Universität Bielefeld

# (some) Recent Developments in Lattice Studies for Dilepton Rates and Transport Coefficients


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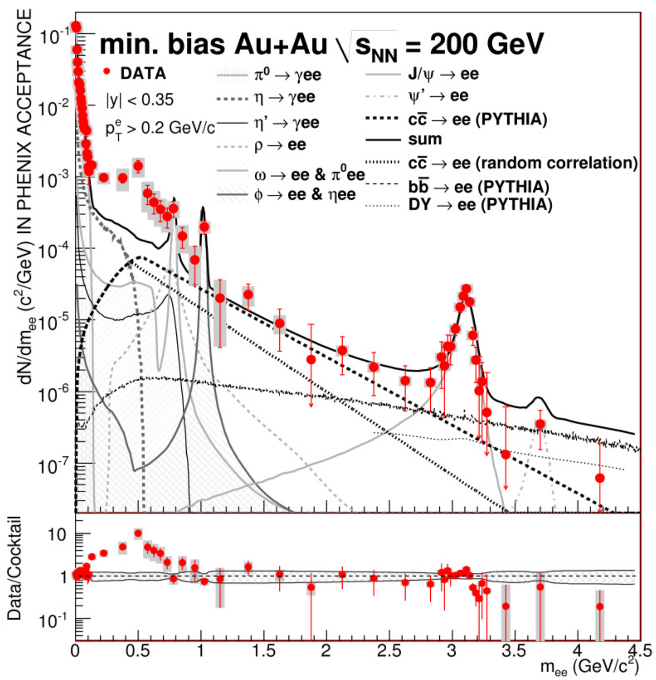
# Motivation – PHENIX/STAR results for the low-mass dilepton rates

pp-data well understood by hadronic cocktail

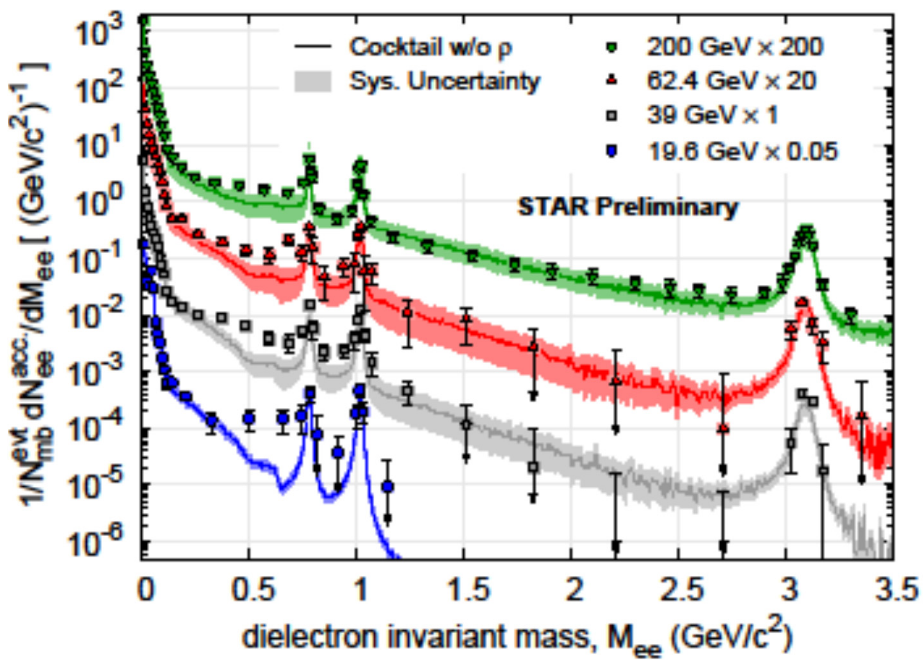
large enhancement in Au+Au between 150-750 MeV

indications for thermal effects!?

Need to understand the contribution from QGP$\rightarrow$ spectral functions from lattice QCD




[PHENIX PRC81, 034911 (2010)]




[STAR preliminary, arXiv:1210.5549]

**Dileptonrate directly related to vector spectral function:**

$$\frac{\mathrm{d}W}{\mathrm{d}\omega \mathrm{d}^3p} = \frac{5\alpha^2}{54\pi^3} \frac{1}{(\omega^2 - \vec{p}^2)(e^{\omega/T} - 1)} \rho_{\mathbf{V}}(\omega, \vec{\mathbf{p}}, \mathbf{T})$$

**Transport Coefficients** are important ingredients into **hydro models for the evolution of the system.**

Usually determined by matching to experiment (see right plot)

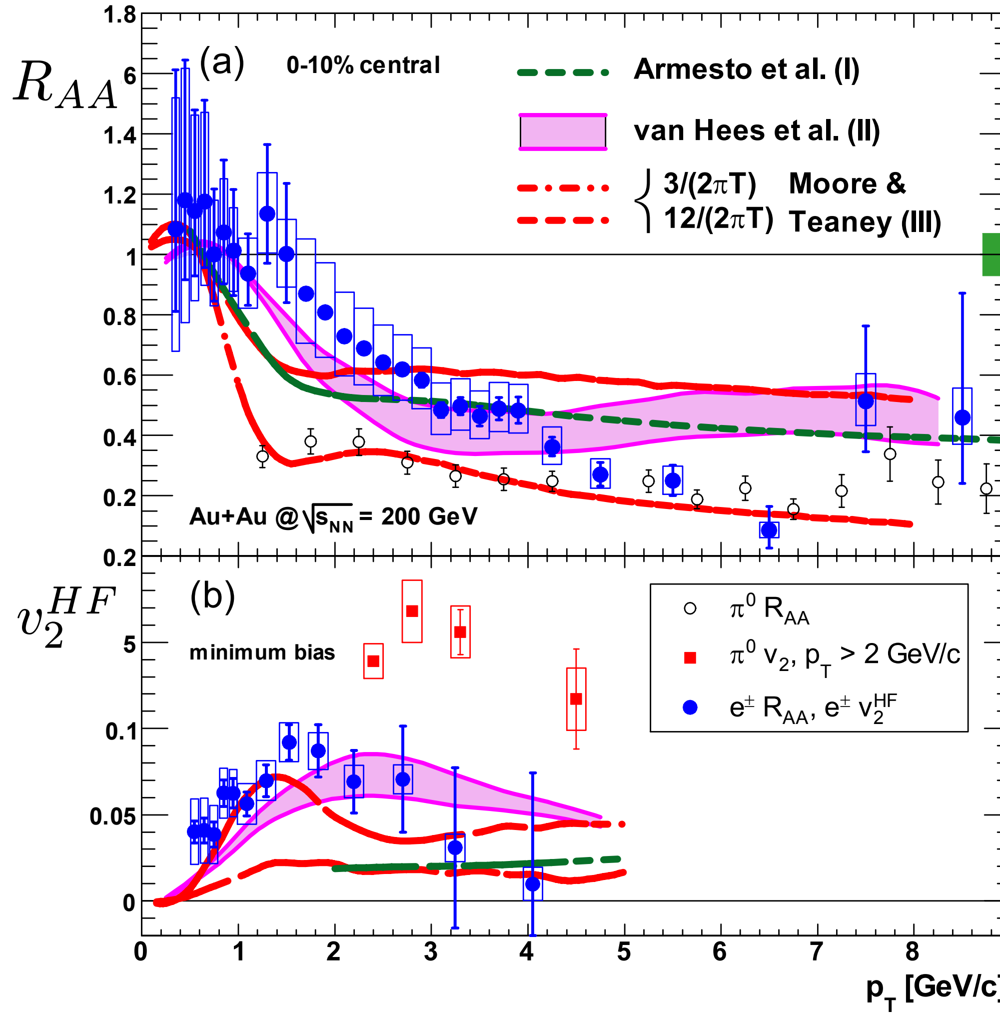
here heavy flavour:

- Heavy Quark Diffusion Constant D
- Heavy Quark Momentum Diffusion

In the following light quarks:

- Light quark flavour diffusion
- Electrical conductivity

**Need to be determined from QCD using first principle lattice calculations!**

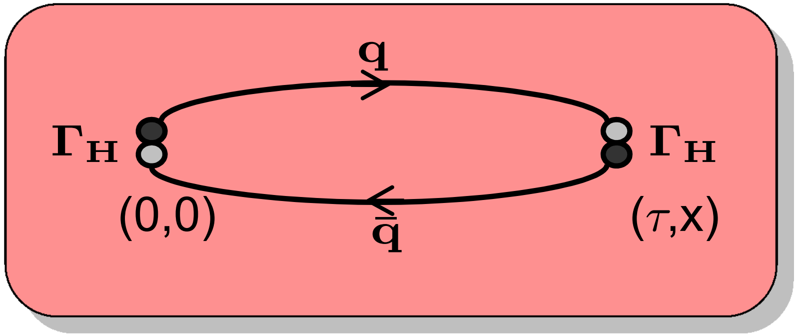



[PHENIX Collaboration, Adare et al.,arXiv:1005.1627 & PRL98(2007)172301]

# Vector correlation functions at high temperature

$$G(\tau,\vec{p},T) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\rho(\omega,\vec{p},T)K(\tau,\omega,T)$$

$$K(\tau,\omega,T) = \frac{\cosh\left(\omega(\tau-\frac{1}{2T})\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$




**Lattice observables:**

$$G_{\mu\nu}(\tau,\vec{x}) = \langle J_\mu(\tau,\vec{x})J_\nu^\dagger(0,\vec{0})\rangle$$

$$J_\mu(\tau,\vec{x}) = 2\kappa Z_V\bar{\psi}(\tau,\vec{x})\Gamma_\mu\psi(\tau,\vec{x})$$

← local, non-conserved current, needs to be renormalized

$$G_{\mu\nu}(\tau,\vec{p}) = \sum_{\vec{x}} G_{\mu\nu}(\tau,\vec{x})e^{i\vec{p}\vec{x}}$$

← only $\vec{p}=0$ used here

**How to extract spectral properties from correlation functions?**

# Spectral functions at high temperature

## Free theory (massless case):

free non-interacting vector spectral function (infinite temperature):

$$\begin{aligned}\rho_{00}^{free}(\omega) &= \boxed{2\pi T^2 \omega \delta(\omega)} \\ \rho_{ii}^{free}(\omega) &= \boxed{2\pi T^2 \omega \delta(\omega)} + \boxed{\frac{3}{2\pi}\,\omega^2 \tanh(\omega/4T)}\end{aligned}$$

$\delta$-functions exactly cancel in $\rho_V(\omega) = -\rho_{00}(\omega) + \rho_{ii}(\omega)$

## With interactions (but without bound states):

while $\rho_{00}$ is protected, the $\delta$-funtion in $\rho_{ii}$ gets smeared:

$$\kappa = \frac{\alpha_s}{\pi}$$

at leading order

**Ansatz**:

$$\begin{aligned}\rho_{00}(\omega) &= 2\pi\chi_q \omega\delta(\omega) \\ \rho_{ii}(\omega) &= 2\chi_q c_{BW} \frac{\omega\Gamma/2}{\omega^2 + (\Gamma/2)^2} + \frac{3}{2\pi}(1+\kappa)\,\omega^2 \tanh(\omega/4T)\end{aligned}$$

