Broad Resonances

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$$\Delta, N^*, \rho, (\omega)$$

- Play important roles in nuclear collisions at relativistic energies
- Questions:
  - Consistent treatment in models?
  - Unique extraction of properties from data?
    (e.g. resonance vs. background)
  - Causality & analyticity

What is a resonance?

- Toy model for Δ:
- Consider $N + \pi \leftrightarrow \Delta$ in a box.
- Hamiltonian: $H = H_B + H_\pi + H_{\text{int}}$
- Bare states: $H_B|N\rangle = m_N|N\rangle$

$$H_\pi|\pi\rangle = \omega_\pi|\pi\rangle$$

- Eigenstates of H: $H|n\rangle = E_n|n\rangle$
- Eigenstates linear combinations of bare states.

Δ Green's function

$$G_\Delta(E, \vec{p}=0) = \sum_n \frac{|\langle n|a^\dagger_\Delta|0\rangle|^2}{E - E_n + i\delta} + \ldots$$

(Recoilless approximation)

Spectral function:

$$S_\Delta(E) = -2ImG_\Delta = 2\pi \sum_n |\langle n|a^\dagger_\Delta|0\rangle|^2 \delta(E - E_n) + \ldots$$

~Probability of finding a "bare" Δ in $|n\rangle$

Analytic structure

- Poles on real axis of complex E-plane located at eigenvalues of Hamiltonian.
- In thermodynamic limit ($V \to \infty$) poles merge into a cut, with a branch point at threshold.
- Resonances then correspond to poles on unphysical Riemann sheets (more later)

Spectral function is not an observable

- Breit-Wigner resonance: $G_R(E) = \dfrac{1}{E - E_R + i\Gamma_R/2}$

$$S_R(E) = \frac{\Gamma_R}{(E - E_R)^2 + (\Gamma_R/2)^2}$$

- Model: $G_\Delta(E) = \dfrac{1}{E - E^0_\Delta - Re\Sigma_\Delta(E) - iIm\Sigma_\Delta(E)}$

$$\simeq \frac{Z_\Delta}{E - E_\Delta + \Gamma_\Delta/2}$$

$$Z_\Delta = (1 - \partial Re\Sigma_\Delta/\partial E)^{-1} \qquad \Gamma_\Delta = -2\, Z_\Delta\, Im\Sigma_\Delta$$

S_Δ is not an observable

- Spectral function cut off (model) dependent

$$S_\Delta(E) \simeq Z_\Delta \frac{\Gamma_\Delta}{(E - E_\Delta)^2 + (\Gamma_\Delta/2)^2}$$

- Cross section not

$$\sigma = \frac{4\pi}{k^2}(2\ell + 1)\sin^2\delta_\ell$$

- $\pi N \to \pi N$ in 33-channel

$$\tan\delta_{33} = \frac{Im\Sigma_\Delta}{E - E_\Delta^0 - Re\Sigma_\Delta} \simeq \frac{\Gamma_\Delta/2}{E - E_\Delta}$$

Cross section independent of Z

- Averaged over spins

$$\sigma = \frac{4\pi}{k^2}\frac{4}{2}\frac{(\Gamma_\Delta/2)^2}{(E-E_\Delta)^2+(\Gamma_\Delta/2)^2}$$

- Mass and width determined by data,

  Σ_Δ, S_Δ not

$$S_\Delta(E) \simeq Z_\Delta \frac{\Gamma_\Delta}{(E-E_\Delta)^2+(\Gamma_\Delta/2)^2}$$

Inelastic processes

- E.g. $\pi N \to e^+ e^- N$
- $\sigma_{BW} = \frac{4\pi}{k^2} \frac{(2J+1)}{(2s_1+1)(2s_2+1)} \frac{\Gamma_{in}/2\,\Gamma_{out}/2}{(E-E_\Delta)^2 + (\Gamma_{\rm tot}/2)^2}$
- In model: $\Sigma_{\pi N}$ $\Sigma_{e^+e^-N}$

$$\sigma = \frac{4\pi}{k^2} \frac{4}{2} \frac{Im\Sigma_{\pi N}\ Im\Sigma_{e^+e^-N}}{(E - E\Delta^0 - Re\Sigma_{tot})^2 + (Im\Sigma_{tot})^2}$$

$$\to \sigma \simeq \frac{4\pi}{k^2} \frac{4}{2} \frac{(\Gamma_{\pi N}/2)\ (-Z_\Delta \Sigma_{e^+e^-N})}{(E - E\Delta)^2 + (\Gamma_\Delta/2)^2}$$

Consistent treatment

- $$\Gamma_{e^+e^-N} = -Z_\Delta Im\Sigma_{e^+e^-N}$$

$$\sigma \simeq \frac{4\pi}{k^2}\frac{4}{2}\frac{(\Gamma_{\pi N}/2)\ (\Gamma_{e^+e^-N}/2)}{(E-E\Delta)^2+(\Gamma_\Delta/2)^2}$$

- or

$$\sigma = \frac{4\pi}{k^2}\frac{4}{2}\frac{1}{2}\ S_\Delta(E)\ |Im\Sigma_{e^+e^-N}|$$

