David Richards

Jefferson Laboratory/Hadron Spectrum Collaboration

EMMI RRTF, GSI, October 2013

LQCD: Issues

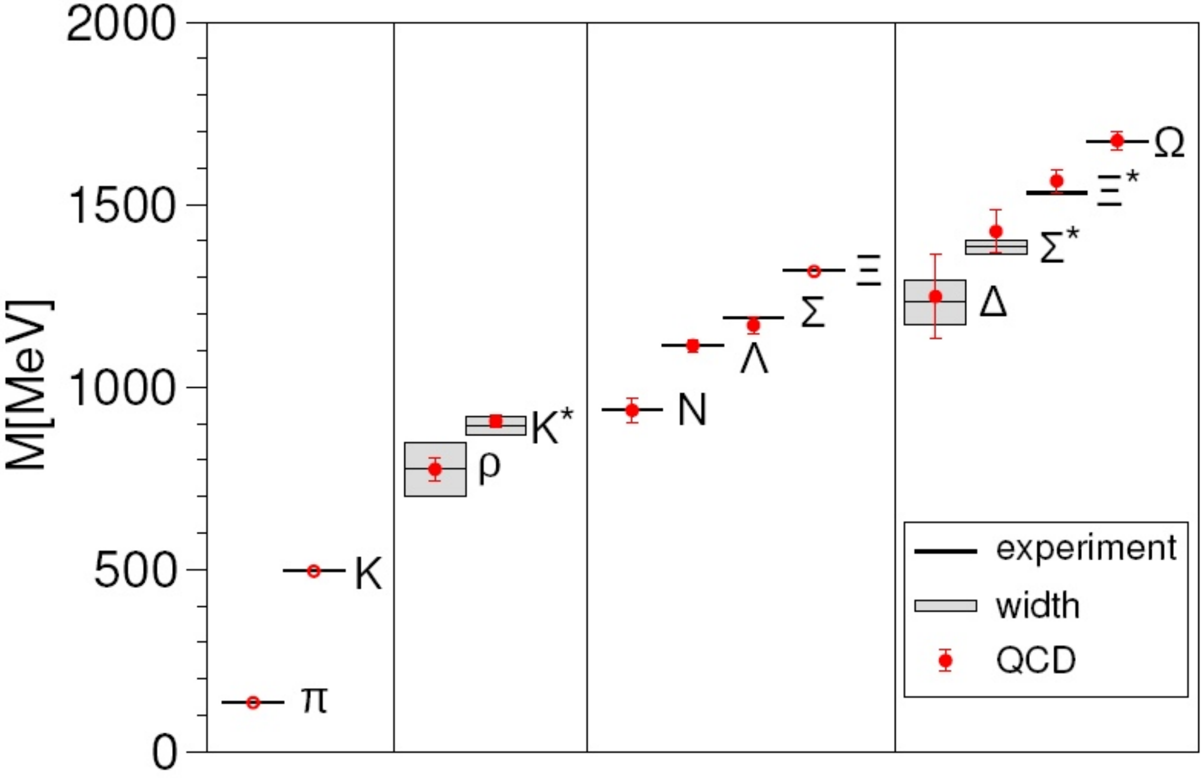
- Spectroscopy Recipe Book
- Spectroscopy
  - Baryons, Mesons, flavor content of isoscalars
  - Resonances in LQCD: *Extraction of Phase Shifts*
  - To-do list - inelastic + multihadron decays
- Form factors of Stable Hadrons
  - Pion form factor
  - Nucleon Form factor
  - In-medium effects....
- Transition form factors
  - “Stable” Delta form factors
  - Photo-couplings between mesons
  - Form Factors of Resonances
  - Time-like form factors - two-photon width?
  - Asymptotic Form Factors - large Q^2

Low-lying Hadron Spectrum

Benchmark of LQCD

$$C(t) = \sum_{\vec{x}} \langle 0 \mid N(\vec{x},t)\bar{N}(0) \mid 0\rangle \;=\; \sum_{n,\vec{x}} \langle 0 \mid e^{ip\cdot x} N(0) e^{-ip\cdot x} \mid n\rangle\langle n \mid \bar{N}(0) \mid 0\rangle$$

$$= \; \mid \langle n \mid N(0) \mid 0\rangle \mid^2 e^{-E_n t} = \sum_n A_n e^{-E_n t}$$



Durr et al., BMW Collaboration

Science 2008

Control over:

- ***Quark-mass dependence***
- ***Continuum extrapolation***
- ***finite-volume effects (pions, resonances)***

Variational Method

Subleading terms → *Excited* states

Construct matrix of correlators with *judicious choice of operators*

$$\begin{aligned} C_{\alpha\beta}(t,t_0) &= \langle 0 \mid \mathcal{O}_\alpha(t)\mathcal{O}^\dagger_\beta(t_0) \mid 0\rangle \\ &\longrightarrow \sum_n Z^n_\alpha Z^{n\dagger}_\beta e^{-M_n(t-t_0)} \end{aligned}$$

Delineate contributions using variational method: solve

$$C(t)u(t,t_0) = \lambda(t,t_0)C(t_0)u(t,t_0)$$

$$\lambda_i(t,t_0) \rightarrow e^{-E_i(t-t_0)}\left(1 + O(e^{-\Delta E(t-t_0)})\right)$$

Eigenvectors, with metric C(t_0), are orthonormal and project onto the respective states

- ➡ Resolve energy dependence - *anisotropic lattice*
- ➡ Judicious construction of interpolating operators - *cubic symmetry*

Challenges

Anisotropic lattices

To appreciate difficulty of extracting excited states, need to understand signal-to-noise ratio in two-point functions. Consider correlation function:

$$C(t) = \langle 0 \mid \mathcal{O}(t)\mathcal{O}(0)^\dagger \mid 0 \rangle \longrightarrow e^{-Et}$$

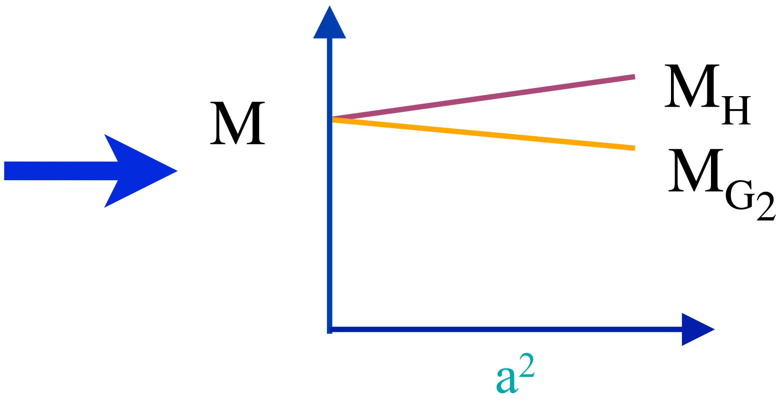
Then the fluctuations behave as **DeGrand, Hecht, PRD46 (1992)**

$$\sigma^2(t) \simeq \left(\langle 0 \mid |\mathcal{O}(t)\mathcal{O}(0)^\dagger|^2 \mid 0 \rangle - C(t)^2 \right) \longrightarrow e^{-2m_\pi t}$$

Signal-to-noise ratio degrades with increasing E - Solution: anisotropic lattice with $a_t < a_s$

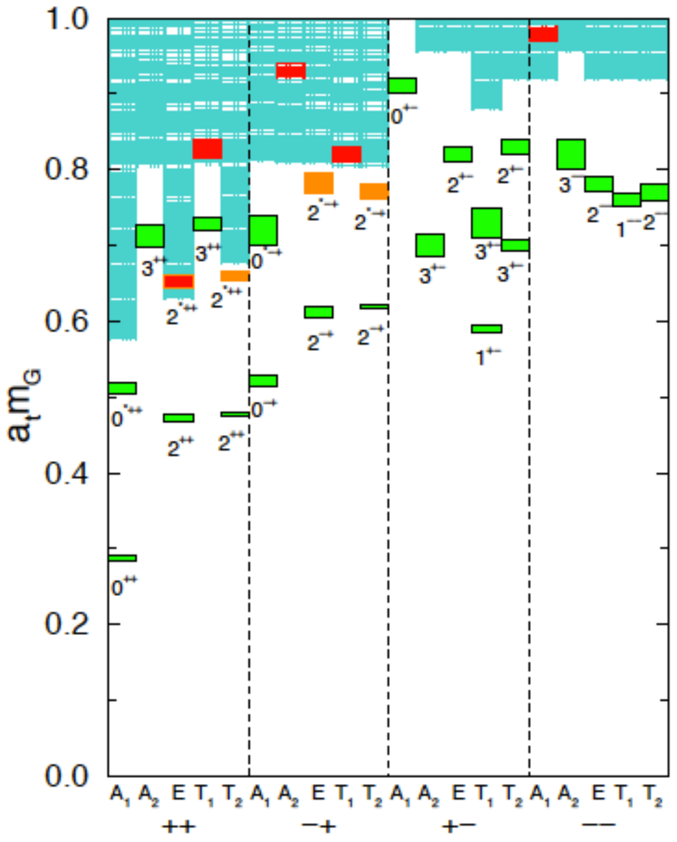
Cubic symmetry of lattices

J	irreps, $\Lambda(\mathrm{dim})$
$\frac{1}{2}$	$G_1(2)$
$\frac{3}{2}$	$H(4)$
$\frac{5}{2}$	$H(4) \oplus G_2(2)$
$\frac{7}{2}$	$G_1(2) \oplus H(4) \oplus G_2(2)$
$\frac{9}{2}$	$G_1(2) \oplus {}^1H(4) \oplus {}^2H(4)$

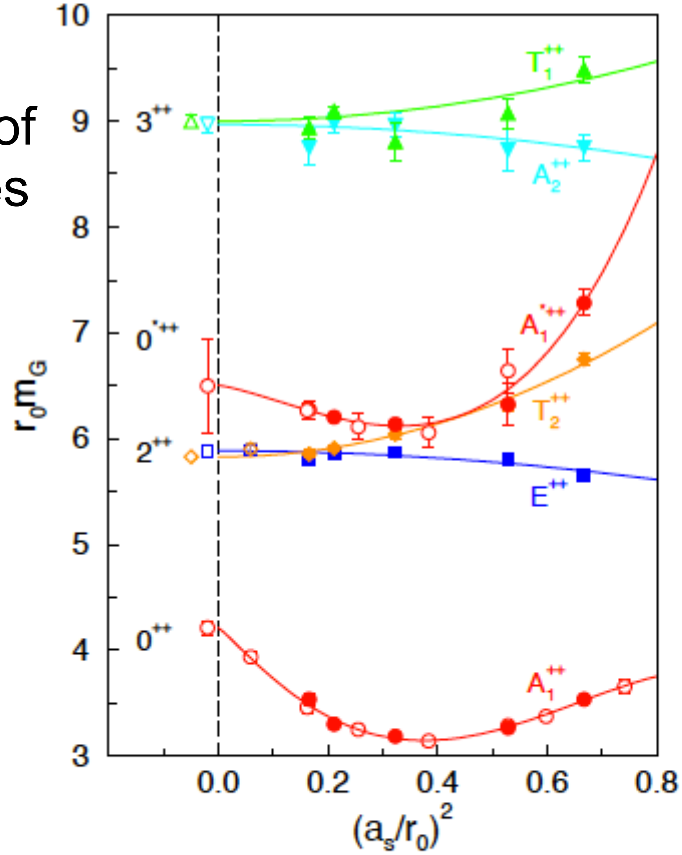


Glueball Spectroscopy - I

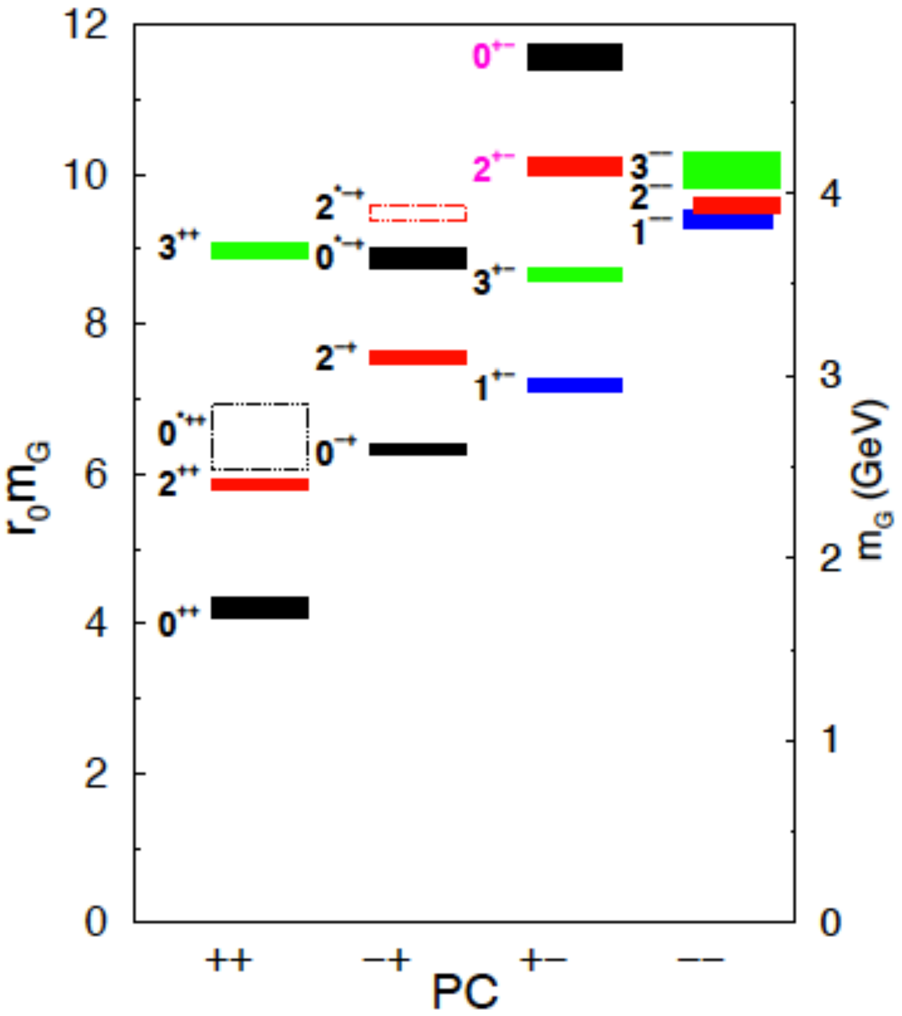
Morningstar, Peardon 97,99



Observe emergence of degeneracies



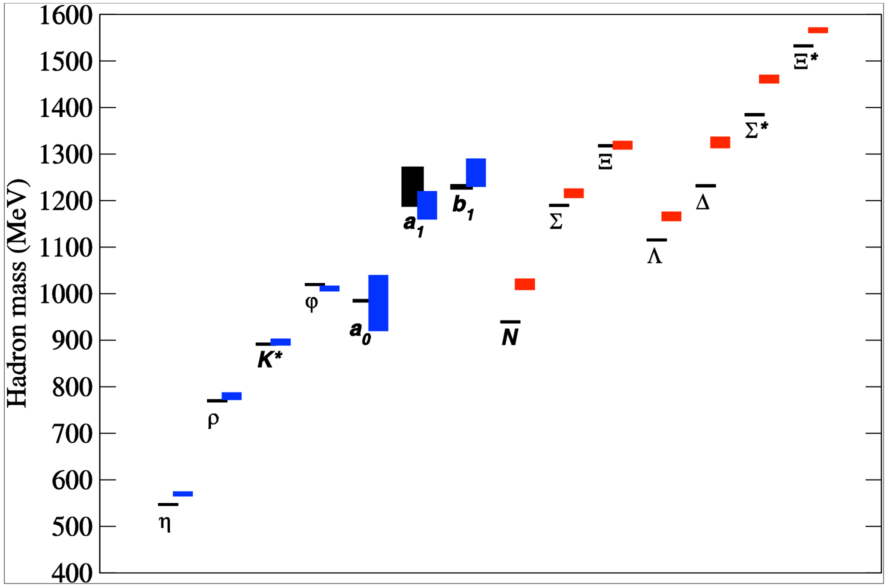
Glueball Spectrum - II



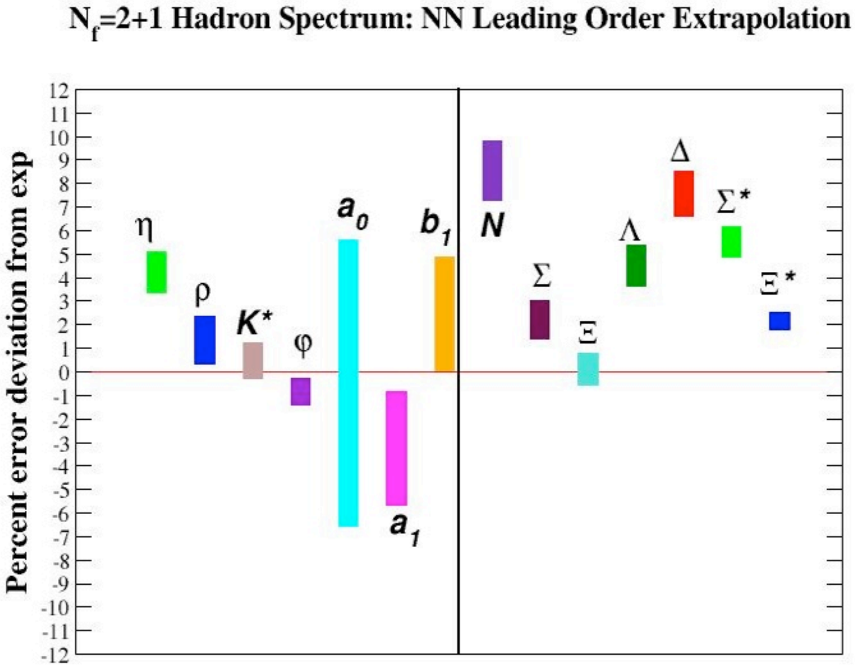
This is the pure Yang-Mills spectrum. Predicts existence of bound states.

2+1 flavor staggered - can mix with two-pi states - *not a smoking gun for gluonic excitations!*

Anisotropic Clover



Low-lying spectrum: *agrees with experiment to 10%*



Meson Operators

Aim: interpolating operators of *definite* (continuum) JM: O^{JM}

$$\langle 0 \mid O^{JM} \mid J', M' \rangle = Z^J \delta_{J,J'} \delta_{M,M'}$$

Starting point $\bar{\psi}(\vec{x},t)\Gamma D_i D_j \dots \psi(\vec{x},t)$

Introduce circular basis:

$$\overleftrightarrow{D}_{m=-1} = \frac{i}{\sqrt{2}} \left(\overleftrightarrow{D}_x - i \overleftrightarrow{D}_y \right)$$

$$\overleftrightarrow{D}_{m=0} = i \overleftrightarrow{D}_z$$

$$\overleftrightarrow{D}_{m=+1} = -\frac{i}{\sqrt{2}} \left(\overleftrightarrow{D}_x + i \overleftrightarrow{D}_y \right).$$

Straighforward to project to definite spin: *J = 0, 1, 2*

$$(\Gamma \times D^{[1]}_{J=1})^{J,M} = \sum_{m_1,m_2} \langle 1, m_1; 1, m_2 \big| J, M \rangle \, \bar{\psi} \Gamma_{m_1} \overleftrightarrow{D}_{m_2} \psi.$$

Use projection formula to find subduction under irrep. of cubic group - operators are closed under rotation!

$$\begin{aligned} O^{[J]}_{\Lambda\lambda}(t,\vec{x}) &= \frac{d_\Lambda}{g_{O_h^D}} \sum_{R \in O_h^D} D^{(\Lambda)*}_{\lambda\lambda}(R) U_R O^{J,M}(t,\vec{x}) U_R^\dagger \\ &= \sum_M S^{J,M}_{\Lambda,\lambda} O^{J,M} \end{aligned}$$

Action of R

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↑ **Irrep, Row**

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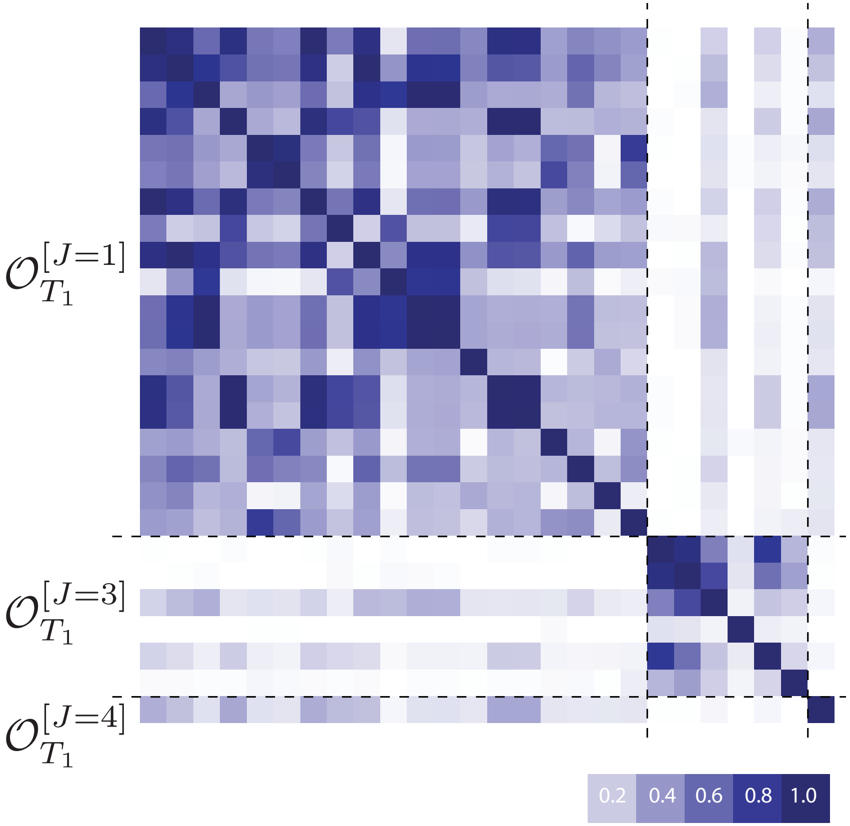
Irrep, Row

