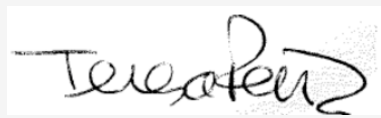
Covariant Spectator Theory and an integrated description of the baryon electromagnetic vertices



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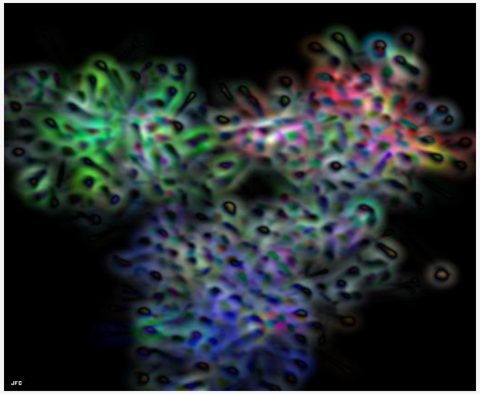
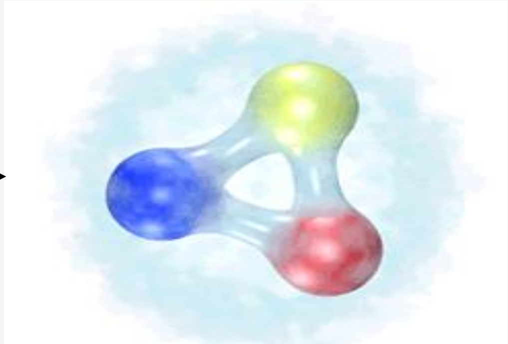
EMMI RRTF 2013

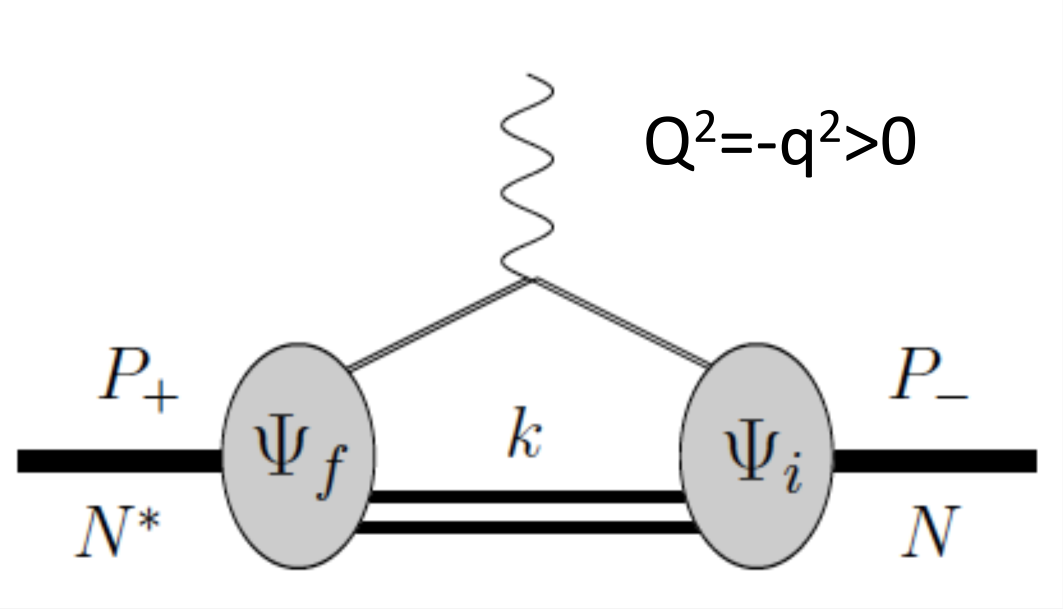




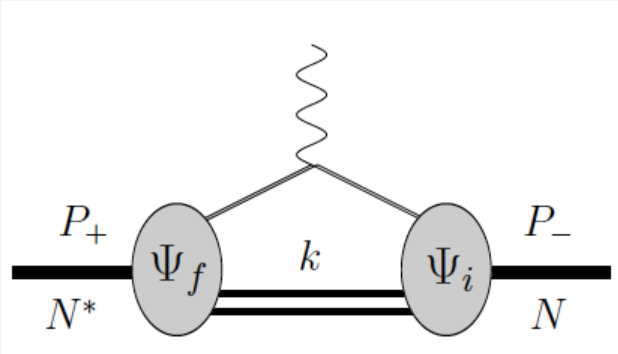
Fock space is truncated

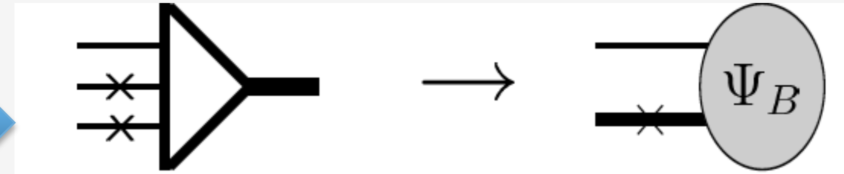
3 constituent quarks with effective size, mass and e.m. form factors, dressed.

E.M. matrix element



E.M. matrix element





$$\int_{k_1 k_2} \equiv \int \frac{d^4k_1 d^4k_2}{(2\pi)^6}\delta_+(m_1^2 - k_1^2)\delta_+(m_2^2 - k_2^2)$$
$$= \int \frac{d^3k_1 d^3k_2}{(2\pi)^6 4E_1 E_2},$$

$$\int_{sk} = \underbrace{\int \frac{d\Omega_{\hat{\mathbf{r}}}}{4(2\pi)^3}\int_{4m_q^2}^{\infty} ds\sqrt{\frac{s-4m_q^2}{s}}}_{\int_s}\underbrace{\int \frac{d^3k}{(2\pi)^3 2E_s},}_{\int_k}$$

- **E.M.** matrix element in terms of **effective** baryon vertices for a quark-diquark structure -- off-mass-shell quark and + on-mass-shell quark pair (diquark)-- with an average mass.

- **Baryon wavefunction** reduced to an effective quark-diquark structure.

Baryon "wavefunction"

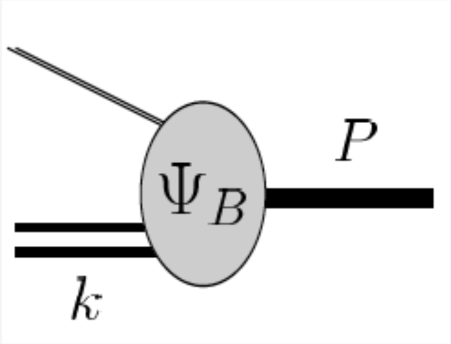
SU(6) × O(3) : impose that the combination of diquark and quark symmetries to be anti-symmetric in the exchange of any pair of quarks

$$\Psi_B = color \otimes flavor \otimes spin \otimes orbital \otimes radial$$

- It is written in a **covariant** form in terms of baryon properties.
- **Extension** to high angular momentum states possible

Nucleon wavefunction

- A quark + **scalar**-diquark component
- A quark+ **axial vector**-diquark component



$$\Psi^S_{N\lambda_n}(P,k) = \frac{1}{\sqrt{2}}[\phi^0_I u_N(P,\lambda_n) - \phi^1_I \varepsilon^{\alpha *}_{\lambda P} U_\alpha(P,\lambda_n)] \times \psi^S_N(P,k).$$

Phenomenological function

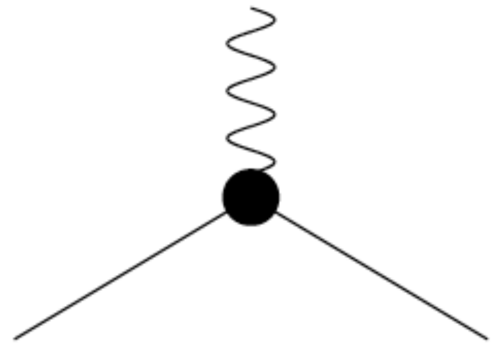
$$U_\alpha(P,\lambda_n) = \frac{1}{\sqrt{3}}\gamma_5\left(\gamma_\alpha - \frac{P_\alpha}{m_H}\right)u_N(P,\lambda_n),$$

Delta wavefunction

- Only quark + **axial vector**-diquark term contributes

$$\Psi^S_\Delta(P,k) = -\psi^S_\Delta(P,k)\tilde{\phi}^1_I \varepsilon^{\beta *}_{\lambda P} w_\beta(P,\lambda_\Delta)$$

E.M. Current



Constituent quarks (quark form factors)

$$j_I^\mu = \left[\frac{1}{6} f_{1+} + \frac{1}{2} f_{1-} \tau_3\right] \gamma^\mu + \left[\frac{1}{6} f_{2+} + \frac{1}{2} f_{2-} \tau_3\right] \frac{i\sigma^{\mu\nu} q_\nu}{2M_N}$$

Vector meson dominance **2** poles

$$f(Q^2) = e + gB(Q^2)e + gB(Q^2)gB(Q^2)e + \cdots = e + \frac{gB(Q^2)e}{1 - gB(Q^2)}$$

if $gB(Q^2) = \frac{\lambda^2}{\Lambda^2 + Q^2}$, then $f(Q^2) = e + \frac{\lambda^2 e}{\Lambda^2 - \lambda^2 + Q^2}$

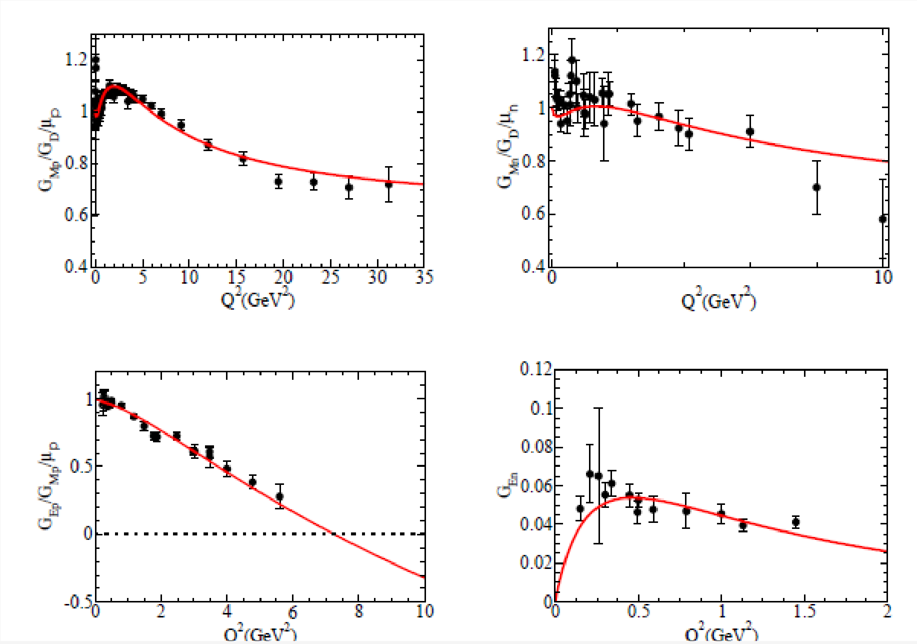
$$f_{1\pm} = \lambda + \frac{1-\lambda}{1 + Q_0^2/m_v^2} + \frac{c_\pm Q_0^2/M_h^2}{\left(1 + Q_0^2/M_h^2\right)^2}$$

$$f_{2\pm} = \kappa_\pm \left(\frac{d_\pm}{1 + Q_0^2/m_v^2} + \frac{(1 - d_\pm)}{1 + Q_0^2/M_h^2} \right)$$

Low-energy behavior encodes high-energy behavior:
DIS used to fixed λ

4 parameters

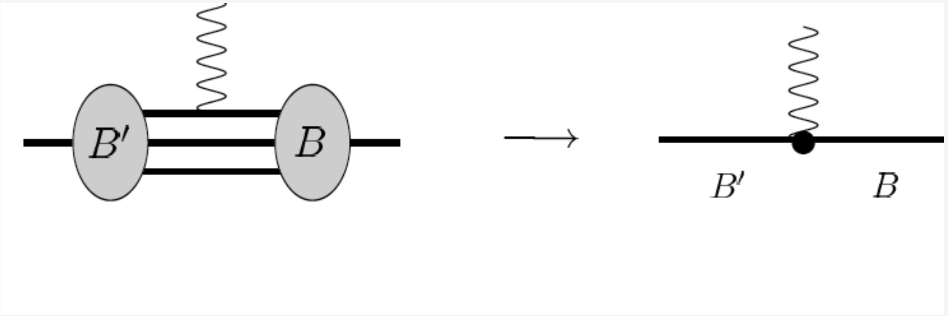
Proton and Neutron form factors $\chi^2 = 1.36$

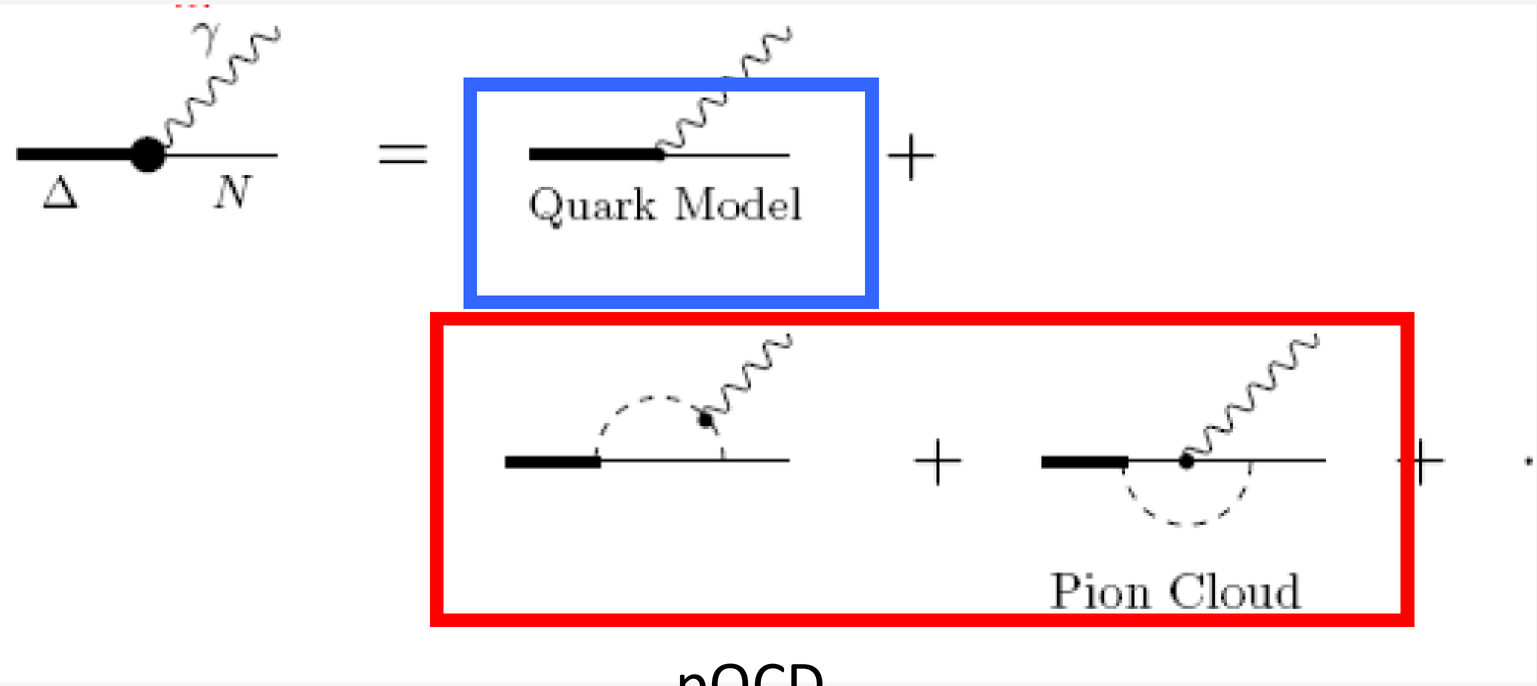


G. Ramalho, M. T. P. and Franz Gross, PHYSICAL REVIEW C 77, 015202 (2008)

Franz Gross, G. Ramalho, M. T. P. PHYSICAL REVIEW D 85, 093006 (2012)

γ -Bare Quark core coupling





pQCD

suppressed with extra $\frac{1}{Q^4}$

C. Carlson, FBS Supp 11 10 (1999)

$$G_M^* = G_M^B + G_M^\pi$$

G. Ramalho, M. T. P. and Gross,
EPJS 36, 329 (2008);
PRD 78, 114017 (2008)

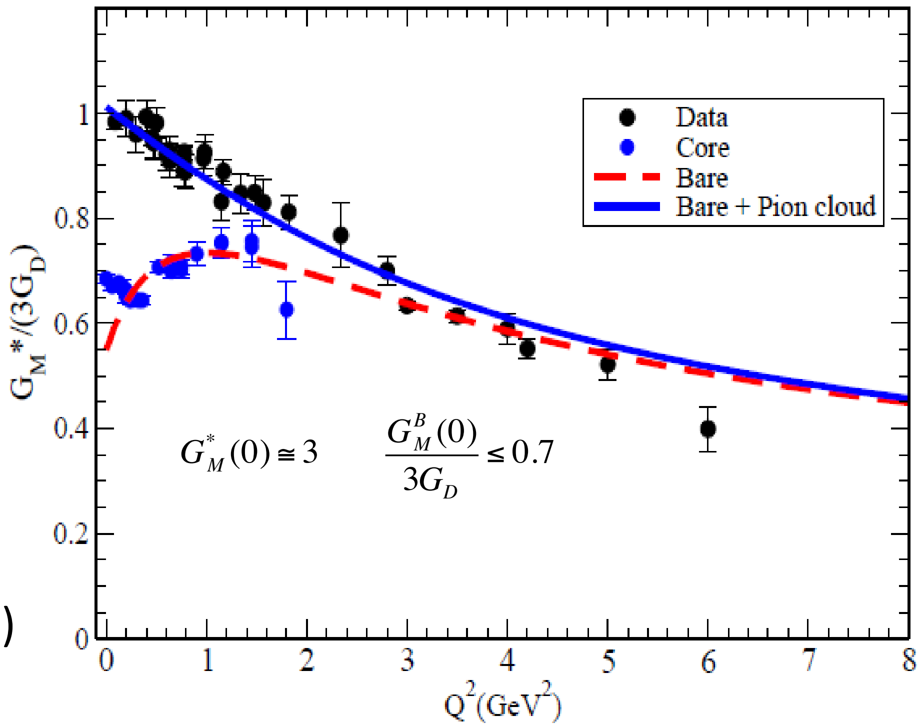
Is this separation supported by experiment?
Best way to determine bare quark core term?

Is this separation supported by experiment?