Closer to reality

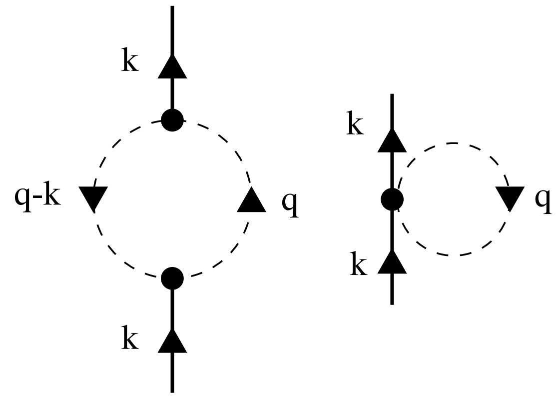
- Scattering amplitude

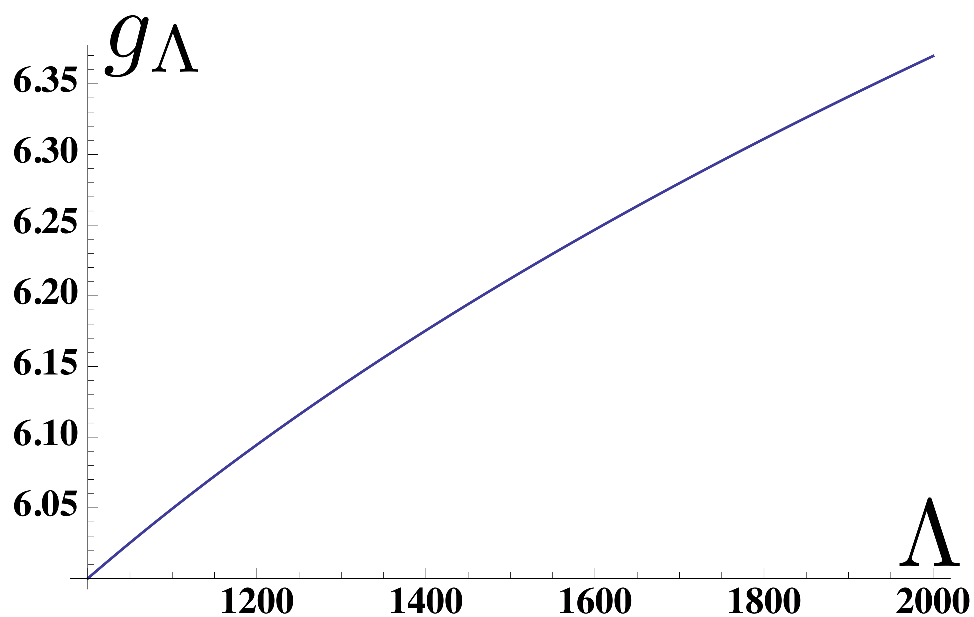
$$f = f_{\Delta} + f_{NRB}$$

- Unless $|f_{\Delta}| >> |f_{NRB}|$
  - Interference terms $\longrightarrow$ difficult to extract f_{Δ}
  - Background different in $\pi N \rightarrow \pi N$ vs. $\pi N \rightarrow e^+ e^- N$

Illustrative example: ρ

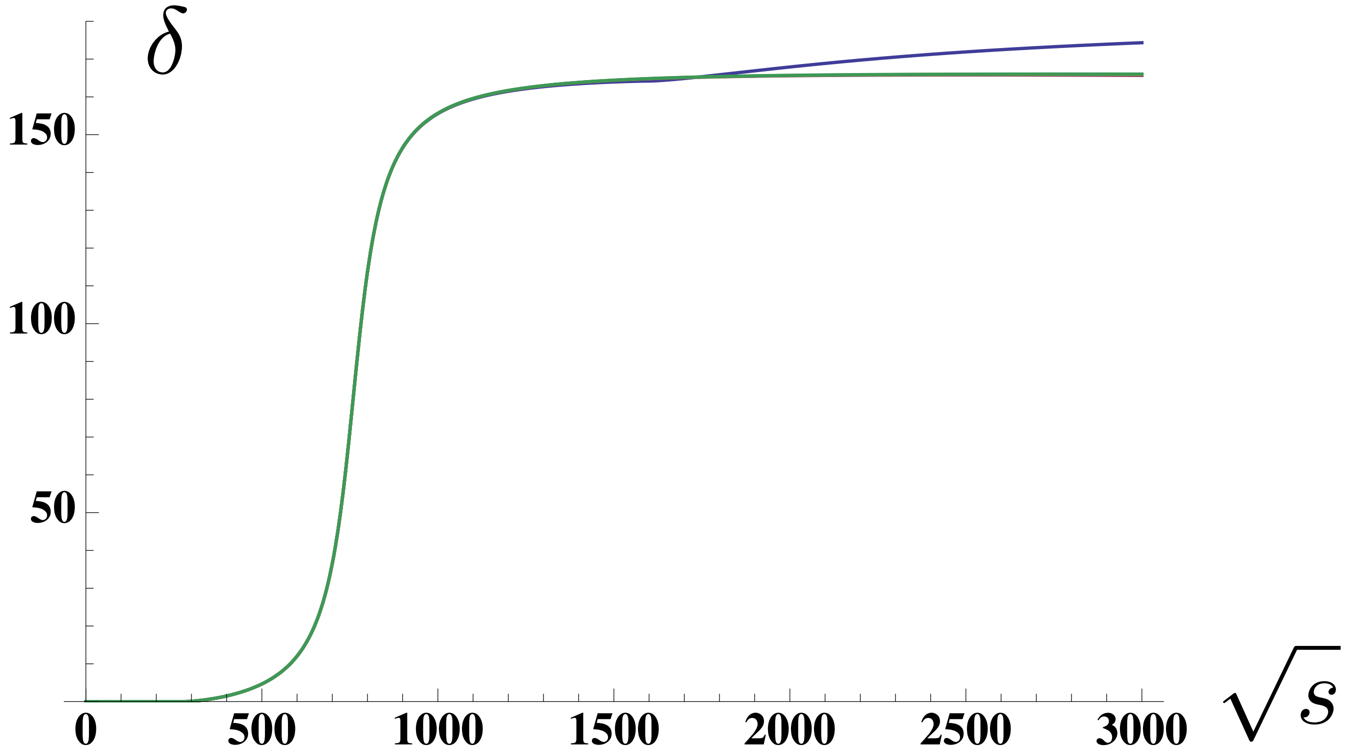
- One-loop model
- Logarthmic divergence
- Fix $\Gamma_\rho = 150\mathrm{MeV}$





