Hadron physics in the Dyson-Schwinger approach

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Motivation

Goal: compute **hadron properties** (ground state & excitations, form factors, scattering amplitudes, etc.) from **quark-gluon substructure in QCD.**

QCD's Green functions $\leftrightarrow$ "Dyson-Schwinger approach":
Nonperturbative, covariant, low and high energies, light and heavy quarks. But: **truncations!**

- **Baryon spectroscopy** from three-body Faddeev equation GE, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010)
- **Elastic & transition form factors** for N and Δ GE, PRD 84 (2011); GE, Fischer, EPJ A48 (2012); GE, Nicmorus, PRD 85 (2012); Sanchis-Alepuz et al., PRD 87 (2013), ...
- **Tetraquark** interpretation for σ meson Heupel, GE, Fischer, PLB 718 (2012)
- **Compton scattering** GE, Fischer, PRD 85 (2012) & PRD 87 (2013)

Here:

- $N \to \Delta\gamma$ transition
- Microscopic coupling of quark to γ and ρ, vector-meson dominance?
- Timelike form factors?
- Compton scattering (also: blueprint for meson electroproduction and $N\pi$ scattering)

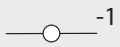
Dyson-Schwinger equations

QCD Lagrangian:
quarks, gluons (+ ghosts)

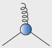
$$\mathcal{L} = \bar{\psi}(x)\,(i\partial\!\!\!/ + g A\!\!\!/ - M)\,\psi(x) - \frac{1}{4}\, F^a_{\mu\nu}\, F^{\mu\nu}_a$$

QCD & hadron properties are encoded in **QCD's Green functions.**
Their quantum equations of motion are the **Dyson-Schwinger equations (DSEs):**

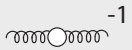
- **Quark propagator:**



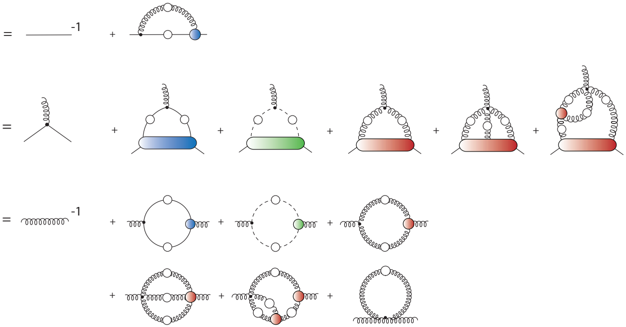
- **Quark-gluon vertex:**



- **Gluon propagator:**



- **Gluon self-interactions, ghosts, . . .**



Dyson-Schwinger equations

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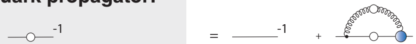

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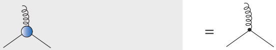

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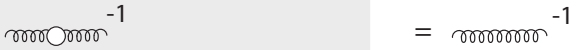


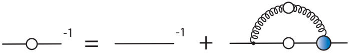
- **Gluon self-interactions, ghosts, . . .**

- **Truncation** ⇒ closed system, solveable.
  Ansätze for Green functions that are **not** solved (based on pQCD, lattice, FRG, ...)

- **Applications:**
  Origin of confinement,
  QCD phase diagram,
  Hadron physics

Dynamical quark mass

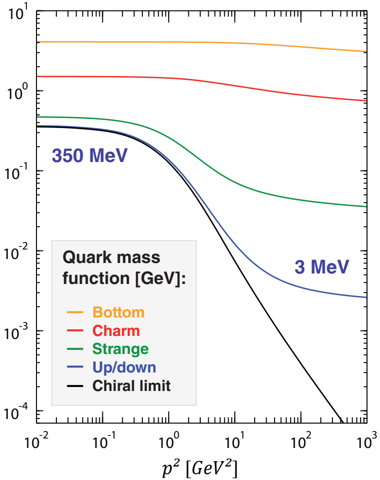
- **Dynamical chiral symmetry breaking:** generates "constituent-quark masses"
- Realized in **quark Dyson-Schwinger eq:**



If (gluon propagator $\times$ quark-gluon vertex) is strong enough ($\alpha > \alpha_{\text{crit}}$): momentum-dependent quark mass $M(p^2)$

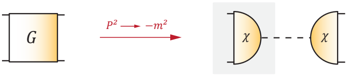
- Already visible in simpler models (NJL, Munczek-Nemirovsky)
- Mass generation for **light hadrons**

Fischer, J. Phys. G 32 (2006)



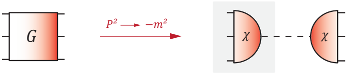
Hadrons: poles in Green functions

- **Quark four-point function:**
  $\langle 0|\mathrm{T}\,\psi(x_1)\,\bar{\psi}(x_2)\,\psi(x_3)\,\bar{\psi}(x_4)|0\rangle$



Bethe-Salpeter WF:
$\langle 0|\,T\,\psi(x_1)\,\bar{\psi}(x_2)\,|H\rangle$

- **Quark six-point function:**



Faddeev WF

Hadrons: poles in Green functions

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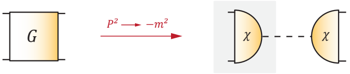

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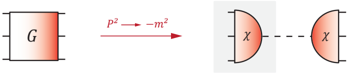
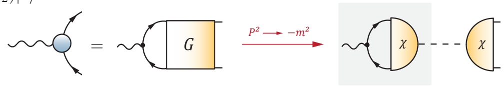


Faddeev WF

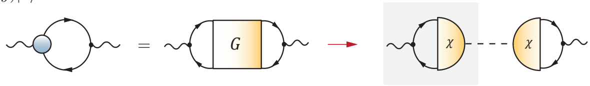
- **Quark-antiquark vertices:** (Currents: $J^\mu = \bar\psi\,\Gamma^\mu\psi$)
  $\langle 0|\mathrm{T}\,J^\mu(x)\,\psi(x_1)\,\bar\psi(x_2)|0\rangle$



Decay constant:
$\langle 0|\,J^\mu\,|H\rangle$

Quark-photon vertex
has ρ-meson poles:
'vector-meson dominance'

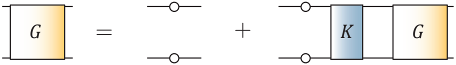
- **Current correlators:**
  $\langle 0|\mathrm{T}\,J^\mu(x)\,J^\nu(y)|0\rangle$



(→ Lattice QCD)

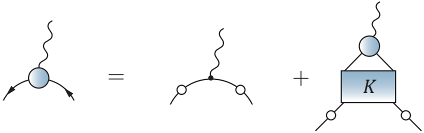
Bethe-Salpeter equations

- Inhomogeneous BSE
  for **quark four-point function:**



- Homogeneous BSE
  for **bound-state wave function:**

- Inhomogeneous BSE
  for **quark-antiquark vertices:**



Analogy: geometric series

$f(x) = 1 + x f(x) \quad \Rightarrow \quad f(x) = \frac{1}{1-x}$

$|x| < 1 \quad \Rightarrow \quad f(x) = 1 + x + x^2 + \ldots$

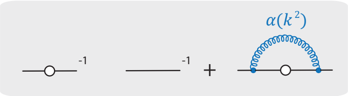
What's the kernel K?