$\gamma N \to \Delta$

GR and MT Peña PRD 80, 013008 (2009)

- **Bare quark core** dominates large Q^2 region
- **Bare quark core results agree with EBAC analysis** : bare quark contributions extracted from the data (meson cloud effects subtracted)

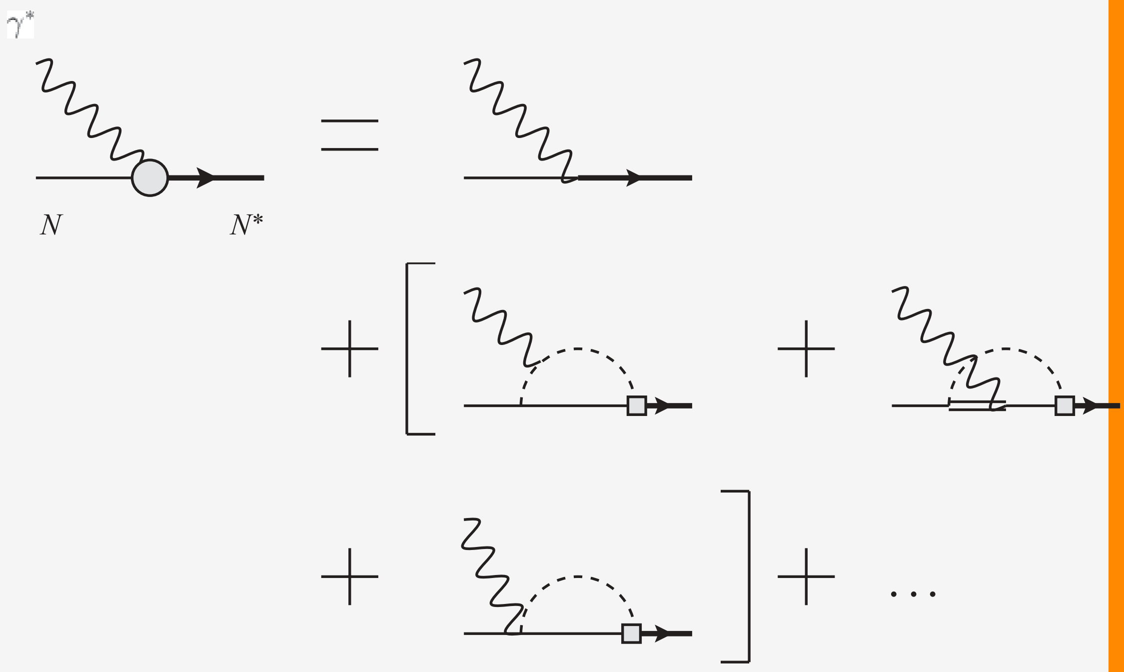


EBAC: Diaz et al., PRC 75, 015205 (2007)

- Bare ≈ Sato-Lee model

$$G_M^\pi = \lambda_\pi \left(\frac{\Lambda_\pi^2}{\Lambda_\pi^2 + Q^2}\right)^2 (3G_D)$$

G. Ramalho, M. T. P. and Franz Gross, PHYSICAL REVIEW D 78, 114017 (2008)



EBAC

Data analysis

Coupled channel dynamical model

$\gamma^* N$, πN, ηN, and $\pi\pi N$ that has $\pi\Delta$, ρN,

and σN components

N-Δ transition (G_M^*)

- Magnetic dipole FF

$$G_M^*(Q^2) = \frac{4}{3\sqrt{3}} \frac{M}{M + M_\Delta} j_- \int \phi_\Delta \phi_N = \overbrace{2.07 \int \phi_\Delta \phi_N}^{Q^2=0}$$

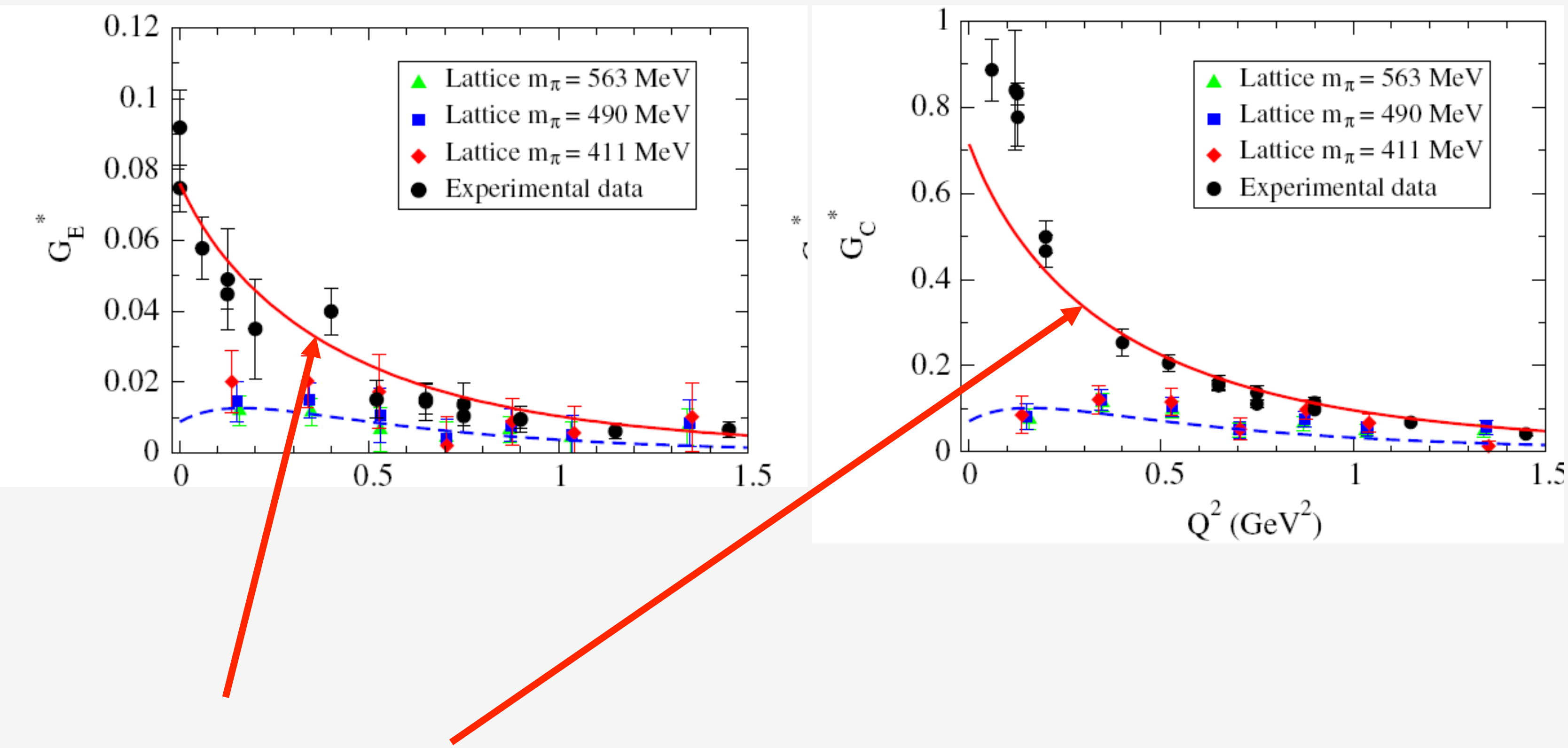
- Cauchy-Schwartz inequality (for $Q^2 = 0$):

$$\int \phi_\Delta \phi_N \leq \sqrt{\int \phi_N^2} \sqrt{\int \phi_\Delta^2} = 1$$

$\Rightarrow G_M^*(0) \leq 2.07$

$\gamma N \rightarrow \Delta$

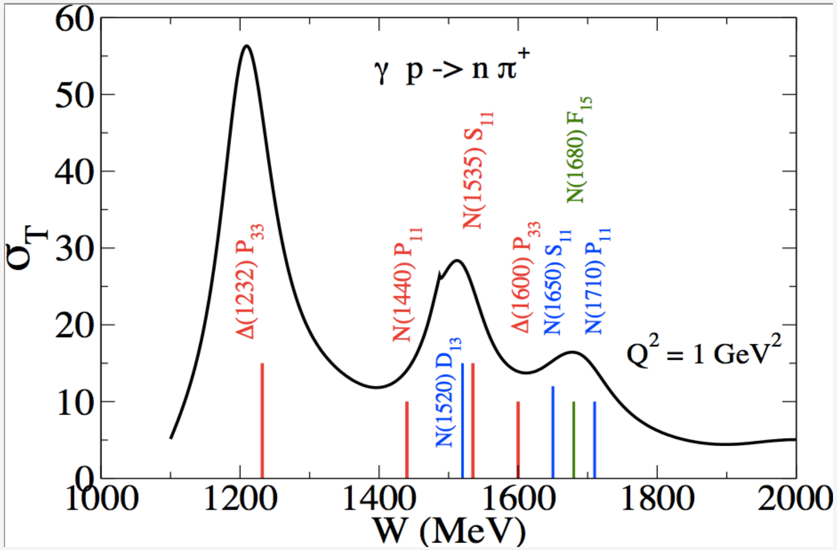
D3 0.72% and D1 0.72 % of the wavefunction



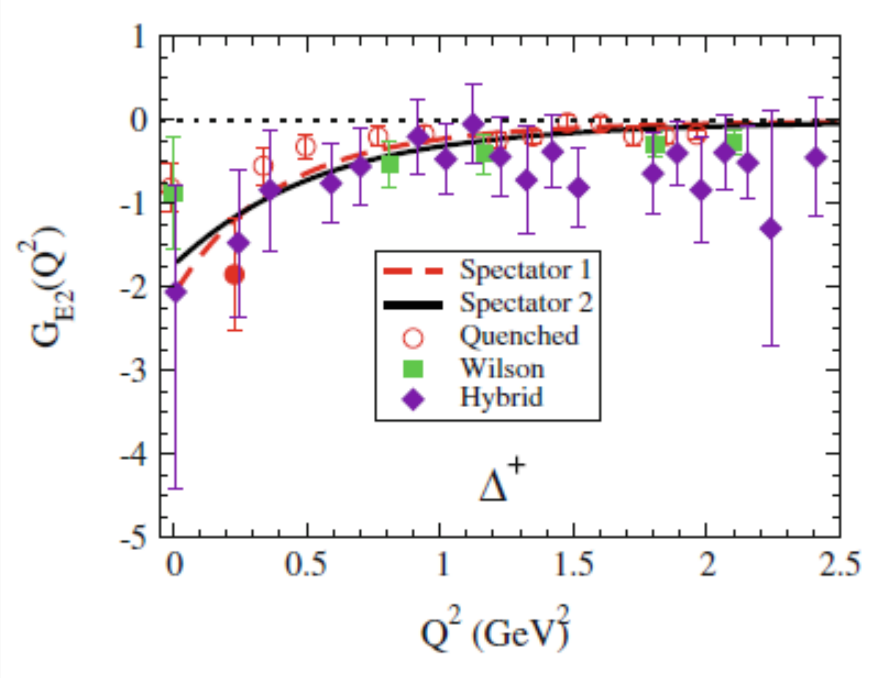
pion cloud : large N_c limit relations Pascalutsa and Vanderhaeghen, PRD76 111501(R) (2007)

G. Ramalho, M. T. P. and Franz Gross, PHYSICAL REVIEW D 80, 013008 (2009)

Predictions



$\gamma\Delta \to \Delta$



LQCD data: C. Alexandrou et al. Phys. Rev.D 79 014507 (2009);

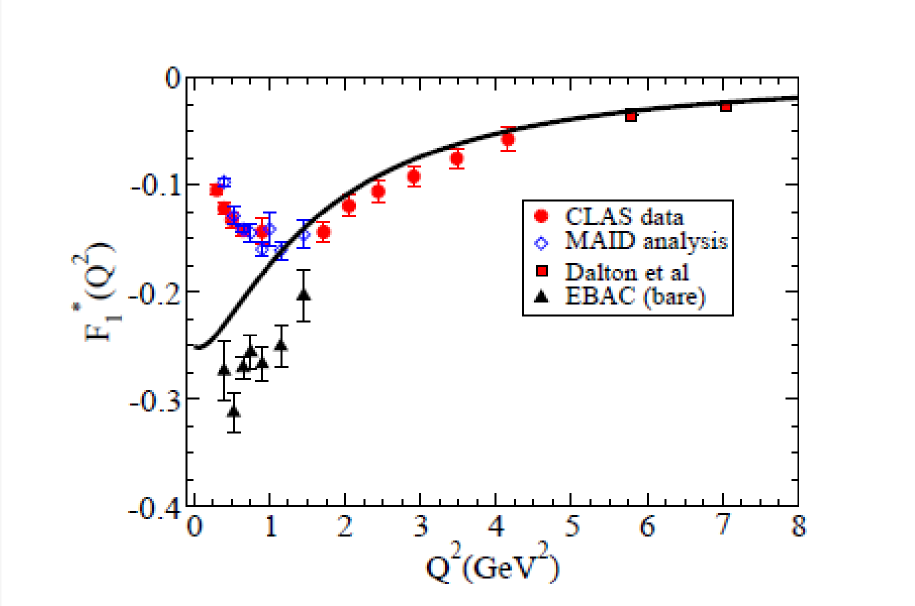
Nucl. Phys. A 825, 115 (2009);

S. Boinepalli et al Phys. Rev. D 80 054505 (2009).

G. Ramalho, M. T. P. and Franz Gross, Physics Letters B 678 (2009) 355–358

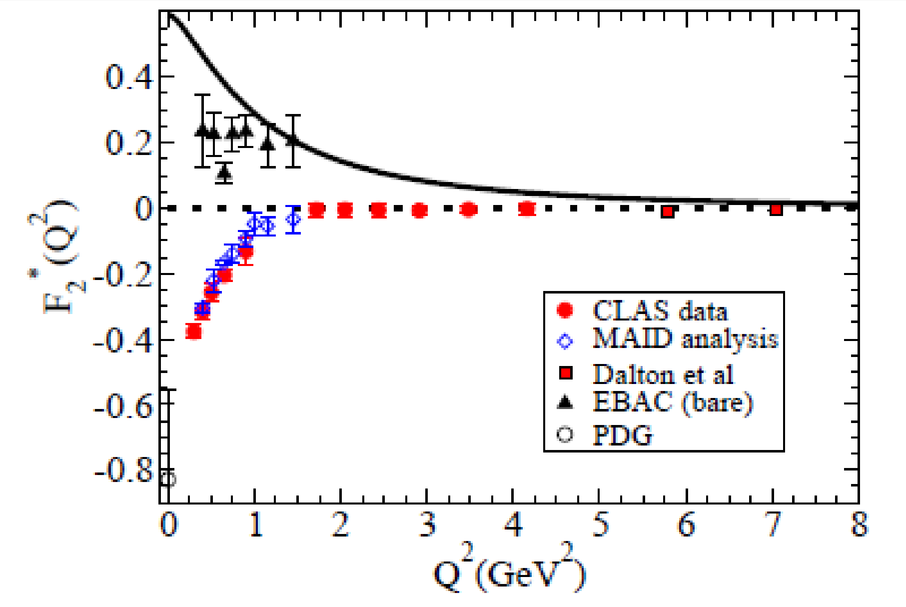
$$N \to N^*(1535)$$

- radial wf identical to nucleon's; angular momentum different (P wave)



- **EBAC (bare)**: bare contributions extracted from the data (meson cloud effects subtracted)

bare quark contribution close to **EBAC** analysis

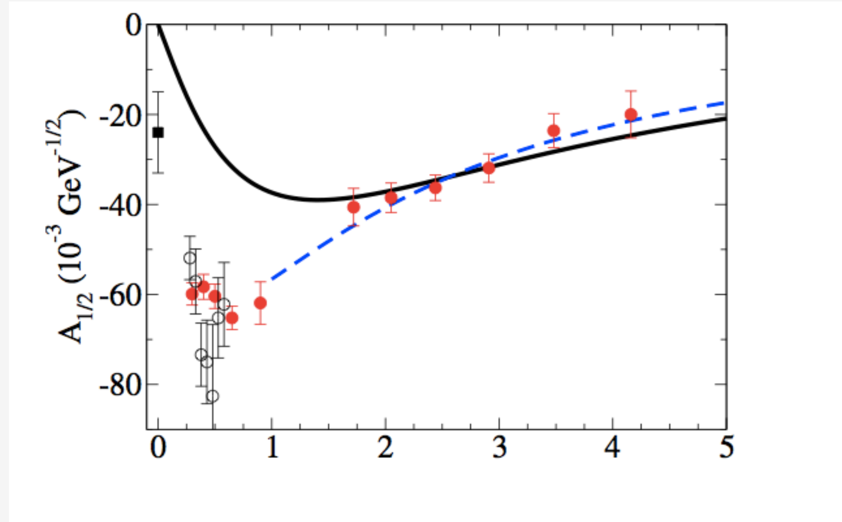


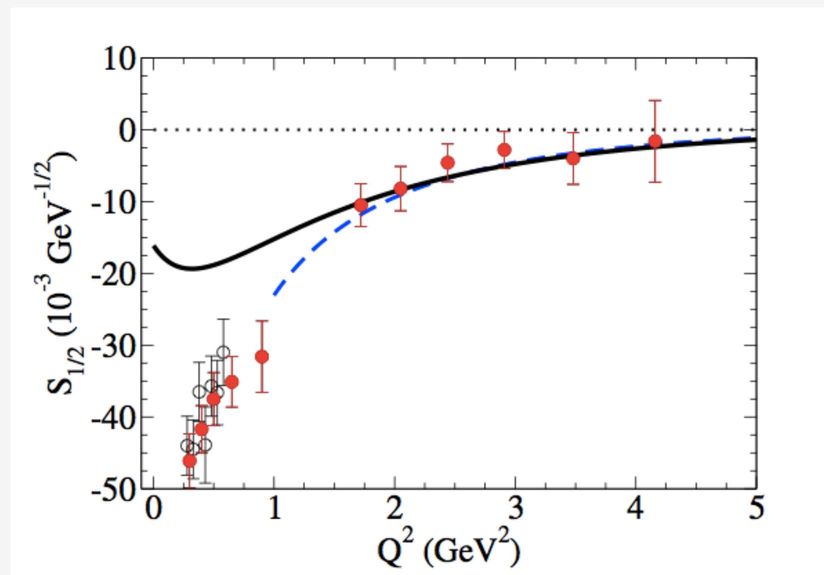
- Meson cloud effects of opposite sign; and above 2 GeV^2 still very important.

G. Ramalho, M. T. P. PHYSICAL REVIEW D 84, 051301(R) (2011)

$N \rightarrow N^*(1520)$

- Radial wf identical to nucleon's; angular momentum different (P wave)
- Good description of high Q^2 region behavior
- Orthogonality through extra term
- One parameter fit to the data for $Q^2 > 1.5$ GeV²





Meson Cloud

G_1, G_4, G_c

$$A_{3/2} = 2\sqrt{3}\mathcal{A}G_4,$$
$$A_{1/2} = 2\mathcal{A}\left\{G_4 - \left[(M_R - M)^2 + Q^2\right]\frac{G_1}{M_R}\right\}$$
$$S_{1/2} = -\frac{1}{\sqrt{2}}\frac{|\mathbf{q}|}{M_R}\mathcal{A}g_C,$$

- **$S_{1/2}$** → meson cloud term to **G_c** is extracted
- **$A_{3/2}$** → meson cloud term to **G_4** is extracted.

- **$A_{1/2}$ mixes** meson contributions to the different form factors (Aznauryan and Burkert, PRC 85 055202 2012)

- **A global fit** of the three amplitudes, indirectly constraining $A_{3/2}$ by $A_{1/2}$, is needed.