Phase shifts

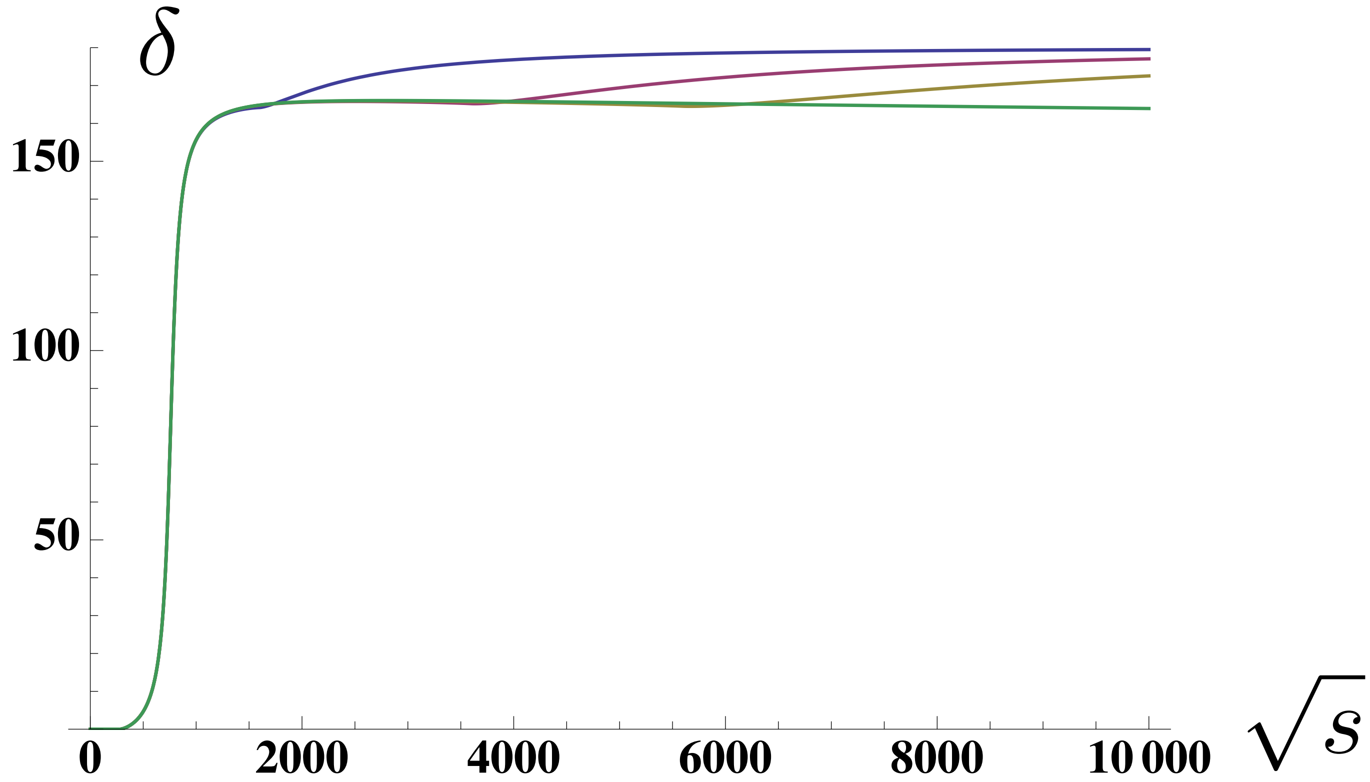
- Cut off $\Lambda = 800, 1800, 2800, 9800\text{MeV}$