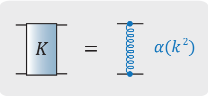
Related to Green functions
via **symmetries:** CVC, PCAC
$\Rightarrow$ vector, axialvector WTIs

Relate **K** with quark propagator
and quark-gluon vertex

Structure of the kernel

Rainbow-ladder: tree-level vertex + effective coupling





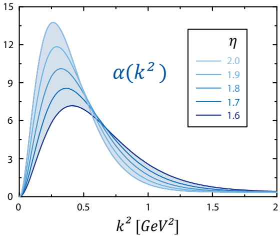
√ **DCSB, CVC, PCAC**

⇒ mass generation

⇒ Goldstone theorem, massless pion in χL

⇒ em. current conservation

⇒ Goldberger-Treiman

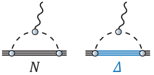


Ansatz for effective coupling:
Maris, Roberts, Tandy, PRC 56 (1997), PRC 60 (1999)

$$\alpha(k^2) = \alpha_{\text{IR}}\left(k^2/\Lambda^2, \eta\right) + \alpha_{\text{UV}}(k^2)$$

Adjust infrared scale Λ to physical observable, keep width η as parameter

No pion cloud, no flavor dependence, no $U_A(1)$ anomaly, no dynamical decay widths



, etc.

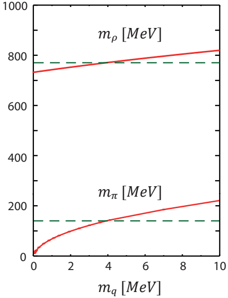
Pion cloud: need infinite summation of t-channel gluons

Mesons

- **Pseudoscalar & vector mesons:** rainbow-ladder is good. Masses, form factors, decays, $\pi\pi$ scattering lengths, PDFs

  Maris, Roberts, Tandy, PRC 56 (1997), PRC 60 (1999); Bashir et al., Commun. Theor. Phys. 58 (2012)

  Pion is Goldstone boson, satisfies GMOR: $m_\pi{}^2 \sim m_q$

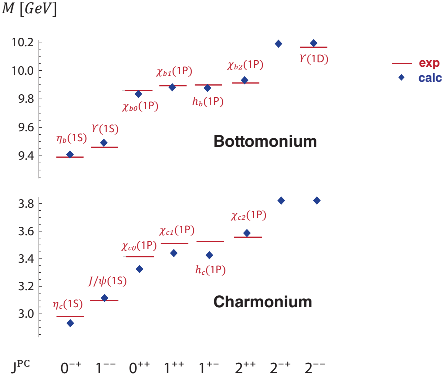


Fischer, J Phys G 32 (2006)

- Need to go **beyond rainbow-ladder** for excited, scalar, axialvector mesons, η-η', etc.

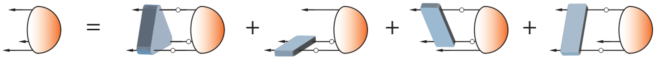
  Fischer, Williams & Chang, Roberts, PRL 103 (2009)
  Alkofer et al., EPJ A38 (2008), Bhagwat et al., PRC 76 (2007)

- **Heavy mesons** Blank, Krassnigg, PRD 84 (2011)



Baryons

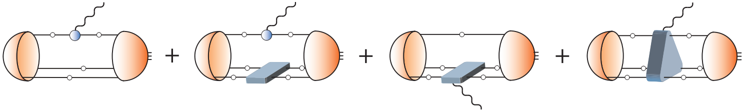
Covariant Faddeev equation: kernel contains 2PI and 3PI parts



Current matrix element: $\langle H|J^\mu|H\rangle = \bar{\chi}\,(G^{-1})^\mu\,\chi$

- Impulse approximation + gauged kernel $(G^{-1})^\mu = (G_0^{-1})^\mu - K^\mu$

'Gauging of equations':
Kvinikhidze, Blankleider, PRC 60 (1999)
Oettel, Pichowsky, von Smekal, EPJ A 8 (2000)



Truncation:

- **Quark-quark correlations** only (dominant structure in baryons?)
- Rainbow-ladder **gluon exchange**
- But **full Poincaré-covariant structure** of Faddeev amplitude retained

$\longrightarrow$ Same input as for mesons, quark from DSE, no additional parameters!

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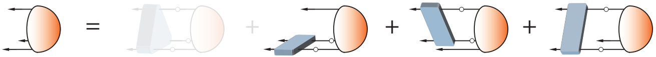

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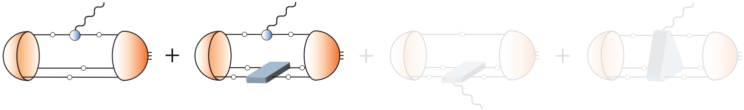

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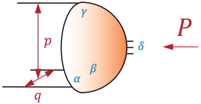
Faddeev wave function