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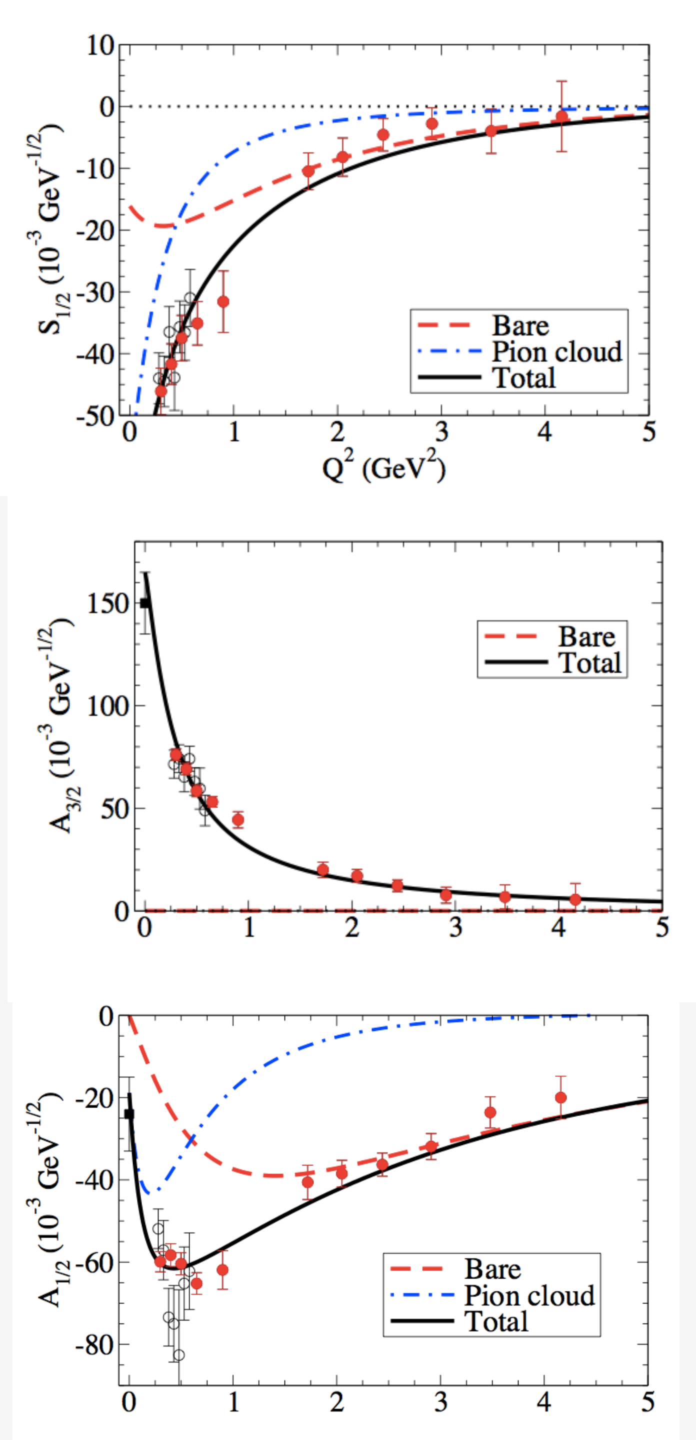


In the timelike region

Delta Dalitz Decay width

F. Dohrmann et al. ERJA 45 401 2010

$$\Gamma_{\gamma^* N}(q;W) = \frac{\alpha}{16}\frac{(W+M)^2}{M^2W^3}\sqrt{y_+y_-}y_-|G_T(q^2,W)|^2$$

$$|G_T(q^2;M_\Delta)|^2 = |G_M^*(q^2;W)|^2 + 3|G_E^*(q^2;W)|^2 + \frac{q^2}{2W^2}|G_C^*(q^2;W)|^2$$

$$\Gamma_{\gamma N}(W) \equiv \Gamma_{\gamma^* N}(0;W)$$

$$\Gamma_{e^+e^-N}(W) = \frac{2\alpha}{3\pi}\int_{2m_e}^{W-M}\Gamma_{\gamma^* N}(q;W)\frac{dq}{q}$$

Running Delta Mass W that may differ from the pole mass

Extension to timelike region

VMD in quark-core current:

$$\frac{m_v^2}{m_v^2 - q^2} \rightarrow \frac{m_\rho^2}{m_\rho^2 - q^2 - i m_\rho \Gamma_\rho}$$

$$\rightarrow \frac{m_\rho^2 \left[(m_\rho^2 - q^2) + i m_\rho \Gamma_\rho \right]}{(m_\rho^2 - q^2)^2 + m_\rho^2 \Gamma_\rho^2}.$$

$$\Gamma_\rho(q^2) = \Gamma_\rho^0 \left(\frac{q^2 - 4m_\pi^2}{m_\rho^2 - 4m_\pi^2} \right)^{3/2} \frac{m_\rho}{q} \theta(q^2 - 4m_\pi^2)$$

H. B. O' Connell, B.C Pearce, A.W. Thomas, A.G Williams, PLB 354 14 (1995)

Model 1 pion CLOUD

$$G_M^\pi(Q^2;W) = 3\lambda_\pi G_D(Q^2)\left(\frac{\Lambda_\pi^2}{\Lambda_\pi^2+Q^2}\right)^2$$

$$G_D(q^2) = \left(\frac{\Lambda_D^2}{\Lambda_D^2-q^2}\right)^2$$

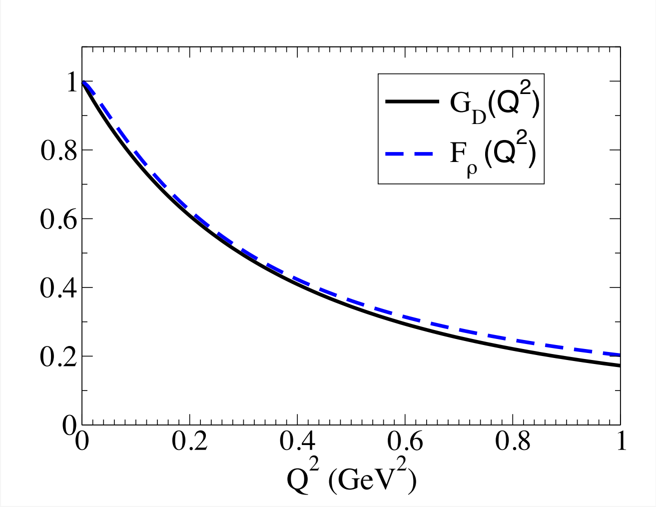
$$G_D(q^2) \;\to\; \left[\frac{\Lambda_D^2}{(\Lambda_D^2-q^2)^2+\Lambda_D^2\Gamma_D^2}\right]^2 \times$$

$$\left[(\Lambda_D^2-q^2)^2-\Lambda_D^2\Gamma_D^2+i2(\Lambda_D^2-q^2)\Lambda_D\Gamma_D\right]$$

Model 2 pion CLOUD

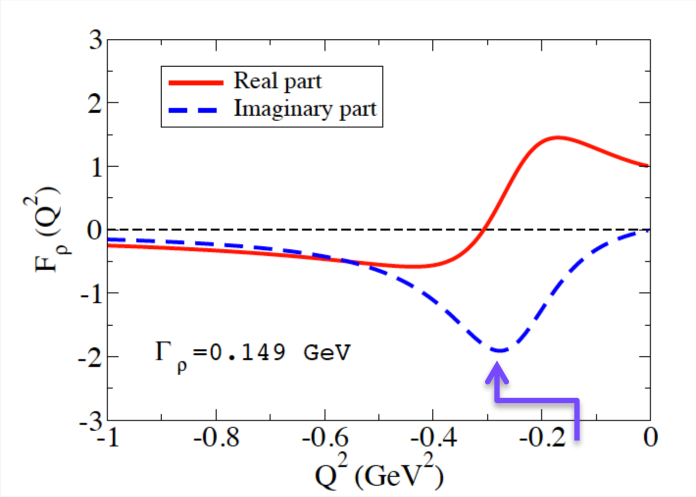
Inspiration: F. Iachello, A.D. Jackson, and Landé, PL 43, 191 (1973
F. Dohrman et al, Eur. Phys. J. A45, 401, (2010)

$$G_M^\pi(Q^2;W) = 3\lambda_\pi G_D(Q^2)\left(\frac{\Lambda_\pi^2}{\Lambda_\pi^2+Q^2}\right)^2$$

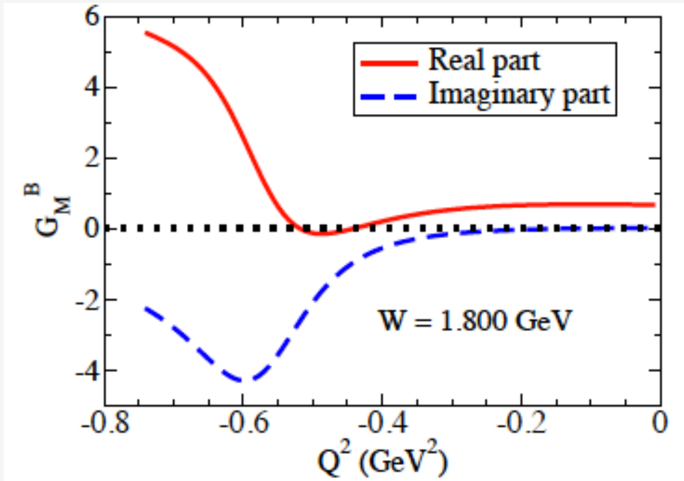


$$G_M^\pi(Q^2) = 3\lambda_\pi F_\rho(q^2)\left(\frac{\Lambda_\pi^2}{\Lambda_\pi^2-q^2}\right)^2$$

$$F_\rho(q^2) = \frac{m_\rho^2}{m_\rho^2 - q^2 - \frac{1}{\pi}\frac{\Gamma_\rho^0}{m_\pi}q^2\log\frac{q^2}{m_\pi^2} + i\frac{\Gamma_\rho^0}{m_\pi}q^2},$$

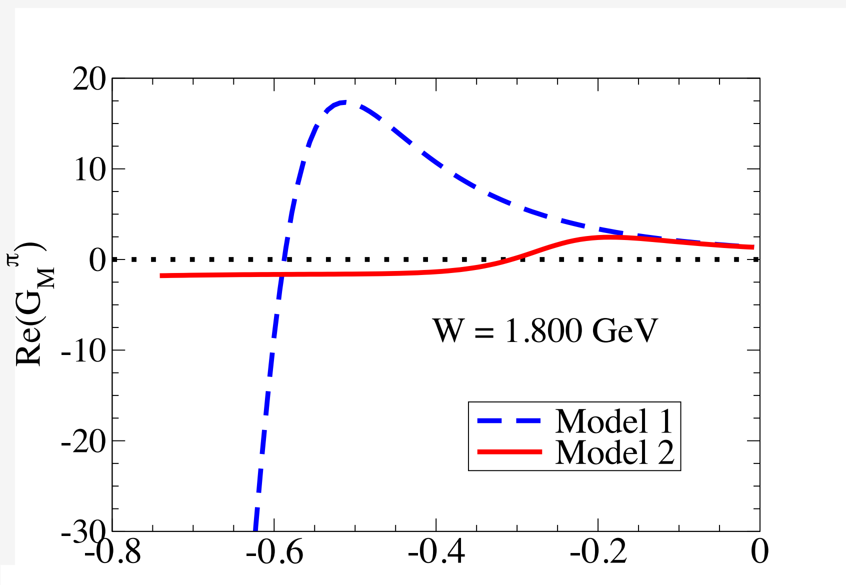


$$Q^2 \geq -(W-M)^2$$

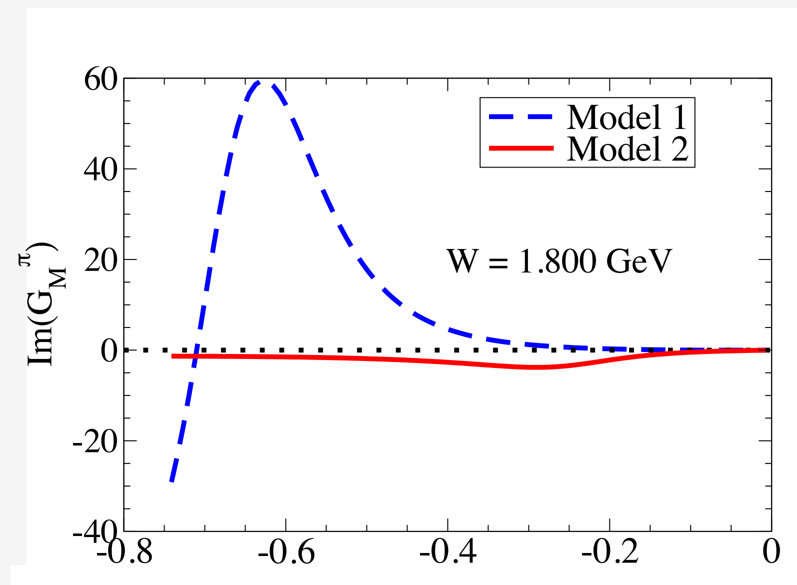


Real part
Imaginary part
W = 1.800 GeV
G_M^B
Q^2 (GeV^2)
6
4
2
0
-2
-4
-0.8
-0.6
-0.4
-0.2
0

+



$Re(G_M^\pi)$
W = 1.800 GeV
Model 1
Model 2
20
10
0
-10
-20
-30
-0.8
-0.6
-0.4
-0.2
0



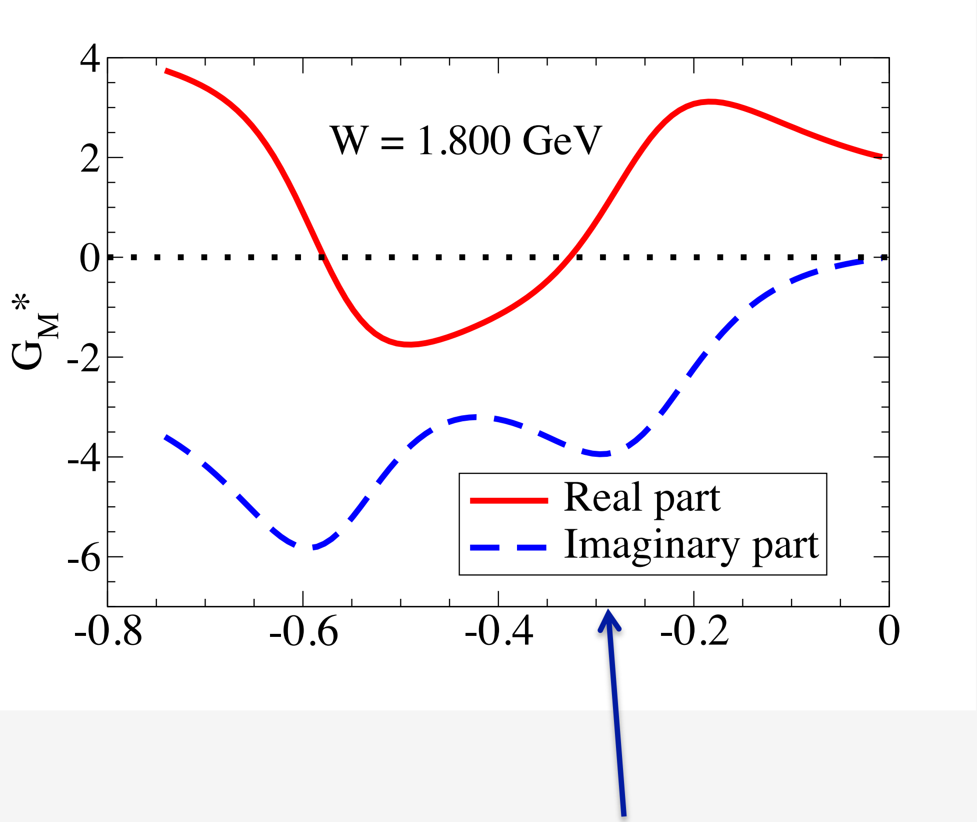
$Im(G_M^\pi)$
Model 1
Model 2
W = 1.800 GeV
60
40
20
0
-20
-40
-0.8
-0.6
-0.4
-0.2
0

=

?

