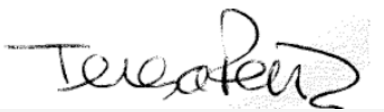
# Covariant Spectator Theory and an integrated description of the baryon electromagnetic vertices



**Gilberto Ramalho** (now UFRN Brasil)
**Alfred Stadler**
**Elmar Biernat**
**Sofia Leitão**
**Franz Gross**

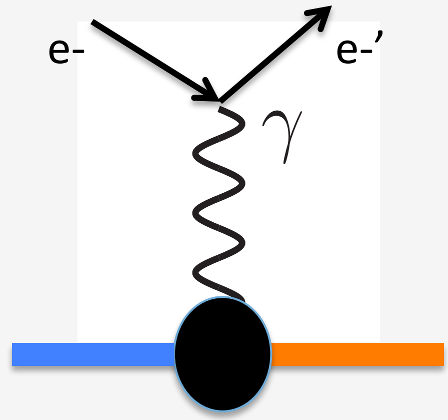

# HAVE YOU CREATED ART IN OR ABOUT AN EXTREME STATE?

# Electro-excitation reactions of N to N*s and Δ*

One tool to reveal hadron structure

*

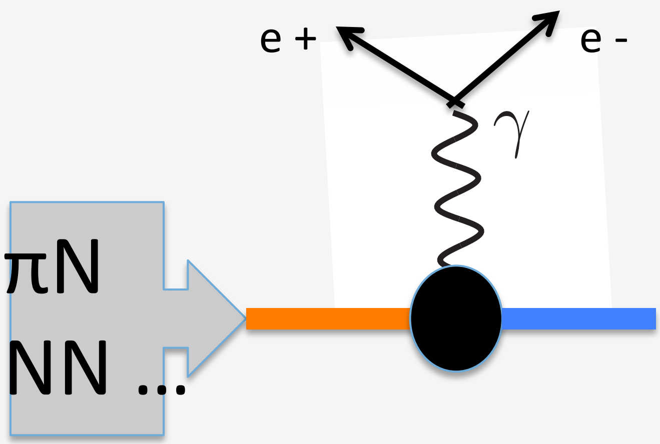
$q^2<0$




Form Factor

Information relevant for interpretation of production processes by strong probes

$q^2>0$




Form Factor

# Theoretical Approaches to study N, N*, Δ*

**1** Constituent Quark Models

**2** Dynamical Coupled Channel Models

**3** Chiral Perturbation Theory

**4** QCD in the Large $N_c$ limit

**5** Dyson-Schwinger

**6** pQCD

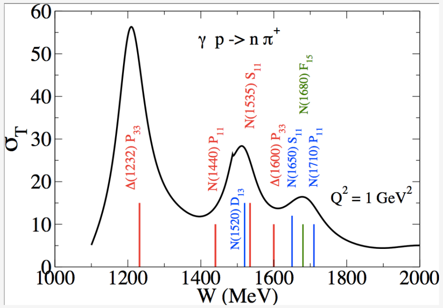
**7** LQCD

## CST©

**C**ovariant **S**pectator **T**heory describes in an efficient way the behavior of $\gamma N \rightarrow N*$ hadronic vertices at high $Q^2$

**1** **Framework: Covariant Spectator Theory**

**2** **Model and results for Baryons**



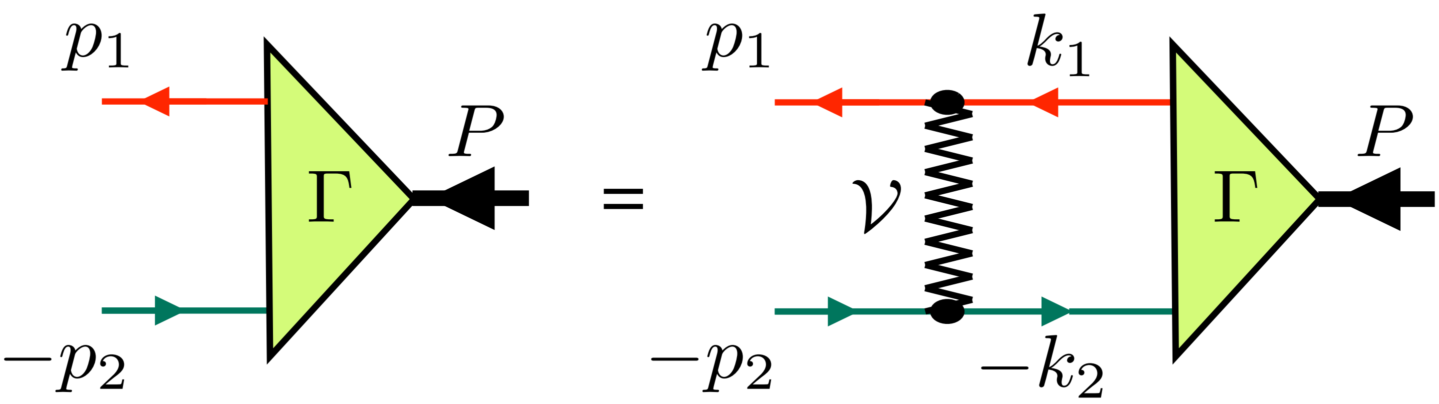

**3** **Towards understand the "pion cloud"?**

**1st results for Pion form factor**

**Connection to LQCD**

# 1 Framework: CST

$$\Gamma_{\rm BS}(p,P) = \mathrm{i}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\, \mathcal{V}(p,k;P)\, S_1(k_1)\, \Gamma_{\rm BS}(k,P)\, S_2(k_2)$$

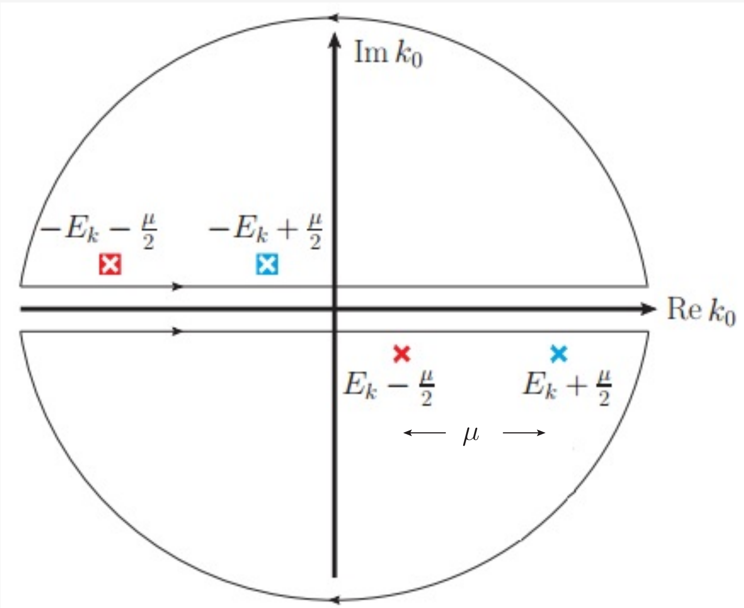



$$S_i(k_i) = \frac{1}{m_{0i} - \not{k}_i + \Sigma_i(\not{k}_i) - \mathrm{i}\epsilon}$$

$$\Sigma_i(\not{k}_i) = A_i(k_i^2) + \not{k}_i B_i(k_i^2)$$

| | |
|---|---|
| $P = k_1 - k_2$ | total momentum |
| $k = (k_1 + k_2)/2$ | relative momentum |
| $\Gamma(k,P)$ or $\Gamma(k_1,k_2)$ | vertex function |
| $\mathcal{V}(p,k)$ | kernel |

# Covariant Spectator Theory

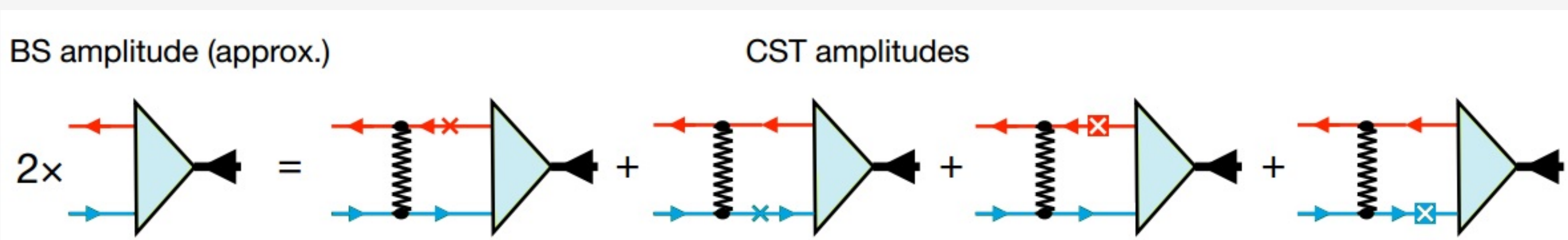


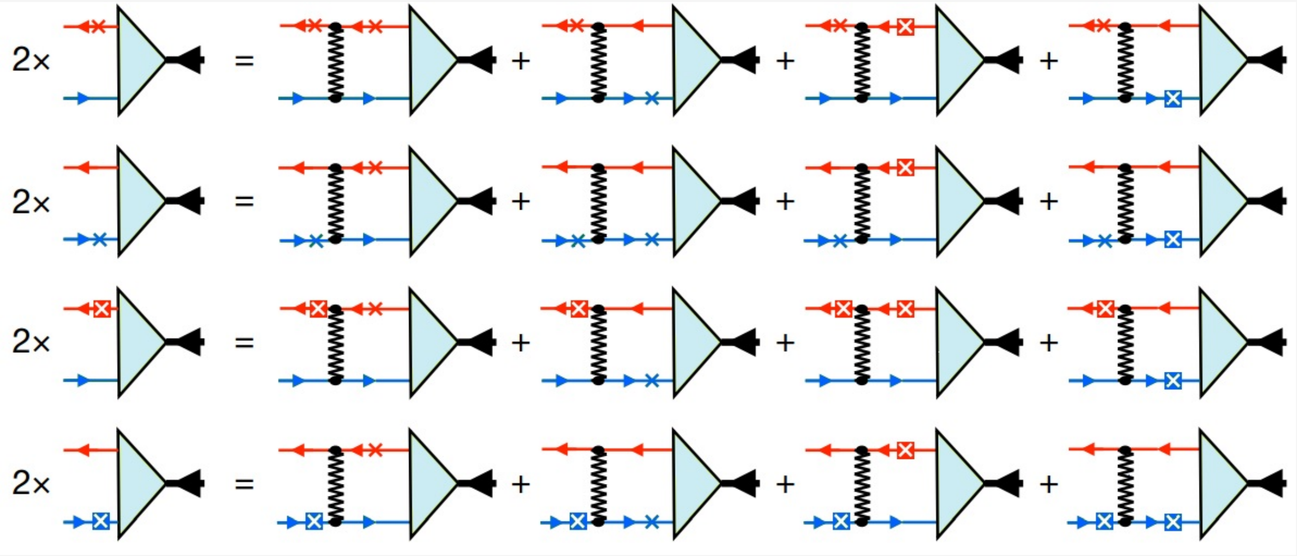

Keep poles from propagators

Reduction the 4D to 3D loop integrations, but **Covariant**

Kernel poles considered together with higher-order irreducible diagrams

Cancellations between ladder and crossed ladder diagrams can occur





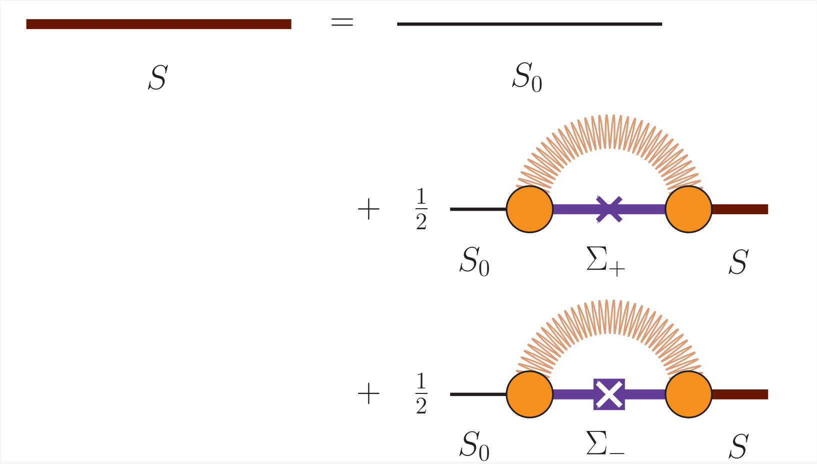

## Charge conjugation invariance of BS preserved in 4 Channel set of CST equations

Solution: bound state mass μ and vertex Γ

In special cases approximations are possible. Eg.:

Large bound state mass ➔ 1 Channel

$$S_i(k_i) = \frac{1}{m_{0i} - \not{k}_i + \Sigma_i(\not{k}_i) - \mathrm{i}\epsilon} \qquad \Sigma_i(\not{k}_i) = A_i(k_i^2) + \not{k}_i B_i(k_i^2)$$



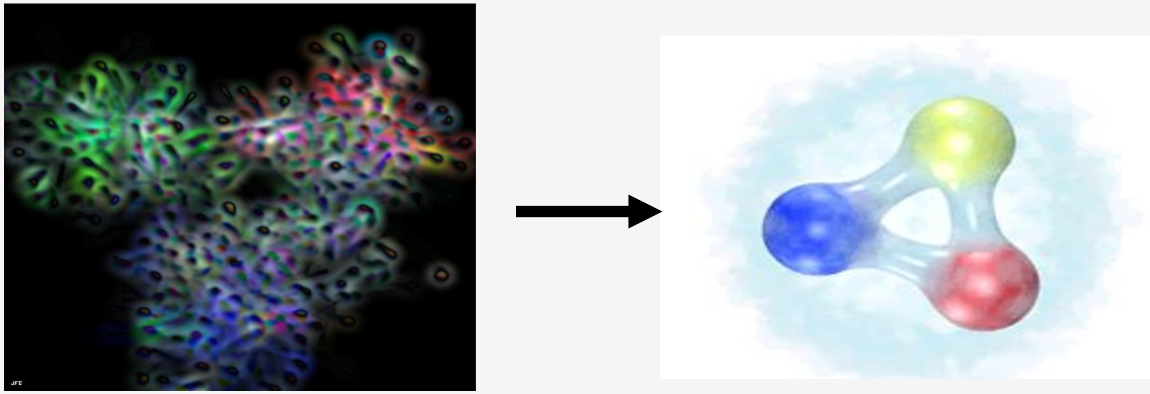

## Chiral symmetry:

Bare Quark mass = 0:

Scalar part of 1 body equation and two body equation are identical.

*A massless pion exists in that limit ( Goldstone boson).*

# 2 Model and results for Baryons

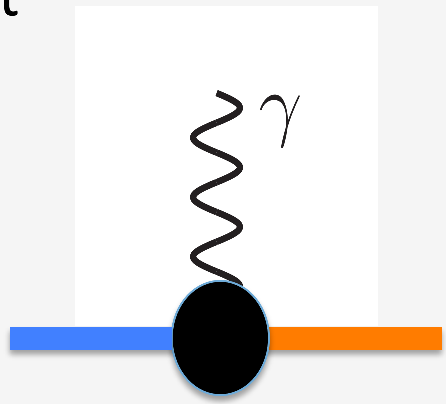


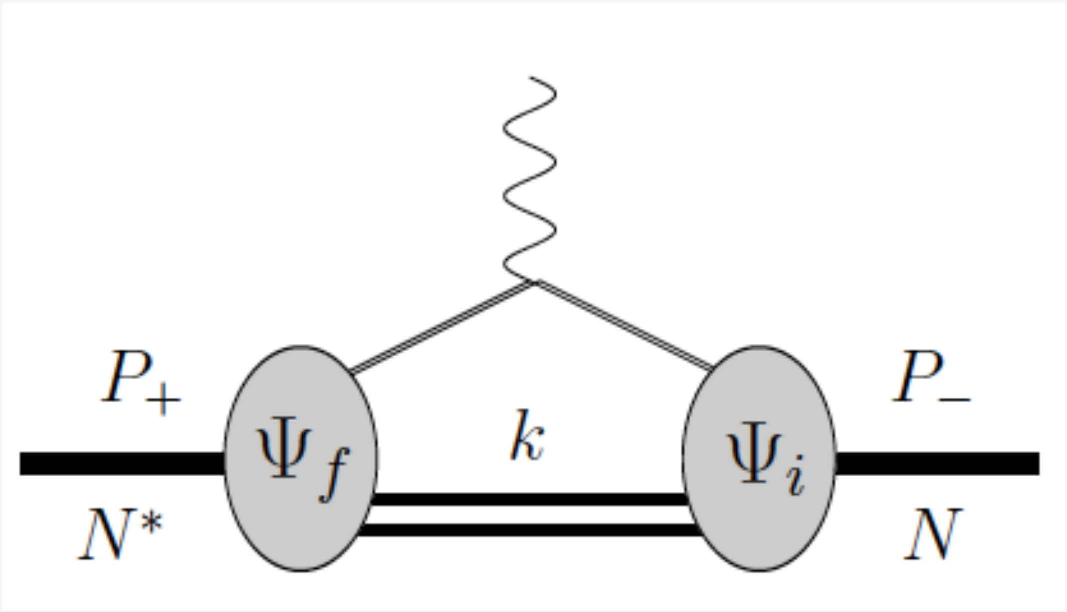
## Fock space is truncated

3 constituent quarks dressed by gluons, and quark-antiquark pairs.

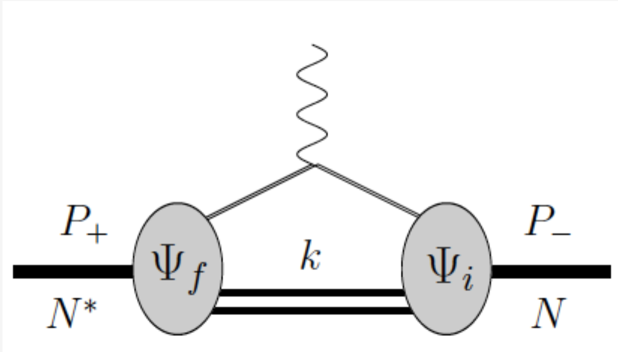
Mass, size, anomolous magnetic moment.

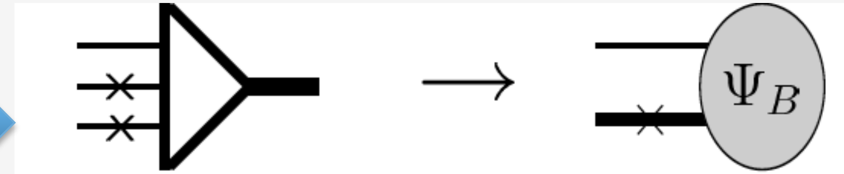
**E.M.** matrix element



# E.M. matrix element (high $Q^2$: impulse approximation)



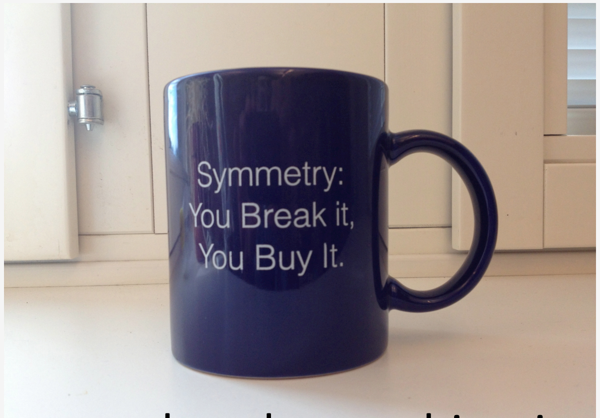

$$\int_{k_1 k_2} \equiv \int \frac{d^4k_1\, d^4k_2}{(2\pi)^6}\,\delta_+(m_1^2 - k_1^2)\,\delta_+(m_2^2 - k_2^2) = \int \frac{d^3k_1\, d^3k_2}{(2\pi)^6\, 4E_1E_2},$$

$$\int_{sk} = \underbrace{\int \frac{d\Omega_{\hat{\mathbf{r}}}}{4(2\pi)^3} \int_{4m_q^2}^{\infty} ds \sqrt{\frac{s - 4m_q^2}{s}}}_{\int_s}\ \underbrace{\int \frac{d^3k}{(2\pi)^3 2E_s}}_{\int_k},$$

- **E.M.** matrix calculation needs only an effective baryon vertex with a **quark-diquark** structure, since diquark internal relative 4-momentum is integrated over.

