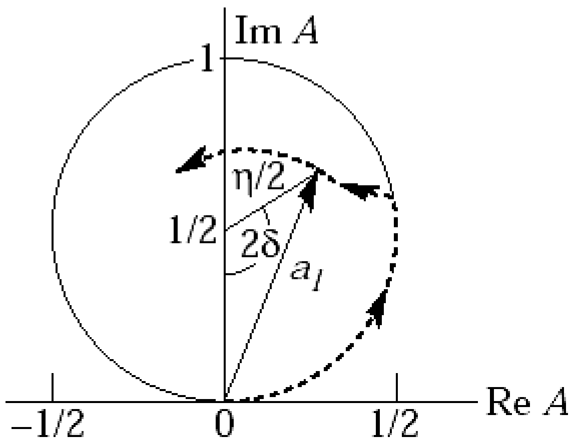
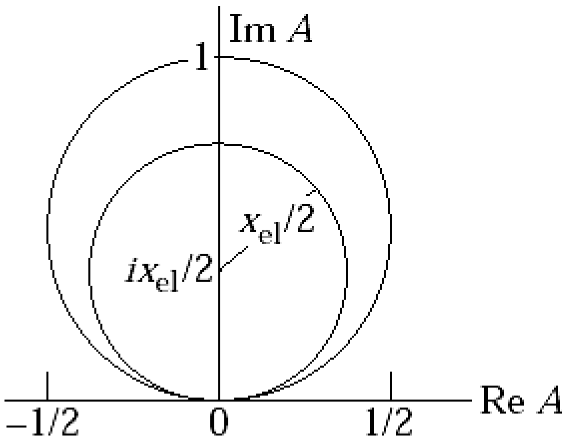
# Features - Argand Diagram



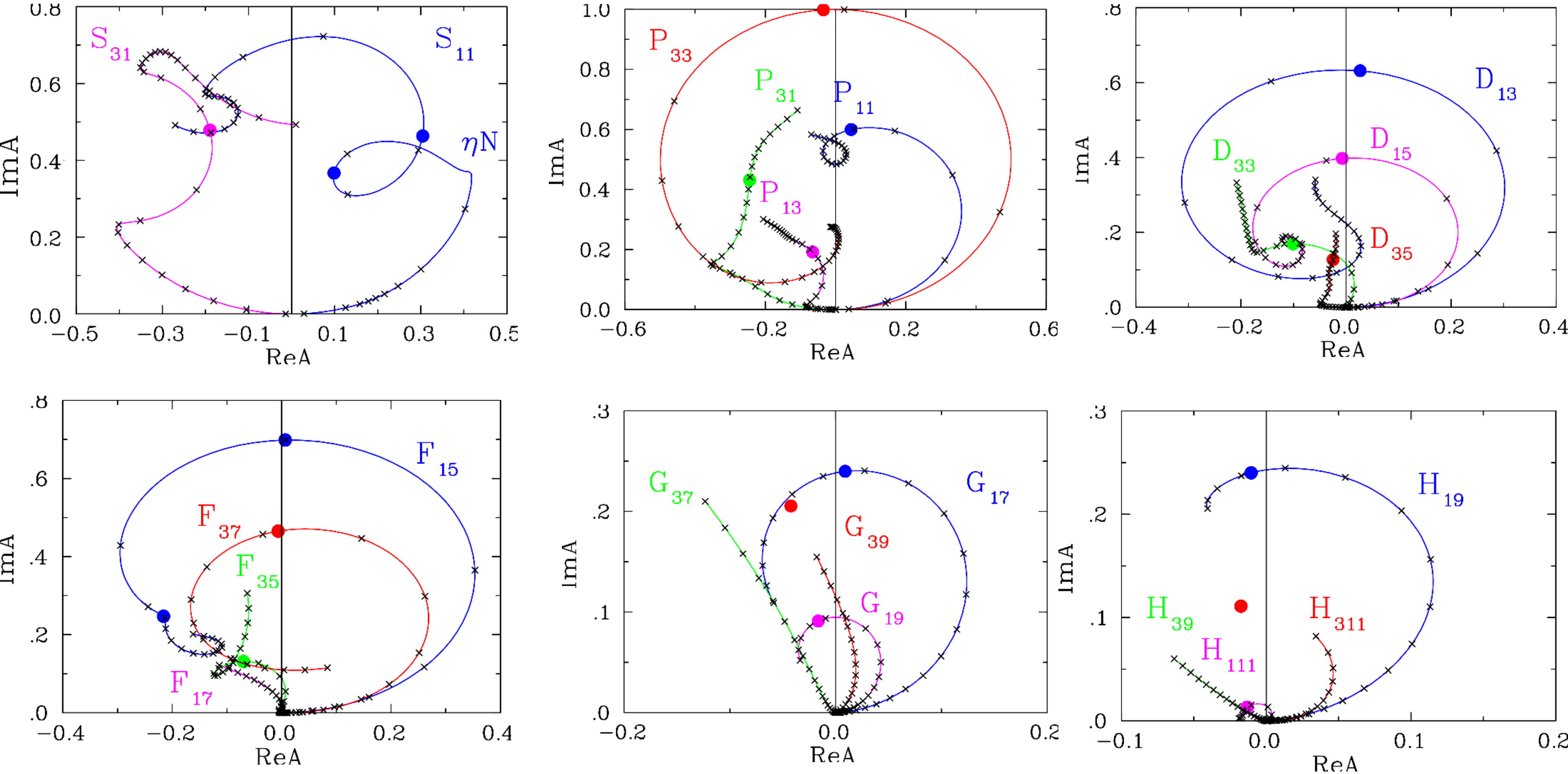




[J.-R. Argand, *Essai sur une maniere de representer les quantites imaginaires dans les constructions geometriques* (Sans nom d'auteur) (Paris, 1806) I vol. petit in-8º, 78 pages]

