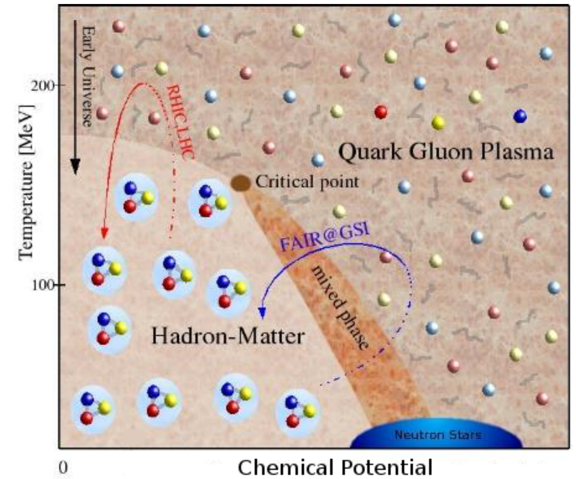
Quarks, gluons and hadrons from Dyson-Schwinger equations

Christian S. Fischer

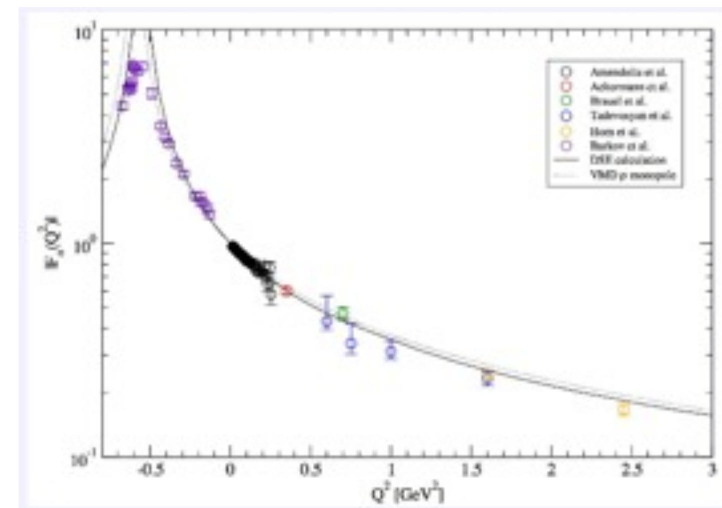
Justus Liebig Universität Gießen

9th of October 2013

Overview

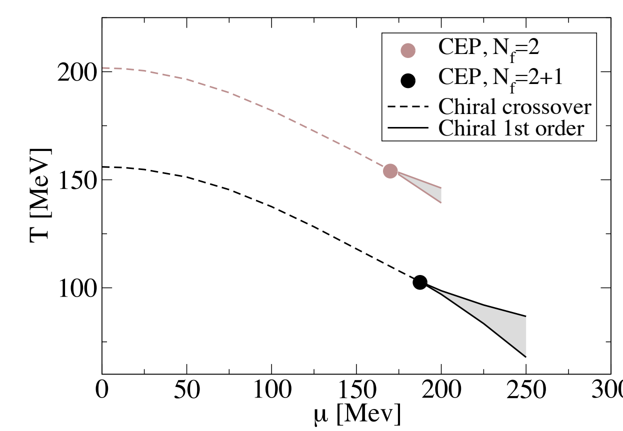


1.Introduction: quarks and gluons



2.Electromagnetic properties of mesons and baryons

3.Gluons, quarks and the QCD phase diagram



Wednesday, October 9, 2013

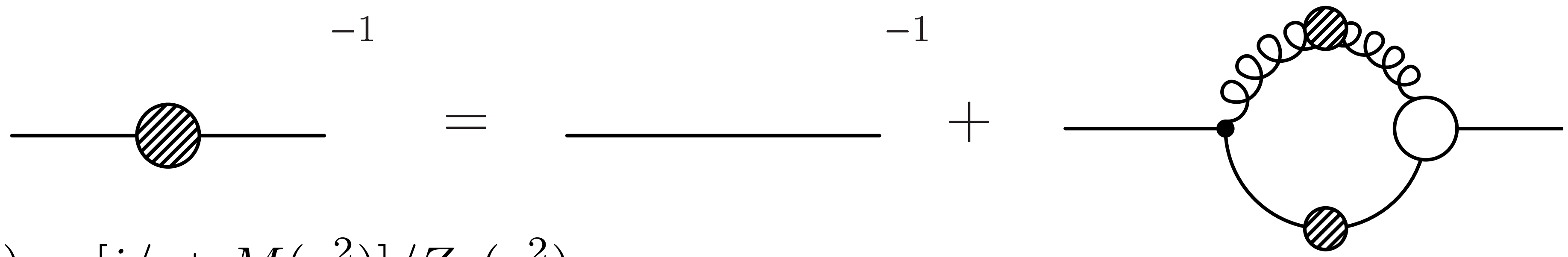
Properties of QCD: Dynamical mass generation

Dynamical quark masses
via weak and strong force

Y.Nambu,
Nobel prize 2008

F. Englert, P. Higgs,
Nobel prize 2013

	u	d	s	c	b	t
M_{weak} $[MeV/c^2]$	3	5	80	1200	4500	176000
M_{strong} $[MeV/c^2]$	350	350	350	350	350	350
M_{total} $[MeV/c^2]$	350	350	450	1500	4800	176000



$$S^{-1}(p) = [i\not{p} + M(p^2)]/Z_f(p^2)$$

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Input parameters in N_f=2+1 QCD

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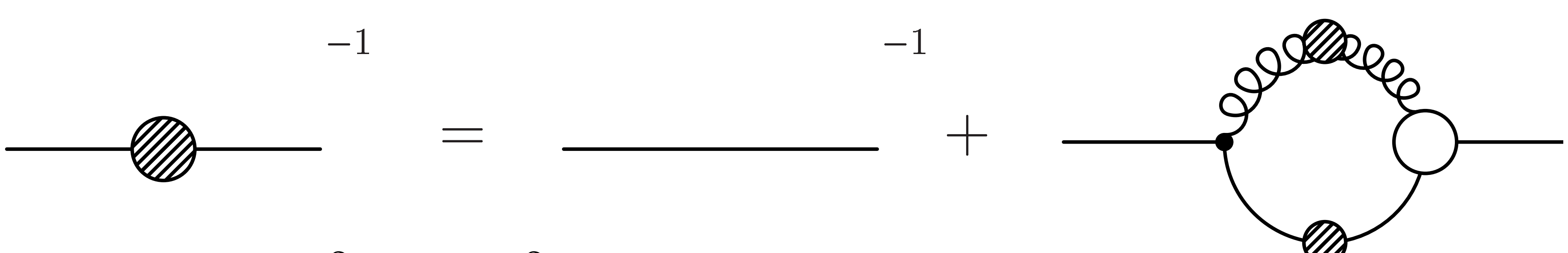
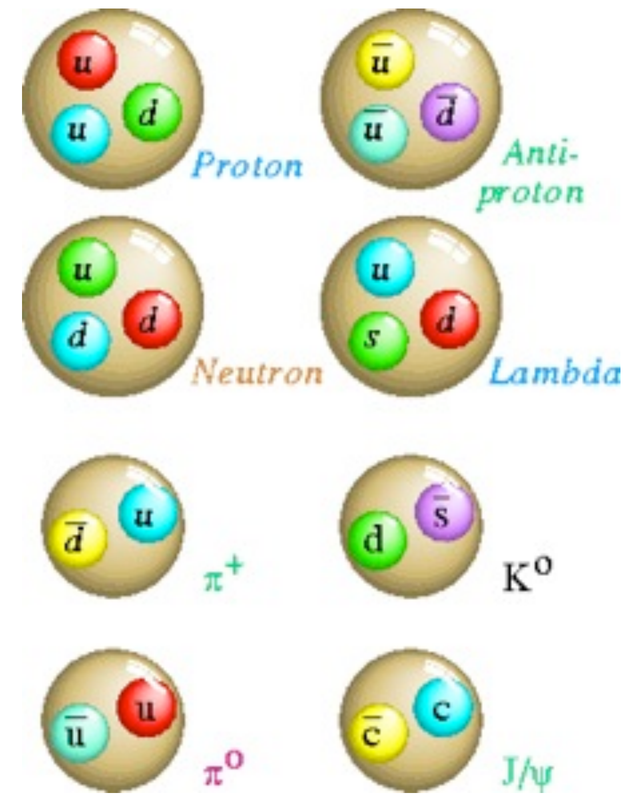


$$S^{-1}(p) = [i\not p + M(p^2)]/Z_f(p^2)$$

Confinement

Color confinement:



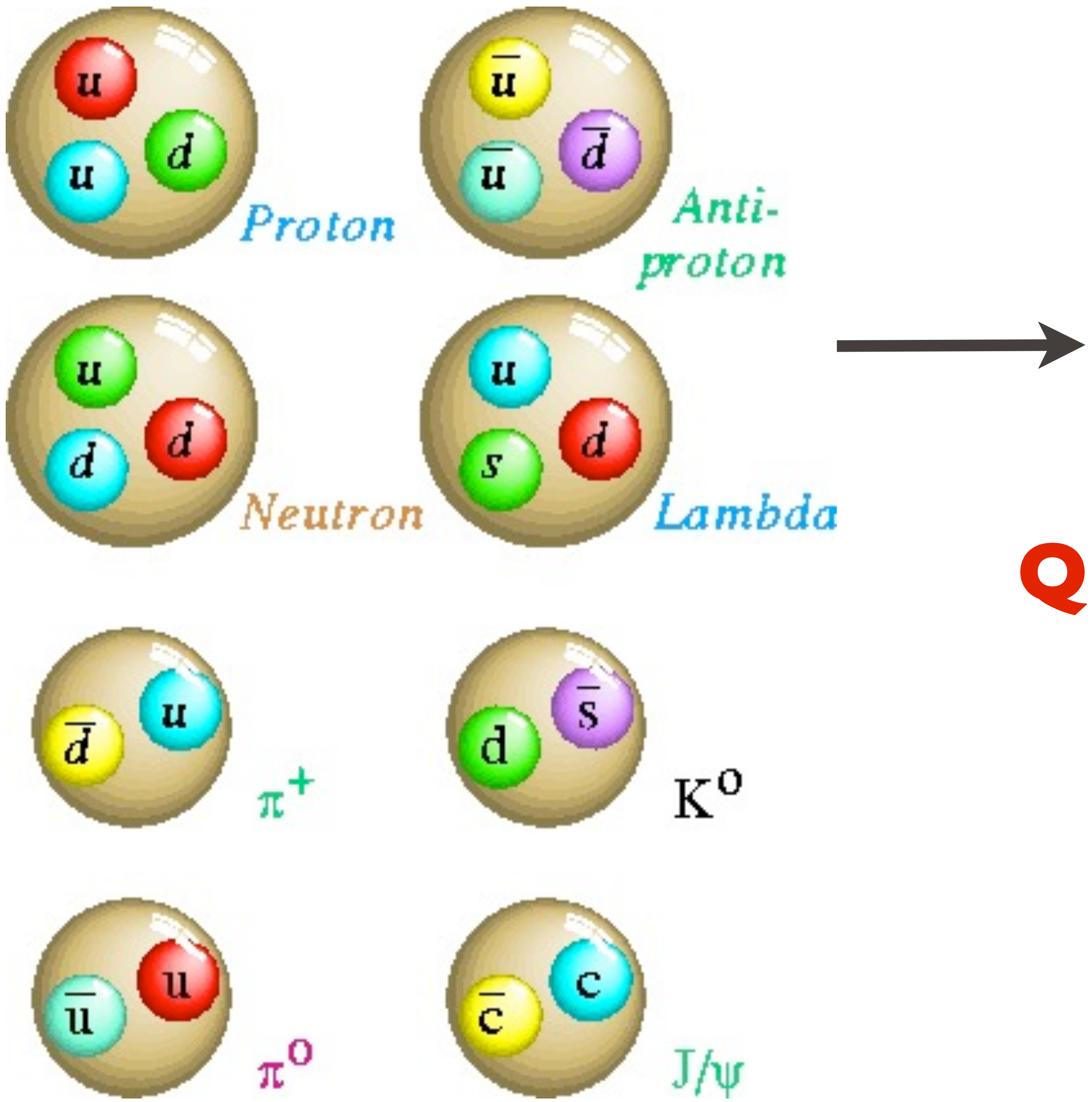
We are not detecting quarks and gluons,
but baryons, mesons, tetraquarks, glueballs....

Strategies to deal with this situation:

- Effective theories in terms of hadrons
- Nonperturbative QCD: Lattice, Functional methods

Chiral symmetry+confinement: meson clouds

Hadrons



Quark configurations
beyond quark model:

Quark core + meson cloud

Baryons:
see Talk of **Gernot Eichmann**
on Saturday

Nonperturbative QCD: Complementary approach

Quarks and gluons

Hadrons

- Lattice simulations
  - Ab initio
  - Gauge invariant
- Effective theories and models (χPT, chiral mod...)
  - Physical degrees of freedom
- Functional approaches (DSE, FRG,Hamilton):
  - Chiral symmetry: physical quark masses
  - Infinite volume and continuum limit
  - Multi-scale problems feasible (e.g. $(g-2)_\mu$)

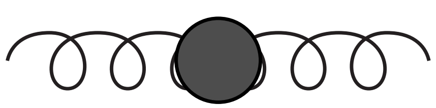
    Goecke, CF, Williams, PRD 87 (2013) 03401
  - Chemical potential: no sign problem

QCD in covariant gauge

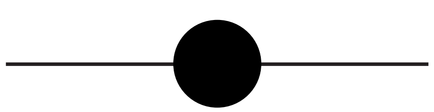
Quarks and Gluons

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \; \exp\left\{ - \int d^4x \Big(\bar{\Psi}\,(i\not{D} - m)\,\Psi - \frac{1}{4}\left(F^a_{\mu\nu}\right)^2 + \text{gauge fixing} \Big) \right\}$$

Landau gauge propagators in momentum space,



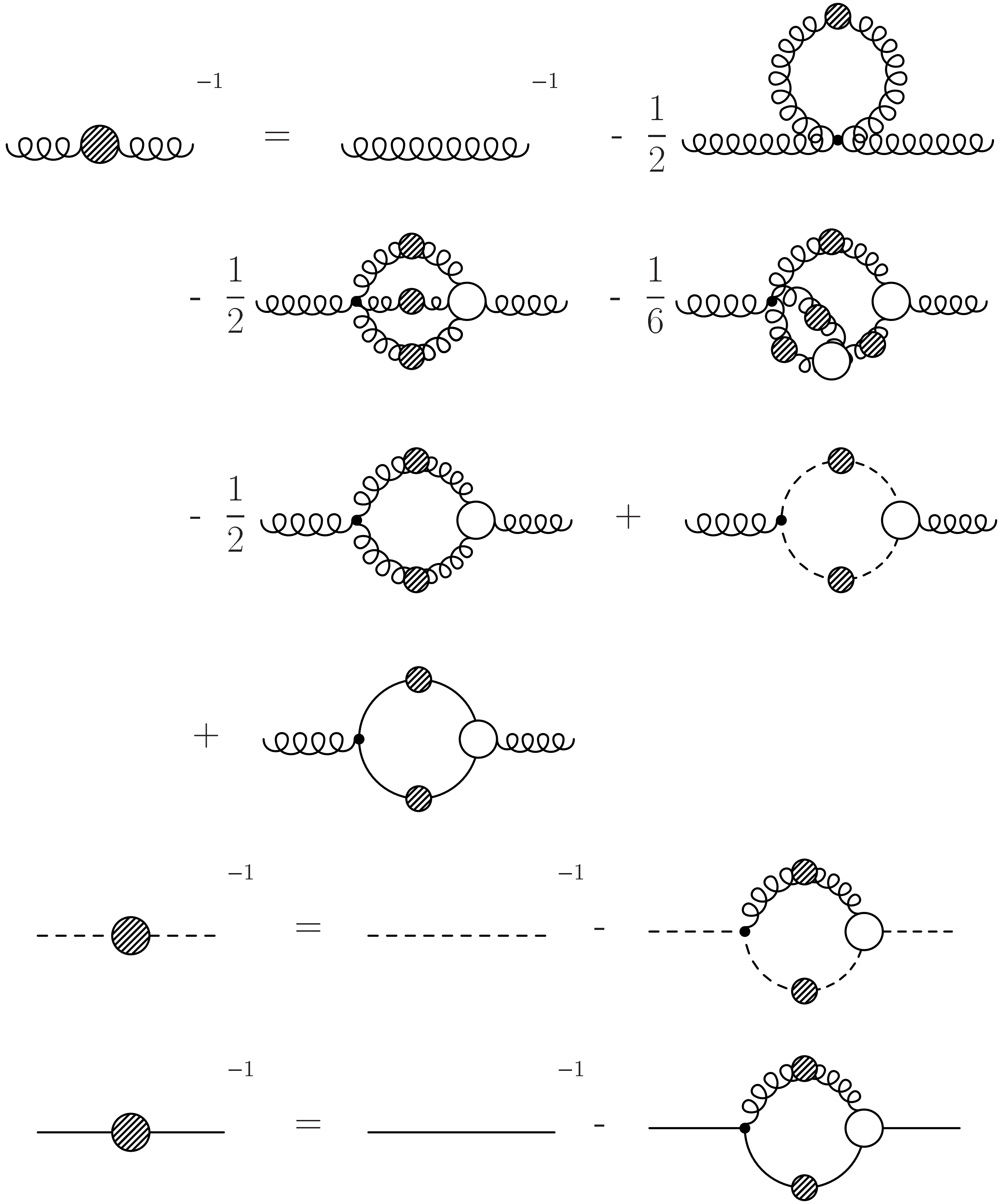
$$D^{Gluon}_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \frac{Z(p^2)}{p^2}$$



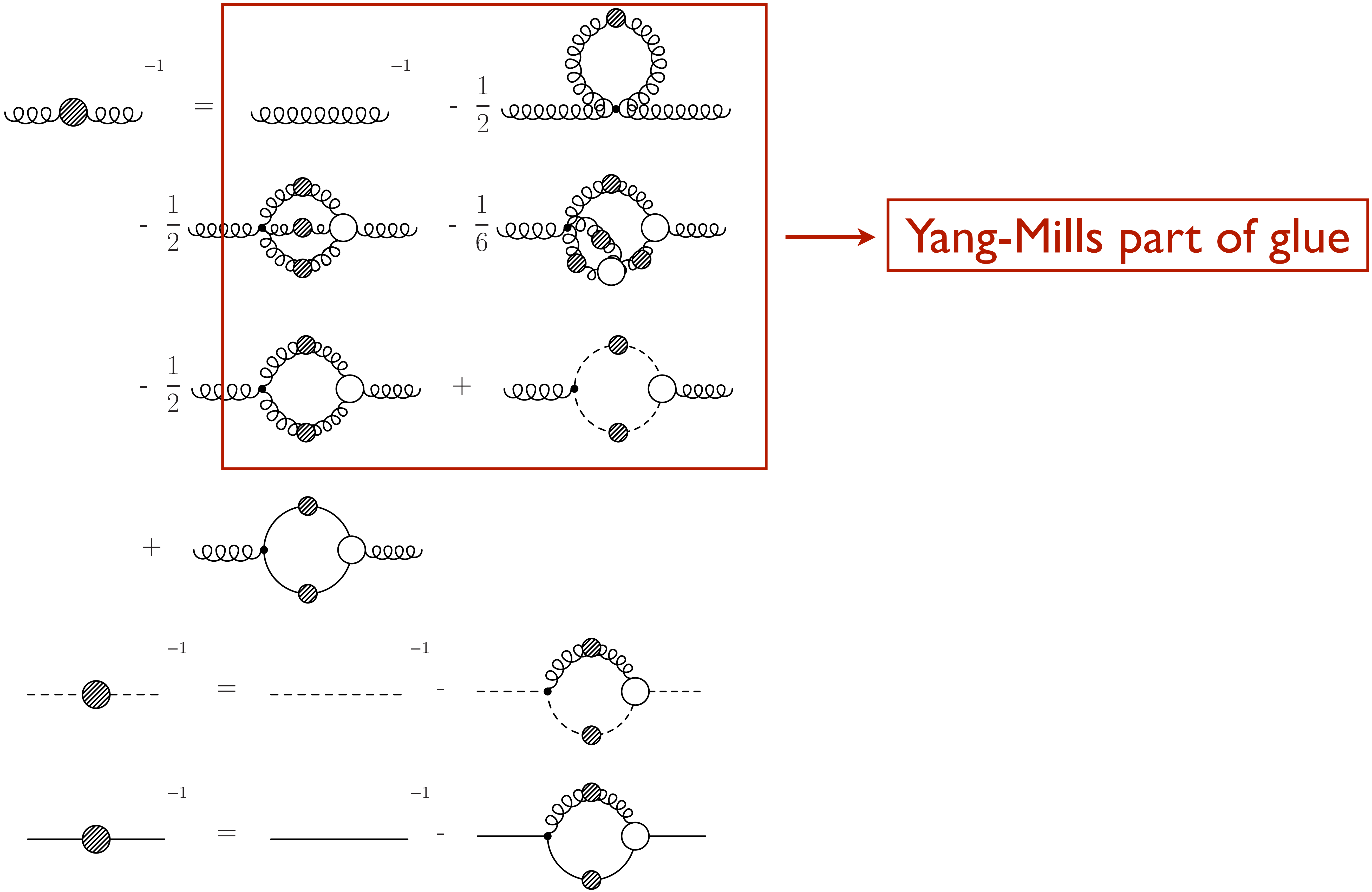
$$S^{Quark}(p) = Z_f(p^2)\left[-i\not{p} + M(p^2)\right]^{-1}$$

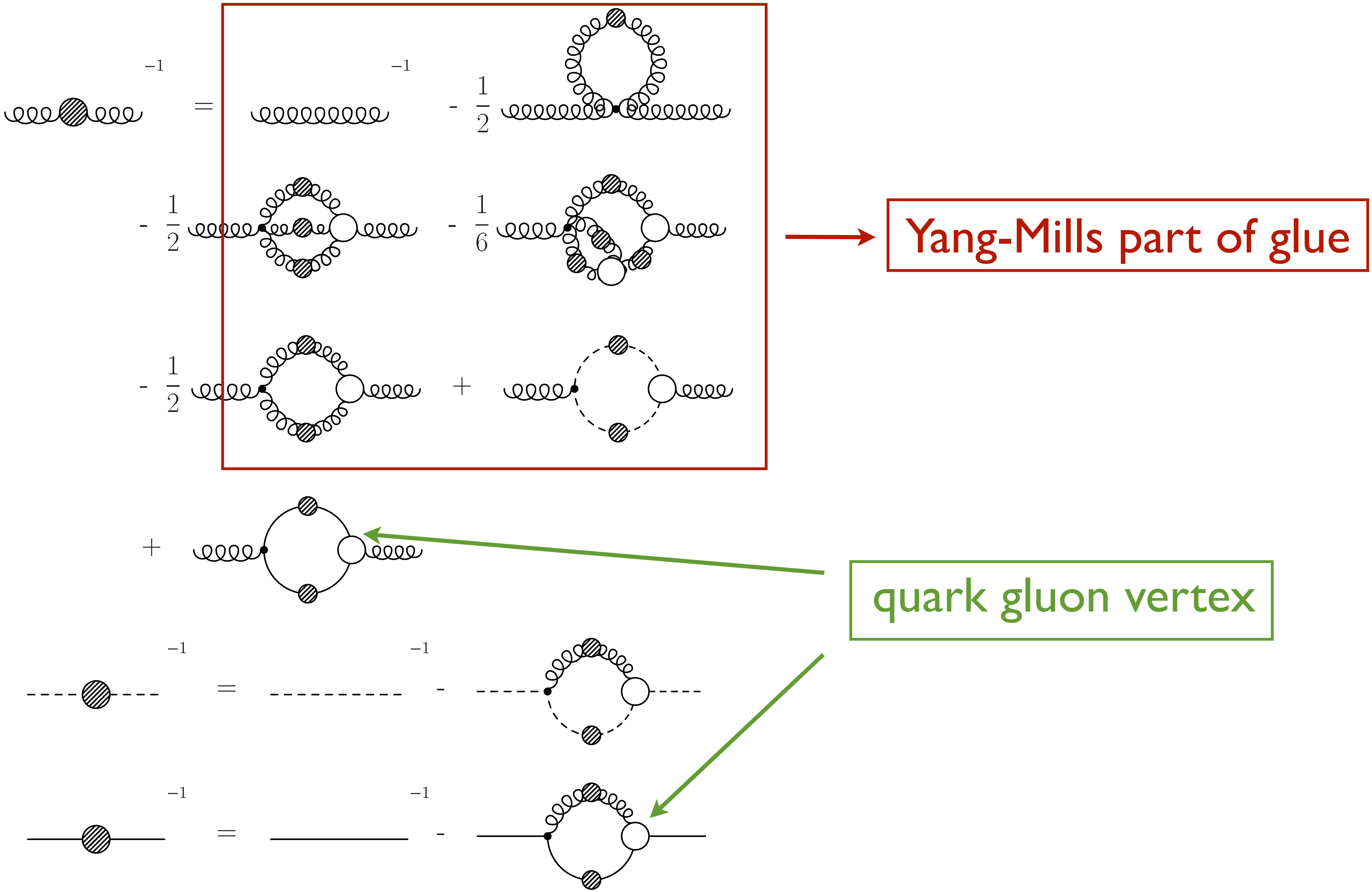
The Goal: gauge invariant information in a gauge fixed approach.