s	l	T_{ij}
		s waves (8)
$1/2$	0	$\mathbb{1} \otimes \mathbb{1}$
$1/2$	0	$\gamma_T^\mu \otimes \gamma_T^\mu$
		p waves (36)
$1/2$	1	$\mathbb{1} \otimes \frac{1}{2}\,[\slashed{p}, \slashed{q}\,]$
$1/2$	1	$\mathbb{1} \otimes \slashed{p}$
$1/2$	1	$\mathbb{1} \otimes \slashed{q}$
$1/2$	1	$\gamma_T^\mu \otimes \gamma_T^\mu\, \frac{1}{2}\,[\slashed{p}, \slashed{q}\,]$
$1/2$	1	$\gamma_T^\mu \otimes \gamma_T^\mu\, \slashed{p}$
$1/2$	1	$\gamma_T^\mu \otimes \gamma_T^\mu\, \slashed{q}$
$3/2$	1	$3\,(\slashed{p} \otimes \slashed{q} - \slashed{q} \otimes \slashed{p}) - \gamma_T^\mu \otimes \gamma_T^\mu\,[\slashed{p}, \slashed{q}\,]$
$3/2$	1	$3\,\slashed{p} \otimes \mathbb{1} - \gamma_T^\mu \otimes \gamma_T^\mu\, \slashed{p}$
$3/2$	1	$3\,\slashed{q} \otimes \mathbb{1} - \gamma_T^\mu \otimes \gamma_T^\mu\, \slashed{q}$
		d waves (20)
$3/2$	2	$3\,\slashed{p} \otimes \slashed{p} - \gamma_T^\mu \otimes \gamma_T^\mu$
$3/2$	2	$\slashed{p} \otimes \slashed{p} + 2\,\slashed{q} \otimes \slashed{q} - \gamma_T^\mu \otimes \gamma_T^\mu$
$3/2$	2	$\slashed{p} \otimes \slashed{q} + \slashed{q} \otimes \slashed{p}$
$3/2$	2	$\slashed{q} \otimes [\slashed{q}, \slashed{p}\,] - \frac{1}{2}\,\gamma_T^\mu \otimes [\gamma_T^\mu, \slashed{p}\,]$
$3/2$	2	$\slashed{p} \otimes [\slashed{p}, \slashed{q}\,] - \frac{1}{2}\,\gamma_T^\mu \otimes [\gamma_T^\mu, \slashed{q}\,]$

$$\chi(x_1, x_2, x_3) = \langle 0|\, T\, \psi(x_1)\, \psi(x_2)\, \psi(x_3)\, |N\rangle$$

Momentum space:
Jacobi coordinates p, q, P
⇒ 5 Lorentz invariants
⇒ 64 Dirac basis elements



$$\chi(p, q, P) = \sum_k \boxed{f_k(p^2,\, q^2,\, p\cdot q,\, p\cdot P,\, q\cdot P) \quad \textbf{Momentum}}$$
$$\boxed{\tau^k_{\alpha\beta\gamma\delta}(p, q, P) \quad \textbf{Dirac}} \; \boxed{\otimes \; \textbf{Flavor} \; \otimes \; \textbf{Color}}$$

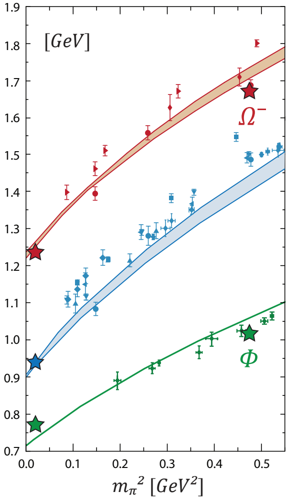
Complete, orthogonal **Dirac tensor basis**
(partial-wave decomposition in nucleon rest frame):
GE, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010)

$$T_{ij}\,(\Lambda_\pm \gamma_5 C \otimes \Lambda_+)$$
$$(\gamma_5 \otimes \gamma_5)\, T_{ij}\,(\Lambda_\pm \gamma_5 C \otimes \Lambda_+)$$

$$(A \otimes B)_{\alpha\beta\gamma\delta} = A_{\alpha\beta}\, B_{\gamma\delta}$$

Baryon masses

- Good agreement with experiment & lattice. Pion mass is also calculated.
- Same kernel as for mesons, scale set by f_π. Full covariant wave functions, no further parameters or approximations.
- Masses not sensitive to effective interaction.
- **Diquark clustering in baryons:** similar results in quark-diquark approach
  Oettel, Alkofer, von Smekal, EPJ A8 (2000)
  GE, Cloet, Alkofer, Krassnigg, Roberts, PRC 79 (2009)
- **Excited baryons** (e.g. Roper): also quark-diquark structure?



Delta mass:
Sanchis-Alepuz *et al.*, PRD 84 (2011)

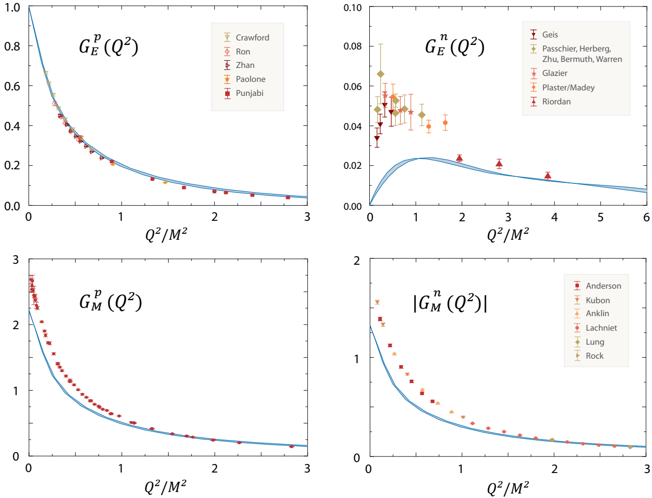
Nucleon mass:
GE, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010);
GE, PRD 84 (2011)

ρ–meson mass:
Maris & Tandy, PRC 60 (1999)

Electromagnetic form factors

FWF
HIC for FAIR
Helmholtz International Center

Nucleon em. FFs
vs. momentum transfer
GE, PRD 84 (2011)



- Agreement with data at larger Q^2 and lattice at larger quark masses

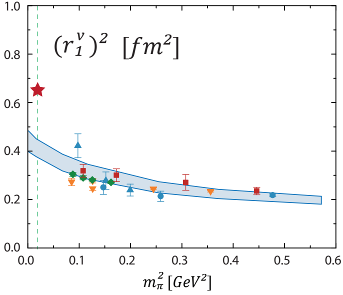
- **Missing pion cloud** below 1–2 GeV^2, in chiral region

$\sim$ **nucleon quark core** without pion effects

Electromagnetic form factors

Nucleon charge radii:

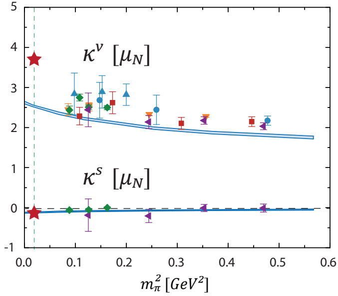
isovector (p-n) Dirac (F1) radius



- **Pion-cloud effects** missing in chiral region (⇒ divergence!), agreement with lattice at larger quark masses.

Nucleon magnetic moments:

isovector (p-n), isoscalar (p+n)



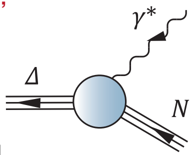
- **But:** pion-cloud **cancels** in κ^s ⇔ **quark core**

Exp: $\kappa^s = -0.12$

Calc: $\kappa^s = -0.12(1)$ **!!**

Nucleon-Δ-γ transition

Electromagnetic transition from **spin-1/2 nucleon** to **spin-3/2 Δ baryon,**
e.g. in pion photo- and electroproduction, Compton scattering.