Irrep of R in Λ

Action of R

Identification of Spin

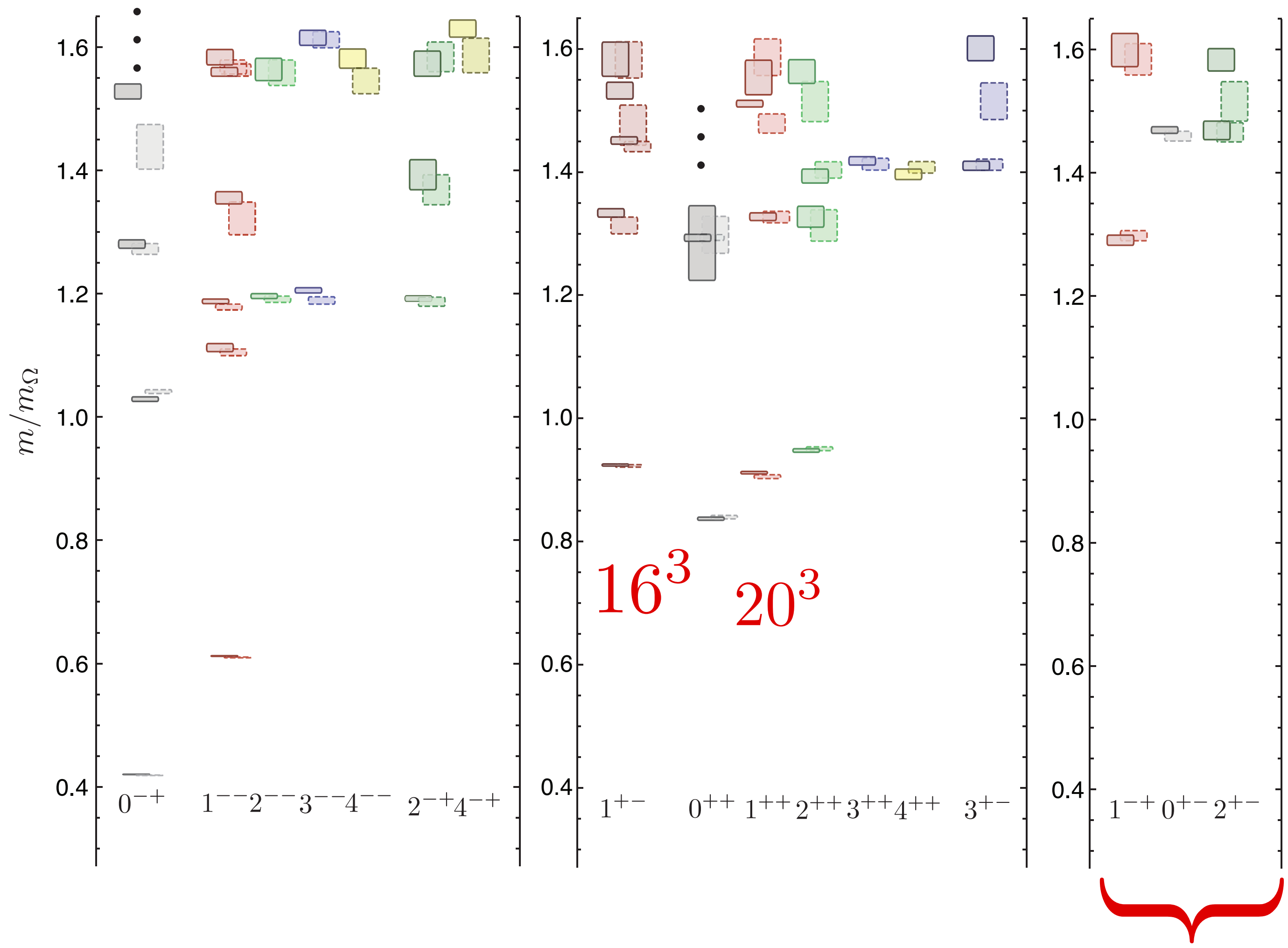
Hadspec collab. (dudek et al), 1004.4930, PRD82, 034508



$$C_{ij}^{\Lambda} = \frac{1}{\dim(\Lambda)} \sum_{\lambda} \langle 0 \mid \mathcal{O}_{i(\Lambda)\lambda}^{[J]} \mathcal{O}_{j(\Lambda)\lambda}^{[J]} \mid 0 \rangle$$

Operators know their parentage

Isovector Meson Spectrum - I

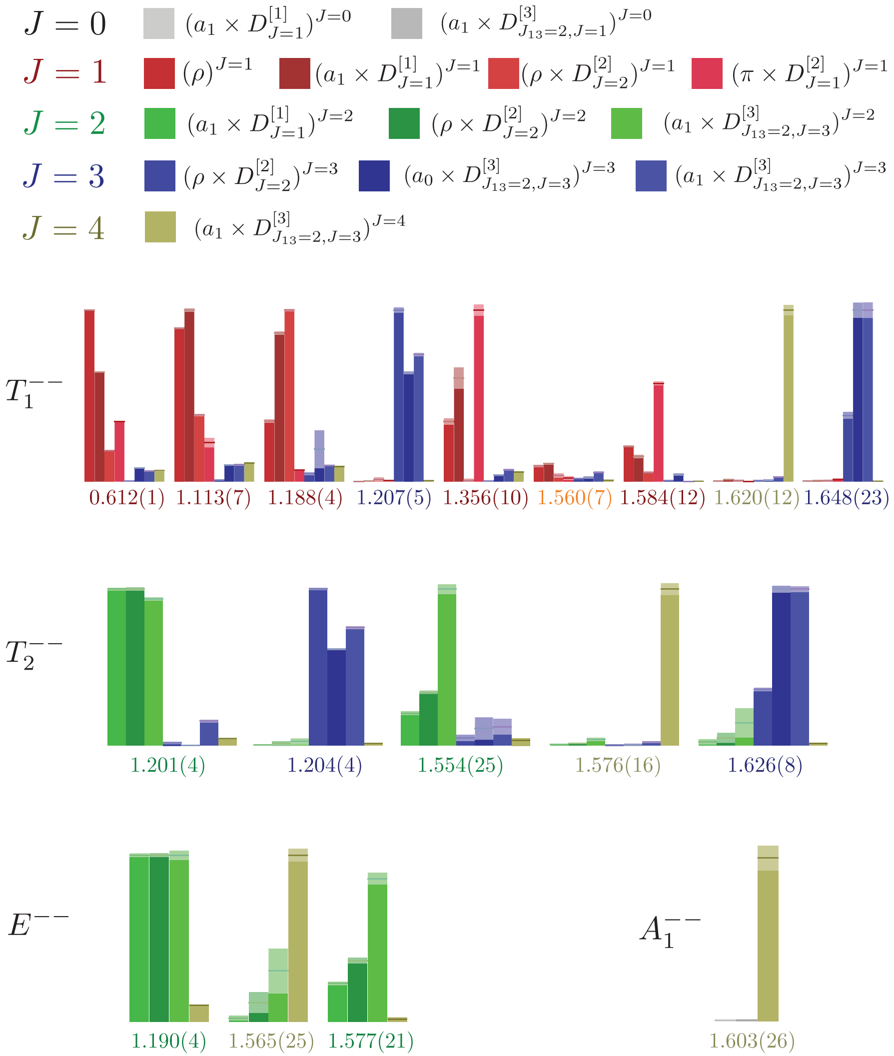


Exotic

PRL 103:262001 (2009)

Isovector spectrum with quantum numbers reliably identified

Interpretation of Meson Spectrum



In each Lattice Irrep, state dominated by operators of particular J

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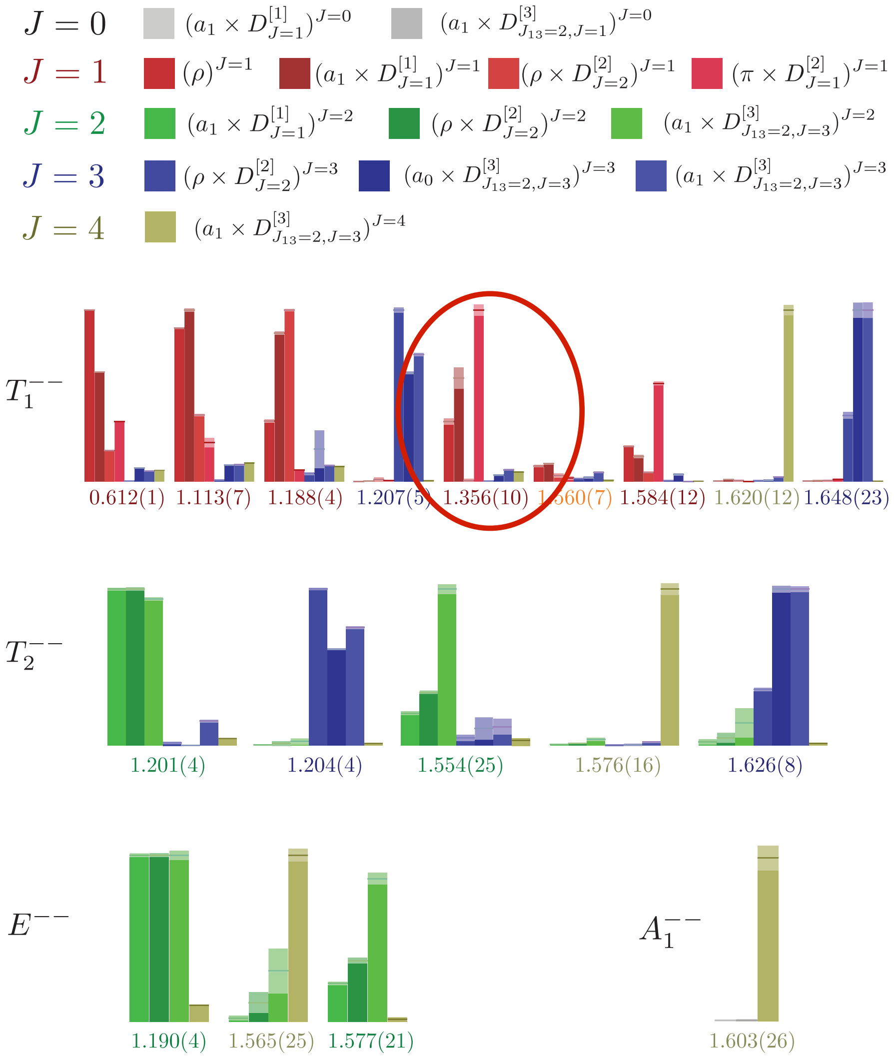

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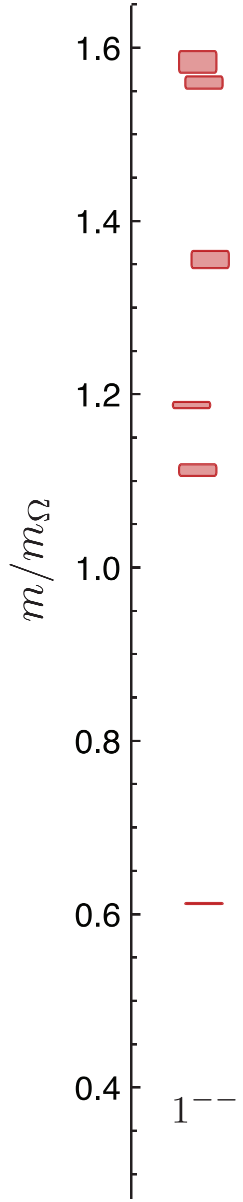
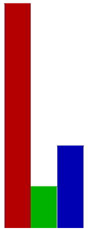
1^{--}





ground state is dominantly $^3S_1$

look at the ‘overlaps’ $Z_n^{\mathbf{\Gamma}} = \langle n | \bar{\psi}\mathbf{\Gamma}\psi | 0 \rangle$

ρ	$^3S_1$
$\left(\rho \times D^{[2]}_{J=2}\right)^{J=1}$	$^3D_1$
$\left(\pi \times D^{[2]}_{J=1}\right)^{J=1}$	hybrid?

Anti-commutator of covariant derivative: vanishes for unit gauge!

Use lattice QCD to build phenomenology of bound states

Dudek, arXiv:1106.5515

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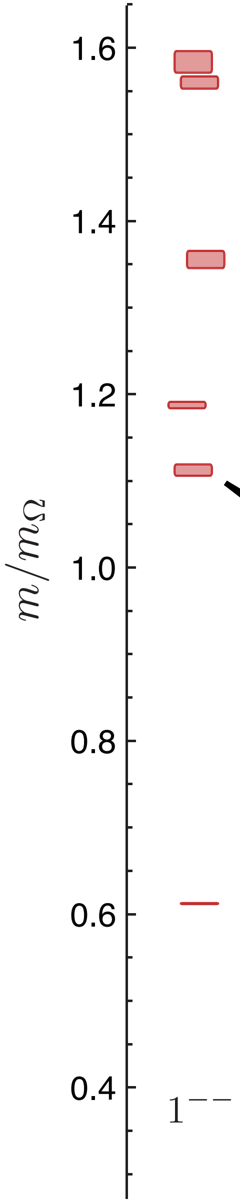
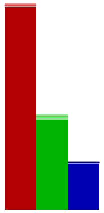
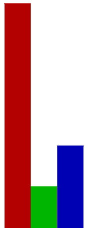




1^{st} excited state is dominantly $^3S_1$ with some $^3D_1$



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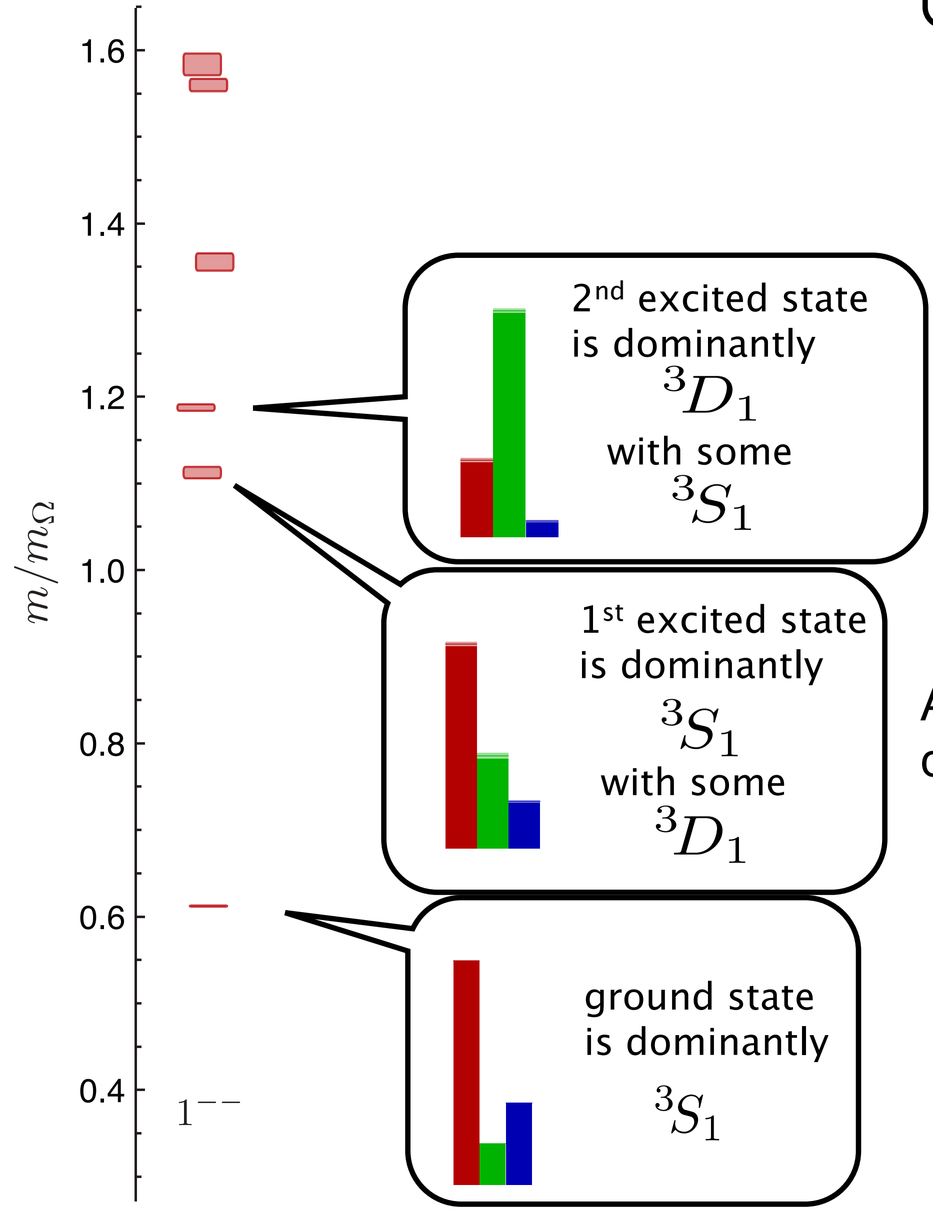

2nd excited state is dominantly $^3D_1$ with some $^3S_1$

1st excited state is dominantly $^3S_1$ with some $^3D_1$

ground state is dominantly $^3S_1$

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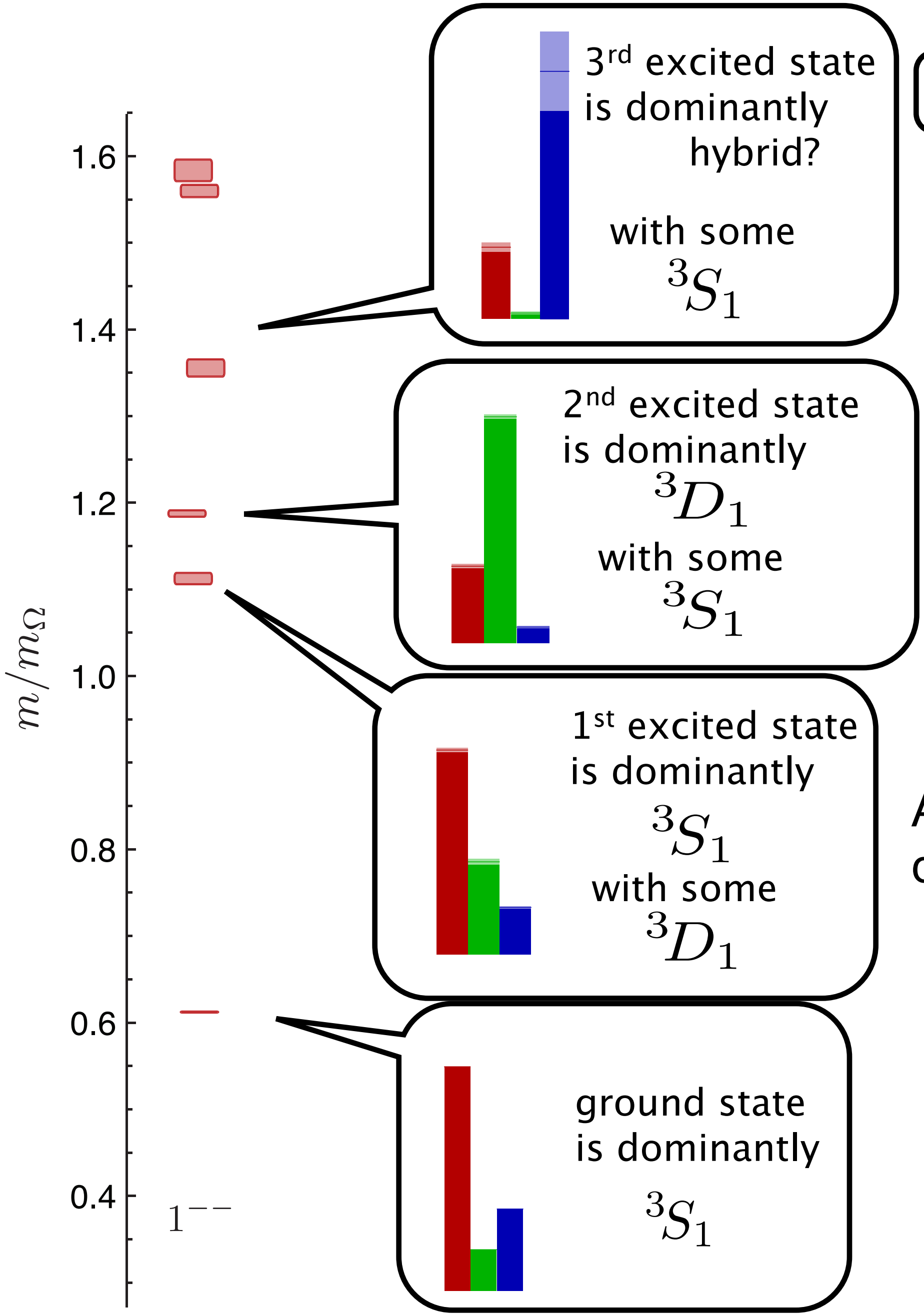

3rd excited state is dominantly hybrid?

with some $^3S_1$

2nd excited state is dominantly $^3D_1$

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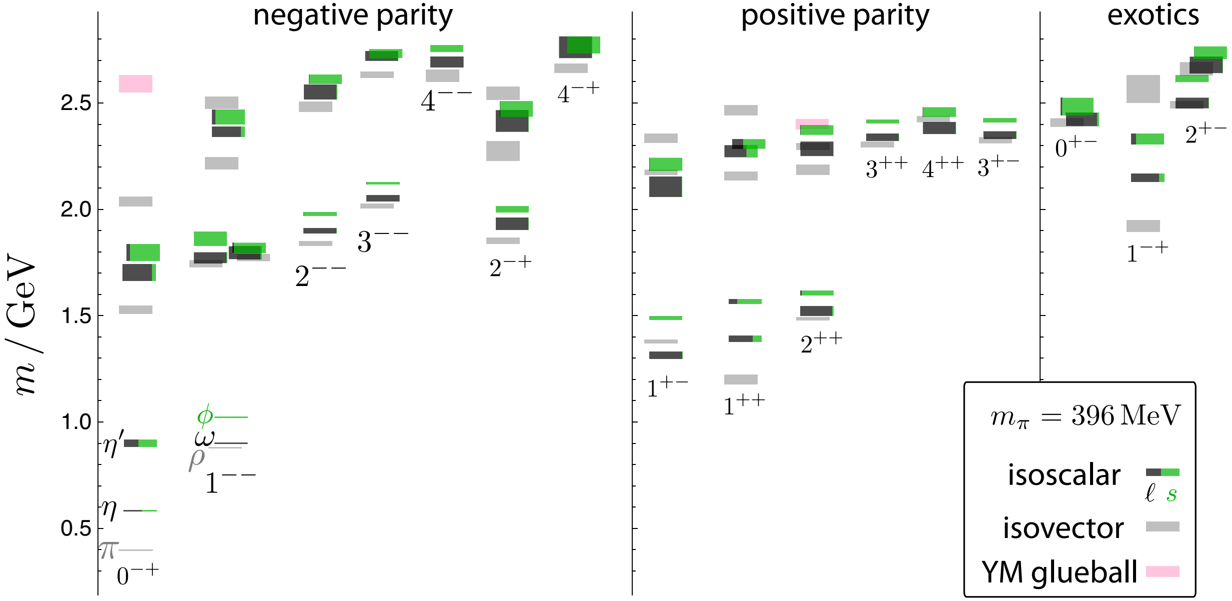
Isoscalar Meson Spectrum



Diagonalize in 2x2 *flavor space*

$$C = \begin{pmatrix} -\mathcal{C}^{\ell\ell} + 2\,\mathcal{D}^{\ell\ell} & \sqrt{2}\,\mathcal{D}^{\ell s} \\ \sqrt{2}\,\mathcal{D}^{s\ell} & -\mathcal{C}^{ss} + \mathcal{D}^{ss} \end{pmatrix}$$

- Spin-identified single-particle spectrum: states of spin as high as four
- Hidden flavor mixing angles extracted - except 0^{-+}, 1^{++} near ideal mixing
- *First determination of exotic isoscalar states: comparable in mass to isovector*