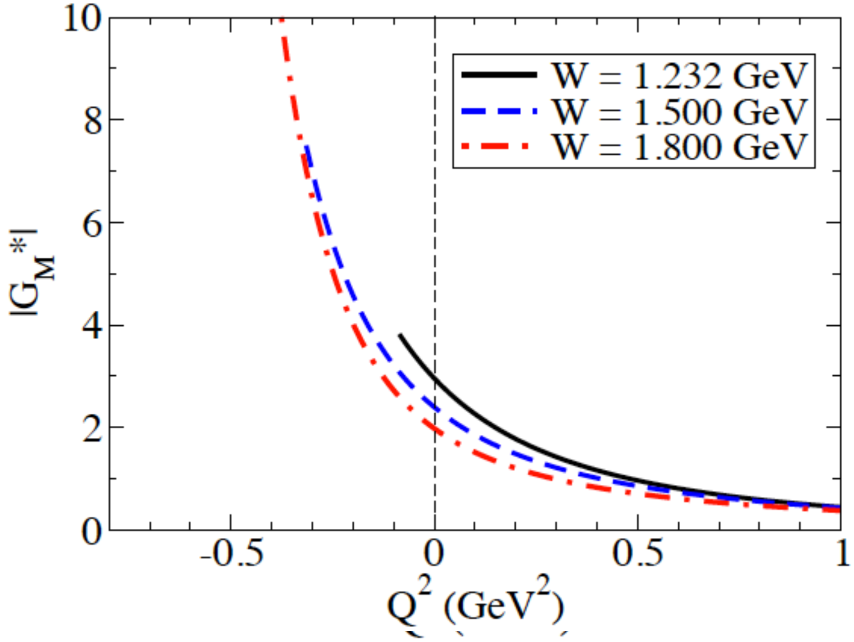
$$G_M^* = G_M^B + G_M^\pi$$

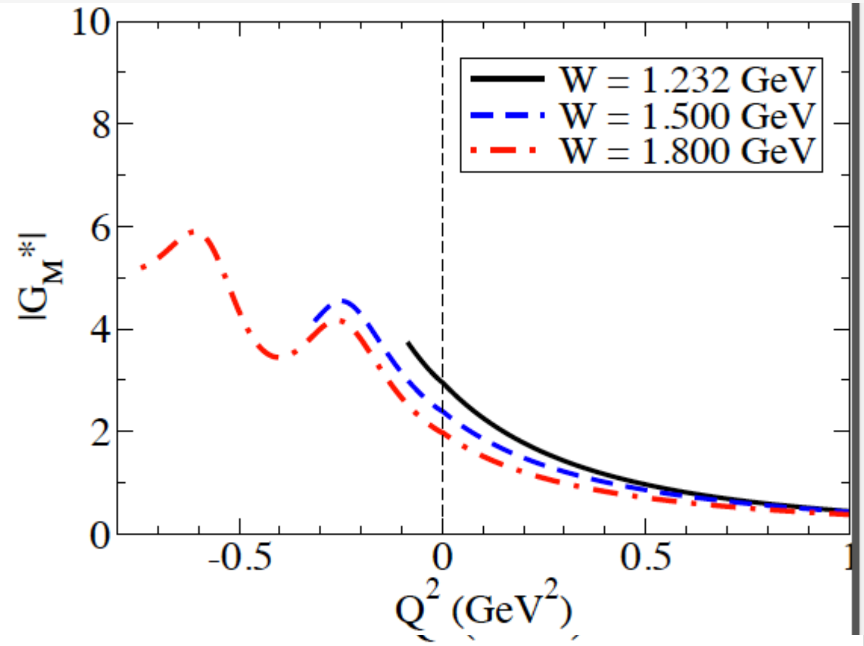
$$Q^2 \geq -(W - M)^2$$



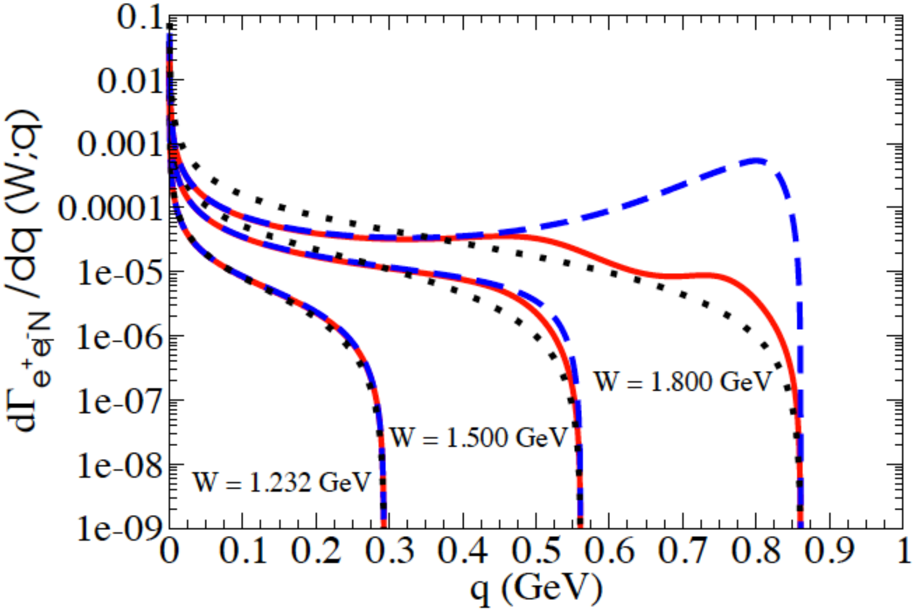
Model 1

Model 2





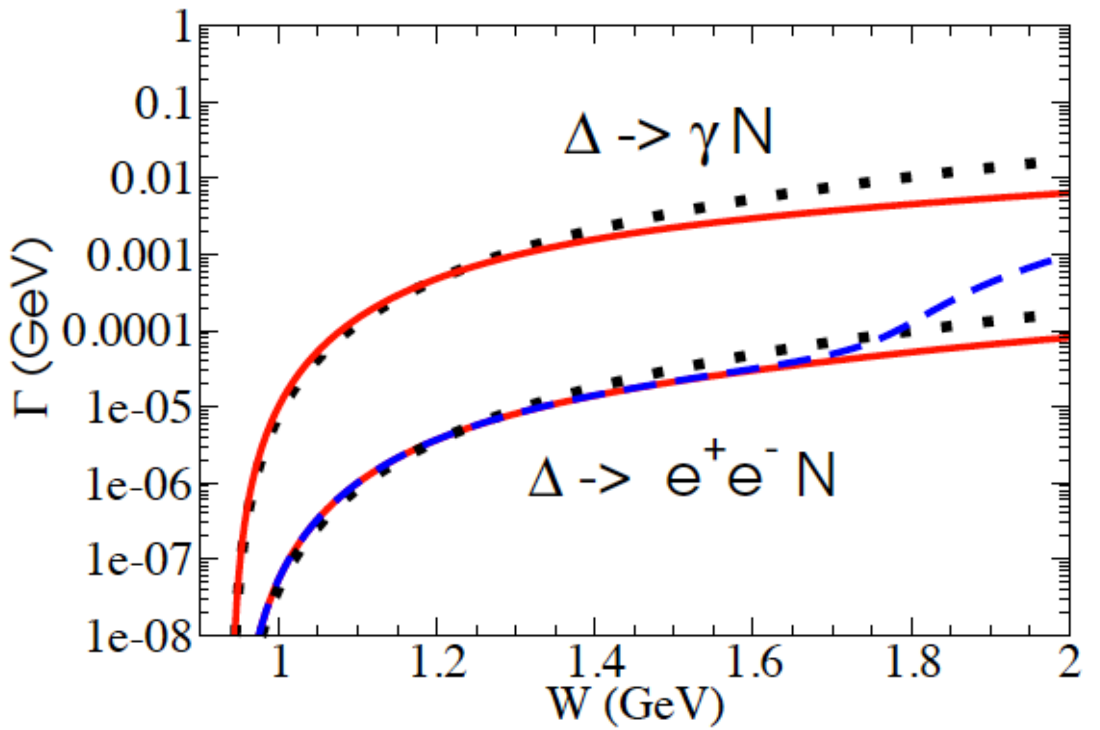
$$Q^2 \geq -(W - M)^2$$



— — — Model 1; —— Model 2; · · · const $G_M^*(q^2;W) \equiv G_M^*(0, M_\Delta)$

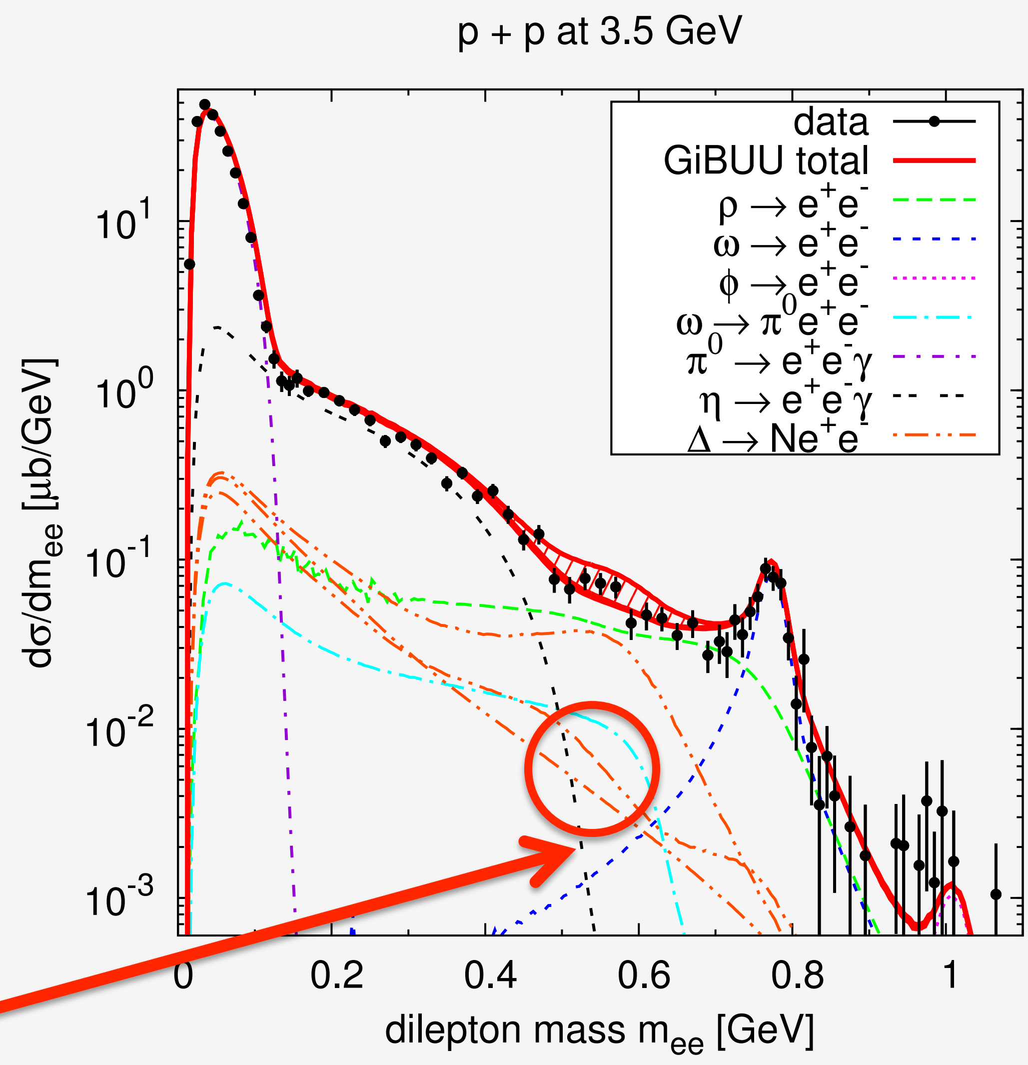
$$g_{\Delta}(W) \approx \frac{W^2 \Gamma_{tot}(W)}{(W^2 - M_{\Delta}^2)^2 + W^2 \left[\Gamma_{tot}(W)\right]^2}$$

$$\Gamma_{tot}(W) = \Gamma_{\pi N}(W) + \Gamma_{\gamma N}(W) + \Gamma_{e^+e^- N}(W)$$



In the timelike region

Courtesy Janus Weil
Giessen



G. Ramalho, M. T. P. , PHYSICAL REVIEW D 85 113014 (2012)

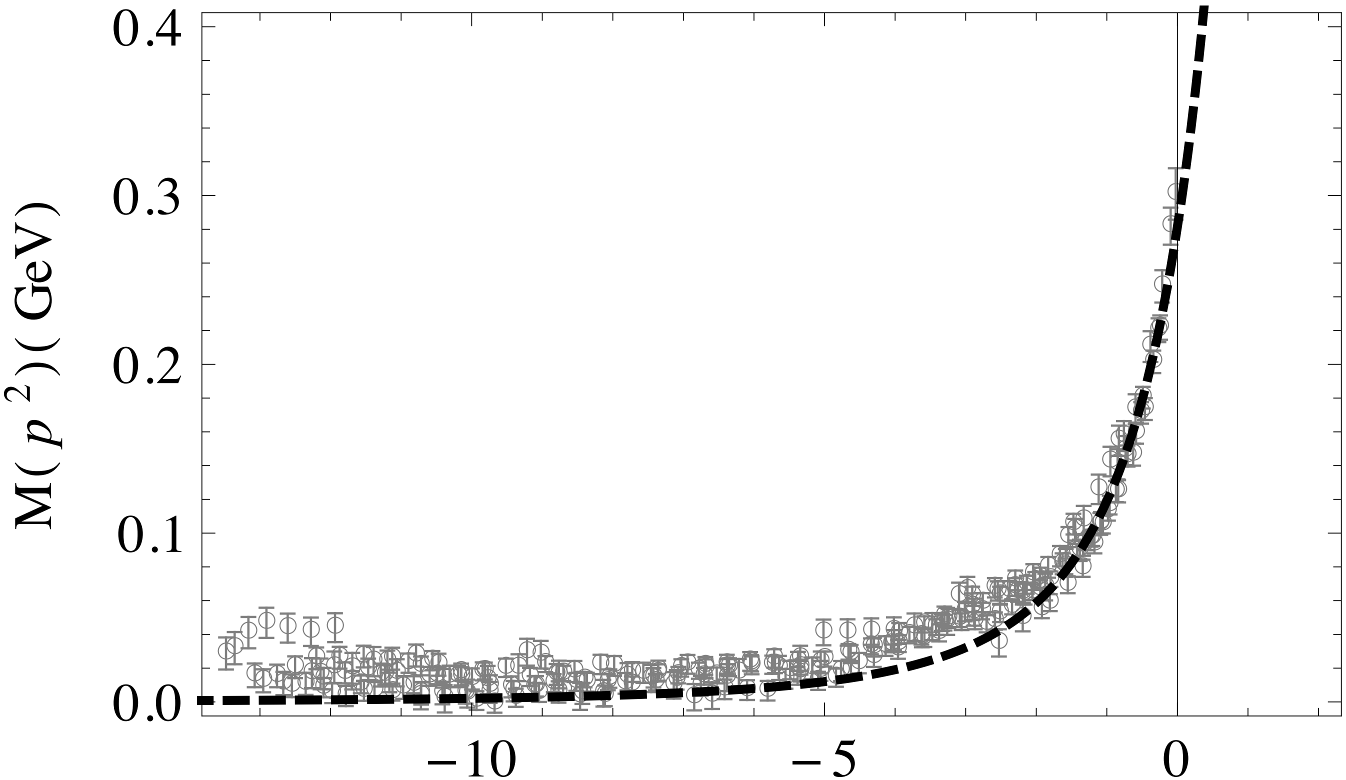
To understand the pion cloud:

Pion form factor

Connection to LQCD

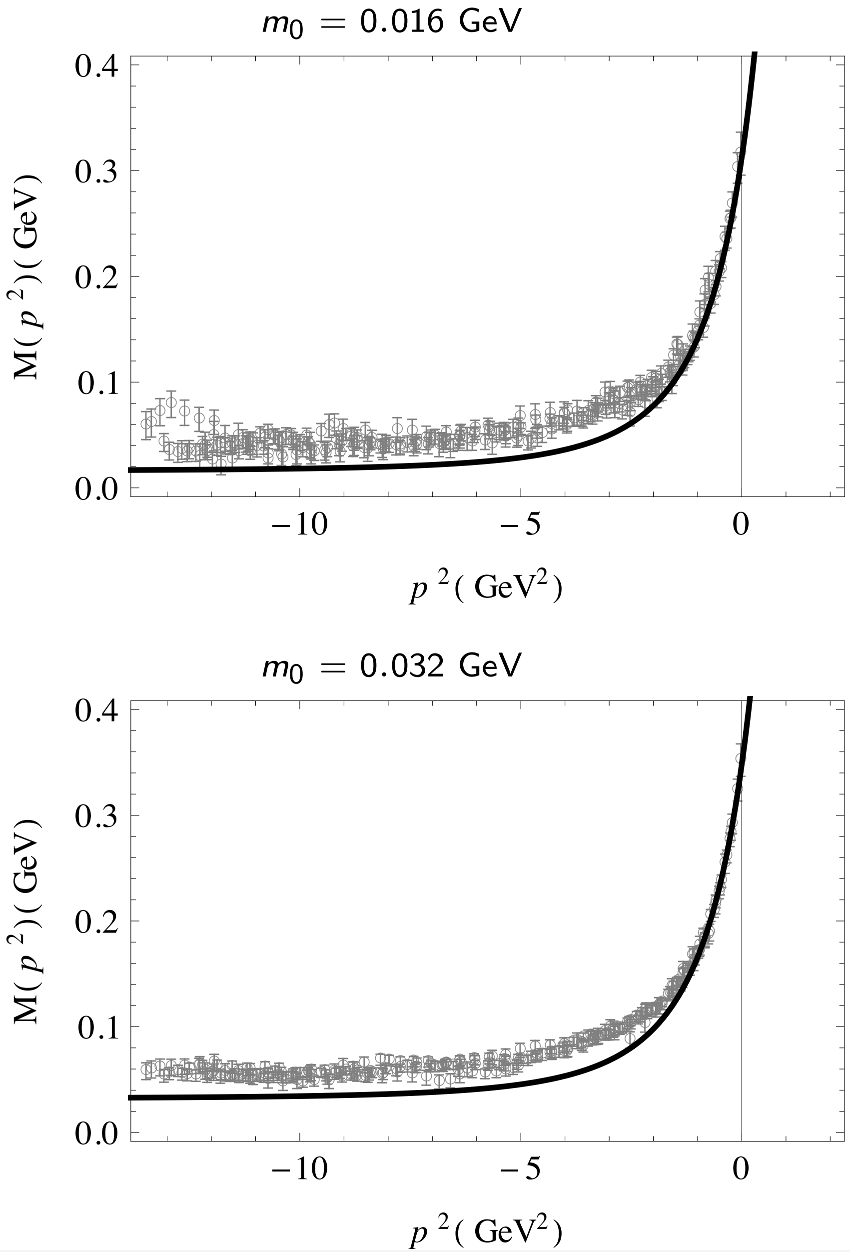
Mass function

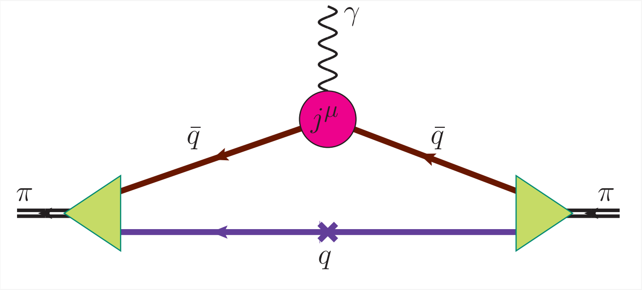
Lattice QCD data from Bowman et al., PRD, 71, 2005 extrapolated to chiral limit

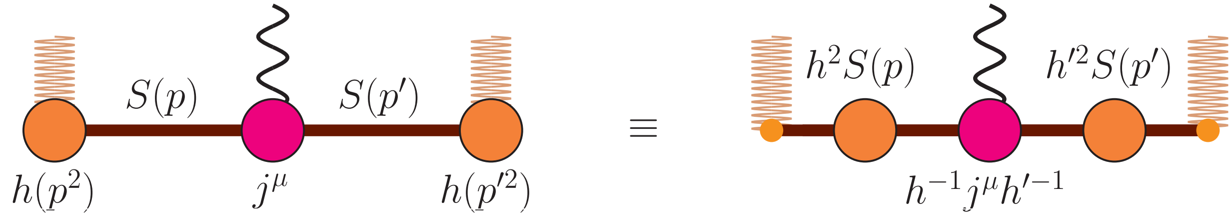


Predictions

Lattice QCD data: [Bowman et al., PRD, 71, 2005]



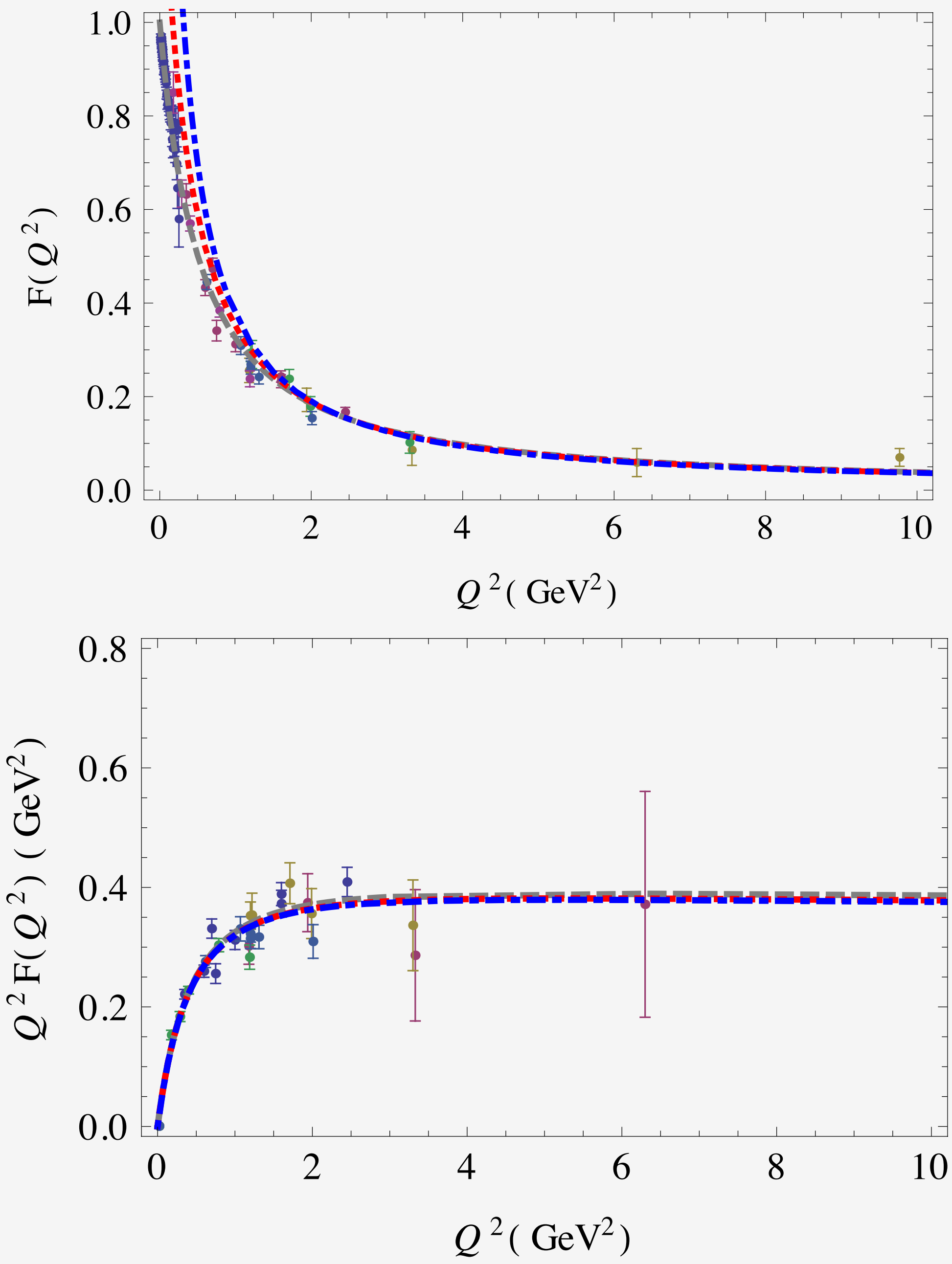




(reduced) off-shell quark current

$j_R^\mu = f(\gamma^\mu + \kappa \frac{i\sigma^{\mu\nu}q_\nu}{2m}) + \delta'\Lambda'\gamma^\mu + \delta\gamma^\mu\Lambda + g\Lambda'\gamma^\mu\Lambda$

$\Lambda^{(\prime)} = \frac{M(p^{(\prime)}) - \not{p}^{(\prime)}}{2M(p^{(\prime)})}$; f, $\delta^{(\prime)}$, g chosen such that j_R^μ satisfies Ward-Takahashi identity



F(Q^2)
Q^2 (GeV^2)
Q^2 F(Q^2) (GeV^2)
Q^2 (GeV^2)

Summary

1 Spectator quark-diquark model : It is covariant and accomodates angular momentum description.

2 At $Q^2 \approx 0$ consistent with EBAC data analysis based on a coupled channel Dynamical Model, and also Large N_C limit.