- Independent of cut off for low energies

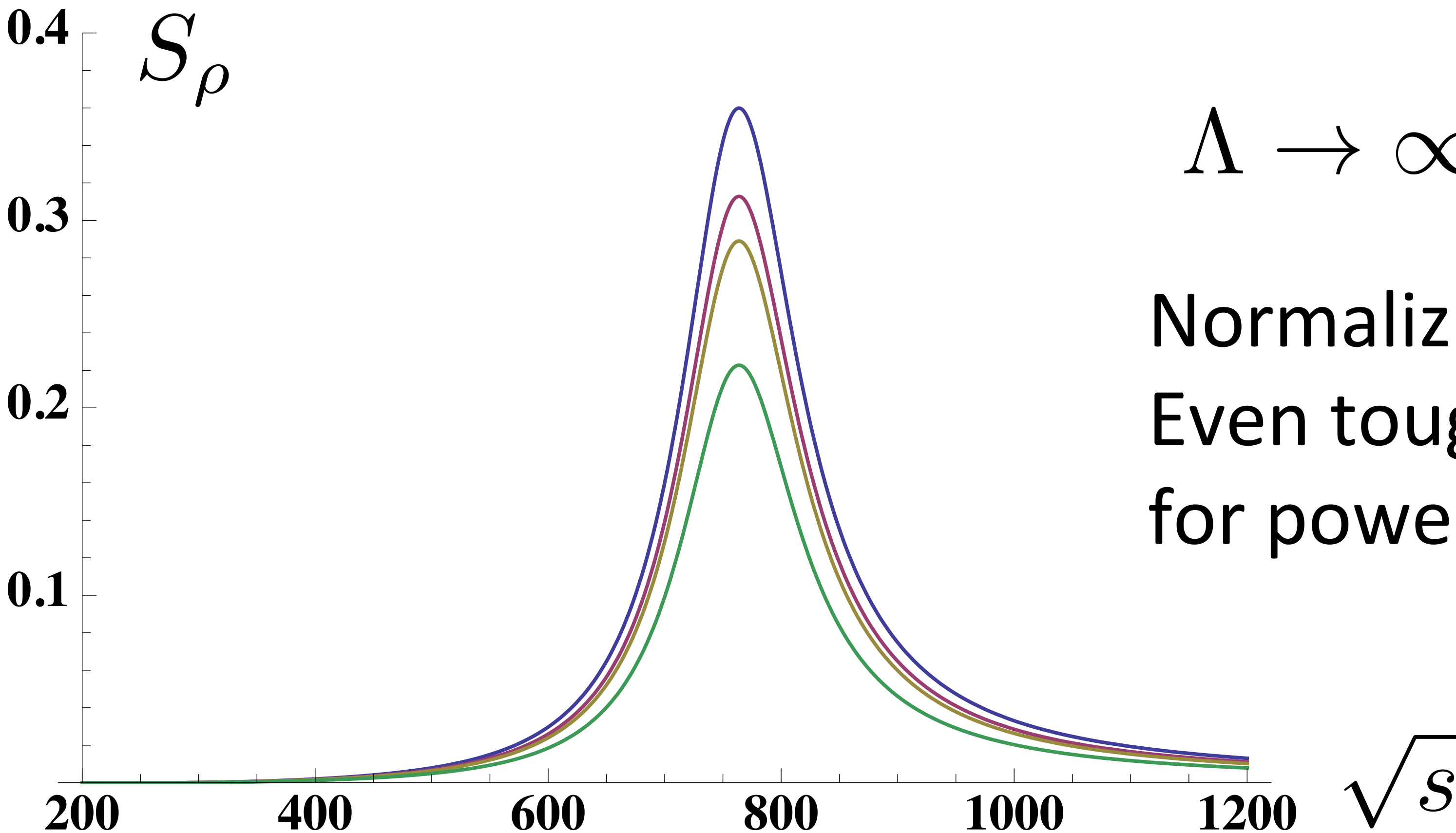
Phase shifts

- Cut off $\Lambda = 800, 1800, 2800, 9800\text{MeV}$



Spectral function

- $\Lambda = 800, 1800, 2800, 9800\text{MeV}$

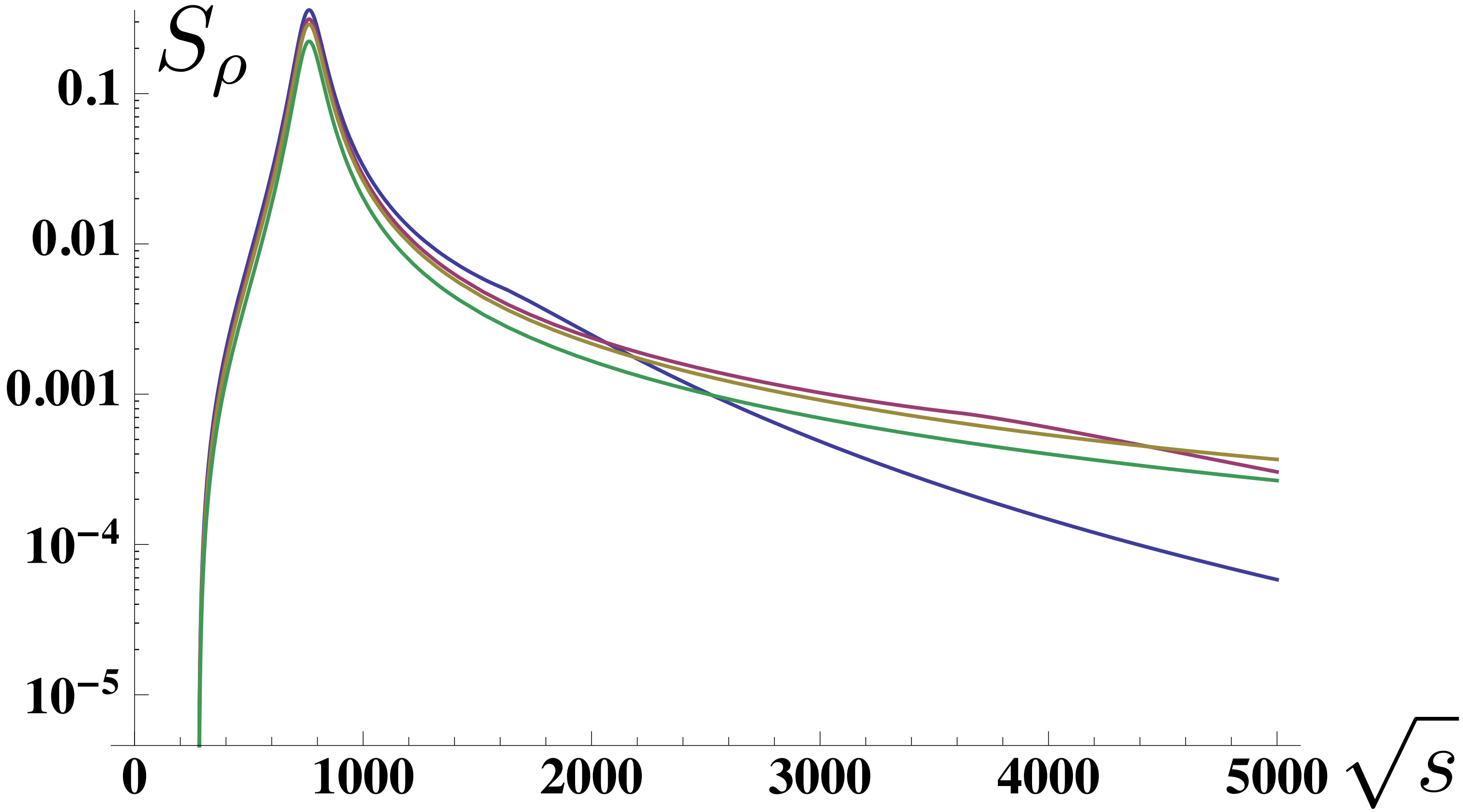


$\Lambda \to \infty?$

Normaliz.?
Even tougher for power-div.