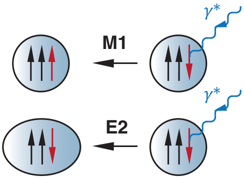
Current matrix element:

$$J^{\mu,\rho}(P,Q) = \mathbb{P}^{\rho\alpha}(P_f)\, i\gamma_5\, \Gamma^{\alpha\mu}(P,Q)\, \Lambda_+(P_i)$$

$$\Gamma^{\alpha\mu} = b\left[\frac{i\omega}{2\lambda_+}\,(G_M^\star - G_E^\star)\,\gamma_5\,\varepsilon^{\alpha\mu\gamma\delta} K^\gamma \widehat{Q}^\delta \;- G_E^\star\, T_Q^{\alpha\gamma}\, T_K^{\gamma\mu} - \frac{i\tau}{\omega}\, G_C^\star\, \widehat{Q}^\alpha K^\mu\right]$$

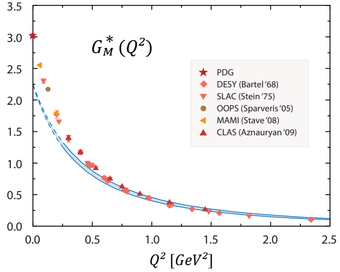
Jones-Scadron form factors:

- **Magnetic dipole form factor** $G_M^\star$
  dominant: quark spin flip
- **Electric & Coulomb quadrupole transitions** $G_E^\star$, $G_C^\star$
  small & negative, encode deformation. Ratios:



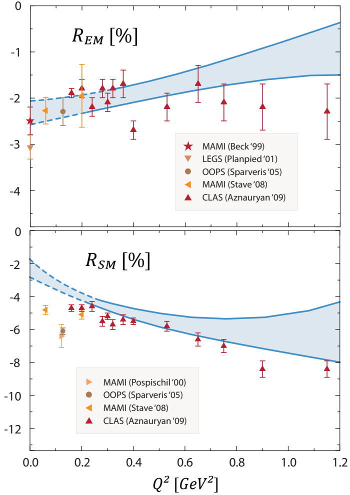
$$R_{EM} = -\frac{G_E^\star}{G_M^\star}\,, \quad R_{SM} = -\frac{|\boldsymbol{Q}|}{2M_\Delta}\,\frac{G_C^\star}{G_M^\star}$$

Nucleon-Δ-γ transition



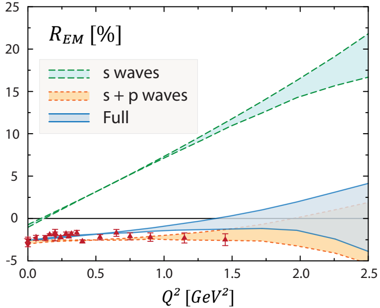
- **Magnetic dipole transition (G_M^*) dominant:** quark spin flip (s wave). “Core + 25% pion cloud”
- **Electric & Coulomb quadrupole transitions** small & negative, encode deformation.

  Quark model: need **d waves** or **pion cloud**.
  Perturbative QCD: $R_{EM} \to 1$, $R_{SM} \to$ const.
- Quark-diquark: ratios reproduced w/o pion cloud?!



Eichmann & Nicmorus, PRD 85 (2012)

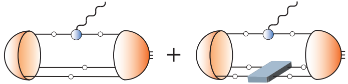
Nucleon-Δ-γ transition

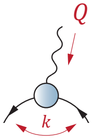


R_{EM} dominated by **p waves!**
Quark OAM in N and Δ wave functions, consequence of Poincaré covariance

s	l	SC	AV	
1/2	0	$\mathbb{1}$		s waves (3)
1/2	0		γ_T^μ	
1/2	0		P^μ	
1/2	1	$\not{q}$		p waves (4)
1/2	1		$\gamma_T^\mu \not{q}$	
1/2	1		$P^\mu \not{q}$	
3/2	1		$\gamma_T^\mu \not{q} - 3q^\mu$	
3/2	2		$\gamma_T^\mu - 3q^\mu \not{q}$	d wave

Quark-photon vertex

Current matrix element: $\langle H|J^\mu|H\rangle =$ 



Vector WTI $Q^\mu\,\Gamma^\mu(k,Q) = S^{-1}(k_+) - S^{-1}(k_-)$
determines vertex up to transverse parts:

$$\Gamma^\mu(k,Q) = \Gamma^\mu_{\mathrm{BC}}(k,Q) + \Gamma^\mu_{\mathrm{T}}(k,Q)$$

- **Ball-Chiu vertex**, completely specified by dressed fermion propagator: Ball, Chiu, PRD 22 (1980)

$$\Gamma^\mu_{\mathrm{BC}}(k,Q) = i\gamma^\mu\,\Sigma_A + 2k^\mu(i\not{k}\,\Delta_A + \Delta_B)$$

$$\Sigma_A := \frac{A(k_+^2)+A(k_-^2)}{2}, \quad \Delta_A := \frac{A(k_+^2)-A(k_-^2)}{k_+^2-k_-^2}, \quad \Delta_B := \frac{B(k_+^2)-B(k_-^2)}{k_+^2-k_-^2}$$

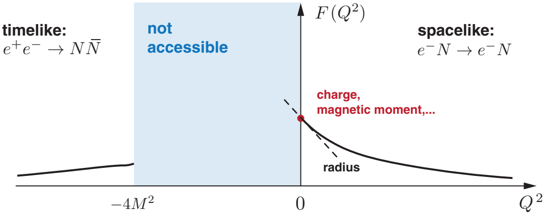
- **Transverse part:** free of kinematic singularities, tensor structures $\sim Q, Q^2, Q^3$, contains meson poles
Kizilersu, Reenders, Pennington, PRD 92 (1995); GE, Fischer, PRD 87 (2013)