Ansatz with 3-4 parameters: $(\chi_q), c_{BW}, \Gamma, \kappa$

["Thermal dilepton rate and electrical conductivity…",
H.T.-Ding, OK et al., PRD83 (2011) 034504]

# Electrical Conductivity

**Electrical Conductivity** $\longleftrightarrow$ slope of spectral function at $\omega$=0 (Kubo formula)

$$\boxed{\frac{\sigma}{T} = \frac{C_{em}}{6} \lim_{\omega \to 0} \frac{\rho_{ii}(\omega, \vec{p}=0, T)}{\omega T}}$$

$$C_{em} = e^2 \sum_{f=1}^{n_f} Q_f^2 \quad = \quad \begin{array}{lll} 5/9\ e^2 & \text{for} & n_f = 2 \\ 6/9\ e^2 & \text{for} & n_f = 3 \end{array}$$

Using our Ansatz for $\rho_{ii}(\omega)$:

$$\boxed{\frac{\sigma}{T} = \frac{2}{3} \frac{\chi_q}{T^2} \frac{T}{\Gamma} c_{BW}\ C_{em}}$$

# Vector correlation function on large & fine lattices

[H.T.-Ding, OK et al., PRD83 (2011) 034504]

**Quenched SU(3) gauge configurations** at $T/T_c$=1.5 (separated by 500 updates)

Lattice size $N_\sigma^3 N_\tau$ with $N_\sigma$ = 32 – 128
$N_\tau$ = 16, 24, 32, 48

Temperature: $T = \frac{1}{aN_\tau}$

**Non-perturbatively O(a) clover improved Wilson fermions**

Non-perturbative renormalization constants

**Quark masses close to the chiral limit,** $\kappa \simeq \kappa_c \Leftrightarrow m_{\overline{MS}}/T[\mu=2\text{GeV}] \approx 0.1$

Volume dependence

| $N_\tau$ | $N_\sigma$ | $\beta$ | $c_{SW}$ | $\kappa$ | $Z_V$ | $1/a[\text{GeV}]$ | $a[\text{fm}]$ | #conf |
|---|---|---|---|---|---|---|---|---|
| 16 | 32 | 6.872 | 1.4124 | 0.13495 | 0.829 | 6.43 | 0.031 | 60 |
| 16 | 48 | 6.872 | 1.4124 | 0.13495 | 0.829 | 6.43 | 0.031 | 62 |
| 16 | 64 | 6.872 | 1.4124 | 0.13495 | 0.829 | 6.43 | 0.031 | 77 |
| 16 | 128 | 6.872 | 1.4124 | 0.13495 | 0.829 | 6.43 | 0.031 | 129 |
| 24 | 128 | 7.192 | 1.3673 | 0.13440 | 0.842 | 9.65 | 0.020 | 156 |
| 32 | 128 | 7.457 | 1.3389 | 0.13390 | 0.851 | 12.86 | 0.015 | 255 |
| 48 | 128 | 7.793 | 1.3104 | 0.13340 | 0.861 | 19.30 | 0.010 | 431 |

cut-off dependence & continuum extrapolation

close to continuum

# New results at 1.1 and 1.2 $T_c$

[M.Müller et al., arXiv:1301.7436]

**PRACE-Project:**
Thermal Dilepton Rates and
Electrical Conductivity in the QGP
(JUGENE Bluegene/P in Jülich)

| | $1.1\ T_c$ | $1.2\ T_c$ | | | | | |
|---|---|---|---|---|---|---|---|
| $N_\sigma$ | $N_\tau$ | $N_\tau$ | $\beta$ | $\kappa$ | $1/a$[GeV] | $a$[fm] | #Confs |
| 96 | 32 | 28 | 7.192 | 0.13440 | 9.65 | 0.020 | 250 |
| 144 | 48 | 42 | 7.544 | 0.13383 | 13.21 | 0.015 | 300 |
| 192 | 64 | 56 | 7.793 | 0.13345 | 19.30 | 0.010 | 240 |

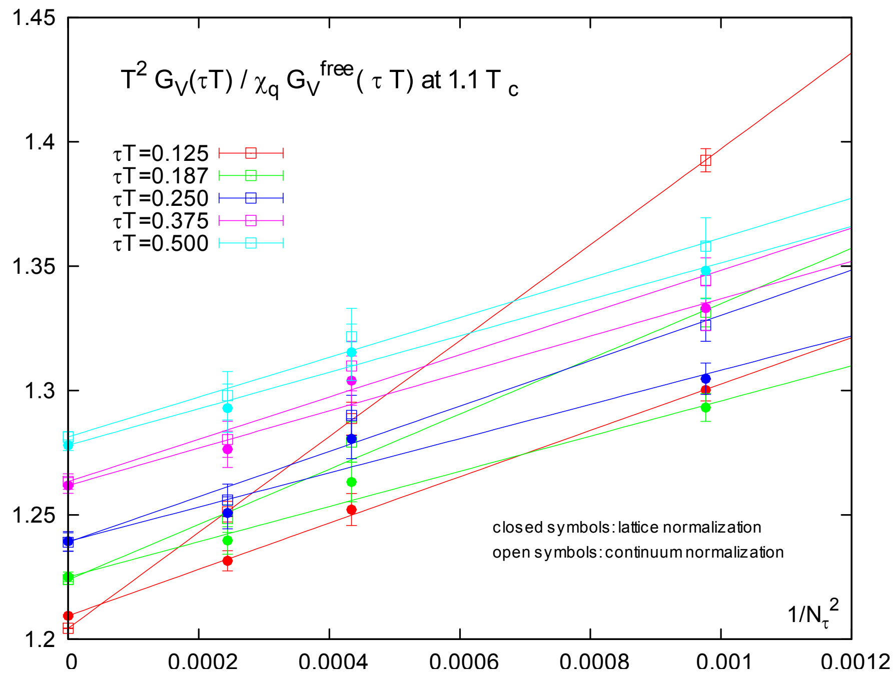
**study of T-dependence of dilepton rates and electrical conductivity**

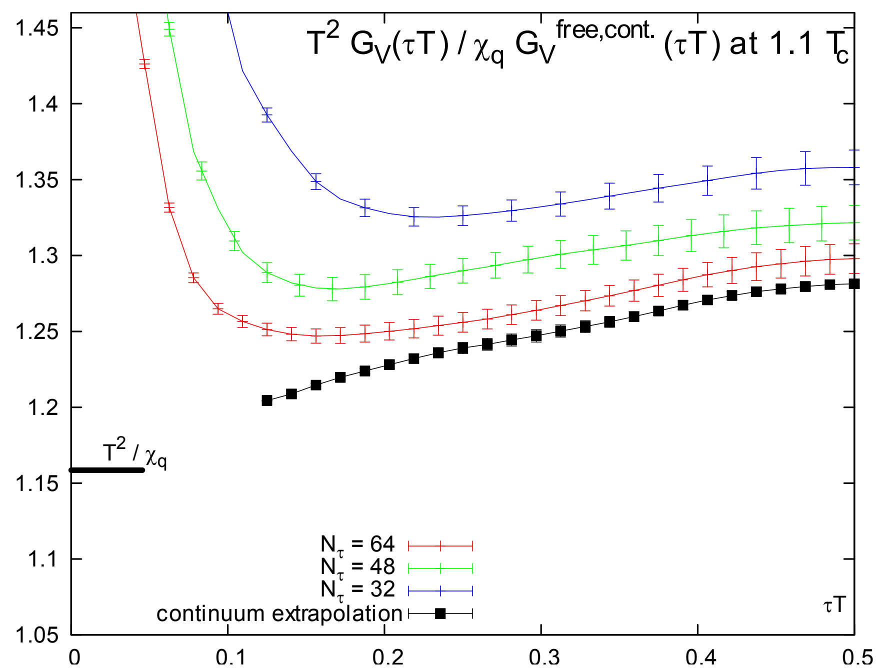
**fixed aspect ratio $N_\sigma/N_\tau = 3$ to allow continuum limit at finite momentum:**

