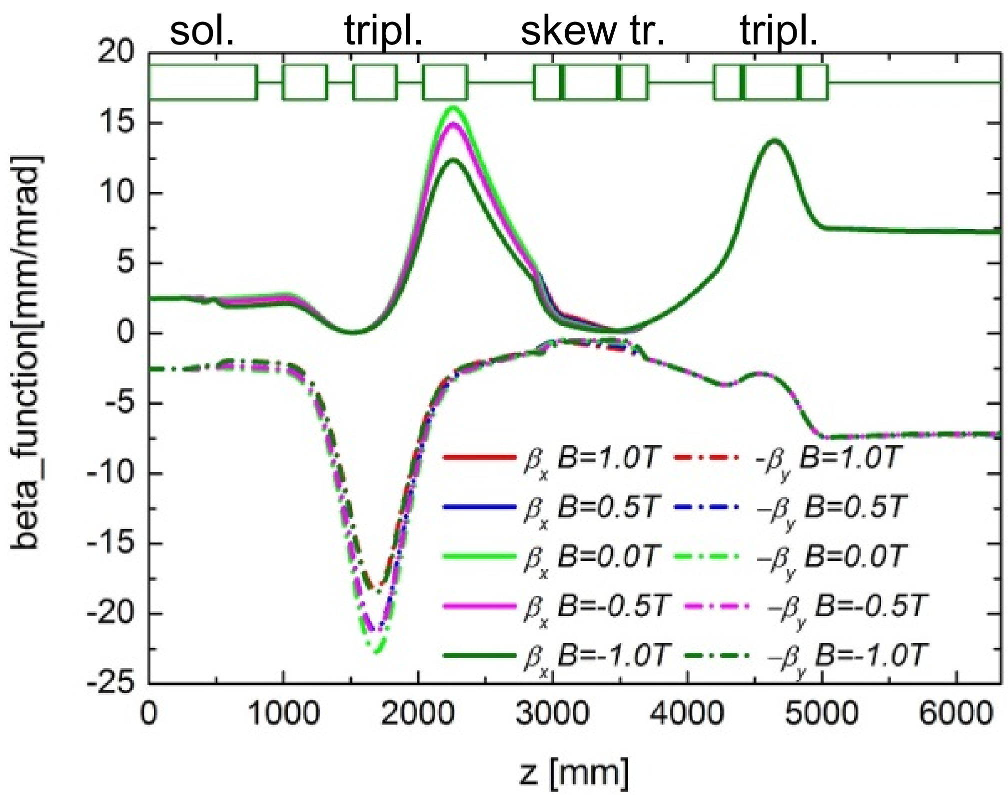
Error Studies



- gradient errors: not critical
- rolls: hor. rms emittances larger than lower eigen-emittance → incomplete decoupling