- **Baryon vertex** is “effective”.

Baryon "wavefunction"



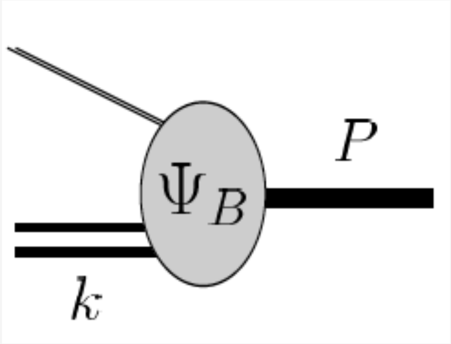

SU(6) × O(3) : impose that the combination of diquark and quark symmetries to be anti-symmetric in the exchange of any pair of quarks

$$\Psi_B = color \otimes flavor \otimes spin \otimes orbital \otimes radial$$

- It is written in a **covariant** form in terms of baryon properties (I,P,J)
- **Treatment of** high angular momentum states possible

# Nucleon wavefunction

- A quark + **scalar**-diquark component
- A quark+ **axial vector**-diquark component




$$\Psi^S_{N\lambda_n}(P,k) = \frac{1}{\sqrt{2}}[\phi^0_I u_N(P,\lambda_n) - \phi^1_I \varepsilon^{\alpha *}_{\lambda P} U_\alpha(P,\lambda_n)] \times \psi^S_N(P,k).$$

Phenomenological function

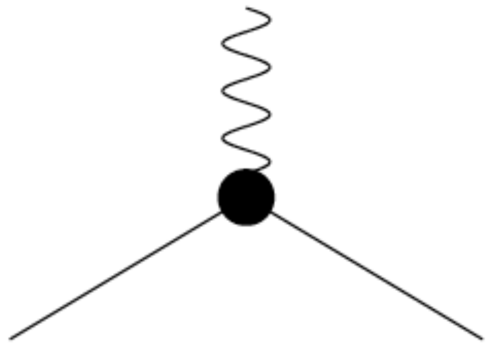
$$U_\alpha(P,\lambda_n) = \frac{1}{\sqrt{3}}\gamma_5\left(\gamma_\alpha - \frac{P_\alpha}{m_H}\right)u_N(P,\lambda_n),$$

# Delta wavefunction

- Only quark + **axial vector**-diquark term contributes

$$\Psi^S_\Delta(P,k) = -\psi^S_\Delta(P,k)\tilde{\phi}^1_I \varepsilon^{\beta *}_{\lambda P} w_\beta(P,\lambda_\Delta)$$

# E.M. Current




Constituent quarks (quark form factors)

$$
\begin{aligned}
j_I^\mu \quad = \quad & \left[\frac{1}{6} f_{1+} + \frac{1}{2} f_{1-} \tau_3\right] \gamma^\mu + \\
& \left[\frac{1}{6} f_{2+} + \frac{1}{2} f_{2-} \tau_3\right] \frac{i\sigma^{\mu\nu} q_\nu}{2M_N}
\end{aligned}
$$

Quark e.m form factor at the quark level

Vector meson dominance **2** poles

$$f(Q^2) = e + gB(Q^2)e + gB(Q^2)gB(Q^2)e + \cdots = e + \frac{gB(Q^2)e}{1 - gB(Q^2)}$$

if $gB(Q^2) = \frac{\lambda^2}{\Lambda^2 + Q^2}$, then $f(Q^2) = e + \frac{\lambda^2 e}{\Lambda^2 - \lambda^2 + Q^2}$

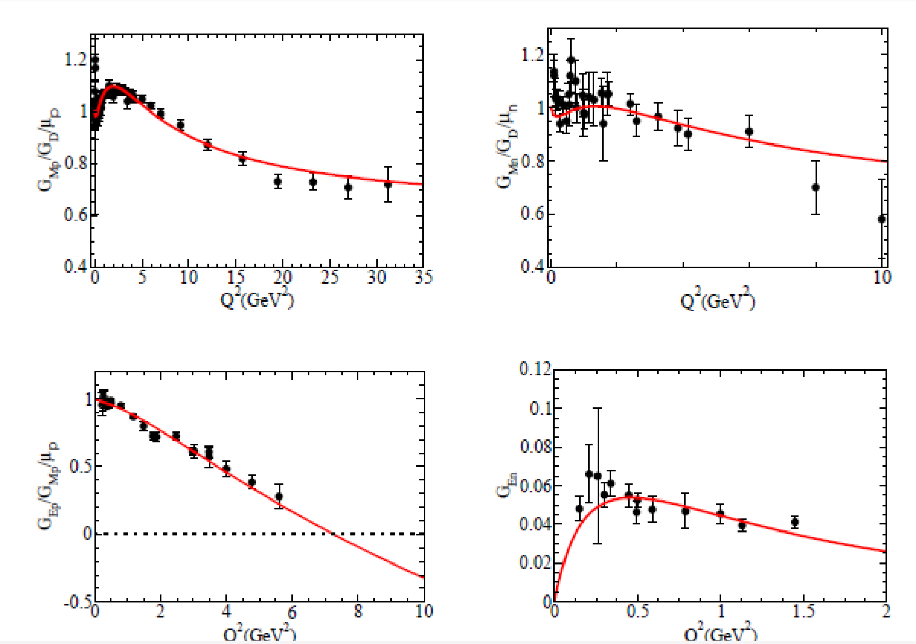
$$f_{1\pm} = \lambda + \frac{1-\lambda}{1+Q_0^2/m_v^2} + \frac{c_\pm Q_0^2/M_h^2}{(1+Q_0^2/M_h^2)^2}$$

$$f_{2\pm} = \kappa_\pm \left( \frac{d_\pm}{1+Q_0^2/m_v^2} + \frac{(1-d_\pm)}{1+Q_0^2/M_h^2} \right)$$

Low-energy behavior encodes high-energy behavior:
DIS used to fixed λ

**4** parameters

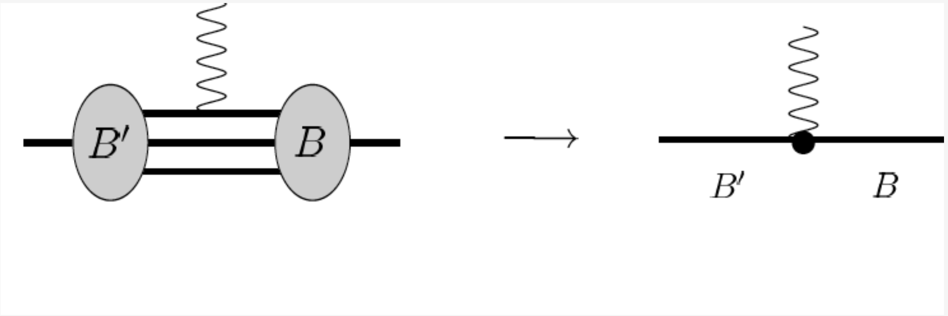
# Proton and Neutron form factors $\chi^2 = 1.36$

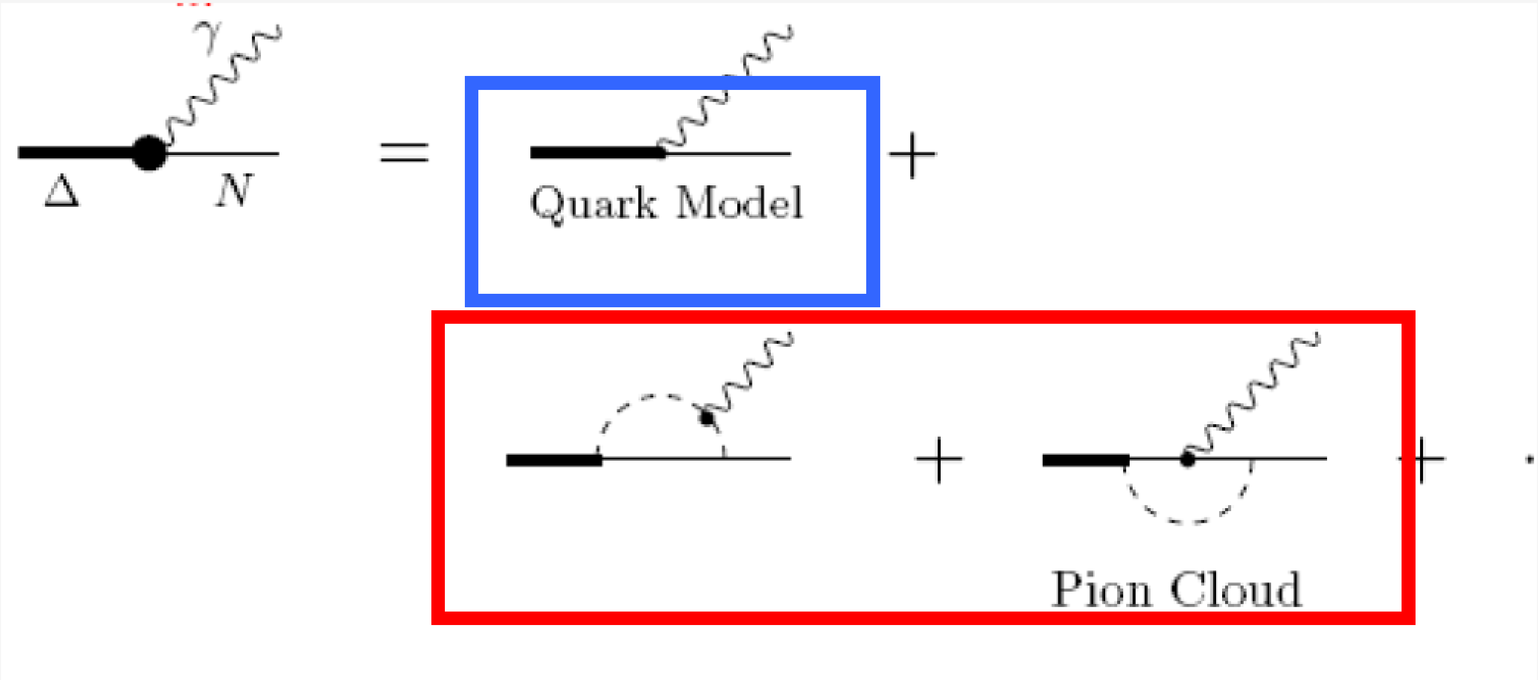



G. Ramalho, M. T. P. and Franz Gross, PHYSICAL REVIEW C 77, 015202 (2008)

Franz Gross, G. Ramalho, M. T. P. PHYSICAL REVIEW D 85, 093006 (2012)

$\gamma$-Bare Quark core coupling




suppressed with extra $\frac{1}{Q^4}$ pQCD

C. Carlson, FBS Supp 11 10 (1999)

$$G_M^* = G_M^B + G_M^\pi$$

G. Ramalho, M. T. P. and Gross,
EPJS 36, 329 (2008);
PRD 78, 114017 (2008)

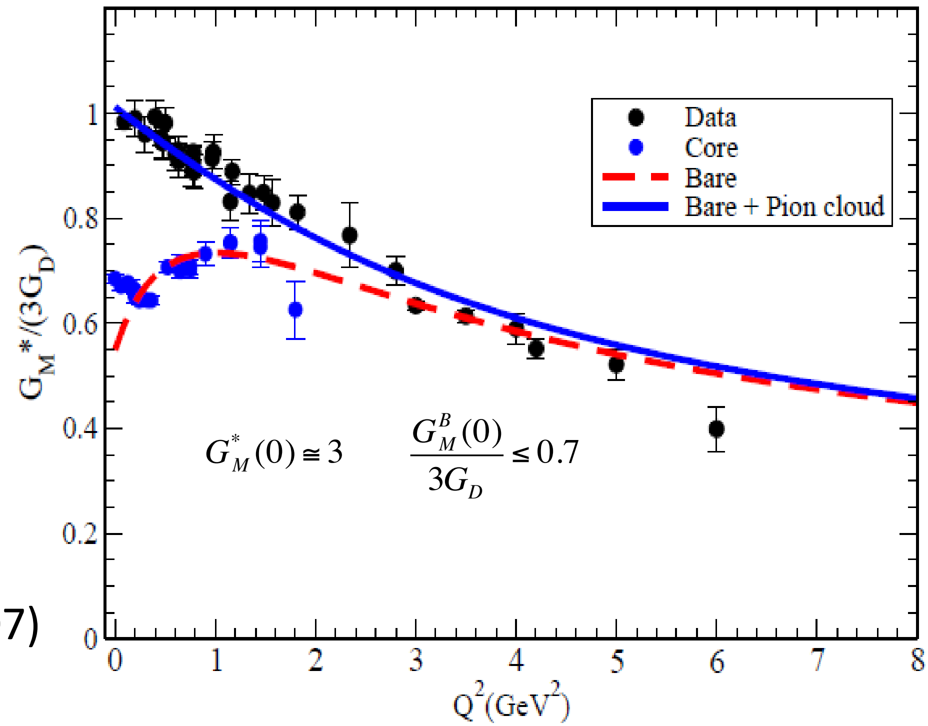
Is this separation supported by experiment?
Best way to determine bare quark core term?

# Is this separation supported by experiment?

GR and MT Peña PRD 80, 013008 (2009)

$$\gamma N \to \Delta$$

- **Bare quark core coupling** dominates in large $Q^2$ region
- **Bare quark core results agree with EBAC analysis** : bare quark contributions extracted from the data (meson cloud effects subtracted)




**EBAC**: Diaz et al., PRC 75, 015205 (2007)

- Bare ≈ Sato-Lee model

$$G_M^\pi = \lambda_\pi \left(\frac{\Lambda_\pi^2}{\Lambda_\pi^2+Q^2}\right)^2 (3G_D)$$

G. Ramalho, M. T. P. and Franz Gross, PHYSICAL REVIEW D 78, 114017 (2008)

# Connection to LQCD

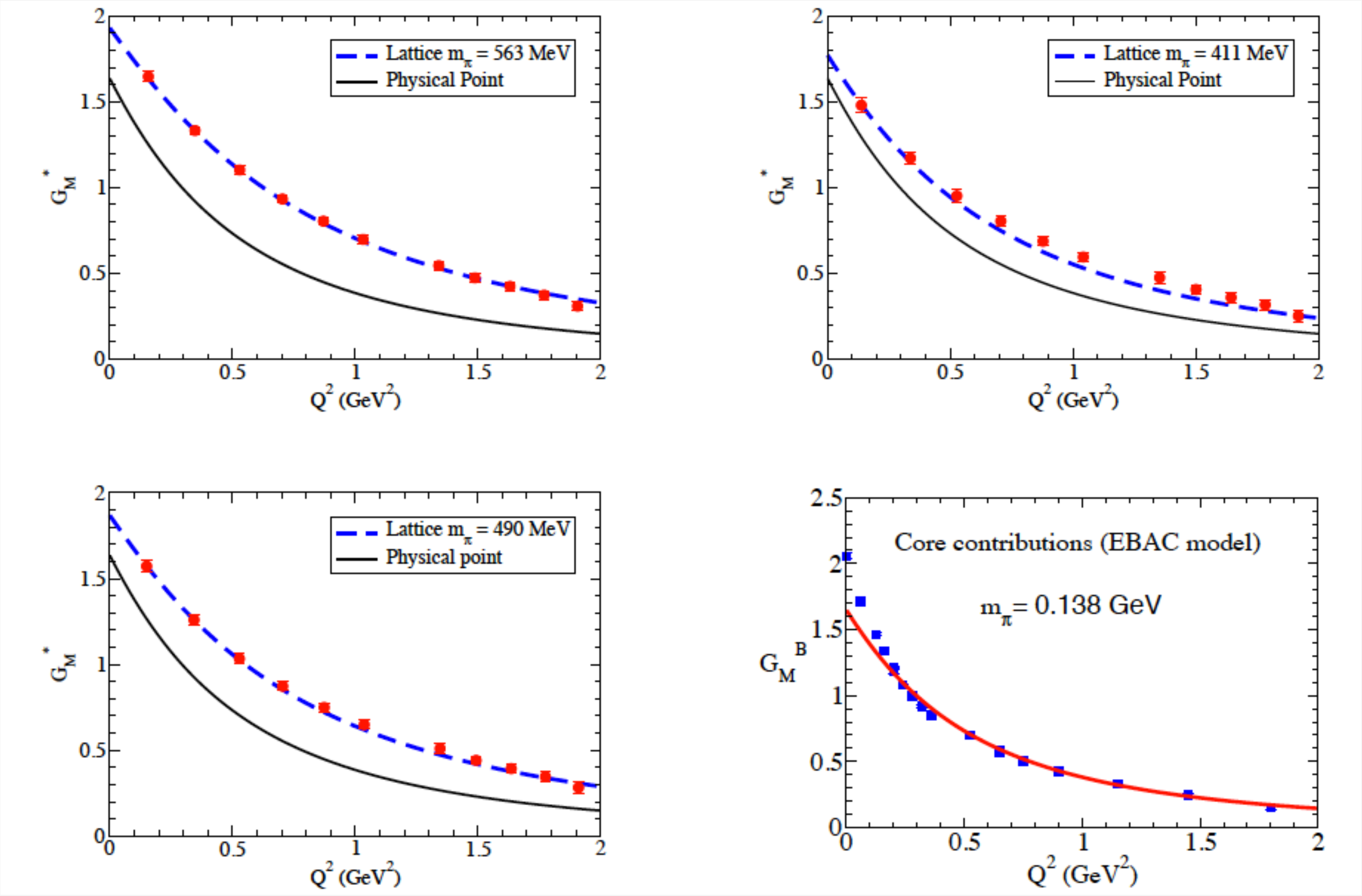
## $\gamma N \to \Delta$ Transition

# Best way to determine bare quark core term?

## Mechanism of **vector meson dominance**

- Vector meson mass function of the **pion mass**.
- Pion cloud contribution negligible for **large pion masses**.