DSEs of QCD



DSEs of QCD





−1
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- 1/2
- 1/2
- 1/6
- 1/2
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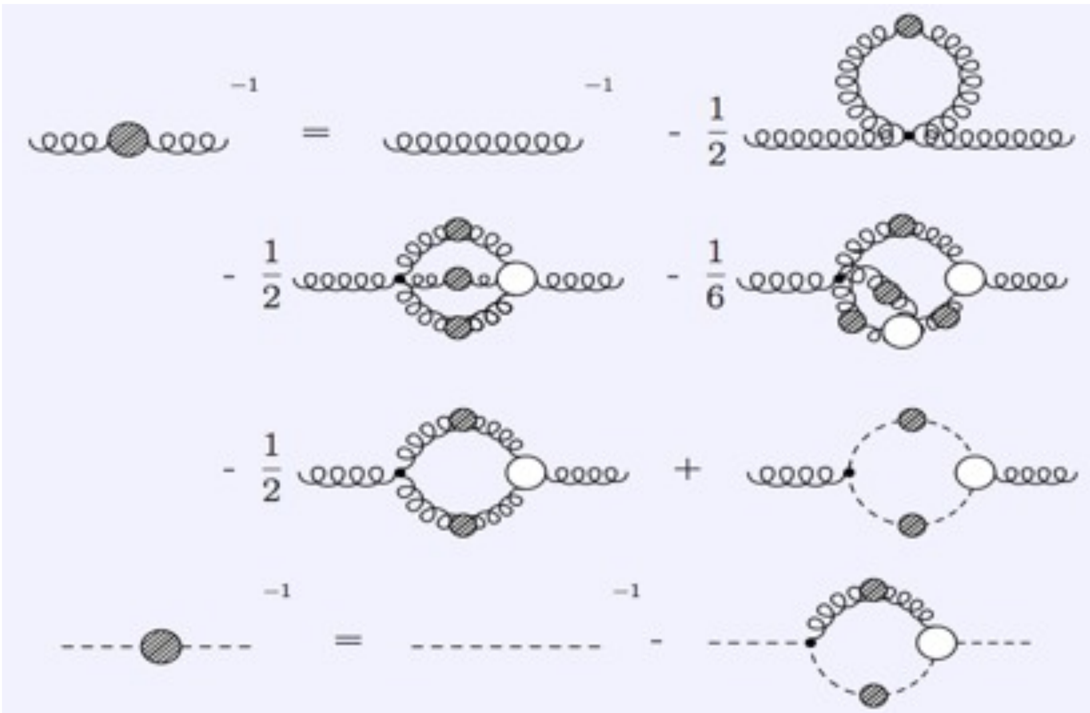
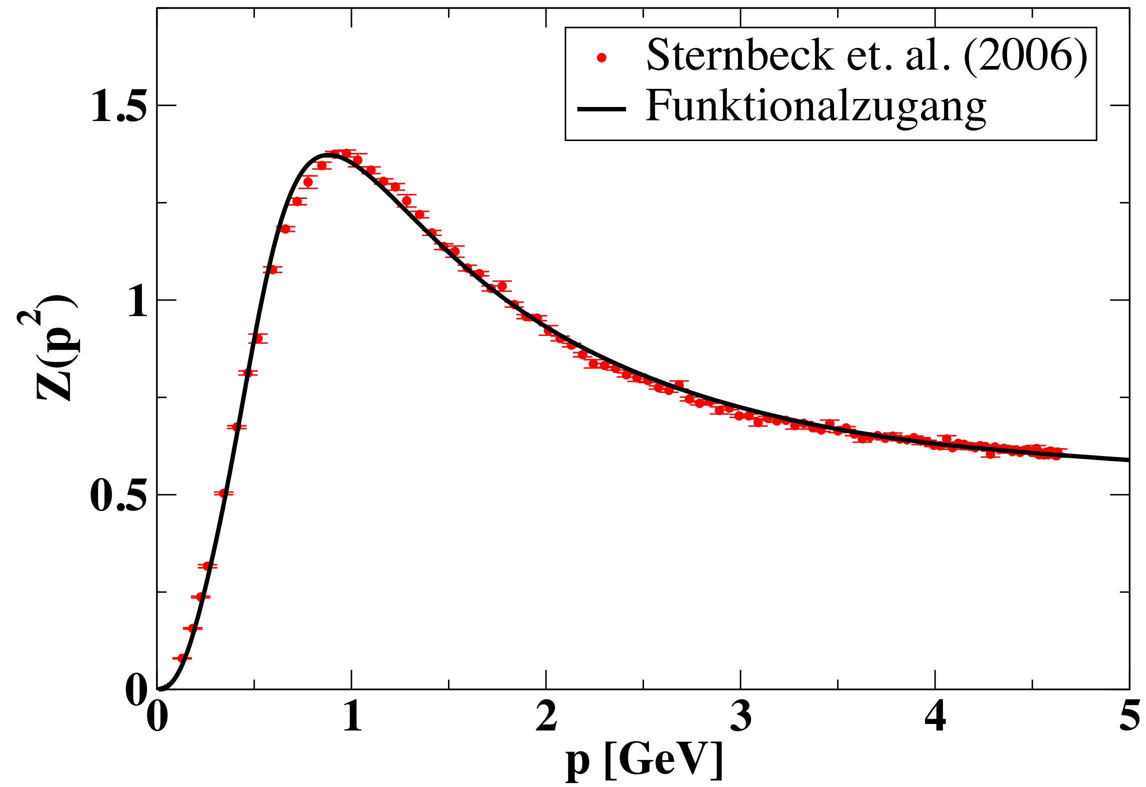
Yang-Mills part of glue
quark gluon vertex

Landau gauge gluon propagator





CF, Maas, Pawlowski, Annals Phys. 324 (2009) 2408.
Huber, von Smekal, JHEP 1304 (2013) 149.

- spacelike momenta: excellent agreement with lattice
- deep infrared: 'massive' behaviour

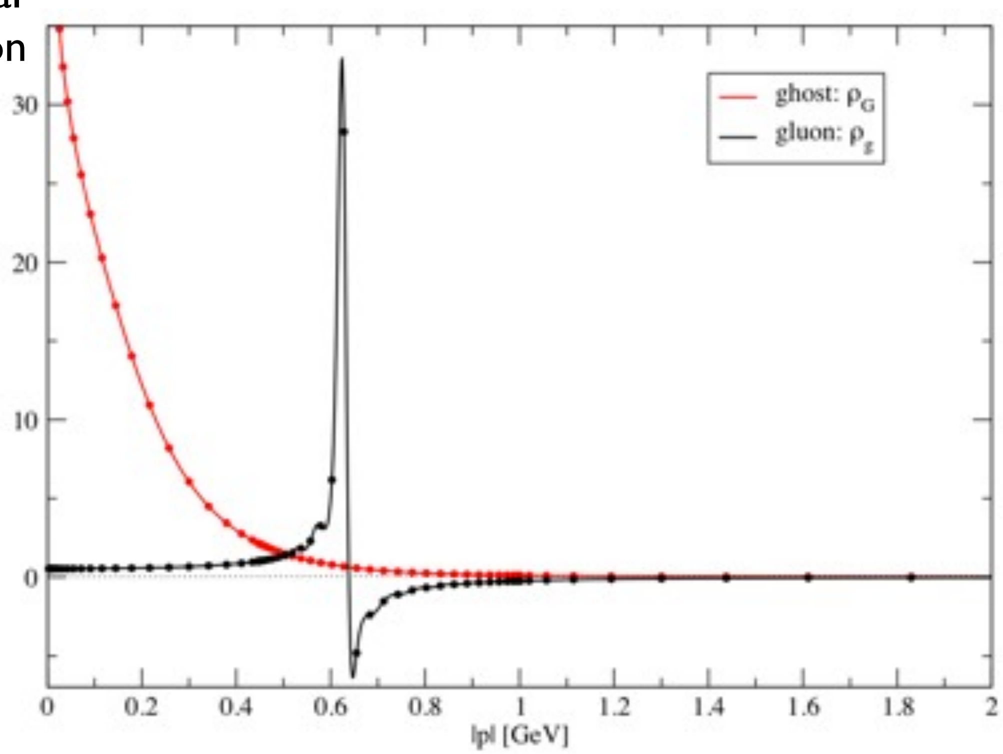
$$Z(p^2)/p^2 \sim const.$$

Aguilar, Binosi, Papavassiliou, PRD **78**, 025010 (2008). Cornwall, PRD **26** (1982) 1453.

- analytic structure: cut at timelike p
- positivity violations!

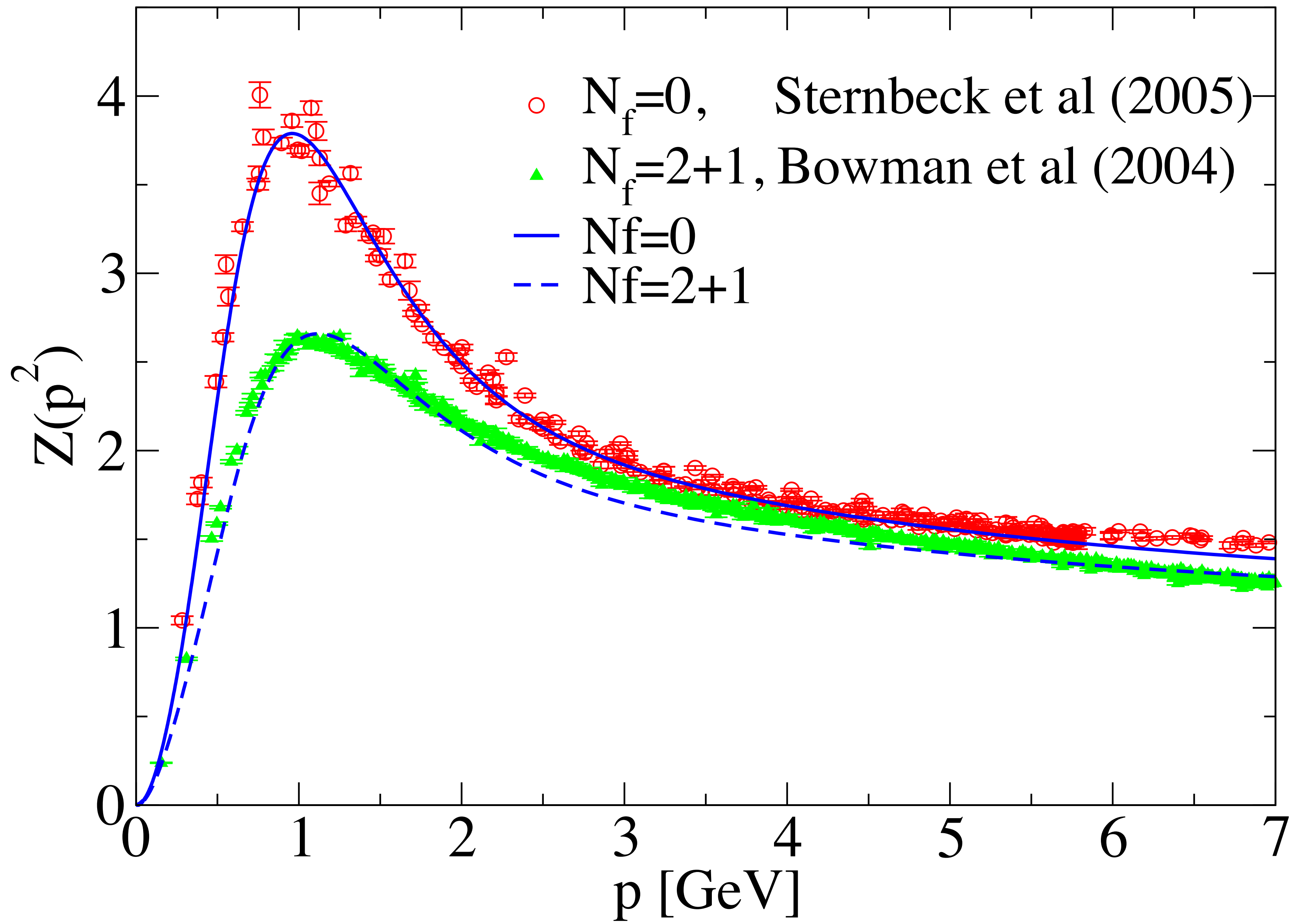
Gluon cannot appear in detector!

spectral function



Strauss, CF, Kellermann, Phys. Rev. Lett. 109, (2012) 252001

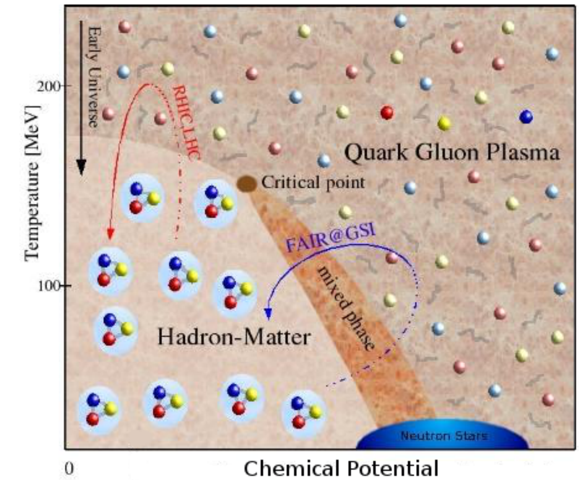
Unquenching effects in gluon



Kubrak, CF, in prep.

- Quantitative agreement with lattice

Based on: CF, Watson and Cassing, PRD 72 (2005) 094025,
Huber, von Smekal, JHEP 1304 (2013) 149



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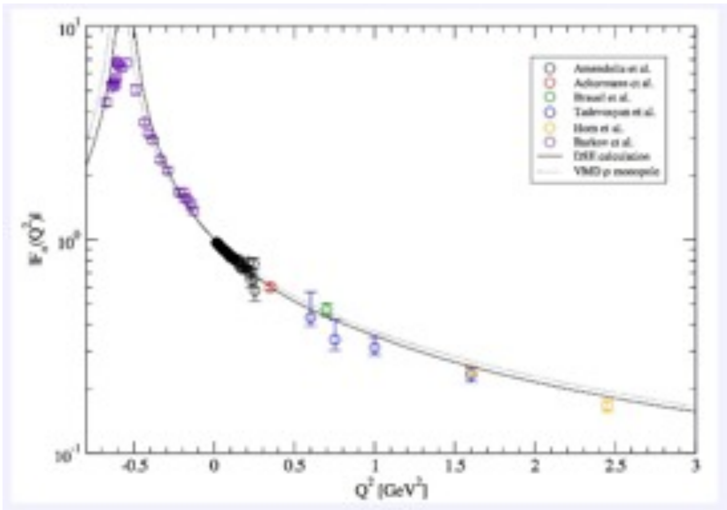

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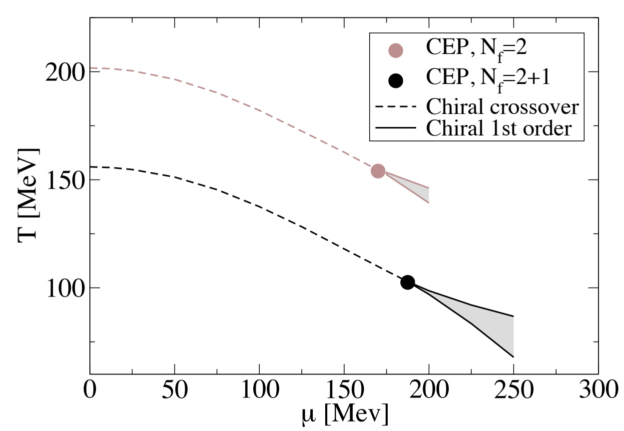
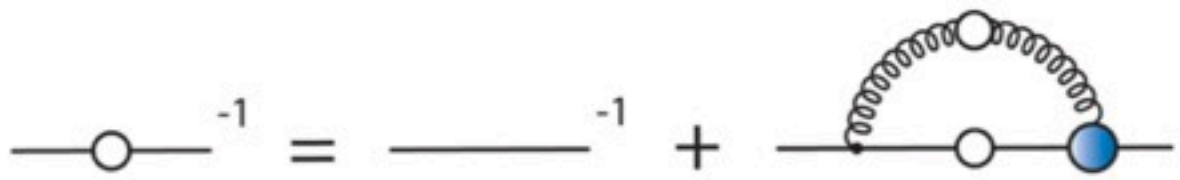


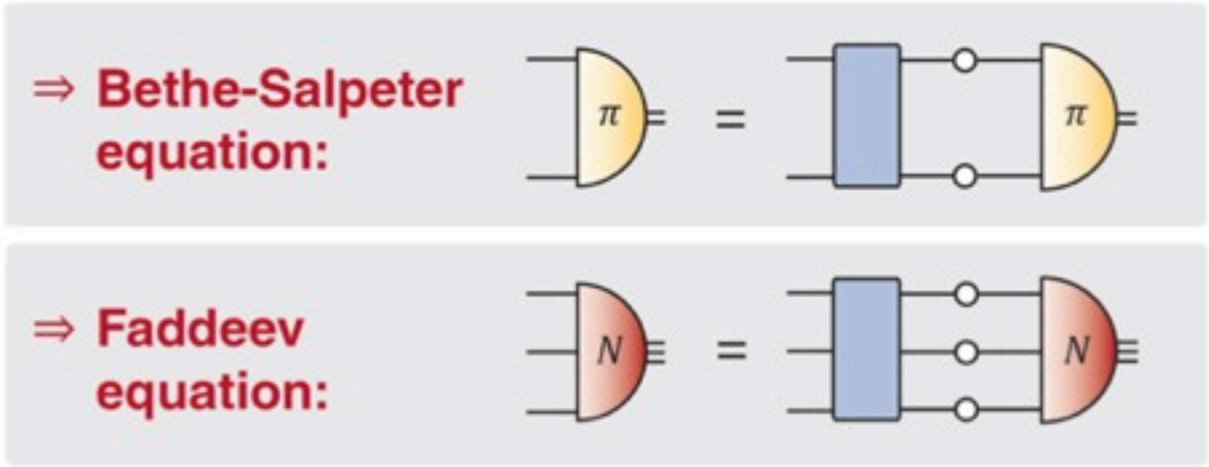
Mesons and Baryons

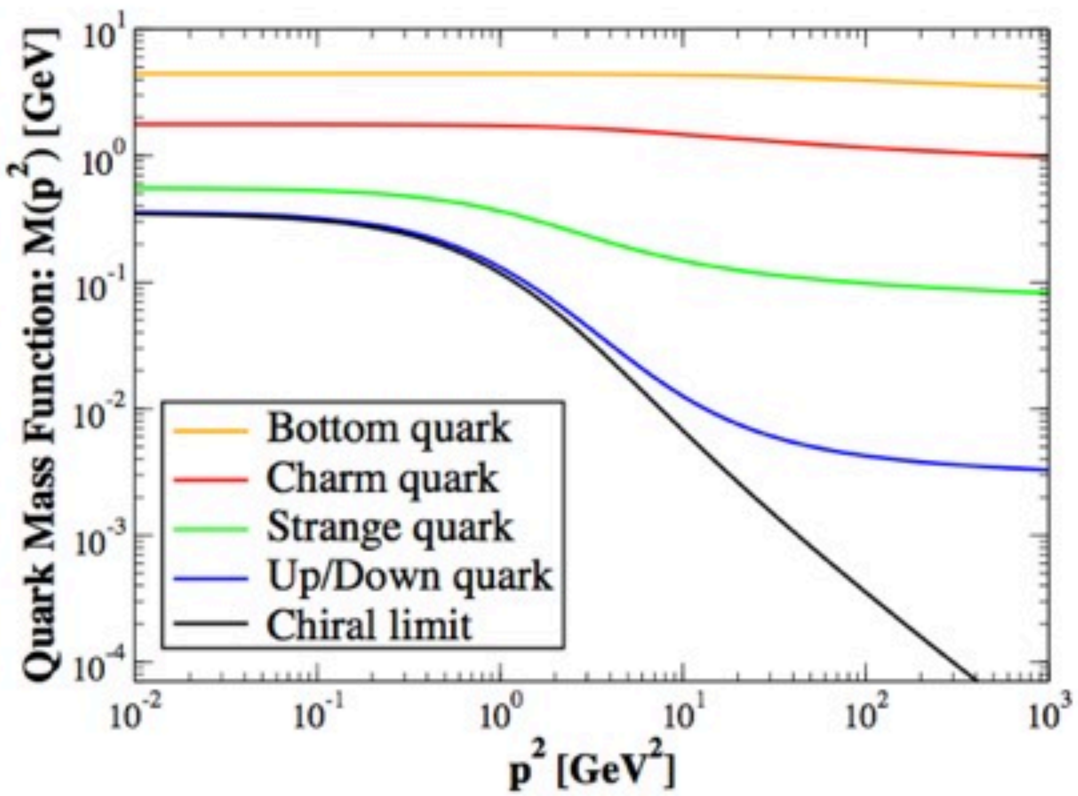
General goal:
Experimental observables from nonperturbative quark and gluon structure of QCD