$$\frac{\vec{p}}{T} = 2\pi\vec{k}\,\frac{N_\tau}{N_\sigma}$$

**constant physical volume** $(1.9\text{fm})^3$

# Continuum extrapolation



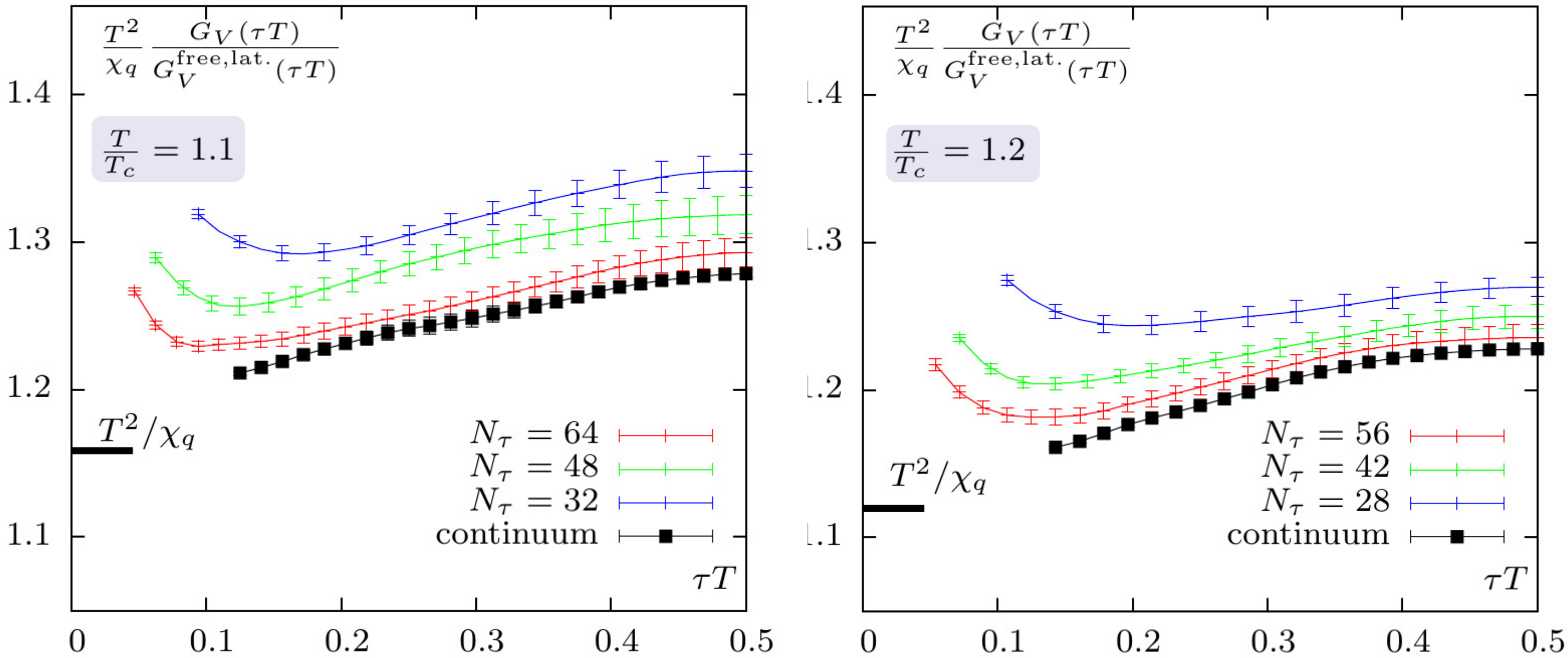

**cut-off effects visible at all distances but**

**well defined continuum limit on the correlator level**

**well behaved continuum correlator down to small distances**

**approaching the correct asymptotic limit for $\tau \to 0$**

# Continuum extrapolation

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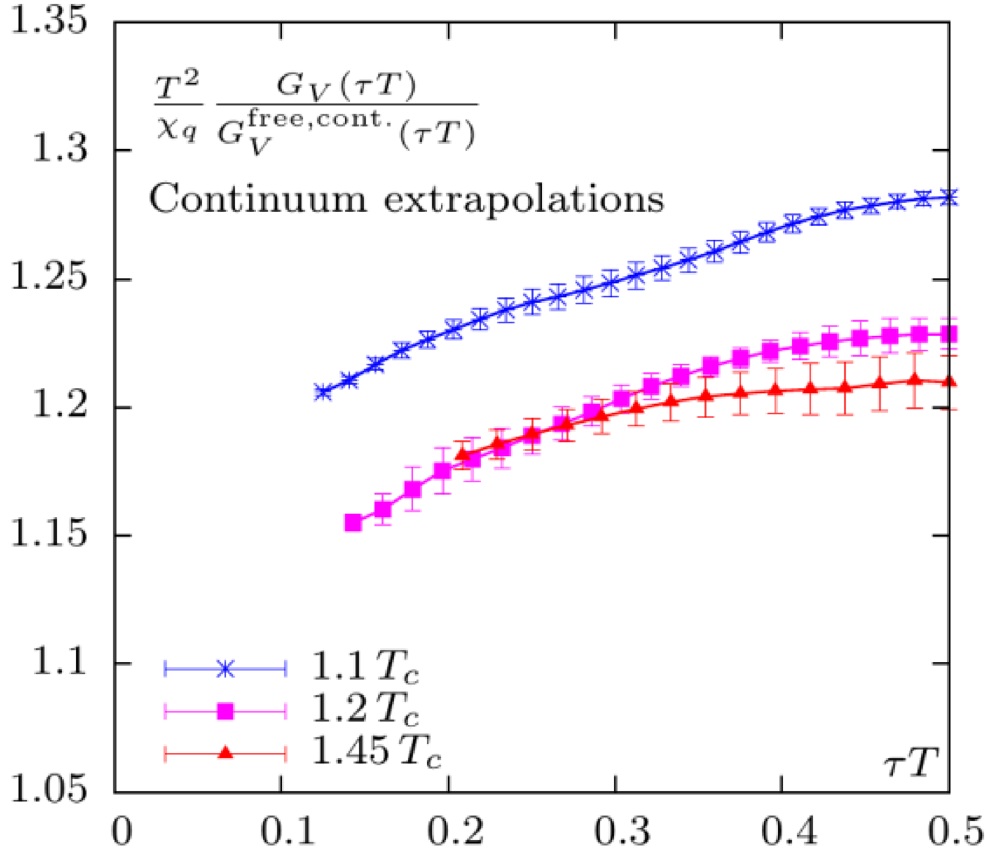
# Continuum extrapolation

## continuum extrapolated correlators



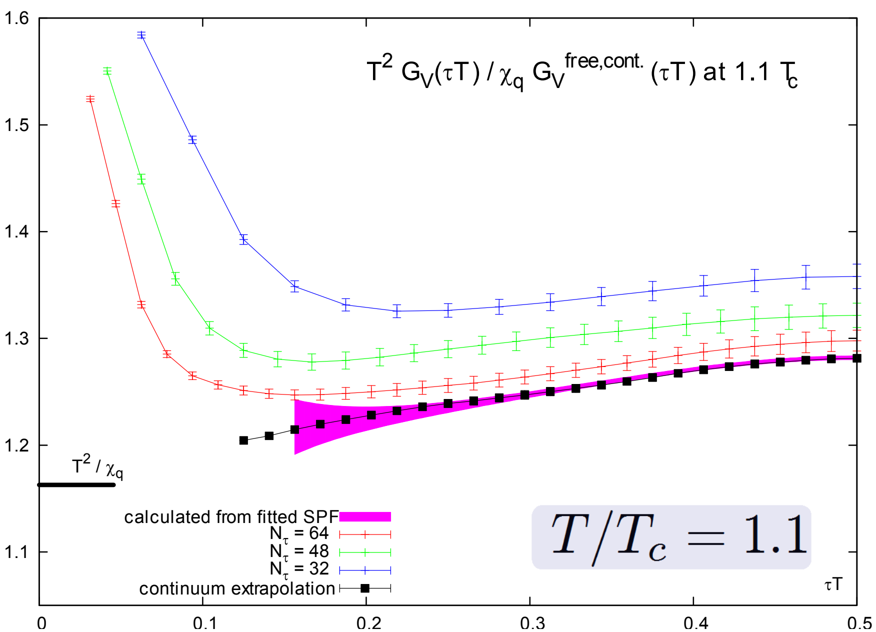
# Spectral function and electrical conductivity

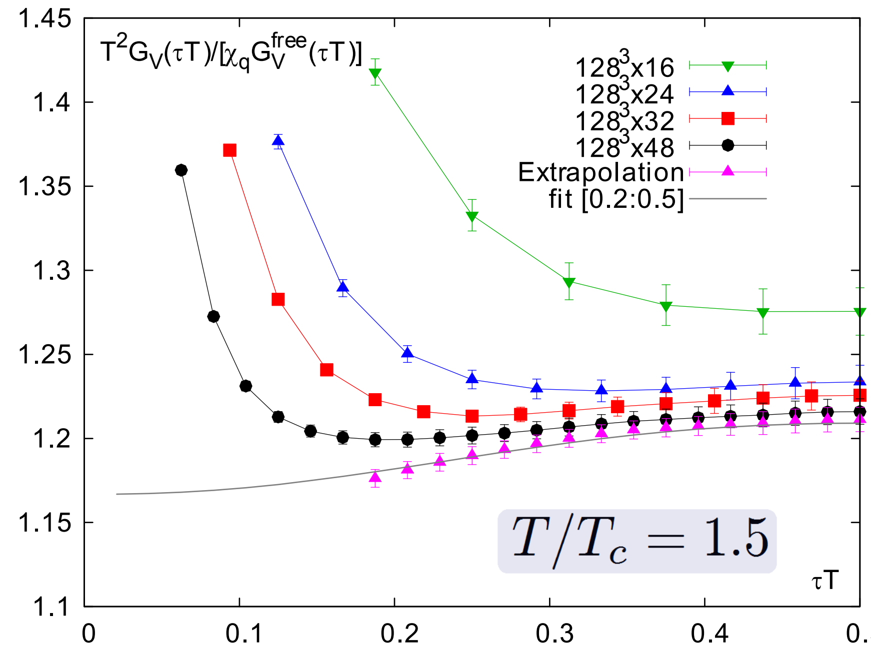
**Use our Ansatz for the spectral function**

$$\rho_{00}(\omega) = 2\pi\chi_q\omega\delta(\omega)$$

$$\rho_{ii}(\omega) = 2\chi_q c_{BW}\frac{\omega\Gamma/2}{\omega^2+(\Gamma/2)^2} + \frac{3}{2\pi}(1+\kappa)\ \omega^2\ \tanh(\omega/4T)$$

**and fit to the continuum extrapolated correlators**




[H.T.-Ding, OK et al., PRD83 (2011) 034504]

**all three temperatures are well described by this rather simple Ansatz**

# Spectral function and electrical conductivity

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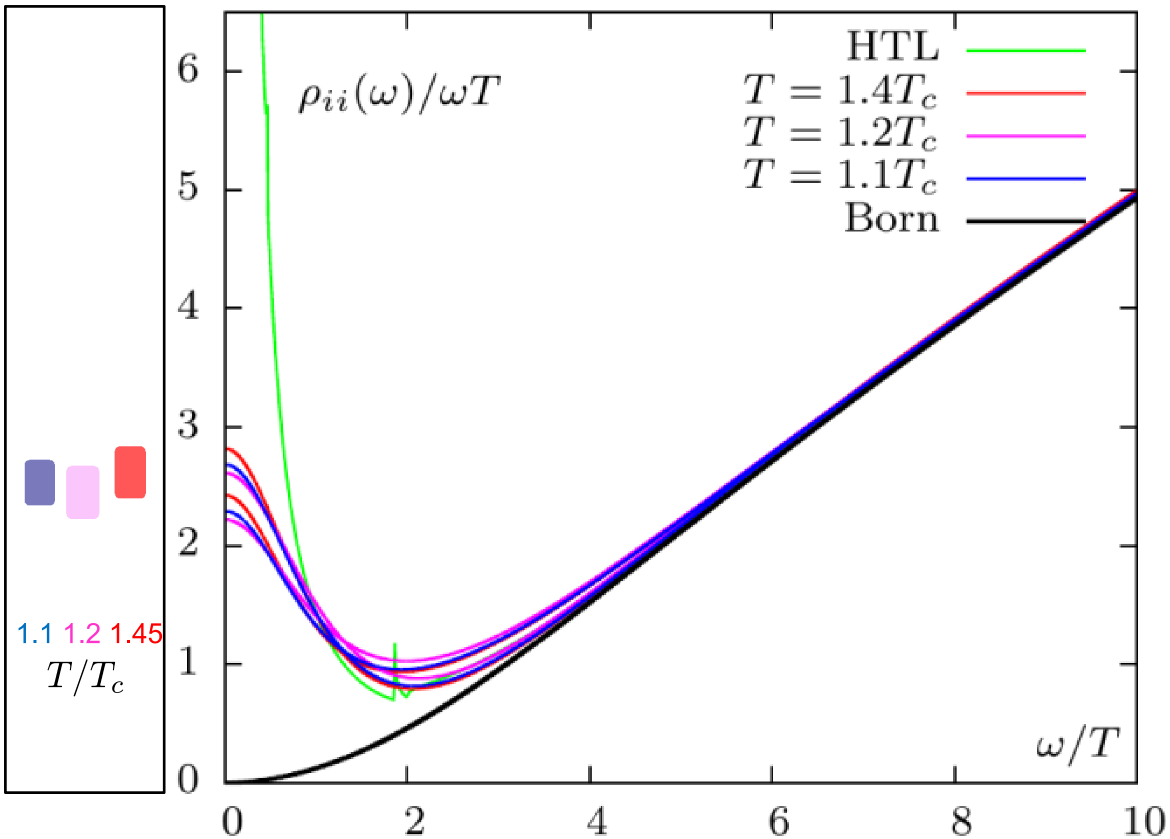