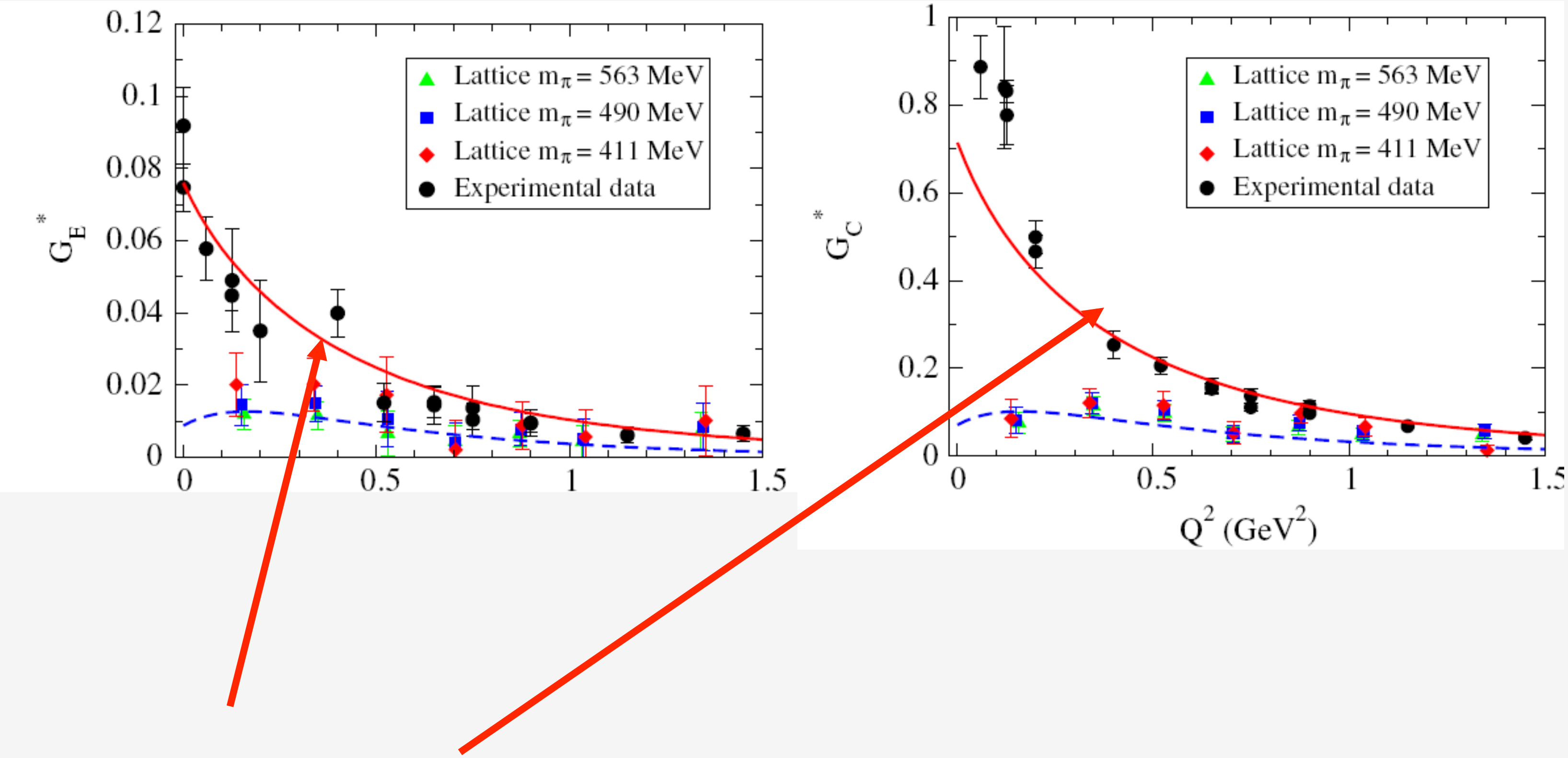
- For large pion masses **bare** quark model calibrated to the **lattice data.**
- After that, in limit of the model to the physical pion mass value the **experimental data** is described, at least in the high $Q^2$ region…




G. Ramalho, M. T. P. and Franz Gross, PHYSICAL REVIEW D 80, 013008 (2009)

# $\gamma N \rightarrow \Delta$

D3 0.72% and D1 0.72 % of the wavefunction

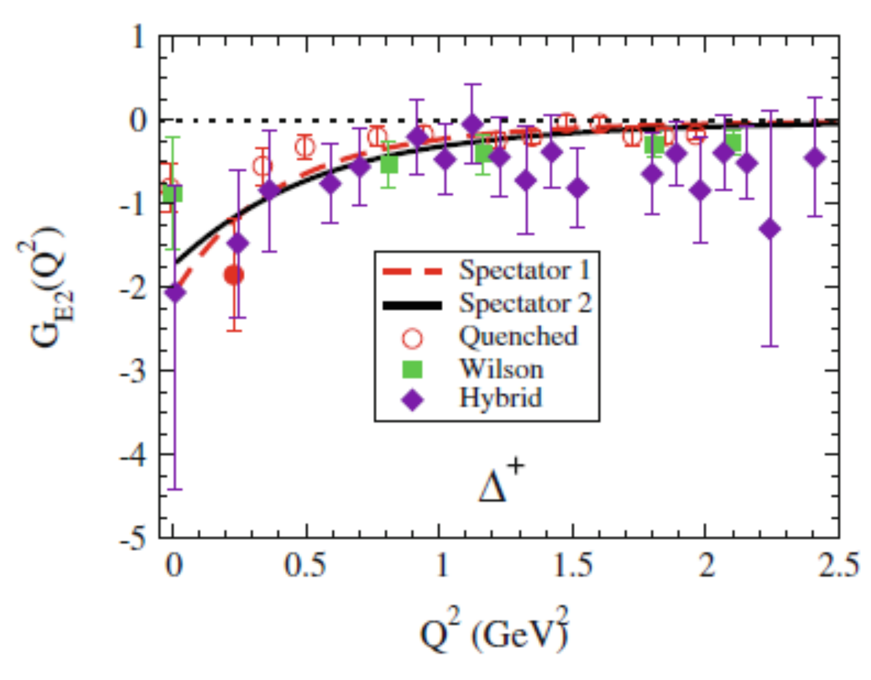



pion cloud : large N_c limit relations Pascalutsa and Vanderhaeghen, PRD76 111501(R) (2007)

G. Ramalho, M. T. P. and Franz Gross, PHYSICAL REVIEW D 80, 013008 (2009)

# Predictions

# $\gamma\Delta \to \Delta$



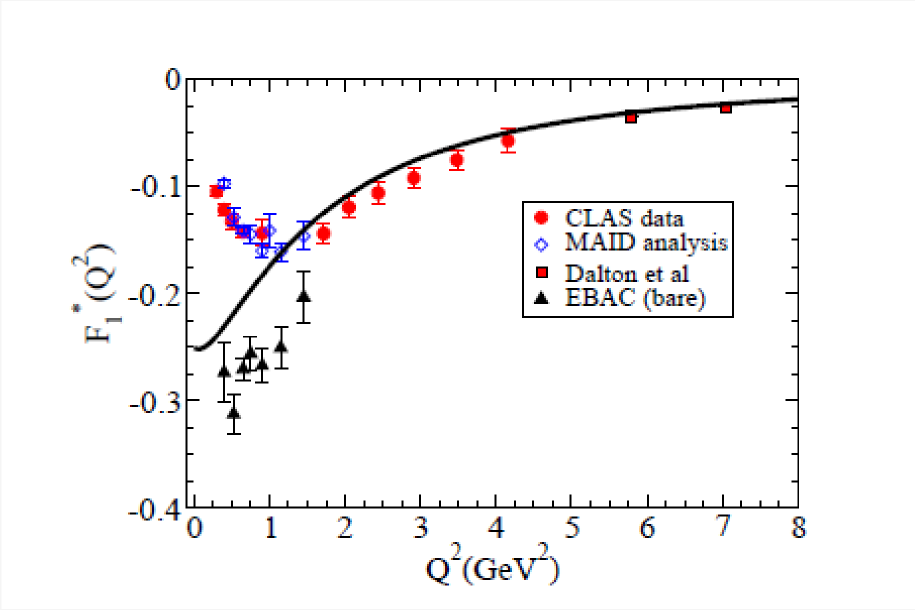

LQCD data: C. Alexandrou et al. Phys. Rev.D 79 014507 (2009);

Nucl. Phys. A 825, 115 (2009);

S. Boinepalli et al Phys. Rev. D 80 054505 (2009).

G. Ramalho, M. T. P. and Franz Gross, Physics Letters B 678 (2009) 355–358

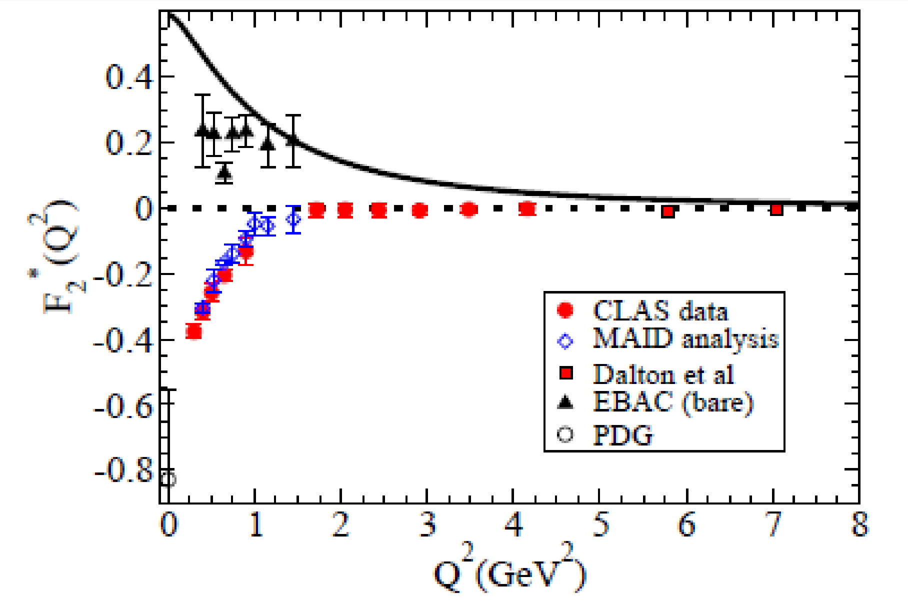
# $N \rightarrow N^*(1535)$




- radial wf identical to nucleon's; angular momentum different (P wave)

- **EBAC (bare)**: bare contributions extracted from the data (meson cloud effects subtracted)

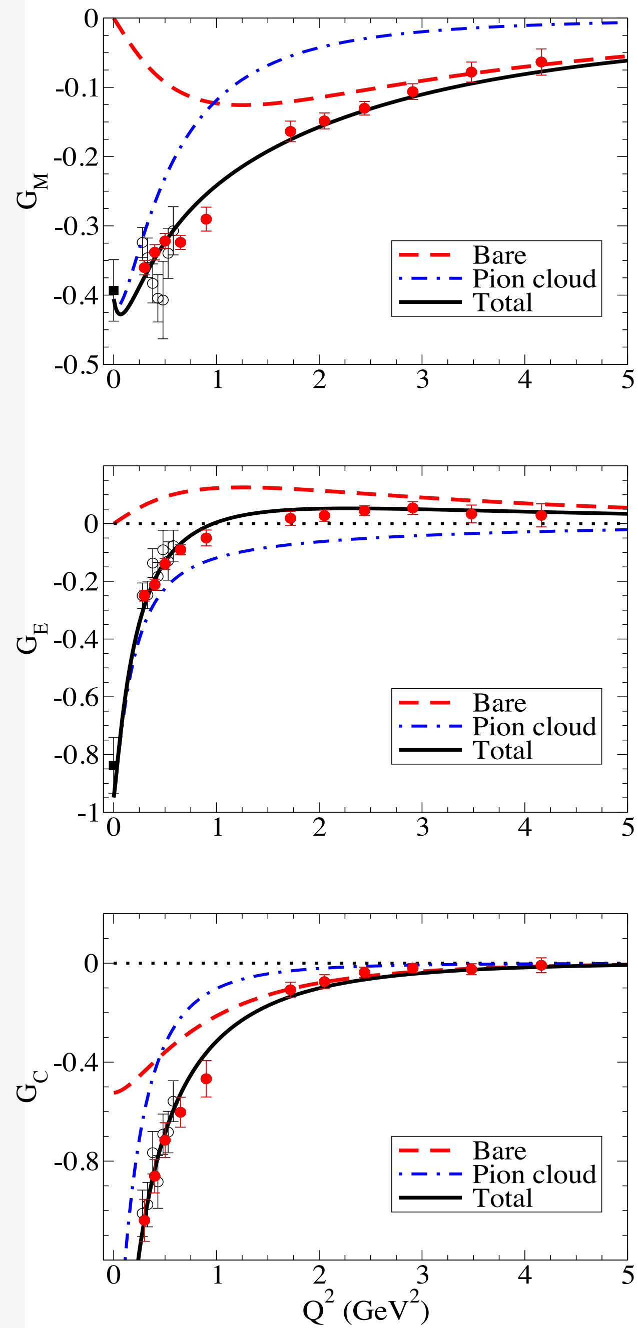
bare quark contribution close to **EBAC** analysis




- Meson cloud effects of opposite sign; and above 2 GeV^2 still very important.

G. Ramalho, M. T. P. PHYSICAL REVIEW D 84, 051301(R) (2011)

# $N \to N^*(1520)$



G_M
G_E
G_C
Bare
Pion cloud
Total
Q^2 (GeV^2)
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.8
-1
1
2
3
4
5

## Extension to timelike region

## $q^2 = -Q^2 > 0$

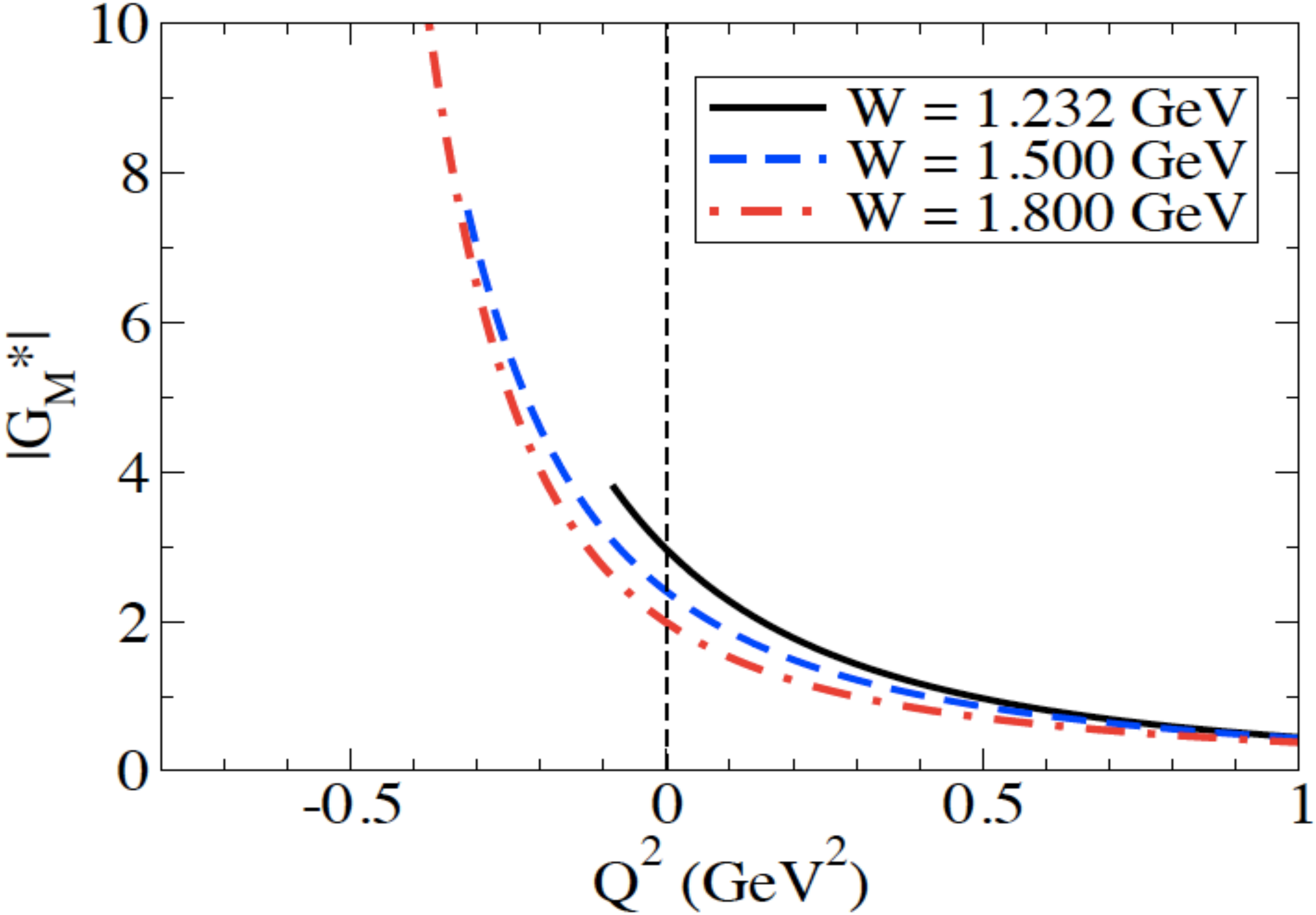
# Delta Dalitz Decay width

F. Dohrmann et al. ERJA 45 401 2010

$$\Gamma_{\gamma^* N}(q;W) = \frac{\alpha}{16}\frac{(W+M)^2}{M^2W^3}\sqrt{y_+y_-}y_-|G_T(q^2,W)|^2$$

$$|G_T(q^2;M_\Delta)|^2 = |G_M^*(q^2;W)|^2 + 3|G_E^*(q^2;W)|^2 + \frac{q^2}{2W^2}|G_C^*(q^2;W)|^2$$

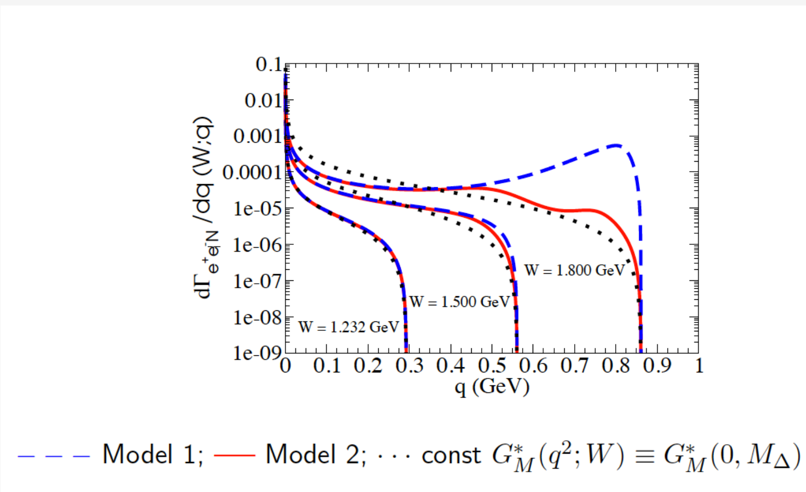
$M_\Delta \longrightarrow$ W running Delta Mass W that may differ from the pole mass; $q^2 \le (W-M)^2$



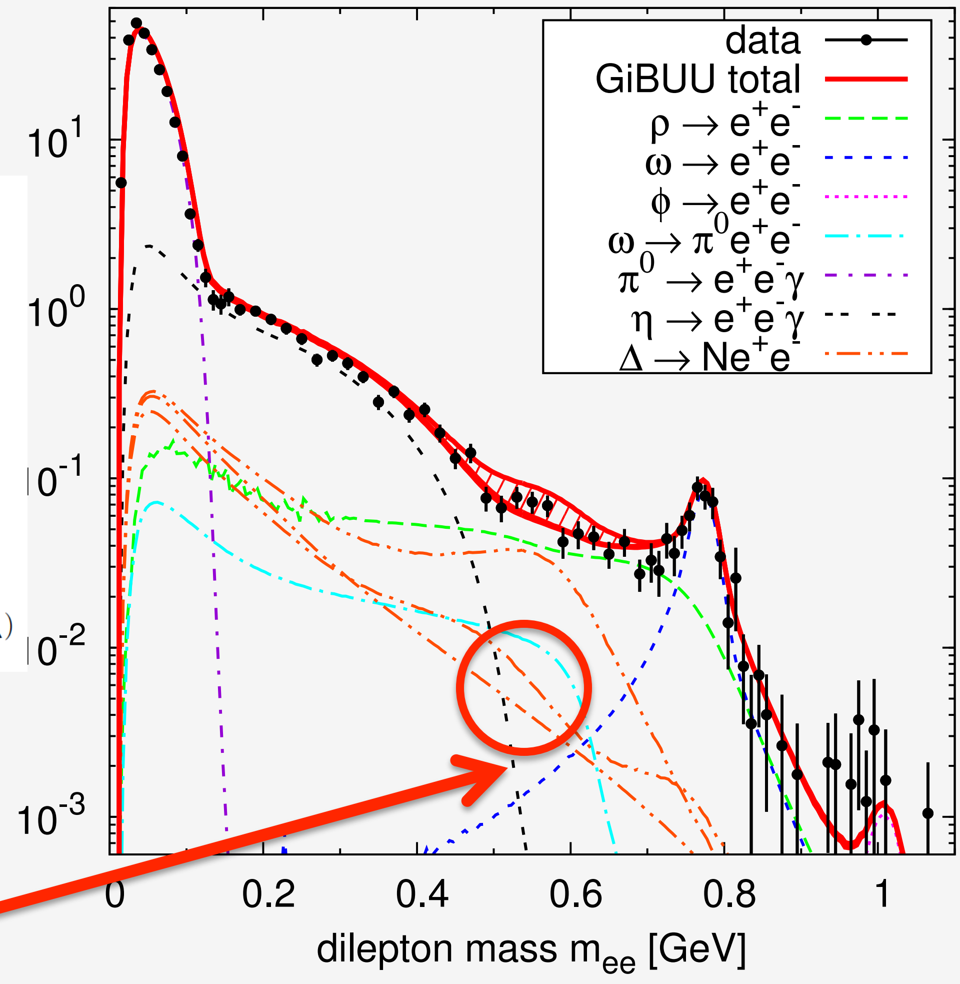

$$Q^2 \geq -(W-M)^2$$

# In the timelike region

Courtesy Janus Weil
Giessen




p + p at 3.5 GeV




G. Ramalho, M. T. P. , PHYSICAL REVIEW D 85 113014 (2012)

# Summary

1 Spectator quark-diquark model : It is covariant and accomodates angular momentum description.

2 At $Q^2 \approx 0$ consistent with EBAC data analysis based on a coupled channel Dynamical Model, and also Large $N_C$ limit.