J. Dudek *et al.*, PRD73, 11502

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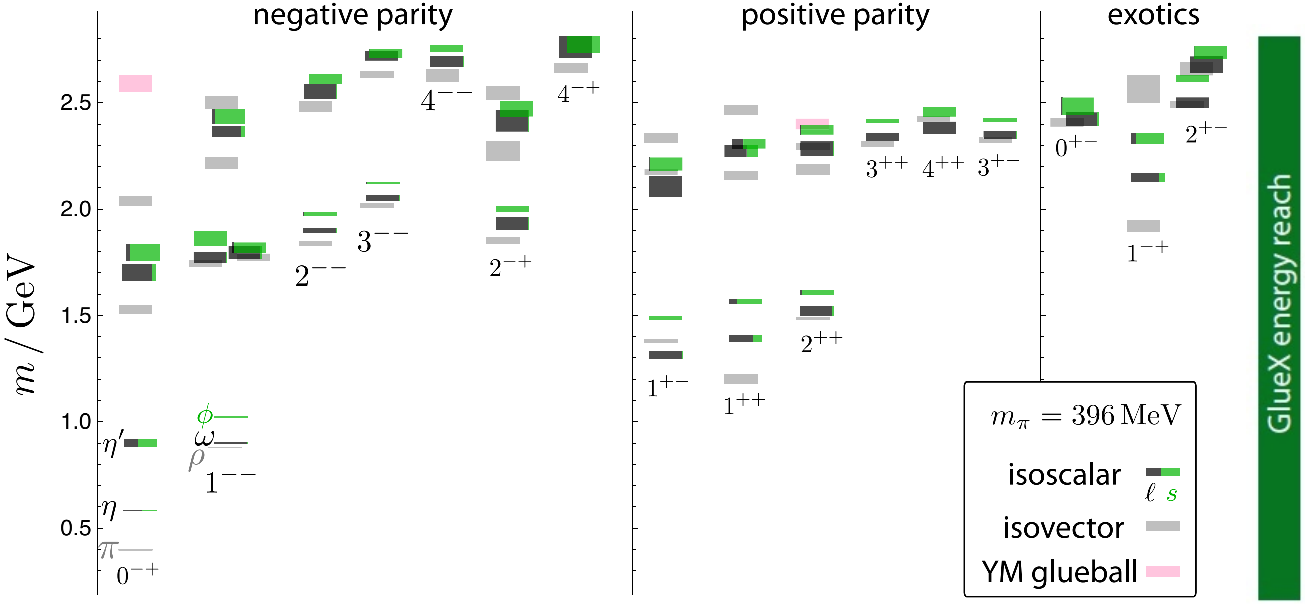

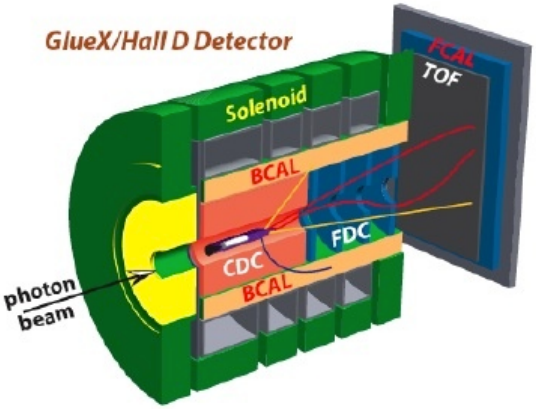

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Excited Baryon Spectrum - I

Construct basis of 3-quark interpolating operators in the continuum:

$$\left(N_{\mathsf{M}} \otimes \left(\tfrac{3}{2}^{-}\right)^{1}_{\mathsf{M}} \otimes D^{[2]}_{L=2,\mathsf{S}} \right)^{J=\frac{7}{2}}$$

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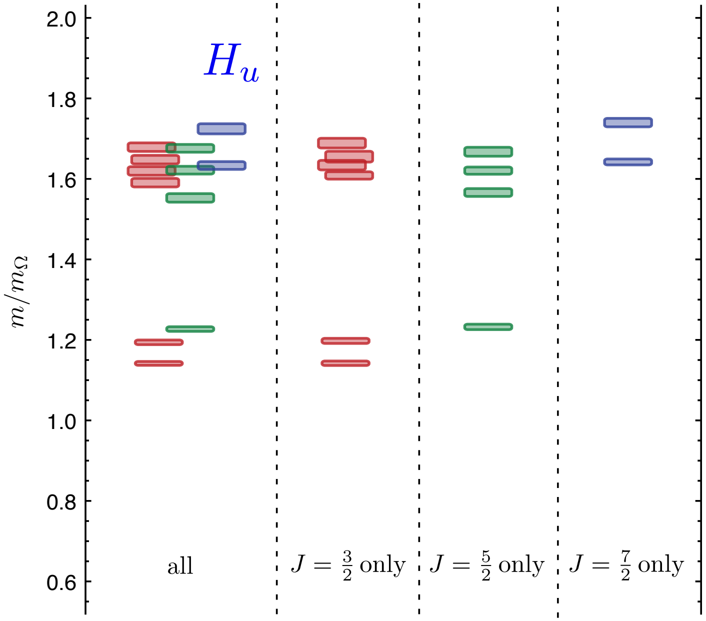

Continuum antecedents

R.G.Edwards et al., arXiv:1104.5152

$16^3 \times 128$ lattices $m_\pi = 524, 444$ and 396 MeV

Observe remarkable realization of rotational symmetry at hadronic scale: *reliably determine spins up to 7/2, for the first time in a lattice calculation*

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"Flavor" x Spin x Orbital

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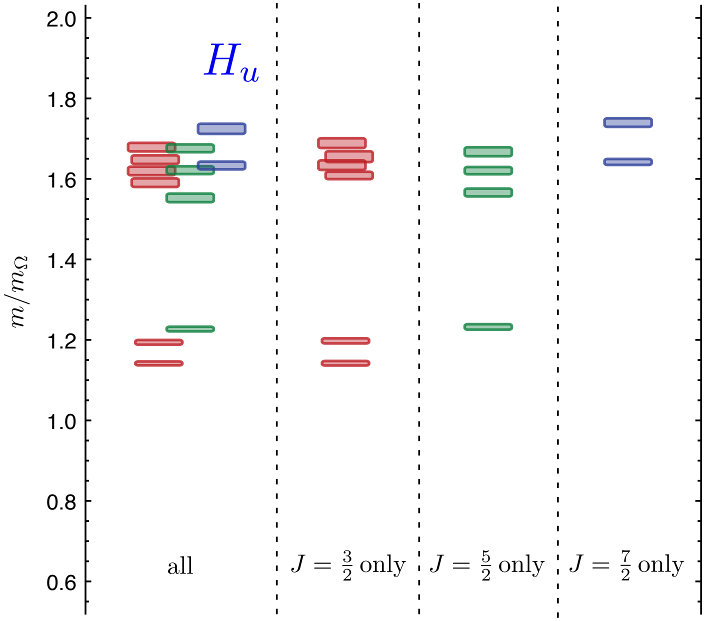

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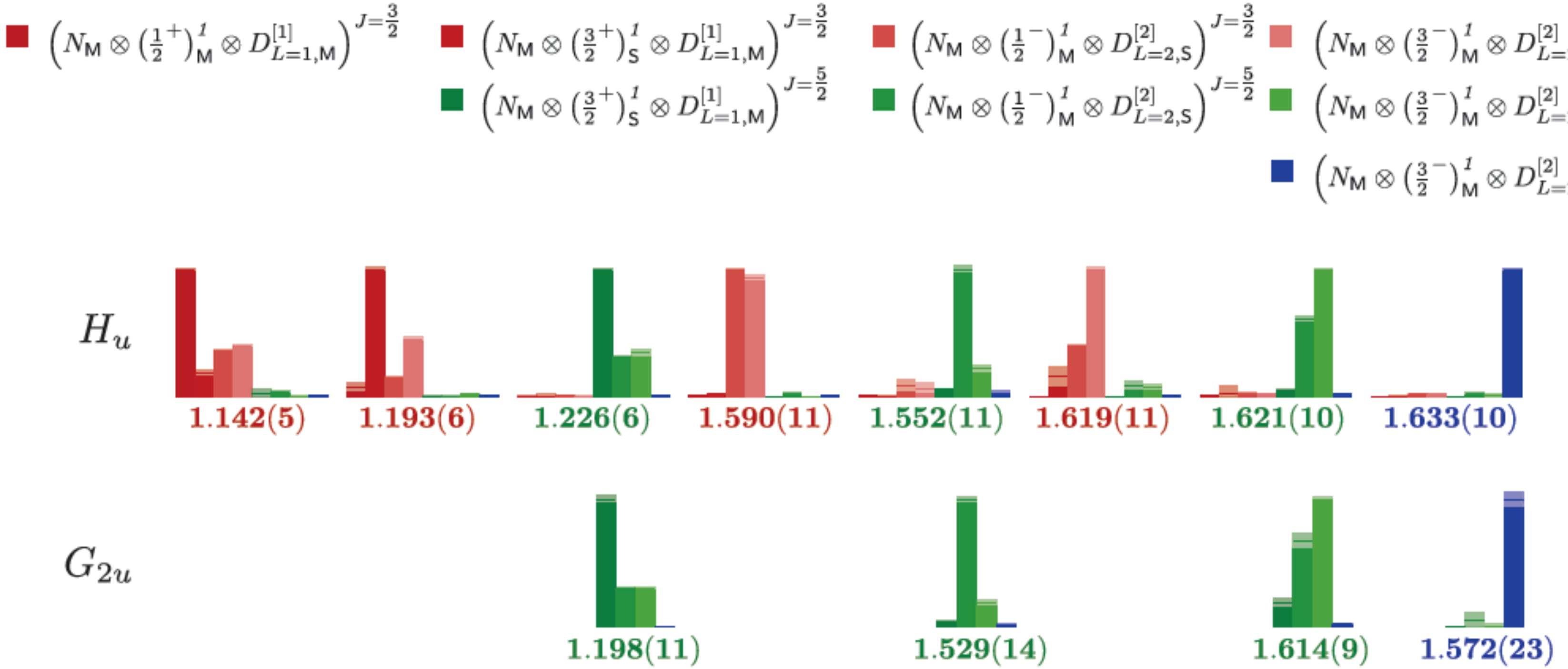
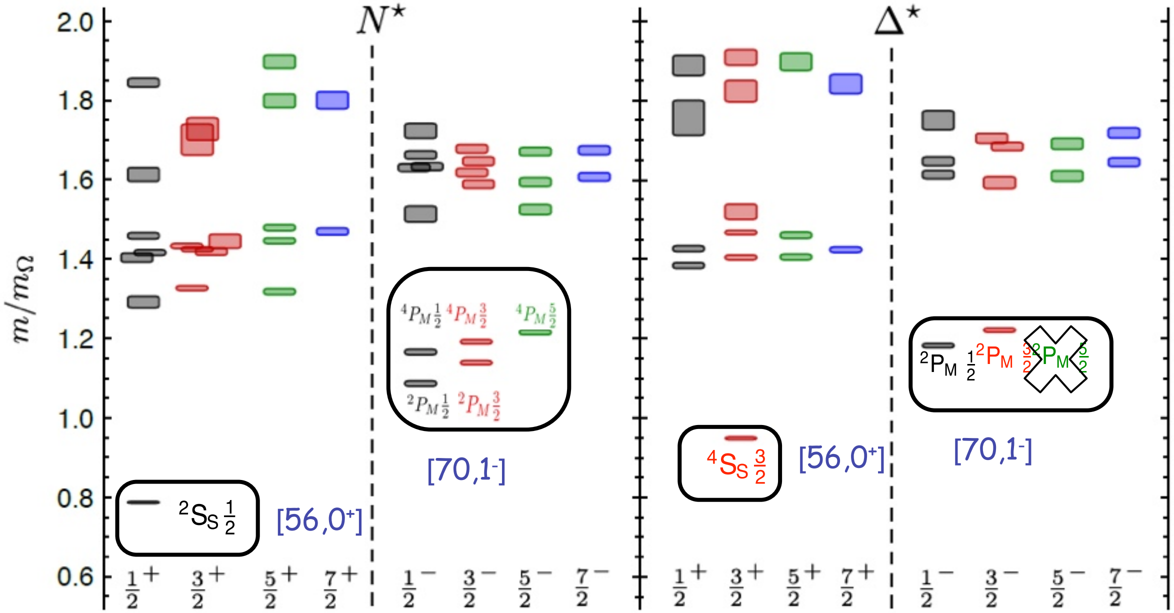
Spectral Overlaps



Excited Baryon Spectrum - II



Broad features of SU(6)xO(3) symmetry.

Counting of states consistent with NR quark model.

Inconsistent with quark-diquark picture or parity doubling.

N $^{1/2+}$ sector: need for complete basis to faithfully extract states

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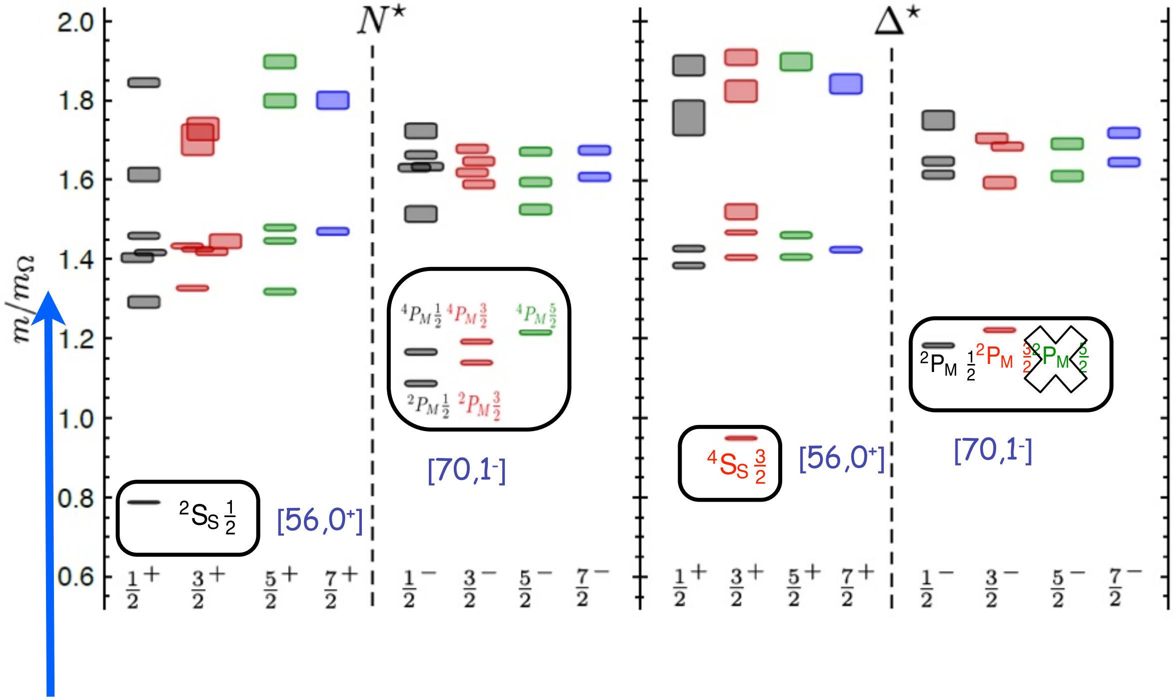

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$$[70,0^+],[56,2^+],[70,2^+],[20,1^+]$$

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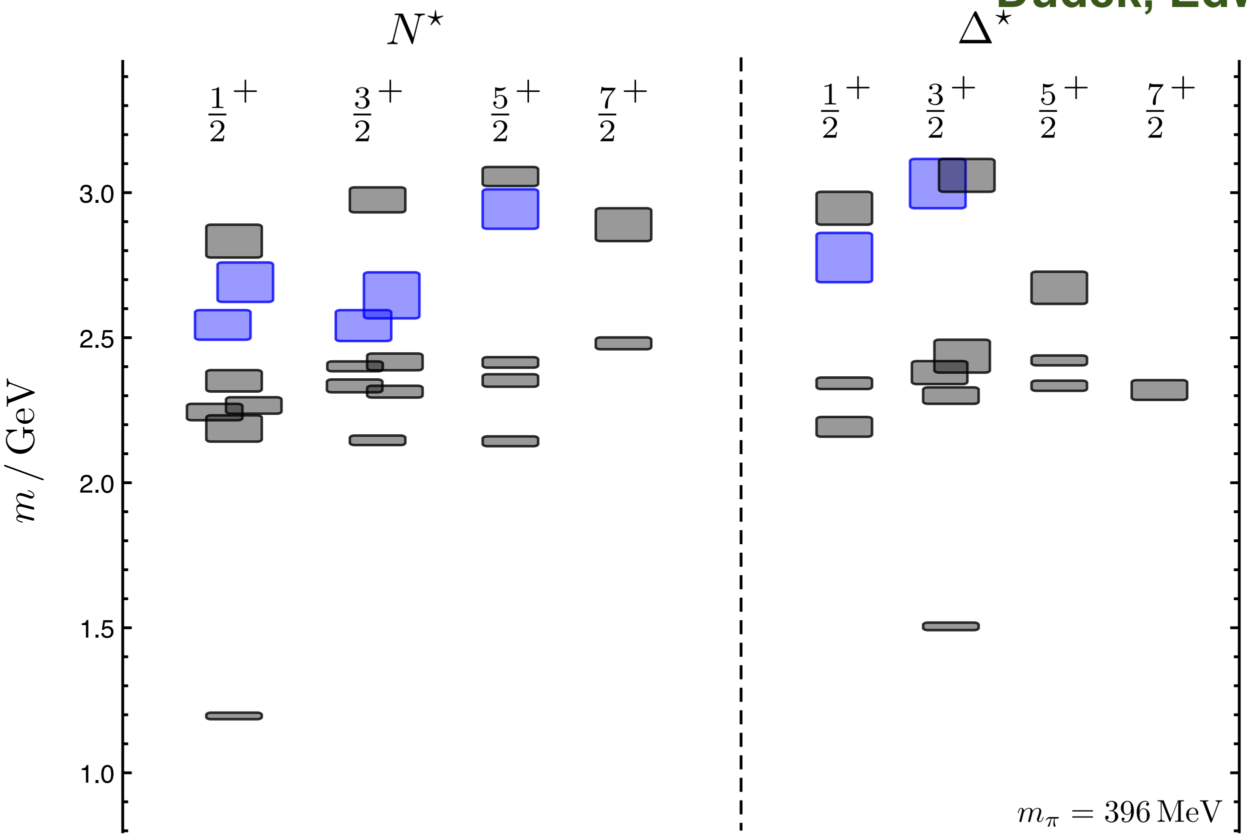
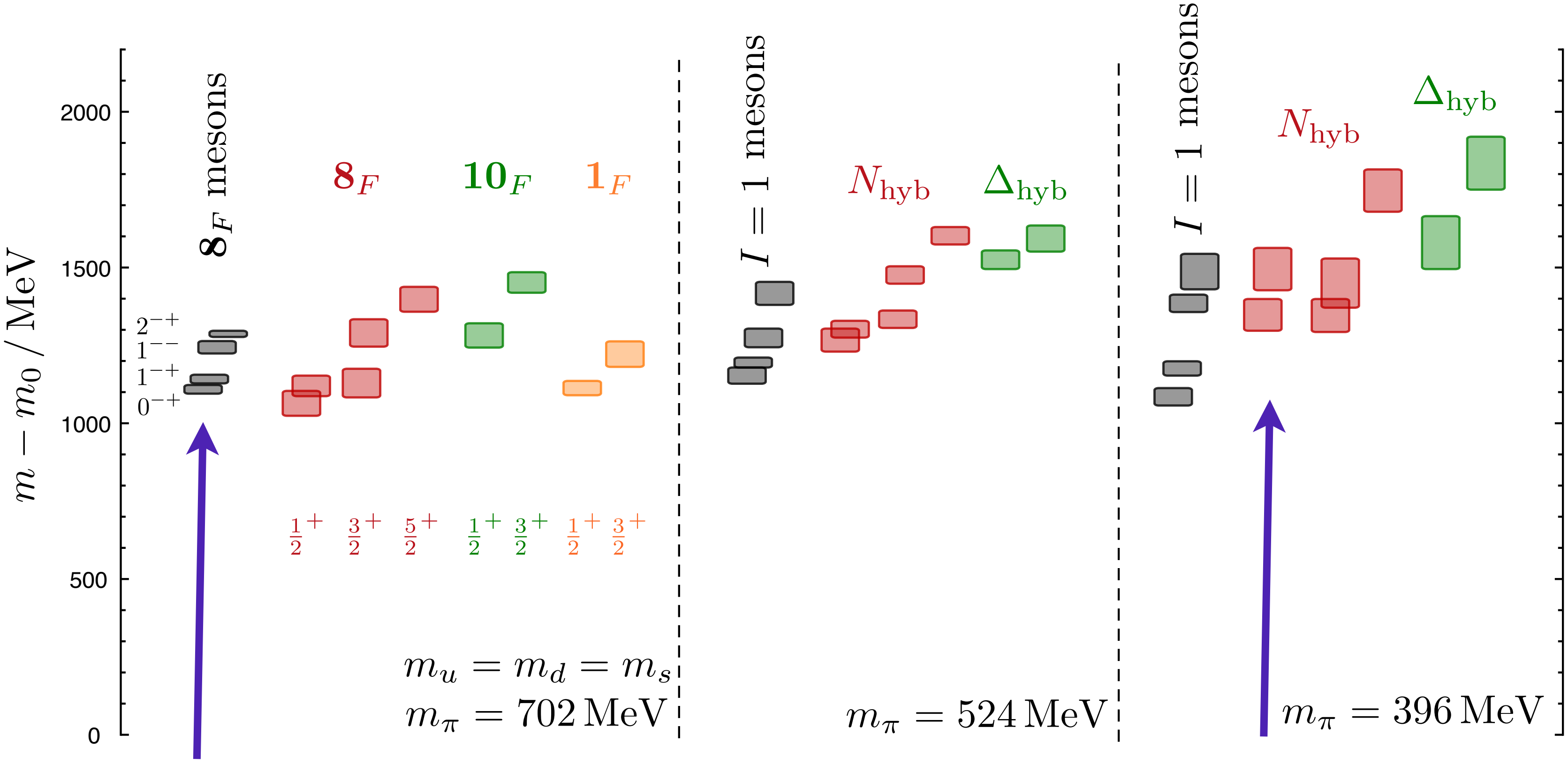
Hybrid Baryon Spectrum