Spectral function

- $\Lambda = 800, 1800, 2800, 9800\text{MeV}$



Summary 1

- Care is needed for consistent treatment of spectral function
- Safer to work with reactions, e.g.

$$\pi N \to e^+ e^- N \qquad \text{(with bckgrnd)}$$

rather than artificial splitting

$$\pi N \to \Delta,\ \Delta \to e^+ e^- N$$

Kinematics of Δ form factor

- $\Delta \to N\gamma^\star \quad \sqrt{p_\Delta^2} = m_\Delta \quad \sqrt{p_N^2} = m_N$

$$q_\gamma = (e_\gamma, \vec{q}_\gamma) \qquad \vec{q}_\gamma^{\,2} = \frac{\lambda(m_\Delta^2, m_N^2, q_\gamma^2)}{4m_\Delta^2}$$

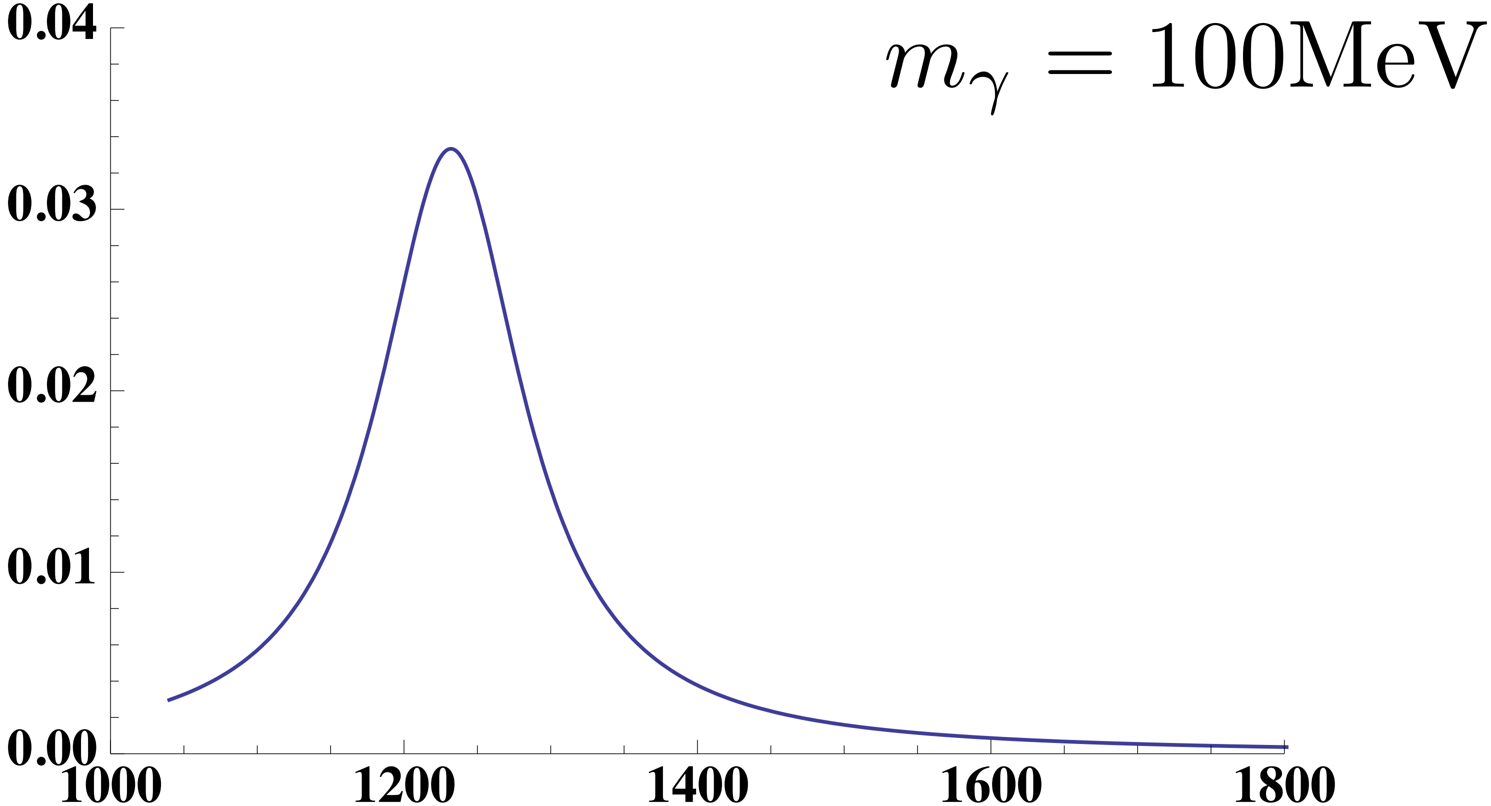
- Determine FF by measuring at fixed q_γ^2

  with m_Δ in the range $(\bar{m}_\Delta - \Gamma_\Delta, \bar{m}_\Delta + \Gamma_\Delta)$