[J. Ashkin and S. H. Vosko, *Graphical method for obtaining phase shifts from the experimental data on meson-nucleon scattering,* Phys Rev 91, 1248 (1953)]

# *Argand Plots*

[R. Arndt, W. Briscoe, IS, R. Workman, Phys Rev C **74**, 045205 (2006)]



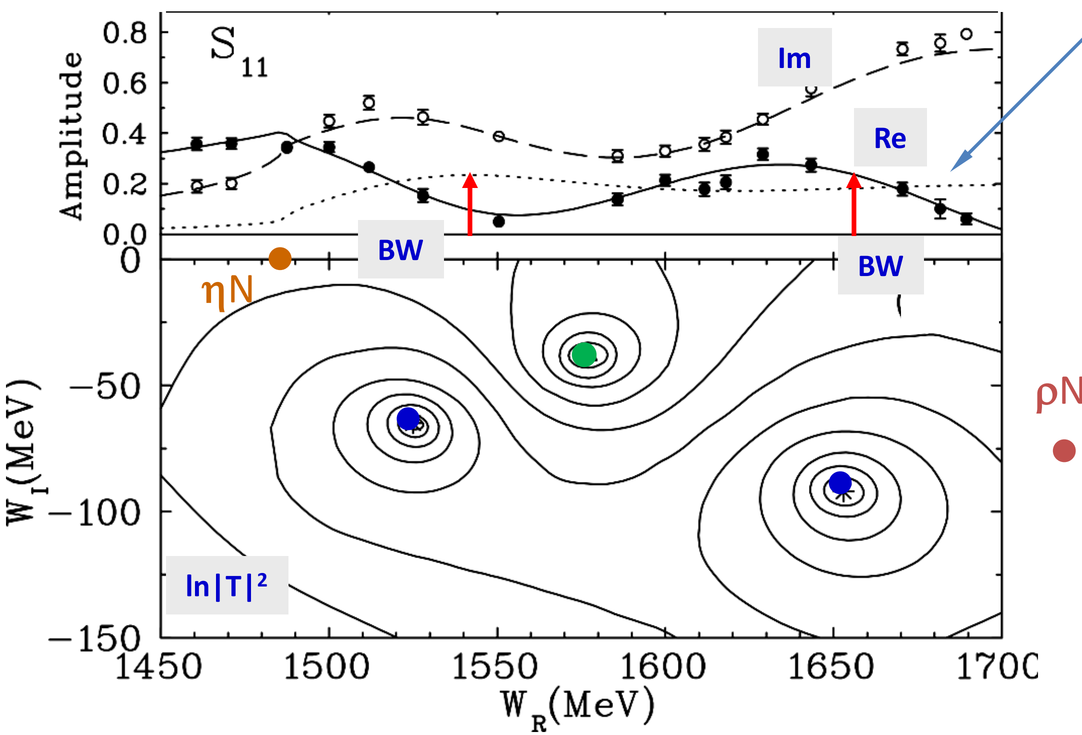

- **Crosses** indicate every 50 MeV step in W
- **Dots** correspond to BW, $W_R = M_R$

- **Every PW has a single BW except $S_{11}$ and $F_{15}$ which have two BWs**

# *Complex Energy Plane for* $S_{11}$

[R. Arndt, W. Briscoe, IS, R. Workman, M. Pavan, Phys Rev C **69**, 035213 (2004)]