3 At high Q^2 consistent with experimental data, and also LQCD in the large pion mass regime.

4 Several applications: **Δ(1232),** N*(1440), N*(1535), N*(1520), Δ(1600), strange sector, DIS.

5 **Dilepton mass spectrum sensitive to momentum dependence of G_M**

To understand the pion cloud is critical

This leads to us to calculate the **Pion form factor**

First results for of CST model in Minkowski space

with dynamical chiral symmetry breaking + covariant generalization of linear & constant vector potential with parameters fixed from Lattice data for mass function.

Pion Form factor independent of pion mass in the high Q^2 region (Chiral symmetry).

The two strategies, models and LQCD, are made **compatible.**

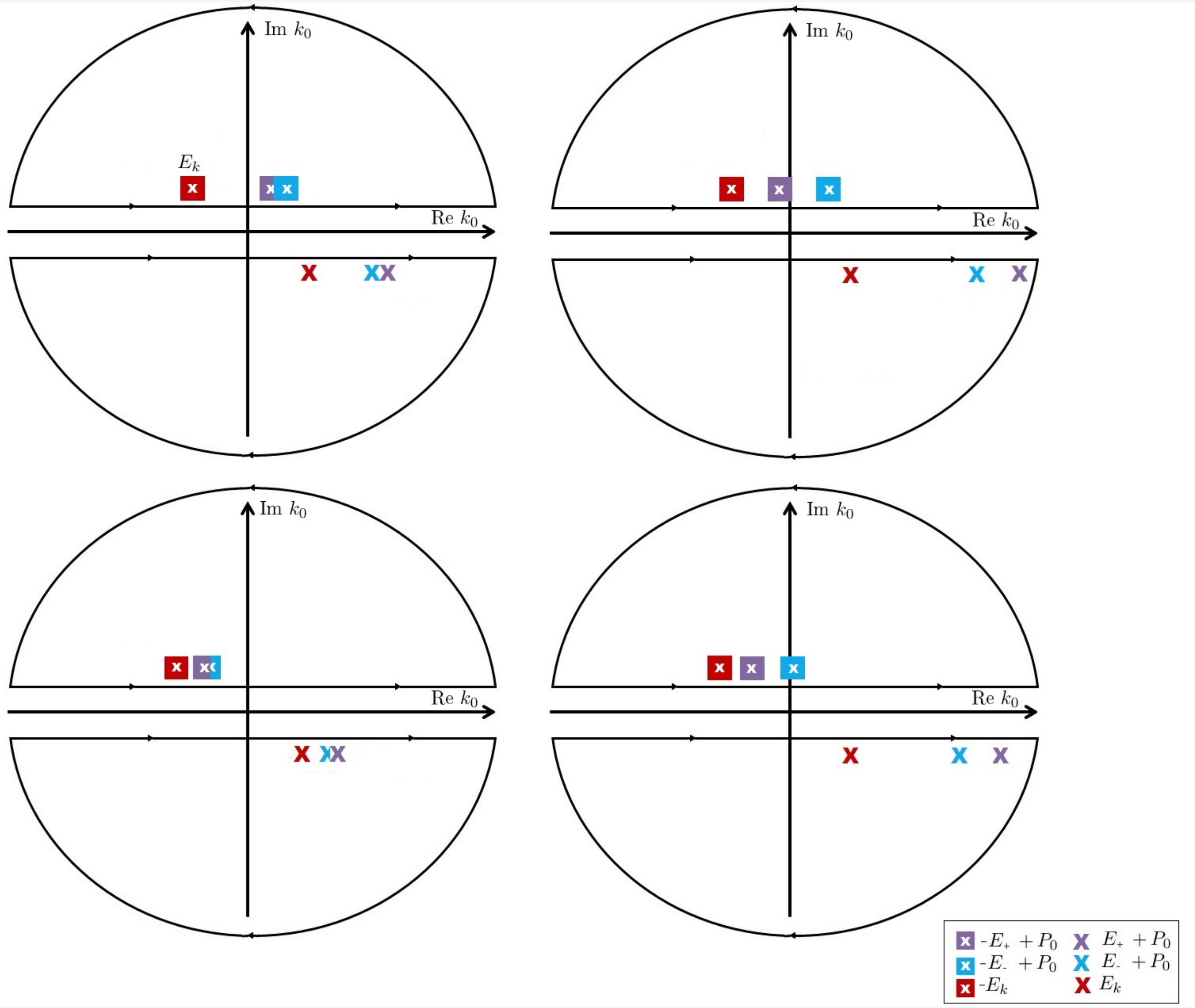
*


Garrett McNamara, Nazaré, Portugal
Photo by Miguel Costa
BILLABONG
XXL
GLOBAL BIG
WAVE AWARDS
not
The end

Small Q^2 Large Q^2

Large M

Small M



$$\psi_S(P,k) = \frac{N_0}{m_s(\beta_1+\chi)(\beta_2+\chi)},$$

where

$$\chi = \frac{(M-m_s)^2-(P-k)^2}{Mm_s} = \frac{2P\cdot k}{Mm_s} - 2$$

$$\begin{aligned}
\chi &= 2\sqrt{1+\frac{\mathbf{k}^2}{m_s^2}}\sqrt{1+\frac{\mathbf{P}^2}{M^2}} - 2\frac{\mathbf{k}\cdot\mathbf{P}}{Mm_s} - 2 \\
&\rightarrow \left(\frac{\mathbf{k}}{m_s}-\frac{\mathbf{P}}{M}\right)^2 \rightarrow \frac{1}{4m_q^2}\left(\mathbf{k}-\frac{2}{3}\mathbf{P}\right)^2 \\
&= \frac{1}{2m_q^2}\chi_{nr}(k,P,0),
\end{aligned}$$

N-Δ transition (G_M^*)

- Magnetic dipole FF

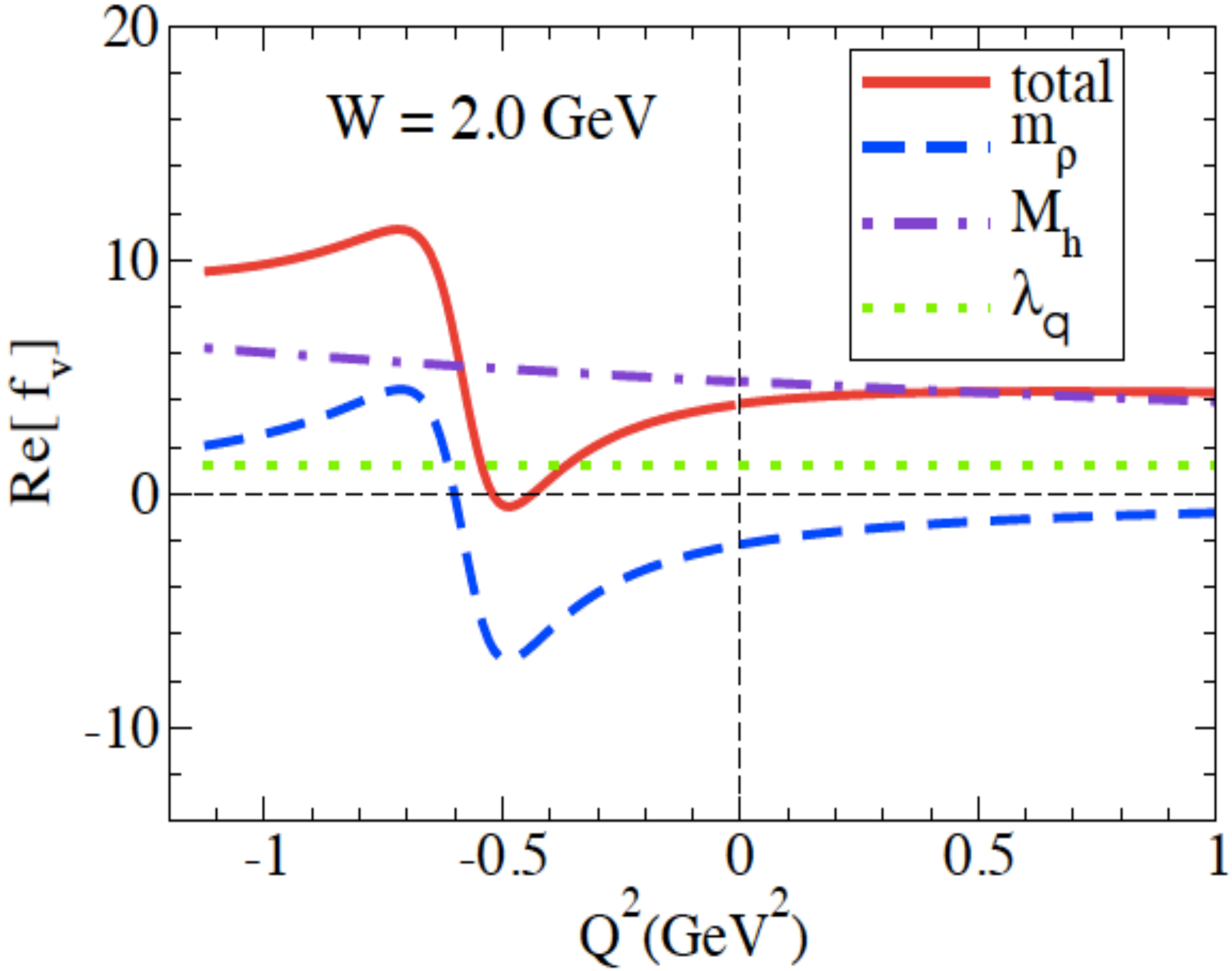
$$G_M^*(Q^2) = \frac{4}{3\sqrt{3}} \frac{M}{M + M_\Delta} j_- \int \phi_\Delta \phi_N = \overbrace{2.07 \int \phi_\Delta \phi_N}^{Q^2=0}$$

- Cauchy-Schwartz inequality (for $Q^2 = 0$):

$$\int \phi_\Delta \phi_N \leq \sqrt{\int \phi_N^2} \sqrt{\int \phi_\Delta^2} = 1$$

$\Rightarrow G_M^*(0) \leq 2.07$

$$G_M^B(Q^2) = \frac{8}{3\sqrt{3}} \frac{M}{M+W} f_v \int_k \psi_\Delta \psi_N \qquad f_v = f_{1-} + \frac{W+M}{2M} f_{2-}$$



Extension to timelike region

Valence quark model applied for $q^2 = -Q^2$ and $M_\Delta \to W$;

$$\frac{m_\rho^2}{m_\rho^2 - q^2} \to \frac{m_\rho^2}{m_\rho^2 - q^2 - i m_\rho \Gamma_\rho}$$

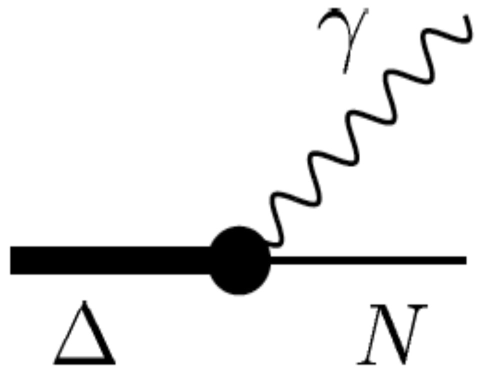
Include ρ–width (2π cut):

$$\Gamma_\rho(q^2) = \Gamma_\rho^0 \left(\frac{q^2 - 4m_\pi^2}{m_\rho^2 - 4m_\pi^2} \right)^{3/2} \frac{m_\rho}{q} \theta(q^2 - 4m_\pi^2)$$

Pion cloud?

$$G_M^\pi(q^2) = \lambda_\pi (3 G_D) \left(\frac{\Lambda_\pi^2}{\Lambda_\pi^2 - q^2} \right)^2$$

$$G_D(q^2) = \left(\frac{\Lambda_N^2}{\Lambda_N^2 - q^2 - i\Gamma_N \Lambda_N} \right)^2, \qquad \Gamma_N \equiv \Gamma_\rho$$



Δ rest frame

$$P_\Delta = (W, 0, 0, 0); \quad P_N = (E_N, 0, 0, -|\mathbf{q}|); \quad q = (\omega, 0, 0, |\mathbf{q}|)$$

Timelike $q^2 > 0$

$$\omega = \frac{W^2 - M^2 + q^2}{2W}$$

$$|\mathbf{q}|^2 = \frac{[(W+M) - q^2][(W-M)^2 - q^2]}{4W^2}$$

$$E_N = \frac{W^2 + M^2 - q^2}{2W}$$

Spacelike $Q^2 > 0$

$$\omega = \frac{W^2 - M^2 - Q^2}{2W}$$

$$|\mathbf{q}|^2 = \frac{[(W+M) + Q^2][(W-M)^2 + Q^2]}{4W^2}$$

$$E_N = \frac{W^2 + M^2 + Q^2}{2W}$$

TL: $q^2 \leq (W - M)^2$

Extension to timelike region $Q^2 < 0$

Valence quark model applied for $q^2 = -Q^2$ and $M_\Delta \to W$;

$$\frac{m_\rho^2}{m_\rho^2 - q^2} \to \frac{m_\rho^2}{m_\rho^2 - q^2 - i m_\rho \Gamma_\rho}$$

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Pion cloud?

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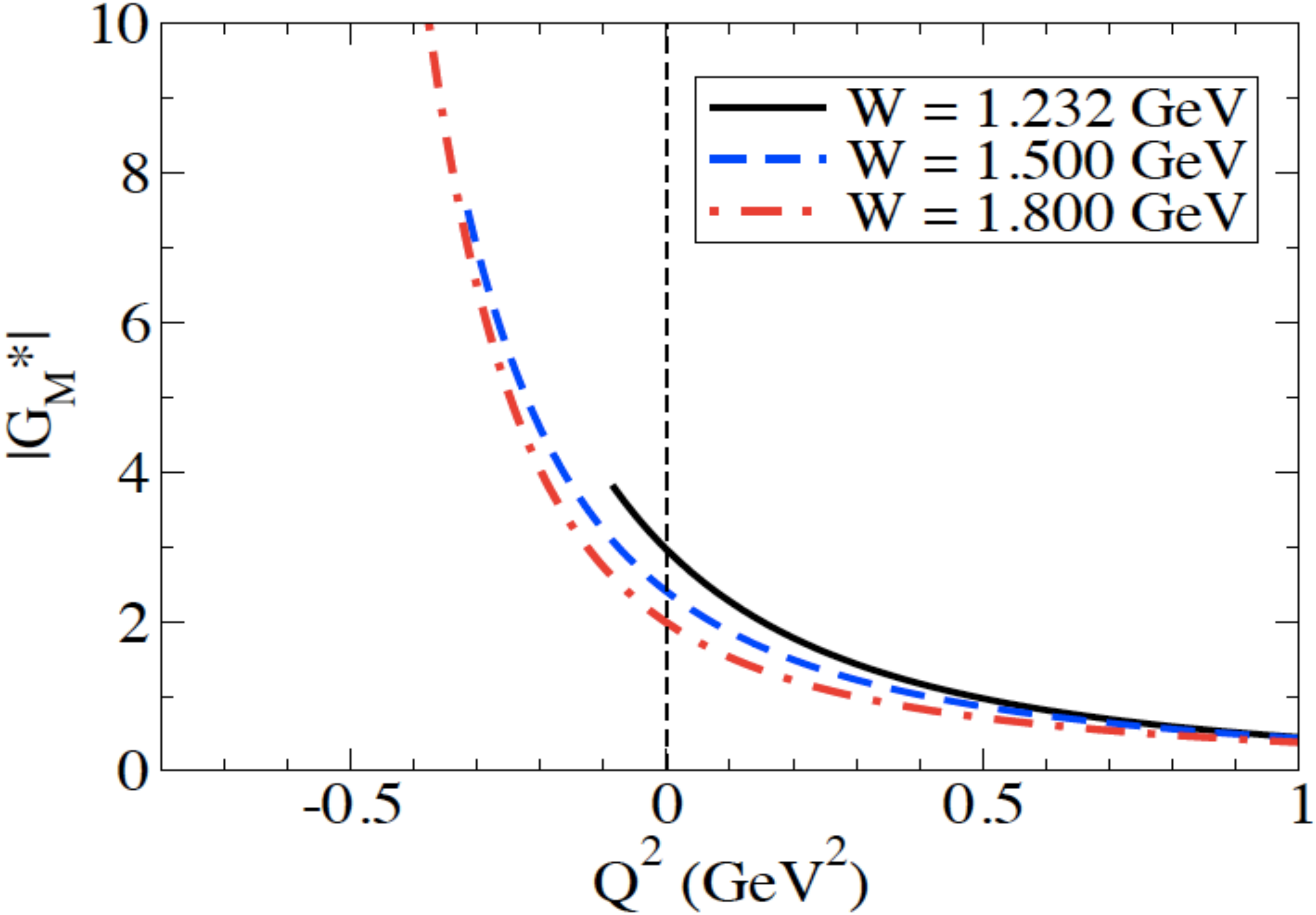
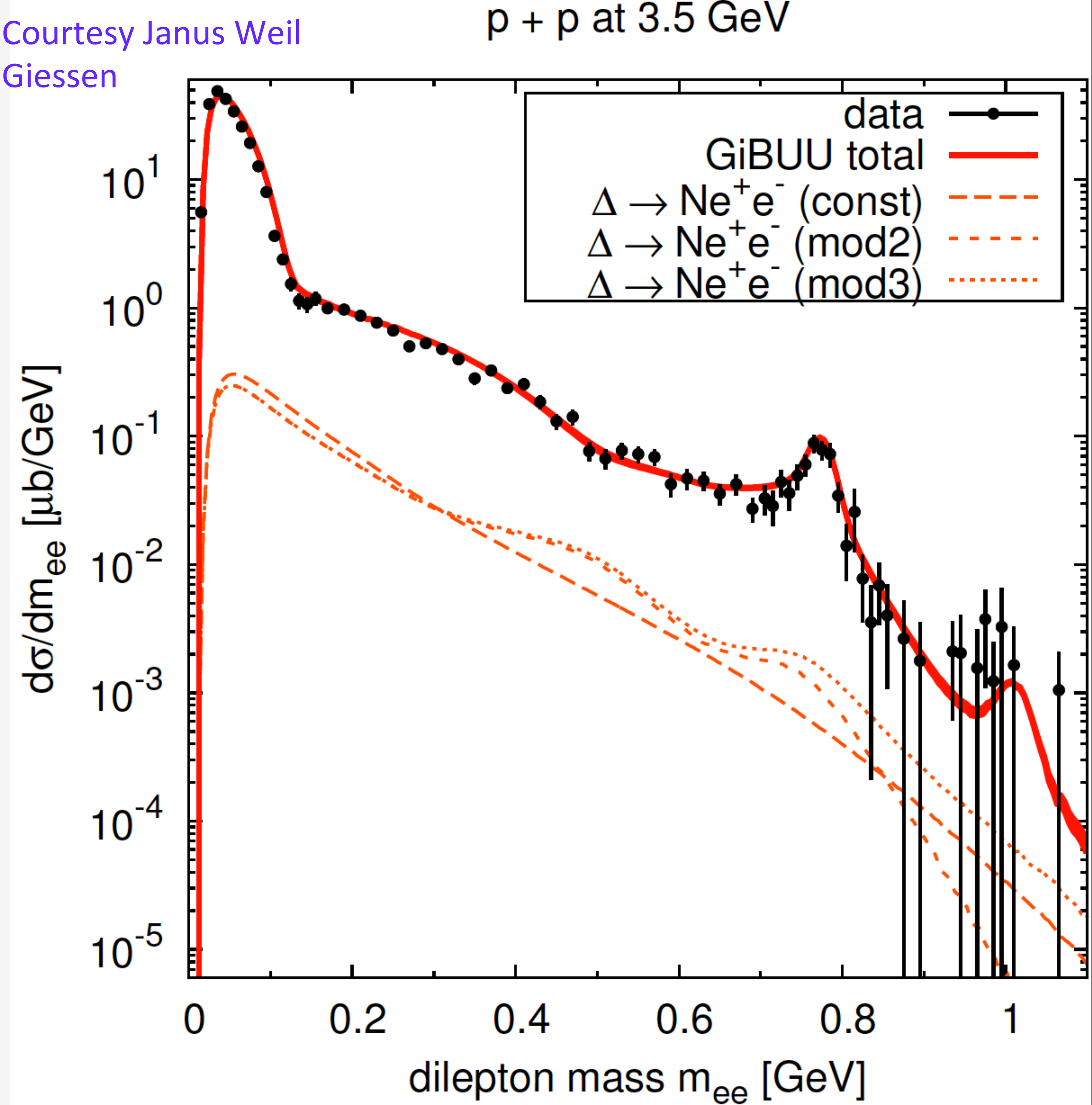