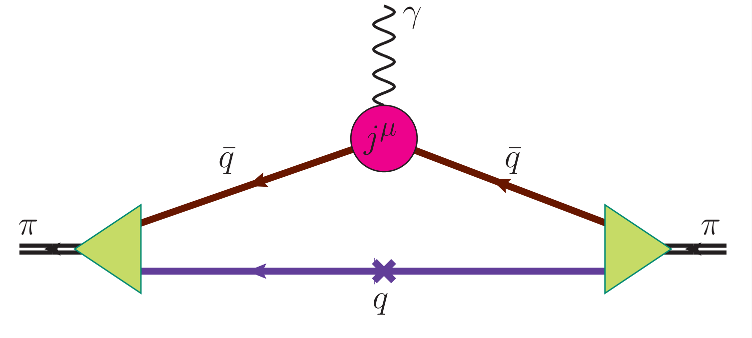
3 At high $Q^2$ consistent with experimental data, and also LQCD in the large pion mass regime.

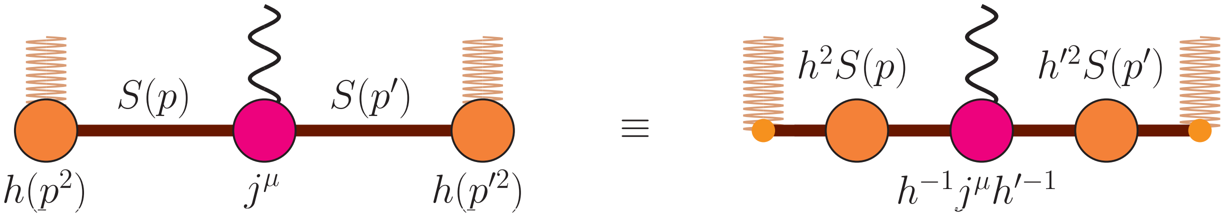
4 Several applications: **Δ(1232),** N*(1440), N*(1535), N*(1520), Δ(1600), strange sector, DIS.

5 **Δ Dalitz Di-lepton** partial decay width sensitive to momentum dependence of $G_M$

# 3 Towards understanding the pion cloud

## 1st results for pion form factor




(reduced) off-shell quark current

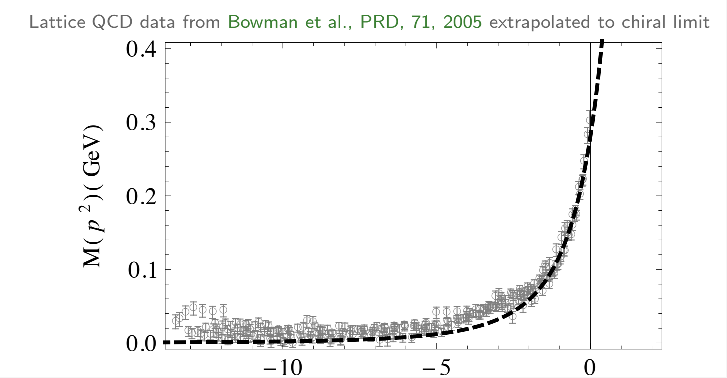
$j_R^\mu = f(\gamma^\mu + \kappa \frac{i\sigma^{\mu\nu}q_\nu}{2m}) + \delta'\Lambda'\gamma^\mu + \delta\gamma^\mu\Lambda + g\Lambda'\gamma^\mu\Lambda$

$\Lambda^{(\prime)} = \frac{M(p^{(\prime)}) - \not{p}^{(\prime)}}{2M(p^{(\prime)})}$; $f$, $\delta^{(\prime)}$, $g$ chosen such that $j_R^\mu$ satisfies Ward-Takahashi identity

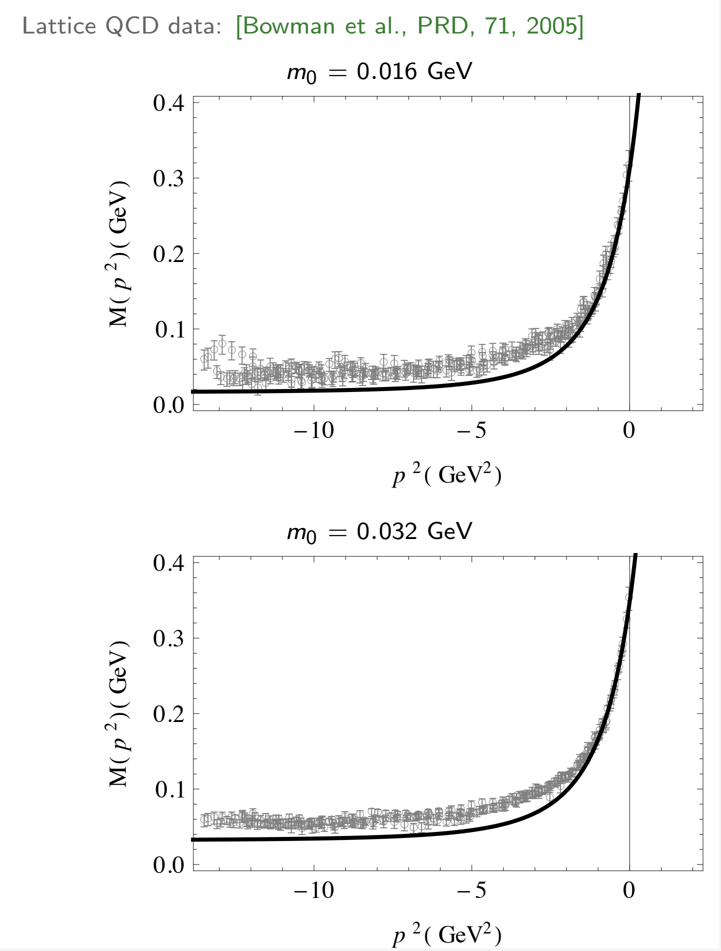
# Connection to LQCD

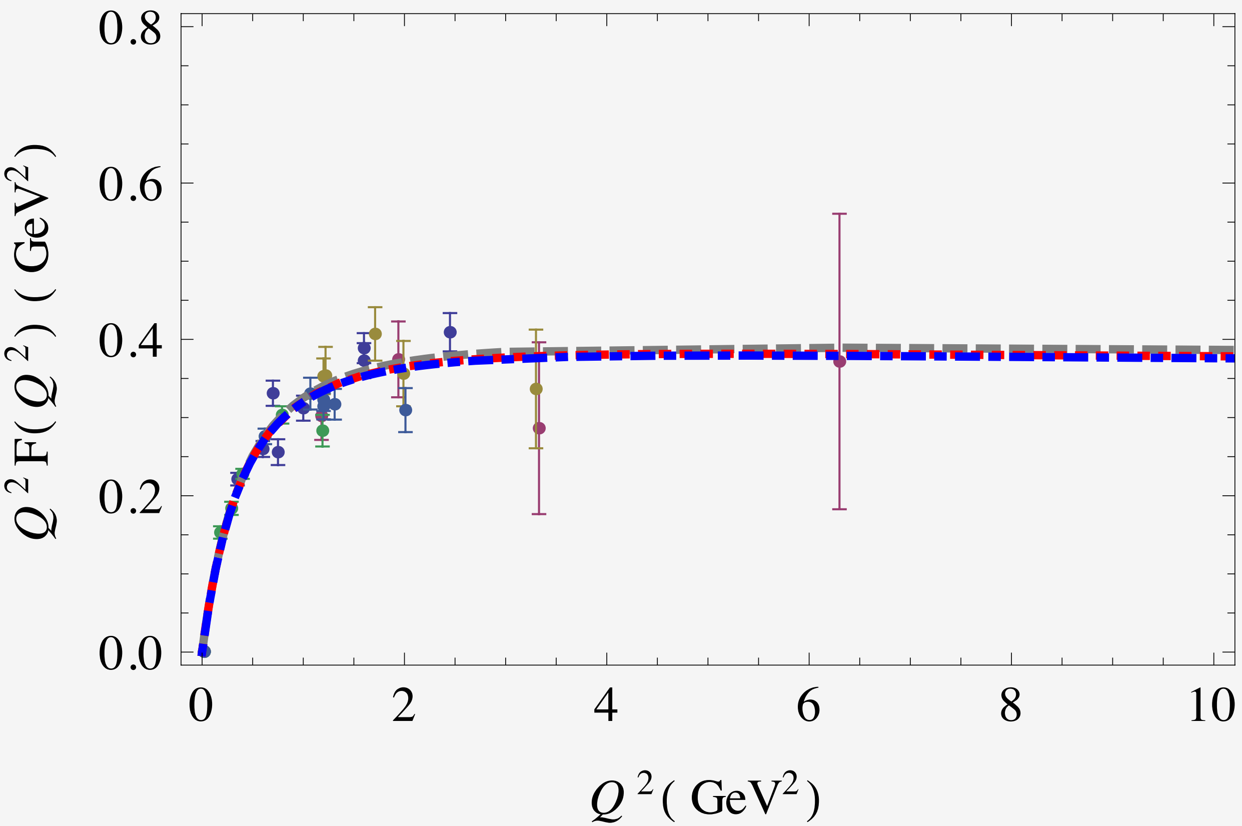
## Mass function

FIT




Prediction





0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10
$Q^2 F(Q^2)$ ( GeV$^2$)
$Q^2$( GeV$^2$)

To understand the pion cloud lead to us to calculate the **Pion form factor**

First results for of **CST** model in Minkowski space

constant vector potential with parameters fixed from Lattice data for mass function.

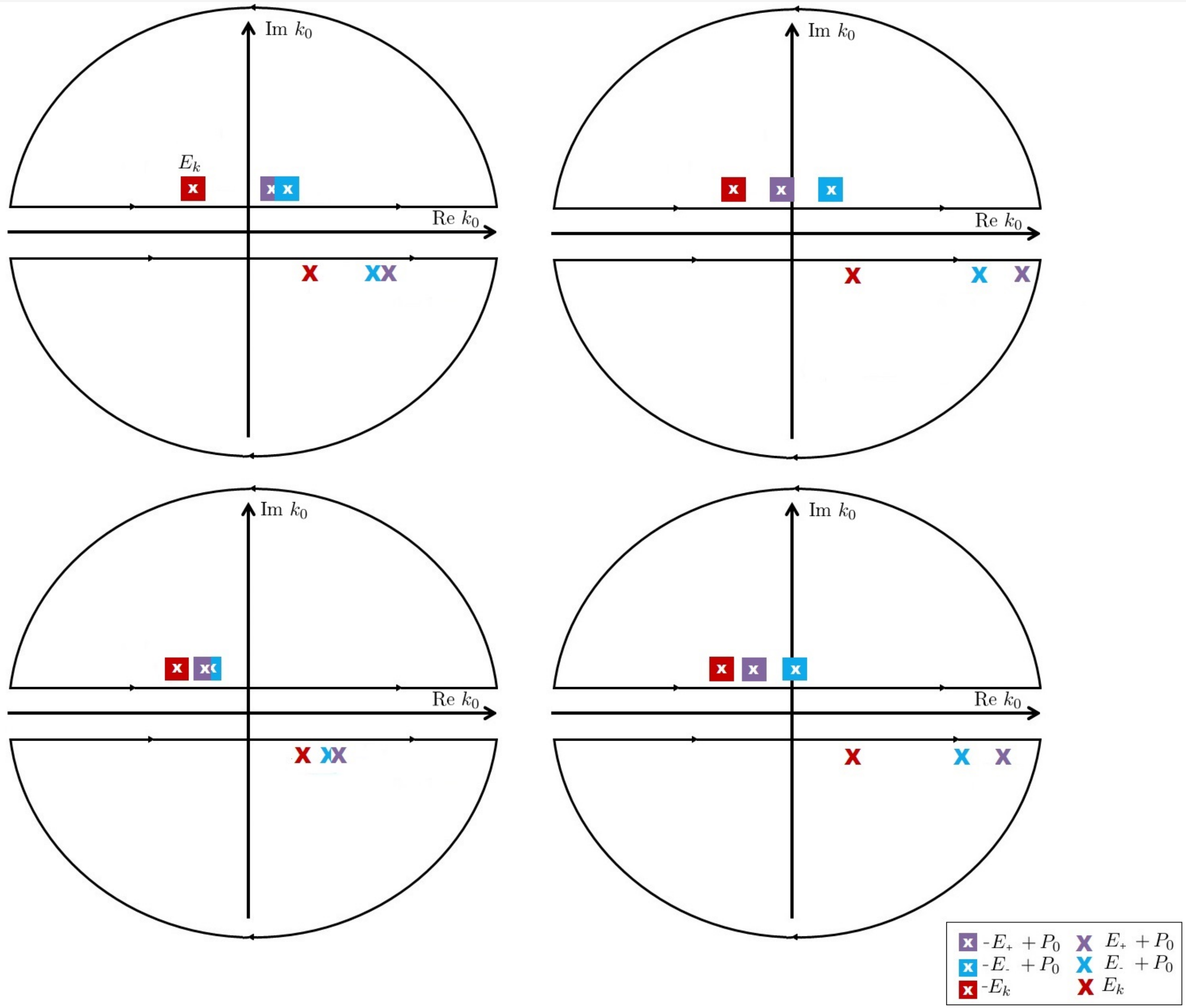
Pion Mass still too large.


Garrett McNamara, Nazaré, Portugal
Photo by Miguel Costa
BILLABONG
XXL
GLOBAL BIG
WAVE AWARDS
TWO THOUSAND THIRTEEN
not
The end

Small $Q^2$ Large $Q^2$

Large M

Small M



$$\psi_S(P,k) = \frac{N_0}{m_s(\beta_1 + \chi)(\beta_2 + \chi)},$$

where

$$\chi = \frac{(M - m_s)^2 - (P - k)^2}{M m_s} = \frac{2P \cdot k}{M m_s} - 2$$

$$\begin{aligned}
\chi &= 2\sqrt{1 + \frac{\mathbf{k}^2}{m_s^2}}\sqrt{1 + \frac{\mathbf{P}^2}{M^2}} - 2\frac{\mathbf{k} \cdot \mathbf{P}}{M m_s} - 2 \\
&\rightarrow \left(\frac{\mathbf{k}}{m_s} - \frac{\mathbf{P}}{M}\right)^2 \rightarrow \frac{1}{4m_q^2}\left(\mathbf{k} - \frac{2}{3}\mathbf{P}\right)^2 \\
&= \frac{1}{2m_q^2}\chi_{nr}(k, P, 0),
\end{aligned}$$

# N-Δ transition ($G_M^*$)

- Magnetic dipole FF

$$G_M^*(Q^2) = \frac{4}{3\sqrt{3}} \frac{M}{M + M_\Delta} j_- \int \phi_\Delta \phi_N = \overbrace{2.07 \int \phi_\Delta \phi_N}^{Q^2=0}$$

- Cauchy-Schwartz inequality (for $Q^2 = 0$):

$$\int \phi_\Delta \phi_N \leq \sqrt{\int \phi_N^2} \sqrt{\int \phi_\Delta^2} = 1$$

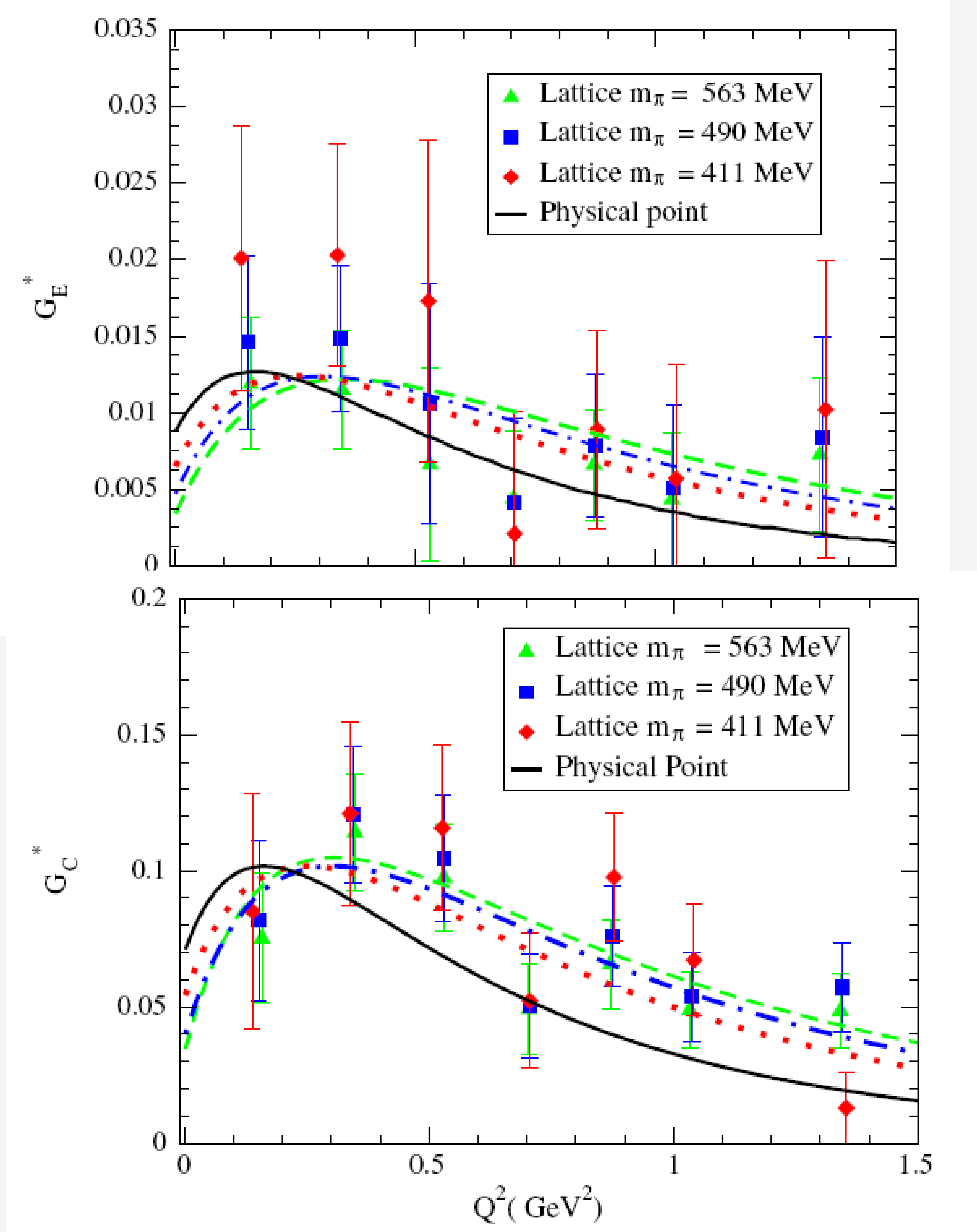
$\Rightarrow G_M^*(0) \leq 2.07$

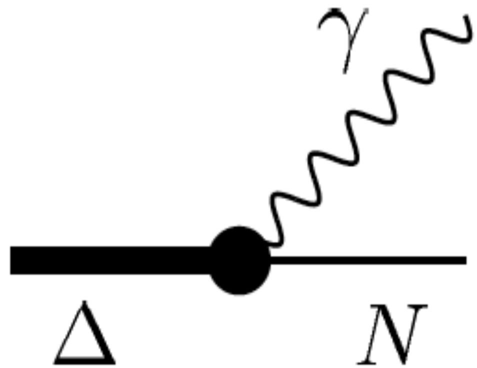
Constraining

the D-states

by the lattice data

PRD80 013008 (2009)






Δ rest frame

$$P_\Delta = (W, 0, 0, 0); \quad P_N = (E_N, 0, 0, -|\mathbf{q}|); \quad q = (\omega, 0, 0, |\mathbf{q}|)$$