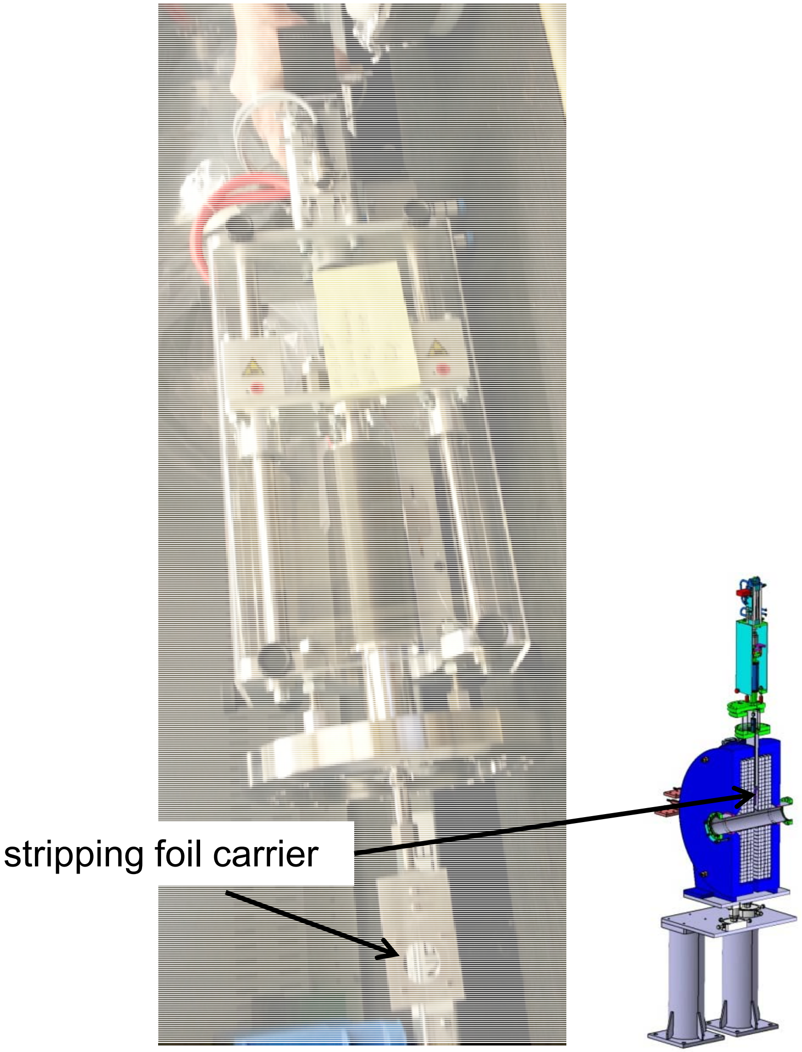
Decoupling & Matching Feature of EMTEX

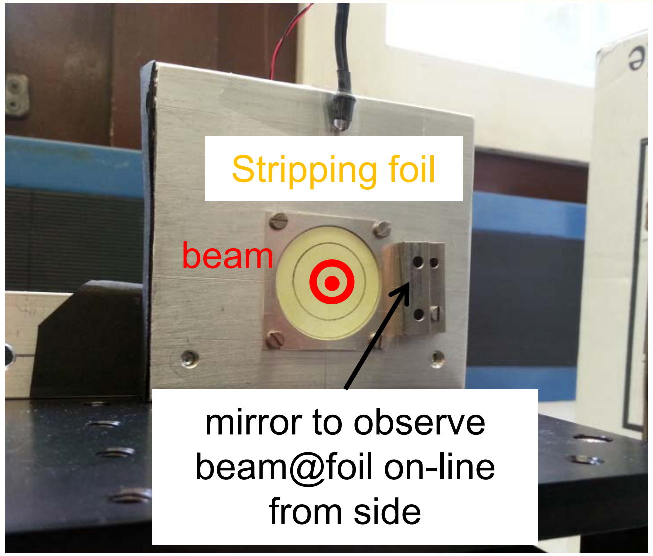
- beta-functions behind the decoupling section do not depend on solenoid field strength (decoupling gradients kept constant)



- EMTEX: after decoupling → beta-functions invariant under solenoid field strength
- achromat: after D & D‘ are closed → beta-functions invariant under momentum spread