- Interpretation of PW amplitudes may appear not simple

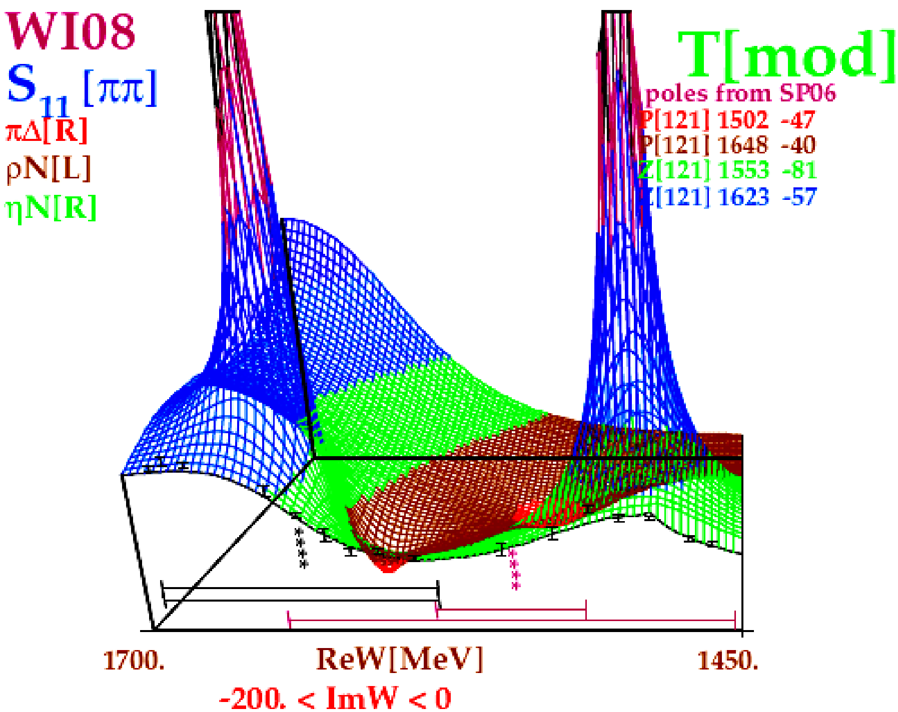


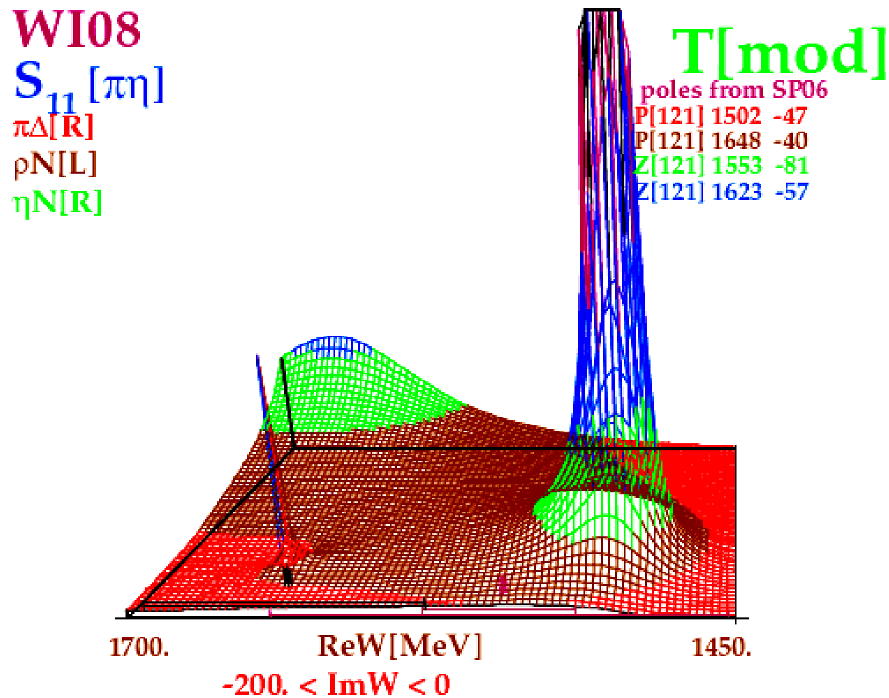

**ImT-T*T ≥ 0 [unitarity boundary]**

- BWs:
  - $W_R$ = 1546.7 ± 2.2 MeV, $\Gamma$ = 178.0 ± 12.0 MeV
  - $W_R$ = 1651.2 ± 4.7 MeV, $\Gamma$ = 130.6 ± 7.0 MeV
- Poles [same, 2nd sheet]:
  - W = 1526 – i65 MeV
  - W = 1653 – i91 MeV
- Zero:
  - W = 1578 – i38 MeV
- Branch-Points:
  - ηN thr: W = 1487 – i0 MeV
  - ρN thr: W = 1715 – i73 MeV