Timelike $q^2 > 0$

$$\omega = \frac{W^2 - M^2 + q^2}{2W}$$

$$|\mathbf{q}|^2 = \frac{[(W+M) - q^2][(W-M)^2 - q^2]}{4W^2}$$

$$E_N = \frac{W^2 + M^2 - q^2}{2W}$$

Spacelike $Q^2 > 0$

$$\omega = \frac{W^2 - M^2 - Q^2}{2W}$$

$$|\mathbf{q}|^2 = \frac{[(W+M) + Q^2][(W-M)^2 + Q^2]}{4W^2}$$

$$E_N = \frac{W^2 + M^2 + Q^2}{2W}$$

TL: $q^2 \leq (W-M)^2$

# VMD quark-core current enables extension:

$$\frac{m_v^2}{m_v^2 - q^2} \to \frac{m_\rho^2}{m_\rho^2 - q^2 - i m_\rho \Gamma_\rho}$$

$$\to \frac{m_\rho^2 \left[(m_\rho^2 - q^2) + i m_\rho \Gamma_\rho\right]}{(m_\rho^2 - q^2)^2 + m_\rho^2 \Gamma_\rho^2}.$$

$$\Gamma_\rho(q^2) = \Gamma_\rho^0 \left(\frac{q^2 - 4m_\pi^2}{m_\rho^2 - 4m_\pi^2}\right)^{3/2} \frac{m_\rho}{q} \theta(q^2 - 4m_\pi^2)$$

H. B. O’ Connell, B.C Pearce, A.W. Thomas, A.G Williams, PLB 354 14 (1995)

$$G_M^{\pi}(Q^2;W) = 3\lambda_\pi G_D(Q^2)\left(\frac{\Lambda_\pi^2}{\Lambda_\pi^2 + Q^2}\right)^2$$

$$G_D(q^2) = \left(\frac{\Lambda_D^2}{\Lambda_D^2 - q^2}\right)^2$$

$$G_D(q^2) \;\to\; \left[\frac{\Lambda_D^2}{(\Lambda_D^2 - q^2)^2 + \Lambda_D^2\Gamma_D^2}\right]^2 \times$$

$$\left[(\Lambda_D^2 - q^2)^2 - \Lambda_D^2\Gamma_D^2 + i2(\Lambda_D^2 - q^2)\Lambda_D\Gamma_D\right]$$

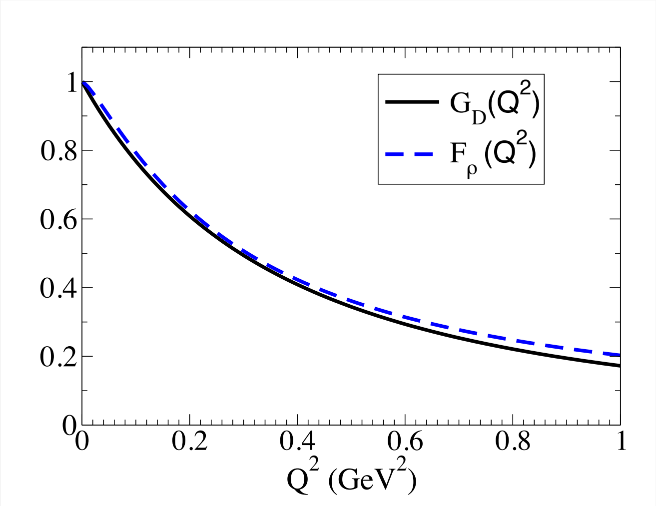
# Model 2 pion CLOUD

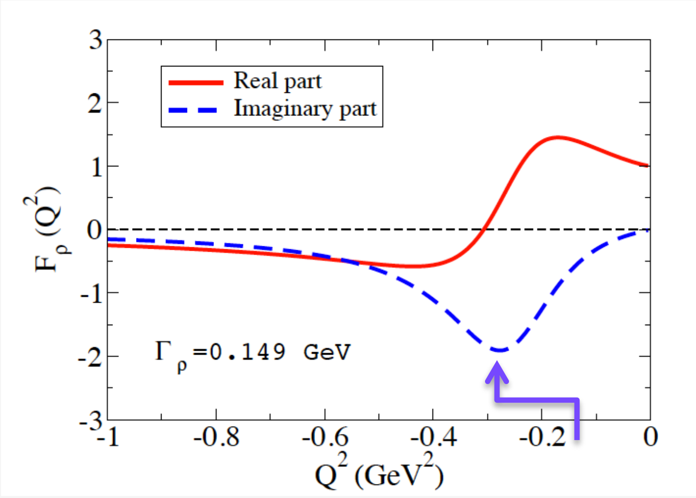
Inspiration: F. Iachello, A.D. Jackson, and Landé, PL 43, 191 (1973
F. Dohrman et al, Eur. Phys. J. A45, 401, (2010)

$$G_M^\pi(Q^2;W) = 3\lambda_\pi G_D(Q^2)\left(\frac{\Lambda_\pi^2}{\Lambda_\pi^2+Q^2}\right)^2$$

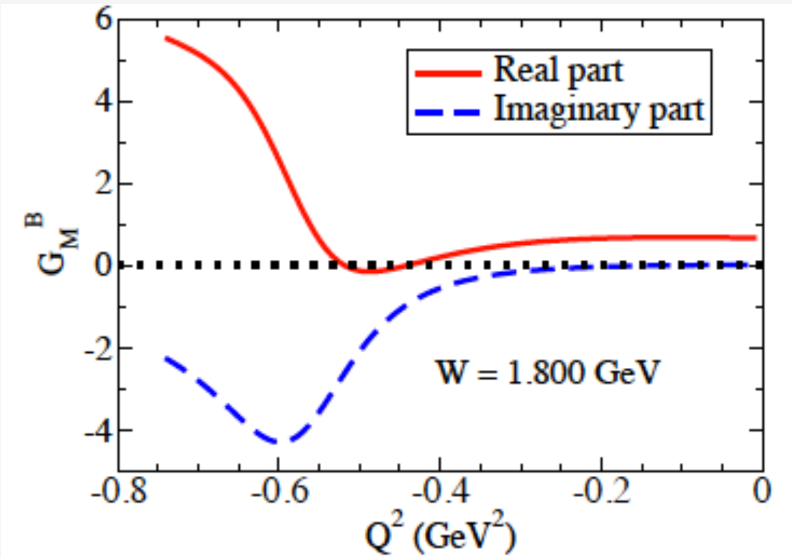
$$G_M^\pi(Q^2) = 3\lambda_\pi F_\rho(q^2)\left(\frac{\Lambda_\pi^2}{\Lambda_\pi^2-q^2}\right)^2$$

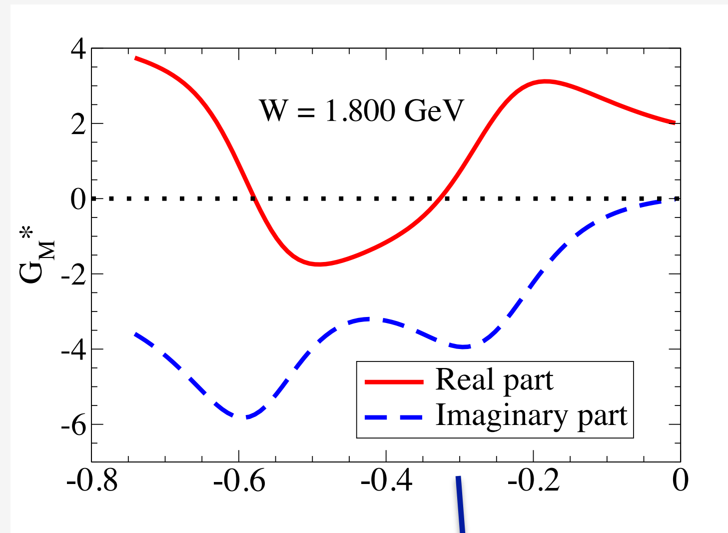
$$F_\rho(q^2) = \frac{m_\rho^2}{m_\rho^2 - q^2 - \frac{1}{\pi}\frac{\Gamma_\rho^0}{m_\pi}q^2\log\frac{q^2}{m_\pi^2} + i\frac{\Gamma_\rho^0}{m_\pi}q^2},$$



$$Q^2 \geq -(W-M)^2$$



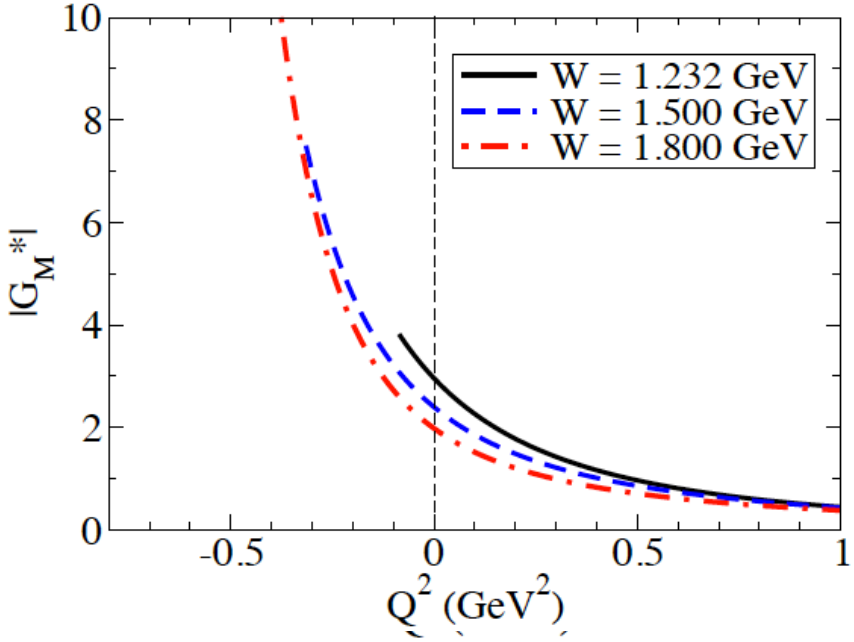




$$q^2 \approx m_\rho^2 \approx 0.3 \quad GeV^2$$

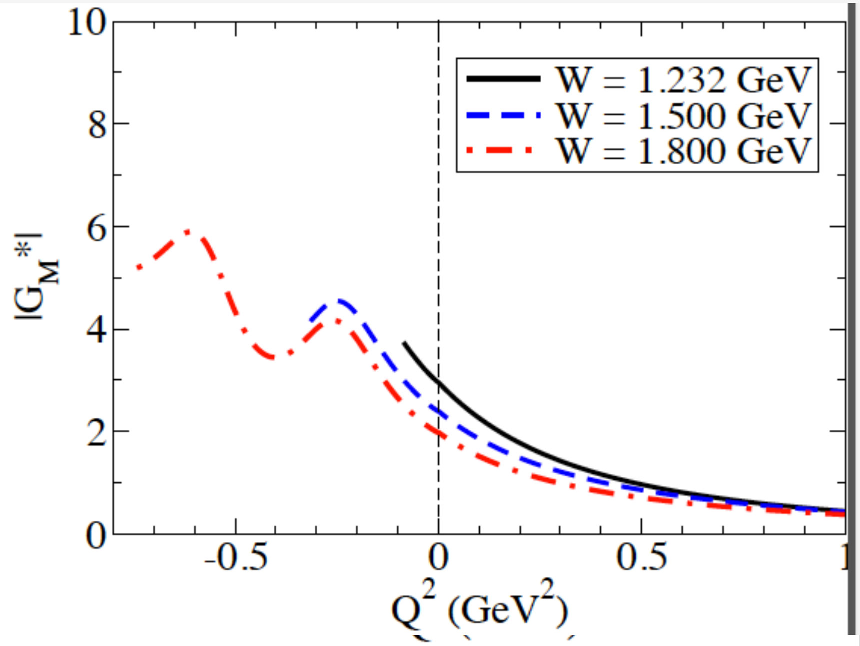
$$q^2 \approx m_\rho^2 \approx 0.6 \quad GeV^2$$

$$G_M^* = G_M^B + G_M^\pi$$

Model 1

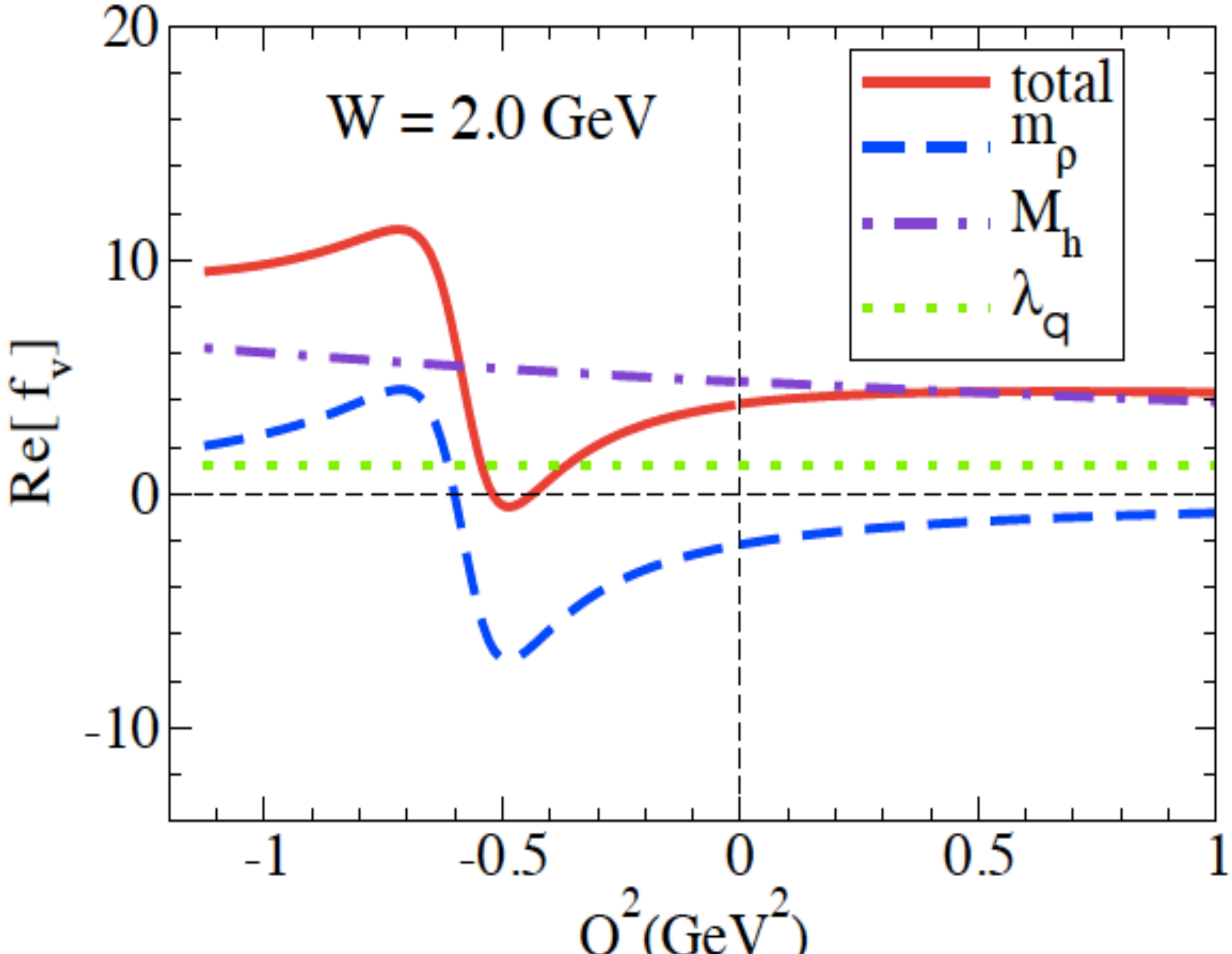



Model 2



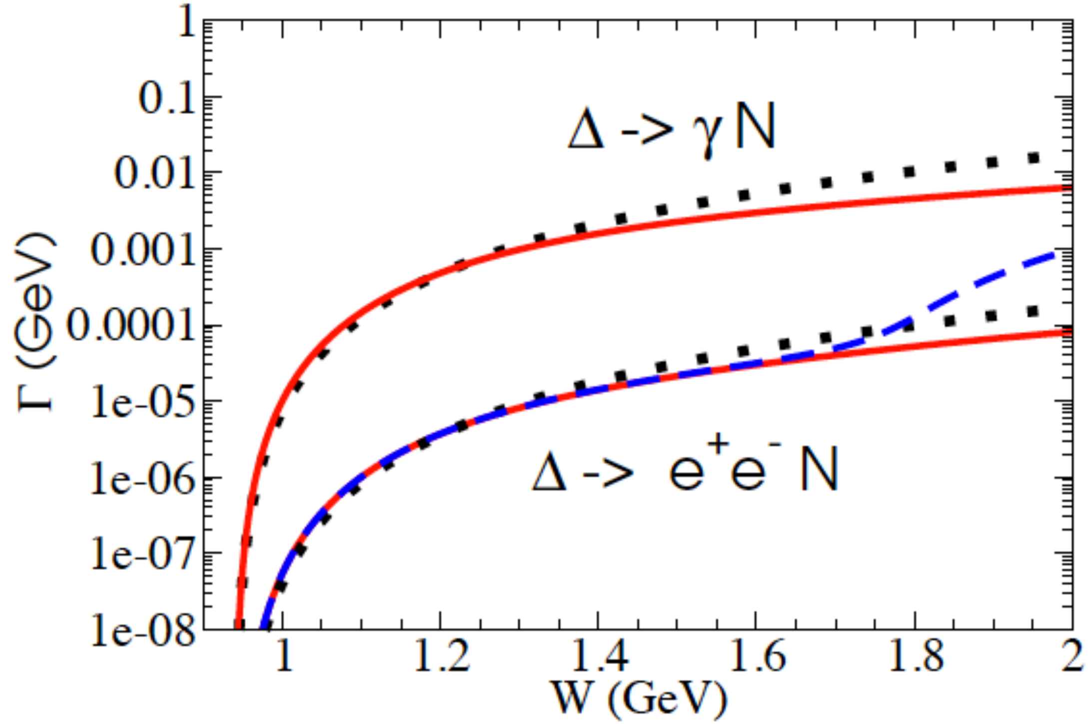

$$Q^2 \geq -(W - M)^2$$

$$G_M^B(Q^2) = \frac{8}{3\sqrt{3}} \frac{M}{M+W} f_v \int_k \psi_\Delta \psi_N \qquad f_v = f_{1-} + \frac{W+M}{2M} f_{2-}$$

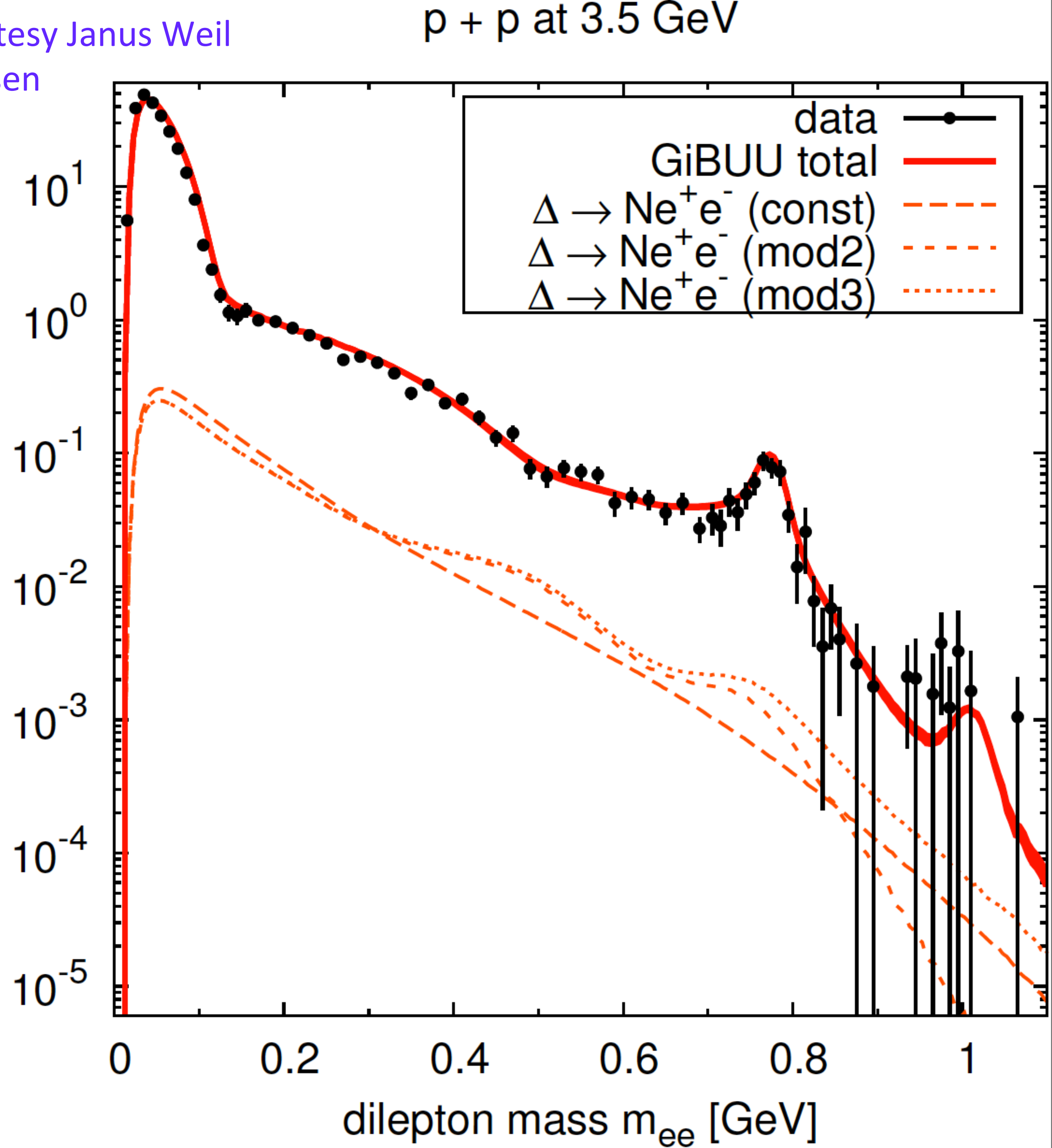


$$g_{\Delta}(W) \approx \frac{W^2\Gamma_{tot}(W)}{(W^2 - M_{\Delta}^2)^2 + W^2\left[\Gamma_{tot}(W)\right]^2}$$

$$\Gamma_{tot}(W) = \Gamma_{\pi N}(W) + \Gamma_{\gamma N}(W) + \Gamma_{e^+e^-N}(W)$$