Framework: DSEs, BSEs, FEs

- Dynamics at perturbative and nonperturbative scales
- Dynamical chiral symmetry breaking: connects dynamically generated 'constituent-quark mass' with current quark mass
- Dynamical realization of Goldstone boson nature of pseudoscalar mesons







The DSE for the quark propagator

$$[S(p)]^{-1} = [-i\not{p} + M(p^2)]/Z_f(p^2)$$

Input: • dressed Gluon propagator

• dressed Quark-Gluon-Vertex

Two strategies: I. calculate gluon and vertex from their DSEs

→mandatory e.g. for QCD phase diagram

CF and Luecker, PLB 718 (2013) 1036-1043

II. use model for quark-gluon interaction

→ok for some phenomenological applications

Strategy II: model for quark-gluon interaction

Combine gluon with quark-gluon vertex:

Maris-Tandy-model

$$\alpha(k^2) = \pi\eta^7 \left(\frac{k^2}{\Lambda^2}\right) e^{-\eta^2\left(\frac{k^2}{\Lambda^2}\right)} + \alpha_{UV}(k^2)$$

Maris, Tandy, 1999

and fix

- two (related) parameters η and Λ from f_π
- α_{UV} from perturbation theory
- masses $m_u = m_d$ from m_π or m_ρ

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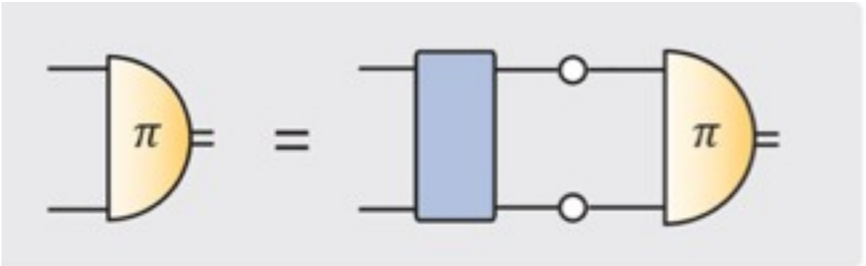


Phenomenology from Maris-Tandy interaction

Summary of light meson results
$m_{u=d} = 5.5$ MeV, $m_s = 125$ MeV at $\mu = 1$ GeV

Pseudoscalar (PM, Roberts, PRC56, 3369)

	expt.	calc.
$-\langle \bar{q}q \rangle^0_\mu$	(0.236 GeV)^3	(0.241†)^3
m_π	0.1385 GeV	0.138†
f_π	0.0924 GeV	0.093†
m_K	0.496 GeV	0.497†
f_K	0.113 GeV	0.109

Charge radii (PM, Tandy, PRC62, 055204)

r^2_π	0.44 fm^2	0.45
$r^2_{K^+}$	0.34 fm^2	0.38
$r^2_{K^0}$	-0.054 fm^2	-0.086

$\gamma\pi\gamma$ transition (PM, Tandy, PRC65, 045211)

$g_{\pi\gamma\gamma}$	0.50	0.50
$r^2_{\pi\gamma\gamma}$	0.42 fm^2	0.41

Weak K_{l3} decay (PM, Ji, PRD64, 014032)

$\lambda_+(e3)$	0.028	0.027
$\Gamma(K_{e3})$	$7.6 \cdot 10^6\ s^{-1}$	7.38
$\Gamma(K_{\mu3})$	$5.2 \cdot 10^6\ s^{-1}$	4.90

Vector mesons (PM, Tandy, PRC60, 055214)

$m_{\rho/\omega}$	0.770 GeV	0.742
$f_{\rho/\omega}$	0.216 GeV	0.207
m_{K^*}	0.892 GeV	0.936
f_{K^*}	0.225 GeV	0.241
m_ϕ	1.020 GeV	1.072
f_ϕ	0.236 GeV	0.259

Strong decay (Jarecke, PM, Tandy, PRC67, 035202)

$g_{\rho\pi\pi}$	6.02	5.4
$g_{\phi KK}$	4.64	4.3
$g_{K^*K\pi}$	4.60	4.1

Radiative decay (PM, nucl-th/0112022)

$g_{\rho\pi\gamma}/m_\rho$	0.74	0.69
$g_{\omega\pi\gamma}/m_\omega$	2.31	2.07
$(g_{K^*K\gamma}/m_K)^+$	0.83	0.99
$(g_{K^*K\gamma}/m_K)^0$	1.28	1.19

Scattering length (PM, Cotanch, PRD66, 116010)

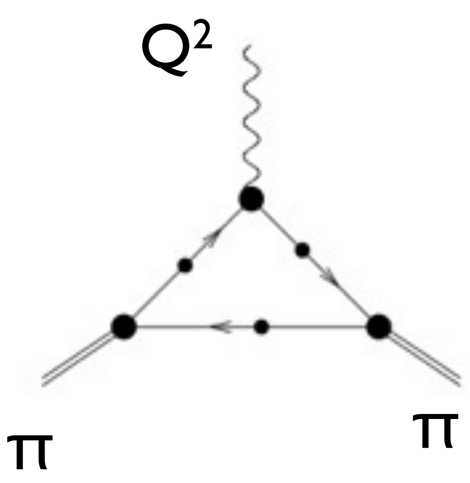
a^0_0	0.220	0.170
a^2_0	0.044	0.045
a^1_1	0.038	0.036

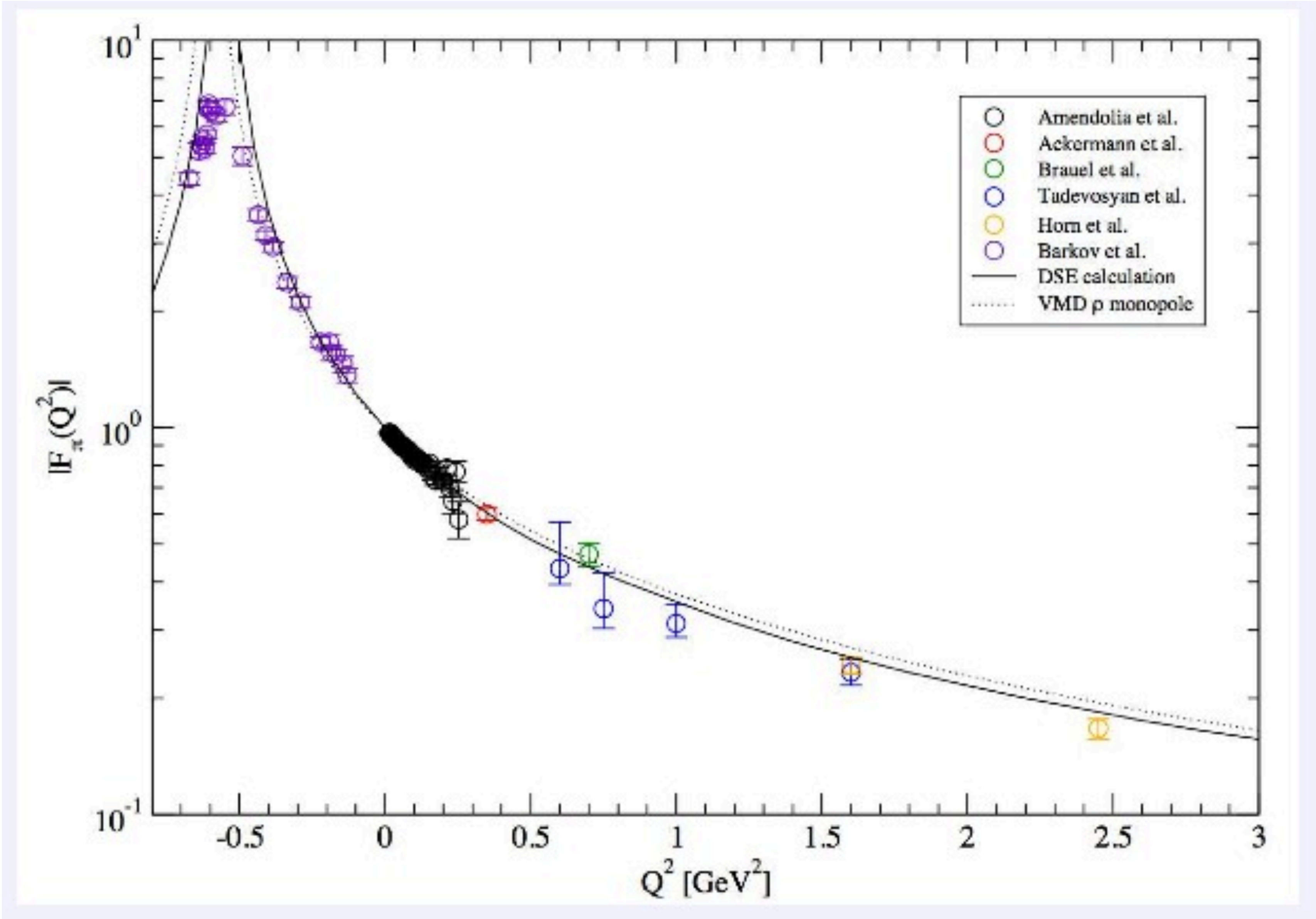
M_ρ, M_ϕ, M_{K^*} good to 5%, f_ρ, f_ϕ, f_{K^*} good to 10%

Slide from Pieter Maris

Quark-photon vertex and pion form factors

Pion form factor:

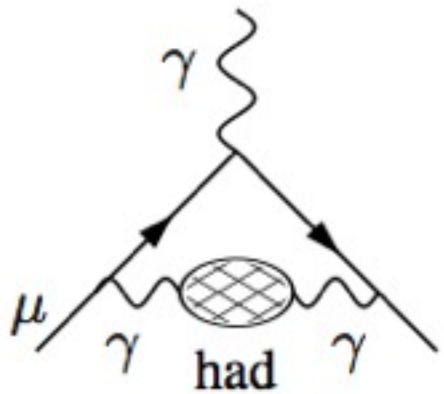


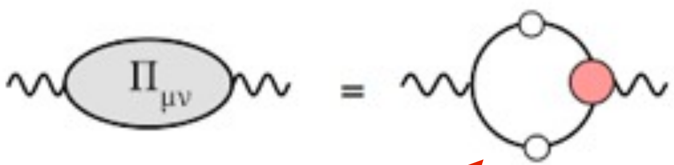


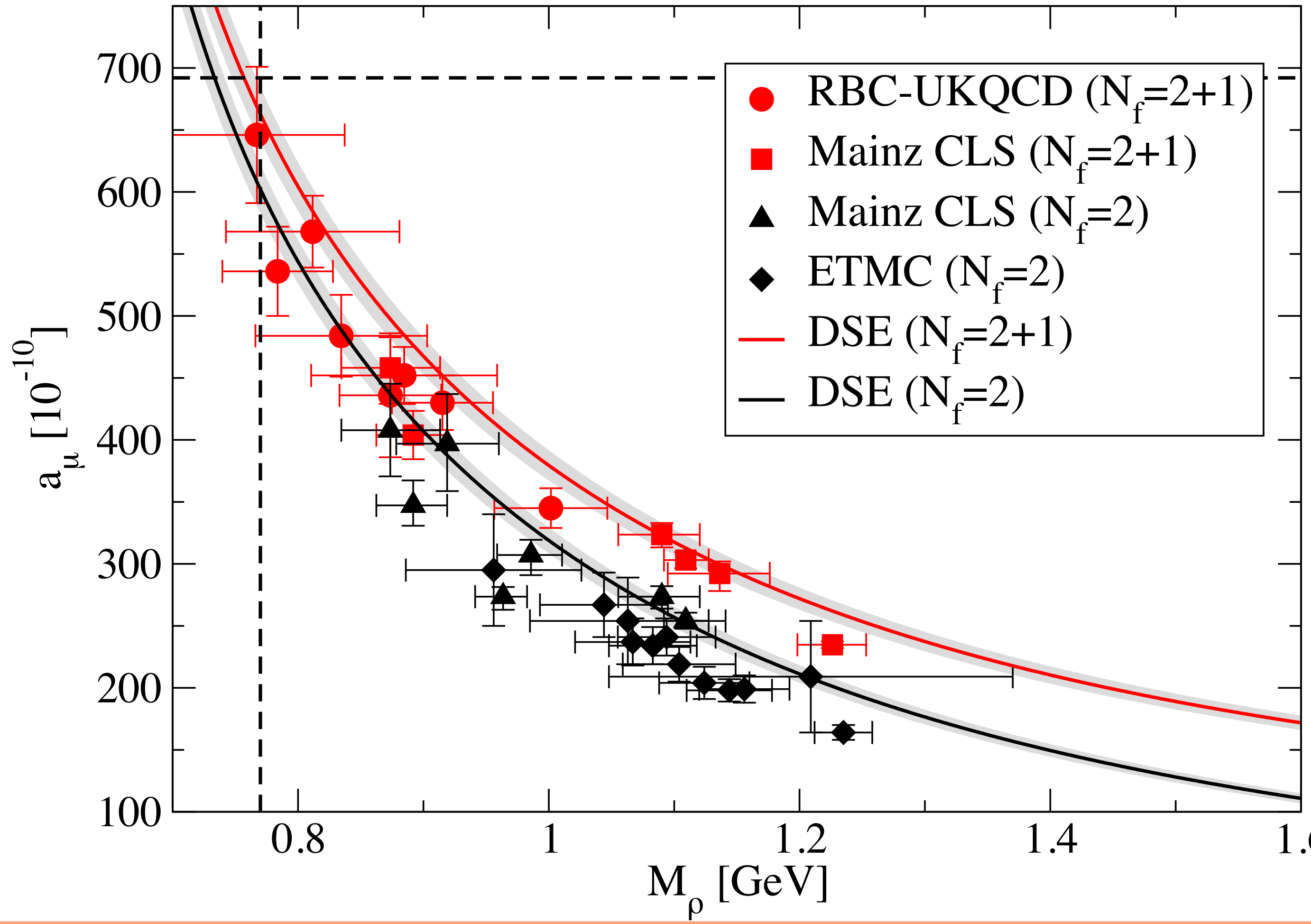
Krassnigg, Schladming 2011;
Maris, Tandy NPPS 161, 2006

- good agreement with data
- rho/omega pole generated dynamically

Results including pion cloud are under way!







five flavors

DSE:

Goecke, CF, Williams, PLB 704 (2011)

$$a_\mu^{had.(1)} = (744.0 \pm 2) \cdot 10^{-10} \quad (m_\pi)$$

$$a_\mu^{had.(1)} = (676.0 \pm 2) \cdot 10^{-10} \quad (m_\rho)$$

Lattice:

Burger et al. arXiv:1308.4327

$$a_\mu^{had.(1)} = (674.0 \pm 39) \cdot 10^{-10}$$

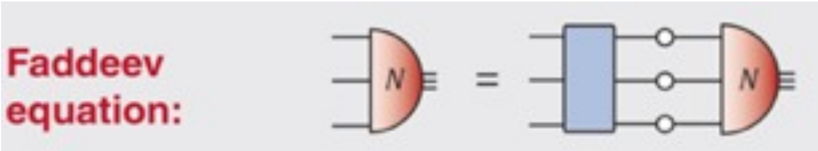
Dispersive analysis:

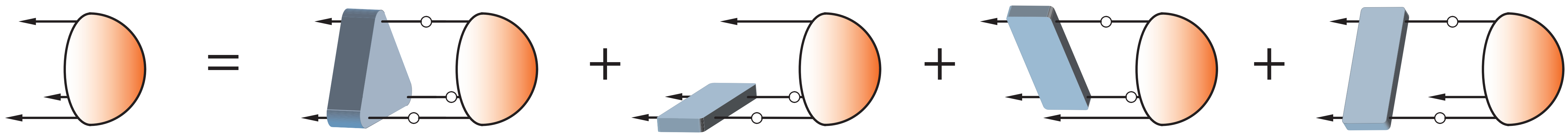
Hagiwara et al. JPG 38 (2011) 085003

$$a_\mu^{had.(1)} = (694.9 \pm 4.3) \cdot 10^{-10}$$

Very reasonable agreement ! → **LBL** Goecke, CF, Williams, PRD87 (2013) 3, 034013

Faddeev - equation



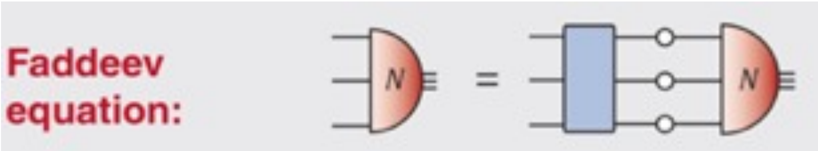


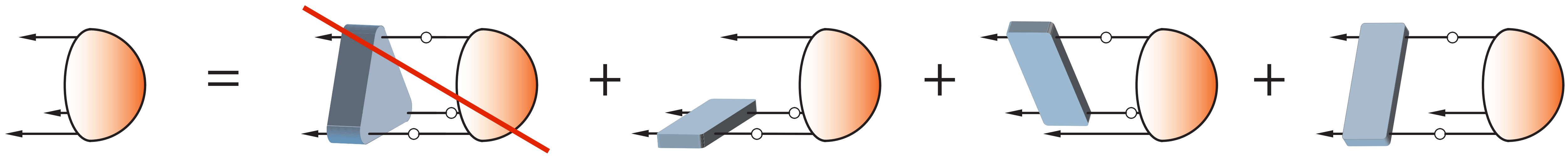
- neglect irreducible three-body forces (three-gluon interaction !)
- approximate two-body interactions by RL-gluon exchange
  - same one-parameter-model (MT) for mesons and baryons
- 64 tensor structures for nucleon: s, p, d - wave
- numerically expensive but manageable !

Eichmann, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010)

Eichmann, PRD 84 (2011)

Sanchis-Alepuz ,Eichmann, Villalba-Chavez, Alkofer, PRD (2012)





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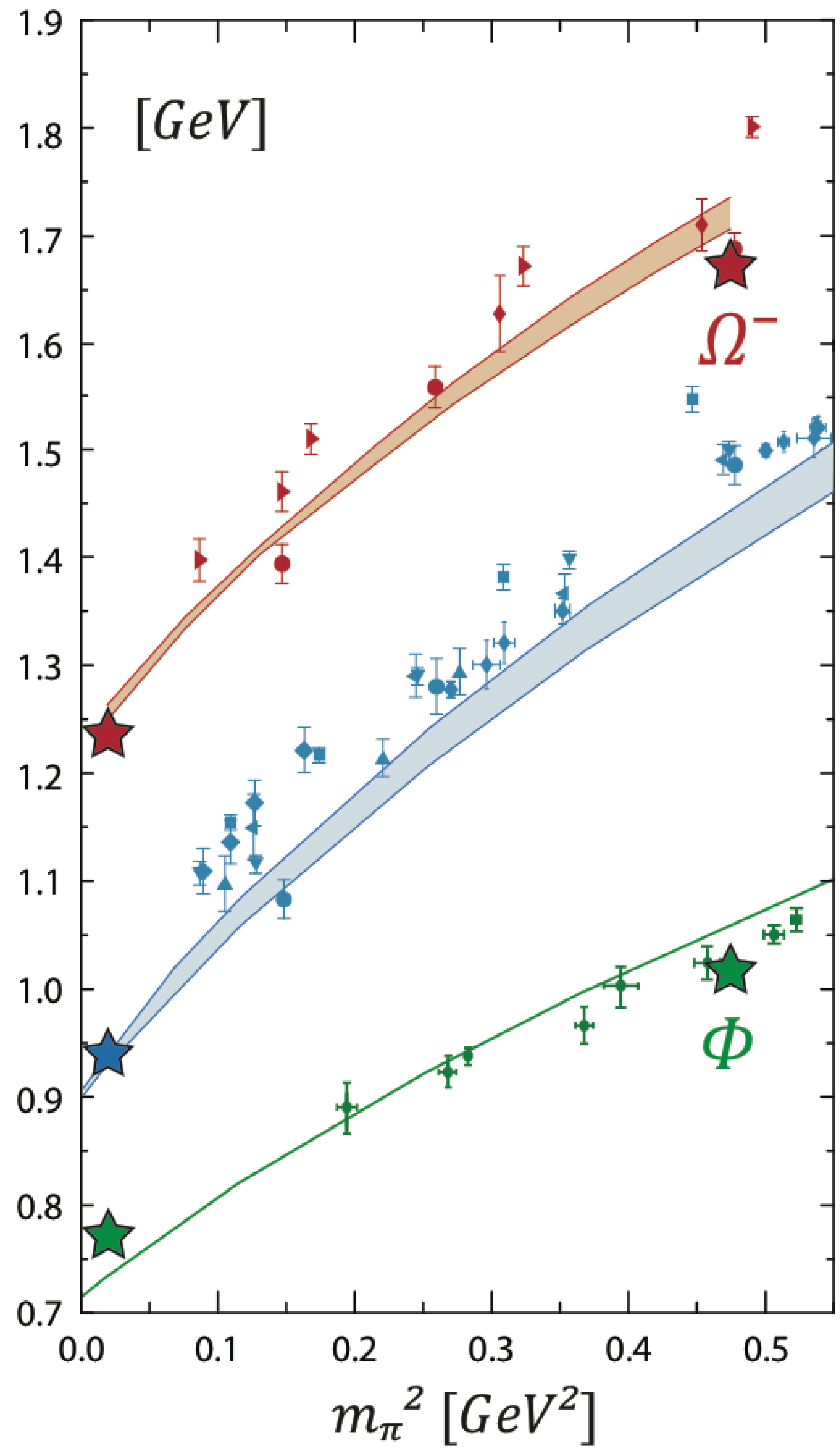
Eichmann, PRD 84 (2011)