Original analysis ignore hybrid operators of form $D^{[2]}_{l=1,M}$

Dudek, Edwards, arXiv:1201.2349



Putting it Together

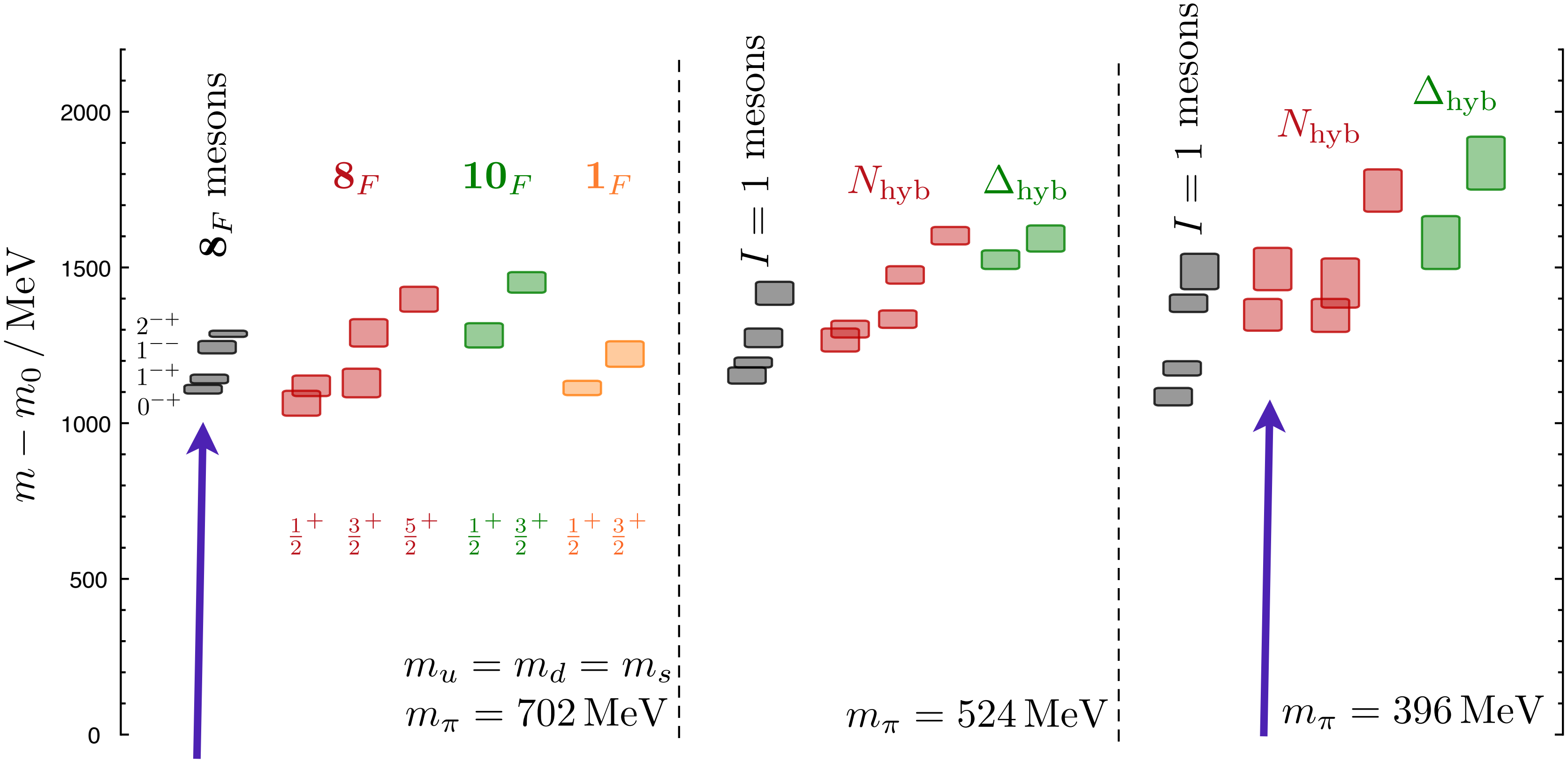


Subtract ρ

Subtract N

Putting it Together

Common mechanism in meson and baryon hybrids: chromomagnetic field with *E_g ~ 1.2 - 1.3 GeV*



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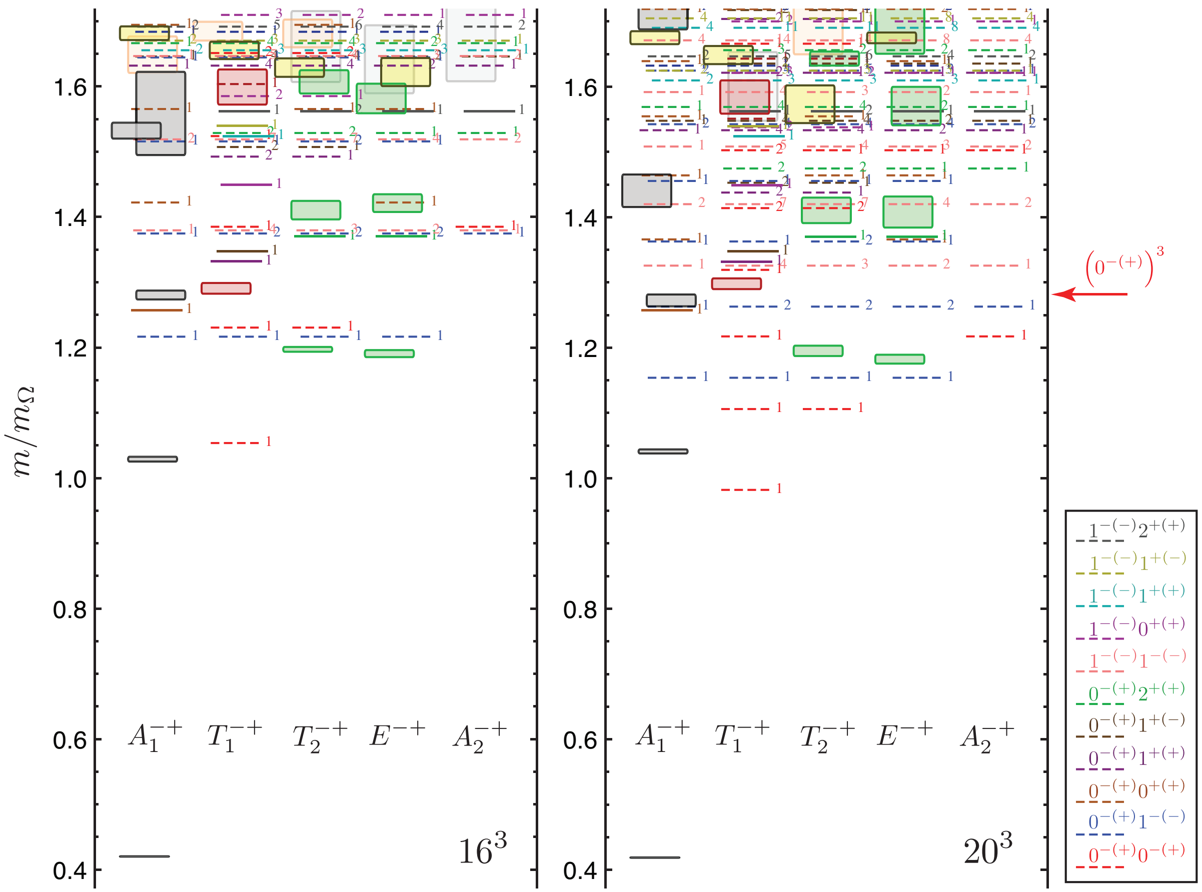
The elephant in the room...



Meson spectrum on two volumes: dashed lines denote expected (non-interacting) multi-particle energies.

Allowed two-particle contributions governed by cubic symmetry of volume

Calculation is incomplete.

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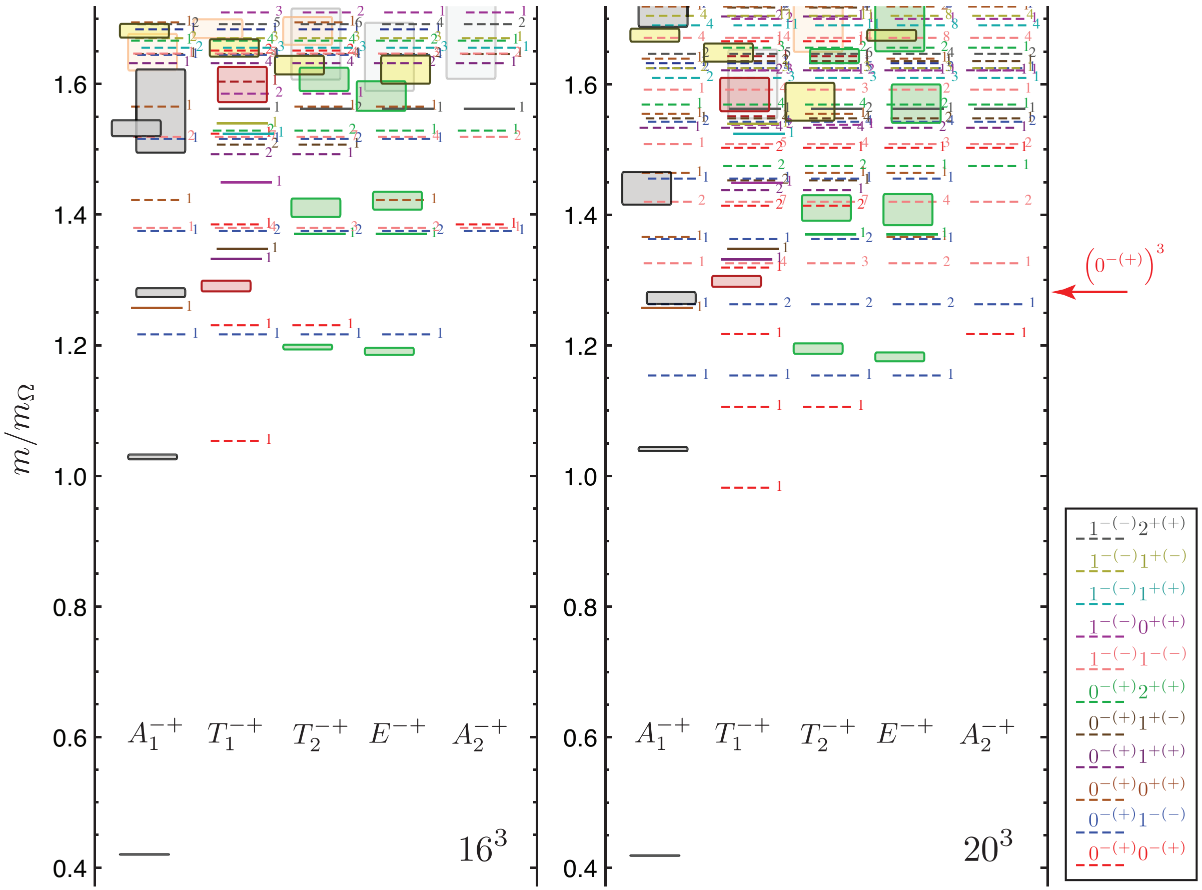

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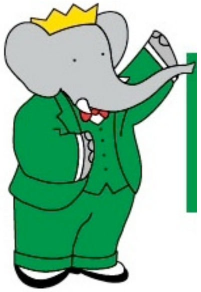
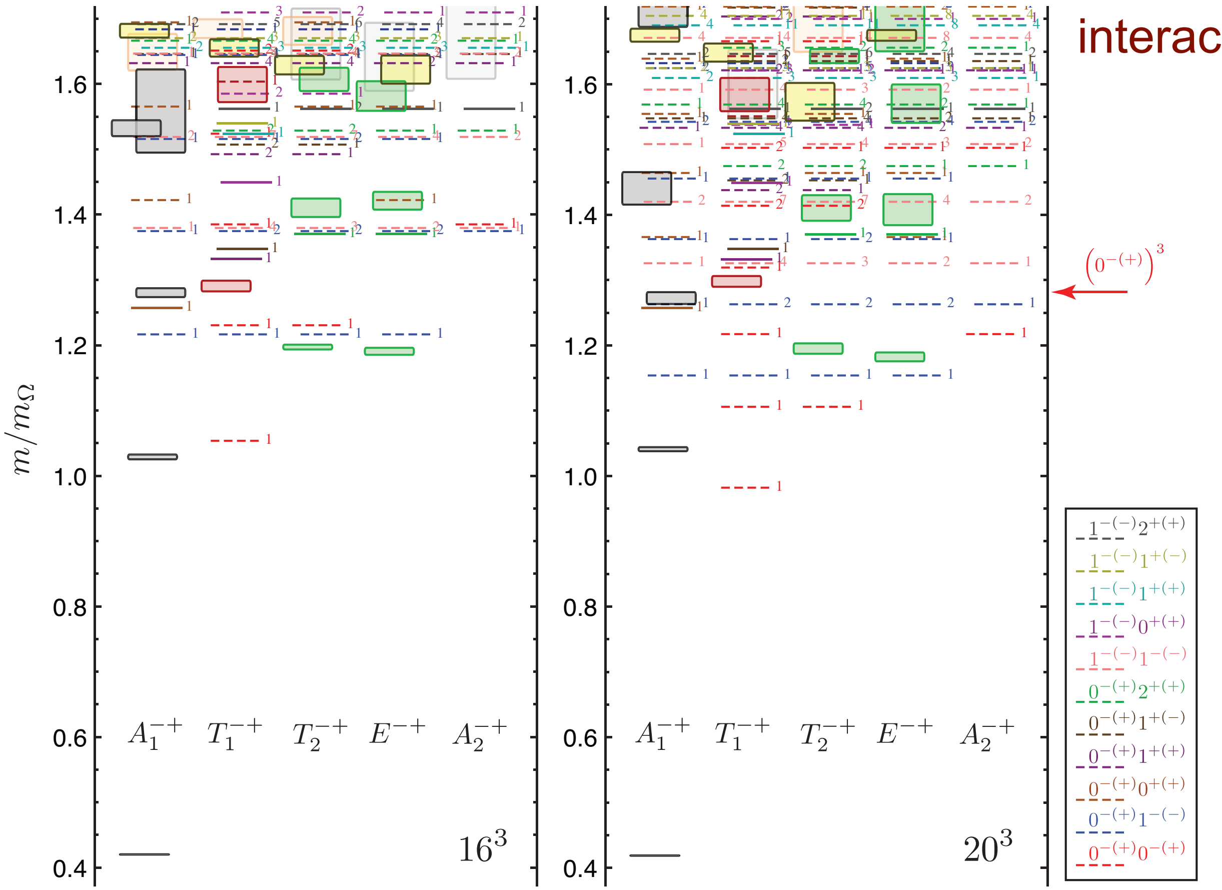


Calculation is incomplete.

The elephant in the room...

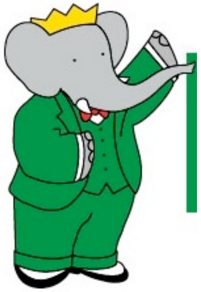


States unstable under strong interactions

Meson spectrum on two volumes: dashed lines denote expected (non-interacting) multi-particle energies.

Allowed two-particle contributions governed by cubic symmetry of volume

Calculation is incomplete.



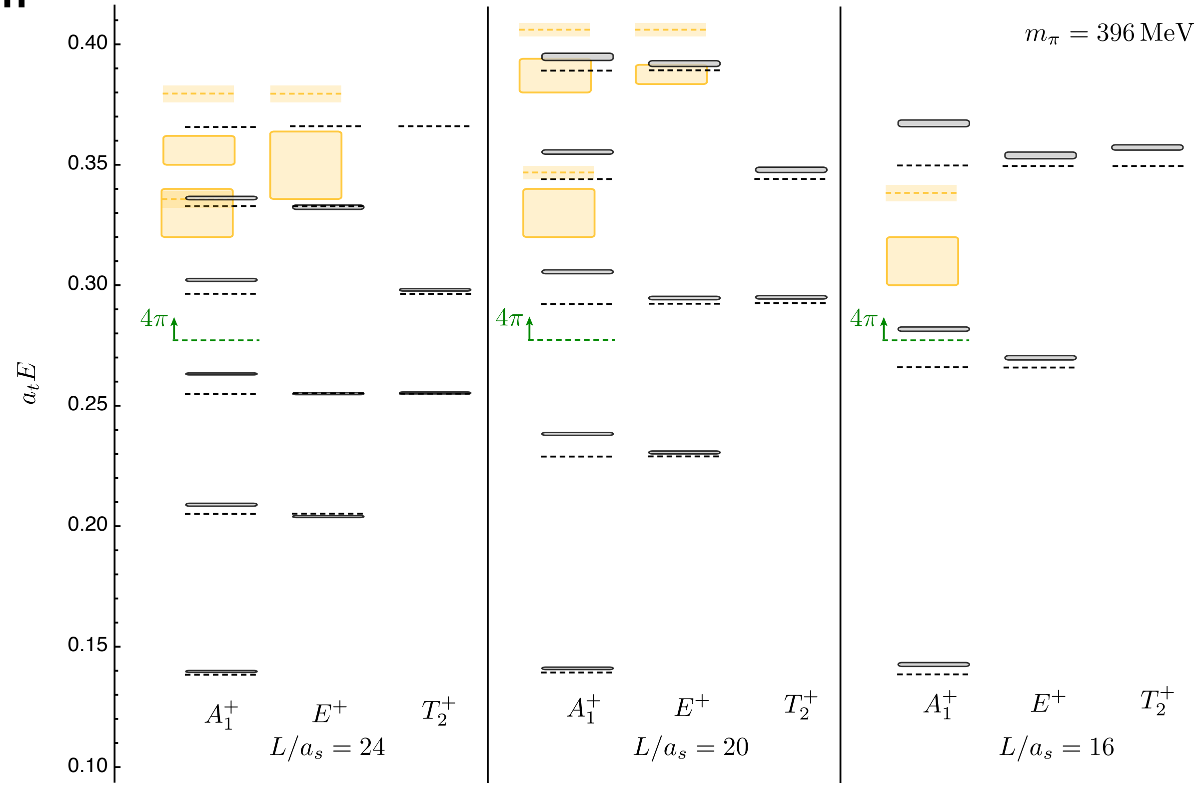
Momentum-dependent I = 2 ππ Phase Shift

Dudek *et al.*, Phys Rev D83, 071504 (2011)

Luescher: energy levels at finite volume ↔ phase shift at corresponding *k*

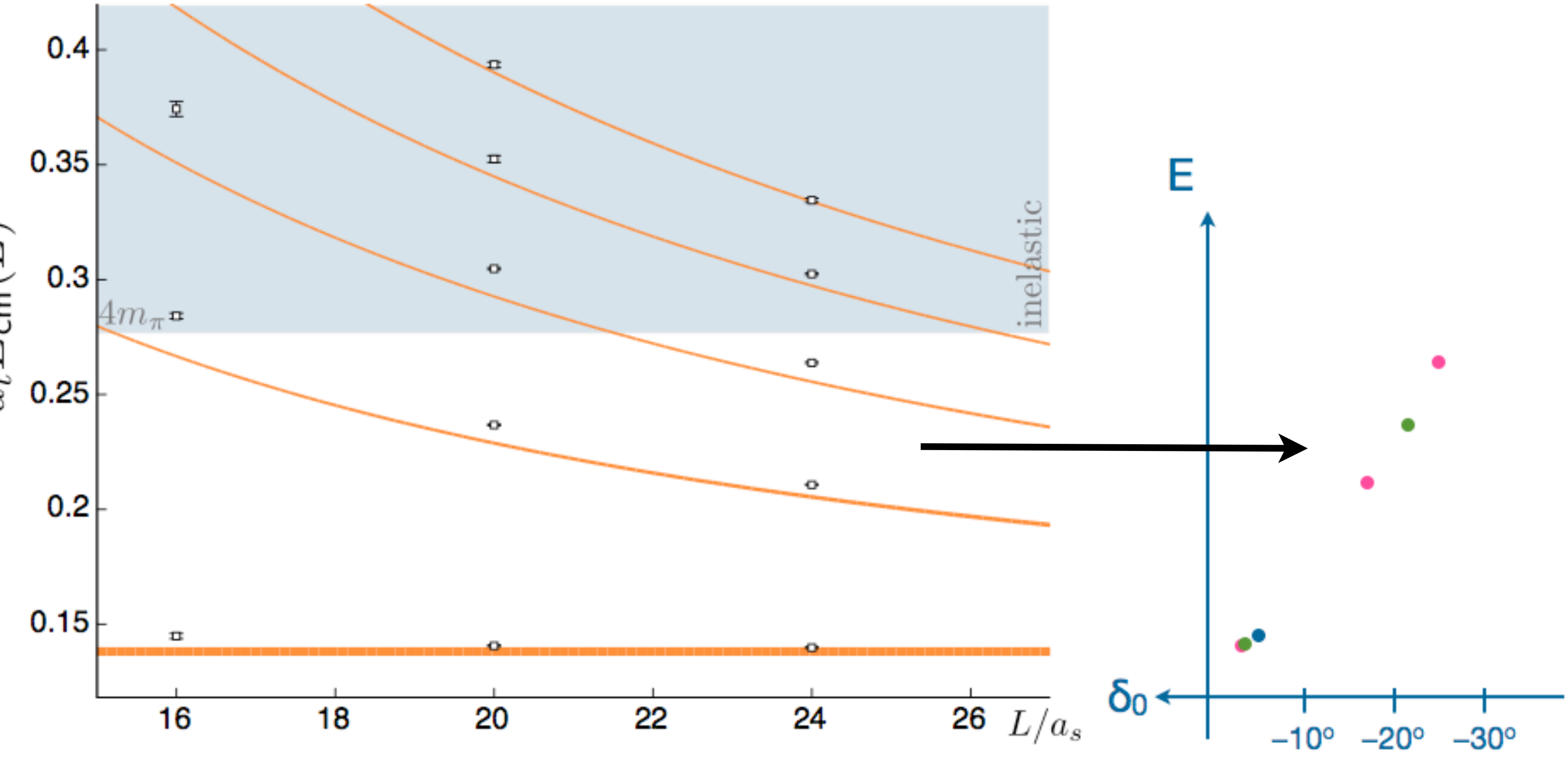
Operator basis $$\mathcal{O}^{\Gamma,\gamma}_{\pi\pi}(|\vec{p}|) = \sum_{m} \mathcal{S}^{\ell,m}_{\Gamma,\gamma} \sum_{\hat{p}} Y^{m}_{\ell}(\hat{p})\, \mathcal{O}_{\pi}(\vec{p})\mathcal{O}_{\pi}(-\vec{p})$$

Total momentum zero - pion momentum ±*p*



Energy Levels for Scattering States

Slide: J. Dudek



Momentum-dependent I = 2 ππ Phase Shift

Dudek *et al.*, Phys Rev D83, 071504 (2011)

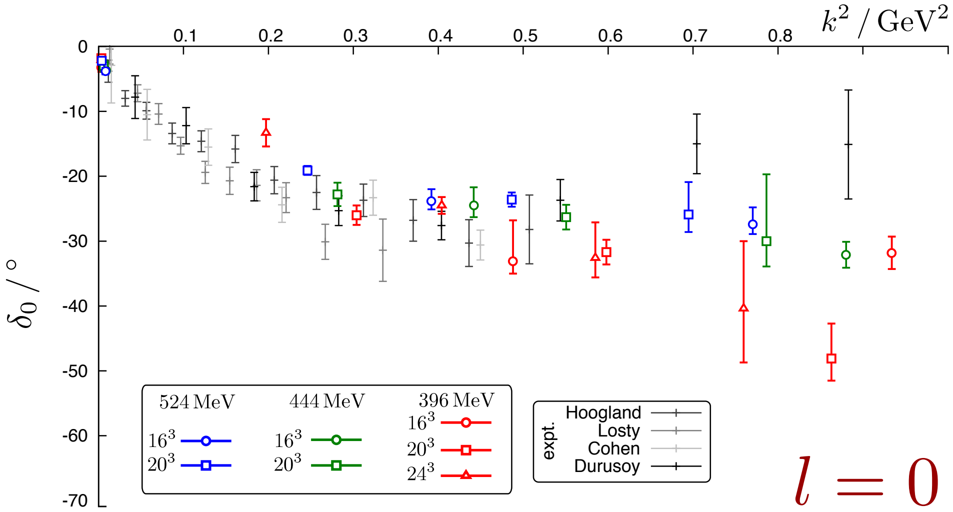
Luescher: energy levels at finite volume ↔ phase shift at corresponding *k*

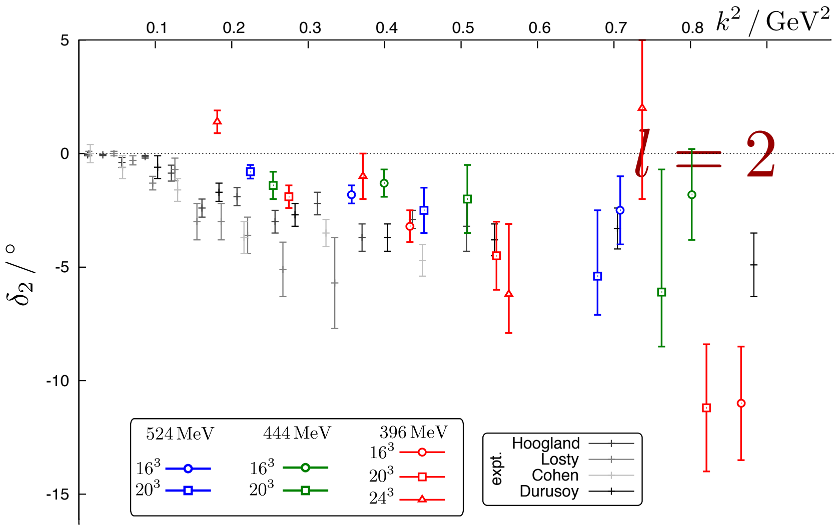
$$\det\left[e^{2i\boldsymbol{\delta}(k)} - \mathbf{U}_\Gamma\left(k\frac{L}{2\pi}\right)\right] = 0$$