Hardware

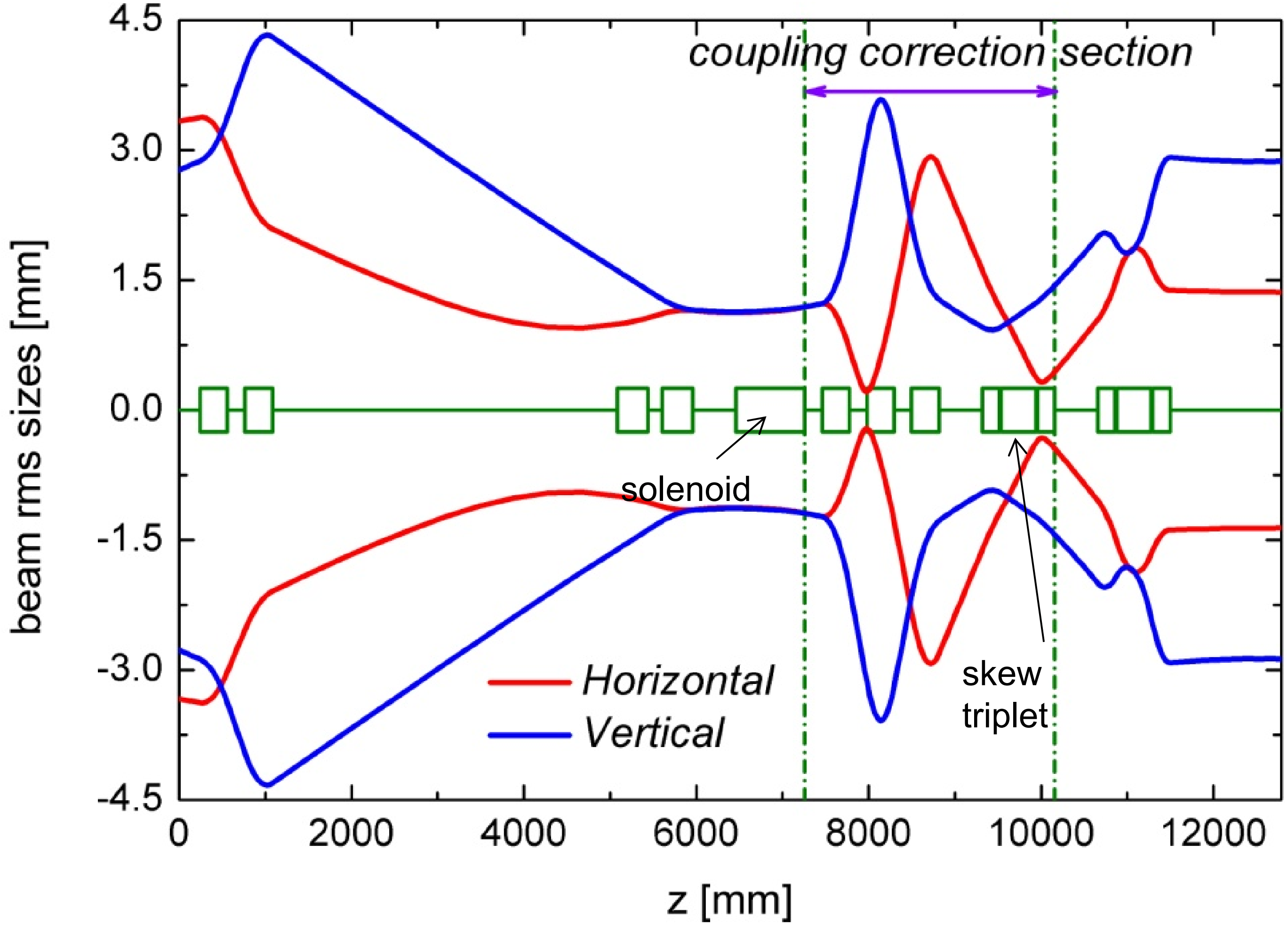

stripping foil carrier


Stripping foil
beam
mirror to observe beam@foil on-line from side


Stripping foil
SOURCE MENU DOWN UP POWER

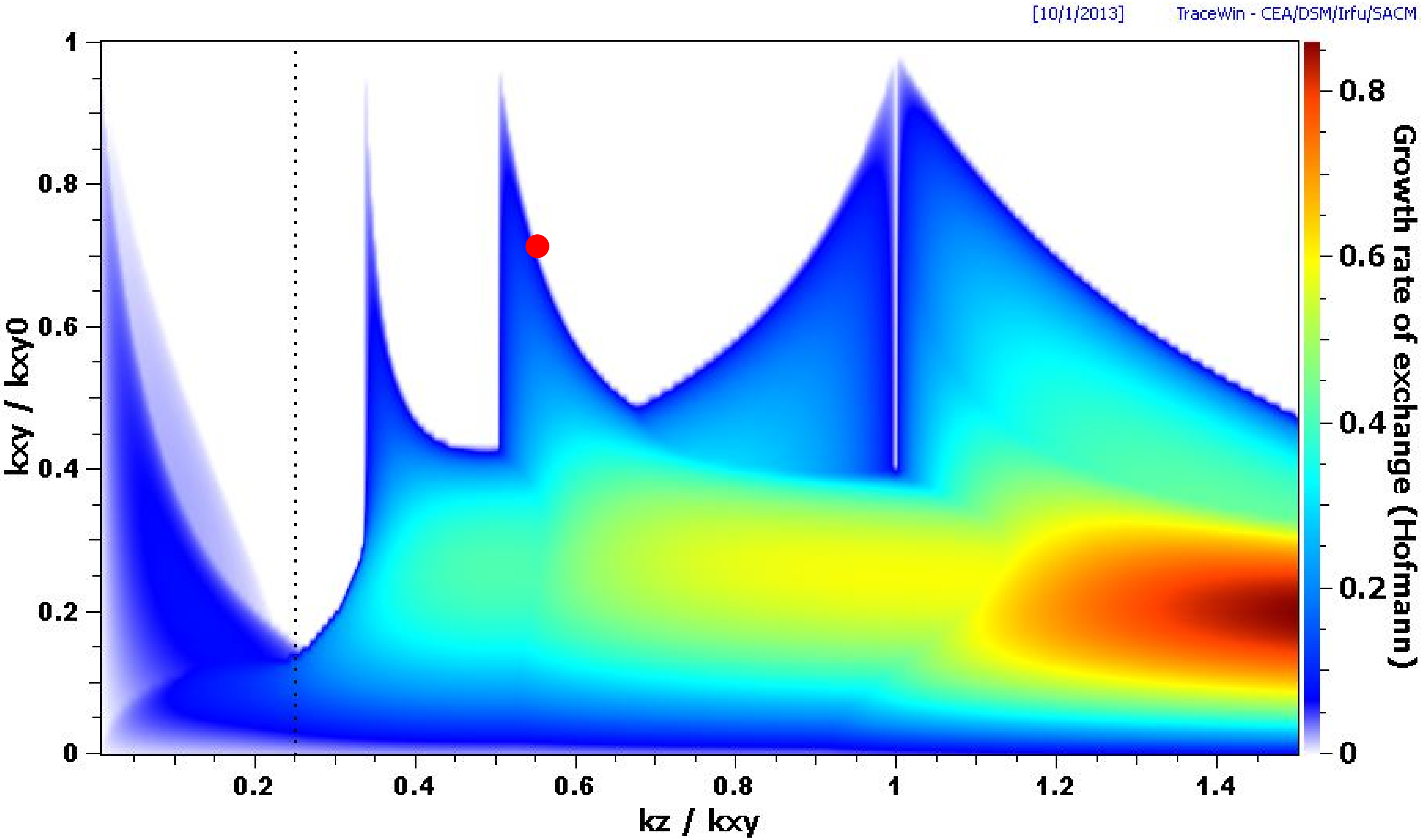
Summary II

- ion beam emittance transfer can be realized by placing a stripper inside a solenoid
- coupling is created non-symplectically .... and removed symplectically
- set-up is single-knob-tool to partition hor. & ver. rms emittances, keeping constant :
  - transv. rms Twiss parameters $\alpha_x, \beta_x, \alpha_y, \beta_y$
  - product of rms emittances $E_x \cdot E_y$
- the single knob is the solenoid field strength
- decoupling is insensitive to :
  - shape of fringe field
  - amount of scattering in stripping foil
  - beam current
  - gradient & *roll errors*



coupling correction section
beam rms sizes [mm]
4.5
3.0
1.5
0.0
-1.5
-3.0
-4.5
0
2000
4000
6000
8000
10000
12000
z [mm]
solenoid
skew
triplet
Horizontal
Vertical

Full Beam Current: Solenoid Field of 0.6 T



Half Beam Current: Solenoid Field of 0.7 T