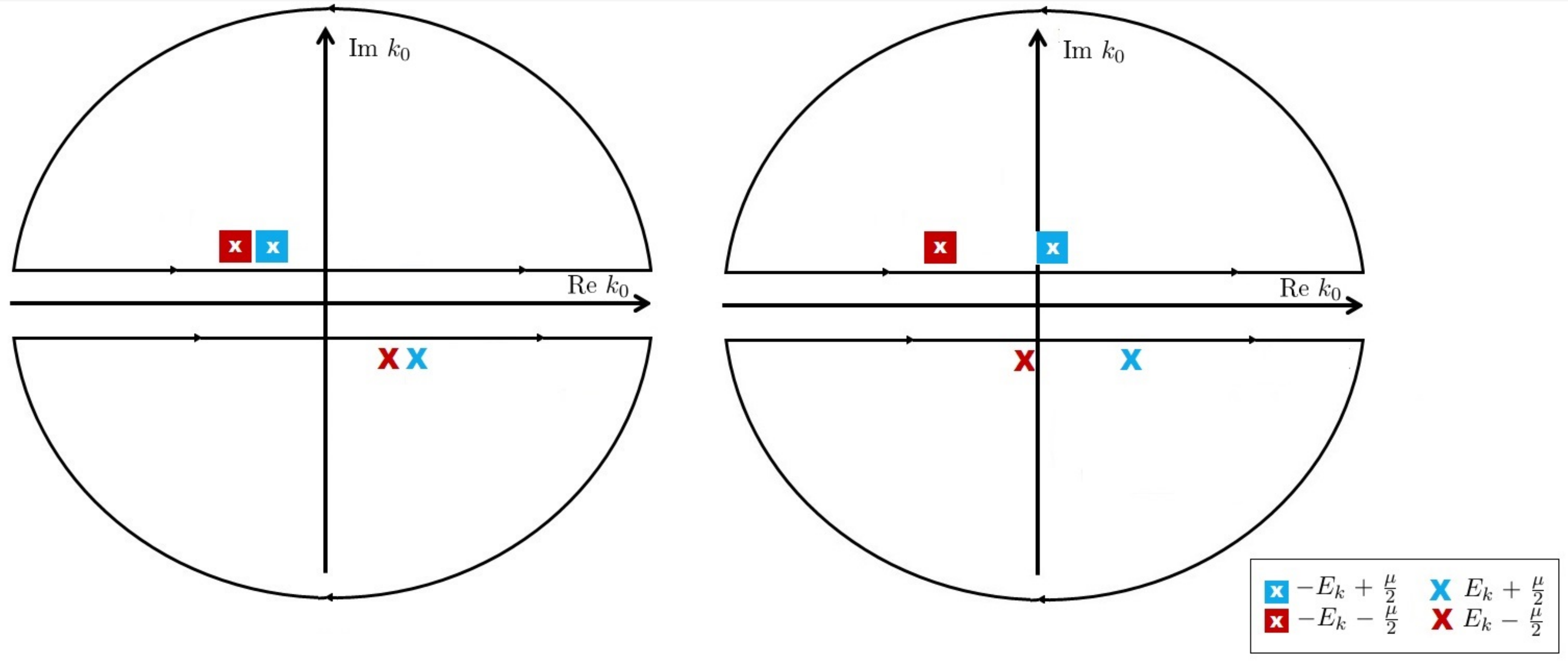
$$Q^2 \geq -(W-M)^2$$

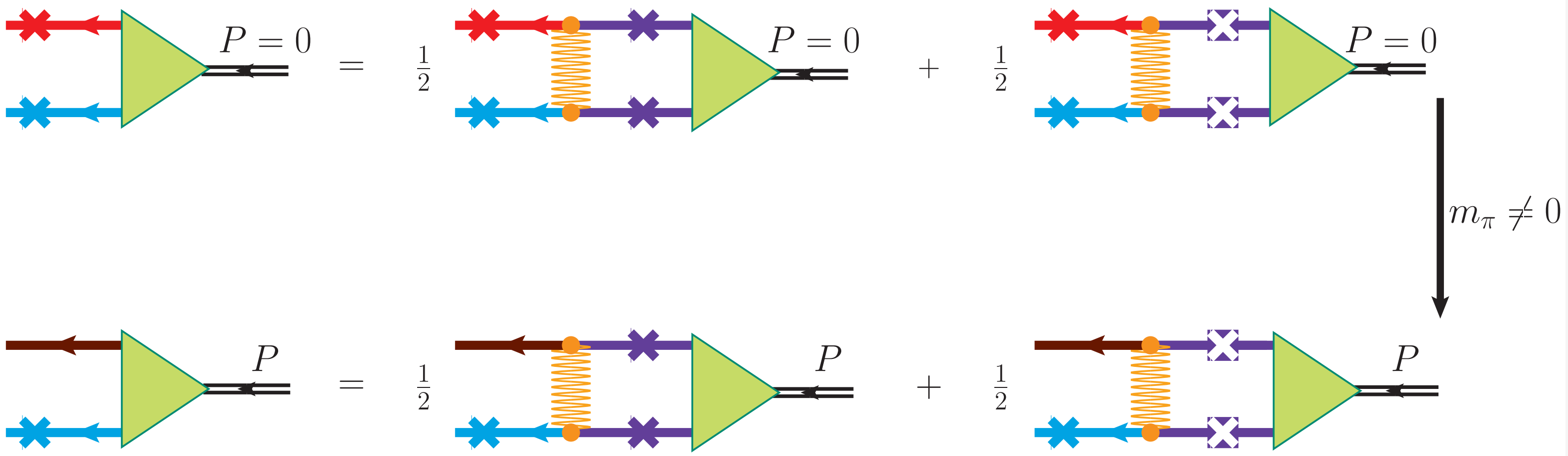
Courtesy Janus Weil
Giessen



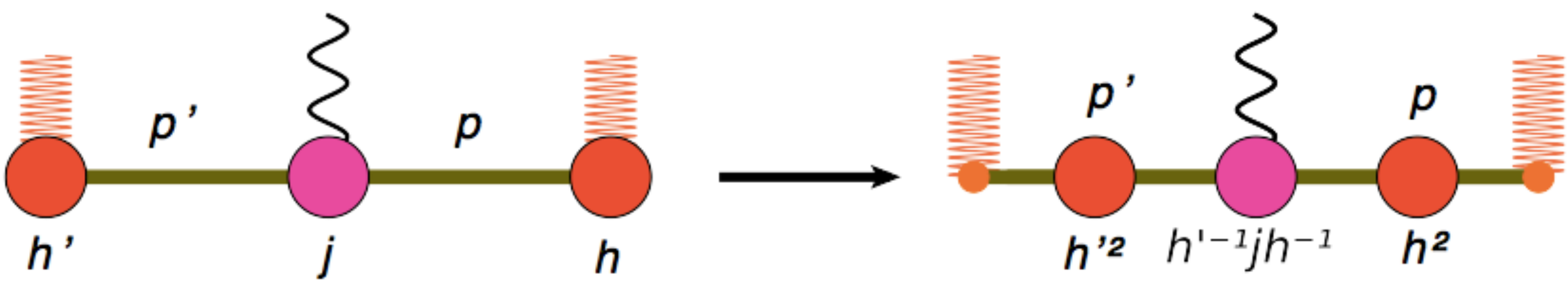
Small μ

Large μ





$\Rightarrow$ approximated pion vertex function $\Gamma(p, P) \sim \gamma^5 h(p^2)$

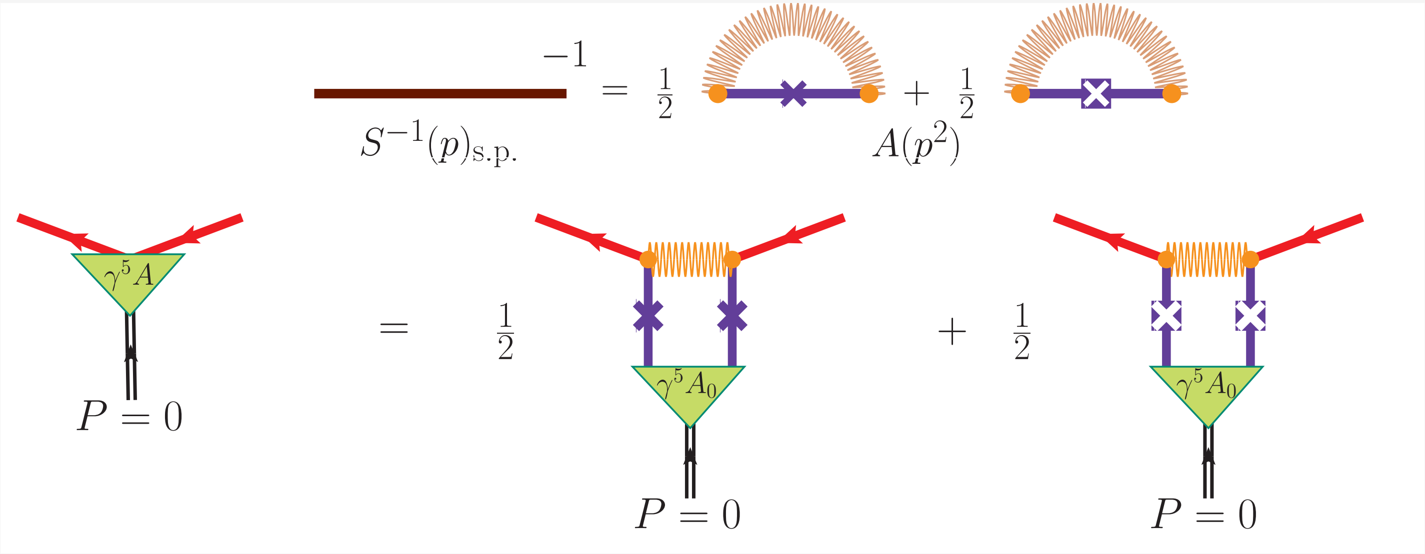


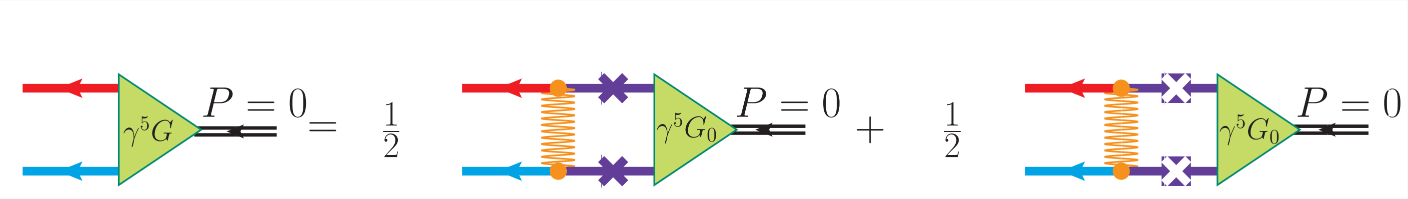
p'
p
h'
j
h
p'
p
h'²
h'⁻¹jh⁻¹
h²

Chiral symmetry

Bare Quark mass =0

Scalar part of 1 body equation and two body equation are identical





A massless pion exists! Goldstone boson.

Kernel

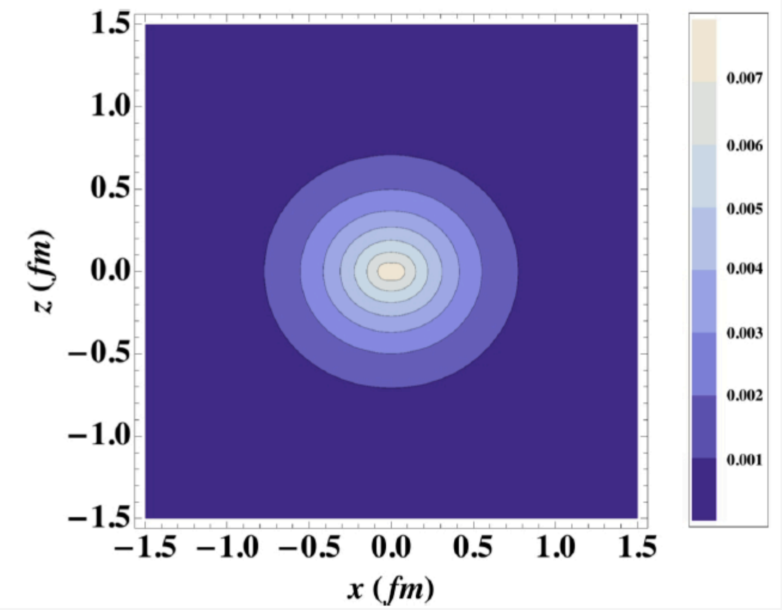
$$\mathcal{V}_L \propto \sigma[\lambda \mathbf{1}_1 \otimes \mathbf{1}_2 - (1-\lambda)\gamma_1^\mu \otimes \gamma_{2\mu}]V_L$$

$$\mathcal{V}_C \propto C\gamma_1^\mu \otimes \gamma_2^\nu \left[g_{\mu\nu} - (1-\xi)\frac{(p-k)_\mu (p-k)_\nu}{(p-k)^2} \right] V_C$$

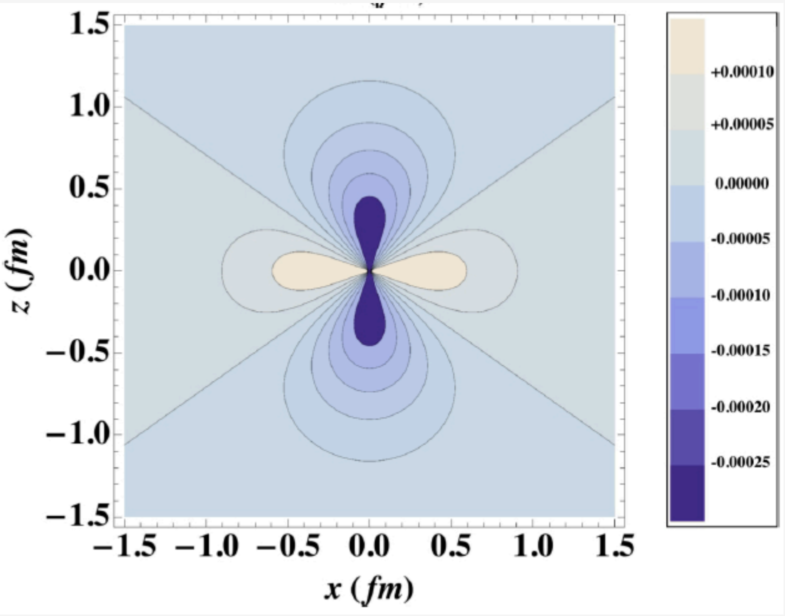
Constituent quark mass **m**

$$M(m^2) = m$$

Coordinate-space charge density in the x-y plane, for spin projection +3/2.



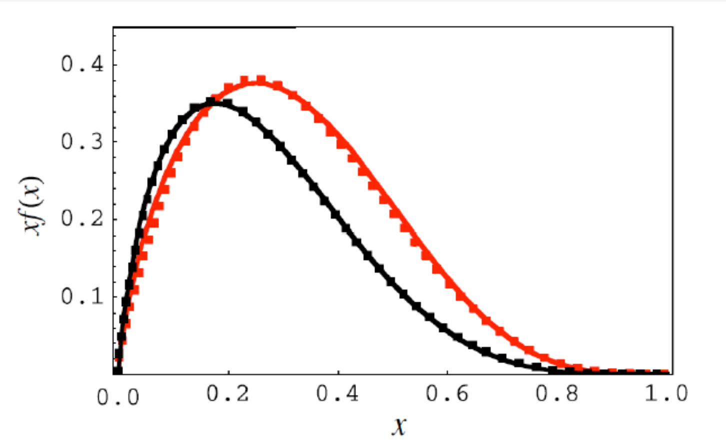
Total



D-states

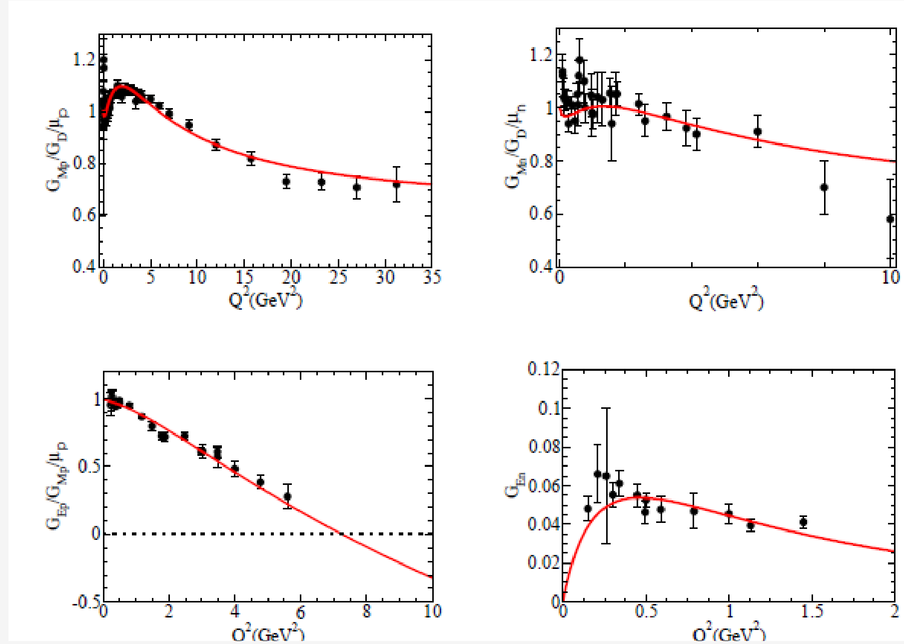
G. Ramalho, M. T. P. and A. Stadler, PHYSICAL REVIEW D 86 093022 (2012)

Structure function



$$f_q(x) = \frac{M m_s \lambda^2}{16\pi^2}\int_\xi^\infty d\chi \left[\psi_q(\chi)\right]^2 \quad \text{with} \quad \xi = \frac{(r+x-1)^2}{r(1-x)}$$

Proton and Neutron form factors



$$\chi^2 = 1.36$$

G. Ramalho, M. T. P. and Franz Gross, PHYSICAL REVIEW C 77, 015202 (2008)

Franz Gross, G. Ramalho, M. T. P. PHYSICAL REVIEW D 85, 093006 (2012)