Matrix in l

lattice irrep

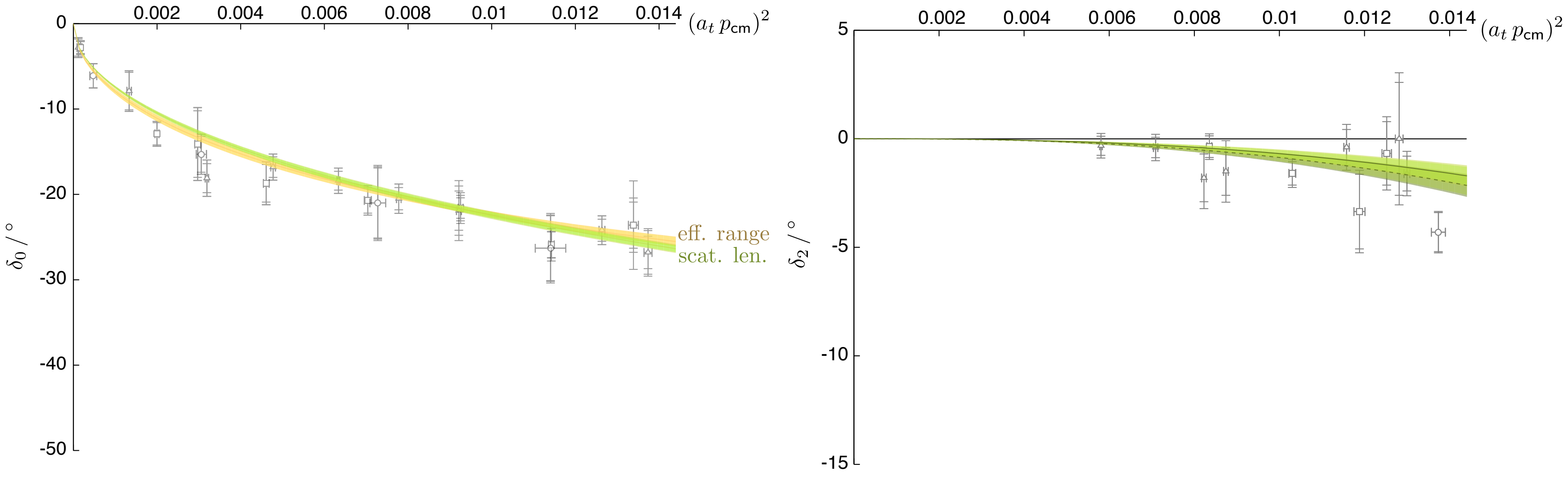
4π at $m_\pi = 396$ MeV





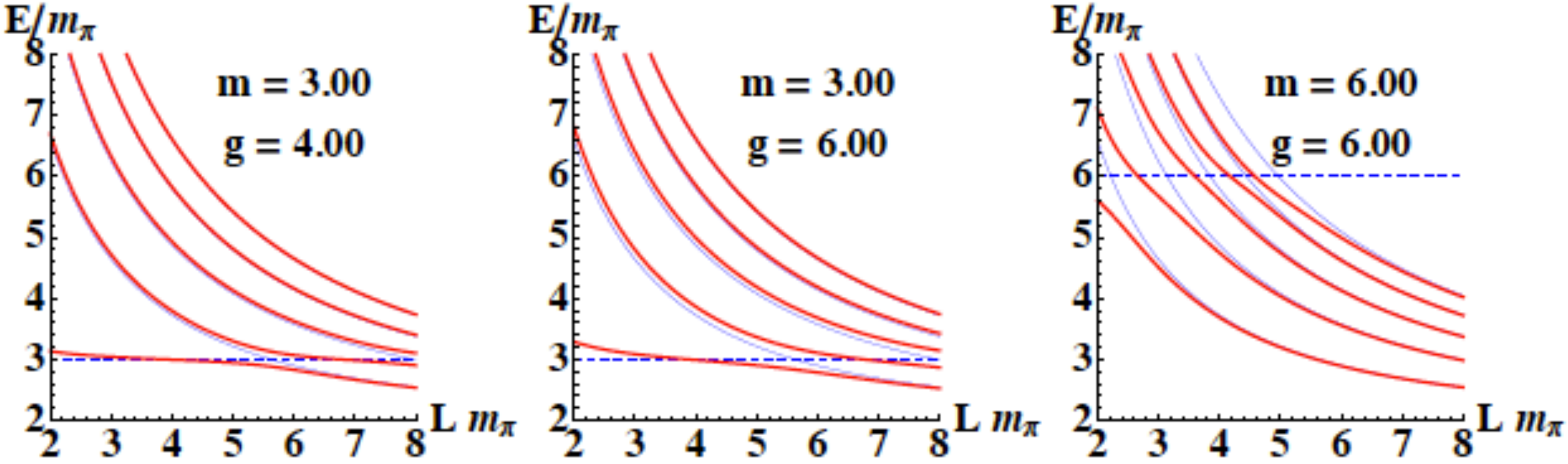
Momentum-dependent I = 2 ππ Phase Shift

- More sophisticated analysis Dudek, Edwards, Thomas, arXiv:1203.6041
  - *Moving ππ system ➙ far more momenta below inelastic threshold*
  - *Optimized single-pion interpolating operators ➙ more precise determination of energies*
  - *Investigation of thermal effects*



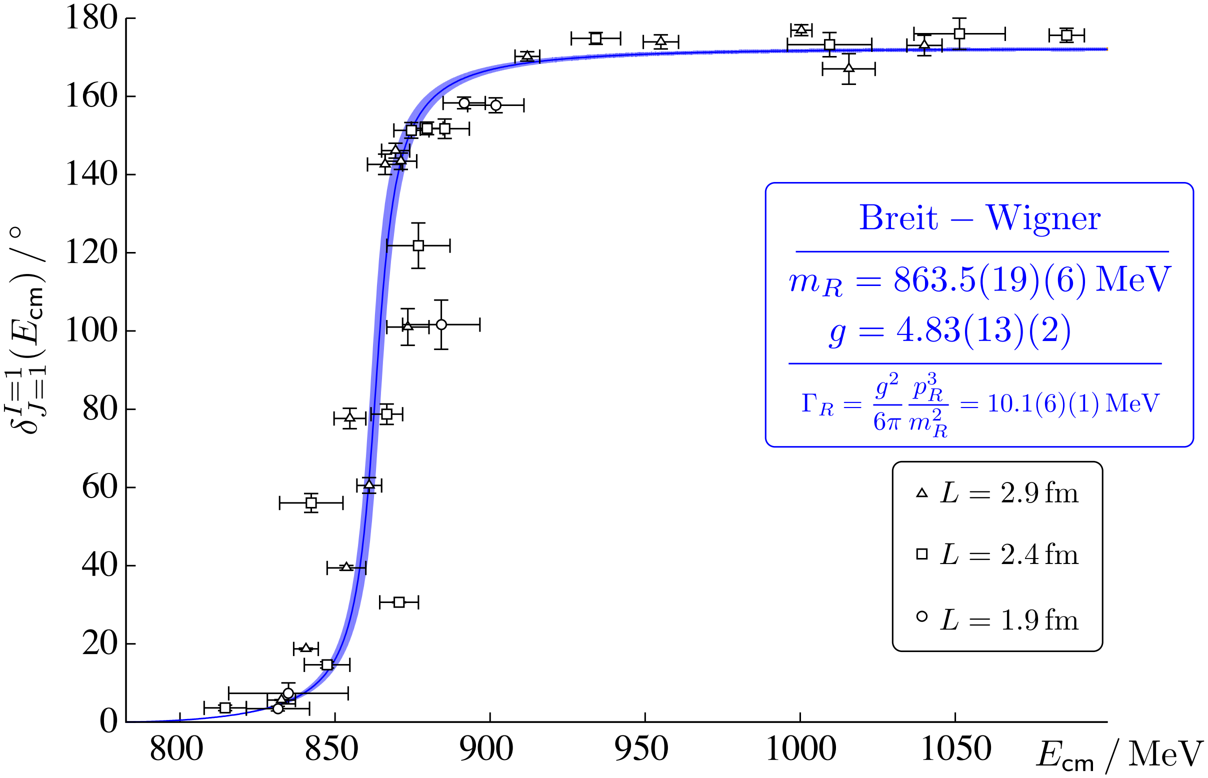
Resonant I = 1 ππ Phase Shift

Avoided level crossings...



Mohler, Lattice 2012

Resonant I = 1 ππ Phase Shift



Feng, Renner, Jansen, PRD83, 094505

PACS-CS, PRD84, 094505

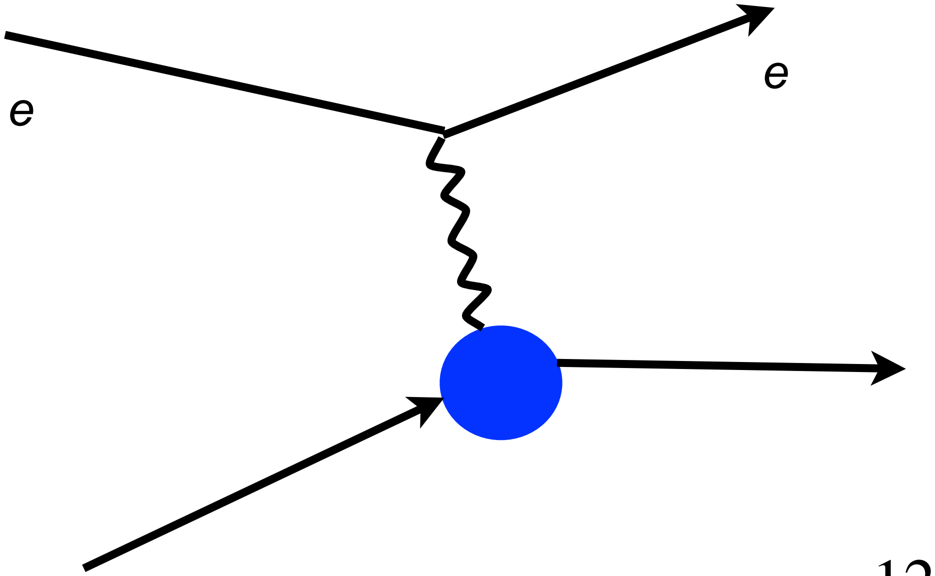
Alexandru et al

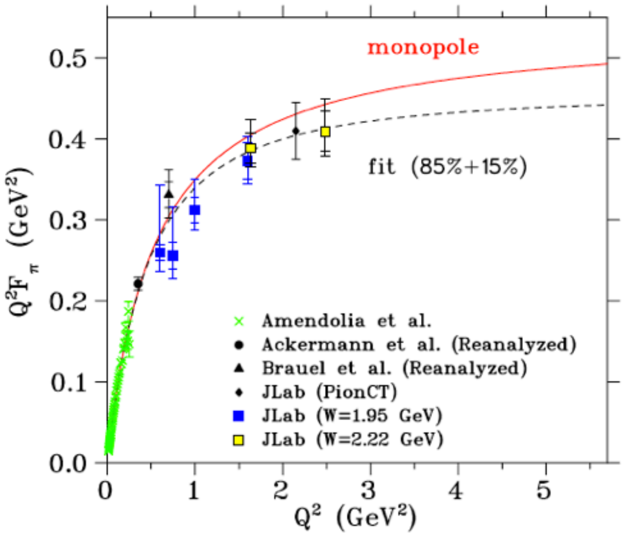
Lang et al., PRD84, 054503

Dudek, Edwards, Thomas, Phys. Rev. D 87, 034505 (2013)

Extend to inelastic channels: Guo et al, Briceno et al.,

Paradigm: Pion EM form factor





12 GeV →

$$\langle \pi(\vec{p}_f) \mid V_\mu(0) \mid \pi(\vec{p}_i) \rangle = (p_i + p_f)_\mu F(Q^2)$$

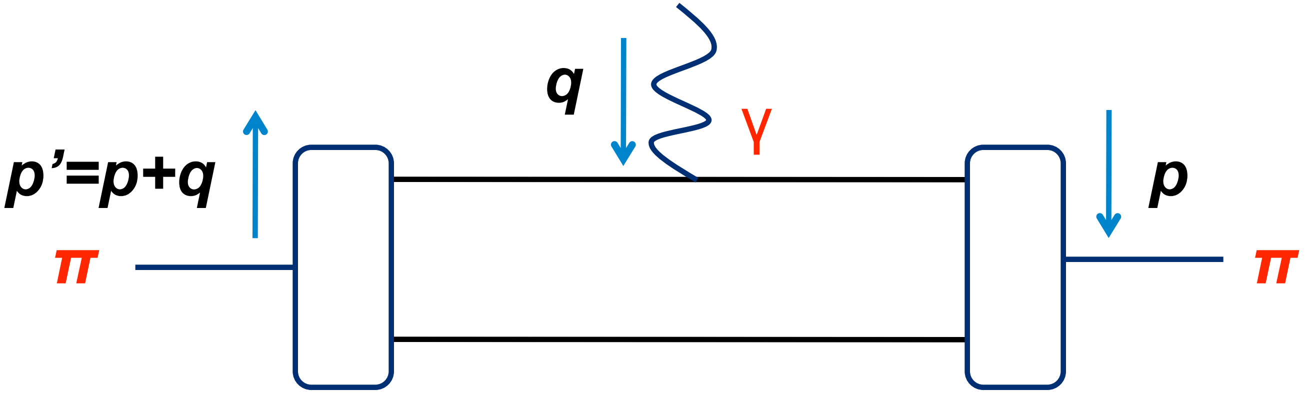
where

$$\begin{aligned} V_\mu &= \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d \\ -Q^2 &= [E_\pi(\vec{p}_f) - E_\pi(\vec{p}_i)]^2 - (\vec{p}_f - \vec{p}_i)^2 \end{aligned}$$

Anatomy of a Matrix Element Calculation - I

Pion Interpolating Operator

$$\left\{\begin{array}{rcl} \phi(x) & = & \bar{d}(x)\gamma_5 u(x) \\ \phi^\dagger(x) & = & -\bar{u}(x)\gamma_5 d(x) \\ V_\mu(x) & = & e_u\bar{u}(x)\gamma_\mu u(x) + e_d\bar{d}(x)\gamma_\mu d(x). \end{array}\right.$$



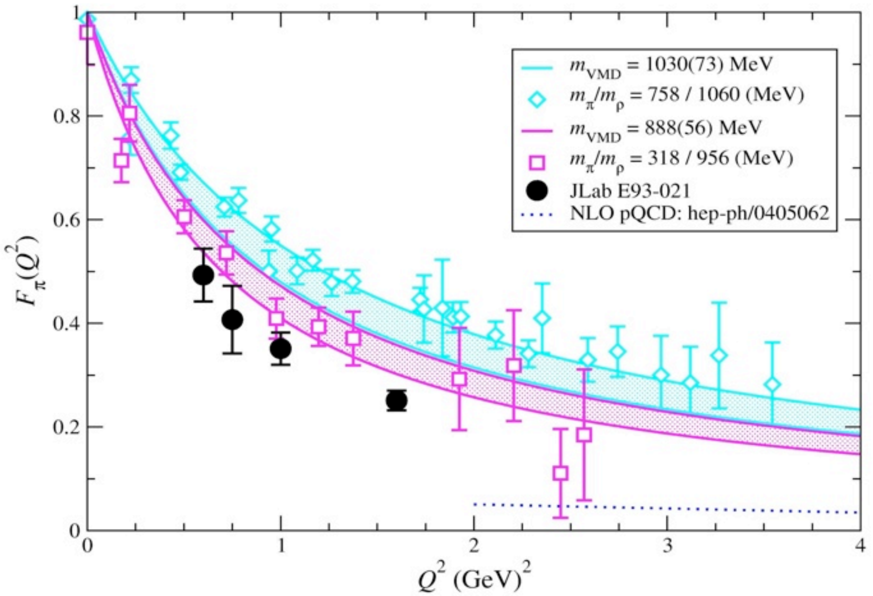
$$\Gamma_{\pi^+\mu\pi^+}(t_f, t; \vec{p}, \vec{q}) = \sum_{\vec{x},\vec{y}} \langle 0|\phi(\vec{x}, t_f)V_\mu(\vec{y}, t)\phi^\dagger(\vec{0}, 0)|0\rangle e^{-i\vec{p}\cdot\vec{x}} e^{-i\vec{q}\cdot\vec{y}},$$

Spacelike form factor

Resolution of unity – insert states

$$\langle 0 \mid \phi(0) \mid \pi, \vec{p}+\vec{q}\rangle\langle \pi, \vec{p}+\vec{q} \mid V_\mu(0) \mid \pi, \vec{p}\rangle\langle \pi, \vec{p} \mid \phi^\dagger \mid 0\rangle e^{-E(\vec{p}(t-t_i)} e^{-E(\vec{p}+\vec{q})(t_f-t)}$$