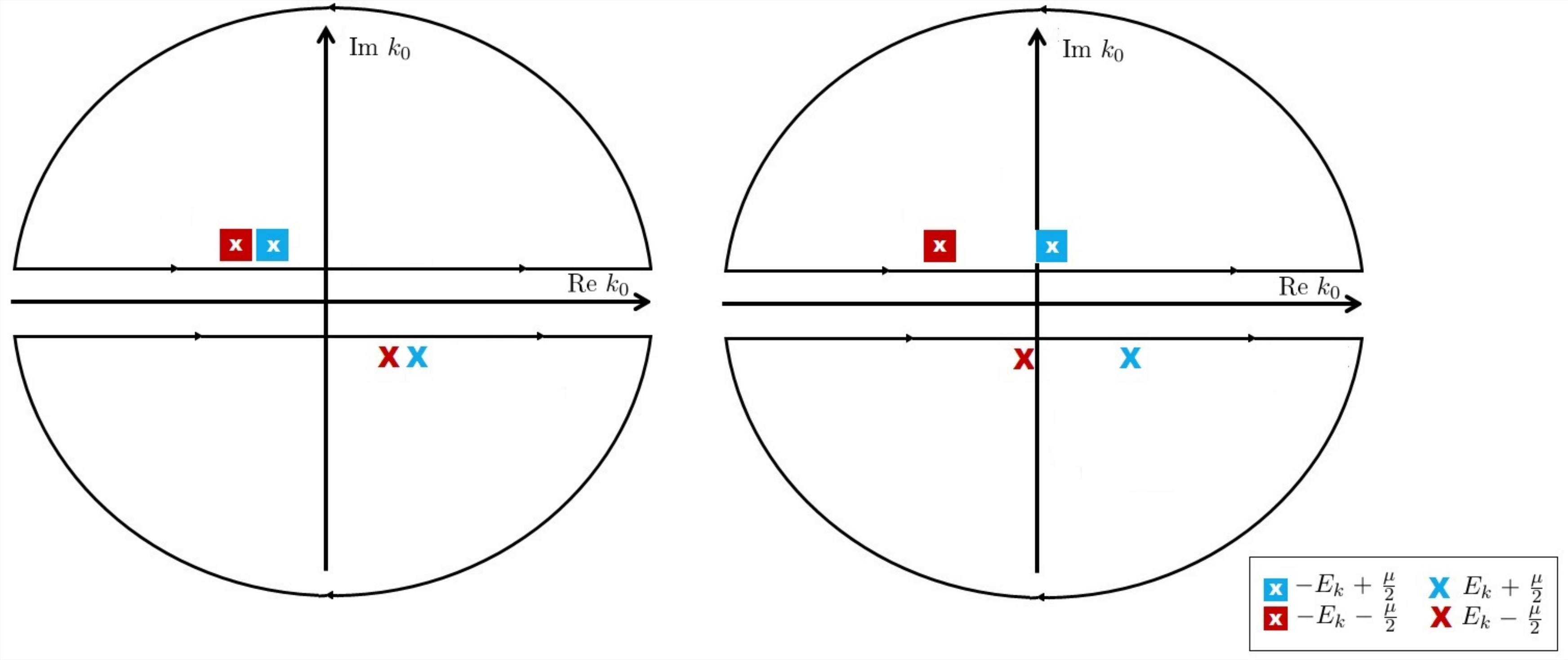


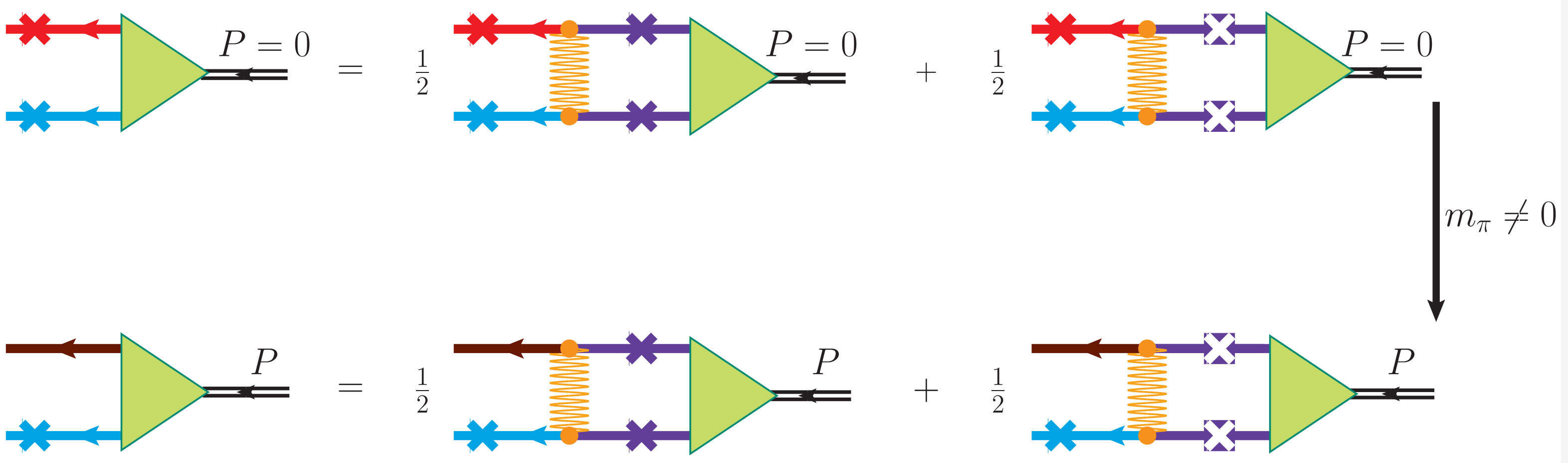
Courtesy Janus Weil
Giessen



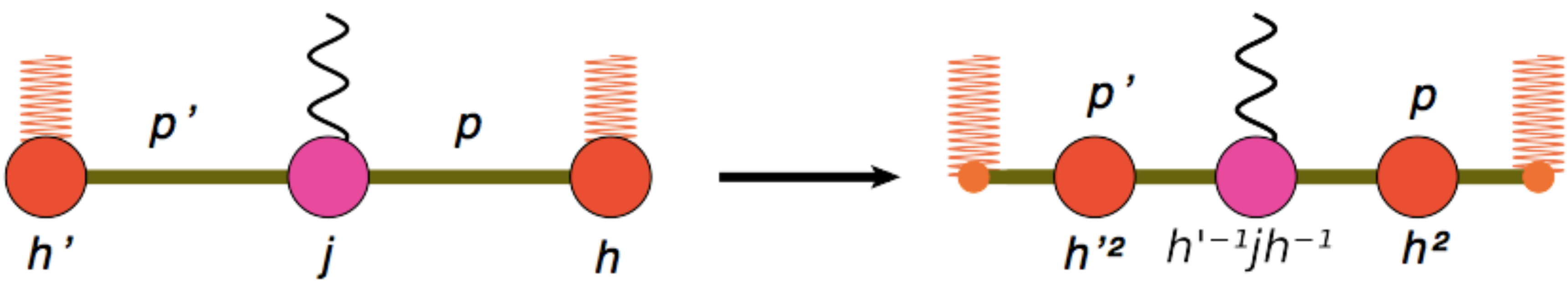
Small μ

Large μ






$\Rightarrow$ approximated pion vertex function $\Gamma(p, P) \sim \gamma^5 h(p^2)$

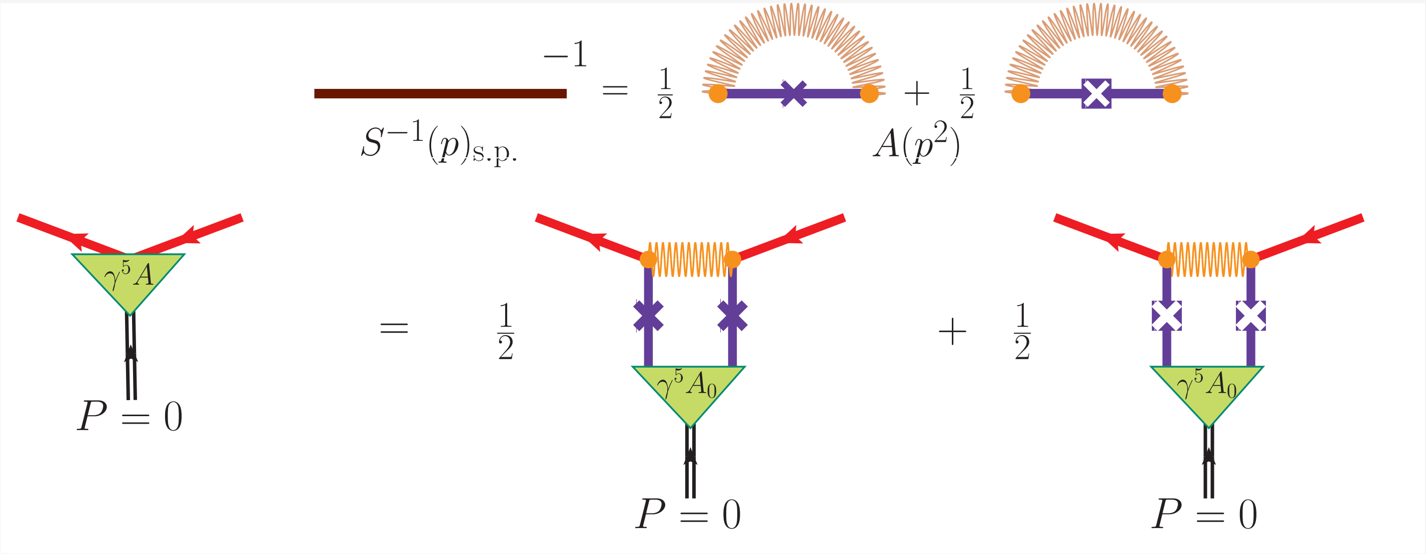


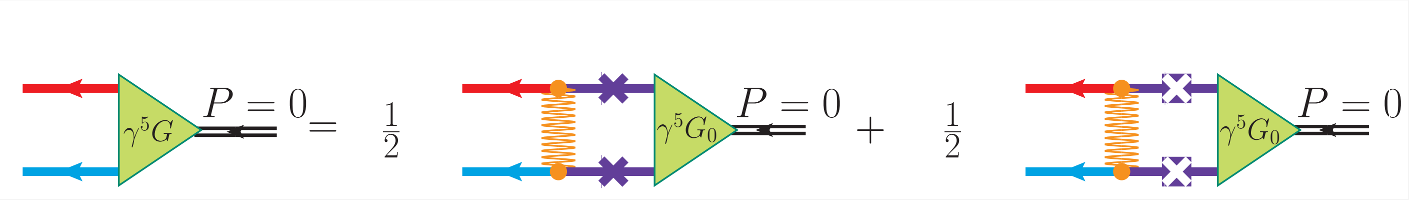
p '
p
h '
j
h
p '
p
h'2
h'−1jh−1
h2

# Chiral symmetry

Bare Quark mass =0

Scalar part of 1 body equation and two body equation are identical




A massless pion exists! Goldstone boson.

# Kernel

$$\mathcal{V}_L \propto \sigma[\lambda \mathbf{1}_1 \otimes \mathbf{1}_2 - (1-\lambda)\gamma_1^\mu \otimes \gamma_{2\mu}] V_L$$

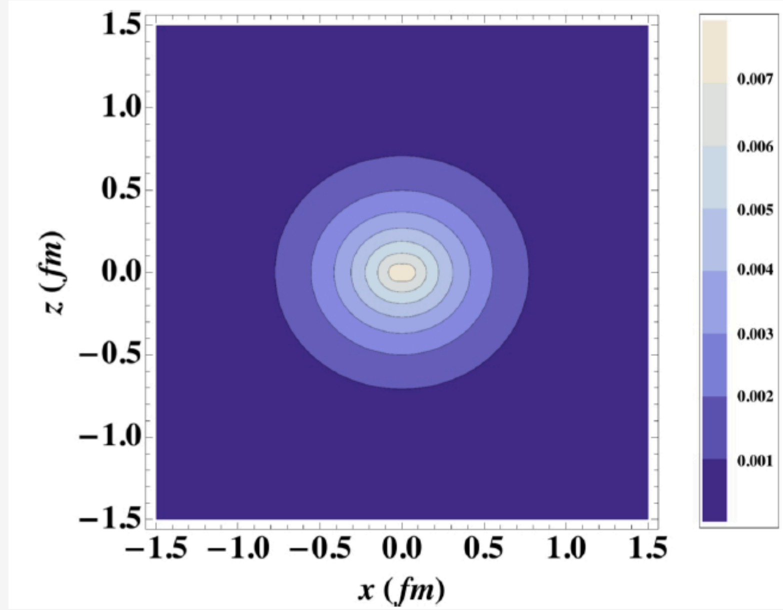
$$\mathcal{V}_C \propto C \gamma_1^\mu \otimes \gamma_2^\nu \left[ g_{\mu\nu} - (1-\xi) \frac{(p-k)_\mu (p-k)_\nu}{(p-k)^2} \right] V_C$$


Constituent quark mass **m**

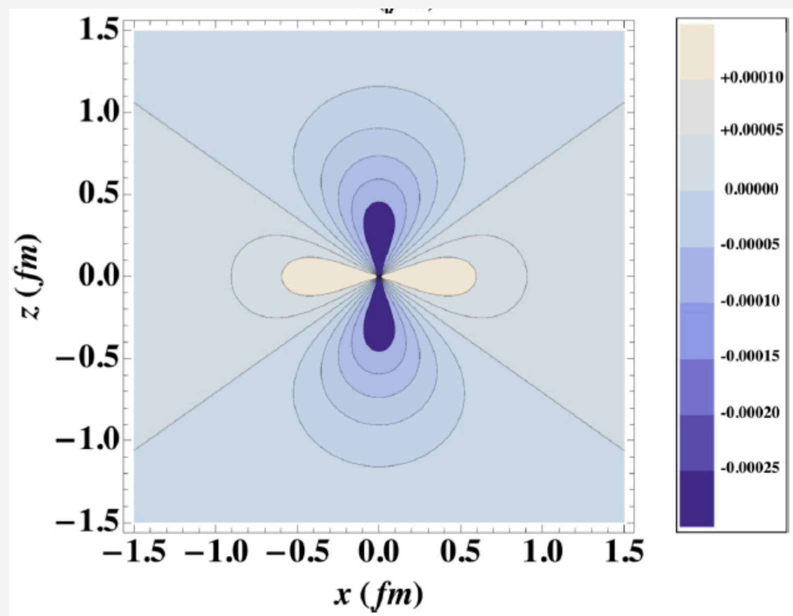
$$M(m^2) = m$$

Coordinate-space charge density in the x-y plane, for spin projection +3/2.

Total

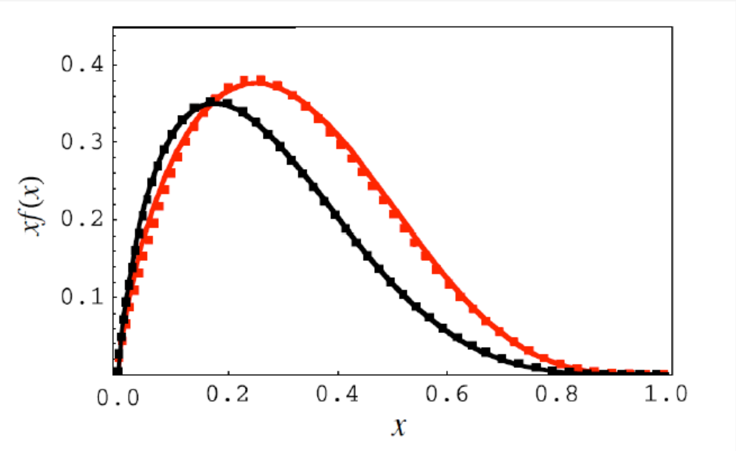



D-states



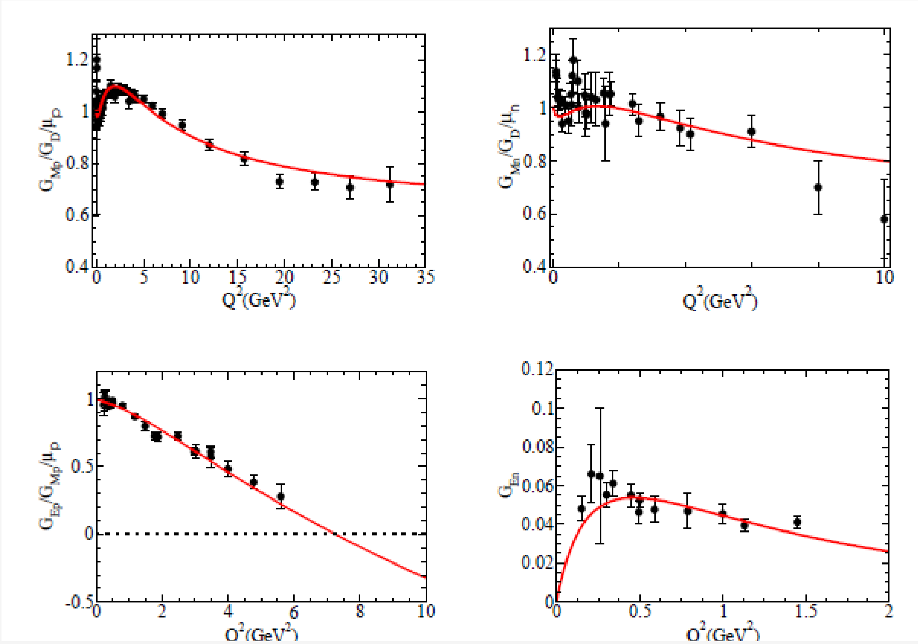

G. Ramalho, M. T. P. and A. Stadler, PHYSICAL REVIEW D 86 093022 (2012)

## Structure function




$$f_q(x) = \frac{M m_s \lambda^2}{16\pi^2} \int_\xi^\infty d\chi \left[\psi_q(\chi)\right]^2 \quad \text{with} \quad \xi = \frac{(r+x-1)^2}{r(1-x)}$$