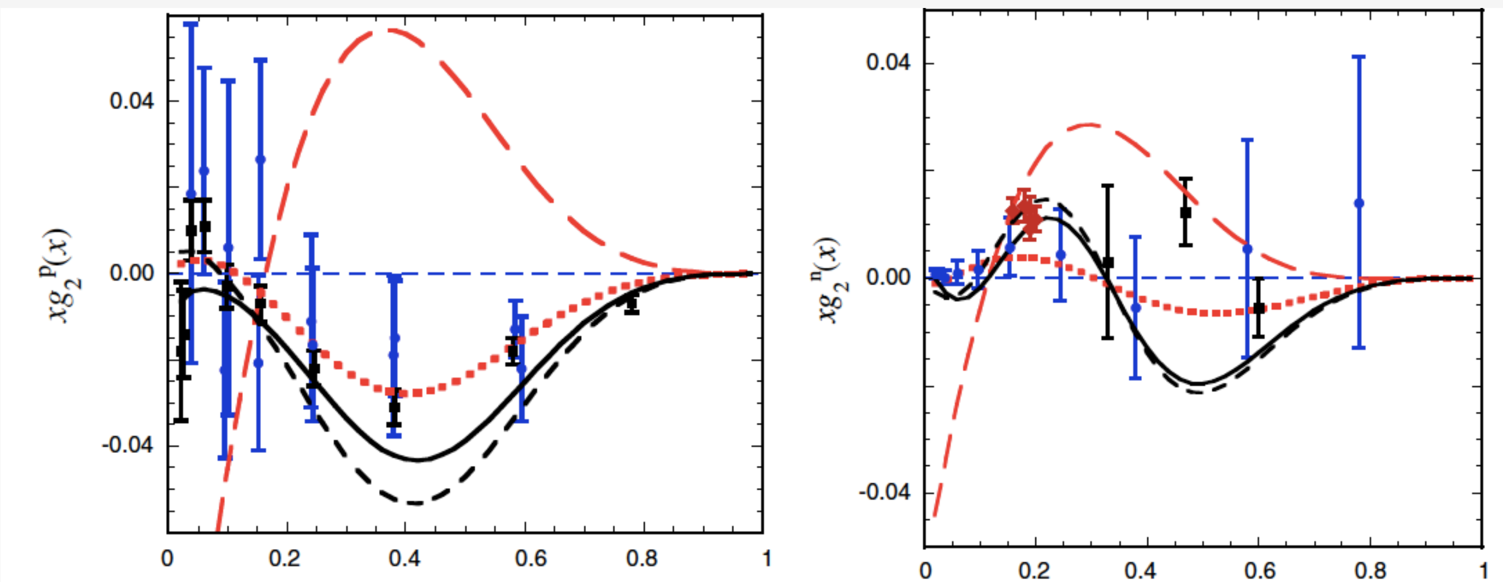
Also:

Δ(1600),
Baryon decuplet
DIS

Description of general size and shape of proton and neutron structure functions

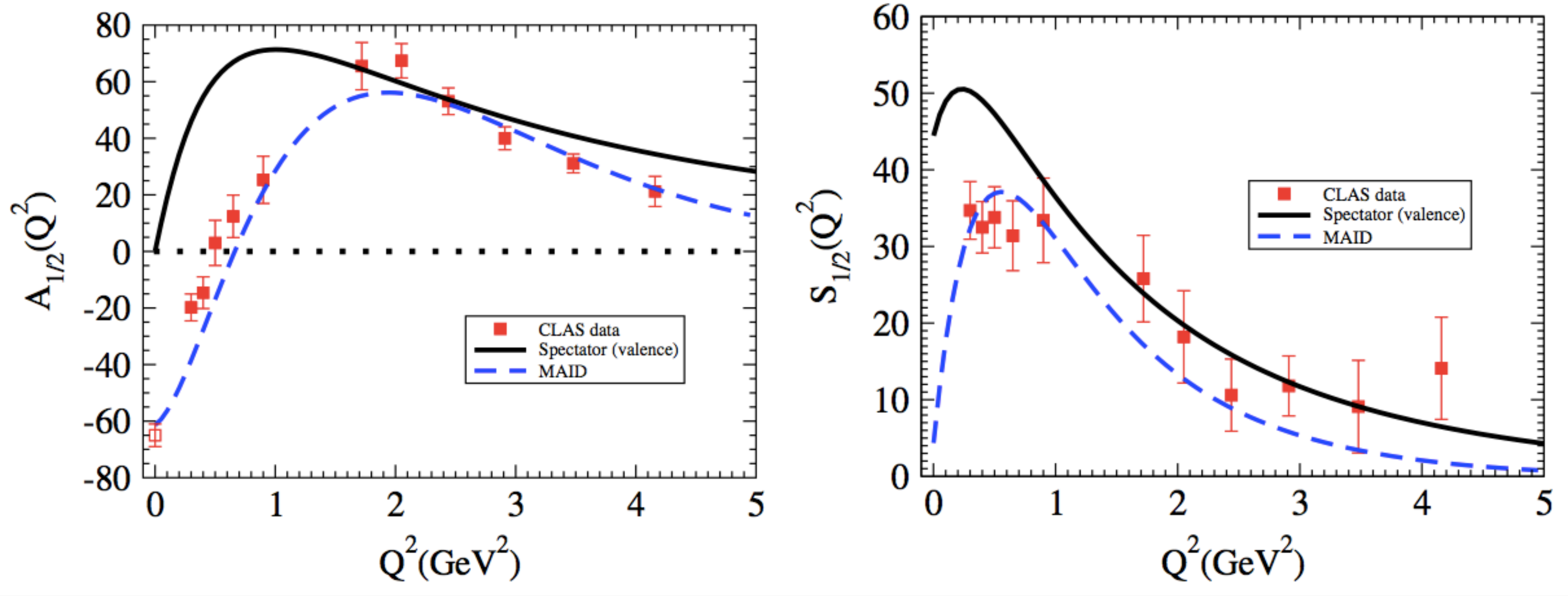
Model 1- P 18% D 3%   ….. No P wave

Model 2- P 0.6% D 35% ---- No P wave



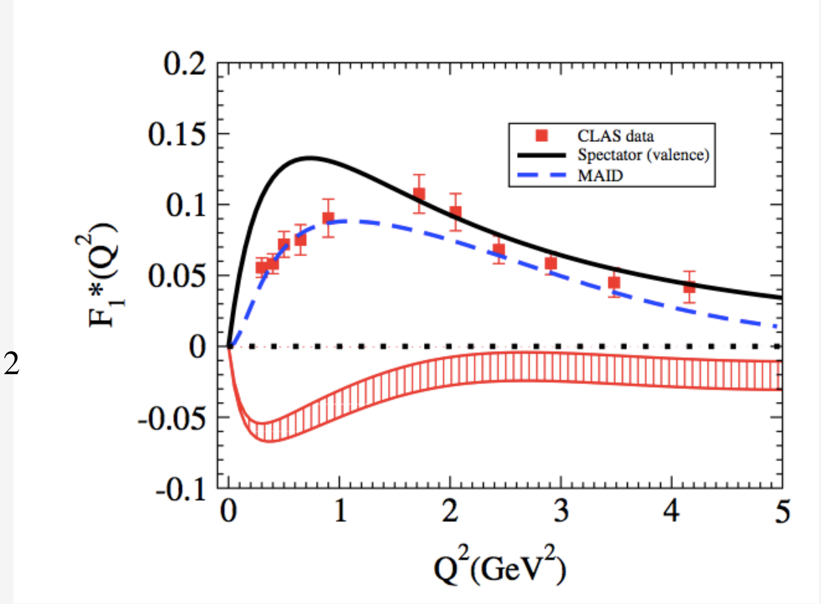
Franz Gross, G. Ramalho, M. T. P.   PHYSICAL REVIEW D 85, 093006 (2012)

$N \rightarrow N^*(1440)$

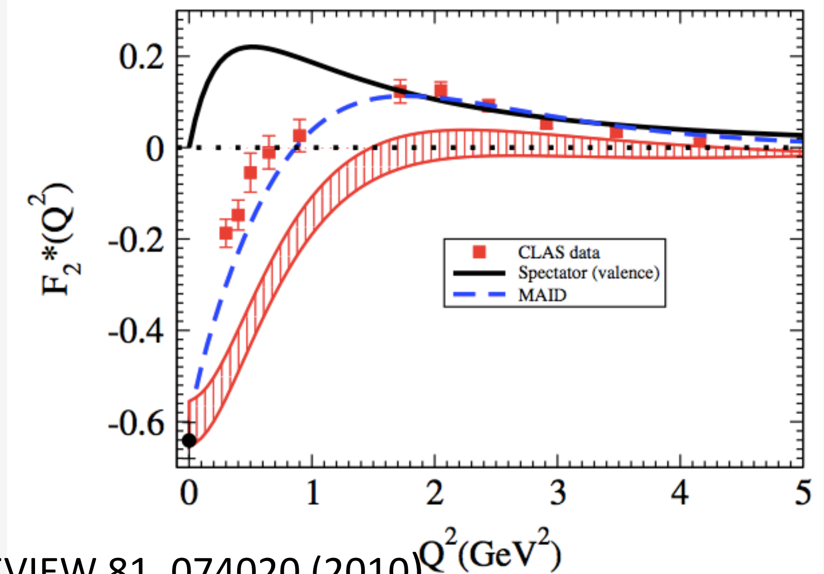


$N \to N^*(1440)$

- The orthogonality condition fixes term of the radial excitation.
- Quark core amplitude describes high Q^2 data.

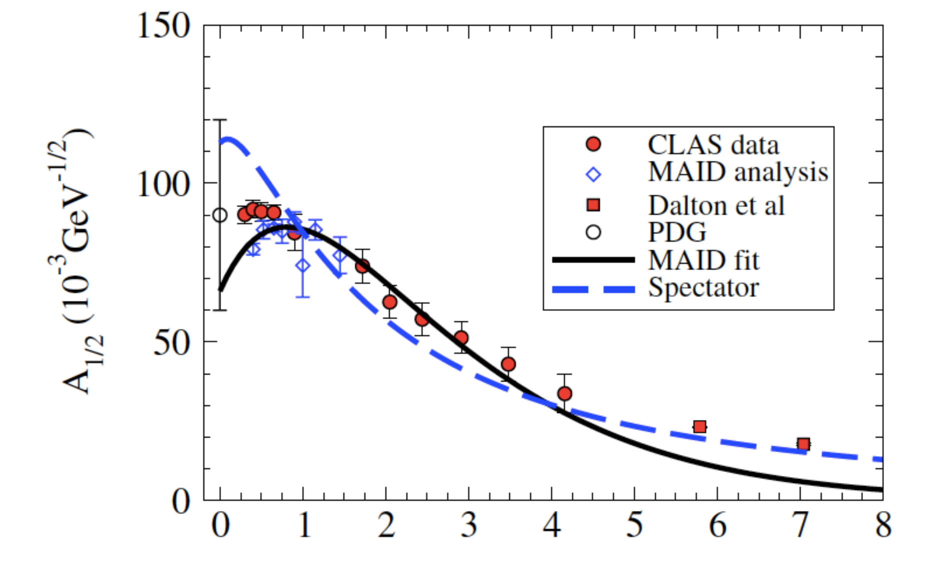


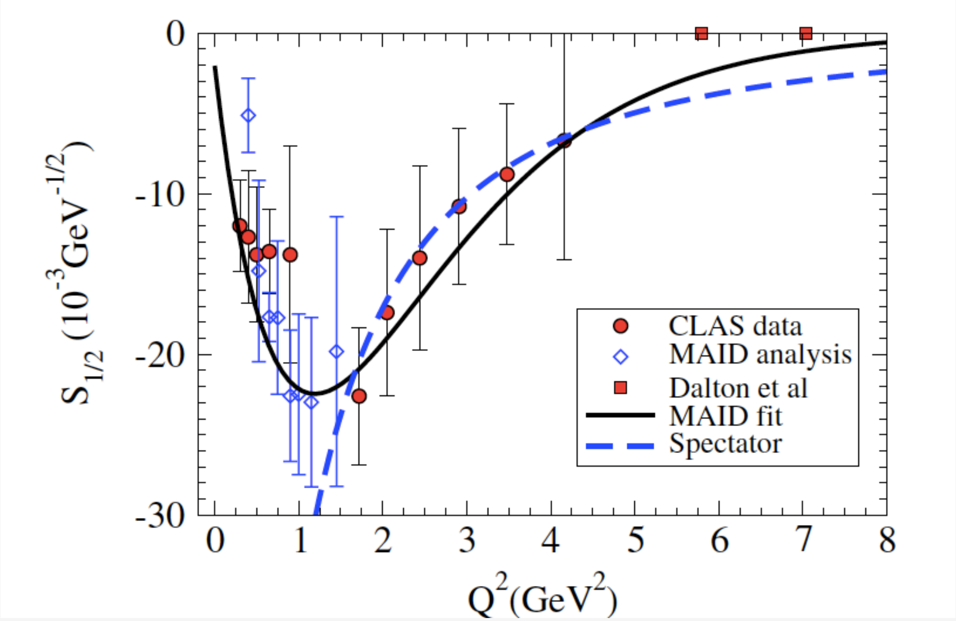
- Pion cloud estimated as difference between MAID fit and the quark core.
- Error bands from error bars in the data



G. Ramalho, K. Tsushima, PHYSICAL REVIEW 81, 074020 (2010)

$N \rightarrow N^*(1535)$





N(1535)

$$F_1^*(Q^2) = \frac{1}{2}(3j_1 + j_3)\mathcal{I}_0$$
$$F_2^*(Q^2) = -\frac{1}{2}(3j_2 - j_4)\frac{M_S + M}{2M}\mathcal{I}_0$$

$$A_{1/2} = -2b\left[F_1^* + \frac{M_S - M}{M_S + M}F_2^*\right]$$
$$S_{1/2} = \sqrt{2}b(M_S + M)\frac{|\mathbf{q}|}{Q^2}\left[\frac{M_S - M}{M_S + M}F_1^* - \tau F_2^*\right]$$

$$\Gamma^{\beta\mu} = G_1 q^\beta \gamma^\mu + G_2 q^\beta P^\mu + G_3 q^\beta q^\mu - G_4 g^{\beta\mu}$$

$N \to N^*(1520)$

$$
\begin{aligned}
G_M &= -F\left(\frac{1}{\sqrt{3}}A_{3/2} - A_{1/2}\right) \\
&= -\mathcal{R}\left[(M_R - M)^2 + Q^2\right]\frac{G_1}{M_R}, \\
G_E &= -F\left(\sqrt{3}A_{3/2} + A_{1/2}\right) \\
&= -\mathcal{R}\left\{2G_4 - \left[(M_R - M)^2 + Q^2\right]\frac{G_1}{M_R}\right\}
\end{aligned}
$$

$$
G_C = 2\sqrt{2}\frac{M_R}{|\mathbf{q}|}F\,S_{1/2} = -\mathcal{R}g_C,
$$

Coupling core spin states
with orbital angular momentum states

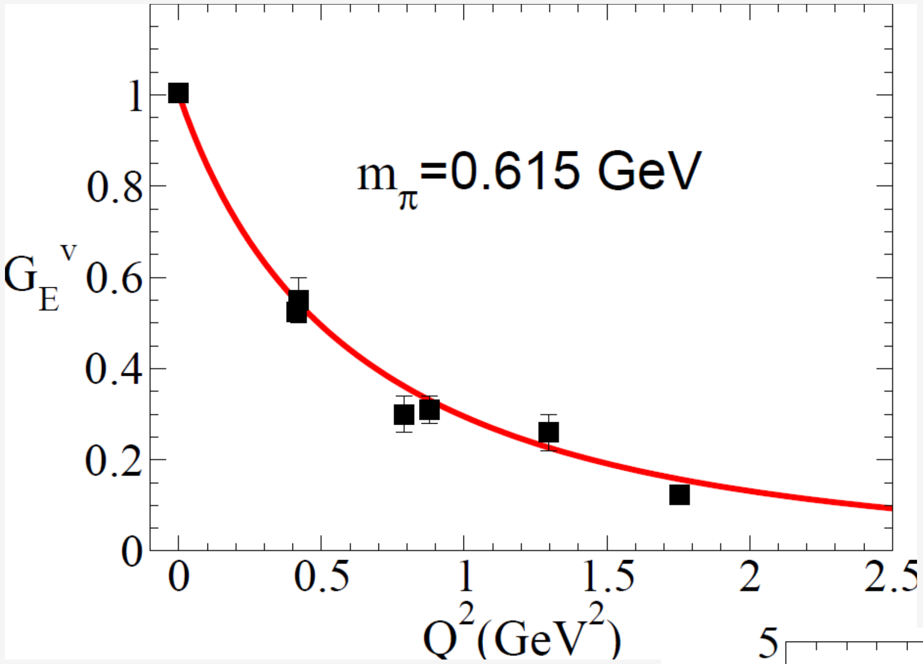
$$V_S^{\alpha}(P, \lambda_s) = \sum_{\lambda} \left\langle \frac{1}{2} \lambda 1 \lambda' | S \lambda_s \right\rangle \varepsilon^{\alpha}_{\lambda' P} u_{\Delta}(P, \lambda),$$

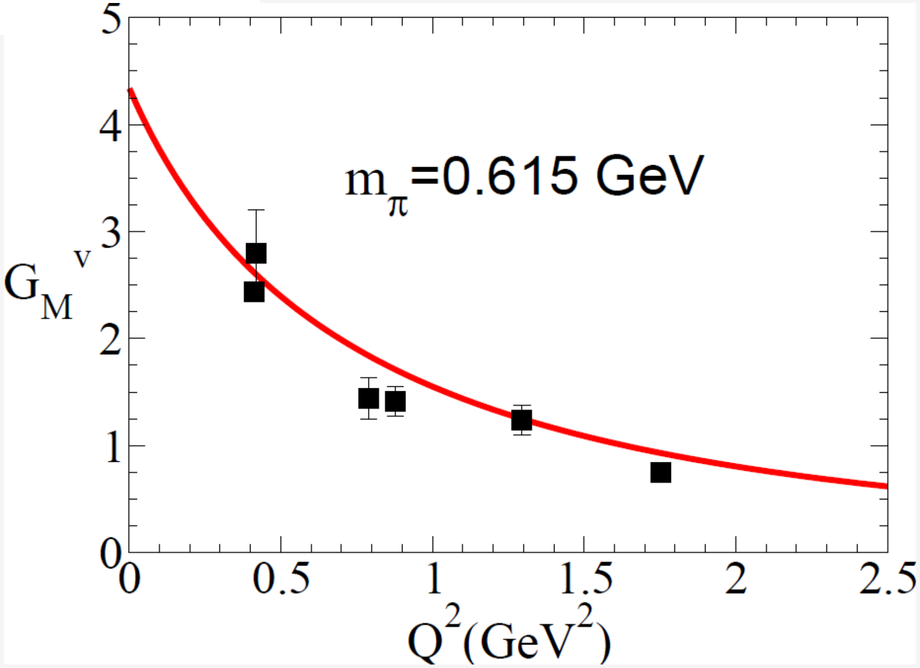
Delta

$$S = \frac{1}{2}, S = \frac{3}{2}$$

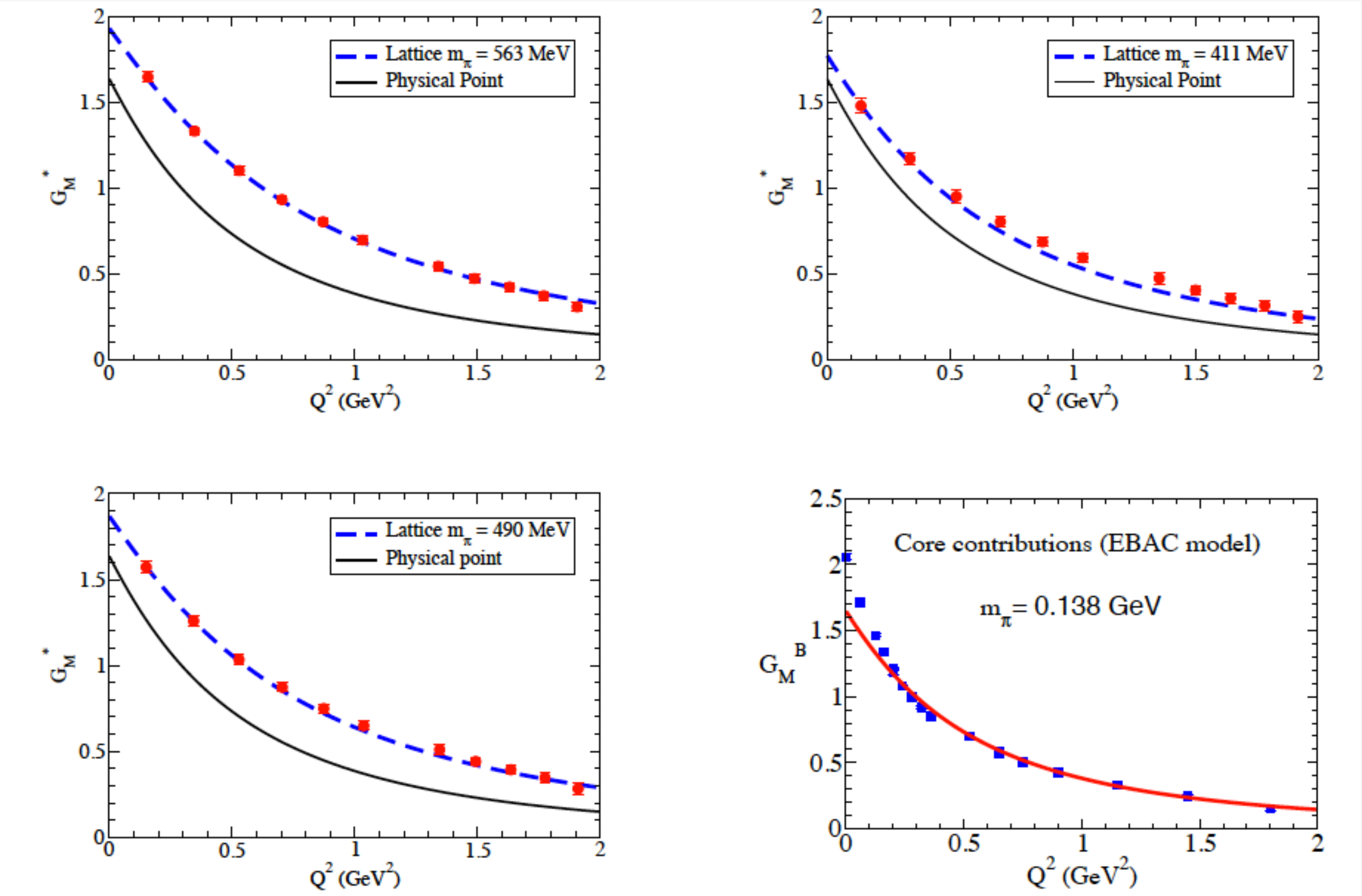
$$J = \frac{3}{2} \rightarrow S = \frac{3}{2} \otimes L = 0;\ S = \frac{3}{2} \otimes L = 2;\ S = \frac{1}{2} \otimes L = 2$$