Pion Form Factor - I



LHPC, Bonnet et al,
Phys.Rev. D72 (2005) 054506

$$F(Q^2) = \frac{1}{1 + Q^2/{M_{\mathrm{VMD}}}^2}$$

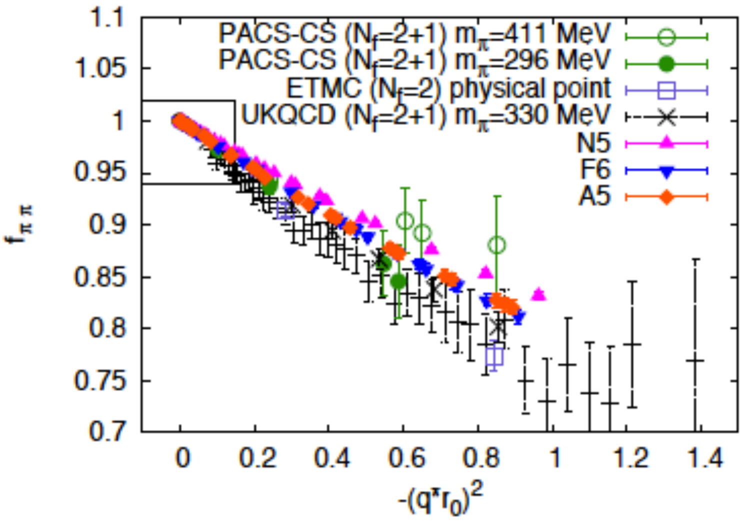
$$Q_{\max} \,|\simeq \frac{1}{a}$$

Quark distribution amplitudes

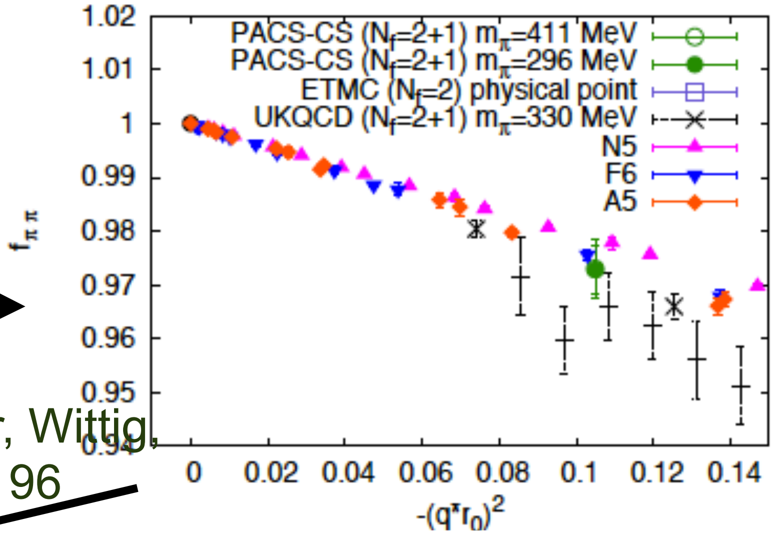
Charge radius

$$\langle r^2 \rangle = 6\,\frac{dF(q^2)}{dq^2}\bigg|_{q^2=0}$$

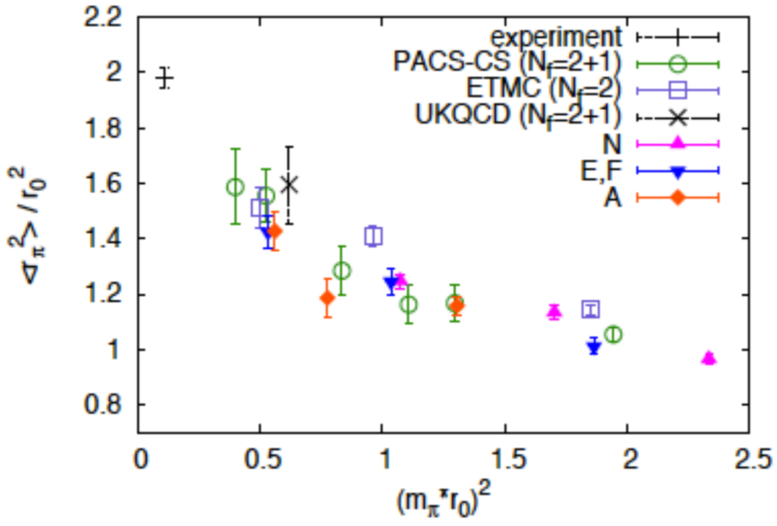
Pion Form Factor - II

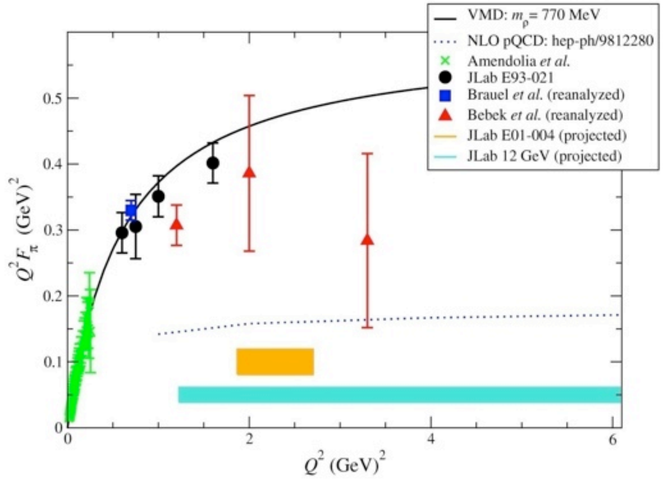


Twisted boundary conditions

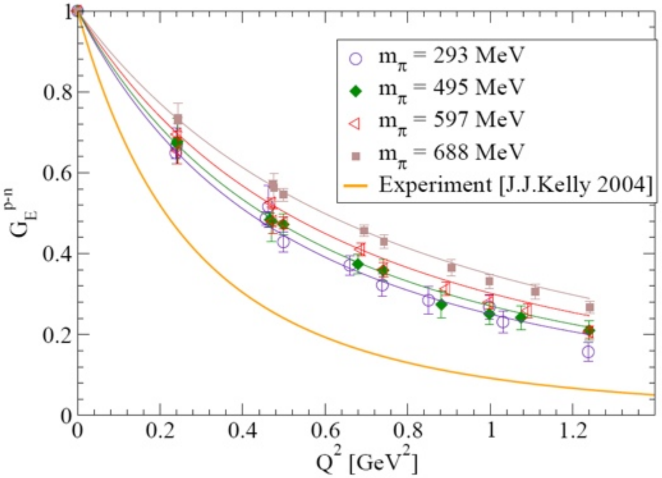


Brandt, Jutter, Wittig, arXiv:1109.0196





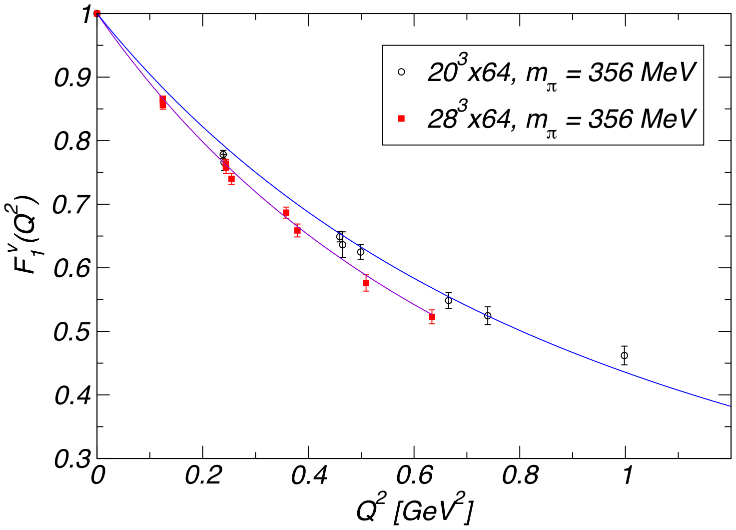
Isovector Form Factor

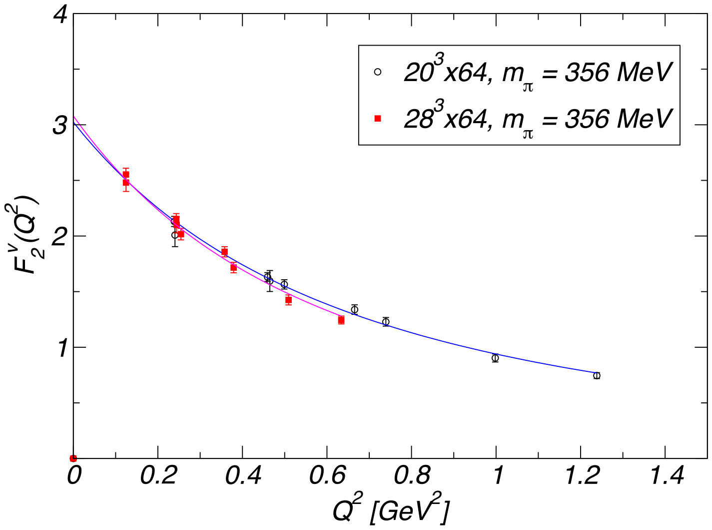


DWF valence/Asqtad sea

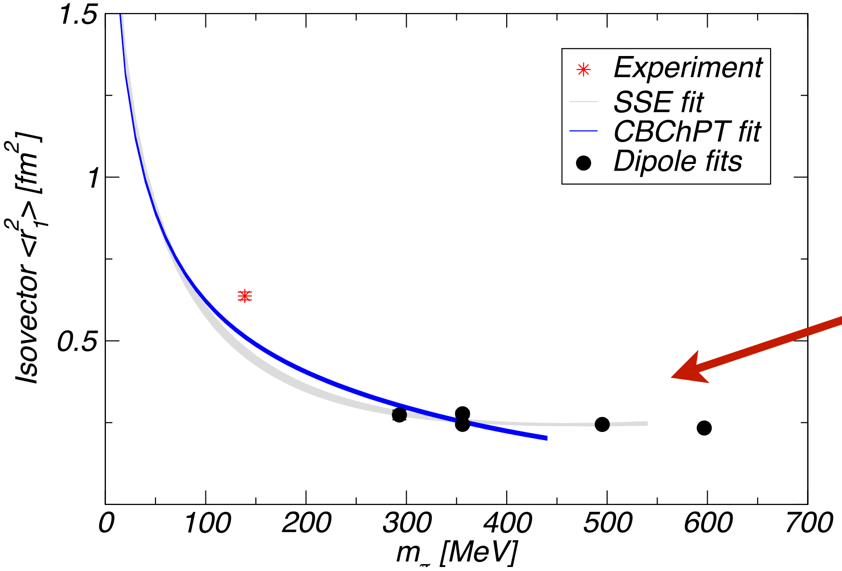
J.D.Bratt et al (LHPC), arXiv:0810.1933

Data well described by dipole form - but example of notable finite-volume effect:





Isovector Charge Radius

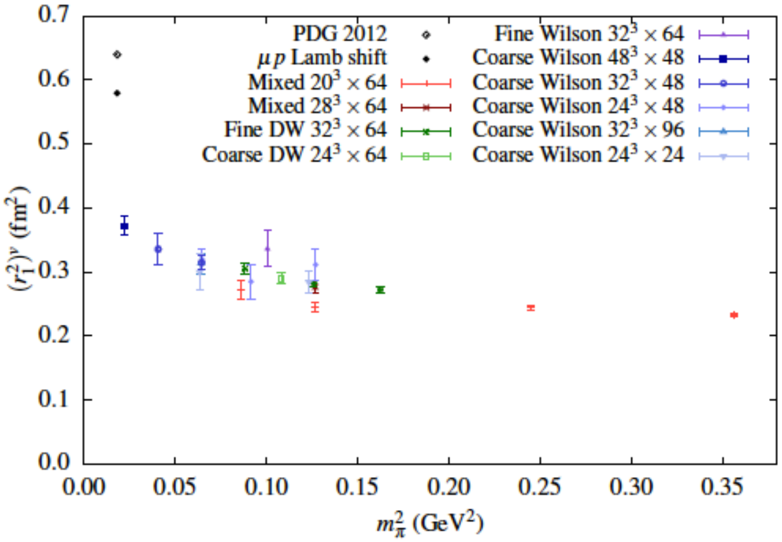


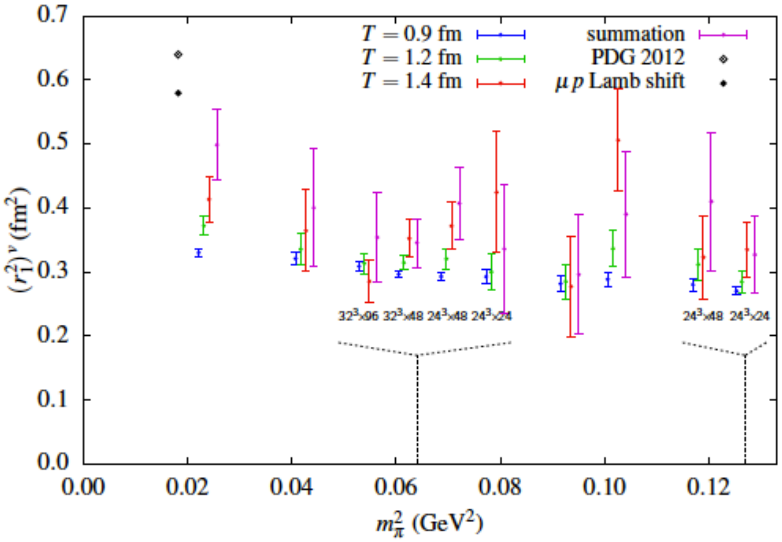
LHPC, arXiv:1001.3620

Dipole fits at each pion mass

Towards physical quark masses

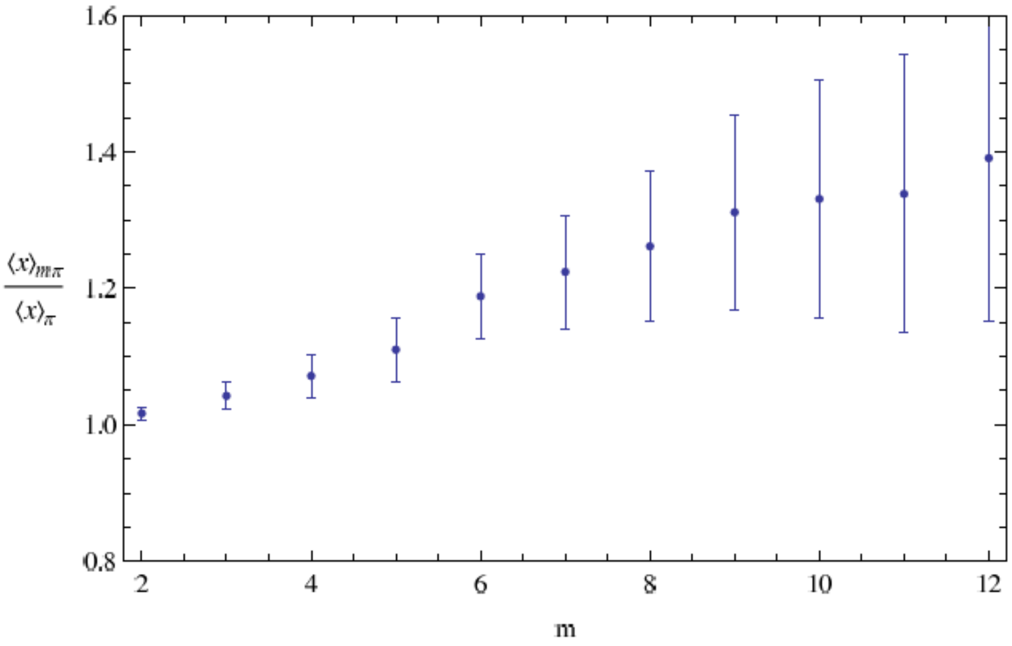
Green et al, arXiv:1211.0253





Medium modification of structure

- How is the structure of a hadron modified "in medium" - EMC effect?
- First attempt - momentum fraction carried by quarks in Bose-condensed pion gas.



W Detmold, H-W Lin, arXiv:1112.5682

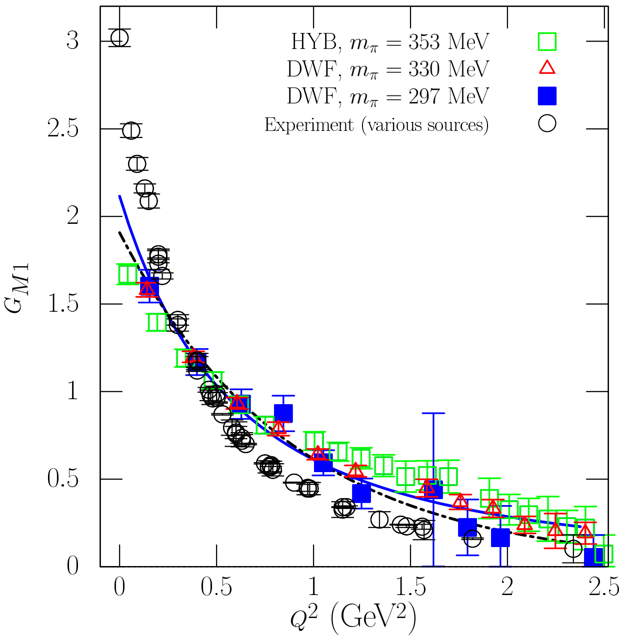
Proof of concept

Transition Form Factors

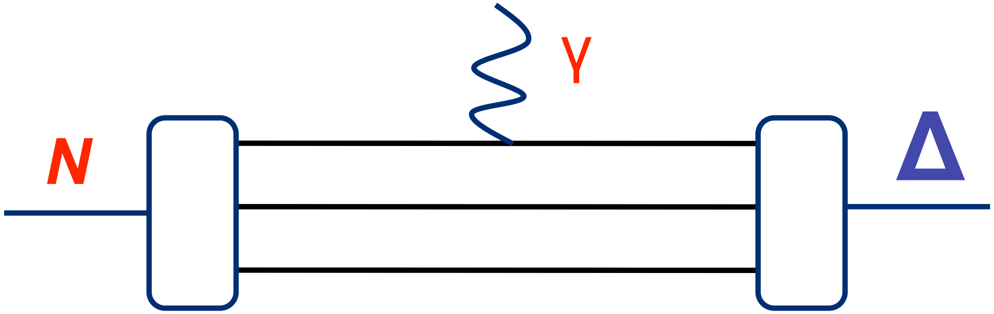
Form factors of excited states, and transition form factors to excited states, provide additional insight into nature of QCD. Precise electro-production data

Program of computations looking at Δ form factor, and Nγ → Δ transition form factors
N.B. Δ → Nπ is p-wave decay, suppressed at zero momentum.

Admits *three* multipoles: magnetic dipole, electric quadrupole and Coulomb quadrupole: G_{M1}, G_{E2}, G_{C2}



Alexandrou et al, DWF + DWF valence/Asqtad sea



Free of disconnected contributions

Transition Form Factors

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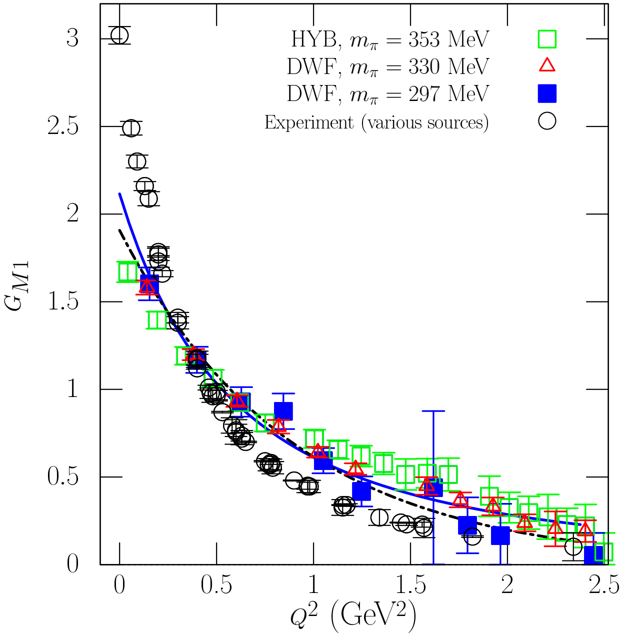

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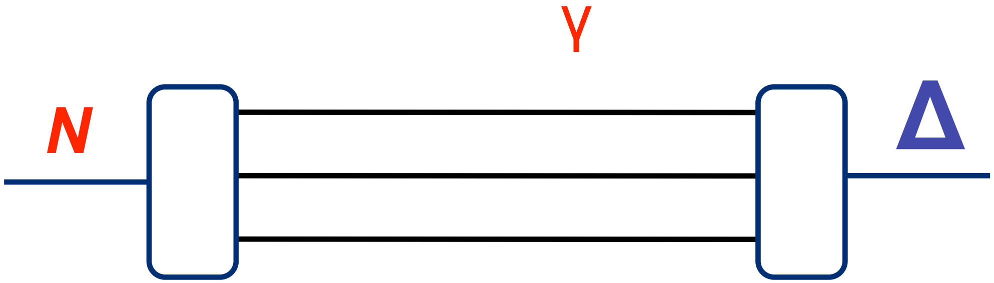
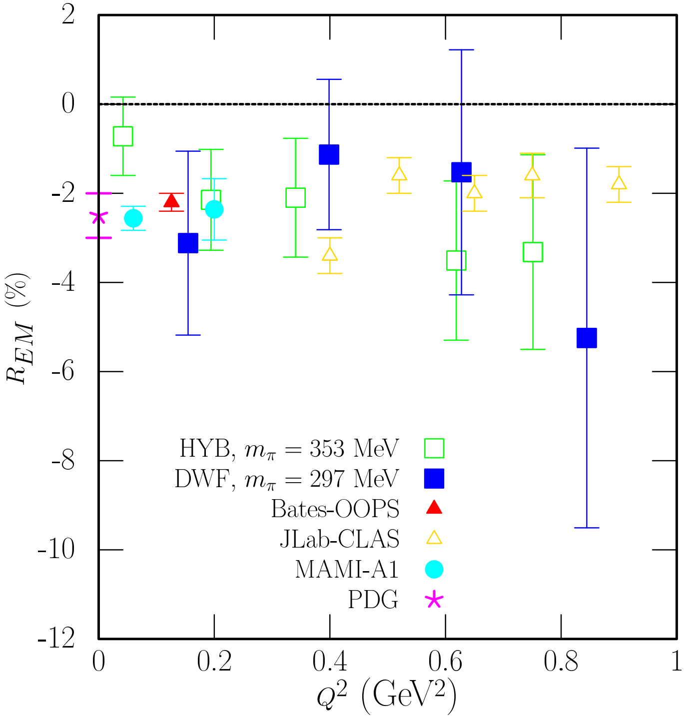


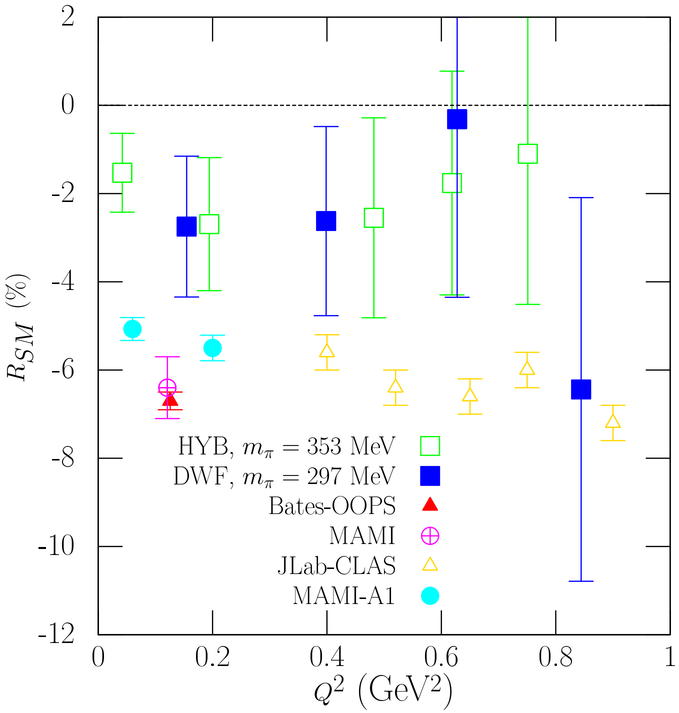
Free of disconnected contributions

N-Δ Transition Form Factor

$$R_{EM} = -\frac{G_{E2}(Q^2)}{G_{M1}(Q^2)}$$

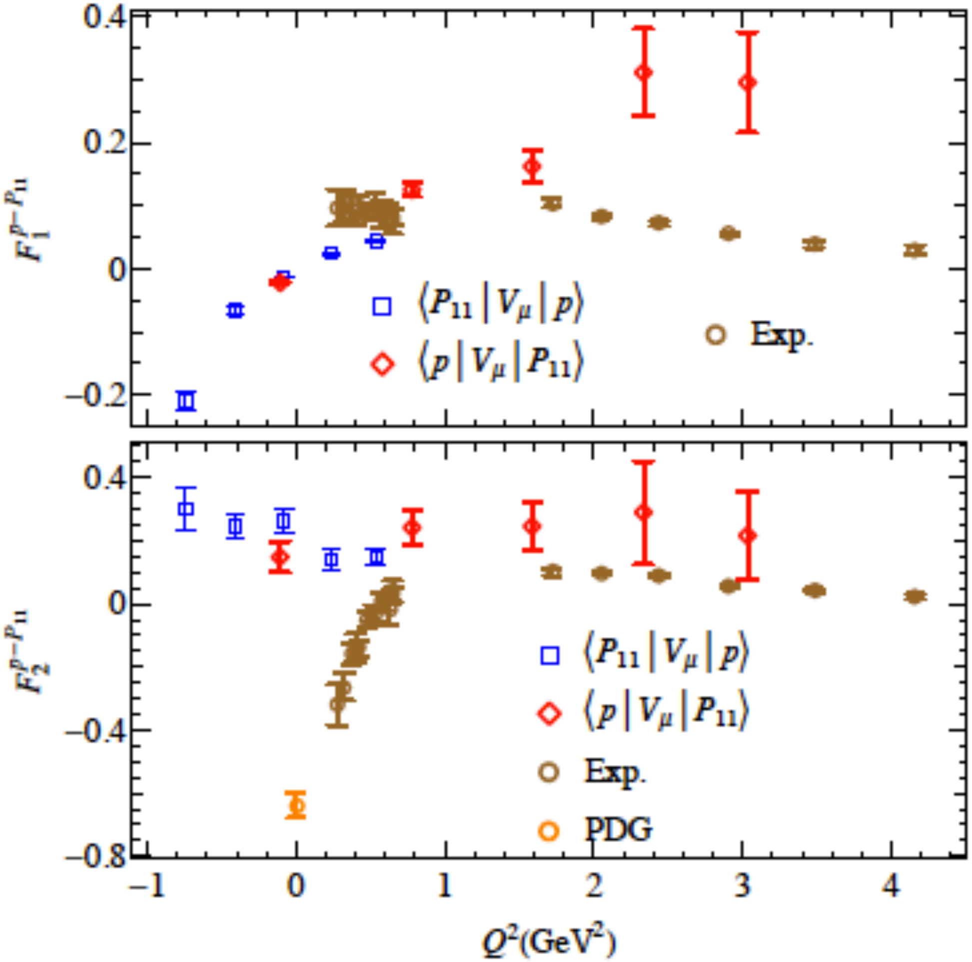
$$R_{SM} = -\frac{|\vec{q}\,|}{2m_\Delta}\frac{G_{C2}(Q^2)}{G_{M1}(Q^2)}$$





Non-zero values: sphericity in either N or Δ - zero quadrupole moment for spin-1/2 system