Sanchis-Alepuz ,Eichmann, Villalba-Chavez, Alkofer, PRD (2012)

Baryon masses

- first covariant three-body calculations !
- grosso modo: consistent description of mesons and baryons
- masses dominated by s-waves



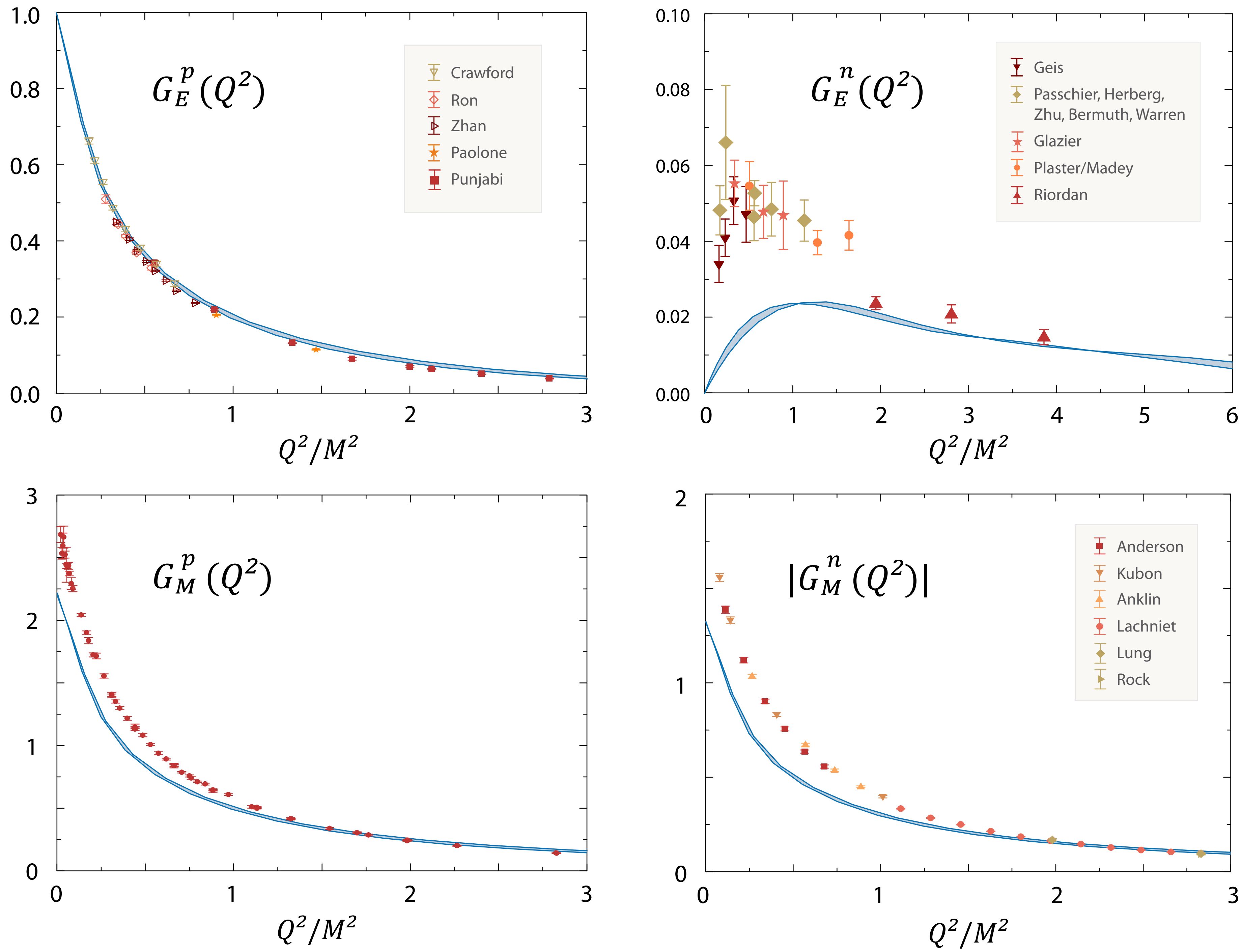
Delta mass:
Sanchis-Alepuz et al., 1109.0199 [hep-ph]

Nucleon mass:
GE, PRD 84 (2011)

ρ–meson mass:
Maris, Tandy, Nucl. Phys. Proc. Suppl. 161 (2006)

Eichmann, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010)
Eichmann, PRD 84 (2011)
Sanchis-Alepuz ,Eichmann, Villalba-Chavez, Alkofer, PRD (2012)

Nucleon EM form factors



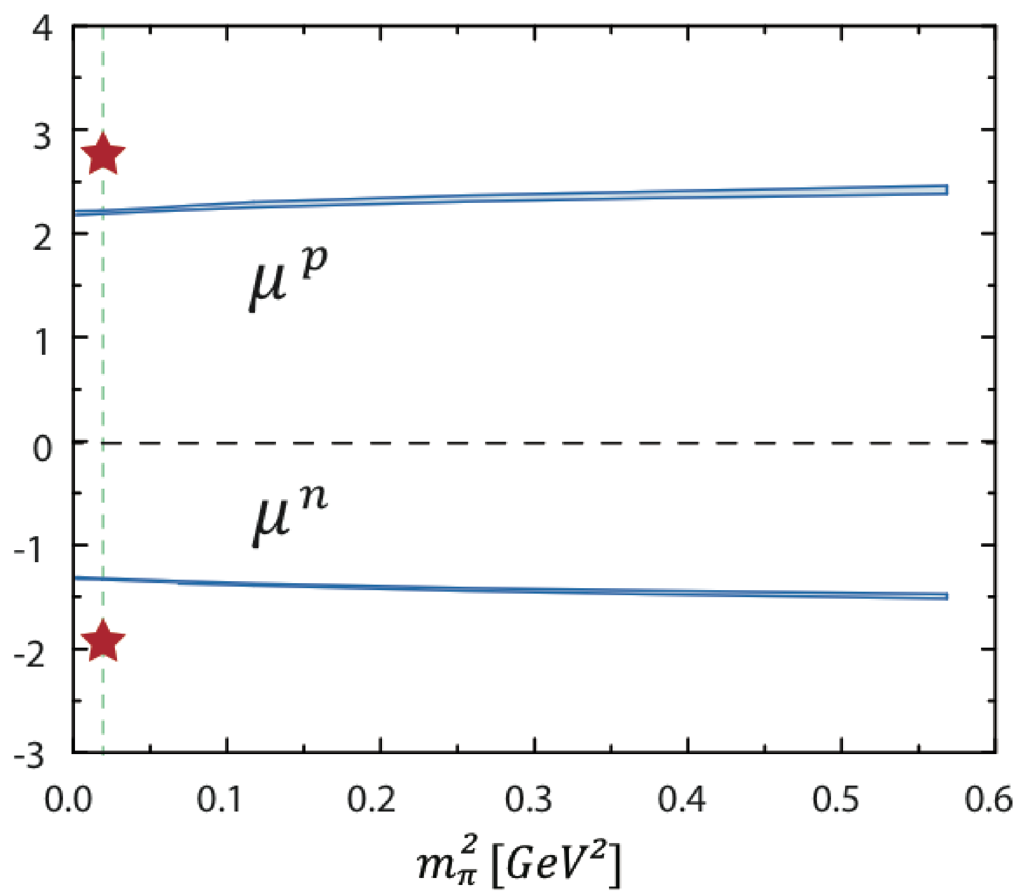
- missing pion cloud effects
- similar for axial form factors

Eichmann, PRD 84 (2011)

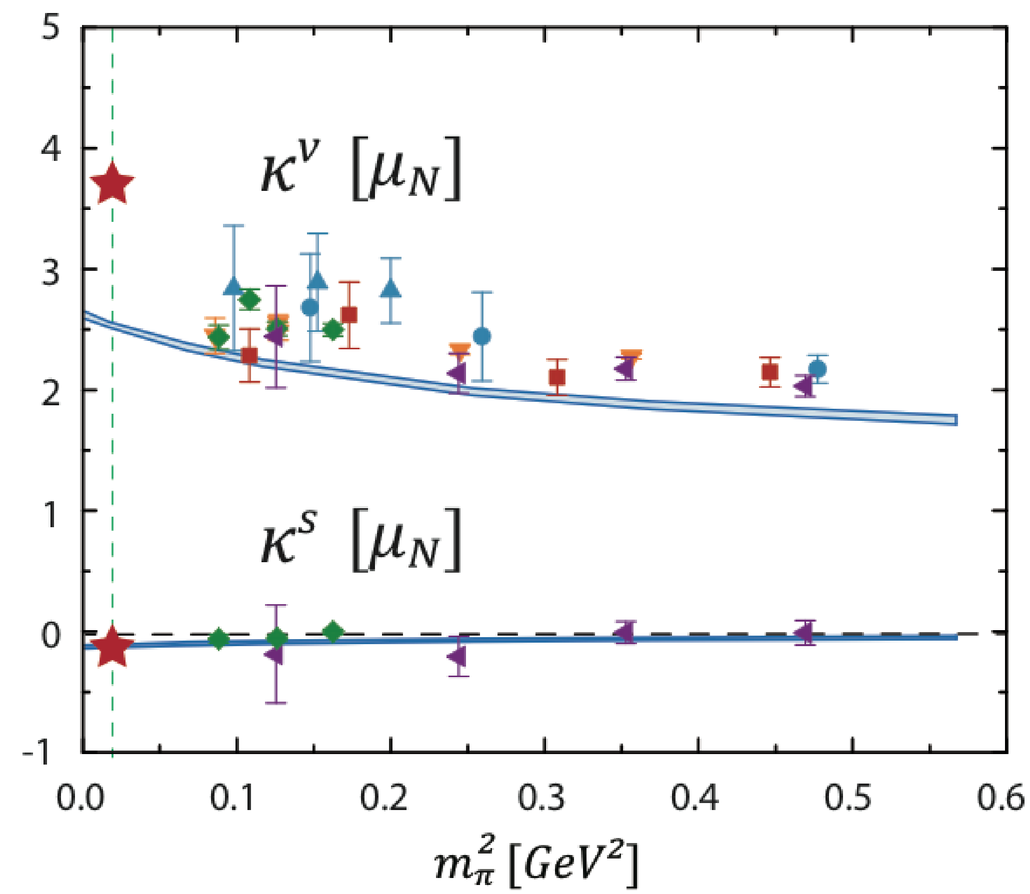
Eichmann and CF, Eur. Phys. J. A48 (2012) 9

Magnetic moments

Magnetic moments (p, n):



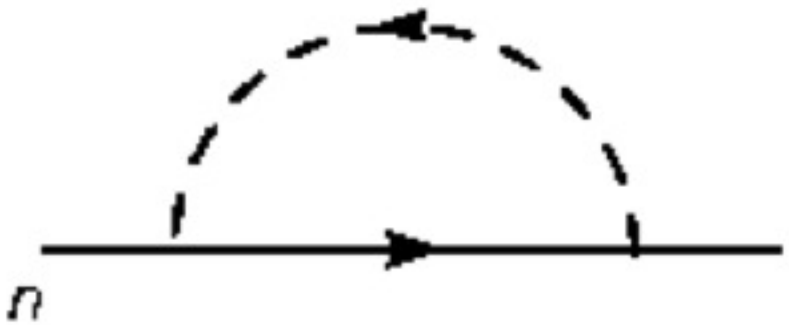
Isovector (p-n), isoscalar (p+n):

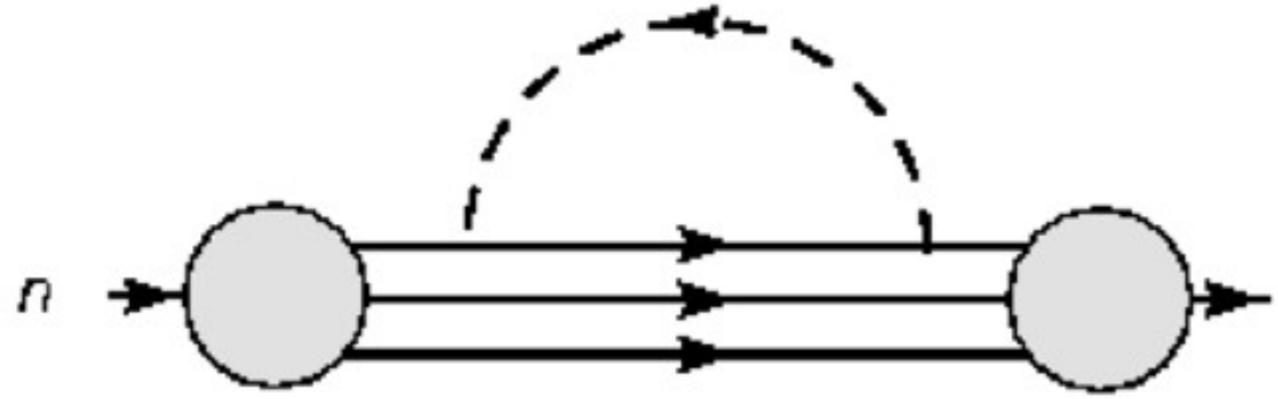


- missing pion cloud effects in isovector moment Kv
- no pion cloud effects in isoscalar moment Ks

Eichmann, PRD 84 (2011)

Strategy I: Including the pion cloud





- Hadron level: πN-contributions to nucleon self-energy
- Quark-level: π-contributions to quark self-energy

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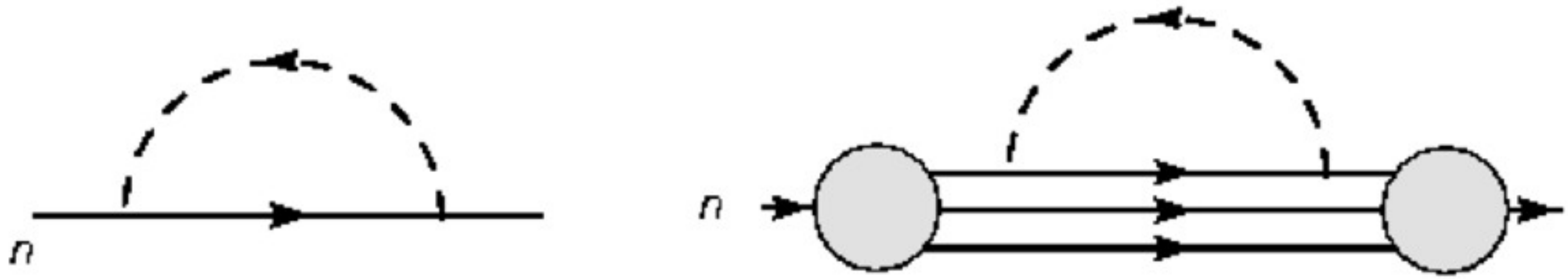

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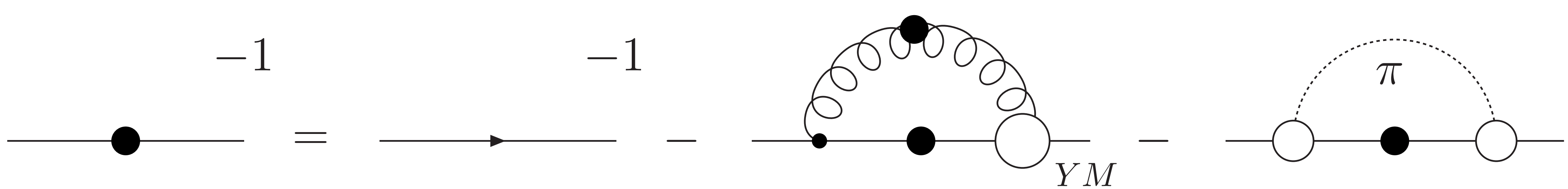
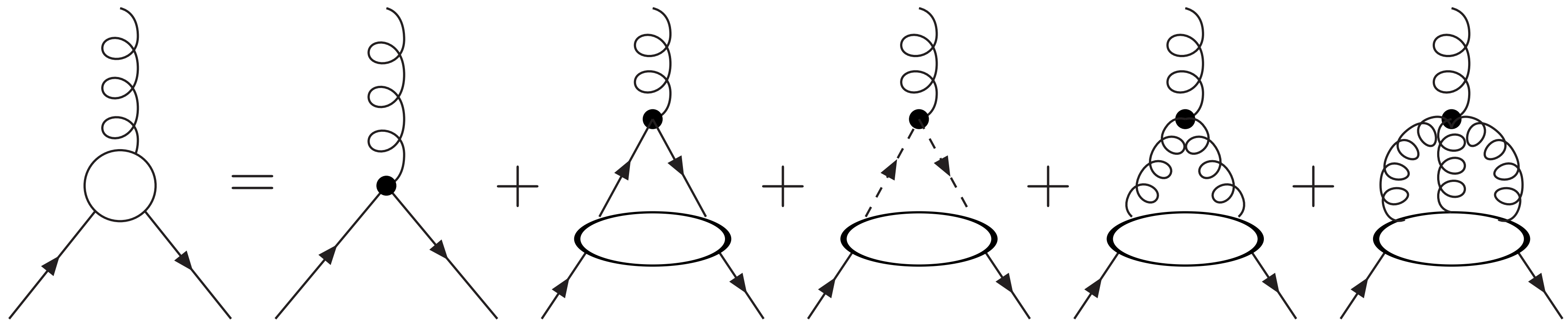
- Quark-level: π-contributions to quark self-energy



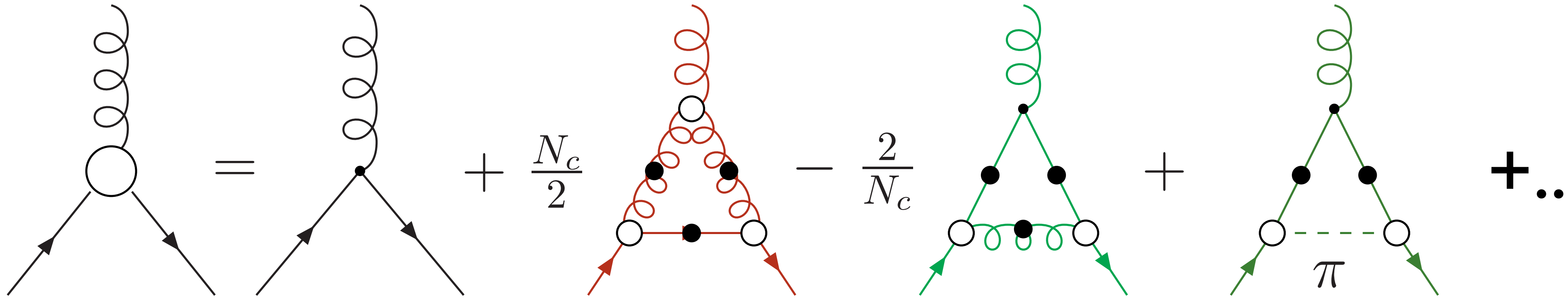
quark-gluon vertex:



CF, Nickel and Wambach, PRD 76 (2007) 094009

quark:

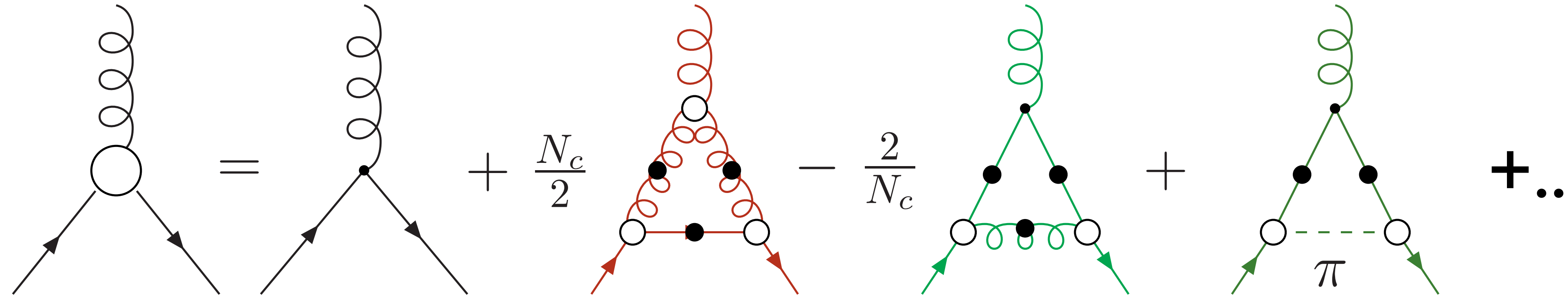
quark-gluon vertex:

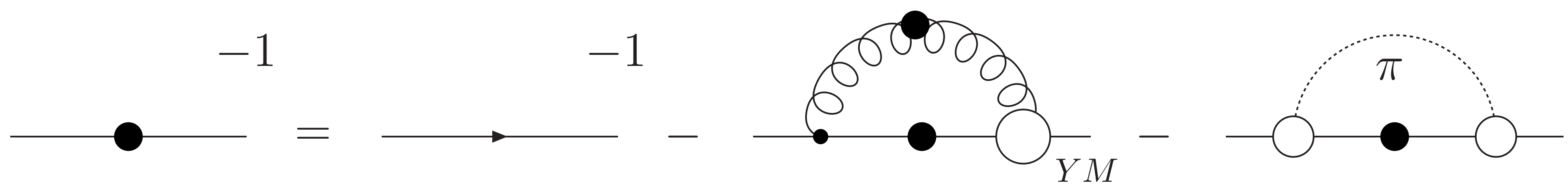


CF, Nickel and Wambach, PRD 76 (2007) 094009

quark:

quark-gluon vertex:



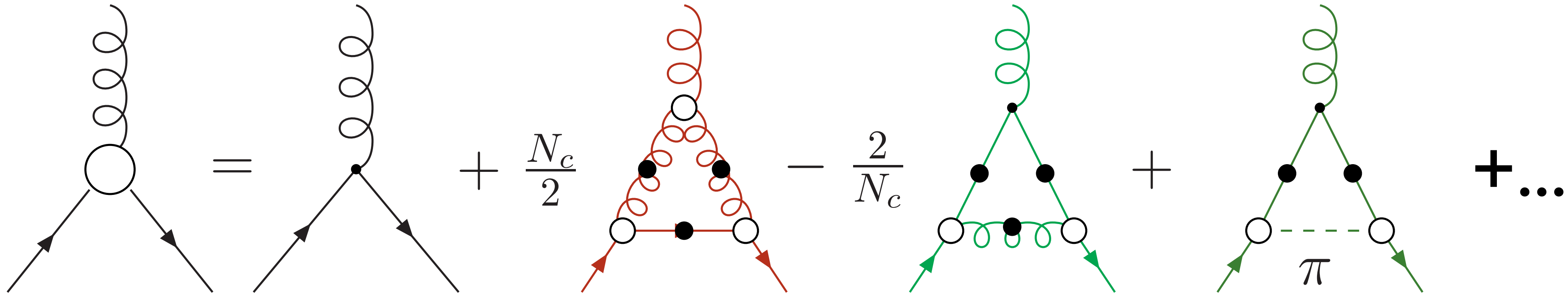


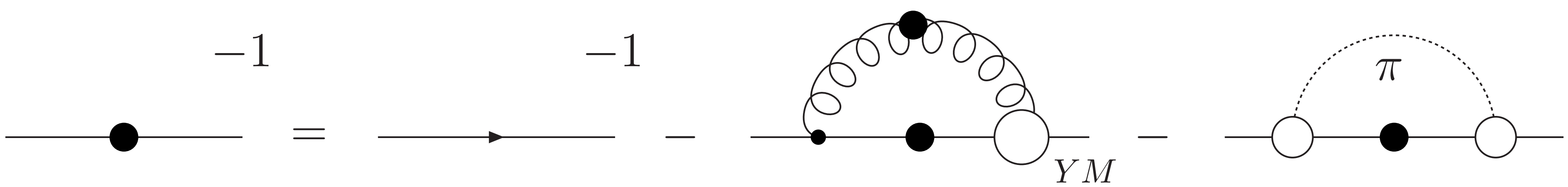
CF, Nickel and Wambach, PRD 76 (2007) 094009

quark:

Strategy I: Pion effects in quark-gluon interaction

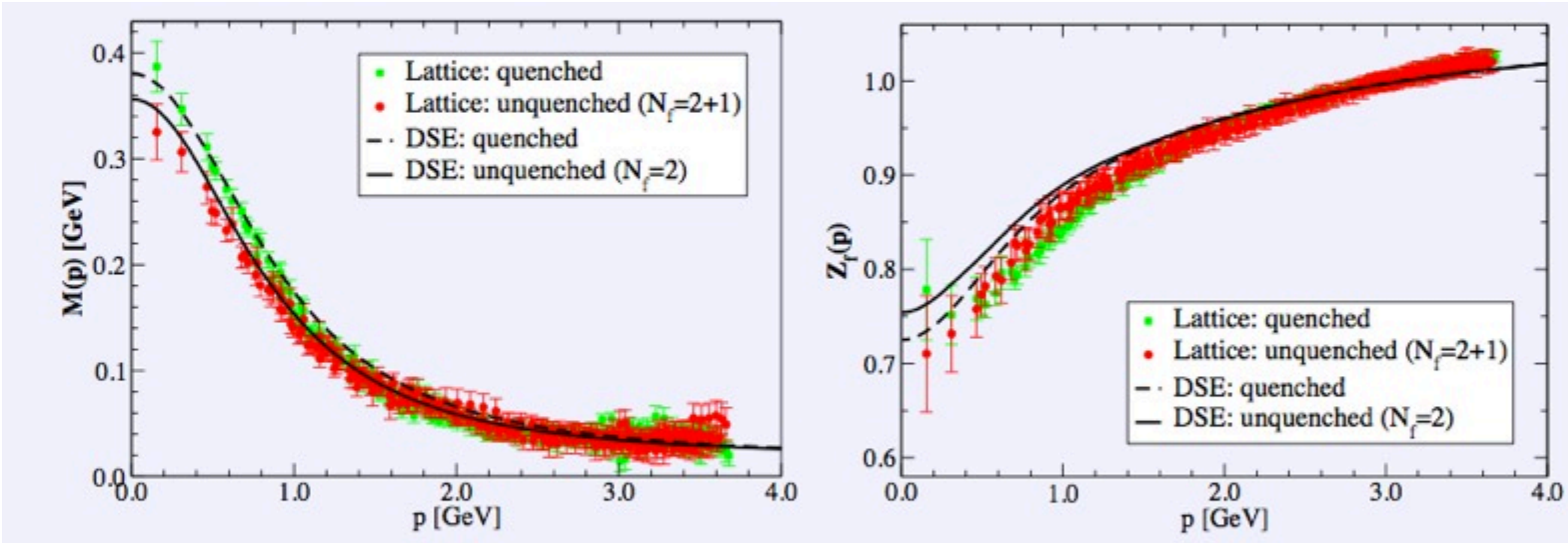
quark-gluon vertex:





CF, Nickel and Wambach, PRD 76 (2007) 094009

quark:



CF, D. Nickel and R. Williams, EPJC **60**, 1434 (2008)

Unquenching effects: Light mesons

	RL	3g	3g+π	Experiment
M_π	138	138	138	138
f_π	94	111	105	93
M_ρ	758	881	805	776
f_ρ	154	176	168	162
M_σ	645	884	820	450
M_{a_1}	926	1055	1040	1230
M_{b_1}	912	972	940	1229

CF, Williams, PRL 103 (2009), PRD 78 (2008)

- Attractive effects of pion cloud
- Scalar too large or ... too low!

cp Parganlija, Kovacs, Wolf, Giacosa and Rischke, PRD 87 (2013) 014011
Heupel, Eichman, CF, PLB 718 (2012) 545-549

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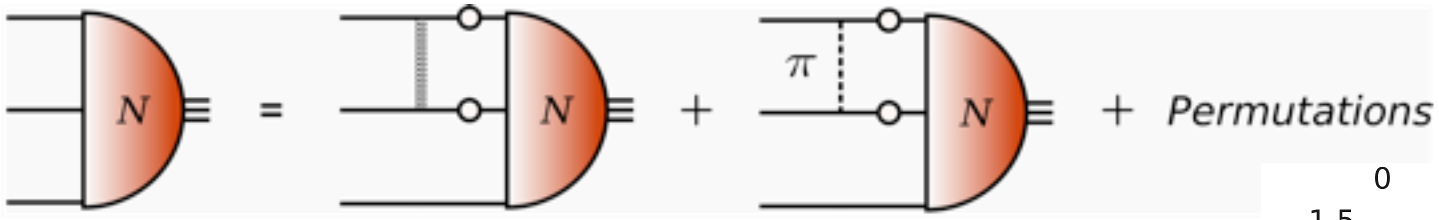
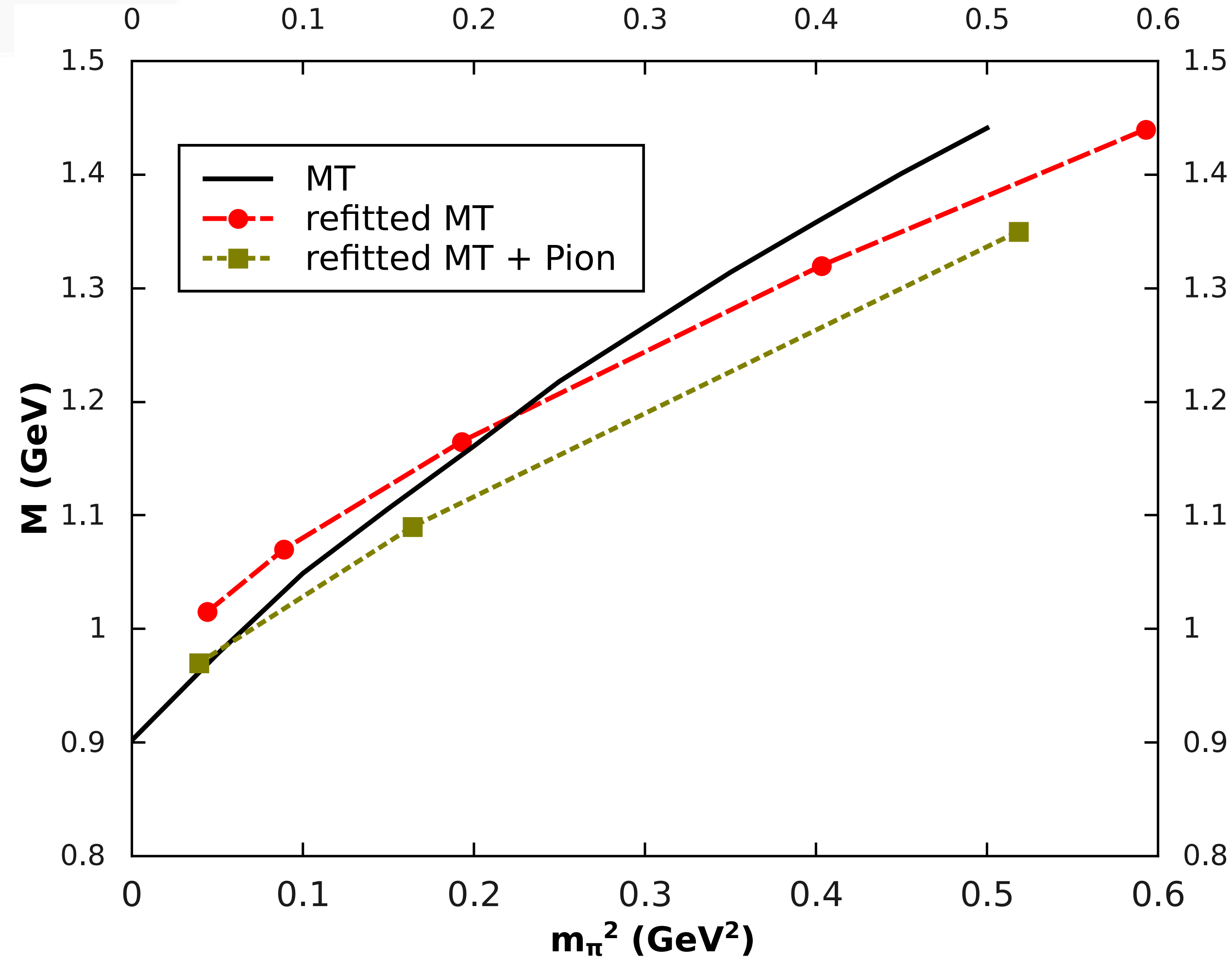
cp Parganlija, Kovacs, Wolf, Giacosa and Rischke, PRD 87 (2013) 014011
Heupel, Eichman, CF, PLB 718 (2012) 545-549

Pion cloud effects in baryons





Sanchis-Alepuz, Kubrak, CF, in prep.

First results:

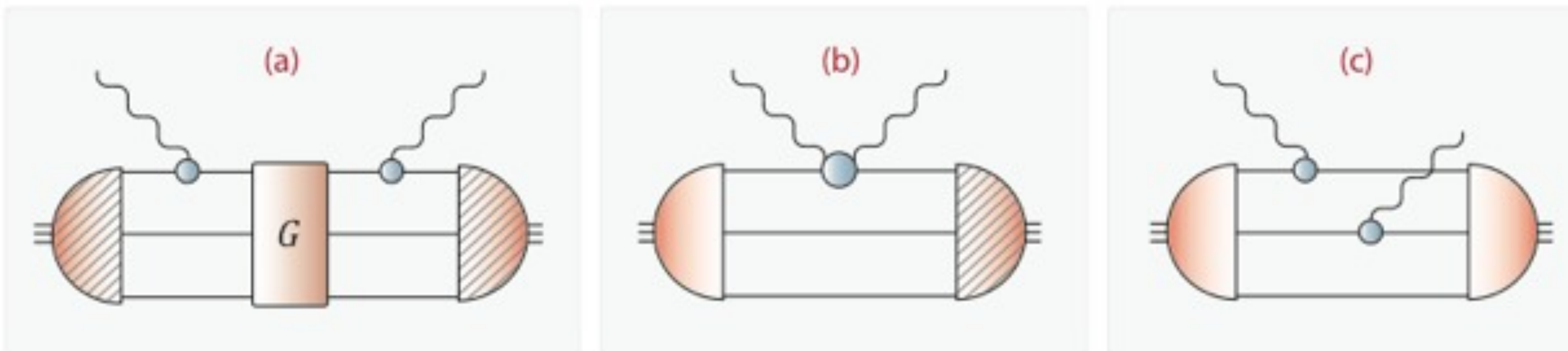
- Nucleon:
  1.05 → 0.94 GeV
- Roper:
  1.60 → 1.35 GeV

Next steps:

- systematics + precision — in progress
- EM form factors w. pion cloud
  - Gauge invariant formalism — ✔
  - Test calculation: pion — in progress
  - Extend calculation to nucleon

Nonperturbative description of hadron-photon and hadron-meson scattering on quark-gluon basis

Eichmann, CF, PRD 85 034015 (2012)



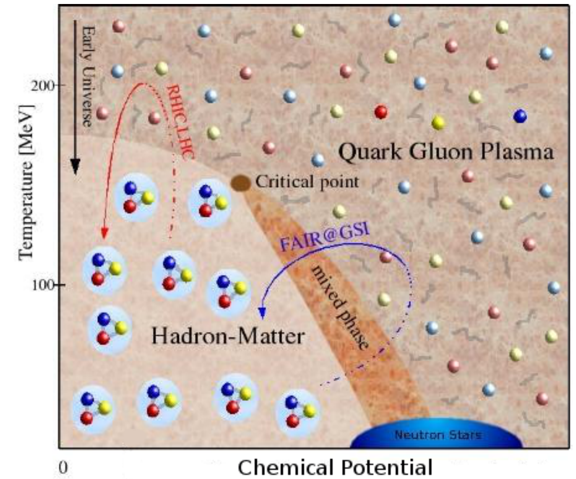
Technical/conceptual progress:

- Derive fermion-two-photon vertex
  - consistent with gauge invariance
  - free of kinematic singularities
  - transverse part: on-shell nucleon Compton amplitude
- Reproduce $\pi\gamma\gamma$ transition form factor on t-channel pole

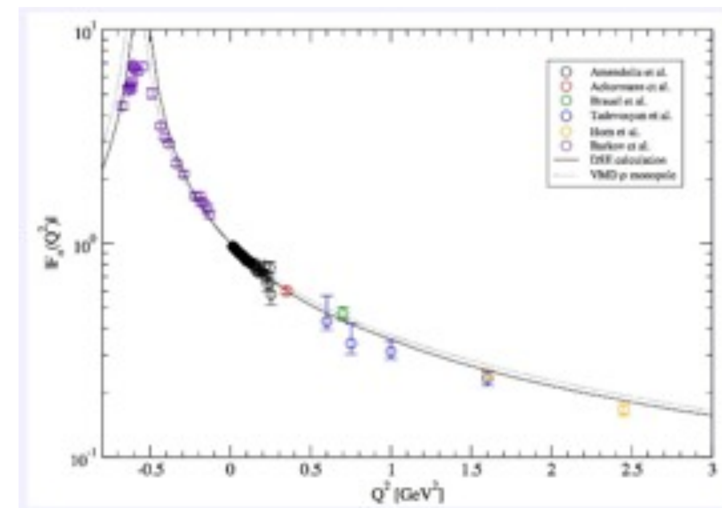
Eichmann and CF, PRD 87 (2013) 036006

Next steps:

- Two-photon contributions to EM form factor
- Polarisabilities

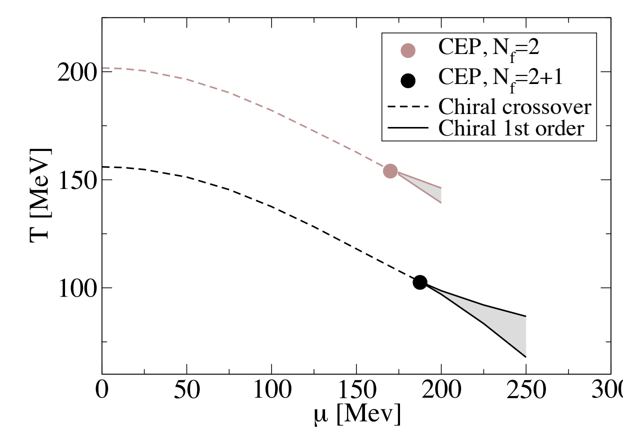


1. Introduction: quarks and gluons

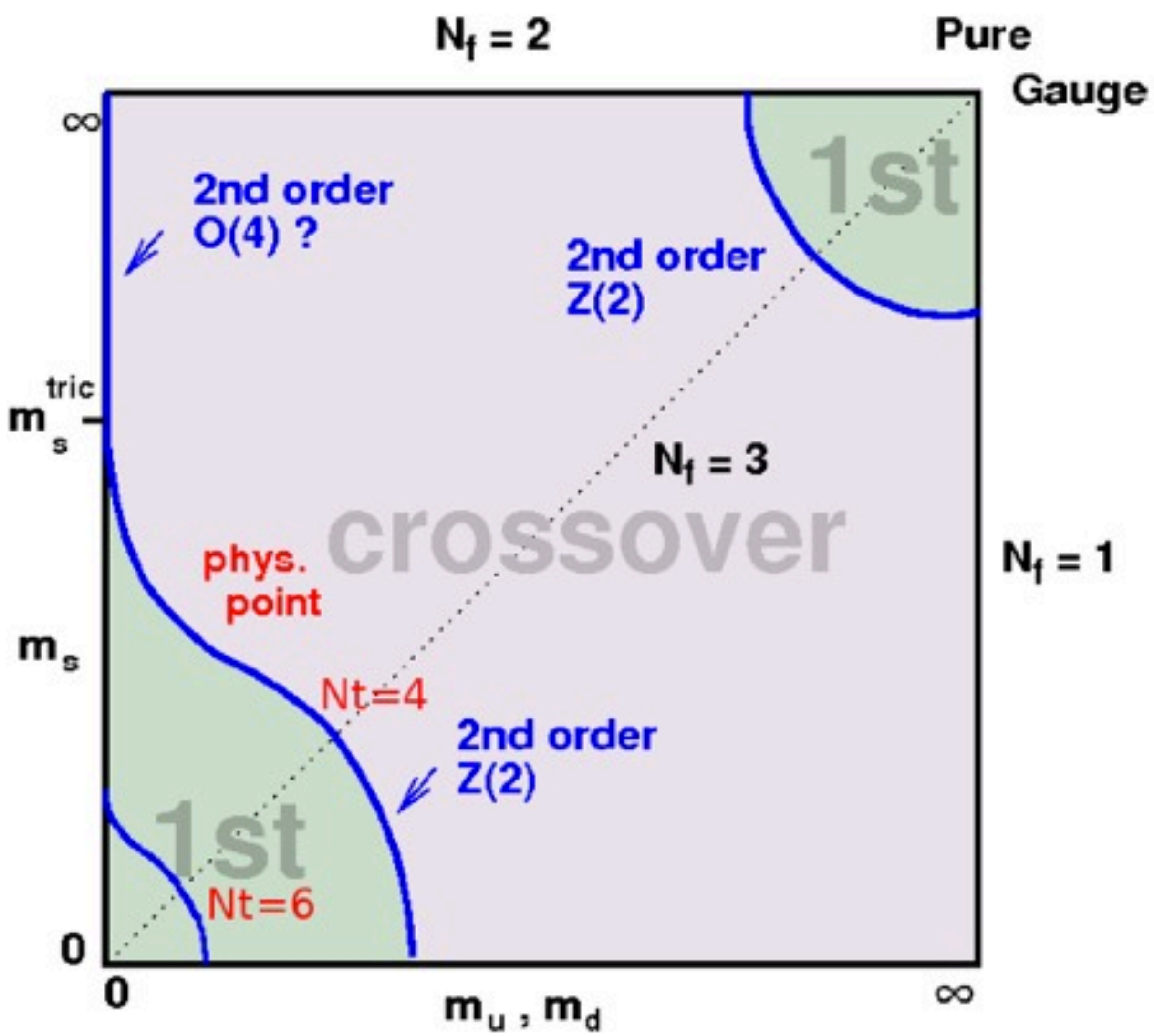


2. Electromagnetic properties of mesons and baryons

3. Gluons, quarks and the QCD phase diagram

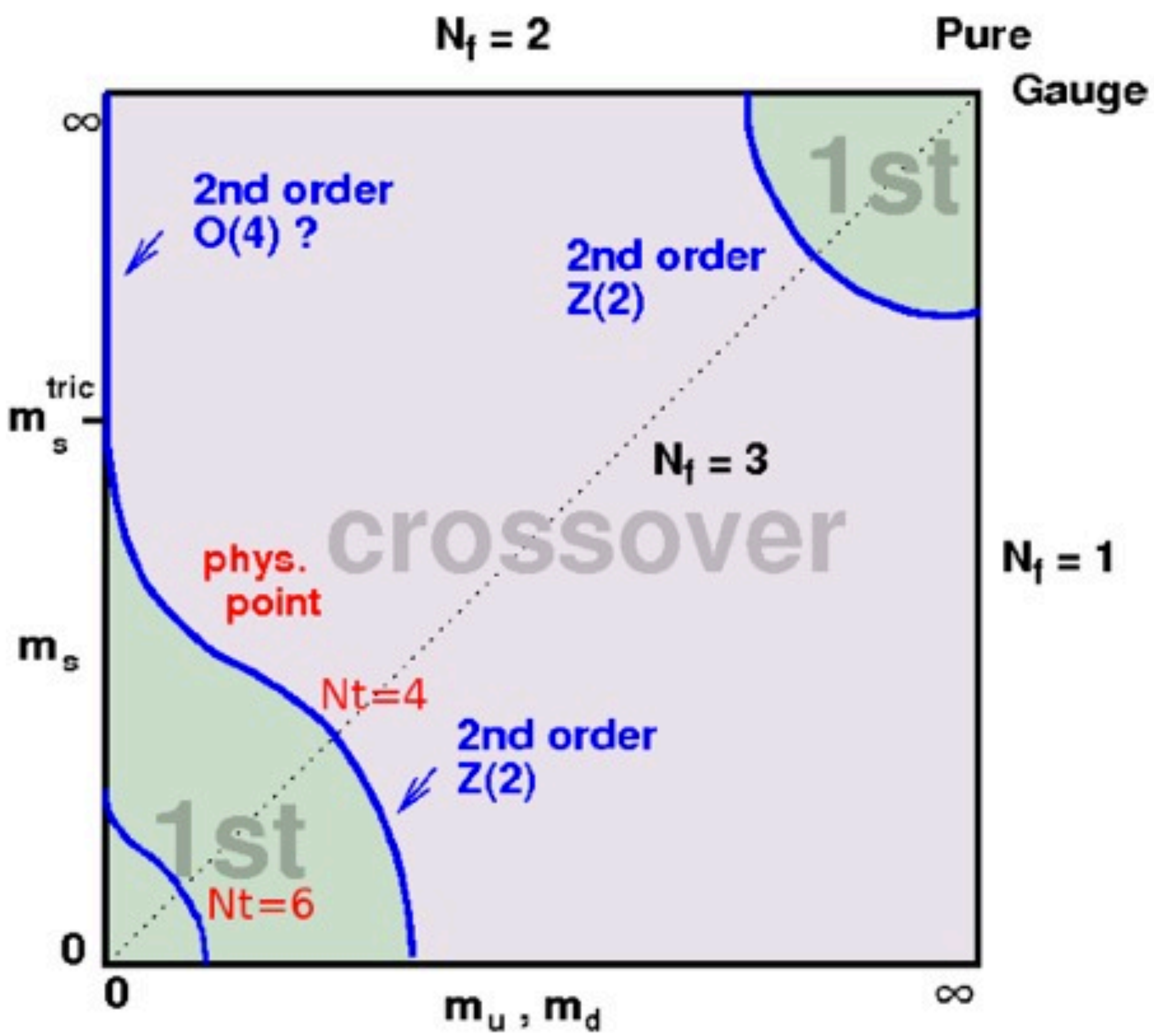


QCD phase transitions



plot by O. Philipsen

QCD phase transitions



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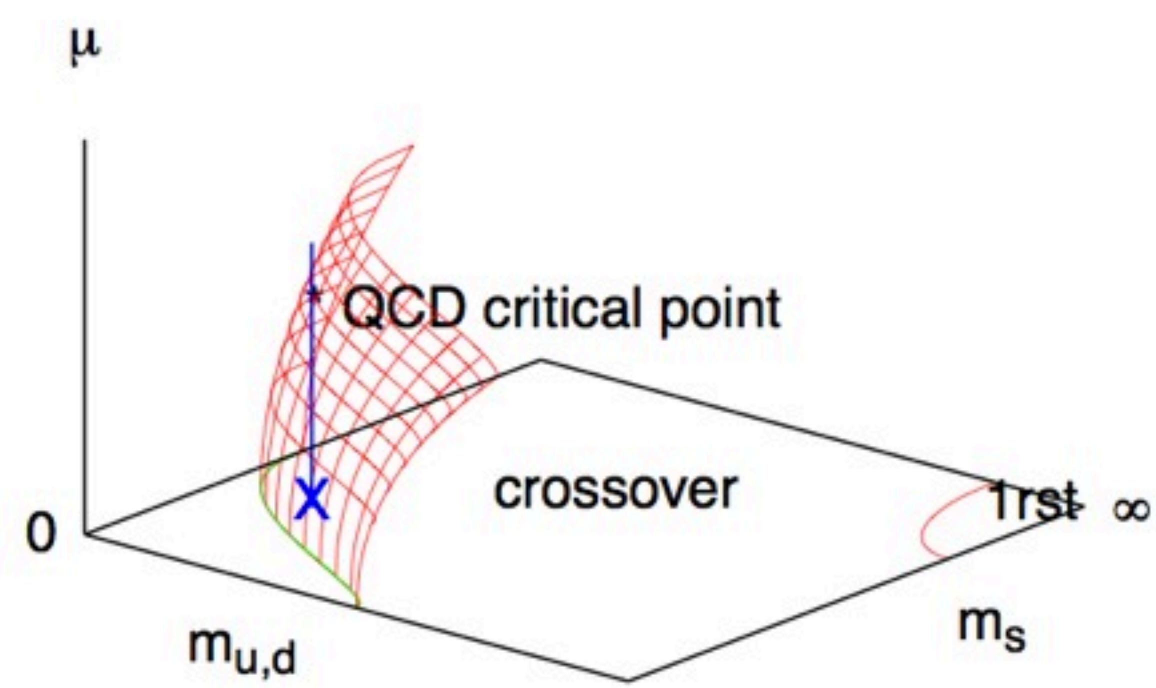

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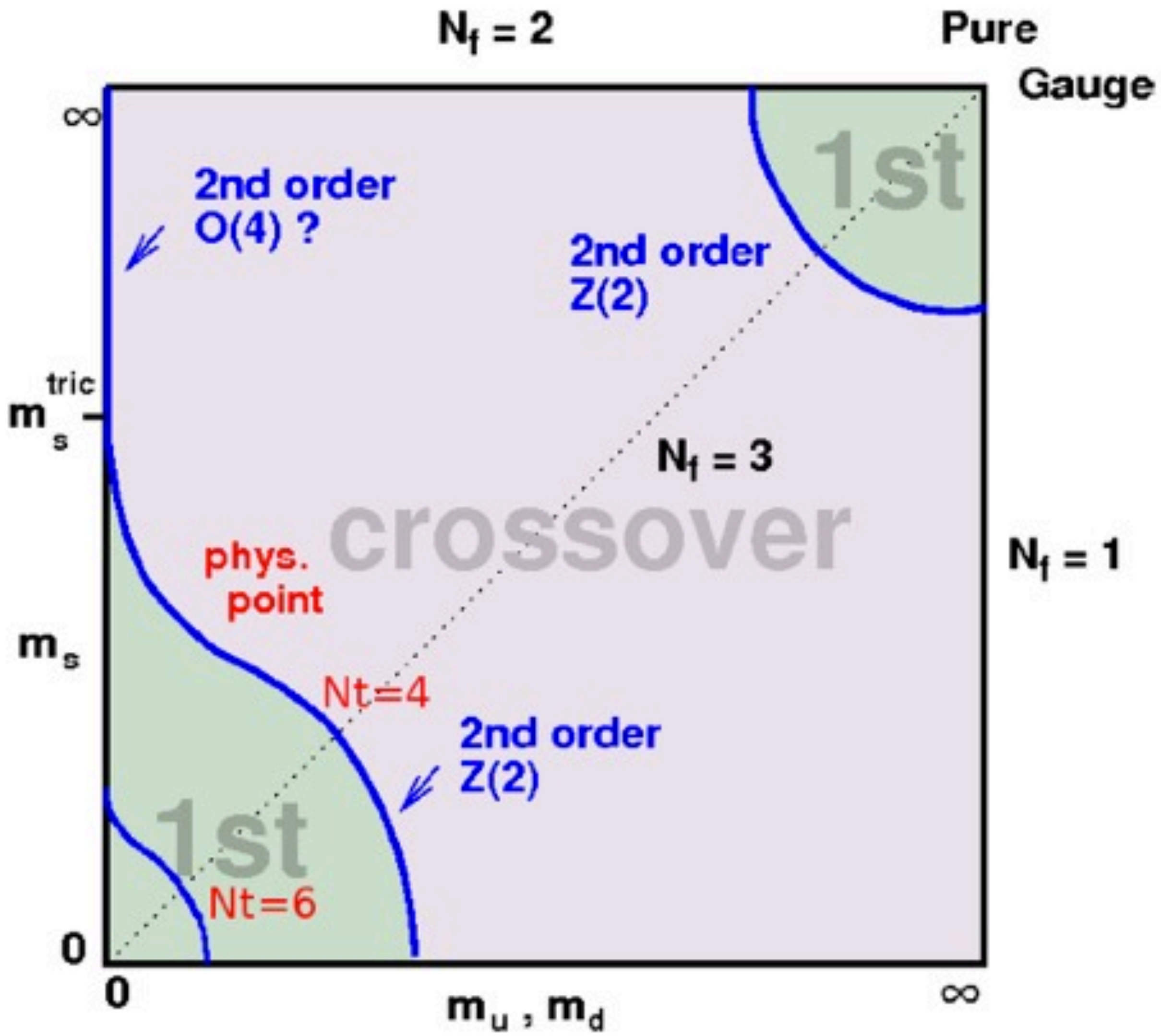

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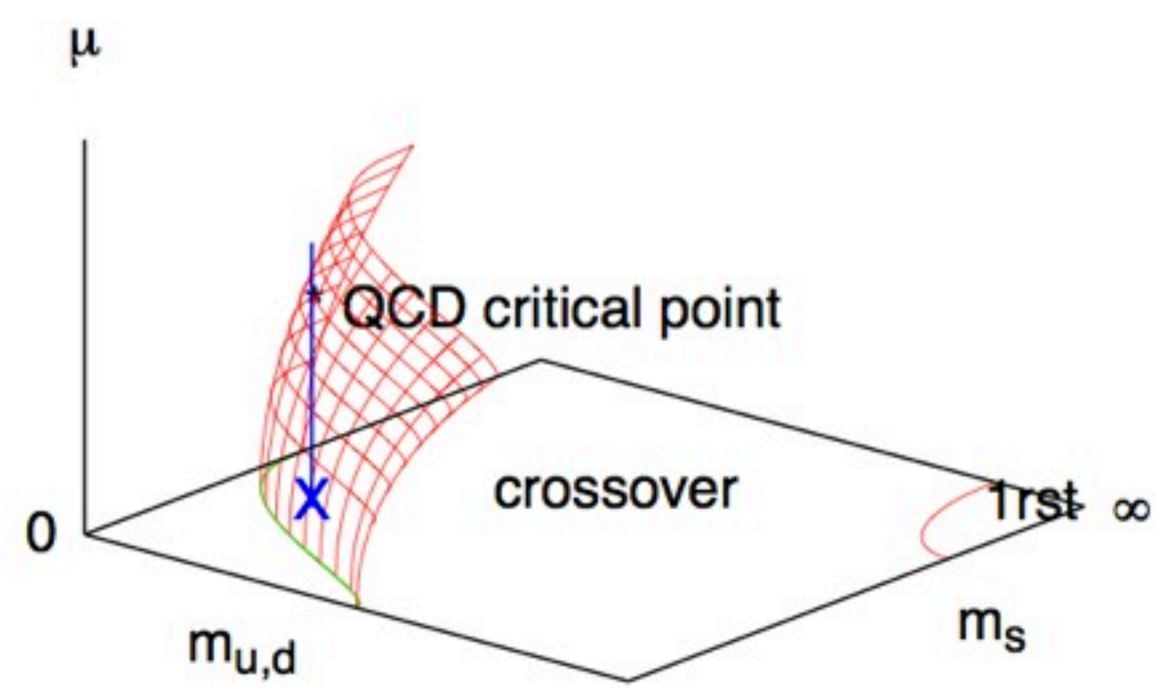
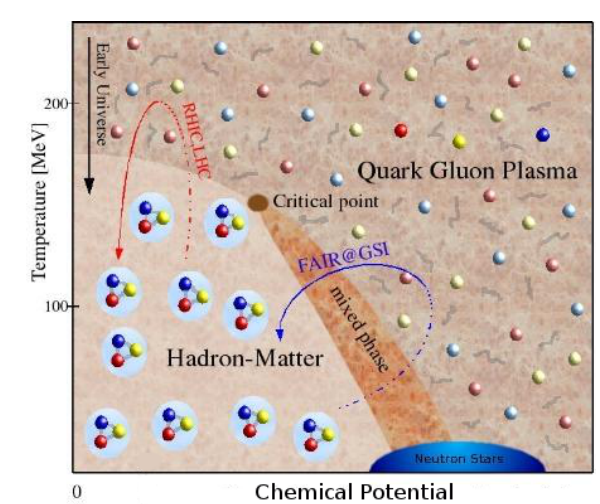


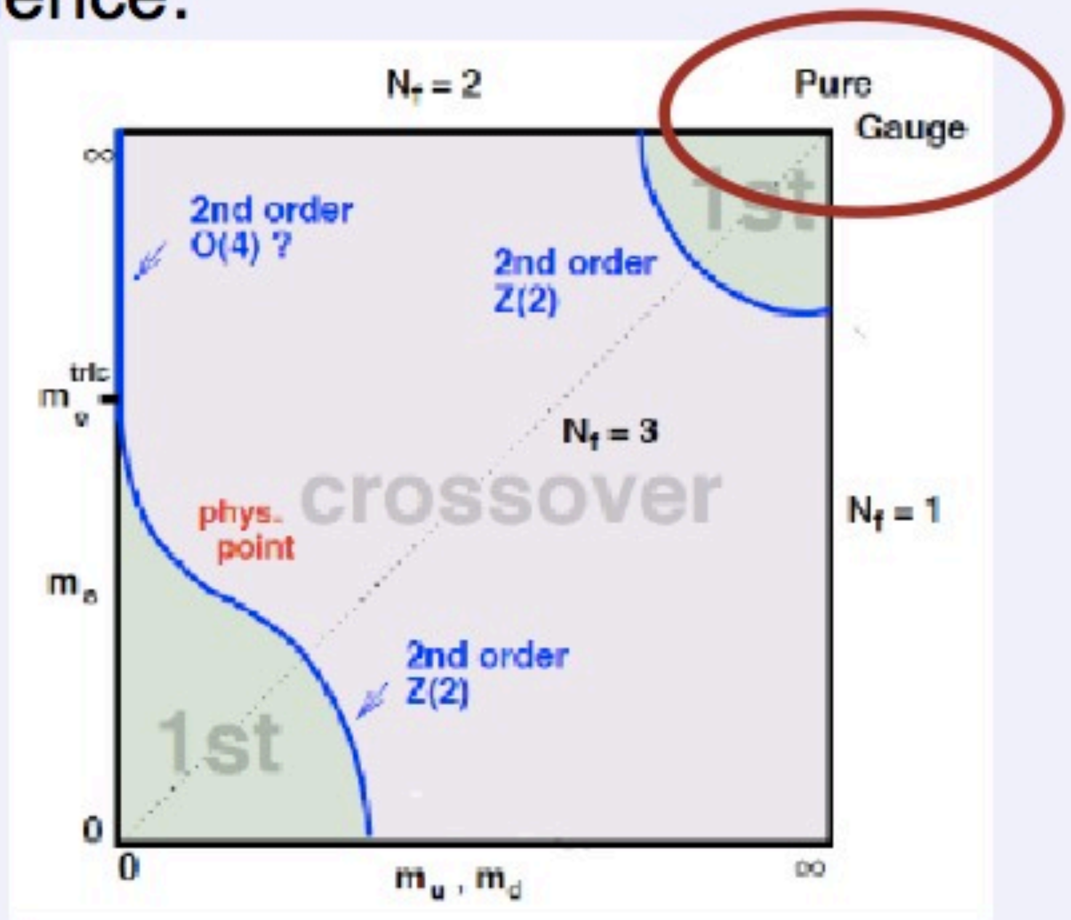
Is this happening ??



Wednesday, October 9, 2013

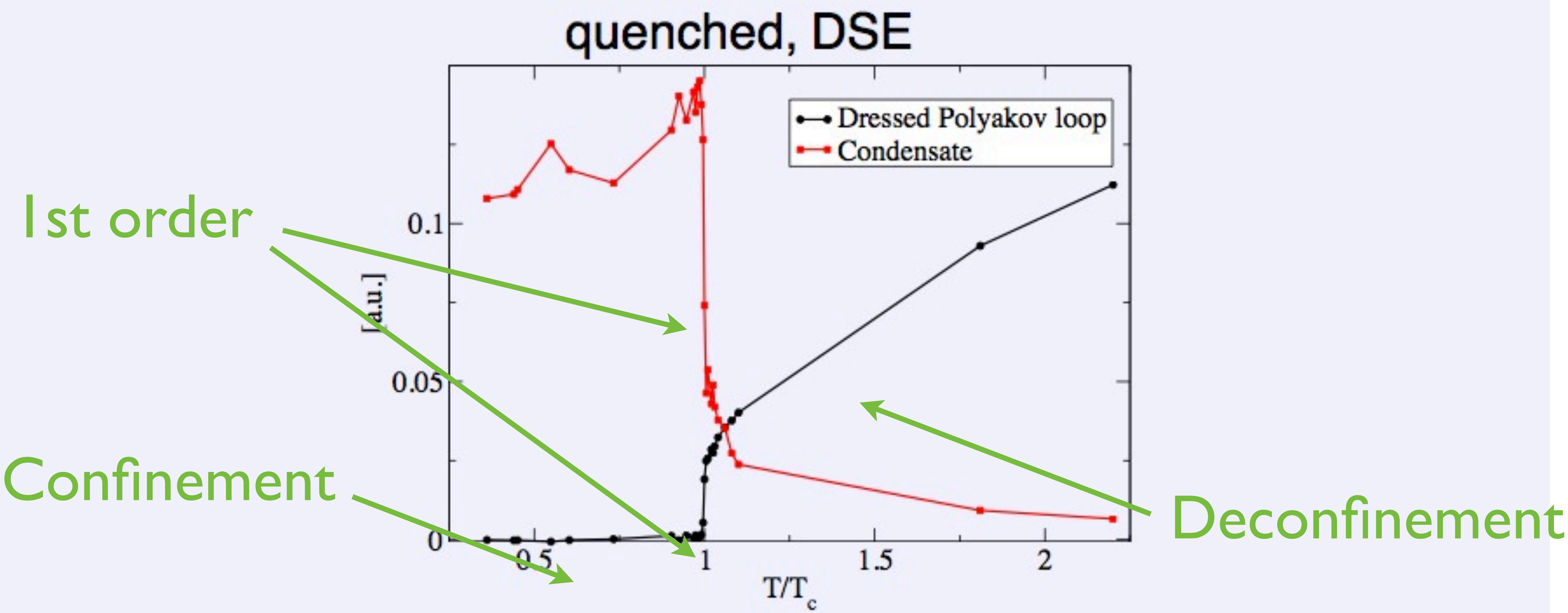
QCD phase transition: heavy quark limit/quenched

Quark mass dependence:



- Expect: Transitions controlled by deconfinement
- SU(2) second order, SU(3) first order

Transition temperatures, quenched



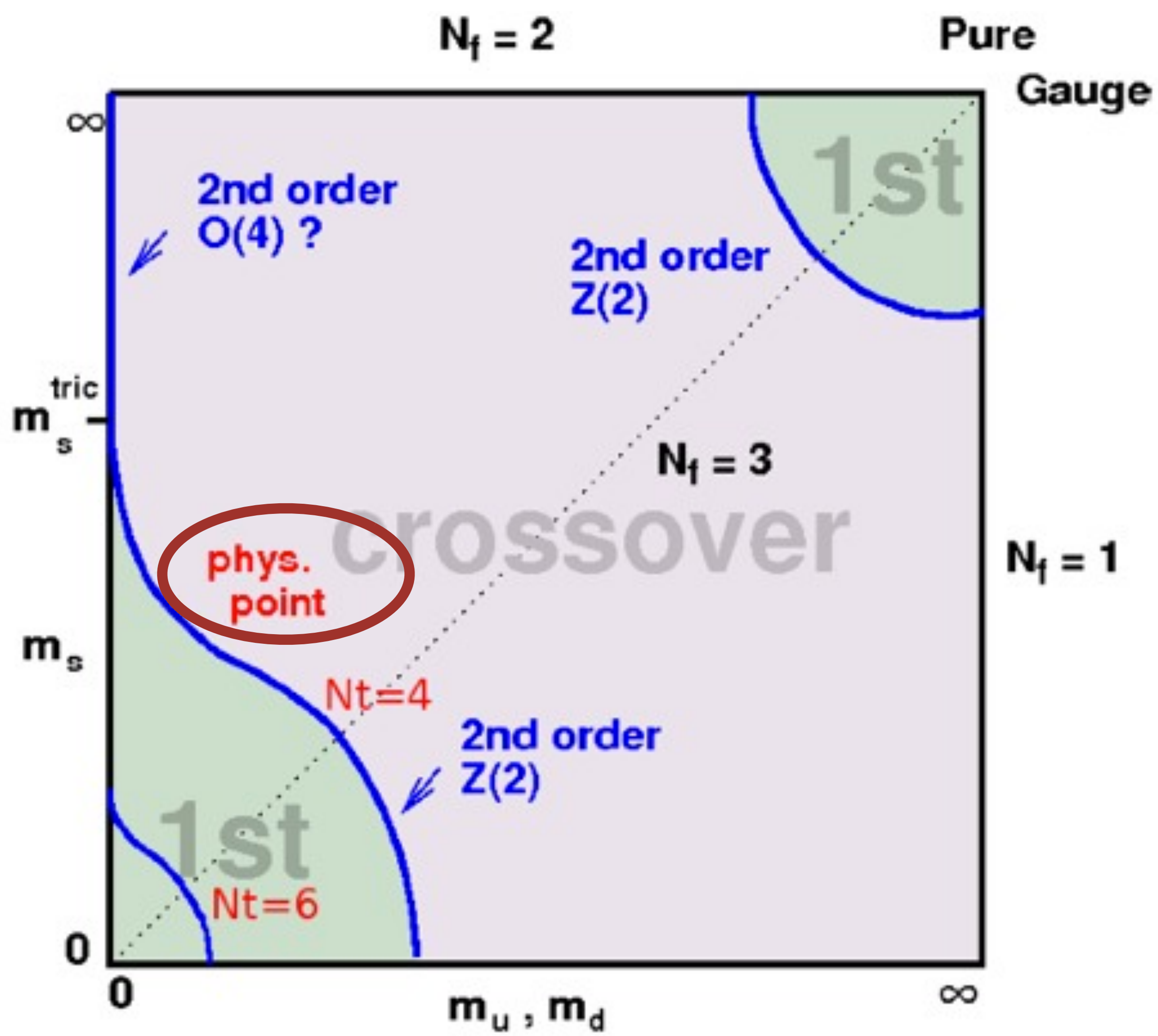
Luecker, C.F., arXiv:1111.0180; C.F., Maas, Mueller, EPJC 68 (2010).

- SU(2): $T_c \approx 305$ MeV
  SU(3): $T_c \approx 270$ MeV
- $T \leq T_c$: increasing condensate due to electric part of gluon

cf. Buividovich, Luschevskaya, Polikarpov, PRD 78 (2008) 074505.
cf. Braun, Gies, Pawlowski, PLB 684 (2010) 262-267.