S state **D3** state **D1** state





LQCD data: Gockeler et al. PRD 71, 034508 (2005)



Lattice m_π = 563 MeV
Physical Point
G_M*
Q² (GeV²)
Lattice m_π = 411 MeV
Physical Point
G_M*
Q² (GeV²)
Lattice m_π = 490 MeV
Physical point
G_M*
Q² (GeV²)
Core contributions (EBAC model)
m_π = 0.138 GeV
G_M^B
Q² (GeV²)

$i = 1, 2,$

$$j_i = \tfrac{1}{6} f_{i+}(Q^2) + \tfrac{1}{2} f_{i-}(Q^2)\tau_3 \tag{21}$$

where $f_{i\pm}$ are the isoscalar and isovector combinations, related to the u and d quark form factors by

$$\begin{aligned} \tfrac{2}{3} f_{iu} &= \tfrac{1}{6} f_{i+} + \tfrac{1}{2} f_{i-} \\ -\tfrac{1}{3} f_{id} &= \tfrac{1}{6} f_{i+} - \tfrac{1}{2} f_{i-} \,. \end{aligned} \tag{22}$$

The form factors are normalized (with $n = \{u, d\}$) to

$$\begin{aligned} f_{1n}(0) &= 1 \qquad\qquad & f_{2n}(0) &= \kappa_n \\ f_{1\pm}(0) &= 1 \qquad\qquad & f_{2\pm}(0) &= \kappa_\pm \end{aligned} \tag{23}$$

where κ_u and κ_d are the u and d quark anomalous magnetic moments (scaled by the quark charges) and

$$\begin{aligned} \kappa_+ &= 2\kappa_u - \kappa_d \\ \kappa_- &= \tfrac{2}{3}\kappa_u + \tfrac{1}{3}\kappa_d \,. \end{aligned} \tag{24}$$

$$\mu_p = 1 + \frac{1}{6}(\kappa_+ + 5\kappa_-)$$

$$\mu_n = -\frac{2}{3} + \frac{1}{6}(\kappa_+ - 5\kappa_-)$$

$$\kappa_+ = 3(\mu_p + \mu_n) - 1 = 1.639$$

$$\kappa_- = \frac{3}{5}(\mu_p - \mu_n) - 1 = 1.823$$

$$G_E(Q^2) = F_1(Q^2) - \tau F_2(Q^2)$$
$$= \frac{1}{2} B(Q^2) \Big\{ (f_{1+} + \tau_3 f_{1-}) - \tau\,(f_{2+} + \tau_3 f_{2-}) \Big\}$$
$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$
$$= \frac{1}{6} B(Q^2) \Big\{ (f_{1+} + 5\tau_3 f_{1-}) + (f_{2+} + 5\tau_3 f_{2-}) \Big\}$$

Vector meson dominance

approximated by a single pole

⇩

$f_{1+} = f_{1-}$ and $f_{2+} = f_{2-}$

G_{En} is identically zero **!**

Model	β_1, β_2	c_+, c_-	d_+, d_-	b_E, b_M	λ, r	N_0^2, χ^2
I(4)	0.057	2.06	-0.444	$--$	1.22	10.87
	0.654	2.06^*	-0.444^*	$--$	0.88	9.26
II(5)	0.049	4.16	-0.686	$--$	1.21	11.27
	0.717	1.56	-0.686^*	$--$	0.87	1.36
III(6)	0.078	1.91	-0.319	0.163	1.27	12.36
	0.598	1.91^*	-0.319^*	0.311	0.89	1.85
IV(9)	0.086	4.48	-0.134	0.079	1.25	8.46
	0.443	2.45	-0.513	0.259	0.89	1.03

Not always # parameters larger means better description

$$\int_{sk} = \underbrace{\int \frac{d\Omega_{\hat{\mathbf{r}}}}{4(2\pi)^3} \int_{4m_q^2}^{\infty} ds \sqrt{\frac{s - 4m_q^2}{s}}}_{\int_s} \underbrace{\int \frac{d^3k}{(2\pi)^3 2E_s}}_{\int_k},$$

$$\sum_{\lambda_1\lambda_2} \int_s \bar{\Psi}_{\lambda_1\lambda_2,\lambda_+}(P_+, k_1k_2) \otimes \Psi_{\lambda_1\lambda_2;\lambda_-}(P_-, k_1k_2)$$

$$\equiv \sum_{\Lambda} \bar{\Psi}_{\Lambda\lambda_+}(P_+, k) \otimes \Psi_{\Lambda\lambda_-}(P_-, k)\big|_{s=m_s^2},$$

For very large masses ($E_s \to m_s$; $s \to 4m_q^2$), we can

$$m_q m_s \int_{sk} \to \frac{1}{16} \int \frac{d^3k}{(2\pi)^3} \int \frac{d\Omega_{\hat{\mathbf{r}}}}{(2\pi)^3} \int_{4m_q^2}^{\infty} ds \sqrt{s - 4m_q^2}$$

$$= \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3r}{(2\pi)^3},$$

NΔ transition: State D1

State $(2, \frac{1}{2})$ is not orthogonal to $(0, \frac{1}{2})$

In principle:
$$q_\mu J^\mu = 3(M_\Delta - M) j_1 \sum_\lambda \int_k \bar{\Psi}_\Delta \Psi_N \neq 0.$$

There is a chance that $G_C^* \neq 0$; but $q_\mu J^\mu \neq 0$

Imposing current conservation

$$J_R^\mu = 3 j_1 \sum_\lambda \int_k \bar{\Psi}_\Delta \left(\gamma^\mu - \frac{\not{q} q^\mu}{q^2} \right) \Psi_N + 3 j_2 \sum_\lambda \int_k \bar{\Psi}_\Delta \frac{i \sigma^{\mu\nu} q_\nu}{2M} \Psi_N$$

$$q_\mu J_R^\mu = 0, \qquad G_C^* \; \alpha \; \frac{1}{Q^2} \sum_\lambda \int_k \bar{\Psi}_\Delta \Psi_N$$

To avoid divergence as $Q^2 \to 0$:

$$\sum_\lambda \int_k \bar{\Psi}_\Delta \Psi_N \sim Q^2 \qquad \text{[Orthogonality]}$$

Delta

$$J^\mu = -\bar{w}_\alpha(P_+)\left\{\left[F_1^*(Q^2)g^{\alpha\beta} + F_3^*(Q^2)\frac{q^\alpha q^\beta}{4M_\Delta^2}\right]\gamma^\mu + \left[F_2^*(Q^2)g^{\alpha\beta} + F_4^*(Q^2)\frac{q^\alpha q^\beta}{4M_\Delta^2}\right]\frac{i\sigma^{\mu\nu}q_\nu}{2M_\Delta}\right\} w_\beta(P_-)$$

$$\begin{aligned}
G_{E0}(Q^2) &= [F_1^* - \tau F_2^*]\left(1 + \frac{2}{3}\tau\right) \\
&\quad -\frac{1}{3}[F_3^* - \tau F_4^*]\,\tau\,(1+\tau) \\
G_{M1}(Q^2) &= [F_1^* + F_2^*]\left(1 + \frac{4}{5}\tau\right) \\
&\quad -\frac{2}{5}[F_3^* + F_4^*]\,\tau\,(1+\tau) \\
G_{E2}(Q^2) &= [F_1^* - \tau F_2^*] \\
&\quad -\frac{1}{2}[F_3^* - \tau F_4^*]\,(1+\tau) \\
G_{M3}(Q^2) &= [F_1^* + F_2^*] \\
&\quad -\frac{1}{2}[F_3^* + F_4^*]\,(1+\tau)
\end{aligned}$$

$\gamma\Delta \rightarrow \Delta$

$$J^\mu = \bar{w}_\alpha(P_+)\Gamma^{\alpha\beta\mu}(P,q)w(P_-)_\beta(P_+)$$

$$\begin{aligned} J^\mu \;=\; & -\bar{w}_\alpha(P_+)\left\{\left[F_1^*(Q^2)g^{\alpha\beta} + F_3^*(Q^2)\frac{q^\alpha q^\beta}{4M_\Delta^2}\right]\gamma^\mu \right. \\ & \left. + \left[F_2^*(Q^2)g^{\alpha\beta} + F_4^*(Q^2)\frac{q^\alpha q^\beta}{4M_\Delta^2}\right]\frac{i\sigma^{\mu\nu}q_\nu}{2M_\Delta}\right\} w_\beta(P_-) \end{aligned}$$

4: $G_{E0}(Q^2)$ $G_{M1}(Q2)$ $G_{E2}(Q2)$ $G_{M3}(Q2)$

PRC77 015202 (2008); PRD78 114017(2008); JPG36 085004 (2009)

$\gamma\Delta \rightarrow \Delta$

a and **b** small

$$G_{E0}(Q^2) = N^2 \tilde{g}_\Delta \mathcal{I}_S$$

$$G_{M1}(Q^2) = N^2 \tilde{f}_\Delta \left[\mathcal{I}_S + \frac{4}{5} a \mathcal{I}_{D3} - \frac{2}{5} b \mathcal{I}_{D1}\right]$$

$$G_{E2}(Q^2) = 3(aN^2)\tilde{g}_\Delta \frac{\mathcal{I}_{D3}}{\tau}$$

$$G_{M3}(Q^2) = \tilde{f}_\Delta N^2 \left[a \frac{\mathcal{I}_{D3}}{\tau} + 2b \frac{\mathcal{I}_{D1}}{\tau}\right]$$

D state corrections

from overlap

Integrals between

S and D states

$\gamma\Delta \to \Delta$

a and **b** small

$$G_{E0}(Q^2) = N^2 \tilde{g}_\Delta \mathcal{I}_S$$

$$G_{M1}(Q^2) = N^2 \tilde{f}_\Delta \left[\mathcal{I}_S + \frac{4}{5} a \mathcal{I}_{D3} - \frac{2}{5} b \mathcal{I}_{D1}\right]$$

$$G_{E2}(Q^2) = 3(aN^2)\tilde{g}_\Delta \frac{\mathcal{I}_{D3}}{\tau}$$

$$G_{M3}(Q^2) = \tilde{f}_\Delta N^2 \left[a \frac{\mathcal{I}_{D3}}{\tau} + 2b \frac{\mathcal{I}_{D1}}{\tau}\right]$$

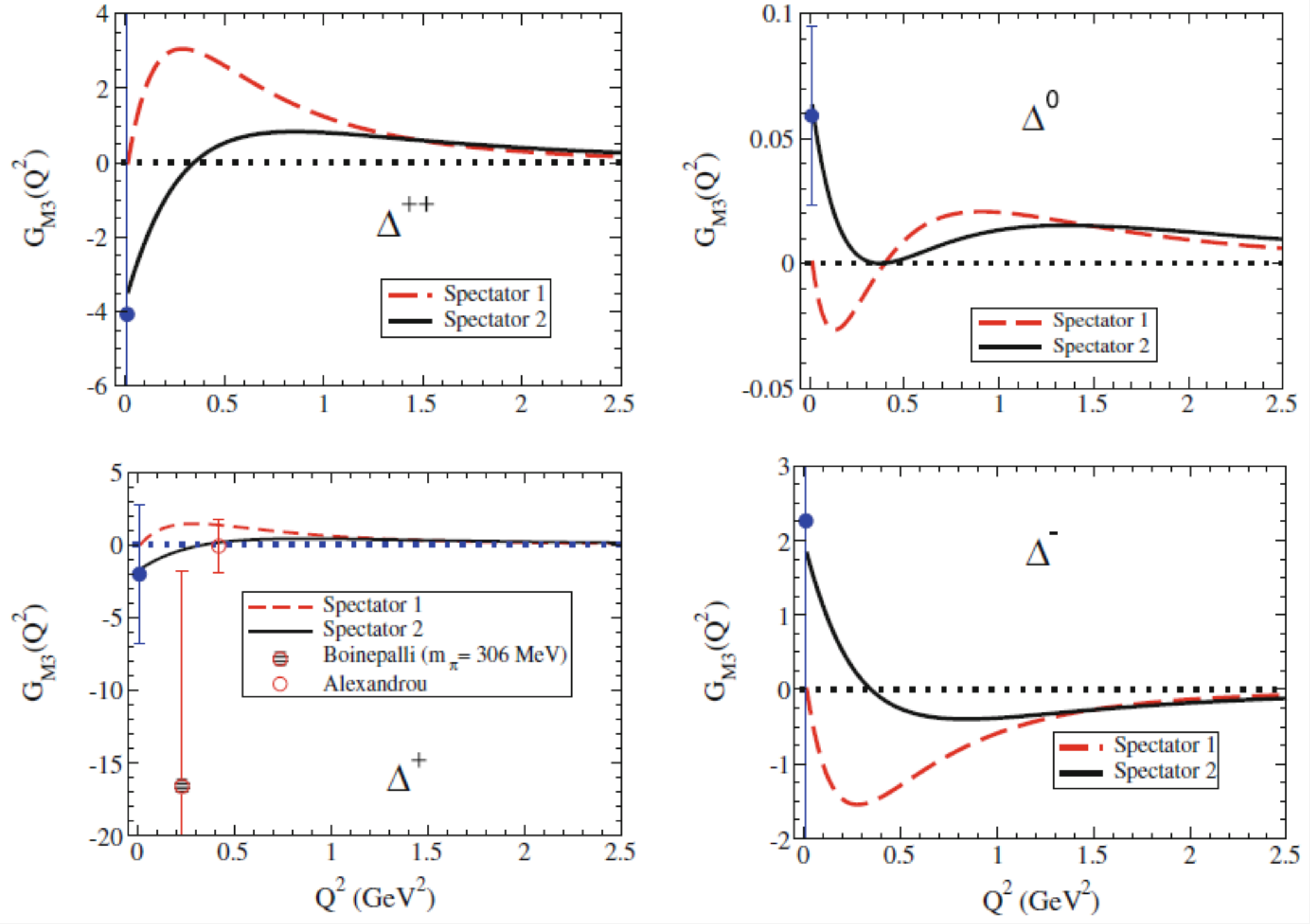
D state corrections

from overlap

Integrals between

S and D states

$\gamma\Delta \rightarrow \Delta$



	$\mathcal{Q}_{\Delta}^{\perp}\,(+\frac{3}{2})$	$\mathcal{O}_{\Delta}^{\perp}\,(+\frac{3}{2})$
Lattice QCD:		
Quenched [6]	0.83±0.21	
Wilson [6]	0.46±0.35	
Hybrid [6]	0.74±0.68	
Spectator quark models:		
Spectator S [15]	0.29	-3.44
Spectator SD [17]	0.92	-3.38

TABLE I: Transverse electric quadrupole moment $\mathcal{Q}_{\Delta}^{\perp}\,(+\frac{3}{2})$ in units of $\frac{e}{M_{\Delta}^2}$, and transverse magnetic octupole moment $\mathcal{O}_{\Delta}^{\perp}\,(+\frac{3}{2})$ in units of $\frac{e}{2M_{\Delta}^3}$, for the Δ^+.

G. Ramalho, M. T. P., A. Stadler arXiv: 1297.4392, Phys. Rev. D (to appear)

- Helicity states are usually used to define polarization.
  In the $x-z$ plane: $k=(E_k, k\cos\theta, 0, k\sin\theta)$

$$\xi(0)=\frac{1}{m}\begin{bmatrix} k \\ E_k\sin\theta \\ 0 \\ E_k\cos\theta \end{bmatrix}, \qquad \xi(\pm)=\mp\frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ \cos\theta \\ \pm i \\ -\sin\theta \end{bmatrix}$$

  $\xi(\lambda)$ is $\theta-$ dependent; $\qquad k\cdot\xi=0$

- Fixed-axis: vector particle is bound to a system with $P=(P_0,0,0,\mathrm{P})$:

$$\varepsilon(0)=\frac{1}{M}\begin{bmatrix} P \\ 0 \\ 0 \\ P_0 \end{bmatrix}, \qquad \varepsilon(\pm)=\mp\frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ \pm i \\ 0 \end{bmatrix}$$

  No angular dependence; $\qquad P\cdot\varepsilon=0$

arXiv:0708.0995 [nucl-th]

By design, a quark-diquark system in S wave

Di-quark first

$$\xi^0 = \frac{1}{\sqrt{2}}(ud - du)$$
$$\xi^1_0 = \frac{1}{\sqrt{2}}(ud + du) = \xi_z$$
$$\xi^1_+ = uu = -\frac{1}{\sqrt{2}}(\xi_x + i\xi_y)$$
$$\xi^1_- = dd = \frac{1}{\sqrt{2}}(\xi_x - i\xi_y)\ .$$

$$\phi^1_{\frac{1}{2}} = \sqrt{\tfrac{2}{3}}\begin{pmatrix}0\\1\end{pmatrix}\xi^1_+ - \sqrt{\tfrac{1}{3}}\begin{pmatrix}1\\0\end{pmatrix}\xi^1_0$$
$$\to \sqrt{\tfrac{1}{6}}\,[2d(uu) - u(ud + du)]\,,$$

$$\phi^0_I = \xi^{0*}\chi^I \qquad (2)$$
$$\phi^1_I = -\frac{1}{\sqrt{3}}\tau\cdot\xi^{1*}\chi^I$$
$$= \frac{1}{\sqrt{6}}\left[\tau_-\xi^1_+ - \tau_+\xi^1_- - \sqrt{2}\tau_3\xi^1_0\right]\chi^I \qquad (3)$$

where $\tau_\pm = \tau_x \pm i\tau_y$ are the isospin raising and lowering operators, $I = \pm 1/2$ is the isospin of the quark (or nucleon)

$$\chi^{+\frac{1}{2}} = \begin{pmatrix}1\\0\end{pmatrix} = u\,(\mathrm{or}\,p) \quad \chi^{-\frac{1}{2}} = \begin{pmatrix}0\\1\end{pmatrix} = d\,(\mathrm{or}\,n), \qquad (4)$$

$$\Phi^i(s) = -\frac{1}{\sqrt{3}}\sigma_i \chi_s$$

$$\Phi^i \longrightarrow u_S^\alpha(s) = -\frac{1}{\sqrt{3}}\gamma_5\gamma^\alpha u(s)$$

$$m_\rho = c_0 + c_1 m_\pi^2 ,$$