M(Pole) = $ReW_p$
$\Gamma$(Pole) = 2*$ImW_p$

# $\pi N$ $S_{11}$


WI08
S11 [ππ]
πΔ[R]
ρN[L]
ηN[R]
T[mod]
poles from SP06
P[121] 1502 -47
P[121] 1648 -40
Z[121] 1553 -81
Z[121] 1623 -57
1700.
ReW[MeV]
1450.
-200. < ImW < 0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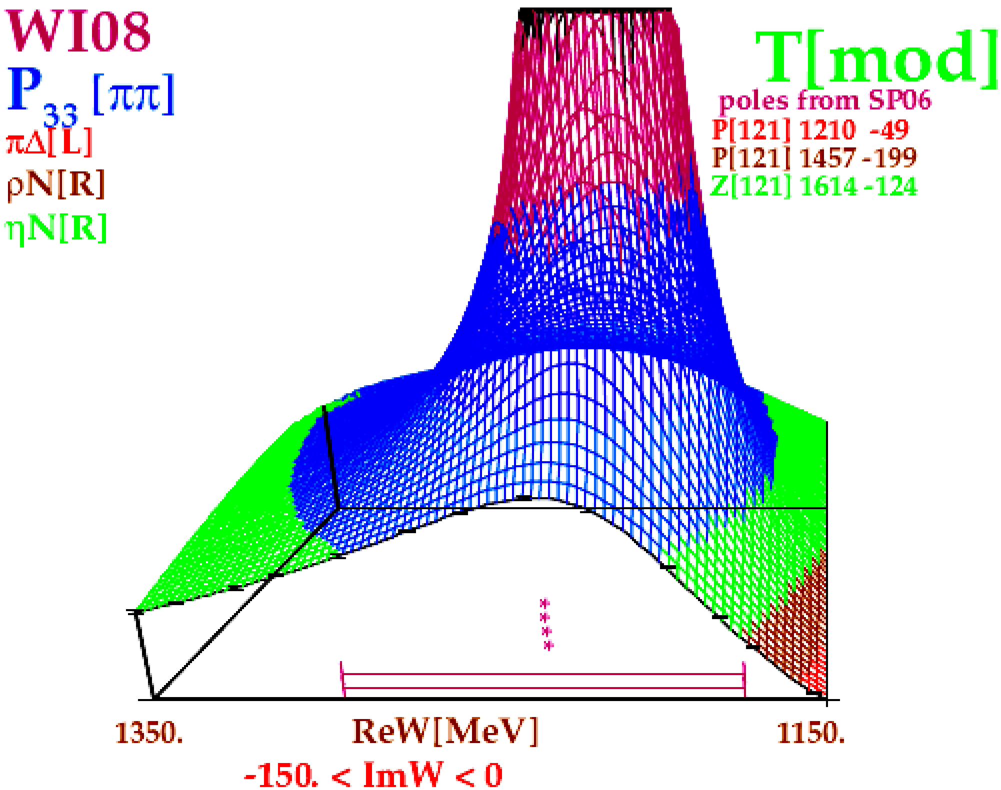
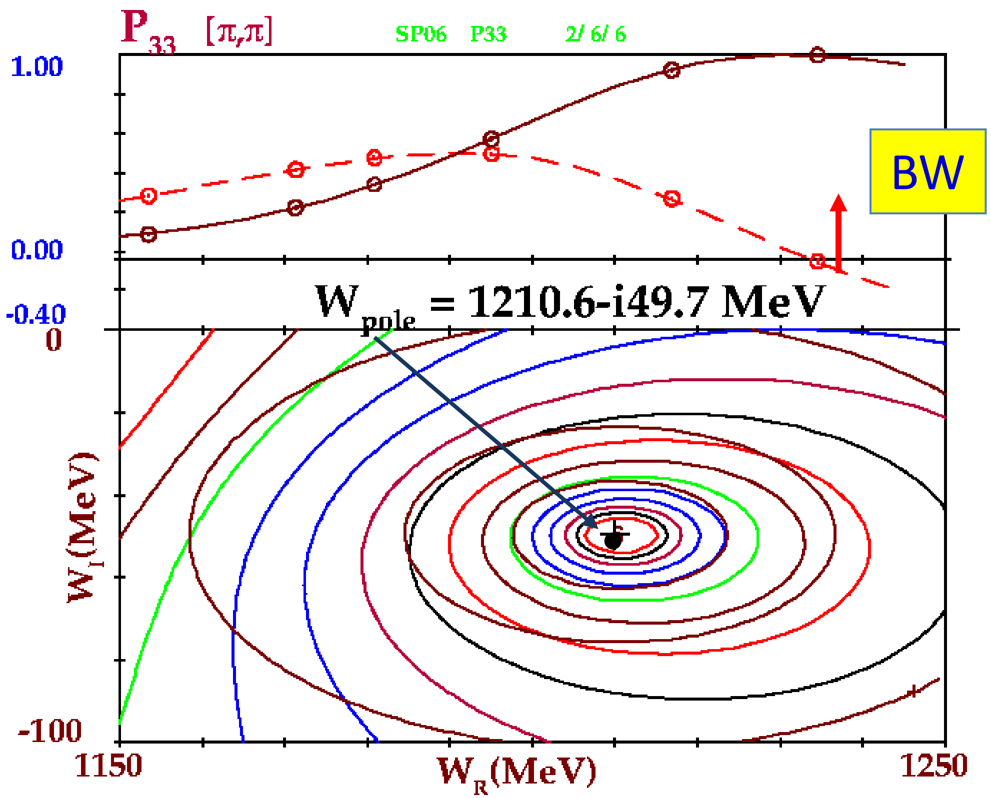
1700.
ReW[MeV]
1450.
-200. < ImW < 0

# $\pi N$ $\mathcal{P}_{33}$



# *Where is Δ(1232)$P_{33}$ ?*




- ReA = 0 at 'crossover' energy
- But crossover energy is NOT mass

| Ampl | Crossover | ImA | $\sigma_{reac}$ |
|---|---|---|---|
| $H^+$ | 1231.32 | 1.000 | 0.00 |
| $\pi^+p$ | 1231.17 | 1.000 | 0.00 |
| $H^-$ | 1231.38 | 0.994 | 1.12 |
| $\pi^-p$ | 1231.38 | 0.994 | 1.12 |

- BW-fit [+ bgrd] yields:
  $M_\Delta$ = 1232.86±0.74 MeV
  $\Gamma_\Delta$ = 118.06±1.20 MeV
- Pole:
  W = 1210.6-i49.7 MeV

# $N(1440)^{****}$ – What is Known

[J. Beringer *et al* [PDG] Phys Rev D **86**, 010001 (2012)]

Two-faced Janus
Roman God of Gates & Doors

- **Dick Arndt:** *``This is one of mysterious Resonances"*

| PWA-Pole Ref | Re(MeV) | −2xIm(MeV) | |
|---|---|---|---|
| BnGa12 | 1370 ± 4 | 190 ± 7 | |
| SAID-SP06 | 1359 | 164 | 1st Riemann sheet |
| | 1388 | 166 | 2nd Riemann sheet |
| KH93 | 1385 | 164 | |
| CMU80 | 1375 ± 30 | 180 ± 40 | |

| PWA-BW Ref | Mass(MeV) | Width(MeV) | BR |
|---|---|---|---|
| BnGa12 | 1430 ± 8 | 365 ± 35 | 0.62 ± 0.03 |
| SAID-SP06 | 1485 ± 1.2 | 284 ± 18 | 0.787 ± 0.016 |
| KSU92 | 1462 ± 10 | 391 ± 34 | 0.69 ± 0.03 |
| CMU80 | 1440 ± 30 | 340 ± 70 | 0.68 ± 0.04 |
| KH79 | 1410 ± 12 | 135 ± 10 | 0.51 ± 0.05 |