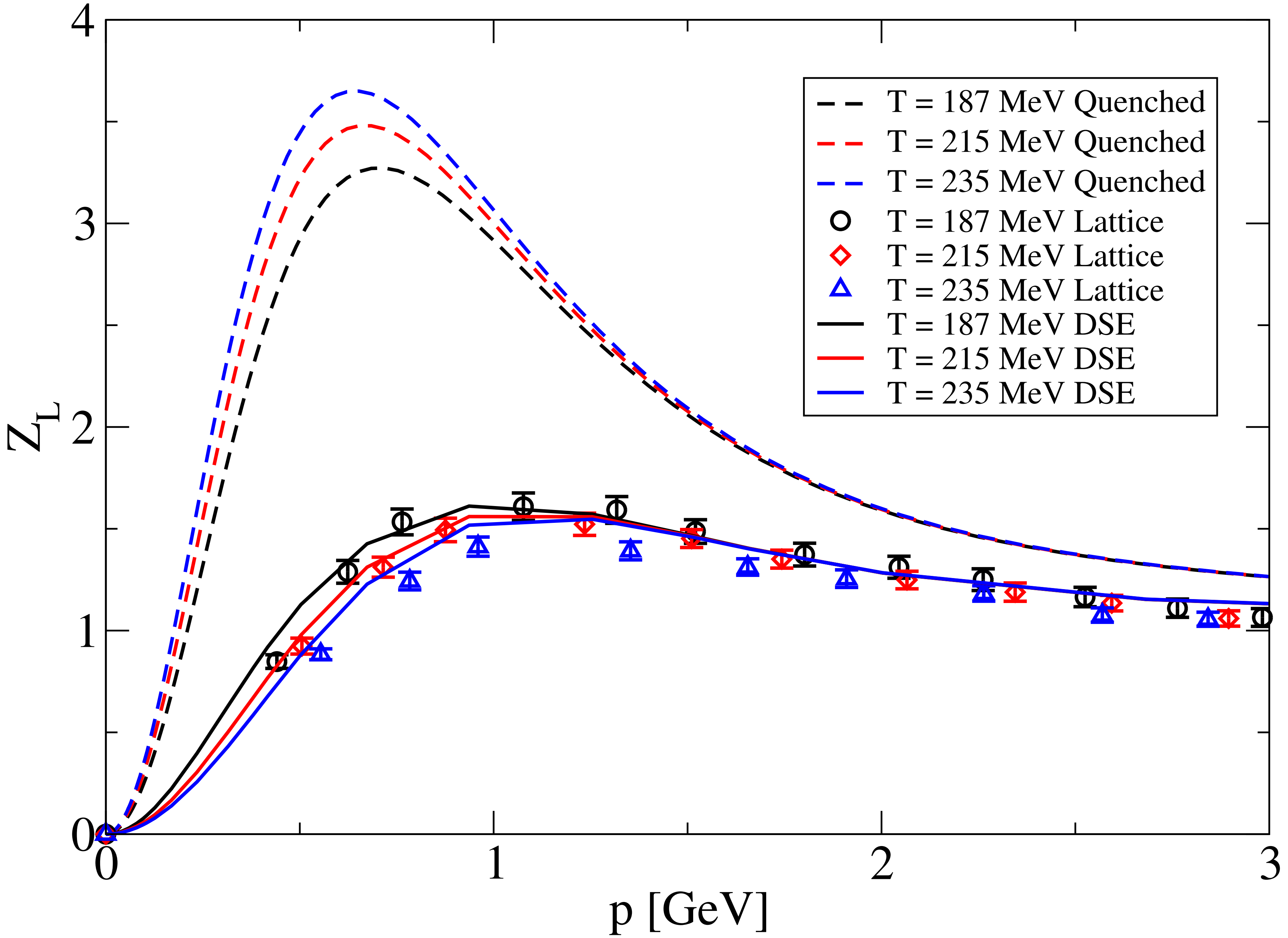
- quark spectral functions   Mueller, CF, Nickel, EPJC 70 (2010) 1037-1049

QCD phase transitions: N_f=2+1



- Physical up/down and strange quark masses
- Transition controlled by chiral dynamics
- at μ=0: compare to available lattice results

Unquenched Gluon DSE vs Lattice

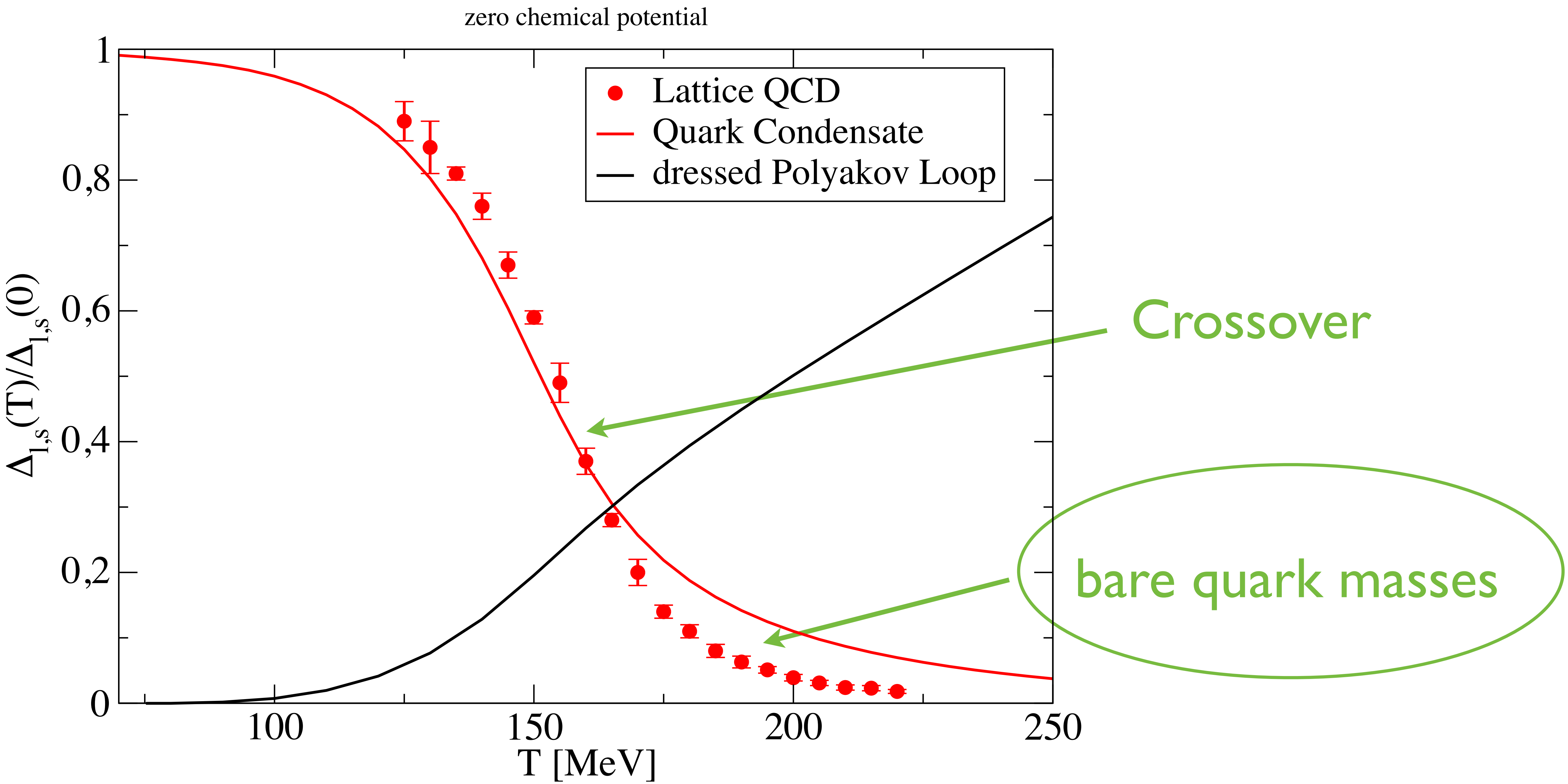


- quantitative agreement: DSE prediction verified by lattice

DSE: CF, Luecker, PLB 718 (2013) 1036 [arXiv:1206.5191]

Lattice: Aouane, Burger, Ilgenfritz, Muller-Preussker and Sternbeck, arXiv:1212.1102

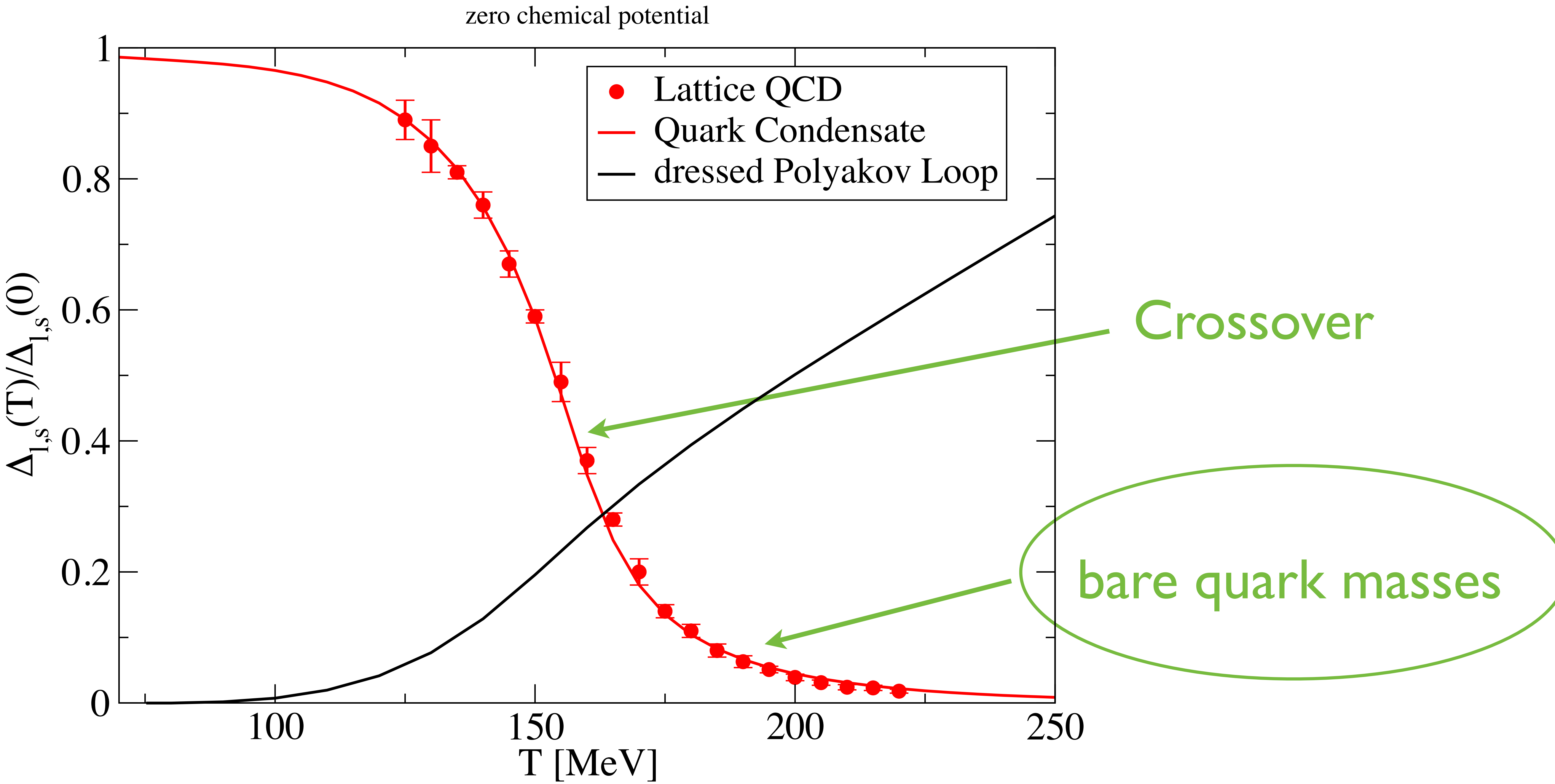
N_f=2+1, zero chemical potential



Lattice: Borsanyi *et al.* [Wuppertal-Budapest Collaboration], JHEP 1009(2010) 073
DSE: CF, Luecker, PLB 718 (2013) 1036, CF, Luecker, Welzbacher, in prep.

- quantitative agreement

N_f=2+1, zero chemical potential



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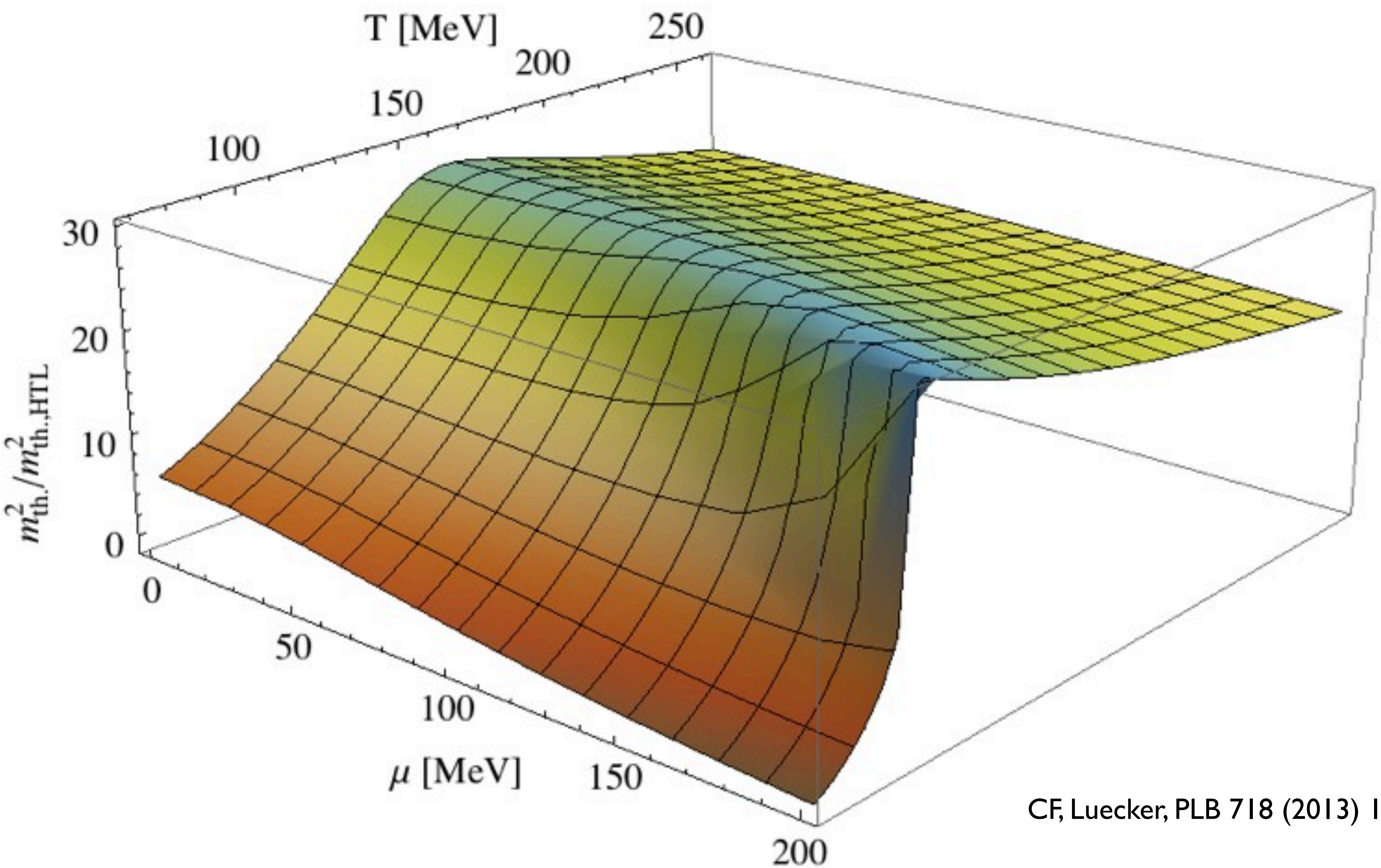
- quantitative agreement

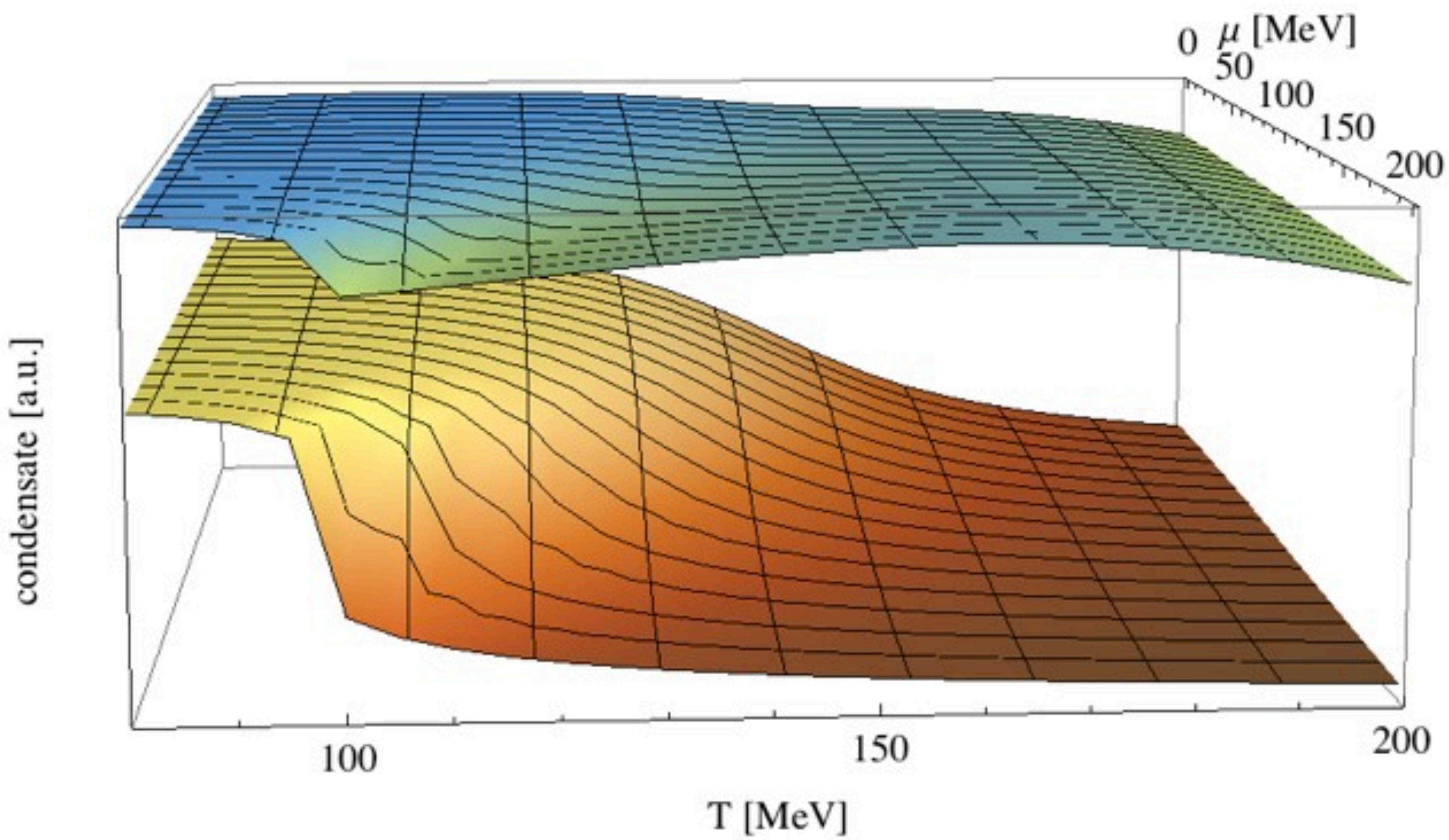
Nf=2+1: thermal electric gluon mass



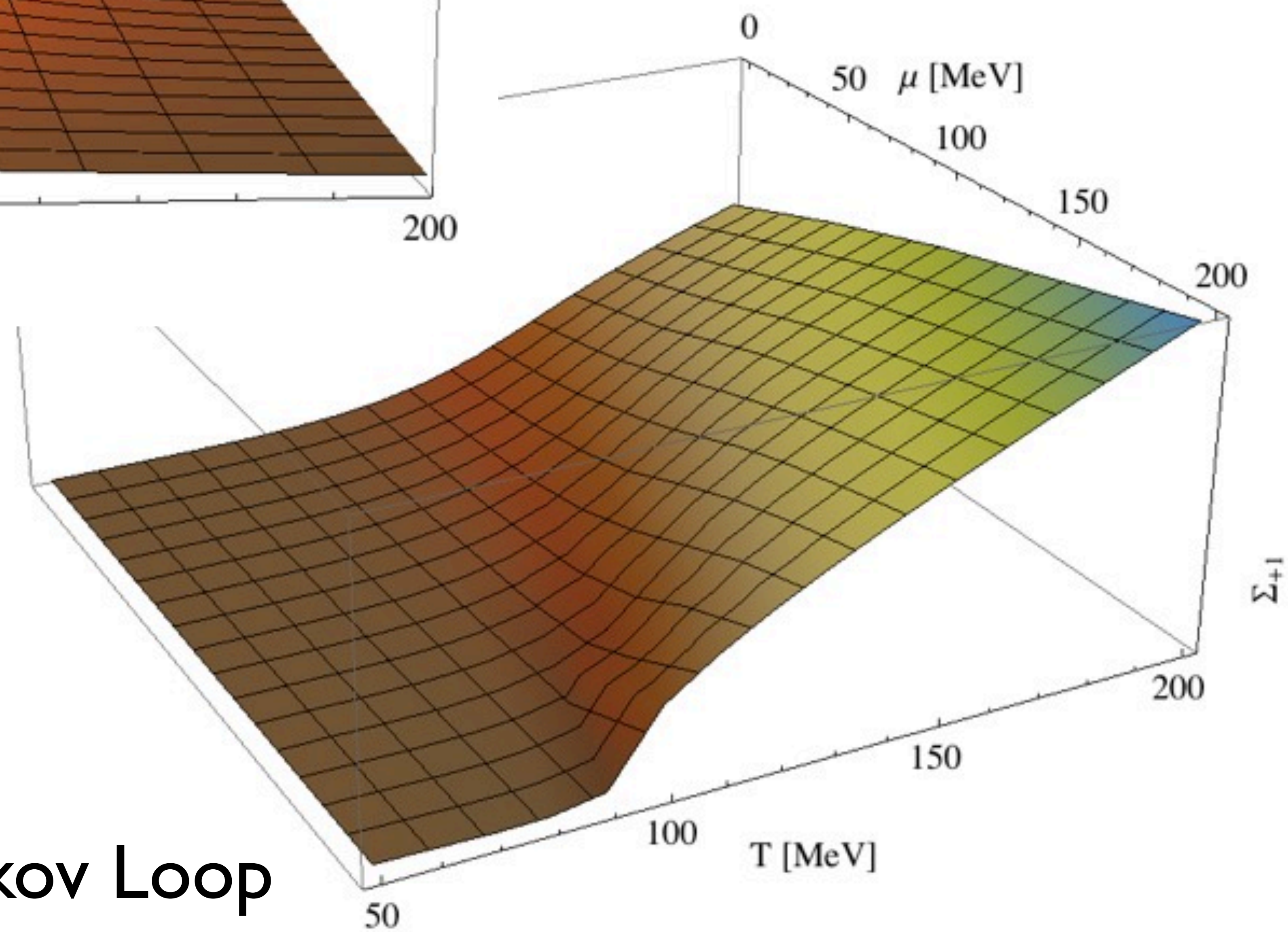
CF, Luecker, PLB 718 (2013) 1036

- large temperatures: behavior as expected from HTL
- first order transition at large chemical potential