**Analysis of the systematic errors**

using truncation of the large $\omega$ contribution

$$\Theta(\omega_0,\Delta_\omega) = \left(1+e^{(\omega_0^2-\omega^2)/\omega\Delta_\omega}\right)^{-1}$$

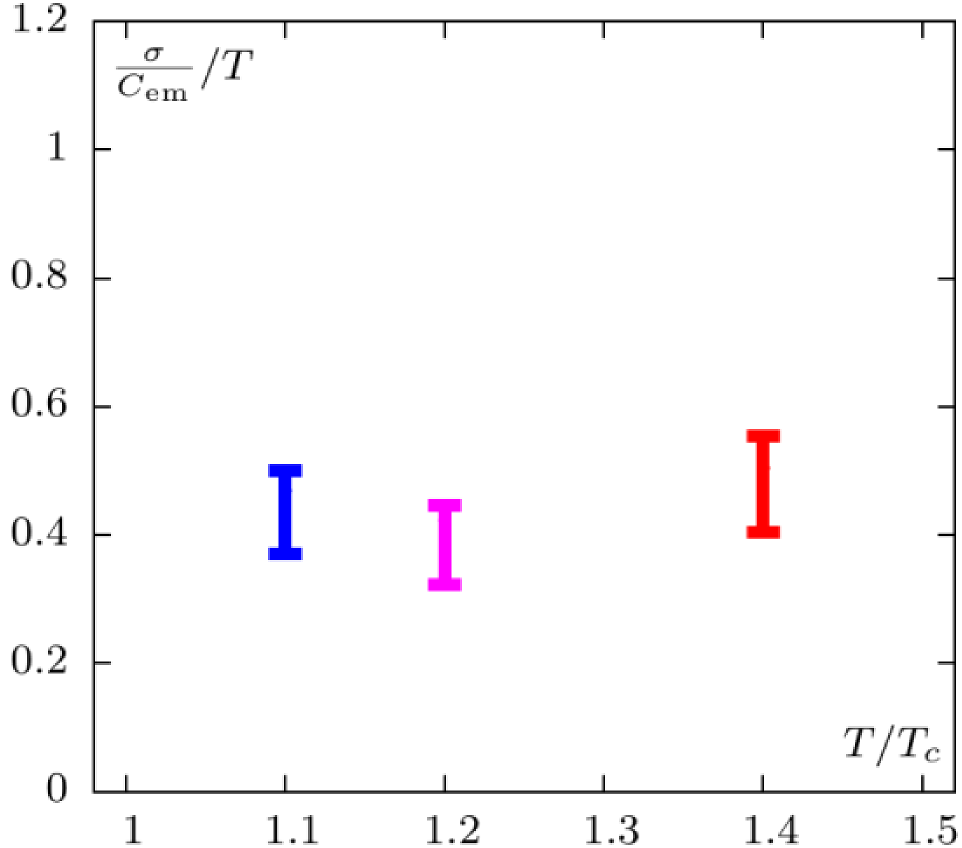
$$\frac{\sigma}{T} = \frac{C_{em}}{6}\lim_{\omega\to 0}\frac{\rho_{ii}(\omega,\vec{p}=0,T)}{\omega T}$$

**electrical conductivity**

# Electrical conductivity

**T-dependence of the electrical conductivity:**

$$\frac{\sigma}{T} = \frac{C_{em}}{6} \lim_{\omega \to 0} \frac{\rho_{ii}(\omega, \vec{p} = 0, T)}{\omega T}$$



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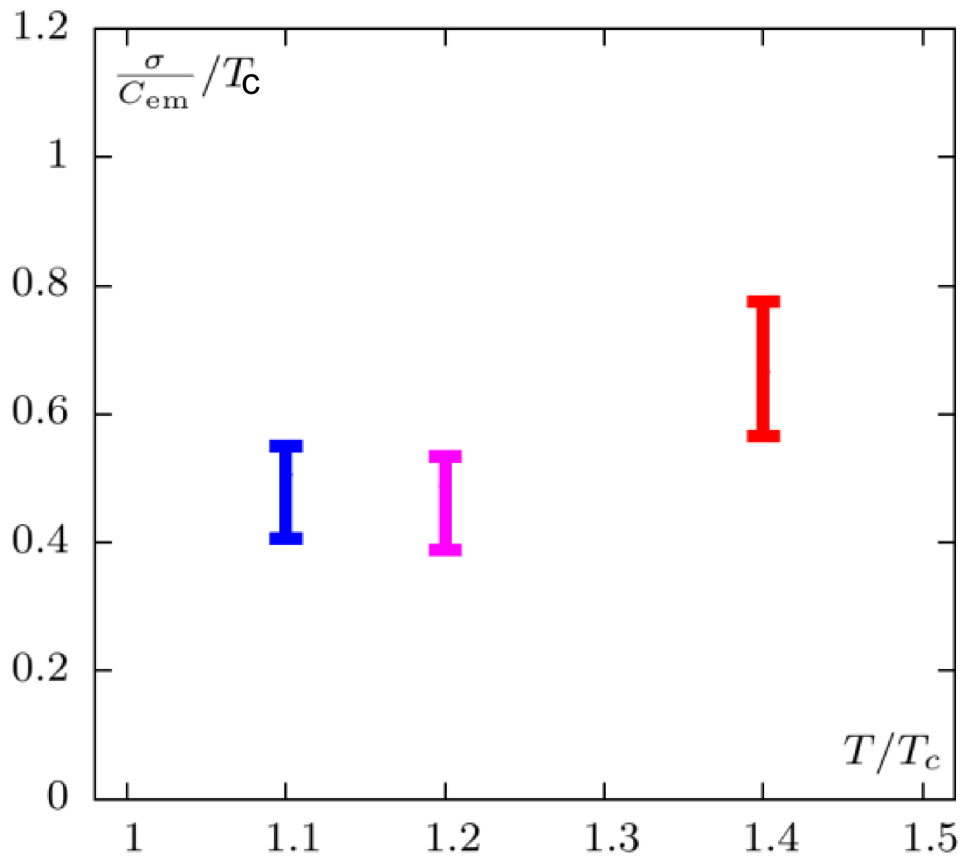

# Dilepton rates

**Dileptonrate directly related to vector spectral function:**

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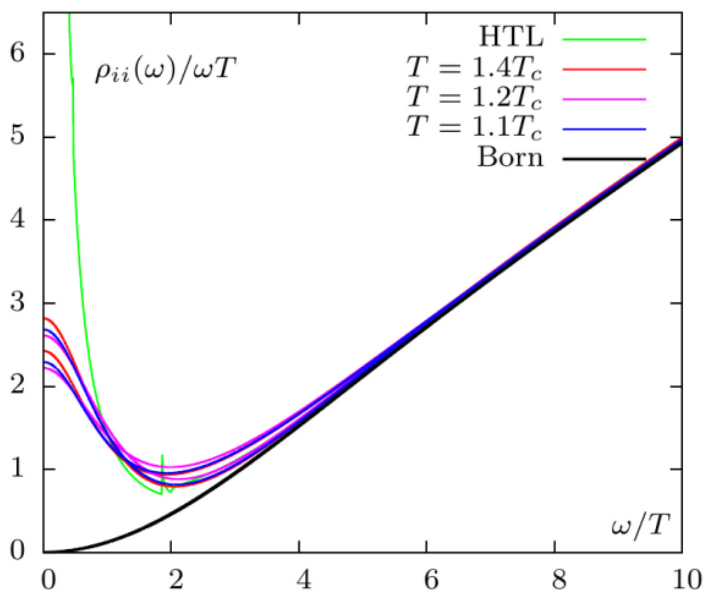