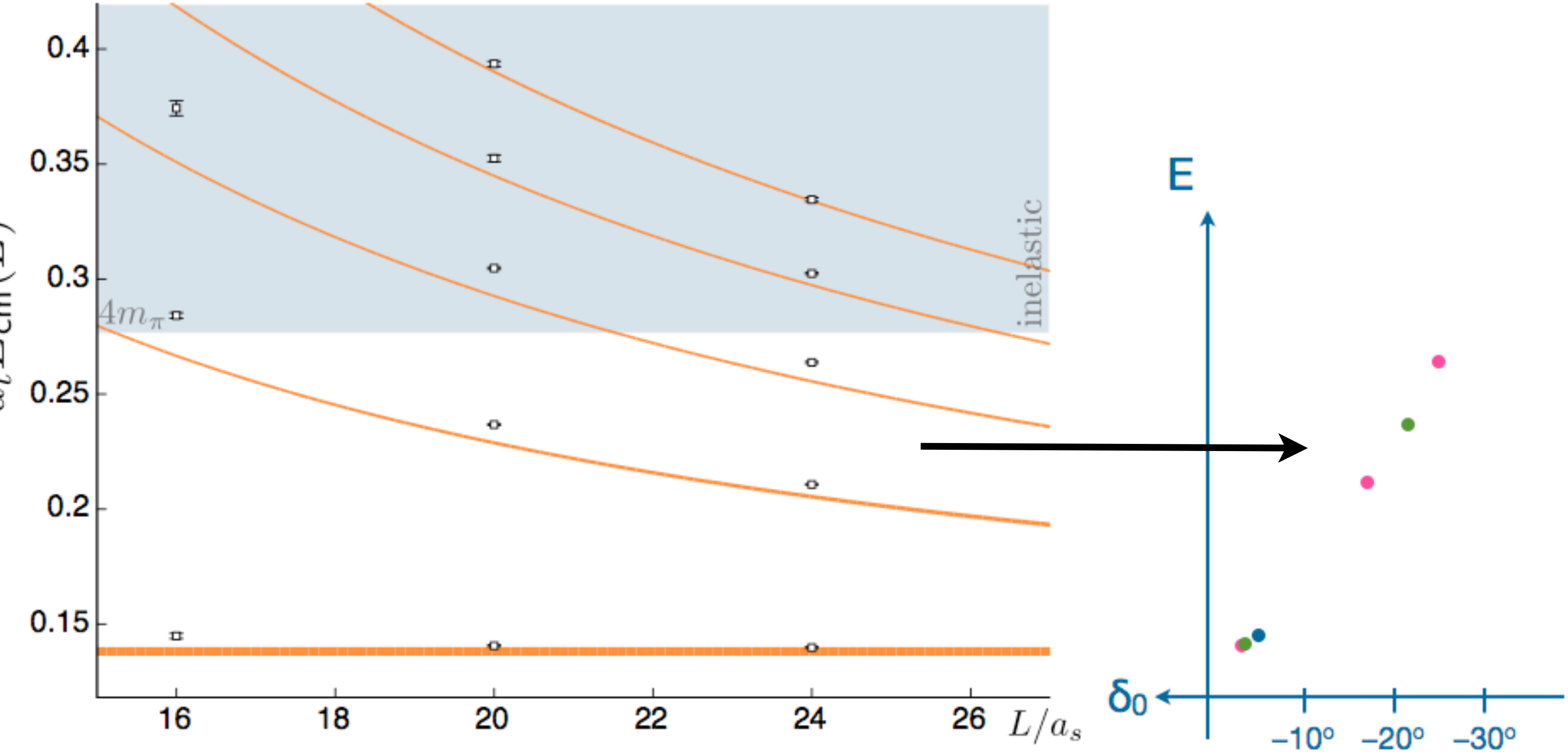
P11-N Form Factors



Lin, Cohen et al., PRD78, 114508 (2008)

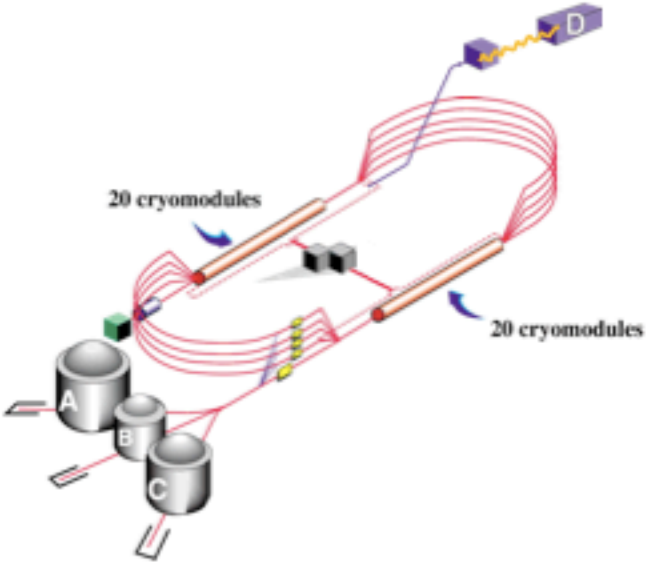
Matrix Elements for Scattering States

Slide: J. Dudek

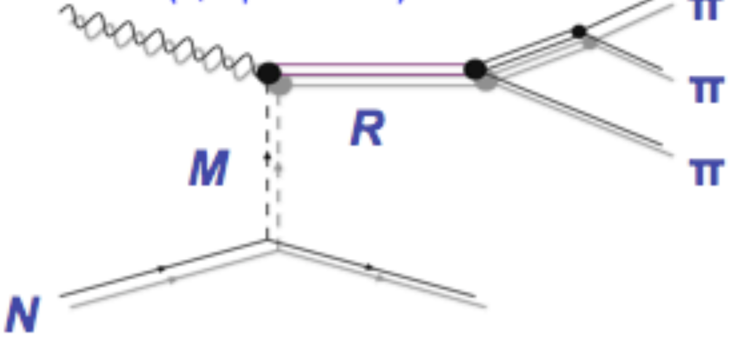


General formalism: Bernard, Hoya, Meisner, Rusetsky, arXiv:1204.4642

Radiative Transitions in Mesons

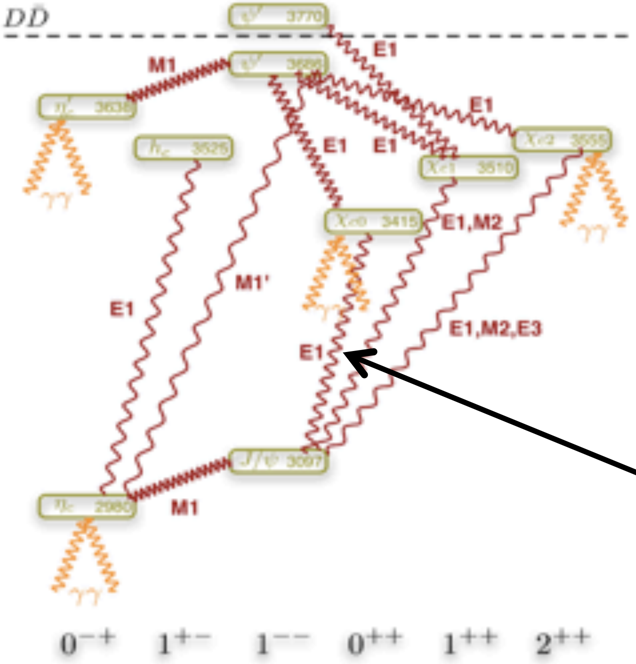


$$\langle\gamma \mid M R\rangle$$



Radiative Transitions in Mesons - II

Look at radiative decays in charmonium - wealth of experimental data. Lots of transitions below threshold!

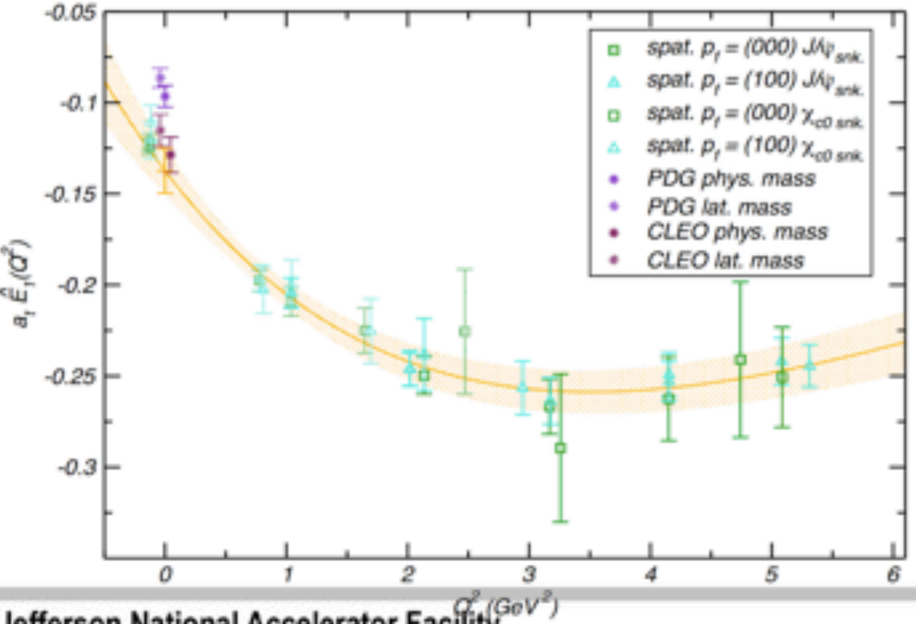


Dudek, Edwards, DGR - 2006
Chen et al (TMQCD), 2011

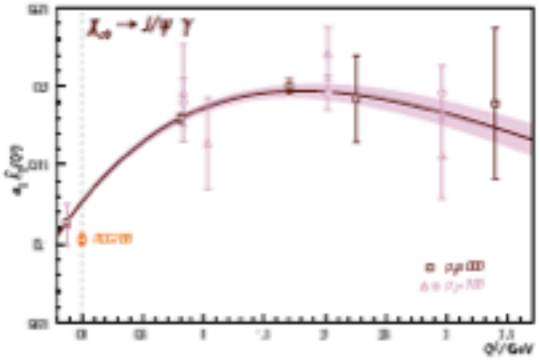
$$\Gamma(\chi_{c0} \to J/\psi\,\gamma) = \frac{1}{8\pi}\frac{|\vec{q}\,|}{m_S^2}\,2\,(2e_c)^2\,|\,E_1(0)\,|^2$$

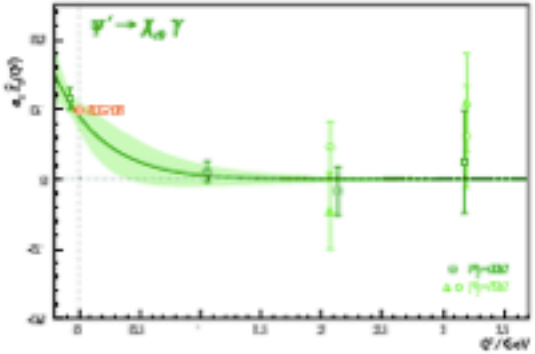
Quenched, anisotropic Wilson-fermion action $a_t m_q < \mathcal{O}(1)$

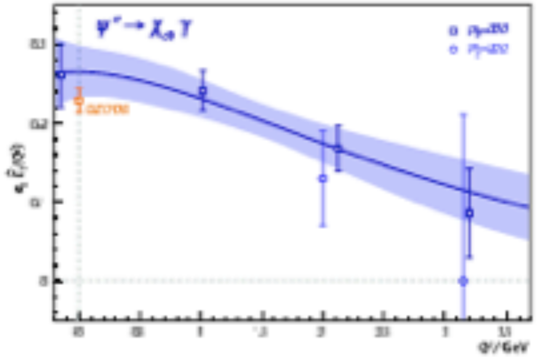
Lattice spacing from static quark potential



Transitions from Excited States?

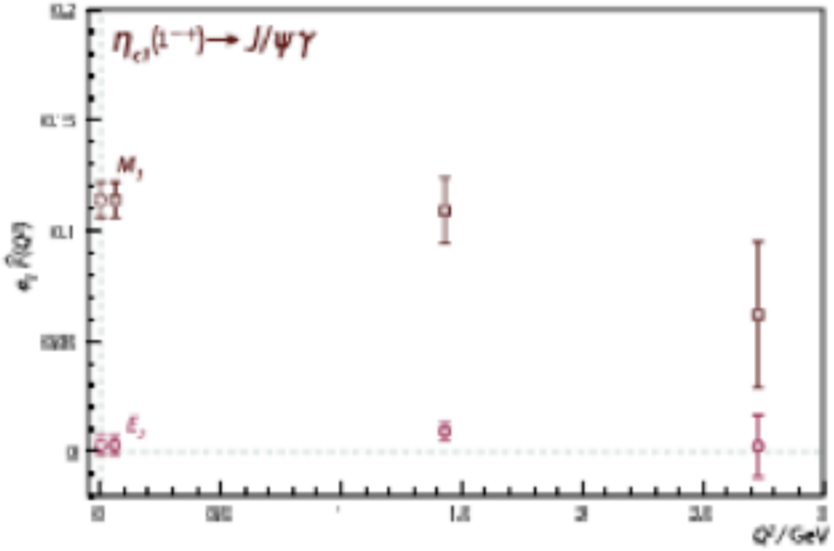






Many of these radiative widths have been measured...

Can Access time-like form factors!



$$\Gamma(\eta_{c1} \to J/\psi\gamma) = 115(16)\ \mathrm{keV}$$

Large for M1 transition - large production of exotics at JLab if true in light-quark sector

Time-Like Form Factors

Ji and Jung, PRL86, 208 (2001) and PRD64, 034506;
Dudek and Edwards, Phys.Rev.Lett. 97, 172001 (2006);
Cohen and Lin, arXiv:1302.0874; Meyer, arXiv:1303.0138

Photon Structure function and two-photon decays of neutral mesons - *defined in Minkowski space:*

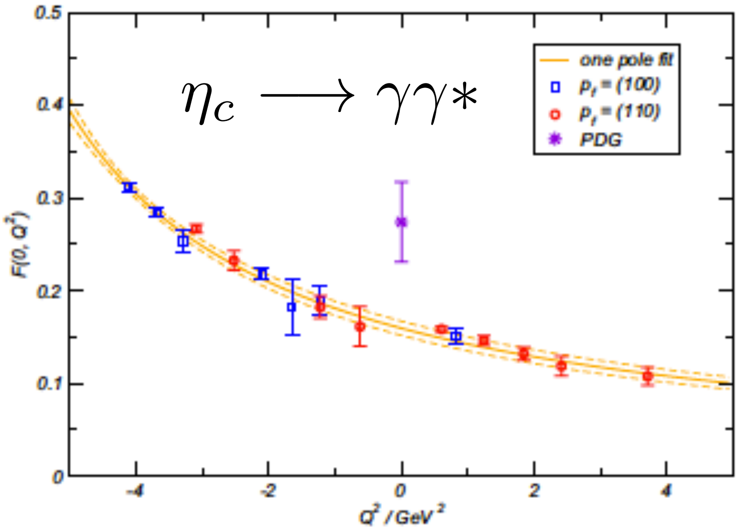
$$\langle\gamma(q_1,\lambda_1)\gamma(q_2,\lambda_2)|M(p)\rangle = -\lim_{\substack{q_1'\to q_1\\ q_2'\to q_2}} \epsilon^*_\mu(q_1,\lambda_1)\epsilon^*_\nu(q_2,\lambda_2)$$

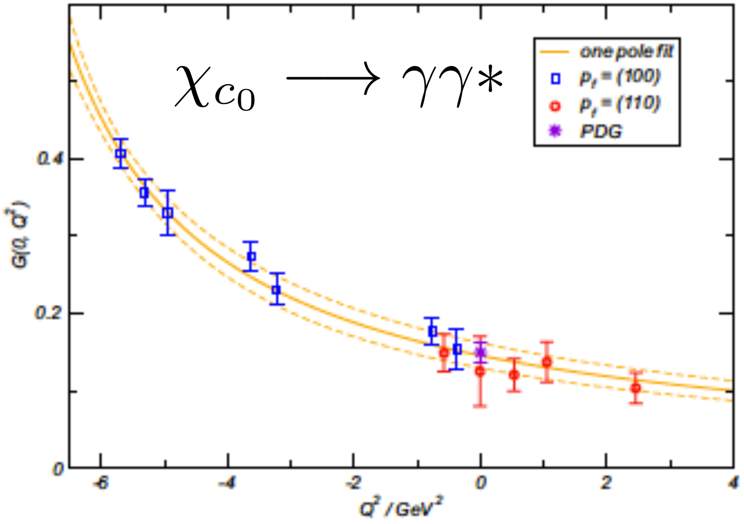
$$\times q_1'^2 q_2'^2 \int d^4x d^4y\, e^{iq_1'\cdot y+iq_2'\cdot x}\langle 0|T\{A^\mu(y)A^\nu(x)\}|M(p)\rangle,$$

Analytically continue to Euclidean space *providing* photons not sufficiently off-shell to produce on-shell hadrons $Q^2 =|\vec{q}|^2 - \omega^2 > -m_H^2$

First example: two-photon with in charmonium - Dudek, Edwards

Time-Like Form Factors - II





Time-like pion form factor: H. Meyer, PRL 107, 072002 (2011)

Form factors at High Q^2

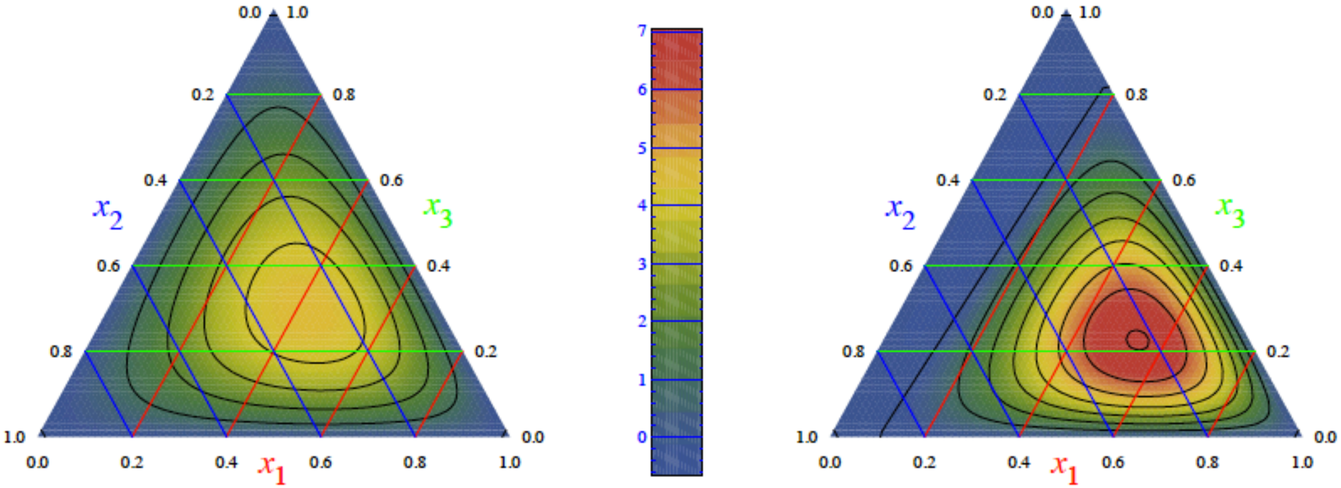
- For exclusive processes at sufficiently high Q^2, can describe processes in terms of quark distribution amplitudes, e.g. for N(*)

$$|N,\uparrow\rangle = f_N \int \frac{[\mathrm{d}x]\varphi(x_i)}{2\sqrt{24x_1x_2x_3}}\{|u^\uparrow(x_1)u^\downarrow(x_2)d^\uparrow(x_3)\rangle - |u^\uparrow(x_1)d^\downarrow(x_2)u^\uparrow(x_3)\rangle\}.$$

- Can compute low moments of quark distribution amplitudes

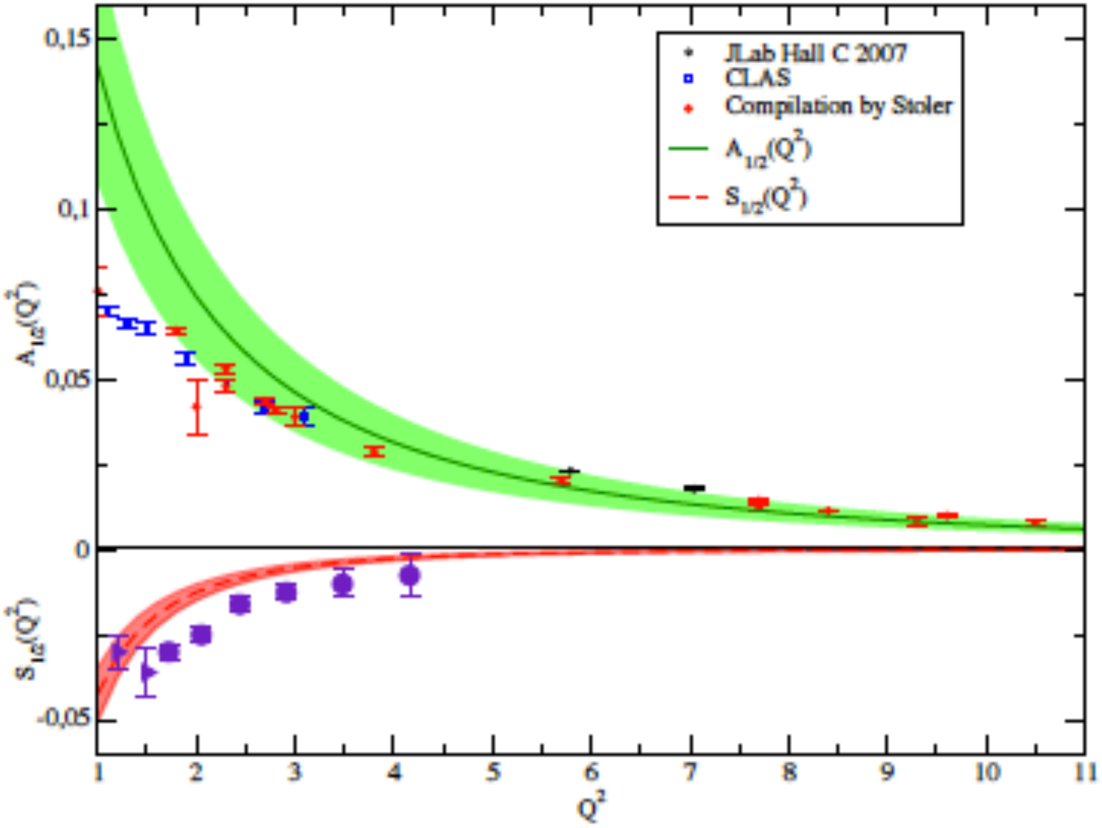
$$\varphi^{lmn} = \int [\mathrm{d}x] x_1^l x_2^m x_3^n \varphi(x_1,x_2,x_3).$$

QCDSF, arXiv:1112.0473



N *N*(1535)*

Form factors at High Q^2



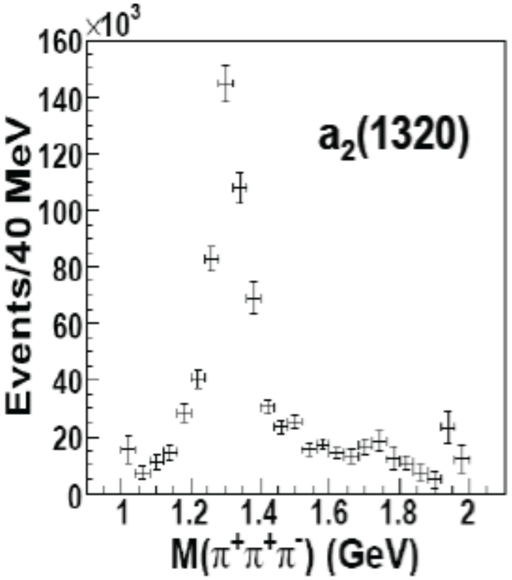
V.Braun, arXiv:1008.5228

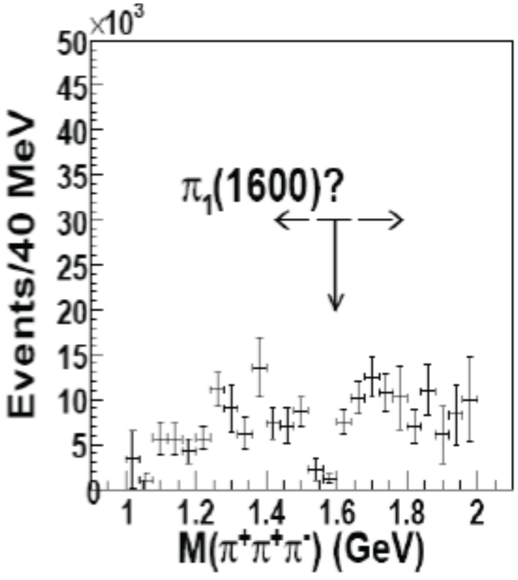
Helicity amplitudes from DA

SUMMARY

- Remarkable progress at understanding the excited-state spectrum of QCD - **resonance properties**
- In principle, extension to EM properties of resonances straightforward, but need to do the work
- I haven’t discussed chiral extrapolations - focus is on understanding decays at calculated masses
- Euclidean-space at finite volume is a blessing, not a curse