Nf=2+1: Condensate and dressed Polyakov Loop

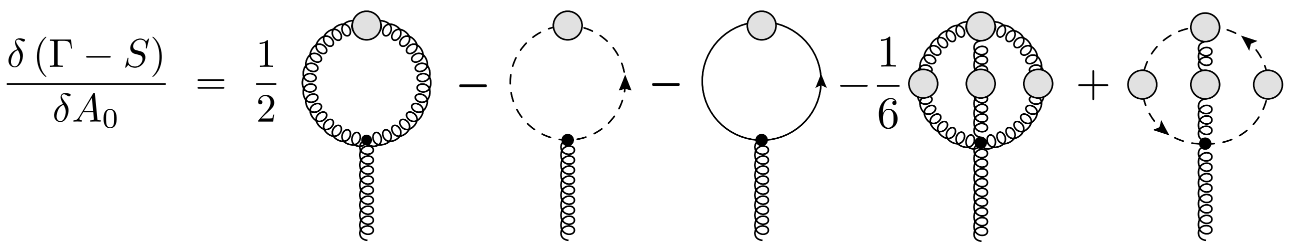


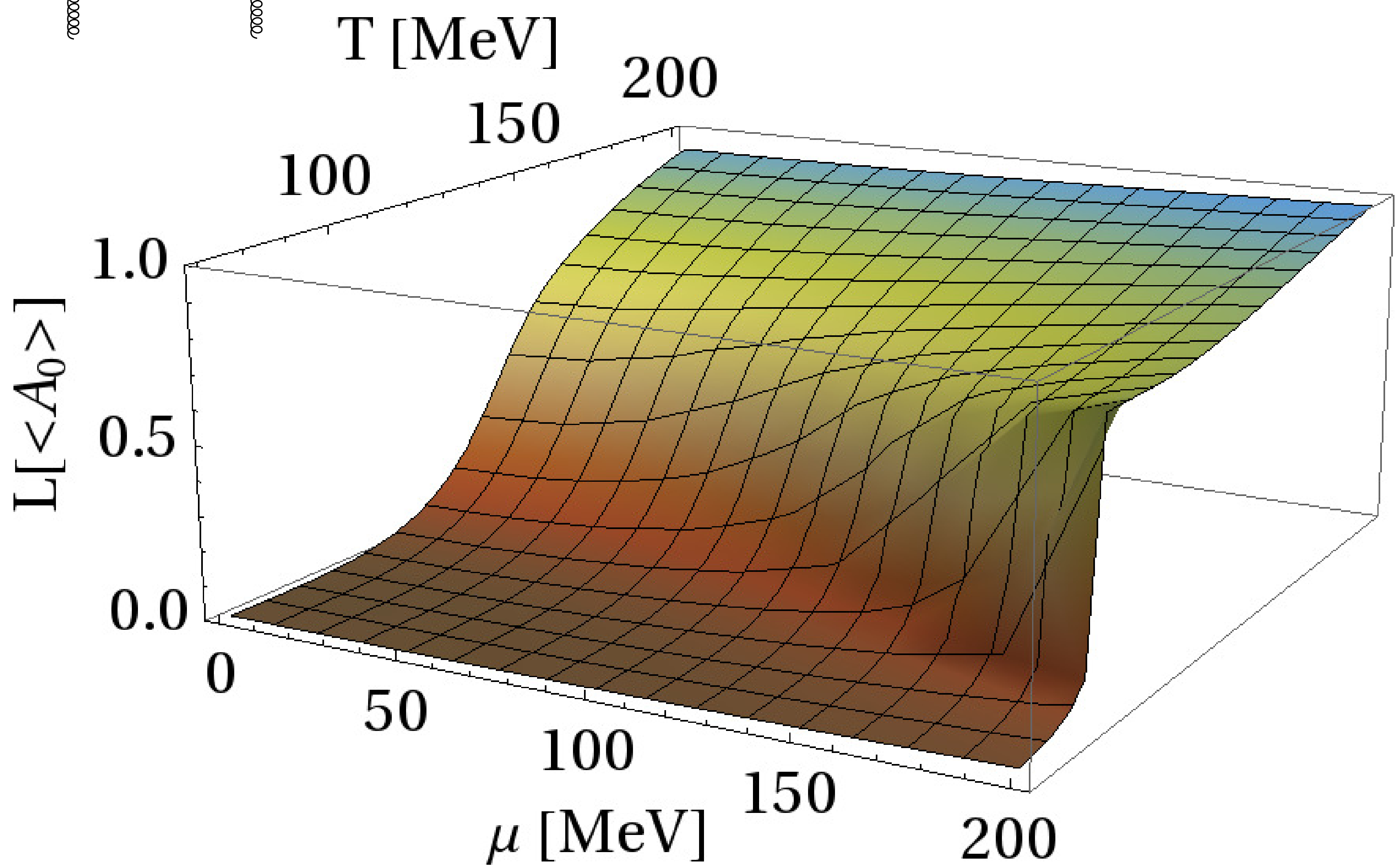
Quark condensate



Dressed Polyakov Loop

N_f=2+1: Polyakov loop potential at finite μ





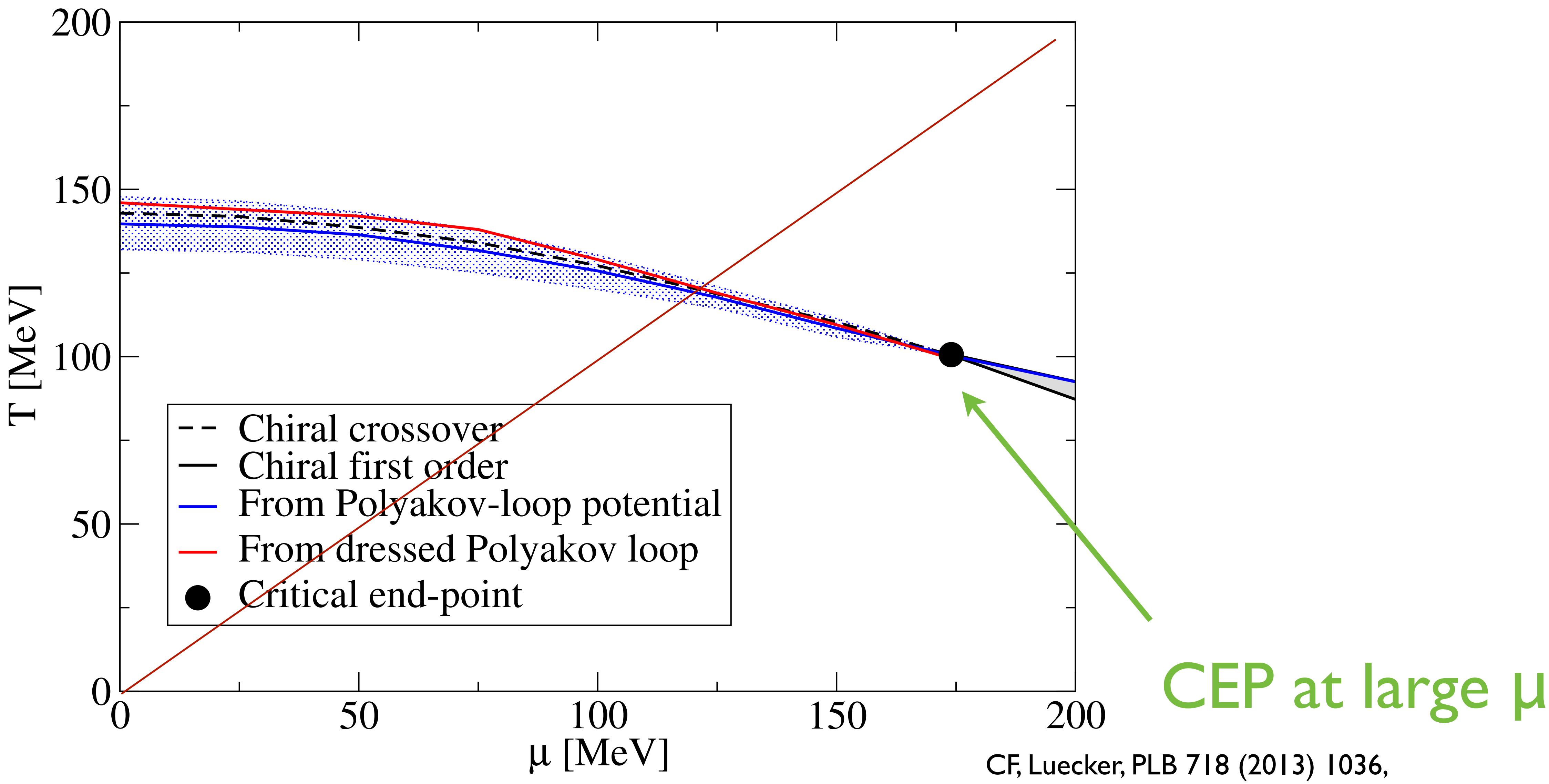
Polyakov-Loop

$$L = \frac{1}{N_c} tr \; e^{ig \int A_0}$$

CF, Fister, Luecker, Pawlowski, arXiv:1306.6022

- evaluated from Polyakov-Loop potential
- important input for P-models: PQM, PNJL !

N_f=2+1: Polyakov loop and phase diagram



CF, Luecker, PLB 718 (2013) 1036,
CF, Fister, Luecker, Pawlowski, arXiv:1306.6022

- no CEP at μ_c/T_c < 1 in agreement with lattice and FRG

de Forcrand, Philipsen, JHEP 0811 (2008) 012; Nucl Phys. B642 (2002) 290-306
Endrodi, Fodor, Katz, Szabo, JHEP 1104 (2011) 001
Braun, Haas, Marhauser and Pawlowski, PRL 106 (2011) 022002

Caveat: baryon effects missing...

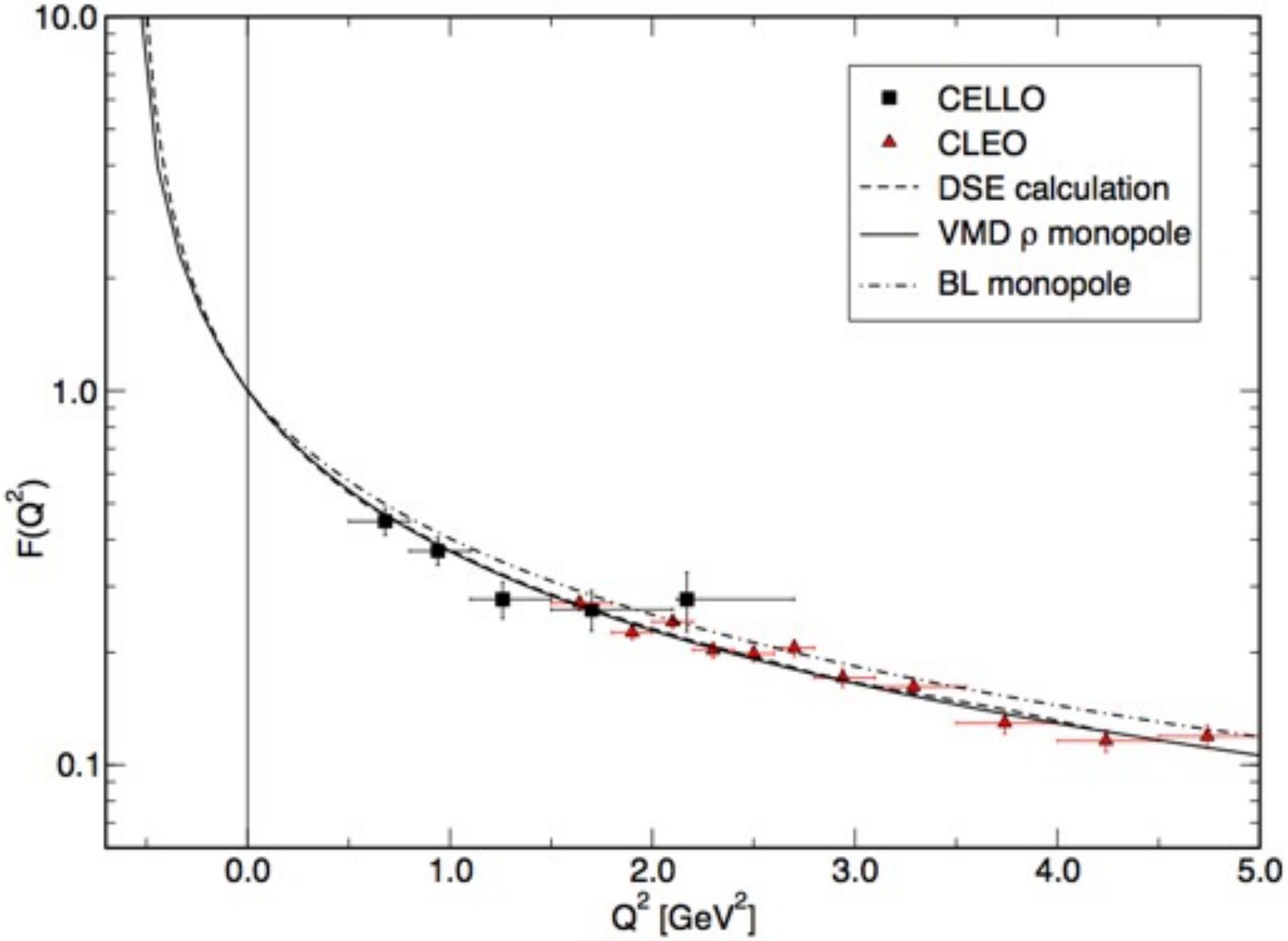
Nc=2, PQM: Strodthoff, Schaefer and Smekal, PRD 85 (2012) 074007

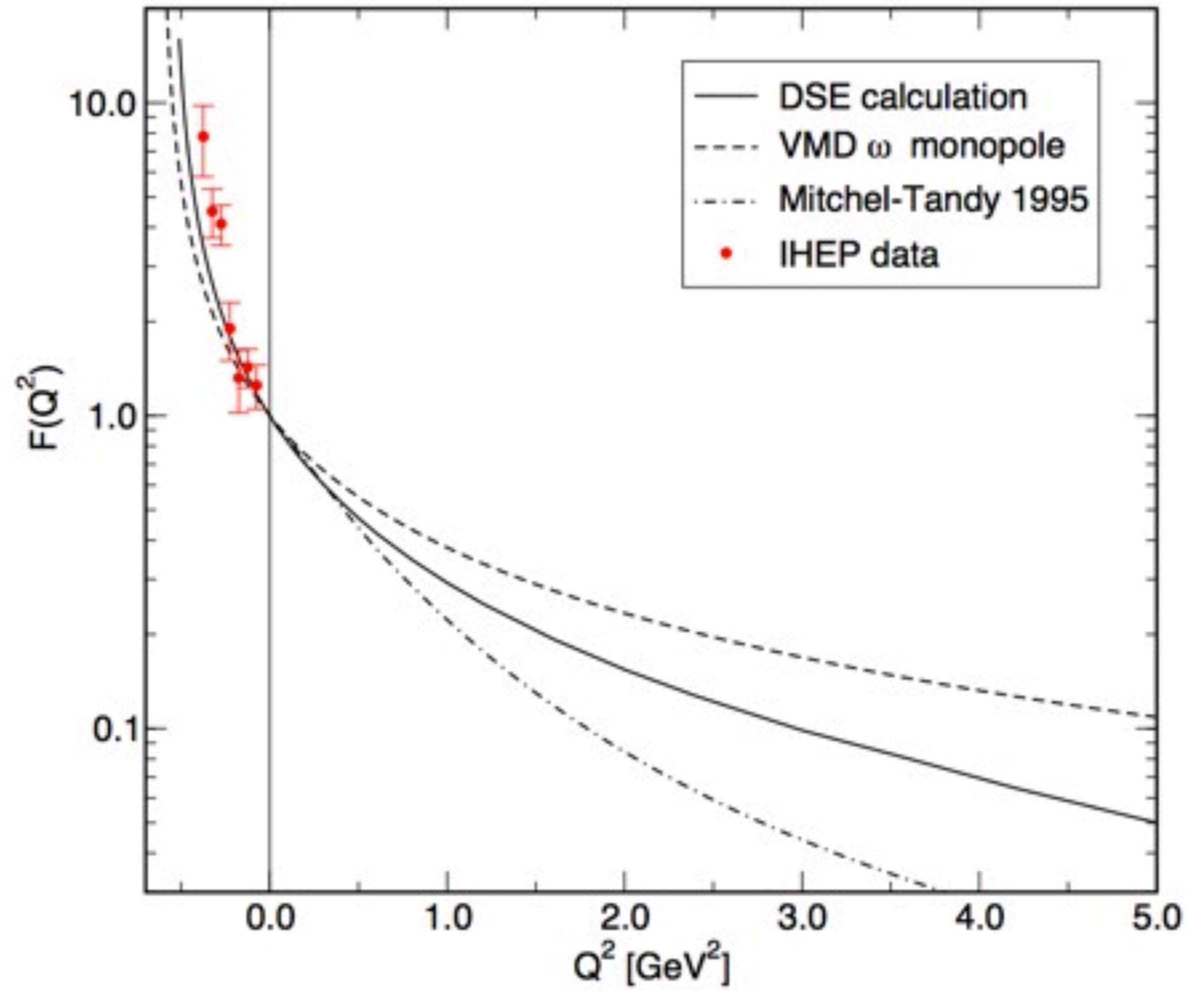
Summary

- Properties of hadrons
  - Pion cloud effects in light mesons
  - Masses and form factors of baryons
  - Current work: charmonia, EM form factors w. pion cloud

- QCD with finite chemical potential
  - temperature dependent gluon propagator
  - first calculation of Polyakov-loop potential at finite μ
  - N_f=2+1: CEP at $\mu_c/T_c > 1$
  - current work: N_f=2+1+1, spectral functions

Backup Slides





$\gamma^*\pi \to \gamma$ $\qquad\qquad$ $\omega\pi\gamma^*$ and $\rho\pi\gamma^*$

- good agreement with data
- rho/omega pole generated dynamically

Maris, Tandy, Phys. Rev. C 65 045211 (2002)

Quenched QCD: quark spectral functions

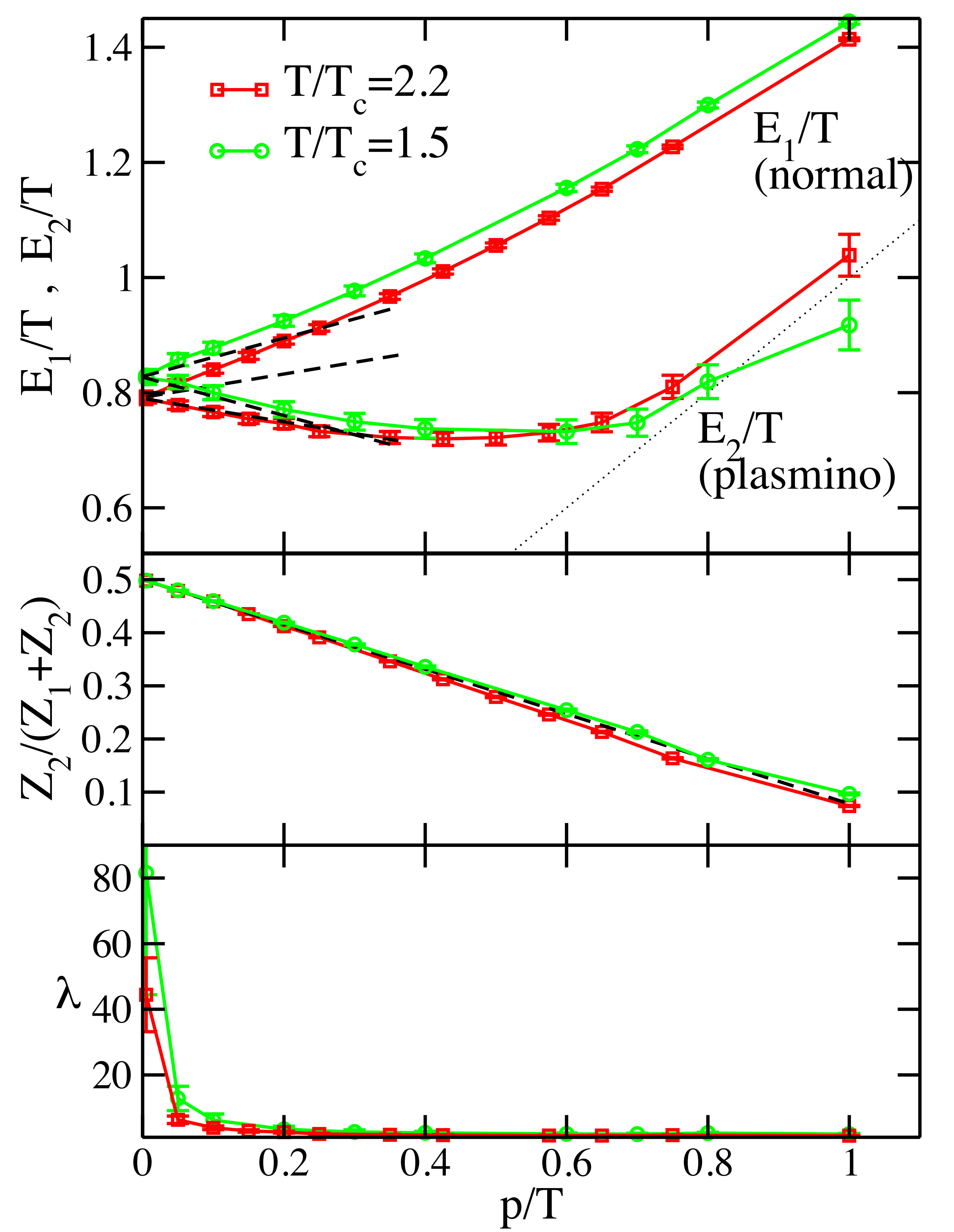
Idea: Fit spectral representation to quark propagator

Karsch and Kitazawa, PRD 80, 056001 (2009)

$$S(p_0,\vec{p}) = \int dp_0' \frac{\rho(p_0',\vec{p})}{p_0-\omega'}$$

$$\rho_\pm(p_0,p) = 2\pi\left[Z_1\delta(p_0\mp E_1) + Z_2\delta(p_0\pm E_2)\right] + \lambda\left(1-\frac{p_0^2}{p^2}\right)e^{-p_0^2}\,\Theta\left(1-\frac{p_0^2}{p^2}\right)$$

- Quark, plasmino and continuum (Landau damping)
- agreement with HTL at p=0



Mueller, CF, Nickel, EPJC 70 (2010) 1037-1049