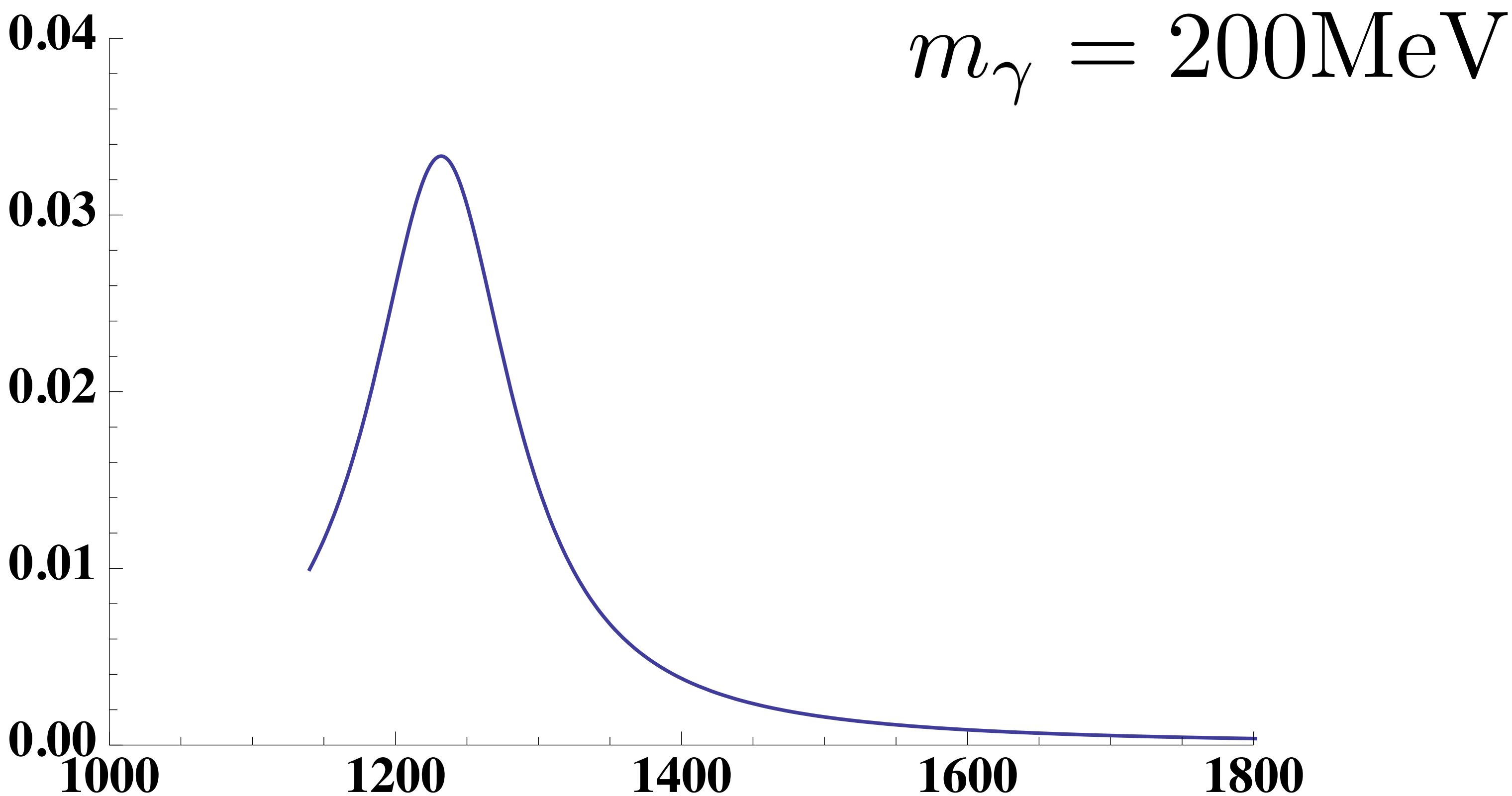
  in order to identify Δ

  (assuming M.E. weakly dependent on $\vec{q}_\gamma$)

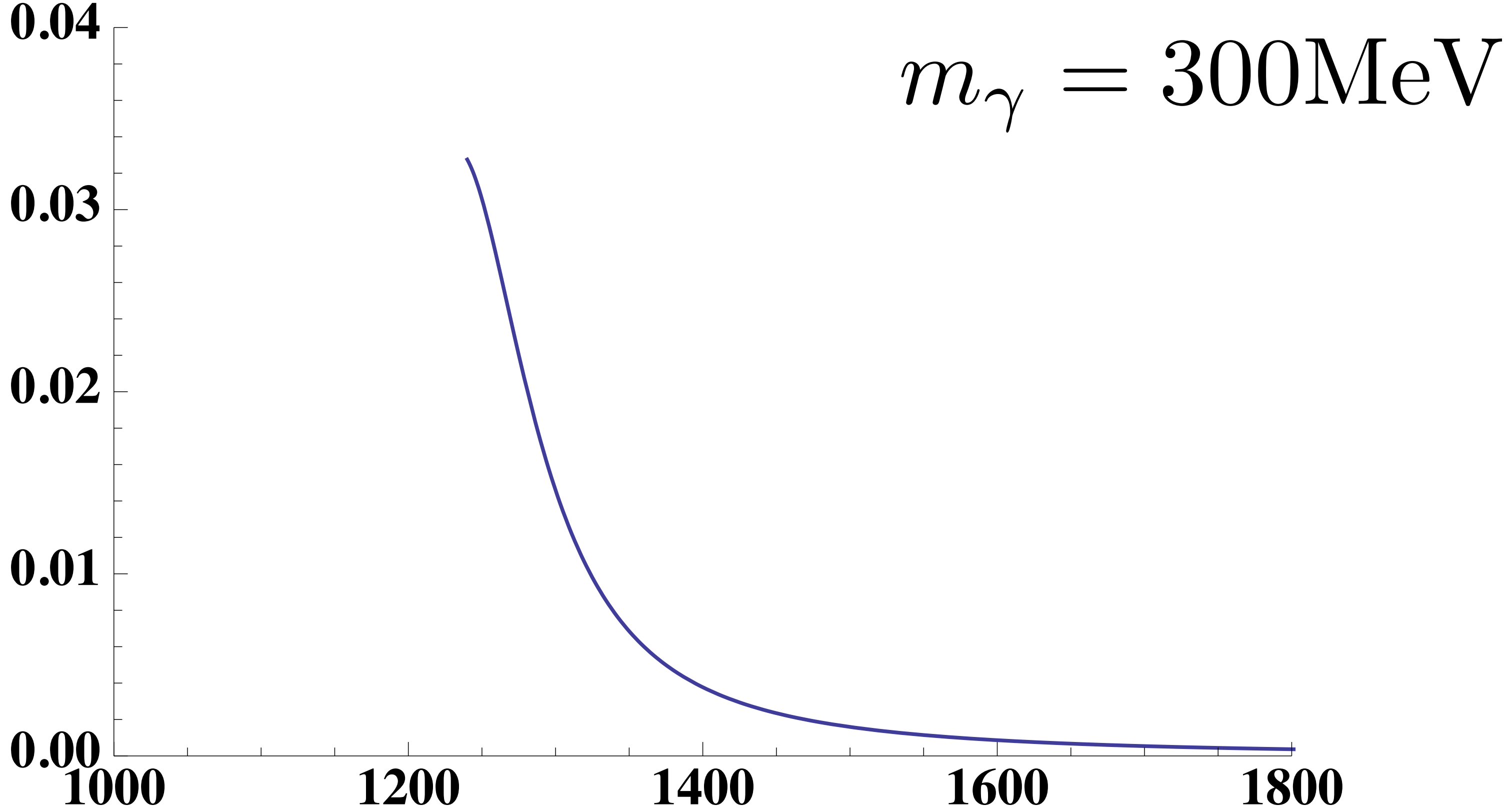
Spectral function probed?