$$f(E) = \frac{k}{(E^2 - M^2)^2 + M^2\Gamma^2}$$

$M = \mathrm{Re}W_p$
$\Gamma = 2 \cdot \mathrm{Im}W_p$

# *Complex Energy Plane for $P_{11}$*

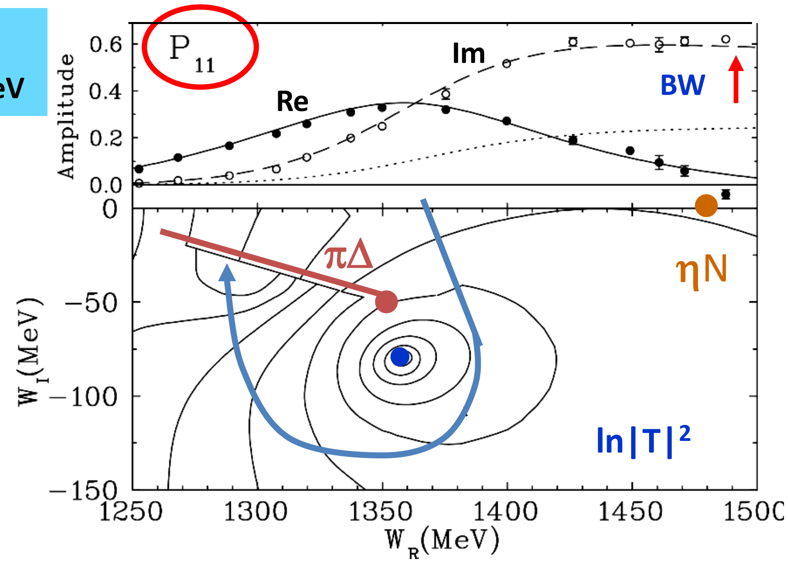
[R. Arndt, W. Briscoe, IS, R. Workman, Phys Rev C **74**, 045205 (2006)]

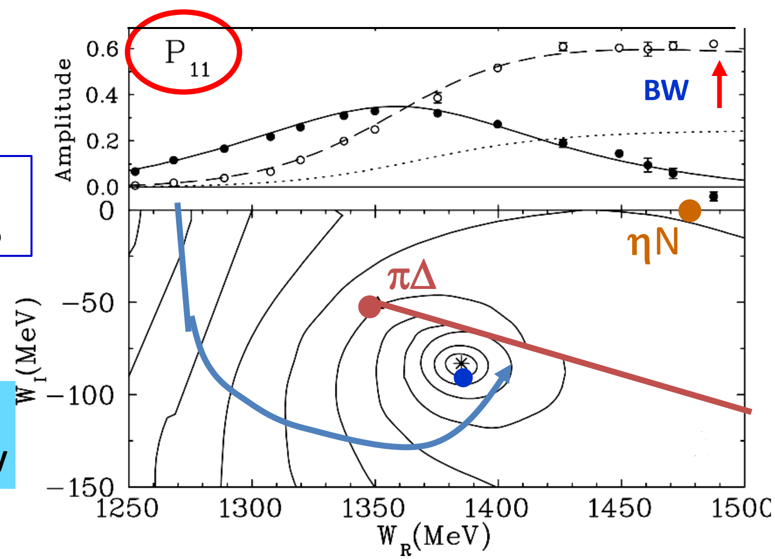
- 1st Riemann sheet
  Pole 1: $W_p$ = 1359–i82 MeV

- There is a shift between Pole positions at two sheets, due to a non-zero jump on the πΔ-cut

- 2nd Riemann sheet
  Pole 2: $W_p$ = 1388–i83 MeV

M = $ReW_p$
Γ = 2*$ImW_p$




- BW: $W_R$ = 1485.0± 1.2 MeV
  Γ = 248 ±18 MeV

**Branch-points:**
- πΔ thr [W = 1350 - i50 MeV]
- ηN thr [W = 1487 - i0 MeV]
- πΔ Branch Cut is two-body and has 2 Riemann sheets

- Sheet 1 is the sheet reached most directly the real axis
- Sheet 2 is behind the πΔ Branch Cut

- N(1440) is a Resonance which manifests itself via 2 Poles at 2 different Riemann sheets (with respect to the πΔ cut)

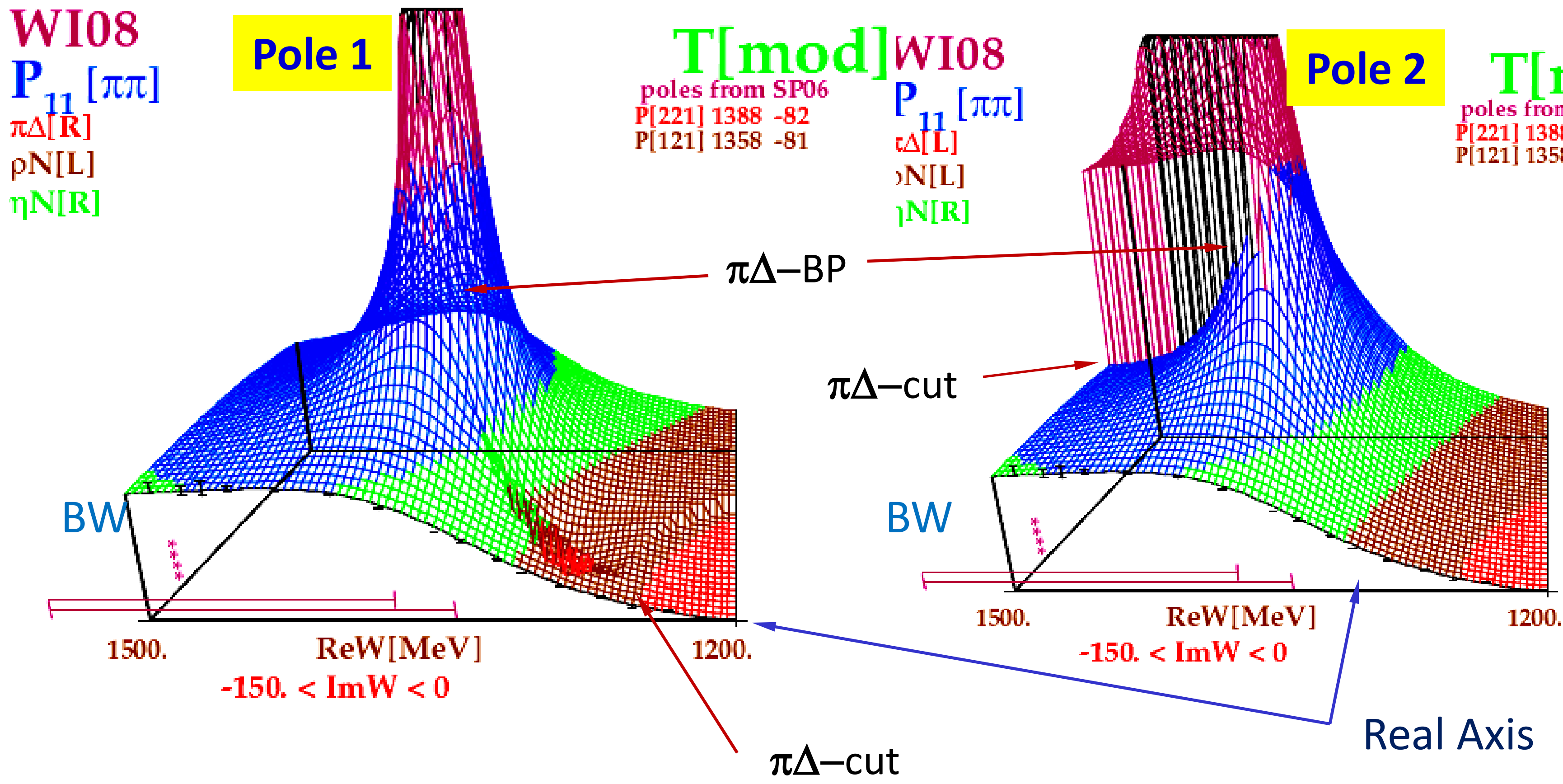
- Due to nearby πΔ Branch Point, both poles are not far from physical region