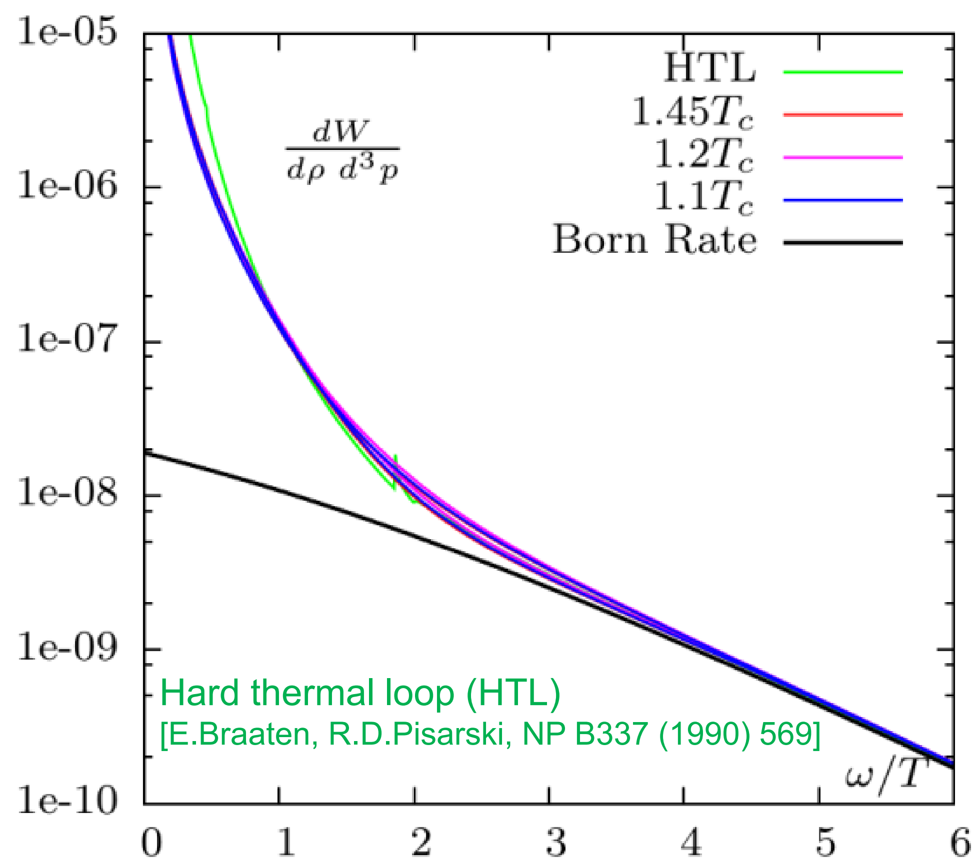

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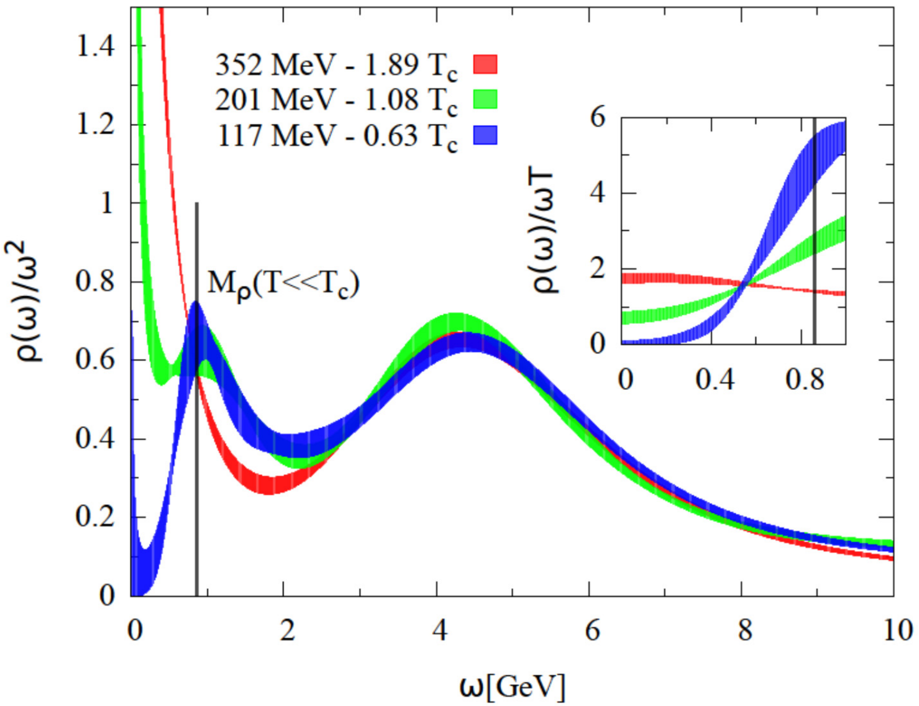
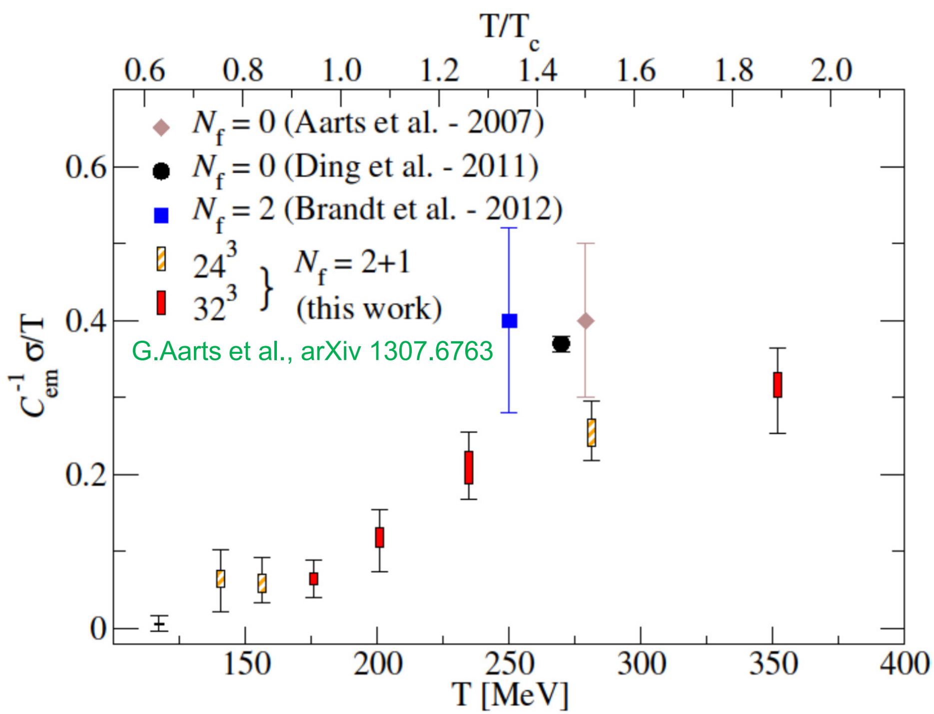
G.Aarts et al., arXiv 1307.6763




**Ding et al.:** **Quenched on isotropic lattices + *continuum limit***

**Aarts et al.:** **2+1-flavor dynamical Wilson fermions on anisotropic lattices (Ns=24-32 Nt=16-48)**
**(cut-off effects and energy resolution determined by spatial lattice spacing)**

**Brandt et al.:** **2-flavour dynamical Wilson fermions on isotropic lattices (Ns=64 Nt=16)**

# Electrical conductivity

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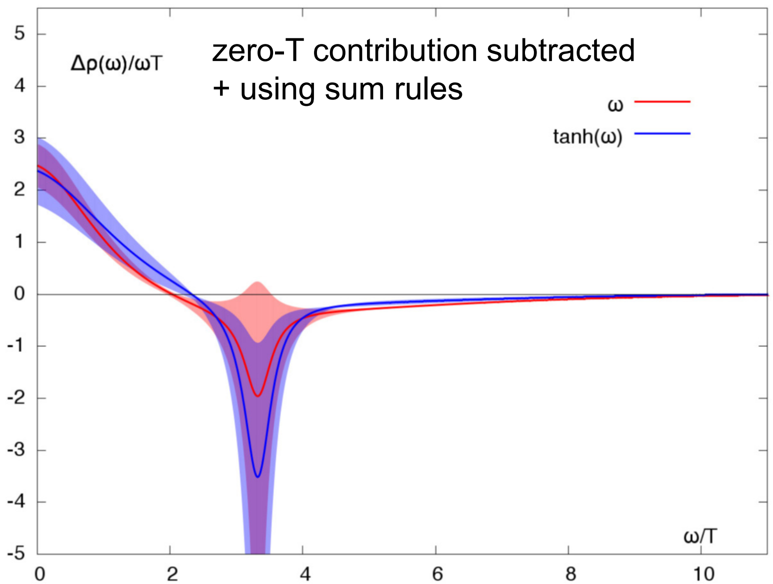
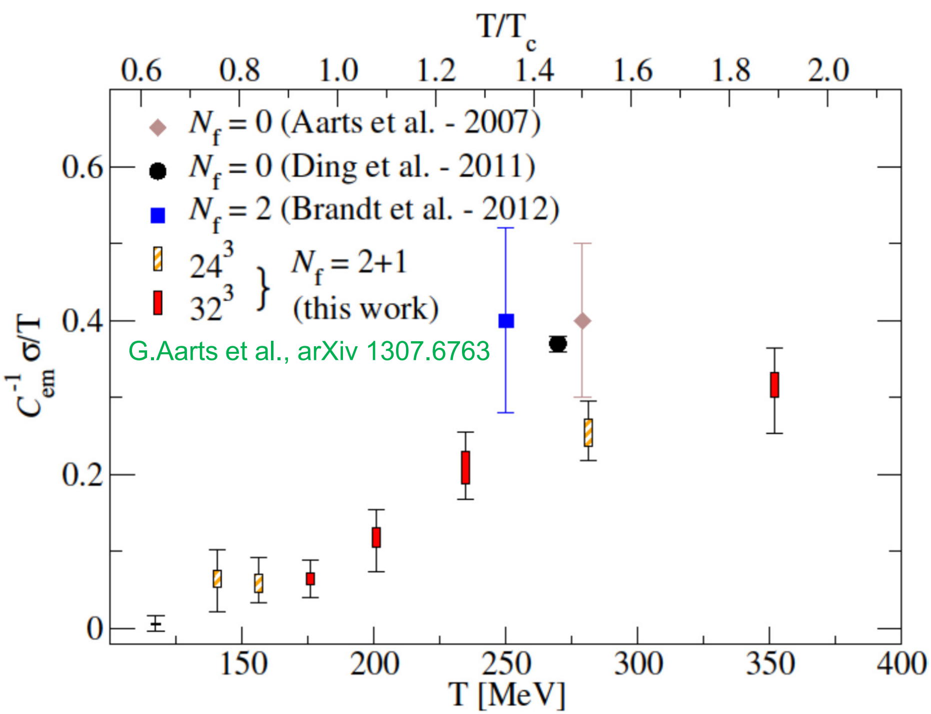
Brandt et al., 2012

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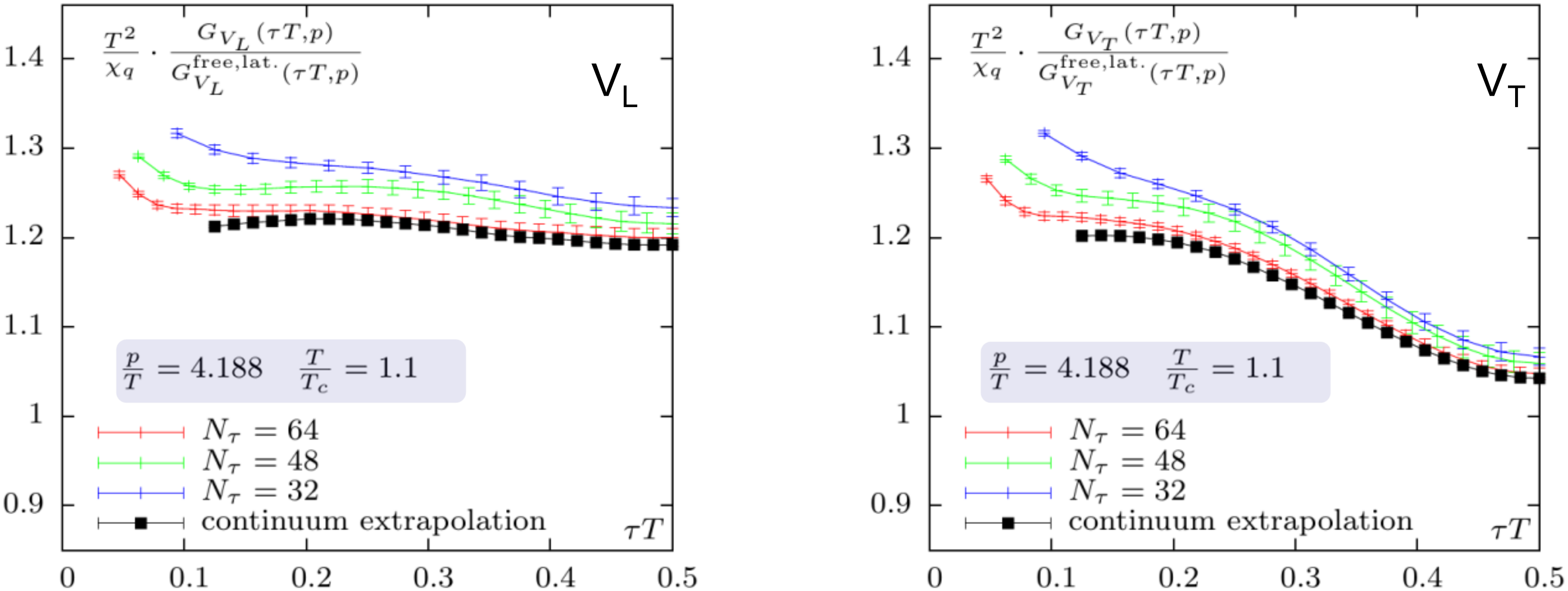
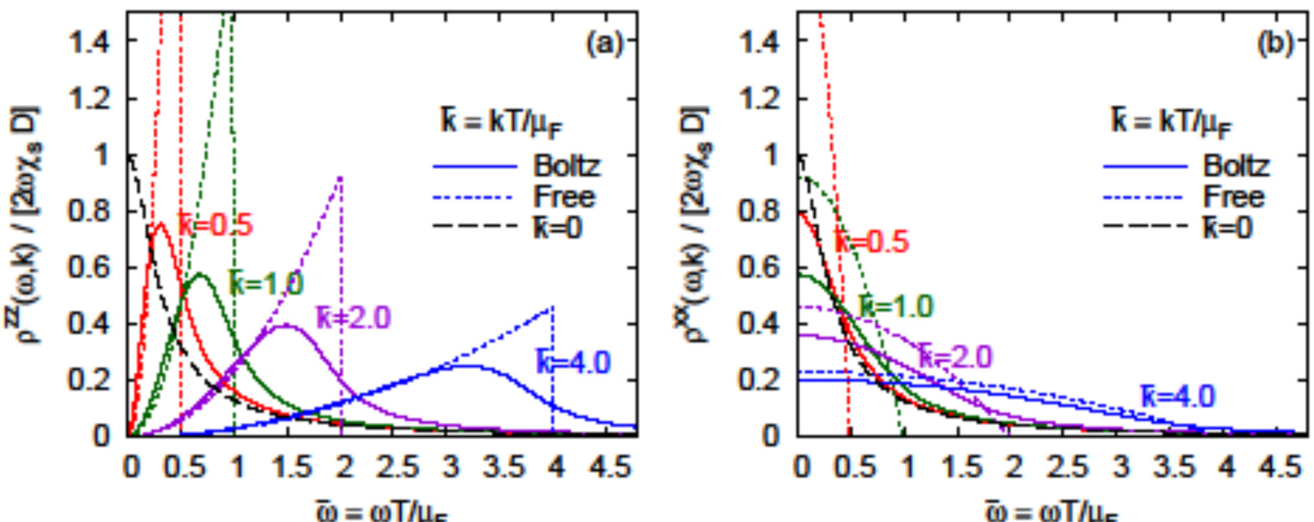
# Non-zero momentum