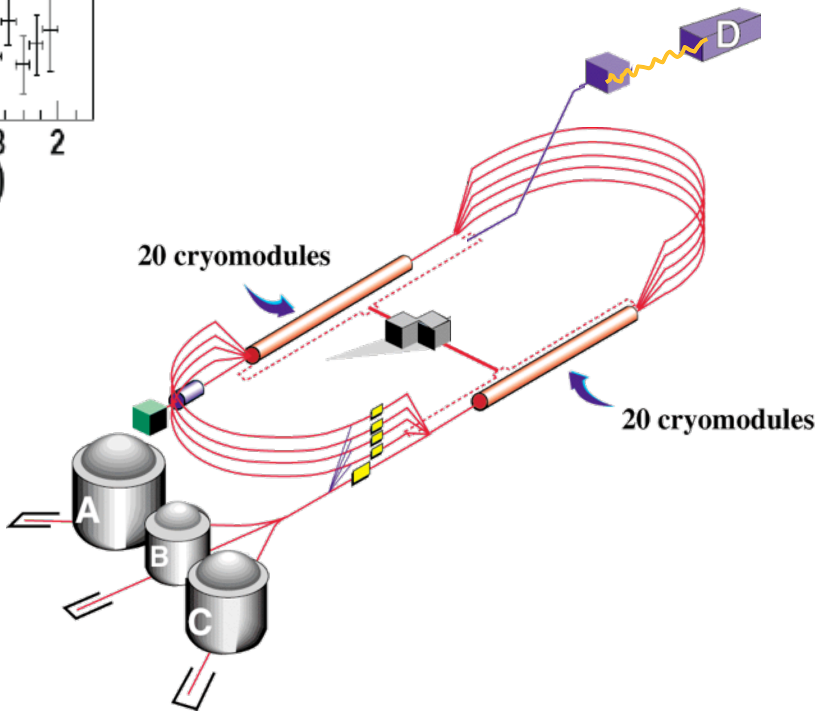
Hybrids - lattice + expt



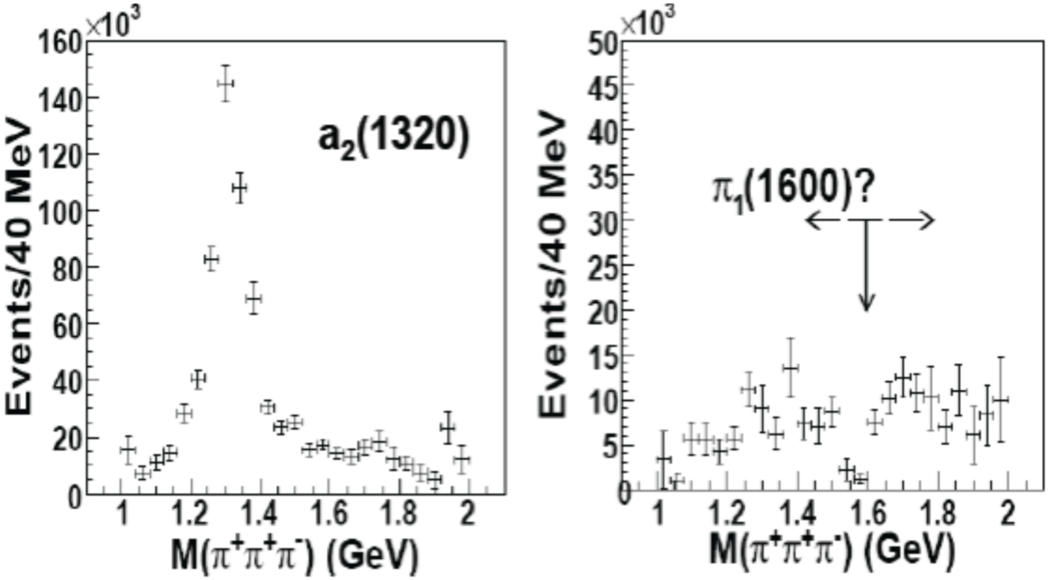


π1(1600) in pion production at BNL

No clear evidence in photoproduction at CLAS



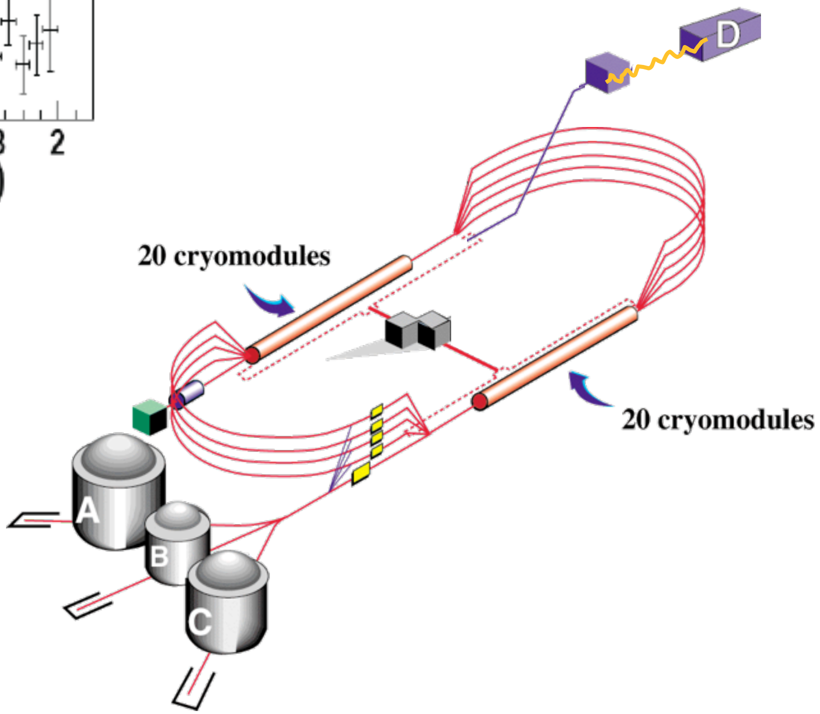
Hybrids - lattice + expt



Beyond "bump hunting"!

π1(1600) in pion production at BNL

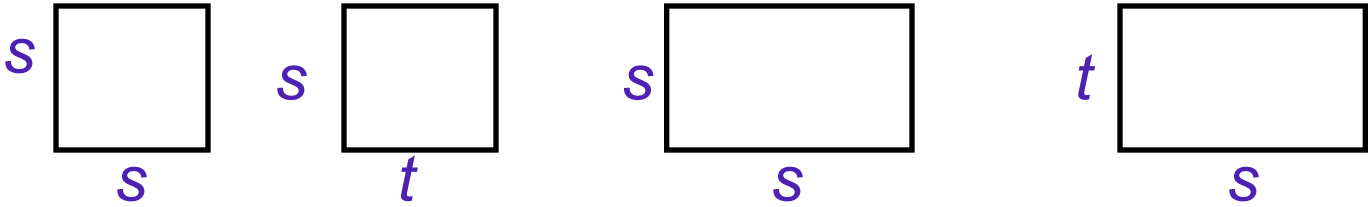
No clear evidence in photoproduction at CLAS



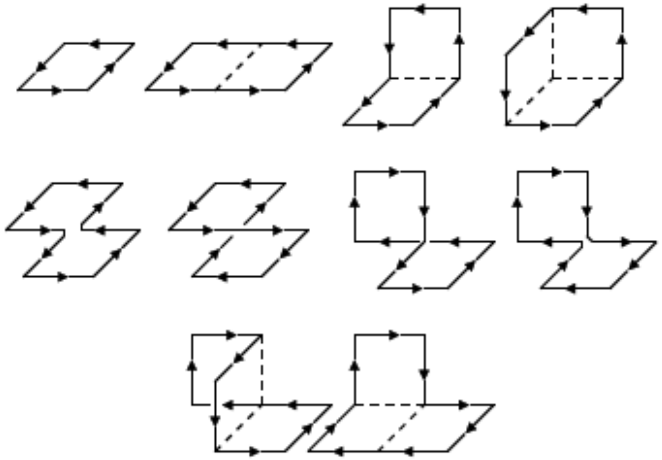
Glueball Spectroscopy - I

Improved anisotropic pure-gauge action Morningstar, Peardon 97,99

$$S[U] = \beta\xi\left\{\frac{5}{3U_s^4}P_{ss'} + \frac{4}{3\xi^2 u_s^2 u_t^2}P_{st} - \frac{1}{12u_s^6}R_{ss'} - \frac{1}{12\xi^2 u_s^4 u_t^2}R_{st}\right\}$$



Operators: closed Wilson loops



ξ is bare anisotropy a_s/a_t

Obtain renormalized anisotropy by comparing different Wilson Loops

$$W_{xt}(Ia_s, Ja_t) \xrightarrow{J\to\infty} Z_{xt}e^{-Ja_tV(Ia_s,0,0)},$$
$$W_{xy}(Ia_s, Ja_s) \xrightarrow{J\to\infty} Z_{xy}e^{-Ja_s[V(Ia_s,0,0)+V_0]}$$

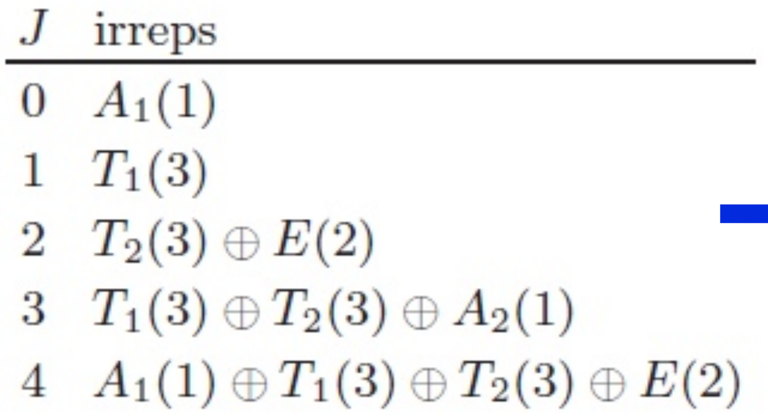
Ratio at large J gives ξ

Morningstar, 96

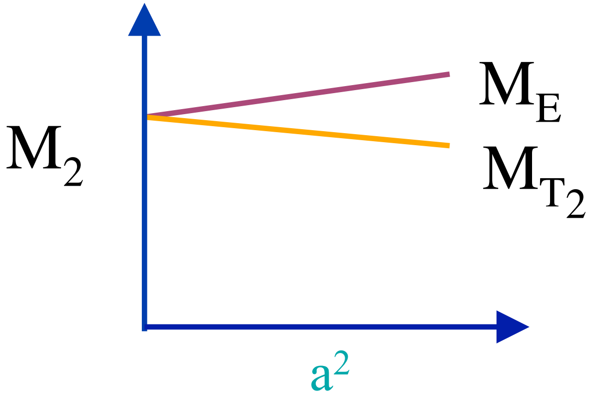
Challenges - II

- States at rest are characterized by their behavior under *rotations - SO(3)*

  Lattice does not possess full symmetry of the continuum - allowed energies characterised by cubic symmetry, or the octahedral point group *O_h*
  - *24 elements*
  - *5 conjugacy classes/5 irreducible representations*
  - *O_h x I_s:* rotations + inversions (parity)



J	irreps
0	$A_1(1)$
1	$T_1(3)$
2	$T_2(3) \oplus E(2)$
3	$T_1(3) \oplus T_2(3) \oplus A_2(1)$
4	$A_1(1) \oplus T_1(3) \oplus T_2(3) \oplus E(2)$



Anisotropic Clover Generation - I

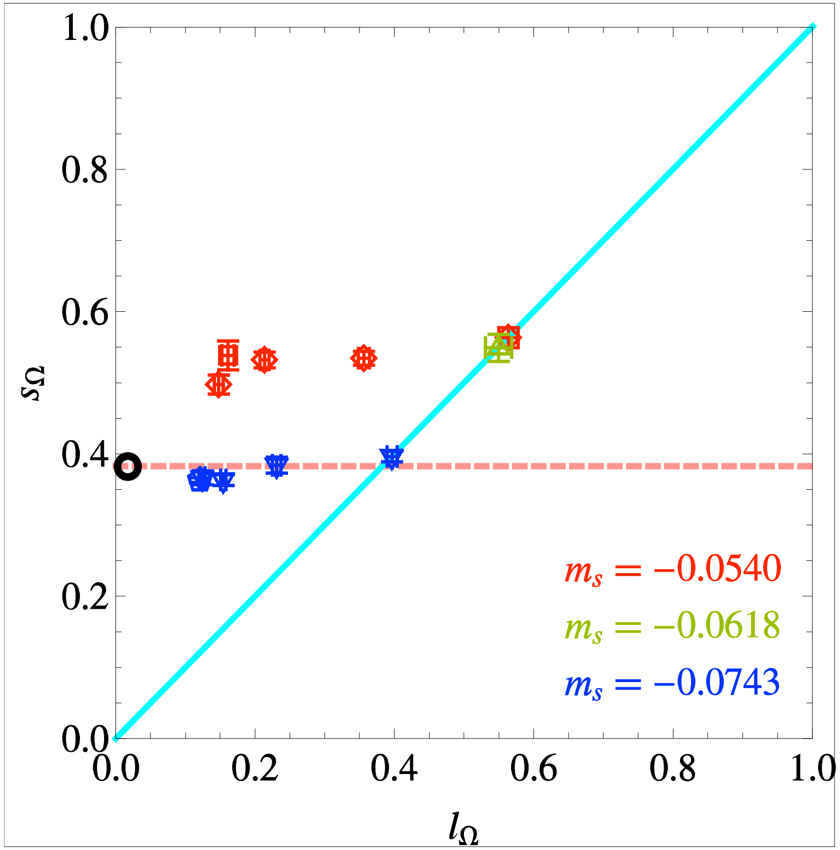
Tuning performed for three-flavor theory

Challenge: setting scale and strange-quark mass

Lattice coupling *fixed*

$$s_X = (9/4)[2m_K^2 - m_\pi^2]/m_X^2$$

Omega



$$l_X = (9/4)m_\pi^2/m_X^2$$

Express physics in (dimensionless) (l,s) coordinates

H-W Lin et al (Hadron Spectrum Collaboration), PRD79, 034502 (2009)

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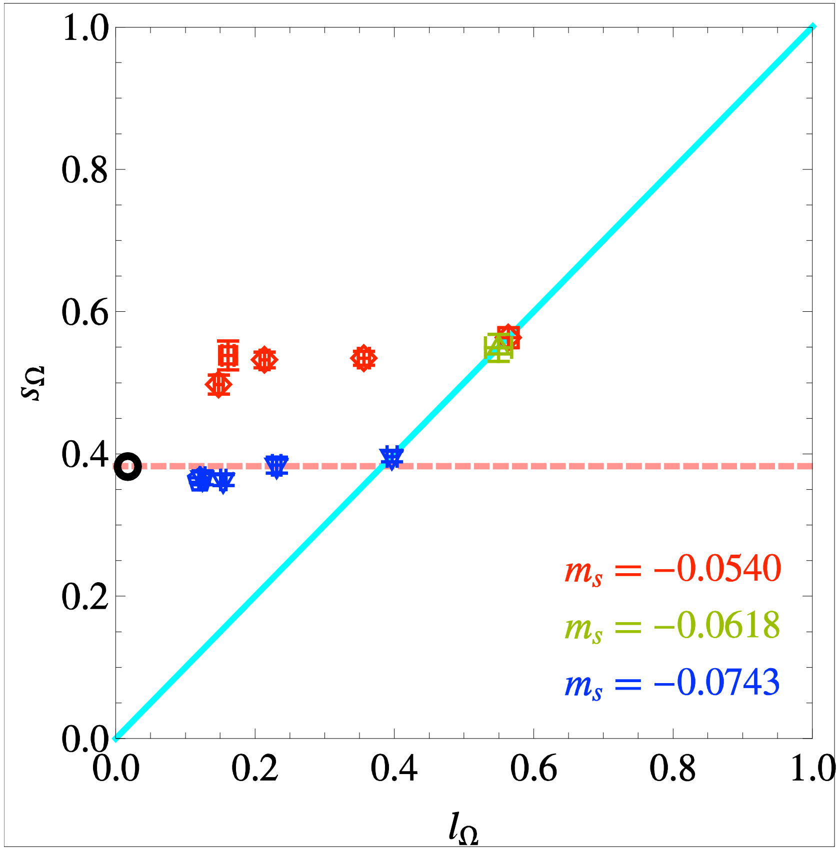

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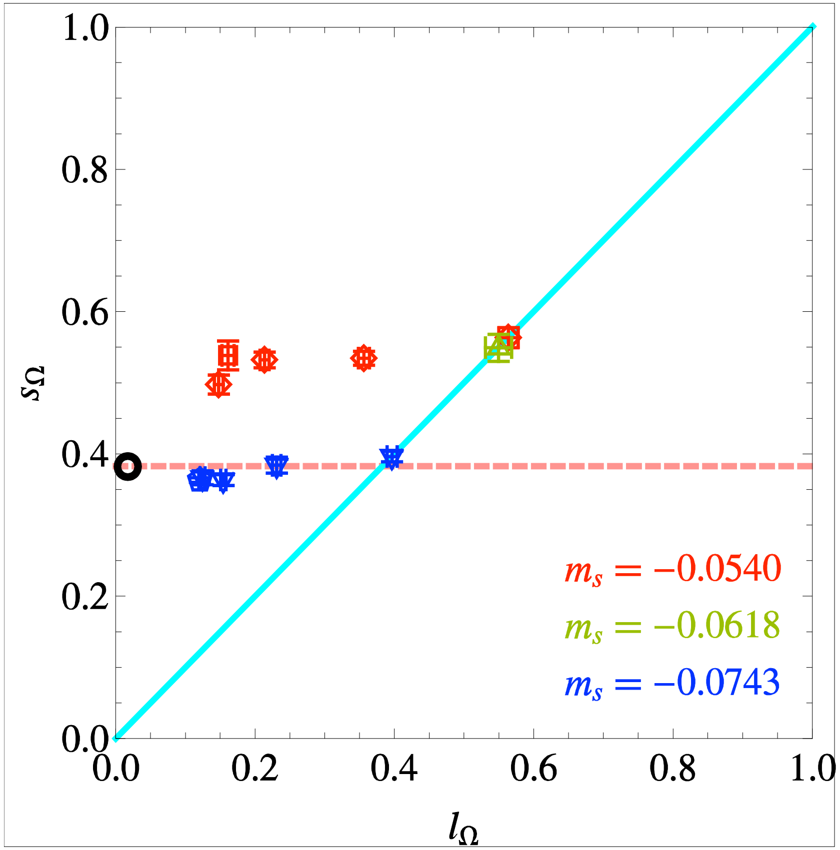

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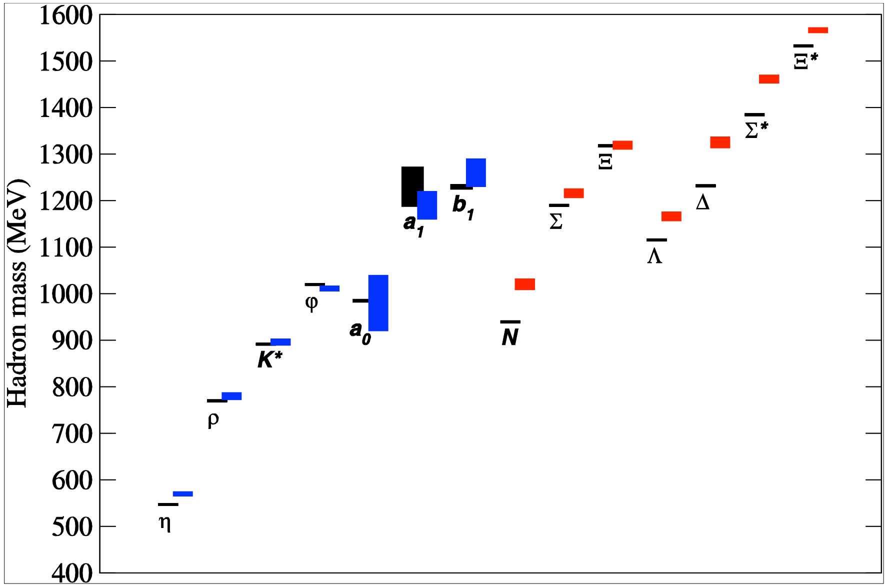
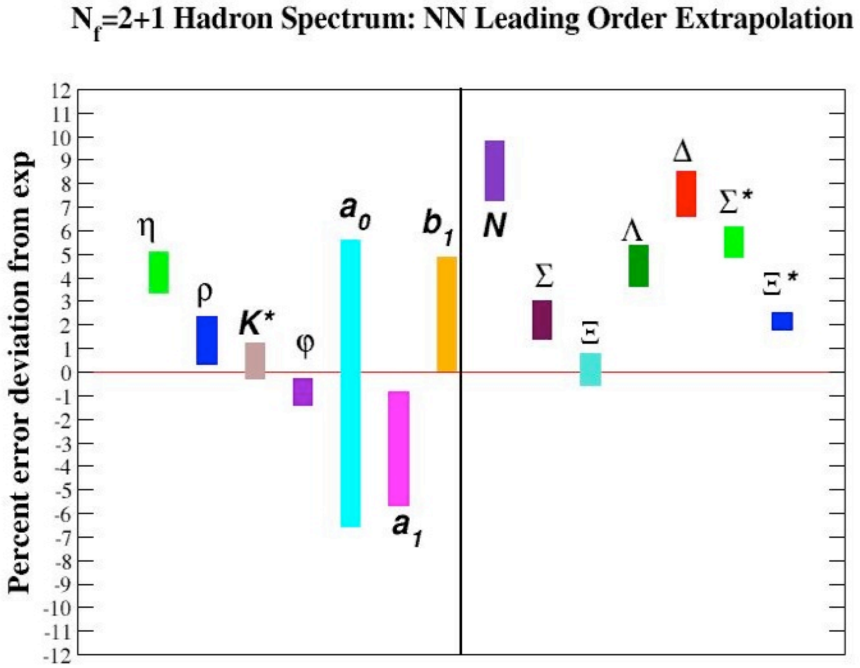
Anisotropic Clover – II



Low-lying spectrum: *agrees with experiment to 10%*



Correlation functions: Distillation

Eigenvectors of Laplacian

- Use the new "distillation" method.
- Observe $$L^{(J)} \equiv (1 - \frac{\kappa}{n}\Delta)^n = \sum_{i=1} f(\lambda_i) v^{(i)} \otimes v^{*(i)}$$
- Truncate sum at sufficient i to capture relevant physics modes – we use 64: set "weights" f to be unity
- Meson correlation function

Includes displacements

$$C_M(t,t') = \langle 0 \mid \bar{d}(t')\Gamma^B(t')u(t')\bar{u}(t)\Gamma^A(t)d(t)|0\rangle$$

- Decompose using "distillation" operator as

M. Peardon *et al.*, PRD80,054506 (2009)

$$C_M(t,t') = \mathrm{Tr}\langle \phi^A(t')\tau(t',t)\Phi^B(t)\tau^\dagger(t',t),\rangle$$

where

$$\Phi^{A,ij}_{\alpha\beta} = v^{*(i)}(t)[\Gamma^A(t)\gamma_5]_{\alpha\beta}v^{(j)}(t')$$

Perambulators $\longrightarrow$ $$\tau^{ij}_{\alpha\beta}(t,t') = v^{*(i)}(t')M^{-1}_{\alpha\beta}(t',t)v^{(j)}(t).$$

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