$$t^{\mu\nu}_{ab} := a\cdot b\,\delta^{\mu\nu} - b^\mu a^\nu$$

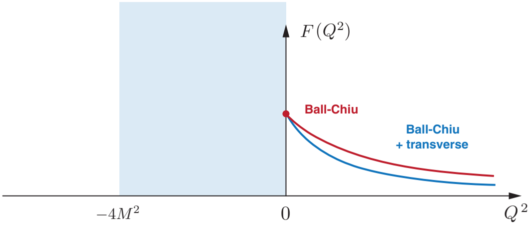
Dominant	$\tau_1^\mu = t^{\mu\nu}_{QQ}\,\gamma^\nu$,	$\tau_5^\mu = t^{\mu\nu}_{QQ}\,ik^\nu$,
	$\tau_2^\mu = t^{\mu\nu}_{QQ}\,k\cdot Q\,\frac{i}{2}[\gamma^\nu,\not{k}]$,	$\tau_6^\mu = t^{\mu\nu}_{QQ}\,k^\nu\not{k}$,
Anomalous magnetic moment	$\tau_3^\mu = \frac{i}{2}[\gamma^\mu,\not{Q}]$,	$\tau_7^\mu = t^{\mu\nu}_{Qk}\,k\cdot Q\,\gamma^\nu$, Curtis, Pennington, PRD 42 (1990)
	$\tau_4^\mu = \frac{1}{6}[\gamma^\mu,\not{k},\not{Q}]$,	$\tau_8^\mu = t^{\mu\nu}_{Qk}\,\frac{i}{2}[\gamma^\nu,\not{k}]$.

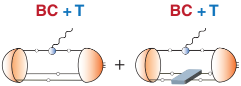
Quark-photon vertex

Structure of quark-photon vertex is reflected in form factors.
Experimentally (sketch):



Calculated:
(Sketch)

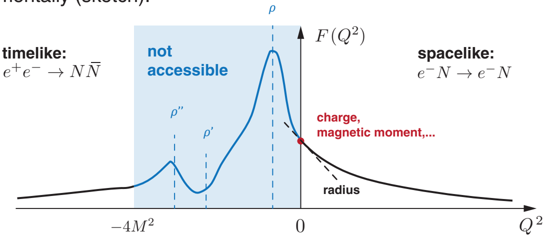




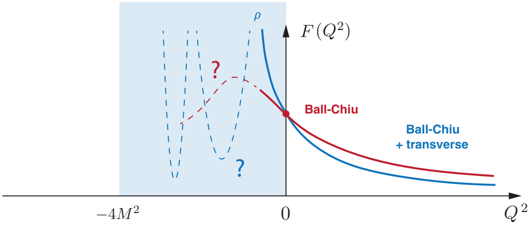
- Ball-Chiu part is dominant **(em. gauge invariance):** charge, magnetic moments
- Transverse part changes slope and charge radii. No pion cloud in RL ⇒ timelike ρ-**meson poles**

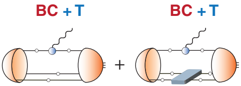
Quark-photon vertex

Structure of quark-photon vertex is reflected in form factors.
Experimentally (sketch):



Calculated:
(Sketch)





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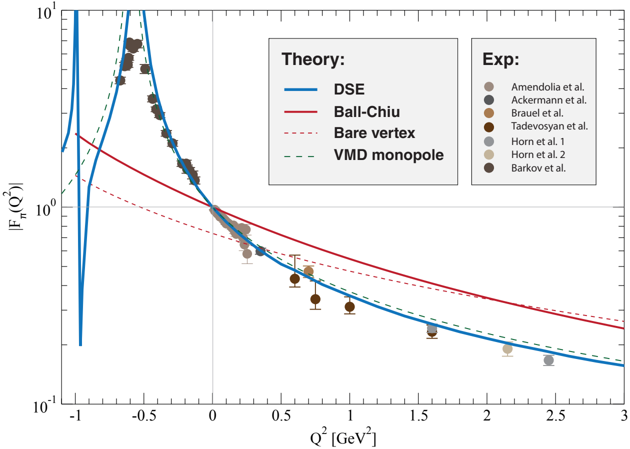
- Transverse part changes slope and charge radii. No pion cloud in RL ⇒ timelike ρ-**meson poles**

Pion form factor



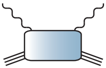
Spacelike and timelike region:

A. Krassnigg
(Schladming 2010)

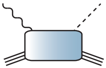
extension of Maris & Tandy, Nucl. Phys. Proc. Suppl. 161 (2006)

Hadron scattering

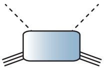
Can we extend this to **four-body scattering** processes?
GE, Fischer, PRD 85 (2012)



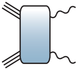
Compton scattering, DVCS, 2γ physics



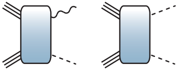
Meson photo- and electroproduction



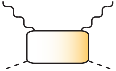
Nucleon-pion scattering



$\bar{p}p \rightarrow \gamma\gamma^*$
annihilation



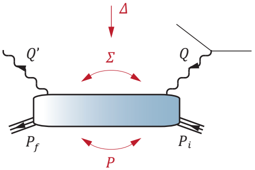
Meson production



Pion Compton scattering

$\Rightarrow$ Nonperturbative description of hadron-photon and hadron-meson scattering

Nucleon Compton scattering

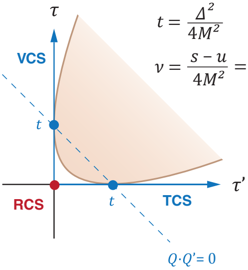


$$\tau = \frac{Q^2}{4M^2}$$

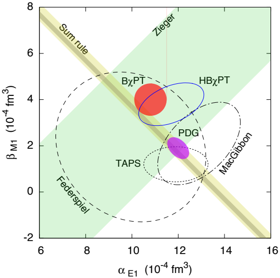
$$\tau' = \frac{Q'^2}{4M^2}$$

$$t = \frac{\Delta^2}{4M^2}$$

$$\nu = \frac{s-u}{4M^2} = -\frac{\Sigma\cdot P}{M^2}$$



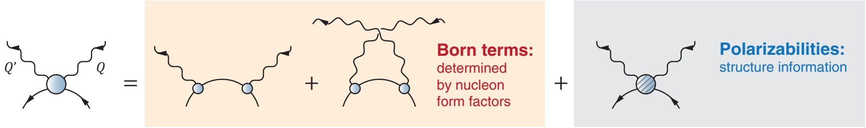
- **RCS, VCS:** nucleon polarizabilities



Krupina & Pascalutsa, PRL 110 (2013)

- **DVCS:** handbag dominance, GPDs
- **Forward limit:** structure functions in DIS
- **Timelike region:** $p\bar{p}$ annhihilation at PANDA
- **Spacelike region:** two-photon corrections to nucleon form factors, proton radius puzzle?