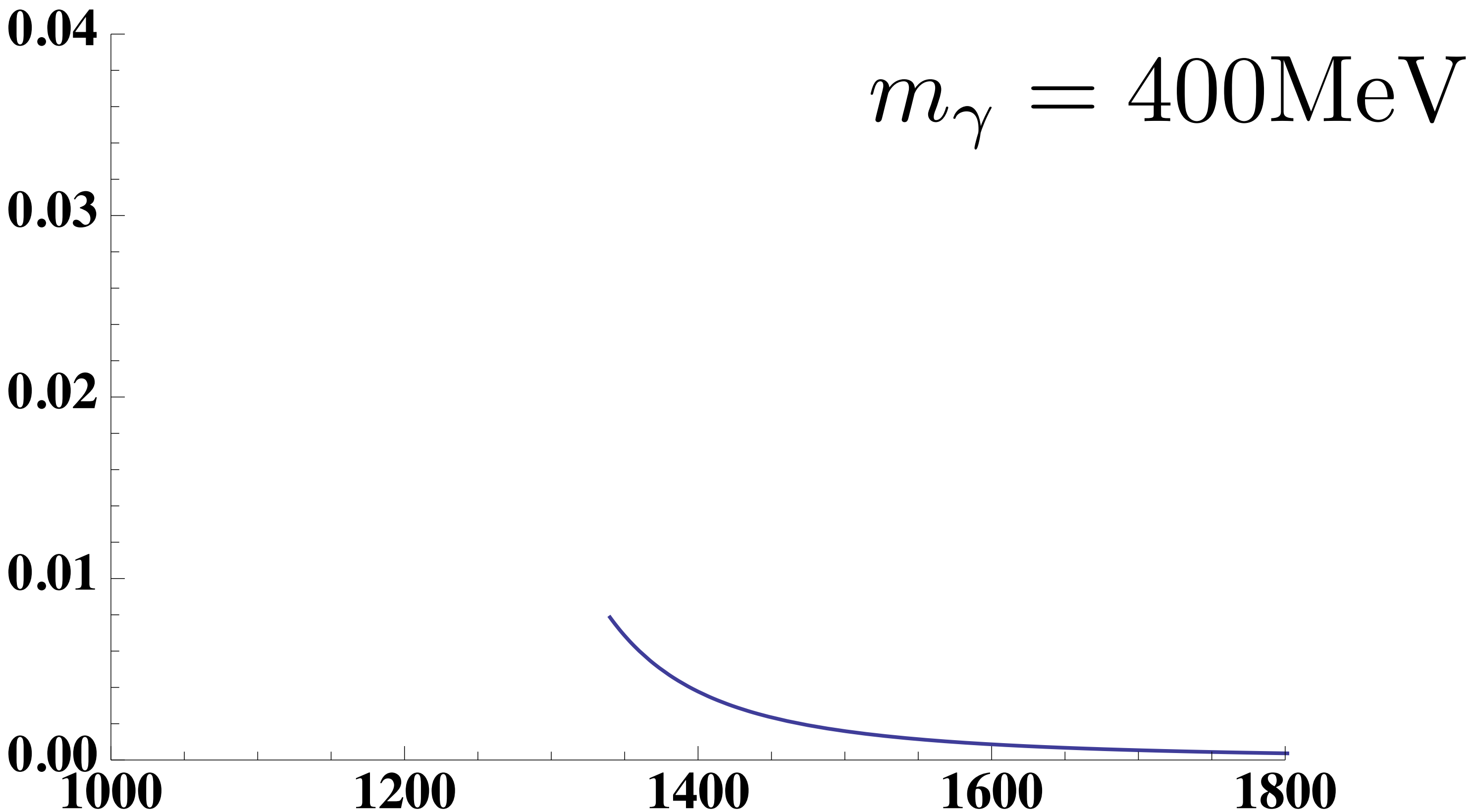
Spectral function probed?