## Proton and Neutron form factors



$$\chi^2 = 1.36$$

G. Ramalho, M. T. P. and Franz Gross, PHYSICAL REVIEW C 77, 015202 (2008)

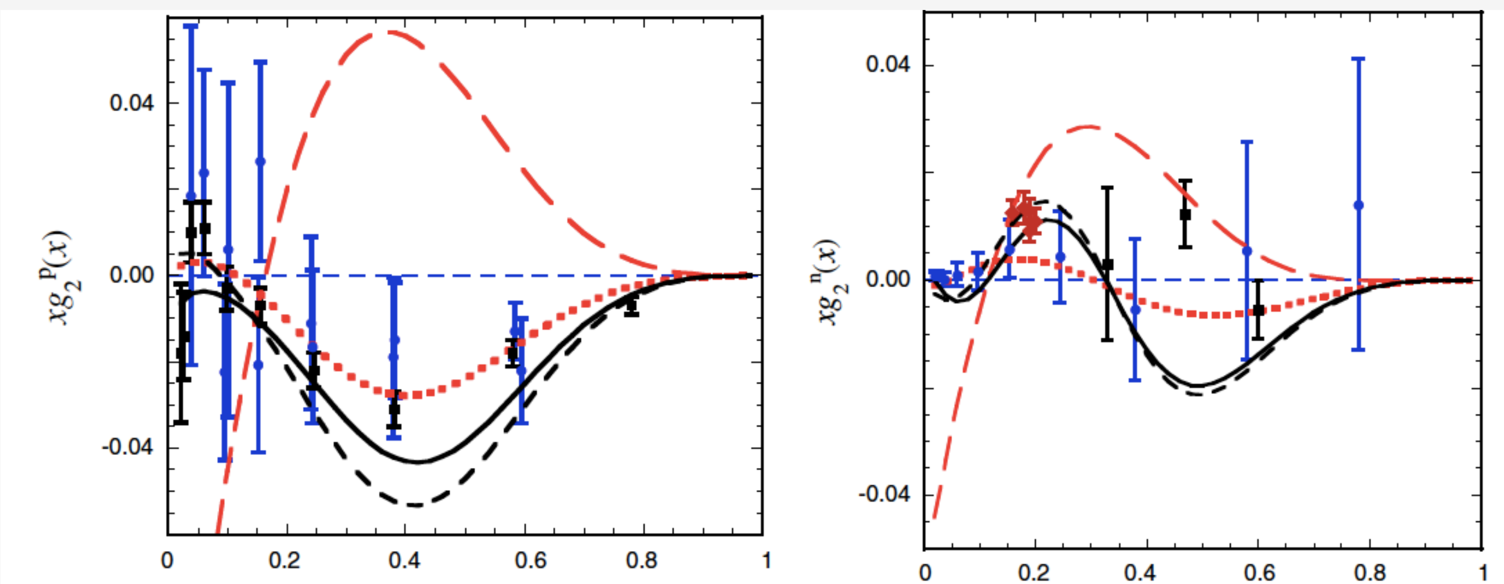
Franz Gross, G. Ramalho, M. T. P. PHYSICAL REVIEW D 85, 093006 (2012)

## Also:

Δ(1600),
Baryon decuplet
**DIS**

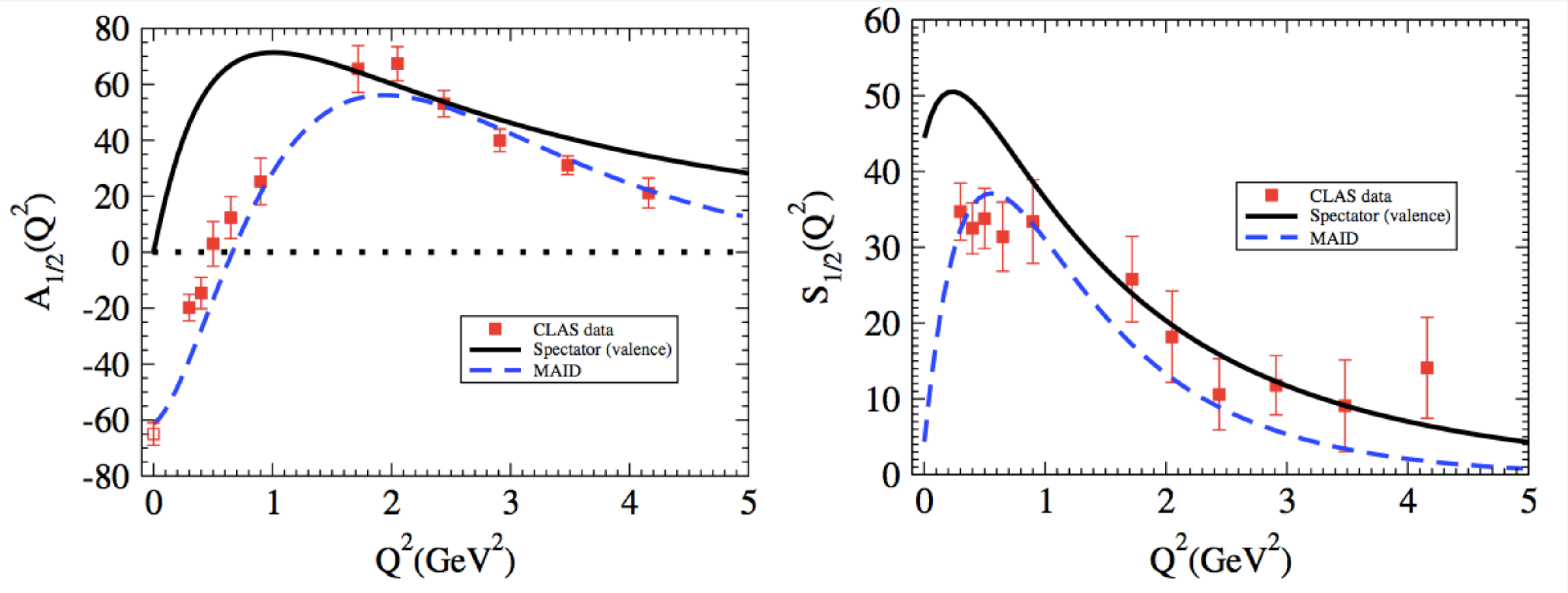
Description of general size and shape of proton and neutron structure functions

**Model 1- P 18%  D 3%   ….. No P wave**
**Model 2- P 0.6% D 35% ---- No P wave**

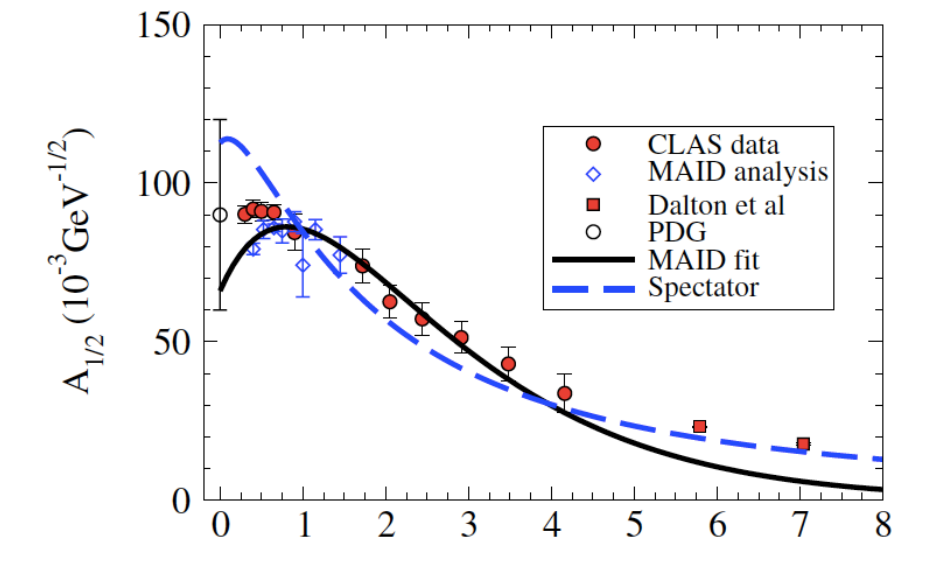


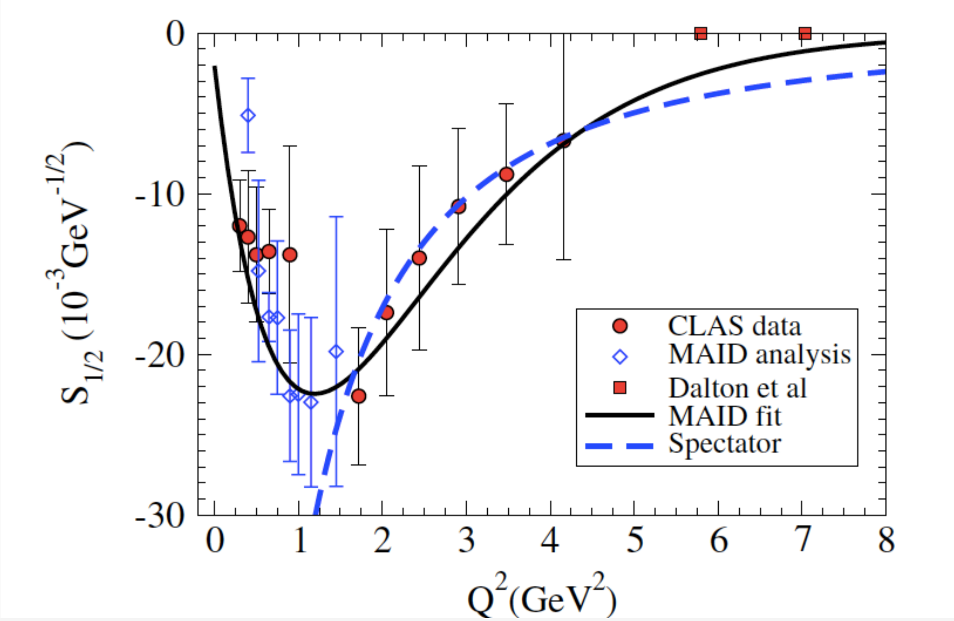

Franz Gross, G. Ramalho, M. T. P.    PHYSICAL REVIEW D 85, 093006 (2012)

# $N \rightarrow N^*(1440)$

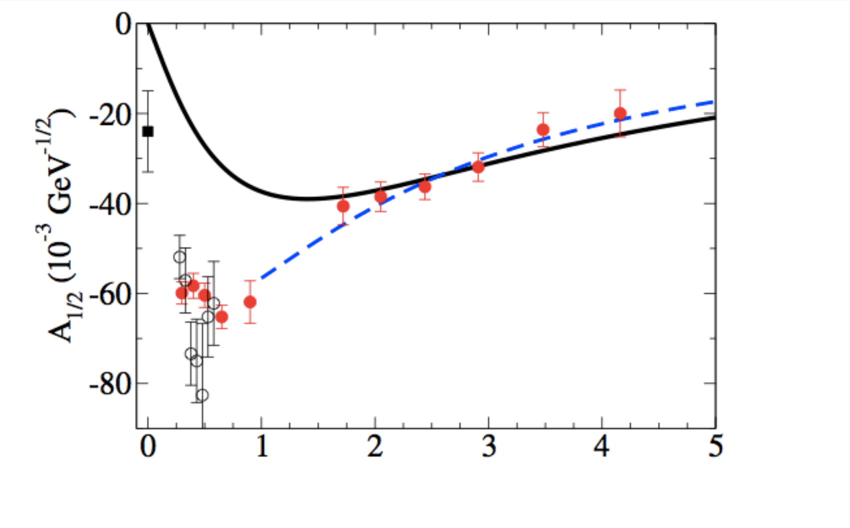


# $N \rightarrow N^*(1535)$

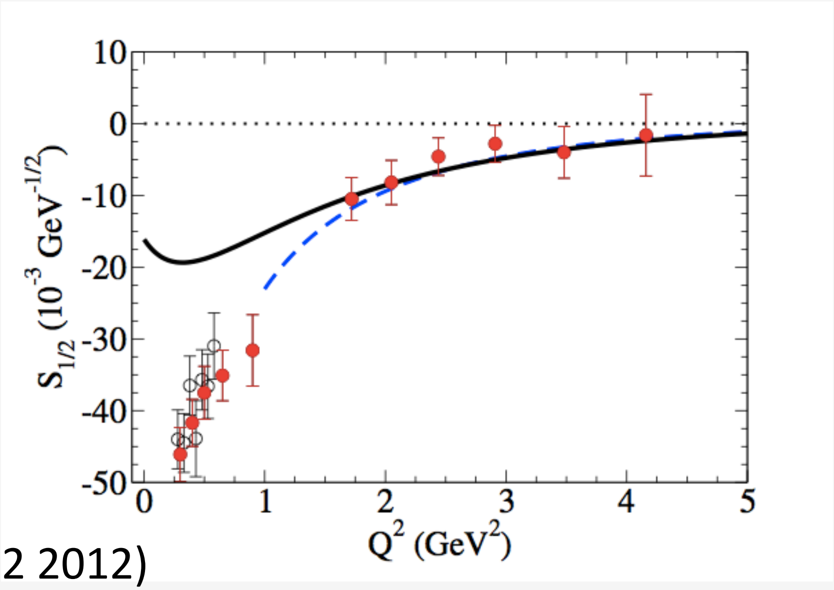


# $N \rightarrow N^*(1520)$




- Radial wf identical to nucleon's; angular momentum different (P wave)
- Good description of high $Q^2$ region behavior
- Orthogonality with nucleon through extra term
- One parameter fit to the data for $Q^2 > 1.5$ GeV²




**$A_{1/2}$ mixes** dominates
(Aznauryan and Burkert, PRC 85 055202 2012)

## Meson Cloud

**$G_1$, $G_4$, $G_c$**

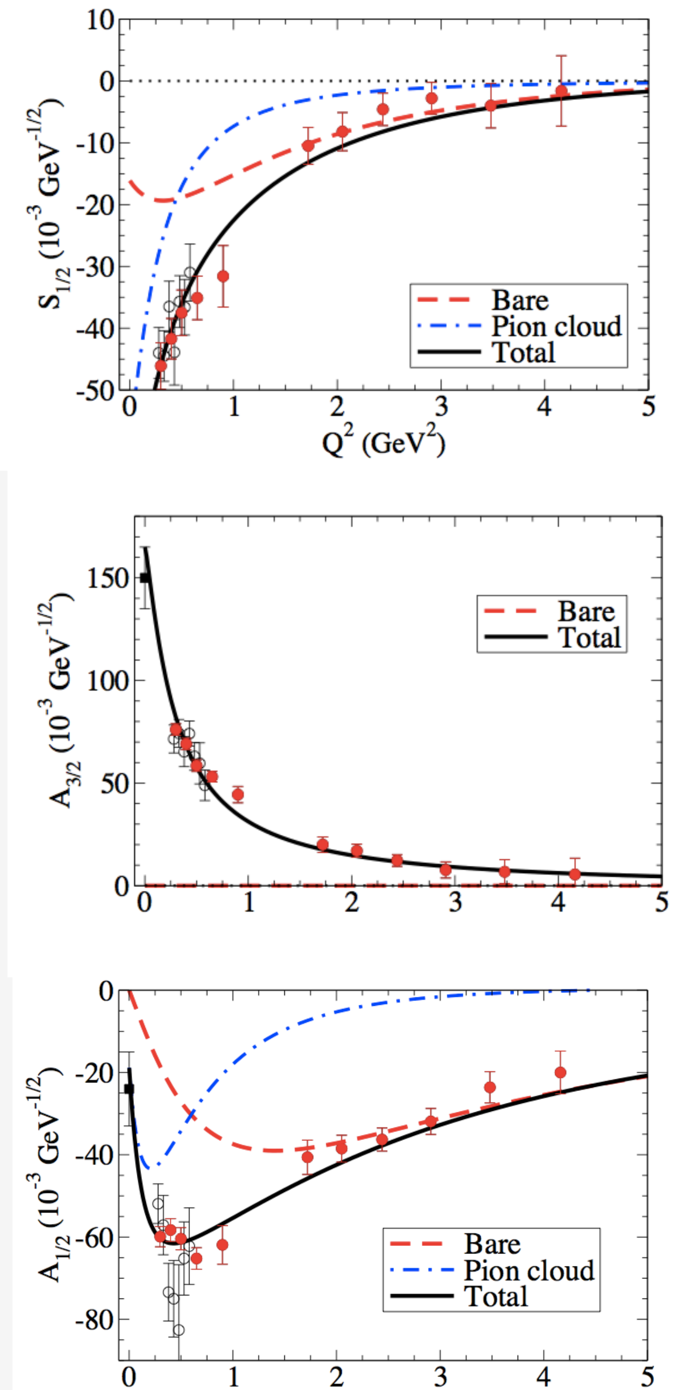
$$A_{3/2} = 2\sqrt{3}\mathcal{A}G_4,$$
$$A_{1/2} = 2\mathcal{A}\left\{G_4 - \left[(M_R - M)^2 + Q^2\right]\frac{G_1}{M_R}\right\}$$
$$S_{1/2} = -\frac{1}{\sqrt{2}}\frac{|\mathbf{q}|}{M_R}\mathcal{A}g_C,$$

- **$S_{1/2}$** → meson cloud term to **$G_c$** is extracted
- **$A_{3/2}$** → meson cloud term to **$G_4$** is extracted.

- **$A_{1/2}$ mixes** meson contributions to the different form factors (Aznauryan and Burkert, PRC 85 055202 2012)

- **A global fit** of the three amplitudes, indirectly constraining $A_{3/2}$ by $A_{1/2}$, is needed.

.



# N(1535)

$$F_1^*(Q^2) = \frac{1}{2}(3j_1 + j_3)\mathcal{I}_0$$
$$F_2^*(Q^2) = -\frac{1}{2}(3j_2 - j_4)\frac{M_S + M}{2M}\mathcal{I}_0$$

$$A_{1/2} = -2b\left[F_1^* + \frac{M_S - M}{M_S + M}F_2^*\right]$$
$$S_{1/2} = \sqrt{2}b(M_S + M)\frac{|\mathbf{q}|}{Q^2}\left[\frac{M_S - M}{M_S + M}F_1^* - \tau F_2^*\right]$$

$$\Gamma^{\beta\mu} = G_1 q^\beta \gamma^\mu + G_2 q^\beta P^\mu + G_3 q^\beta q^\mu - G_4 g^{\beta\mu}$$

## $N \rightarrow N^*(1520)$

$$
\begin{aligned}
G_M &= -F\left(\frac{1}{\sqrt{3}}A_{3/2} - A_{1/2}\right) \\
&= -\mathcal{R}\left[(M_R - M)^2 + Q^2\right]\frac{G_1}{M_R}, \\
G_E &= -F\left(\sqrt{3}A_{3/2} + A_{1/2}\right) \\
&= -\mathcal{R}\left\{2G_4 - \left[(M_R - M)^2 + Q^2\right]\frac{G_1}{M_R}\right\}
\end{aligned}
$$

$$
G_C = 2\sqrt{2}\frac{M_R}{|\mathbf{q}|}F\,S_{1/2} = -\mathcal{R}g_C,
$$

Coupling core spin states
with orbital angular momentum states

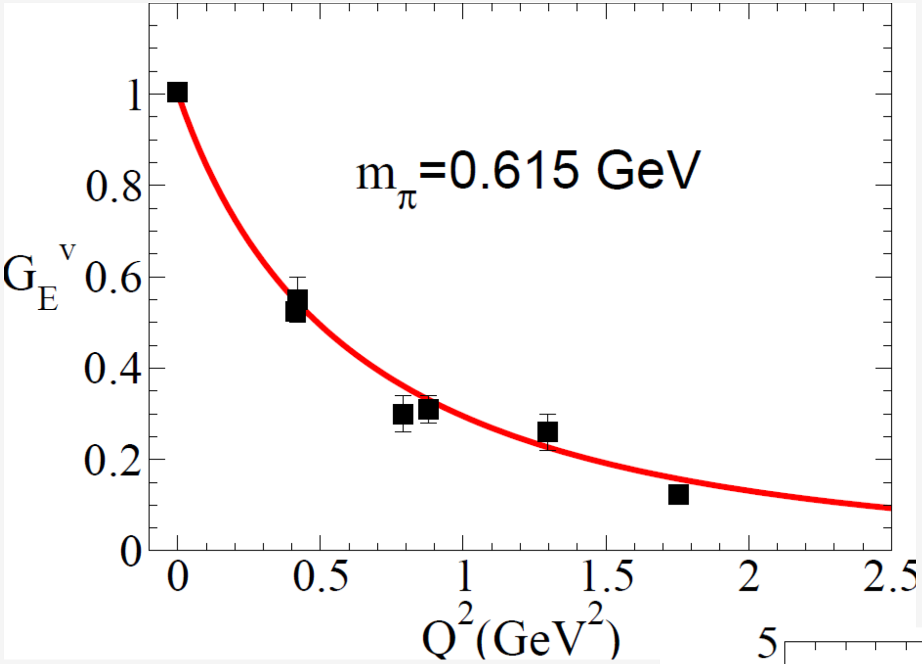
$$V_S^{\alpha}(P,\lambda_s) = \sum_{\lambda}\left\langle \frac{1}{2}\lambda 1 \lambda' | S\lambda_s \right\rangle \varepsilon^{\alpha}_{\lambda' P} u_{\Delta}(P,\lambda),$$