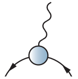
Compton scattering



- All direct measurements in kinematic limits (RCS, VCS, forward limit).
- Em. gauge invariance $\Rightarrow$ Compton amplitude is **fully transverse.** **Analyticity** constrains 1PI part in these limits (low-energy theorem).
- Polarizabilities = coefficients of tensor structures that vanish like $\sim Q^\mu Q'^\nu,\; Q^\mu Q^\nu,\; Q'^\mu Q'^\nu, \dots$
- Need tensor basis free of kinematic singularities (18 elements). Complicated...

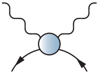
Bardeen, Tung, Phys. Rev. 173 (1968)
Perrottet, Lett. Nuovo Cim. 7 (1973)
Tarrach, Nuovo Cim. 28 A (1975)
Drechsel et al., PRC 57 (1998)
L'vov et al., PRC 64 (2001)
Gorchtein, PRC 81 (2010)
Belitsky, Mueller, Ji, 1212.6674 [hep-ph]
...

Tensor basis?



$$\tau_1 = \gamma^\mu$$
$$\tau_2 = \sigma^{\mu\nu}\, Q^\nu$$

Transversality, analyticity and **Bose symmetry** makes the construction extremely difficult...



Tarrach, Nuovo Cim. 28 (1975)

$$\begin{aligned}
T_1 &= g_{\nu\mu}\,, & T_{18} &= (P_\nu k_\mu - P_\mu k'_\nu)\hat{K}\,,\\
T_2 &= k_\nu k'_\mu\,, & T_{19} &= (P_\nu k'_\mu + P_\mu k_\nu)\hat{K}\,,\\
T_3 &= k'_\nu k_\mu\,, & T_{20} &= (P_\nu k'_\mu - P_\mu k_\nu)\hat{K}\,,\\
T_4 &= k_\nu k_\mu + k'_\nu k'_\mu\,, & T_{21} &= P_\nu \gamma_\mu + P_\mu \gamma_\nu\,,\\
T_5 &= k_\nu k_\mu - k'_\nu k'_\mu\,, & T_{22} &= P_\nu \gamma_\mu - P_\mu \gamma_\nu\,,\\
T_6 &= P_\nu P_\mu\,, & T_{23} &= k_\nu \gamma_\mu + k'_\mu \gamma_\nu\,,\\
T_7 &= P_\nu k_\mu + P_\mu k'_\nu\,, & T_{24} &= k_\nu \gamma_\mu - k'_\mu \gamma_\nu\,,\\
T_8 &= P_\nu k_\mu - P_\mu k'_\nu\,, & T_{25} &= k'_\nu \gamma_\mu + k_\mu \gamma_\nu\,,\\
T_9 &= P_\nu k'_\mu + P_\mu k_\nu\,, & T_{26} &= k'_\nu \gamma_\mu - k_\mu \gamma_\nu\,,\\
T_{10} &= P_\nu k'_\mu - P_\mu k_\nu\,, & T_{27} &= (P_\nu \gamma_\mu + P_\mu \gamma_\nu)\hat{K} - \hat{K}(P_\nu \gamma_\mu + P_\mu \gamma_\nu)\,,\\
T_{11} &= g_{\nu\mu}\hat{K}\,, & T_{28} &= (P_\nu \gamma_\mu - P_\mu \gamma_\nu)\hat{K} - \hat{K}(P_\nu \gamma_\mu - P_\mu \gamma_\nu)\,,\\
T_{12} &= k_\nu k'_\mu \hat{K}\,, & T_{29} &= (k_\nu \gamma_\mu + k'_\mu \gamma_\nu)\hat{K} - \hat{K}(k_\nu \gamma_\mu + k'_\mu \gamma_\nu)\,,\\
T_{13} &= k'_\nu k_\mu \hat{K}\,, & T_{30} &= (k_\nu \gamma_\mu - k'_\mu \gamma_\nu)\hat{K} - \hat{K}(k_\nu \gamma_\mu - k'_\mu \gamma_\nu)\,,\\
T_{14} &= (k_\nu k_\mu + k'_\nu k'_\mu)\hat{K}\,, & T_{31} &= (k'_\nu \gamma_\mu + k_\mu \gamma_\nu)\hat{K} - \hat{K}(k'_\nu \gamma_\mu + k_\mu \gamma_\nu)\,,\\
T_{15} &= (k_\nu k_\mu - k'_\nu k'_\mu)\hat{K}\,, & T_{32} &= (k'_\nu \gamma_\mu - k_\mu \gamma_\nu)\hat{K} - \hat{K}(k'_\nu \gamma_\mu - k_\mu \gamma_\nu)\,,\\
T_{16} &= P_\nu P_\mu \hat{K}\,, & T_{33} &= \gamma_\nu \gamma_\mu - \gamma_\mu \gamma_\nu\,,\\
T_{17} &= (P_\nu k_\mu + P_\mu k'_\nu)\hat{K}\,, & T_{34} &= (\gamma_\nu \gamma_\mu - \gamma_\mu \gamma_\nu)\hat{K} + \hat{K}(\gamma_\nu \gamma_\mu - \gamma_\mu \gamma_\nu)\,,
\end{aligned}$$