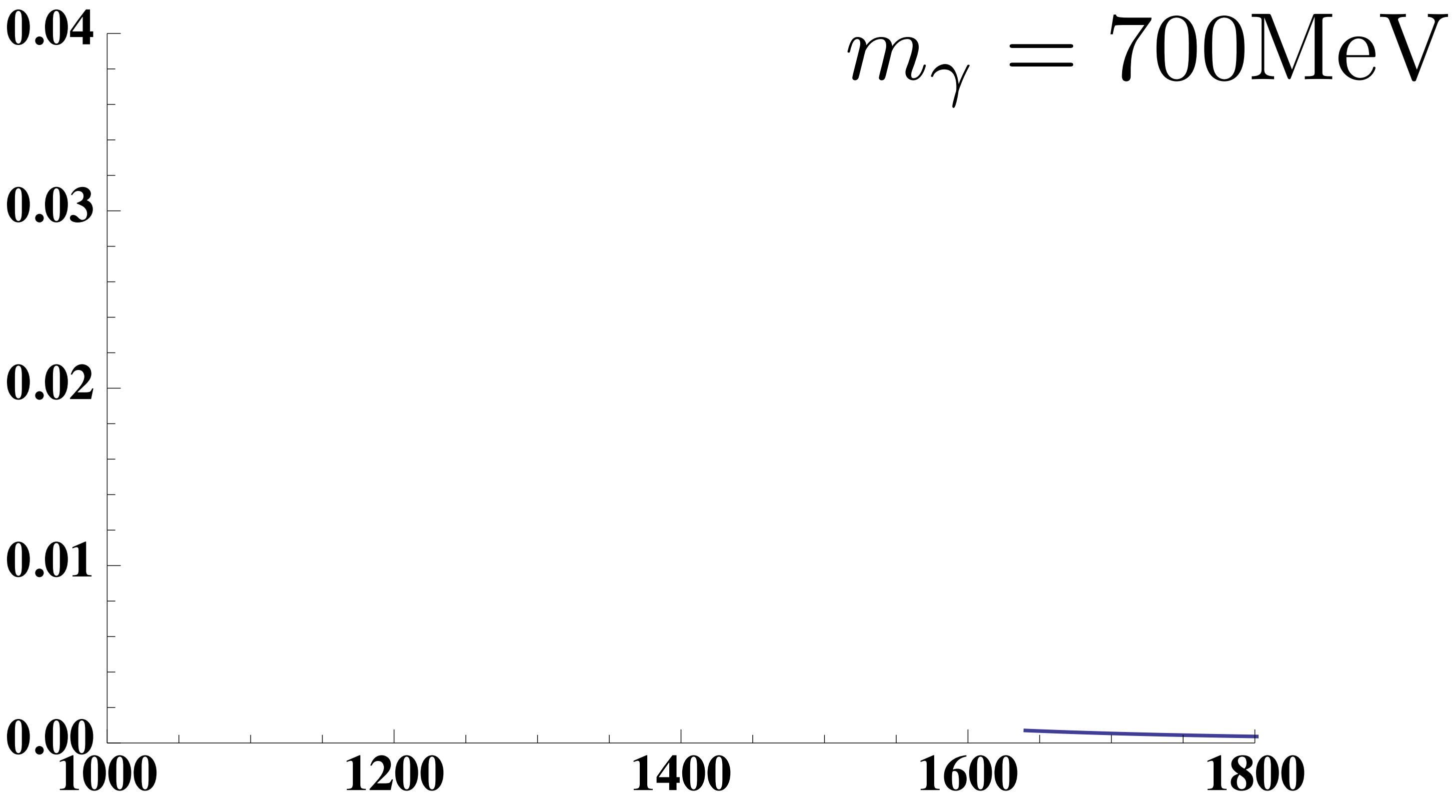
Spectral function probed?



Spectral function probed?



Spectral function probed?



What about N*(1520)?

Summary 2

- Expect only weak constraint on Δ FF
- Dileptons from Δ-Dalitz with $m \simeq m_\rho$ probe extreme tail of Δ strength
- Can experiment constrain FF in relevant mass range?
- Better off using elementary reactions, e.g.

$$\pi N \to e^+ e^- N$$

Causality and analyticity

- Retarded Green's function $G_R(t' - t)$
- Causality: $G_R(t' - t < 0) = 0$
- F.T.: $$G_R(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G_R(t) e^{i\omega t}$$
$$= \frac{1}{2\pi}\int_{0}^{\infty} G_R(t) e^{i\omega t}$$
- Defines $G_R(\omega)$ for complex ω

C & A

- $$G_R(\omega) = \frac{1}{2\pi}\int_0^\infty G_R(t)e^{i\omega t}$$
- If $|G_R(t)| < \infty$ then $G_R(\omega)$ analytic for
  $$Im\ \omega > 0$$
- Causality $\rightarrow G_R(\omega)$ analytic in upper half plane
- Similarly advanced $G_A(\omega)$ analytic in lower half plane

Dispersion relation

- Analyticity in upper half plane implies

$$G_R(\omega) = \frac{-1}{\pi} \int_{-\infty}^{\infty} \frac{Im\, G_R(\omega')}{\omega - \omega' + i\delta}$$

Kramers-Kronig

- Time ordered (Feynman) Green's function

$$G_F(\omega) = \frac{-1}{\pi} \left(\int_0^{\infty} \frac{Im\, G_R(\omega')}{\omega - \omega' + i\delta} + \int_{-\infty}^{0} \frac{Im\, G_R(\omega')}{\omega - \omega' - i\delta} \right)$$

$$G_F(z) = G_R(z) \;\; (Im\, z > 0)$$

$$G_F(z) = G_A(z) \;\; (Im\, z < 0)$$

No poles in complex plane (physical R.S.)

Poles on unphysical Riemann sheet

- BW Green's function: $G(E) = \frac{1}{E - E_R + i\Gamma/2}$

$$Im\,G(E) = -\frac{1}{2i}\left(\frac{1}{E - E_R - i\Gamma/2} - \frac{1}{E - E_R + i\Gamma/2}\right)$$

$$G(z) = \frac{-1}{\pi}\int_{-\infty}^{\infty}\frac{Im\,G(E')}{z - E'}$$

- $$G(E + i\delta) = \frac{1}{E - E_R + i\Gamma/2}$$
- $$G(E - i\delta) = \frac{1}{E - E_R - i\Gamma/2}$$