[M.Müller et al., preliminary 2013]




**indications for non-trivial behavior of spectral functions at small frequencies:**




[Hong+Teaney, PRC82 (2010)044908]

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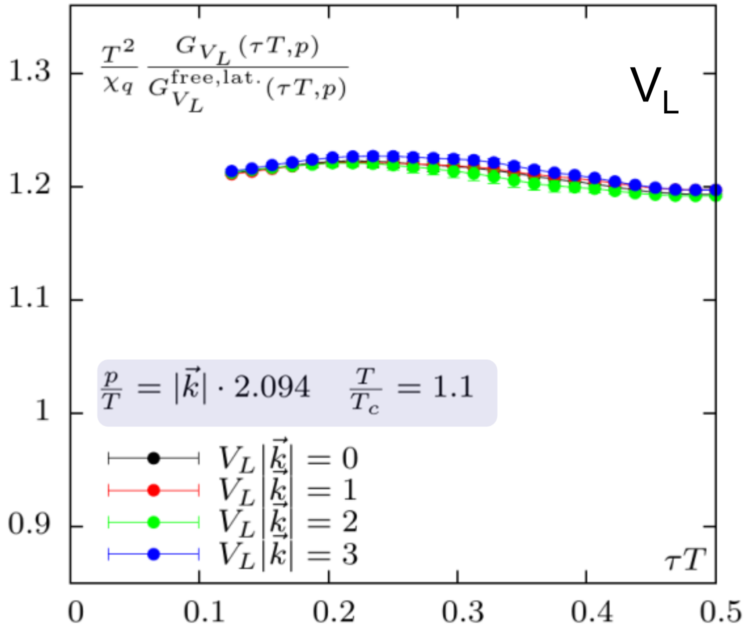


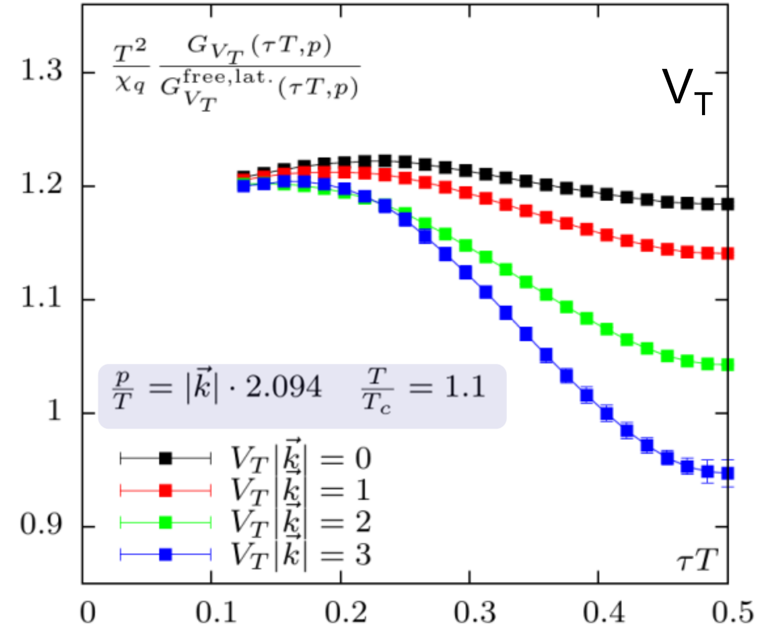


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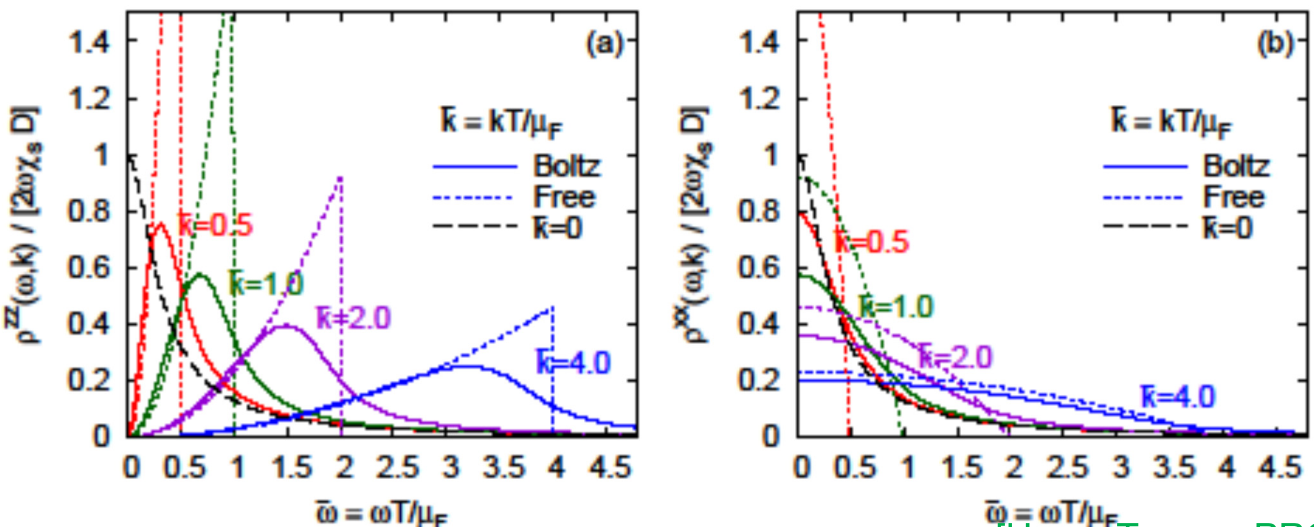
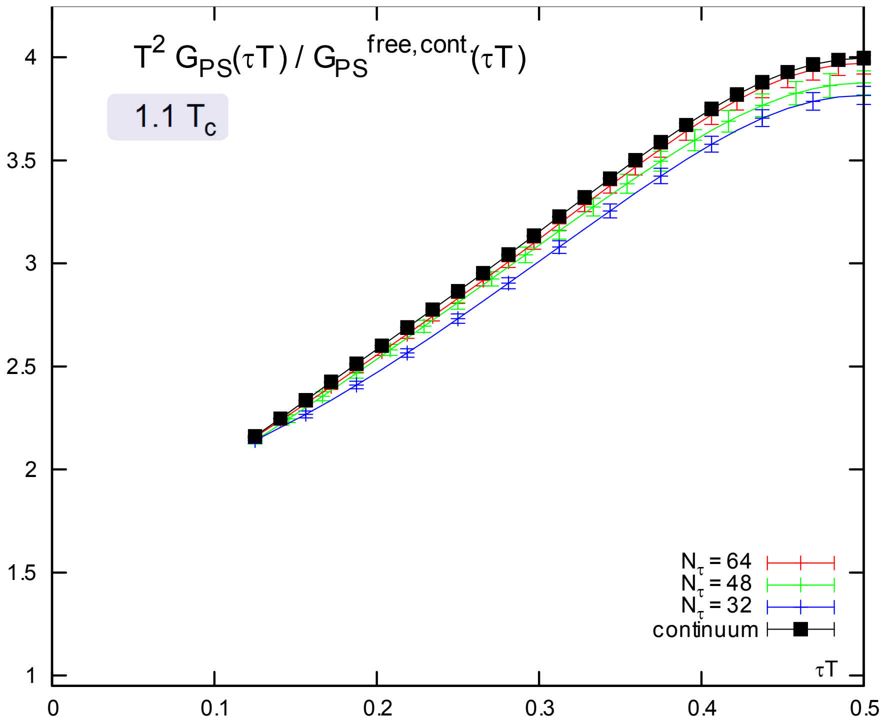
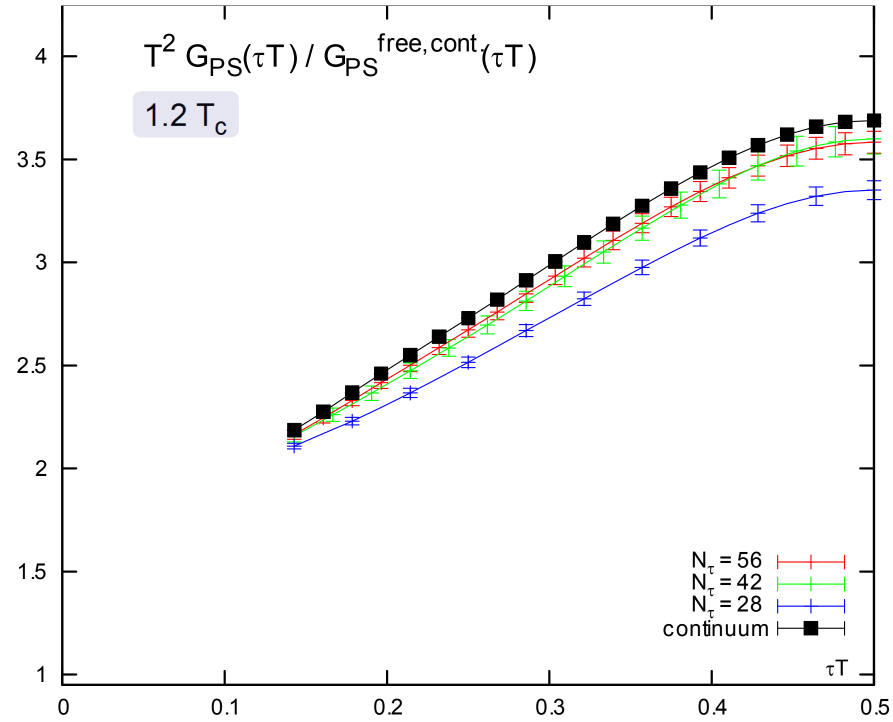