- Simple BW is not adequate to such a complex structure [2 Poles & 2 Branch-Points πΔ & ηN]

# $\pi N\ \mathcal{P}_{11}$

**Sheet 1** is the sheet reached most directly the **real axis**.

**Sheet 2** is behind the **πΔ** Branch Cut.



# $N(1710)^{***}$ – What was Known

[J. Beringer *et al* [RPP] Phys Rev D **86**, 010001 (2012)]

PDG particle data group

The latest GWU analysis (ARNDT 06) finds no evidence for this resonance. [R. Arndt, W. Briscoe, IS, R. Workman, Phys Rev C **74**, 045205 (2006)]

**No Pole, No BW, No Sp(W), No Δt(W)**

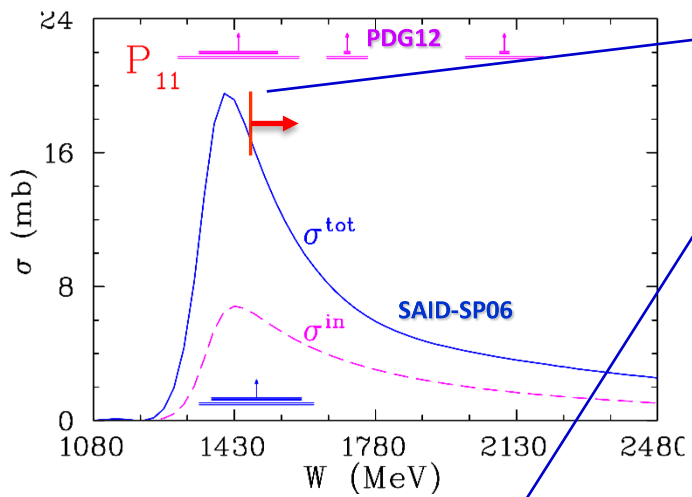
| PWA-Pole | Ref | Re(MeV) | −2xIm(MeV) |
|---|---|---|---|
| | BnGa12 | 1687 ± 17 | 200 ± 25 |
| | SAID-SP06 | | not seen |
| | KH93 | 1690 | 200 |
| | CMU90 | 1698 | 88 |
| | CMU80 | 1690 ± 20 | 80 ± 20 |

| PWA-BW | Ref | Mass(MeV) | Width(MeV) | BR |
|---|---|---|---|---|
| | BnGa12 | 1710 ± 20 | 200 ± 18 | 0.05 ± 0.04 |
| | SAID-SP06 | | not seen | |
| | CMU80 | 1700 ± 50 | 90 ± 30 | 0.20 ± 0.04 |
| | KH79 | 1723 ± 9 | 120 ± 15 | 0.12 ± 0.04 |

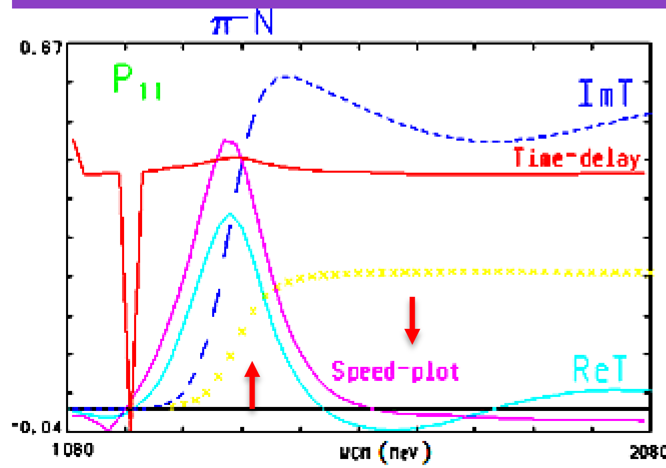
- Spread of $\Gamma$, $\Gamma_\pi/\Gamma$, & $\Gamma_\eta/\Gamma$, selected by **PDG**, is very large
- **Total width** is too large, ≥ **100** MeV