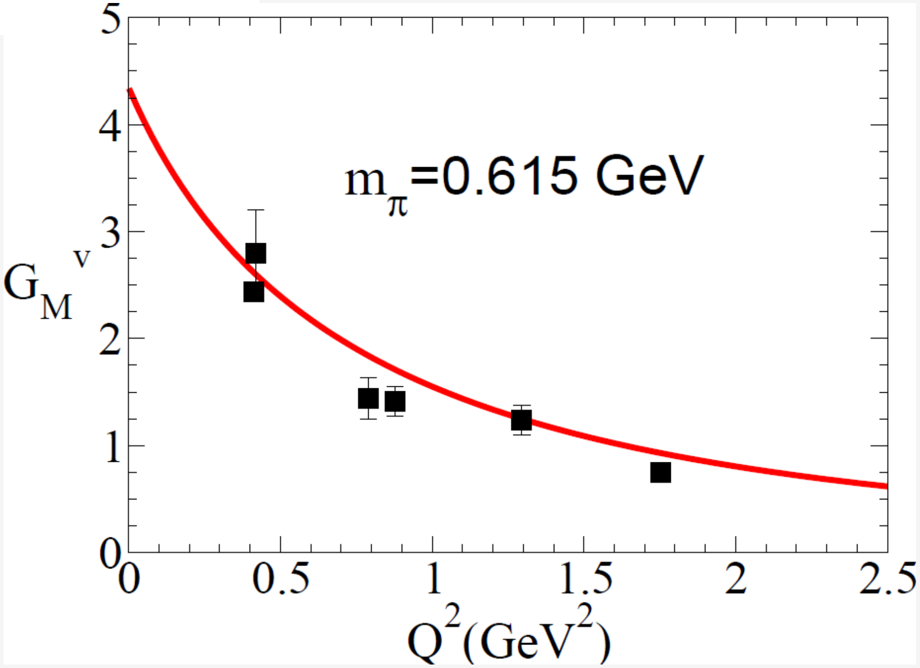
**Delta**

$$S=\frac{1}{2}, S=\frac{3}{2}$$

$$J=\frac{3}{2} \rightarrow S=\frac{3}{2}\otimes L=0;\; S=\frac{3}{2}\otimes L=2;\; S=\frac{1}{2}\otimes L=2$$

**S** state **D3** state **D1** state




LQCD data: Gockeler et al. PRD 71, 034508 (2005)

$i = 1, 2,$

$$j_i = \tfrac{1}{6} f_{i+}(Q^2) + \tfrac{1}{2} f_{i-}(Q^2)\tau_3 \tag{21}$$

where $f_{i\pm}$ are the isoscalar and isovector combinations, related to the $u$ and $d$ quark form factors by

$$\begin{aligned} \tfrac{2}{3} f_{iu} &= \tfrac{1}{6} f_{i+} + \tfrac{1}{2} f_{i-} \\ -\tfrac{1}{3} f_{id} &= \tfrac{1}{6} f_{i+} - \tfrac{1}{2} f_{i-} \,. \end{aligned} \tag{22}$$

The form factors are normalized (with $n = \{u, d\}$) to

$$\begin{aligned} f_{1n}(0) &= 1 \qquad & f_{2n}(0) &= \kappa_n \\ f_{1\pm}(0) &= 1 \qquad & f_{2\pm}(0) &= \kappa_\pm \end{aligned} \tag{23}$$

where $\kappa_u$ and $\kappa_d$ are the $u$ and $d$ quark anomalous magnetic moments (scaled by the quark charges) and

$$\begin{aligned} \kappa_+ &= 2\kappa_u - \kappa_d \\ \kappa_- &= \tfrac{2}{3}\kappa_u + \tfrac{1}{3}\kappa_d \,. \end{aligned} \tag{24}$$

$$\mu_p = 1 + \frac{1}{6}(\kappa_+ + 5\kappa_-)$$
$$\mu_n = -\frac{2}{3} + \frac{1}{6}(\kappa_+ - 5\kappa_-)$$

$$\kappa_+ = 3(\mu_p + \mu_n) - 1 = 1.639$$
$$\kappa_- = \frac{3}{5}(\mu_p - \mu_n) - 1 = 1.823$$

$$G_E(Q^2) = F_1(Q^2) - \tau F_2(Q^2)$$
$$= \frac{1}{2}B(Q^2)\Big\{ (f_{1+} + \tau_3 f_{1-}) - \tau\,(f_{2+} + \tau_3 f_{2-}) \Big\}$$
$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$
$$= \frac{1}{6}B(Q^2)\Big\{ (f_{1+} + 5\tau_3 f_{1-}) + (f_{2+} + 5\tau_3 f_{2-}) \Big\}$$

**Vector meson dominance**

approximated by a single pole

⇩

$f_{1+} = f_{1-}$ and $f_{2+} = f_{2-}$

$G_{En}$ is identically zero **!**

| Model | $\beta_1, \beta_2$ | $c_+, c_-$ | $d_+, d_-$ | $b_E, b_M$ | $\lambda, r$ | $N_0^2, \chi^2$ |
|---|---|---|---|---|---|---|
| I(4) | 0.057 | 2.06 | −0.444 | −− | 1.22 | 10.87 |
| | 0.654 | 2.06* | −0.444* | −− | 0.88 | 9.26 |
| II(5) | 0.049 | 4.16 | −0.686 | −− | 1.21 | 11.27 |
| | 0.717 | 1.56 | −0.686* | −− | 0.87 | 1.36 |
| III(6) | 0.078 | 1.91 | −0.319 | 0.163 | 1.27 | 12.36 |
| | 0.598 | 1.91* | −0.319* | 0.311 | 0.89 | 1.85 |
| IV(9) | 0.086 | 4.48 | −0.134 | 0.079 | 1.25 | 8.46 |
| | 0.443 | 2.45 | −0.513 | 0.259 | 0.89 | 1.03 |

**Not always # parameters larger means better description**

$$\int_{sk} = \underbrace{\int \frac{d\Omega_{\hat{\mathbf{r}}}}{4(2\pi)^3} \int_{4m_q^2}^{\infty} ds \sqrt{\frac{s - 4m_q^2}{s}}}_{\int_s} \underbrace{\int \frac{d^3k}{(2\pi)^3 2E_s}}_{\int_k},$$

$$\sum_{\lambda_1\lambda_2} \int_s \bar{\Psi}_{\lambda_1\lambda_2,\lambda_+}(P_+, k_1k_2) \otimes \Psi_{\lambda_1\lambda_2;\lambda_-}(P_-, k_1k_2)$$

$$\equiv \sum_{\Lambda} \bar{\Psi}_{\Lambda\lambda_+}(P_+, k) \otimes \Psi_{\Lambda\lambda_-}(P_-, k)\big|_{s=m_s^2},$$

For very large masses ($E_s \to m_s$; $s \to 4m_q^2$), we can

$$m_q m_s \int_{sk} \to \frac{1}{16} \int \frac{d^3k}{(2\pi)^3} \int \frac{d\Omega_{\hat{\mathbf{r}}}}{(2\pi)^3} \int_{4m_q^2}^{\infty} ds \sqrt{s - 4m_q^2}$$

$$= \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3r}{(2\pi)^3},$$

# NΔ transition: State D1

## State $(2, \frac{1}{2})$ is not orthogonal to $(0, \frac{1}{2})$

In principle: $$q_\mu J^\mu = 3(M_\Delta - M) j_1 \sum_\lambda \int_k \bar{\Psi}_\Delta \Psi_N \neq 0.$$

There is a chance that $G_C^* \neq 0$; but $q_\mu J^\mu \neq 0$

Imposing current conservation

$$J_R^\mu = 3 j_1 \sum_\lambda \int_k \bar{\Psi}_\Delta \left( \gamma^\mu - \frac{\not{q} q^\mu}{q^2} \right) \Psi_N + 3 j_2 \sum_\lambda \int_k \bar{\Psi}_\Delta \frac{i \sigma^{\mu\nu} q_\nu}{2M} \Psi_N$$

$$q_\mu J_R^\mu = 0, \qquad G_C^* \; \alpha \; \frac{1}{Q^2} \sum_\lambda \int_k \bar{\Psi}_\Delta \Psi_N$$

To avoid divergence as $Q^2 \to 0$:

$$\sum_\lambda \int_k \bar{\Psi}_\Delta \Psi_N \sim Q^2 \qquad \text{[Orthogonality]}$$

Delta

$$J^\mu = -\bar{w}_\alpha(P_+)\left\{\left[F_1^*(Q^2)g^{\alpha\beta} + F_3^*(Q^2)\frac{q^\alpha q^\beta}{4M_\Delta^2}\right]\gamma^\mu + \left[F_2^*(Q^2)g^{\alpha\beta} + F_4^*(Q^2)\frac{q^\alpha q^\beta}{4M_\Delta^2}\right]\frac{i\sigma^{\mu\nu}q_\nu}{2M_\Delta}\right\}w_\beta(P_-)$$

$$G_{E0}(Q^2) = [F_1^* - \tau F_2^*]\left(1+\frac{2}{3}\tau\right) - \frac{1}{3}[F_3^* - \tau F_4^*]\,\tau\,(1+\tau)$$

$$G_{M1}(Q^2) = [F_1^* + F_2^*]\left(1+\frac{4}{5}\tau\right) - \frac{2}{5}[F_3^* + F_4^*]\,\tau\,(1+\tau)$$

$$G_{E2}(Q^2) = [F_1^* - \tau F_2^*] - \frac{1}{2}[F_3^* - \tau F_4^*](1+\tau)$$

$$G_{M3}(Q^2) = [F_1^* + F_2^*] - \frac{1}{2}[F_3^* + F_4^*](1+\tau)$$

# $\gamma\Delta \rightarrow \Delta$

$$J^\mu = \bar{w}_\alpha(P_+)\Gamma^{\alpha\beta\mu}(P,q)w(P_-)_\beta(P_+)$$

$$\begin{aligned} J^\mu \;=\; & -\bar{w}_\alpha(P_+)\left\{\left[F_1^*(Q^2)g^{\alpha\beta} + F_3^*(Q^2)\frac{q^\alpha q^\beta}{4M_\Delta^2}\right]\gamma^\mu \right. \\ & \left. + \left[F_2^*(Q^2)g^{\alpha\beta} + F_4^*(Q^2)\frac{q^\alpha q^\beta}{4M_\Delta^2}\right]\frac{i\sigma^{\mu\nu}q_\nu}{2M_\Delta}\right\} w_\beta(P_-) \end{aligned}$$

4: $G_{E0}(Q^2)$ $G_{M1}(Q2)$ $G_{E2}(Q2)$ $G_{M3}(Q2)$

PRC77 015202 (2008); PRD78 114017(2008); JPG36 085004 (2009)

## $\gamma\Delta \to \Delta$

**a** and **b** small

$$G_{E0}(Q^2) = N^2 \tilde{g}_\Delta \mathcal{I}_S$$

$$G_{M1}(Q^2) = N^2 \tilde{f}_\Delta \left[\mathcal{I}_S + \frac{4}{5} a \mathcal{I}_{D3} - \frac{2}{5} b \mathcal{I}_{D1}\right]$$

$$G_{E2}(Q^2) = 3(aN^2)\tilde{g}_\Delta \frac{\mathcal{I}_{D3}}{\tau}$$

$$G_{M3}(Q^2) = \tilde{f}_\Delta N^2 \left[a\frac{\mathcal{I}_{D3}}{\tau} + 2b\frac{\mathcal{I}_{D1}}{\tau}\right]$$

D state corrections

from overlap

Integrals between

S and D states

$\gamma\Delta \to \Delta$

**a** and **b** small

$$G_{E0}(Q^2) = N^2 \tilde{g}_\Delta \mathcal{I}_S$$

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$$G_{E2}(Q^2) = 3(aN^2)\tilde{g}_\Delta \frac{\mathcal{I}_{D3}}{\tau}$$

$$G_{M3}(Q^2) = \tilde{f}_\Delta N^2 \left[a \frac{\mathcal{I}_{D3}}{\tau} + 2b \frac{\mathcal{I}_{D1}}{\tau}\right]$$

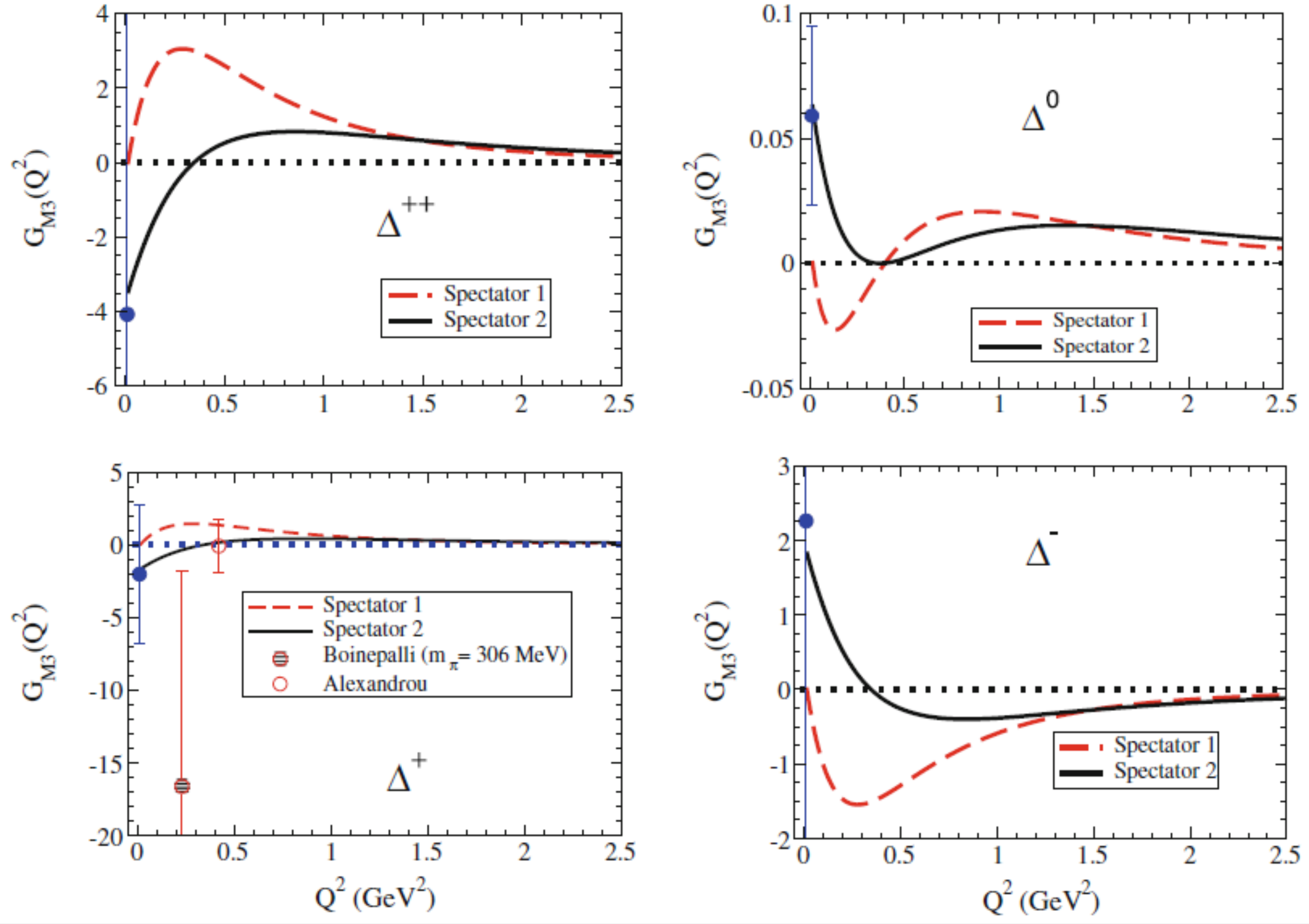
D state corrections

from overlap

Integrals between

S and D states

# $\gamma\Delta \rightarrow \Delta$



| | $\mathcal{Q}^{\perp}_{\Delta}\,(+\frac{3}{2})$ | $\mathcal{O}^{\perp}_{\Delta}\,(+\frac{3}{2})$ |
|---|---|---|
| Lattice QCD: | | |
| Quenched [6] | 0.83±0.21 | |
| Wilson [6] | 0.46±0.35 | |
| Hybrid [6] | 0.74±0.68 | |
| Spectator quark models: | | |
| Spectator S [15] | 0.29 | -3.44 |
| Spectator SD [17] | 0.92 | -3.38 |

TABLE I: Transverse electric quadrupole moment $\mathcal{Q}^{\perp}_{\Delta}\,(+\frac{3}{2})$ in units of $\frac{e}{M^2_{\Delta}}$, and transverse magnetic octupole moment $\mathcal{O}^{\perp}_{\Delta}\,(+\frac{3}{2})$ in units of $\frac{e}{2M^3_{\Delta}}$, for the $\Delta^+$.

G. Ramalho, M. T. P., A. Stadler arXiv: 1297.4392, Phys. Rev. D (to appear)

- Helicity states are usually used to define polarization.
  In the $x - z$ plane: $k = (E_k, k\cos\theta, 0, k\sin\theta)$

$$\xi(0) = \frac{1}{m}\begin{bmatrix} k \\ E_k \sin\theta \\ 0 \\ E_k \cos\theta \end{bmatrix}, \qquad \xi(\pm) = \mp\frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ \cos\theta \\ \pm i \\ -\sin\theta \end{bmatrix}$$

  $\xi(\lambda)$ is $\theta-$ dependent; $\quad k \cdot \xi = 0$

- Fixed-axis: vector particle is bound to a system with $P = (P_0, 0, 0, \mathrm{P})$:

$$\varepsilon(0) = \frac{1}{M}\begin{bmatrix} P \\ 0 \\ 0 \\ P_0 \end{bmatrix}, \qquad \varepsilon(\pm) = \mp\frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ \pm i \\ 0 \end{bmatrix}$$

  No angular dependence; $\quad P \cdot \varepsilon = 0$

arXiv:0708.0995 [nucl-th]

**By design, a quark-diquark system in S wave**

**Di-quark first**

$$\xi^0 = \frac{1}{\sqrt{2}}(ud - du)$$
$$\xi_0^1 = \frac{1}{\sqrt{2}}(ud + du) = \xi_z$$
$$\xi_+^1 = uu = -\frac{1}{\sqrt{2}}(\xi_x + i\xi_y)$$
$$\xi_-^1 = dd = \frac{1}{\sqrt{2}}(\xi_x - i\xi_y) \; .$$

$$\phi_{\frac{1}{2}}^1 = \sqrt{\tfrac{2}{3}}\begin{pmatrix}0\\1\end{pmatrix}\xi_+^1 - \sqrt{\tfrac{1}{3}}\begin{pmatrix}1\\0\end{pmatrix}\xi_0^1$$
$$\to \sqrt{\tfrac{1}{6}}\,[2d(uu) - u(ud + du)]\,,$$

$$\phi_I^0 = \xi^{0*}\chi^I \tag{2}$$
$$\phi_I^1 = -\frac{1}{\sqrt{3}}\tau\cdot\xi^{1*}\chi^I$$
$$= \frac{1}{\sqrt{6}}\left[\tau_-\xi_+^1 - \tau_+\xi_-^1 - \sqrt{2}\tau_3\xi_0^1\right]\chi^I \tag{3}$$

where $\tau_\pm = \tau_x \pm i\tau_y$ are the isospin raising and lowering operators, $I = \pm 1/2$ is the isospin of the quark (or nucleon)

$$\chi^{+\frac{1}{2}} = \begin{pmatrix}1\\0\end{pmatrix} = u\,(\text{or}\,p) \quad \chi^{-\frac{1}{2}} = \begin{pmatrix}0\\1\end{pmatrix} = d\,(\text{or}\,n), \tag{4}$$

$$\Phi^i(s) = -\frac{1}{\sqrt{3}}\sigma_i \chi_s$$

$$\Phi^i \longrightarrow u_S^\alpha(s) = -\frac{1}{\sqrt{3}}\gamma_5\gamma^\alpha u(s)$$

$$m_\rho = c_0 + c_1 m_\pi^2 ,$$