[Hong+Teaney, PRC82 (2010)044908]

# Pseudo-scalar channel




**in contrast to the vector channel**

**no transport peak expected in the pseudo-scalar channel**

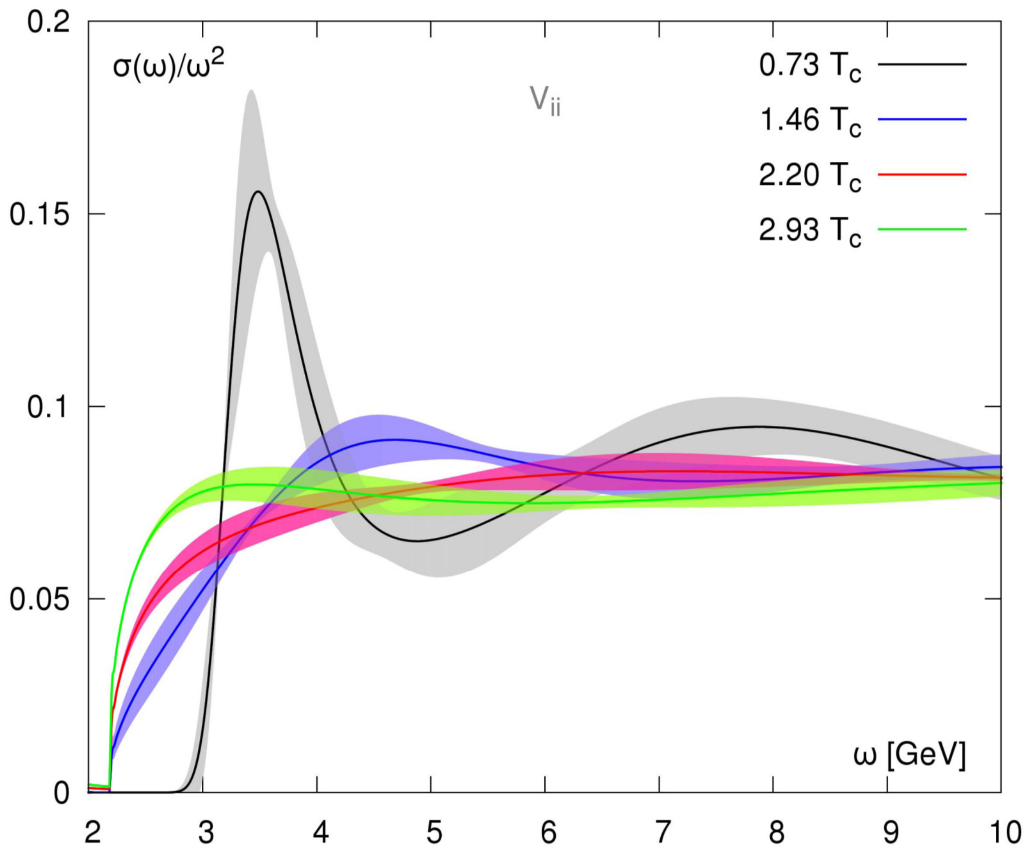
**still strong correlations visible in the pseudo-scalar channel**

**spectral function still needs to be determined!**

# Charmonium Spectral function

[H.T.Ding, OK et al., PRD86(2012)014509]

from sophisticated Maximum Entropy Method analysis:

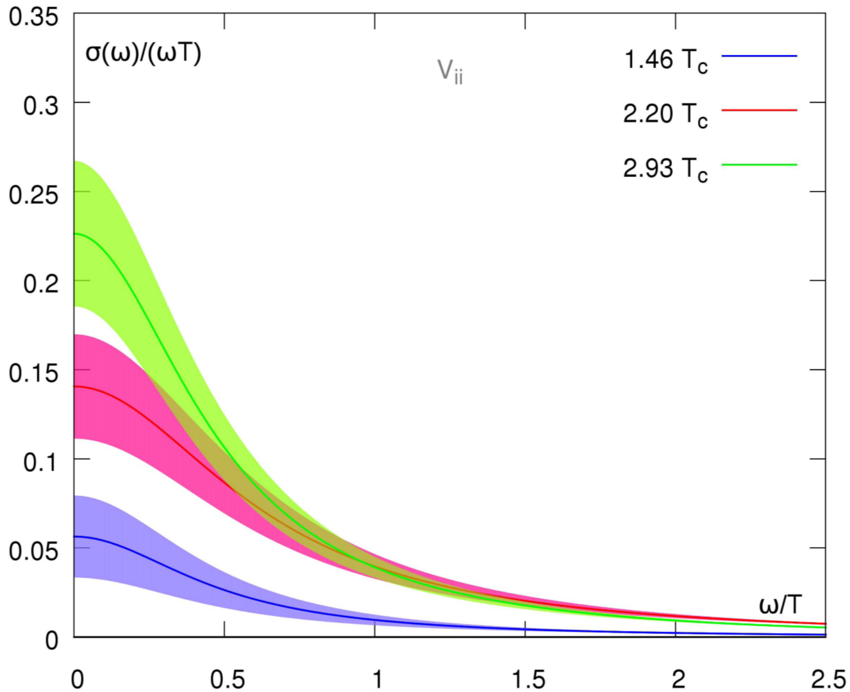



**statistical error band from Jackknife analysis**

**no clear signal for bound states at and above 1.46 $T_c$**

**study of the interesting region closer to $T_c$ on the way!**

# Charmonium Spectral function – Transport Peak

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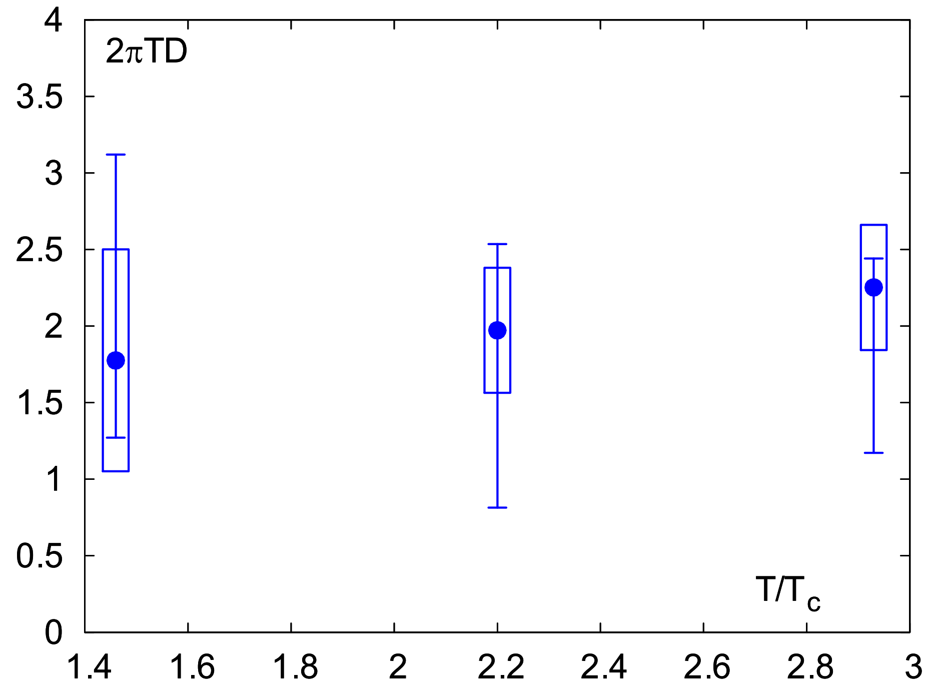



$$D = \frac{\pi}{3\chi_{00}} \lim_{\omega\to 0} \frac{\rho_{ii}(\omega, \vec{p}=0, T)}{\omega T}$$

Perturbative estimate ($\alpha_s$~0.2, g~1.6):

LO: $2\pi TD \simeq 71.2$

NLO: $2\pi TD \simeq 8.4$

[Moore&Teaney, PRD71(2005)064904, Caron-Huot&Moore, PRL100(2008)052301]

Strong coupling limit:

$2\pi TD = 1$

[Kovtun, Son & Starinets,JHEP 0310(2004)064]

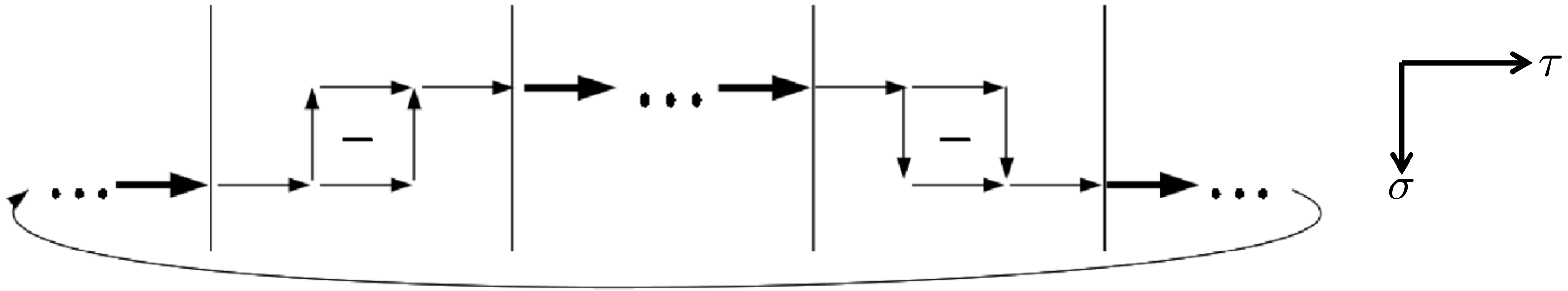
# Heavy Quark Momentum Diffusion Constant

[A.Francis,OK,M.Laine,J.Langelage, arXiv:1109.3941]

Heavy Quark Effective Theory (HQET) in the large quark mass limit

leads to a (pure gluonic) “color-electric correlator”

[J.Casalderrey-Solana, D.Teaney, PRD74(2006)085012, S.Caron-Huot,M.Laine,G.D. Moore,JHEP04(2009)053]