# $P_{11}$ Puzzle above $N(1440)$

[R. Arndt, W. Briscoe, M. Paris, IS, R. Workman, Chinese Phys C **33**, 1063 (2009)]



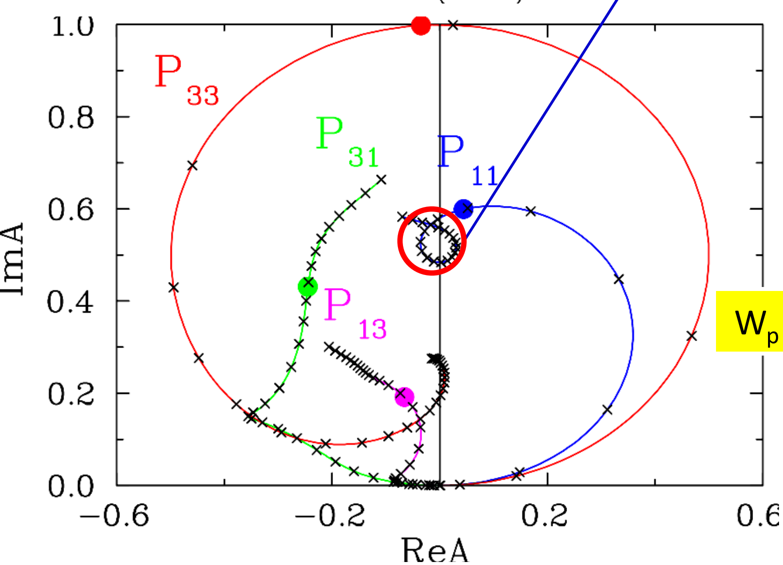

- Above W = 1500 MeV: $\sigma^t \cong 2\sigma^{el} \cong 2\sigma^{in}$ [$\sigma^t = \sigma^{el} + \sigma^{in}$]
- It means: $P_{11}$ is strongly inelastic: $\eta \rightarrow \infty$, $S \cong 0$, $A \cong i/2$, and $\delta$ is badly defined




ImT~0.5

$ImT - T^*T \geq 0$ [unitarity boundary]