$$\begin{aligned}
\tau_1 &= k\cdot k'\, T_1 - T_3\,,\\
\tau_2 &= k^2 k'^2 T_1 + k\cdot k'\, T_2 - \frac{k^2+k'^2}{2} T_4 + \frac{k^2-k'^2}{2} T_5\,,\\
\tau_3 &= (P\cdot K)^2 T_1 + k\cdot k'\, T_6 - P\cdot K T_7\,,\\
\tau_4 &= P\cdot K(k^2+k'^2) T_1 - P\cdot K T_4 - \frac{k^2+k'^2}{2} T_7 + \frac{k^2-k'^2}{2} T_8 + k\cdot k'\, T_9\,,\\
\tau_5 &= -P\cdot K(k^2-k'^2) T_1 + P\cdot K T_5 + \frac{k^2-k'^2}{2} T_7 - \frac{k^2+k'^2}{2} T_8 + k\cdot k'\, T_{10}\,,\\
\tau_6 &= P\cdot K T_2 - \frac{k^2+k'^2}{4} T_9 - \frac{k^2-k'^2}{4} T_{10} - M T_{12} + M\frac{k^2+k'^2}{4} T_{23} -\\
&\quad - M\frac{k^2-k'^2}{4} T_{24} + \frac{k^2-k'^2}{8} T_{29} - \frac{k^2+k'^2}{8} T_{30} - \frac{k^2 k'^2}{4} T_{33}\,,\\
\tau_7 &= 8T_{16} - 4P\cdot K T_{21} + P\cdot K T_{34}\,,\\
\tau_8 &= T_{19} + \frac{k^2-k'^2}{2} T_{22} - P\cdot K T_{23} + \frac{k^2+k'^2}{8} T_{34}\,,\\
\tau_9 &= T_{20} - \frac{k^2+k'^2}{2} T_{22} + P\cdot K T_{24} - \frac{k^2-k'^2}{8} T_{34}\,,\\
\tau_{10} &= -8k\cdot k'\, T_6 + 4P\cdot K T_7 + 4Mk\cdot k'\, T_{21} - 4MP\cdot K T_{25} -\\
&\quad -2P\cdot K T_{32} - 2k\cdot k'\, P\cdot K T_{33} + Mk\cdot k'\, T_{34}\,,\\
\tau_{11} &= T_{18} - k\cdot k'\, T_{22} + P\cdot K T_{26}\,,\\
\tau_{12} &= P\cdot K T_4 - \frac{k^2-k'^2}{2} T_8 - k\cdot k'\, T_9 - M T_{14} + Mk\cdot k'\, T_{23} -\\
&\quad - M\frac{k^2-k'^2}{2} T_{26} - \frac{k^2+k'^2}{4} T_{32} - k\cdot k' \frac{k^2+k'^2}{4} T_{33}\,,\\
\tau_{13} &= P\cdot K T_5 - \frac{k^2+k'^2}{2} T_8 + k\cdot k'\, T_{10} - M T_{15} + Mk\cdot k'\, T_{24} -\\
&\quad - M\frac{k^2+k'^2}{2} T_{26} - \frac{k^2-k'^2}{4} T_{32} - k\cdot k' \frac{k^2-k'^2}{4} T_{33}\,,
\end{aligned}$$

$$\begin{aligned}
\tau_{14} &= 2P\cdot K T_8 - 2Mk\cdot k'\, T_{22} + 2MP\cdot K T_{26} - k\cdot k'\, T_{27} + P\cdot K T_{31}\,,\\
\tau_{15} &= -(k^2-k'^2) T_7 + (k^2+k'^2) T_8 - 2k\cdot k'\, T_{10} - 2Mk\cdot k'\, T_{24} +\\
&\quad + M(k^2-k'^2) T_{25} + M(k^2+k'^2) T_{26} - k\cdot k'\, T_{29} +\\
&\quad + \frac{k^2+k'^2}{2} T_{31} + \frac{k^2-k'^2}{2} T_{32}\,,\\
\tau_{16} &= -(k^2+k'^2) T_7 + (k^2-k'^2) T_8 + 2k\cdot k'\, T_9 - 2Mk\cdot k'\, T_{23} +\\
&\quad + M(k^2+k'^2) T_{25} + M(k^2-k'^2) T_{26} - k\cdot k'\, T_{30} +\\
&\quad + \frac{k^2-k'^2}{2} T_{31} + \frac{k^2+k'^2}{2} T_{32}\,,\\
\tau_{17} &= -4P\cdot K T_1 + 2T_7 + 4MT_{11} - 2MT_{25} + T_{32} + k\cdot k'\, T_{33}\,,\\
\tau_{18} &= 4T_{17} - 4P\cdot K T_{25} + k\cdot k'\, T_{34}\,.\\
\tau_{19} &= \frac{1}{k\cdot k'}\left[2(P\cdot K)^2 \tau_2 + 2k^2 k'^2 \tau_3 - P\cdot K(k^2+k'^2)\tau_4 - P\cdot K(k^2-k'^2)\tau_5\right] =\\
&= 2(P\cdot K)^2 T_2 + 2k^2 k'^2 T_6 - P\cdot K(k^2+k'^2) T_9 - P\cdot K(k^2-k'^2) T_{10}\,,\\
\tau_{20} &= \frac{1}{4k\cdot k'}\left[(k^2-k'^2)\tau_{10} - 2(k^2+k'^2)\tau_{14} + 4P\cdot K \tau_{15}\right] =\\
&= -2(k^2-k'^2) T_6 - 2P\cdot K T_{10} + M(k^2-k'^2) T_{21} + M(k^2+k'^2) T_{22} -\\
&\quad -2MP\cdot K T_{24} + \frac{k^2+k'^2}{2} T_{27} - P\cdot K T_{29} -\\
&\quad - P\cdot K \frac{k^2-k'^2}{2} T_{33} + M\frac{k^2-k'^2}{4} T_{34}\,,\\
\tau_{21} &= \frac{1}{4k\cdot k'}\left[(k^2+k'^2)\tau_{10} - 2(k^2-k'^2)\tau_{14} + 4P\cdot K \tau_{16}\right] =\\
&= -2(k^2+k'^2) T_6 + 2P\cdot K T_9 + M(k^2+k'^2) T_{21} + M(k^2-k'^2) T_{22} -\\
&\quad -2MP\cdot K T_{23} + \frac{k^2-k'^2}{2} T_{27} - P\cdot K T_{30} -\\
&\quad - P\cdot K \frac{k^2+k'^2}{2} T_{33} + M\frac{k^2+k'^2}{4} T_{34}\,.
\end{aligned}$$

Transverse tensor basis for $\Gamma^{\mu\nu}(p,Q,Q')$

- Generalize transverse projectors: $t^{\mu\nu}_{ab} := a\cdot b\,\delta^{\mu\nu} - b^\mu a^\nu$ $\quad a,b\in\{p,Q,Q'\}$
  $\varepsilon^{\mu\nu}_{ab} := \gamma_5\,\varepsilon^{\mu\nu\alpha\beta} a^\alpha b^\beta$ (exhausts all possibilities)

- Apply Bose-(anti-)symmetric combinations

$$\mathsf{E}^{\mu\alpha,\beta\nu}_\pm(a,b) := \tfrac{1}{2}\left(\varepsilon^{\mu\alpha}_{Q'a'}\,\varepsilon^{\beta\nu}_{bQ} \pm \varepsilon^{\mu\alpha}_{Q'b'}\,\varepsilon^{\beta\nu}_{aQ}\right)$$
$$\mathsf{F}^{\mu\alpha,\beta\nu}_\pm(a,b) := \tfrac{1}{2}\left(t^{\mu\alpha}_{Q'a'}\,t^{\beta\nu}_{bQ} \pm t^{\mu\alpha}_{Q'b'}\,t^{\beta\nu}_{aQ}\right)$$
$$\mathsf{G}^{\mu\alpha,\beta\nu}_\pm(a,b) := \tfrac{1}{2}\left(\varepsilon^{\mu\alpha}_{Q'a'}\,t^{\beta\nu}_{bQ} \pm t^{\mu\alpha}_{Q'b'}\,\varepsilon^{\beta\nu}_{aQ}\right)$$

to structures independent of Q, Q':