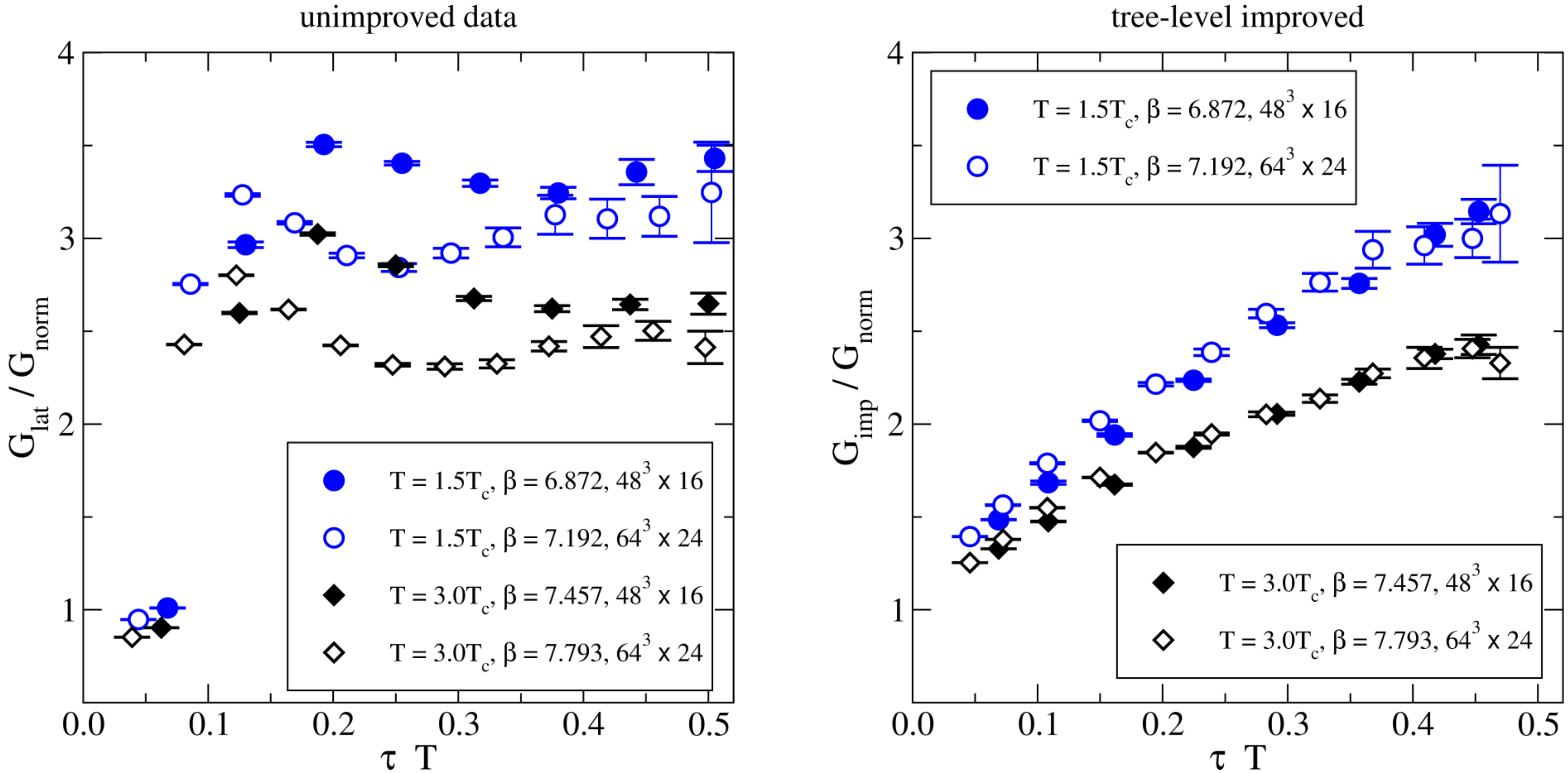

$$G_{\mathrm{E}}(\tau) \equiv -\frac{1}{3}\sum_{i=1}^{3}\frac{\left\langle \mathrm{Re}\,\mathrm{Tr}\left[U(\tfrac{1}{T};\tau)\,gE_i(\tau,\mathbf{0})\,U(\tau;0)\,gE_i(0,\mathbf{0})\right]\right\rangle}{\left\langle \mathrm{Re}\,\mathrm{Tr}[U(\tfrac{1}{T};0)]\right\rangle}$$

Heavy quark (momentum) diffusion:

$$\kappa = \lim_{\omega\to 0}\frac{2T\rho_{\mathrm{E}}(\omega)}{\omega} \quad , \quad D = \frac{2T^2}{\kappa}$$

# Heavy Quark Momentum Diffusion Constant

[A.Francis,OK,M.Laine,J.Langelage, arXiv:1109.3941]




due to the gluonic nature of the operator, signal is extremely noisy

→ multilevel combined with link-integration techniques used to improve the signal

→ tree-level improvement (right figure) to reduce discretization effects

[similar studies by H.B.Meyer, New J.Phys.13 (2011) 035008 and D.Banerjee,S.Datta,R.Gavai,P.Majumdar,PRD85(2012)014510]

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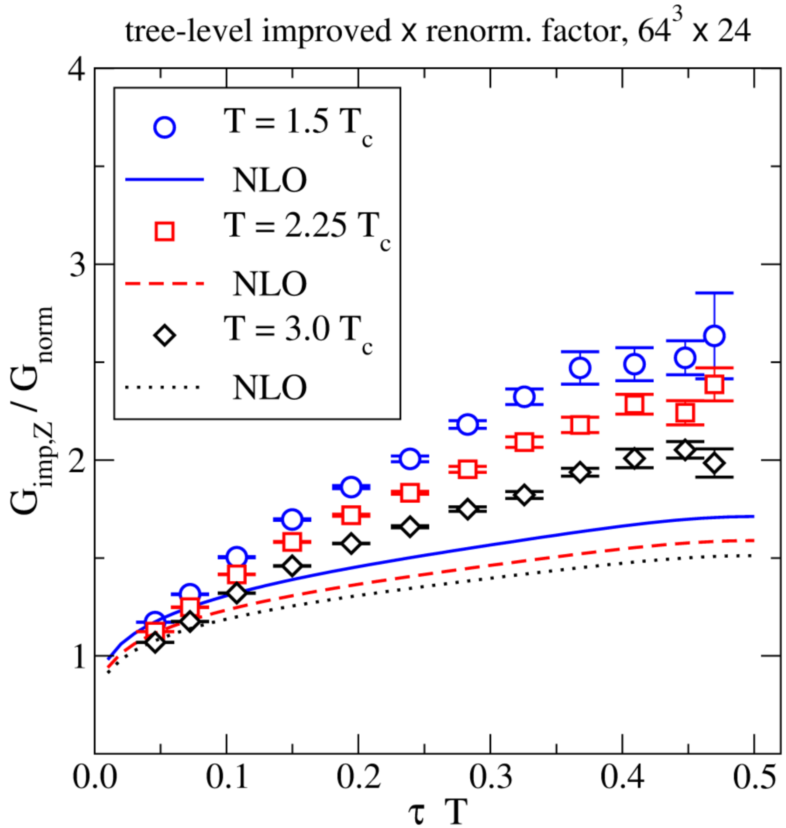


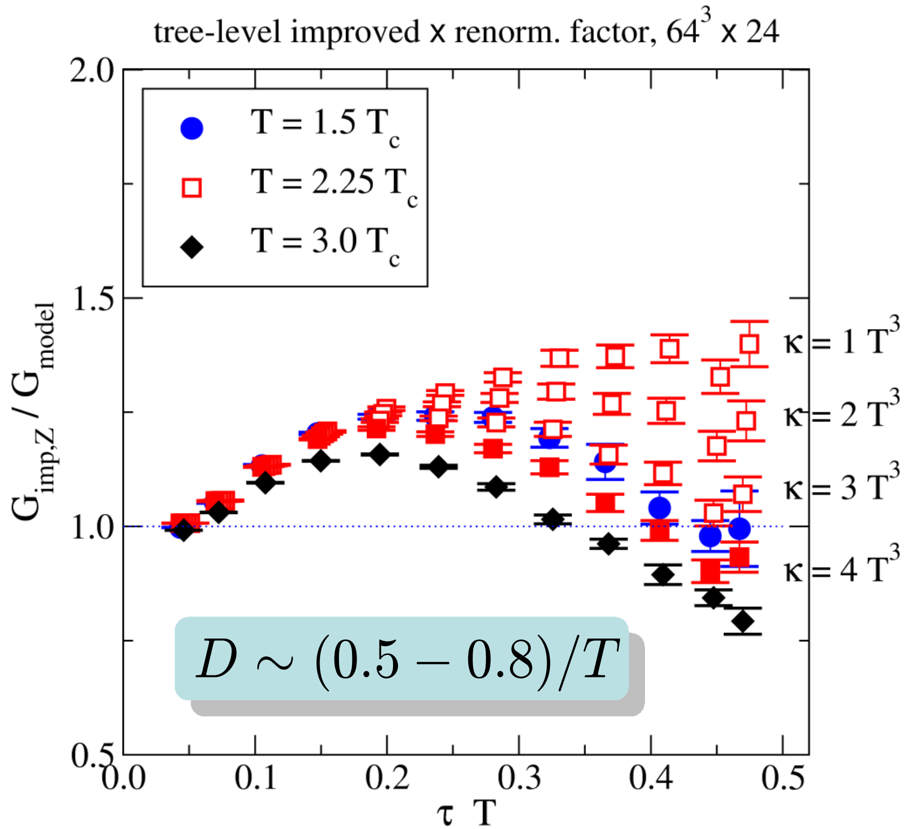


Model spectral function: transport contribution + NLO [Y.Burnier et al. JHEP 1008 (2010) 094)]

$$\rho_{\rm model}(\omega) \equiv \max\left\{\rho_{\rm NLO}(\omega), \frac{\omega\kappa}{2T}\right\}$$

$$G_{\rm model}(\tau) \equiv \int_0^\infty \frac{d\omega}{\pi}\rho_{\rm model}(\omega)\frac{\cosh\left(\frac{1}{2}-\tau T\right)\frac{\omega}{T}}{\sinh\frac{\omega}{2T}}$$

**Still large uncertainties but very promising**

**→ thermodynamic+continuum limit needed**

**→ more constraints on the spectral function**

**→ other operators and observables from EFT?**

# Conclusions and Outlook

**Conclusions:**

Detailed knowledge of the **vector correlation function** in the region $1.1 \leq T/T_c \leq 1.5$

→ **continuum extrapolation** of correlation function and thermal moments

continuum $G_V(\tau T)$ well reproduced by **Breit-Wigner plus continuum** Ansatz for $\sigma_V(\omega)$ in the temperature region $1.1 \leq T/T_c \leq 1.5$

→ **electrical conductivity** $\sigma/T$ shows only small temperature effects

→ **Dilepton rate** approaches leading order Born rate for $\omega/T \geq 4$
enhancement at small $\omega/T$

**Outlook:**

include HTL result for $\sigma_V(\omega)$ at large $\omega/T$ in the Ansatz

vector correlation function at **non-zero momentum**

especially close to $T_c$ effects of dynamical quarks need to be included