$\delta^{\alpha\beta}$	$p^\alpha\gamma^\beta + \gamma^\alpha p^\beta$
$\delta^{\alpha\beta}\,\not{p}$	$p^\alpha\gamma^\beta - \gamma^\alpha p^\beta$
$[\gamma^\alpha,\gamma^\beta]$	$[p^\alpha\gamma^\beta + \gamma^\alpha p^\beta, \not{p}]$
$[\gamma^\alpha,\gamma^\beta,\not{p}]$	$[p^\alpha\gamma^\beta - \gamma^\alpha p^\beta, \not{p}]$
	$p^\alpha p^\beta$
	$p^\alpha p^\beta\,\not{p}$

- obtain
  16 quadratic,
  40 cubic
  16 quartic terms
  $\Rightarrow$ **72 in total** √
- no kinematic singularities √

- **Transverse onshell basis:** GE, Fischer, PRD 87 (2013) & PoS Conf. X (2012)

$\mathsf{E}_+(P,P)$ $(++)$	$\widetilde{\mathsf{E}}_+(P,P)$ $(-+)$
$\mathsf{F}_+(P,P)$ $(++)$	$\widetilde{\mathsf{F}}_+(P,P)$ $(-+)$
$\mathsf{G}_+(P,P)$ $(++)$	$\widetilde{\mathsf{G}}_+(P,P)$ $(--)$
$\mathsf{G}_-(P,P)$ $(--)$	$\widetilde{\mathsf{G}}_-(P,P)$ $(++)$

$\mathsf{F}_+(P,Q)$ $(-+)$	$\widetilde{\mathsf{F}}_+(P,Q)$ $(++)$
$\mathsf{G}_+(P,Q)$ $(-+)$	$\widetilde{\mathsf{G}}_+(P,Q)$ $(+-)$
$\mathsf{F}_-(P,Q)$ $(+-)$	$\widetilde{\mathsf{F}}_-(P,Q)$ $(--)$
$\mathsf{G}_-(P,Q)$ $(+-)$	$\widetilde{\mathsf{G}}_-(P,Q)$ $(-+)$
$\mathsf{F}_+(Q,Q)$ $(++)$	$\widetilde{\mathsf{F}}_+(Q,Q)$ $(-+)$

VCS | VVCS | RCS | Scalar vertex

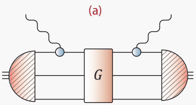
- Simple
- analytic in all limits
- manifest crossing and charge-conjugation symmetry
- scalar & pion pole only in a few Compton form factors
- Tarrach's basis can be cast in a similar form

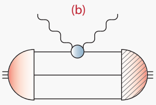
Compton amplitude at quark level

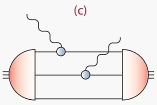
Baryon's **Compton scattering amplitude,** consistent with Faddeev equation:
GE, Fischer, PRD 85 (2012)

$$\langle H|J^\mu J^\nu|H\rangle = \bar{\chi}\left(G^{-1^\mu} G\, G^{-1^\nu} + G^{-1^\nu} G\, G^{-1^\mu} - (G^{-1})^{\mu\nu}\right)\chi$$

In rainbow-ladder (+ crossing & permutation):

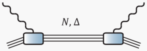






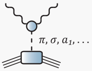
√ crossing symmetry
√ em. gauge invariance
√ perturbative processes included
√ s, t, u channel poles generated in QCD

- **Born (handbag) diagrams:** G = **1** + T
- all s- and u-channel **nucleon resonances:**



1PI **quark 2-photon vertex:**

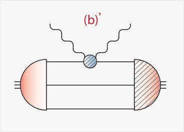
all t-channel **meson poles**



cat's ears diagrams

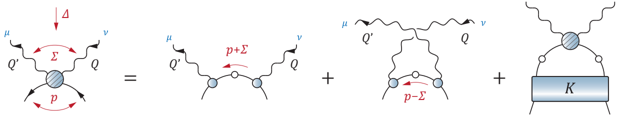
Compton amplitude at quark level

Collect all (nonperturbative!) **'handbag' diagrams**, where photon couples to same quark:
no nucleon resonances, no cat's ears



- **not electromagnetically gauge invariant**, but comparable to 1PI ‚structure part' at nucleon level?
- reduces to **perturbative handbag** at large photon momenta
- but also all **t-channel poles** included!

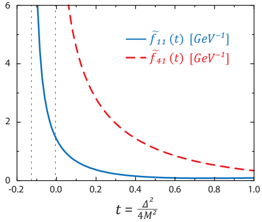
Represented by full **quark Compton vertex**, including Born terms.
Satisfies inhomogeneous BSE:



Solved in rainbow-ladder: 128 tensor structures (72 transverse).
Simplifies dramatically by choice of convenient basis!

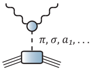
t-channel poles

Quark Compton vertex:
recovers t-channel poles,
e. g. **scalar** and **pion** √

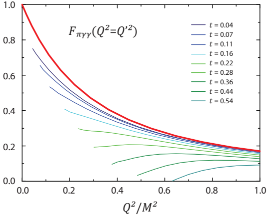


GE & Fischer, PRD 87 (2013)

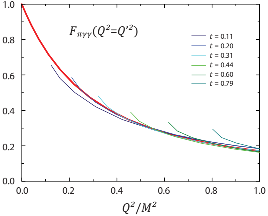
Quark Compton vertex and **nucleon Compton amplitude:** residues at pion pole recover
***$\pi\gamma\gamma$* transition form factor** √



Rainbow-ladder result:
Maris & Tandy, PRC 65 (2002)



(extracted from
quark Compton vertex)



(extracted from
nucleon Compton amplitude)

Summary

Systematic description of QCD phenomenology from quark & gluon substructure:

- **Nonperturbative:** factorization property not necessary
- **Dynamical chiral symmetry breaking:** mass generation for quarks and light hadrons
- **Poincaré covariance:** quark orbital angular momentum via p waves
- **Quark-quark interaction** dominant in ground-state baryons
- **Pion cloud** essential for chiral and low-momentum structure
- **Tetraquark** identification for light scalar mesons plausible

Need to improve **truncations** (pion cloud, decay channels, 3- and 4-quark interactions) and **kinematical coverage**

Interplay between experiment and theory:

- Hadron **masses, wave functions, form factors and scattering amplitudes from QCD**
- Refined tools for understanding fundamental **properties of QCD from experiment**

Thanks for your attention.

Cheers to my collaborators:

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