ReT<|0.1|



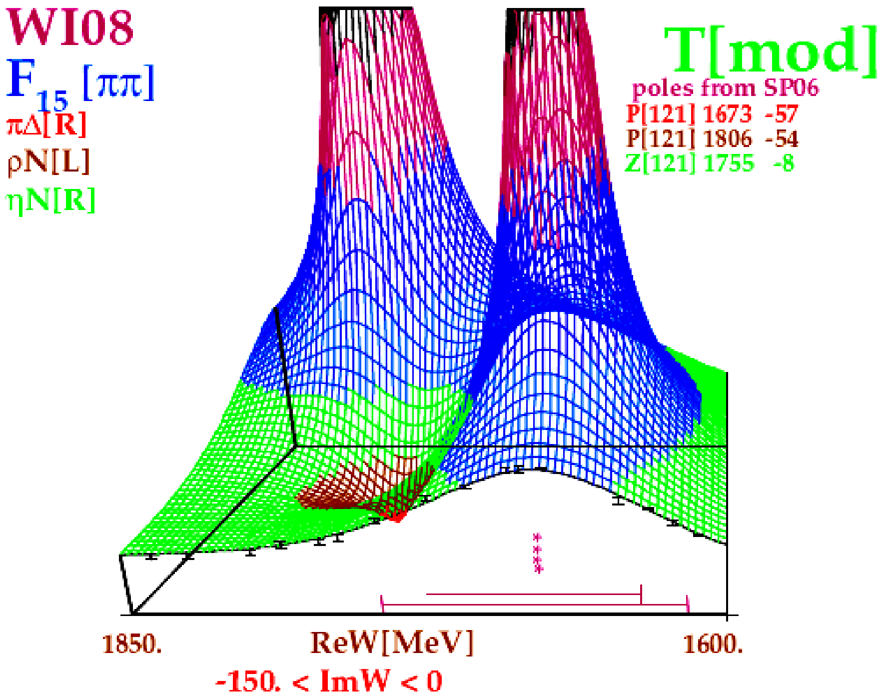

$W_p = M_R$

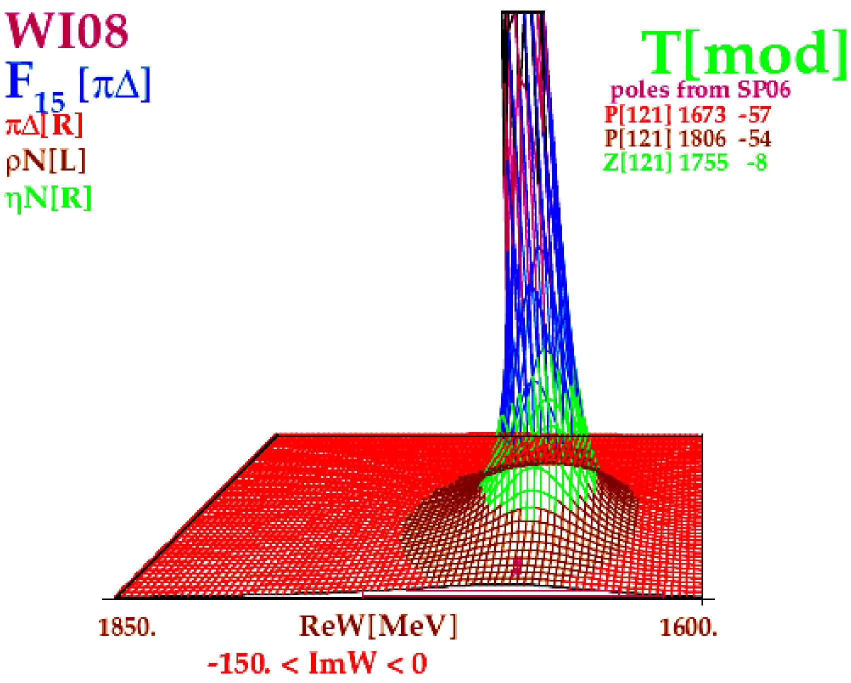
x W = 1080 [50] 2480 MeV

- Above W = 1.5 GeV, Sp(W) & Δt(W) are flat
- Sp(W) = |dT/dW| → peak at W=M (pole) at NonRes→0 [G. Hoehler, πN Newslett **7**, 94 (1992)]
- Δt(W) = dδ/dW → peak at W=M (pole) L. Eisenbud, Ph.D. Thesis, 1948

- There is no `standard' Res in $P_{11}$ above W=1500 MeV, except possible state(s) with small $\Gamma_{el}$

# πN $\mathcal{F}_{15}$

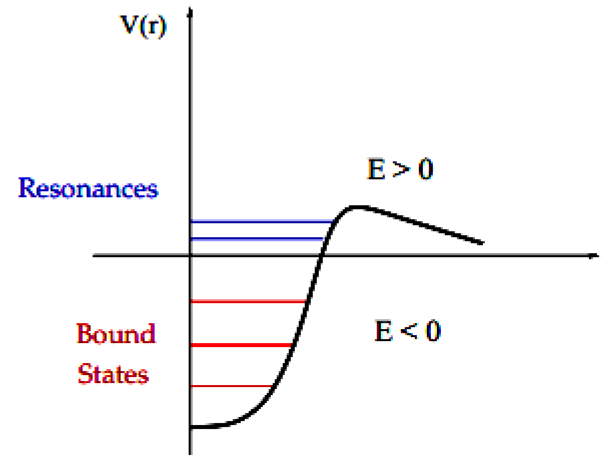


# Breit-Wigner Resonances

Negative energy state (**bound states**) are stationary states and obey the stationary Schroedinger equation.
Consider the expansion of the **phase shift** **tan $\delta_l$** for **$p \to 0$**

$$\tan\delta_\ell \xrightarrow{p\to 0} \frac{(\ell+1) - R\,\gamma_0(R)}{\ell + R\,\gamma_0(R)}\;\frac{(pR)^{2\ell+1}}{[1\cdot 3\cdot 5\cdots(2\ell-1)]^2\;(2\ell+1)}$$




The denominator vanishes for **$R\,\gamma_0(R) = -l$**.
Thus **tan $\delta_l$** $\to \propto$, i.e., **tan $\delta_l$ = $\pi/2 + n\pi$.**
This condition occurs for a specific momentum **$p_R$** at a specific energy **$E_R = p_R/2\mu$.**

Or $\tan\delta_\ell = \frac{1}{E - E_R}\frac{\Gamma_\ell}{2}$ with $\Gamma_\ell = \frac{2(pR)^{2\ell+1}}{[(2\ell-q)!!]^2\frac{d(\gamma R)}{dE}}$.

This leads to the **BW** resonance form of the amplitude $k f_\ell = e^{i\delta_\ell}\sin\delta_\ell = \frac{\Gamma_\ell/2}{E_R - E - i\Gamma_\ell/2}$

Physically, a sharp peak in the energy dependence of the cross section Indicates a dynamical origin, such as a strong attraction at that energy.
If the phase shift passes rapidly through **$\pi/2$** (**$|\pi|$**), this probably means a resonance, i.e., beam and target particle binding temporarily and then breaking up again.
Resonances are poles in **$f_l(E)$** at $E = E_R - i\Gamma_l/2$. This means $p = \sqrt{2\mu E} \approx p_r - i\frac{\mu\Gamma_l/2}{p_r}$ (for small $\Gamma_l$)

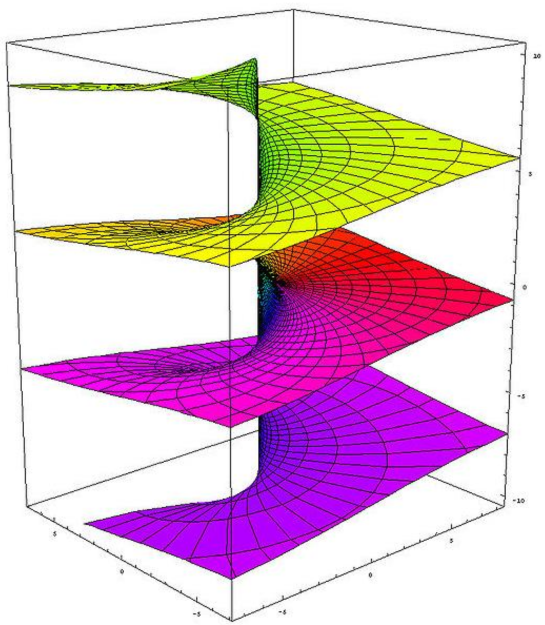
If $\Gamma_l$ is small, the pole is right below the real positive **p**-axis. When taking $E = p^2/2\mu$, bound state poles map to the **first** Riemann sheet, resonance poles move into the lower half of the **second** (unphysical) Riemann energy sheet.

# Breit-Wigner Resonances

The resonance corresponding to the field **S** is related to a pole on the (unphysical) **second Riemann sheet**. We denote this pole as

$$z = M_{pole} - i\Gamma_{pole}/2 \,,$$

where $M_{pole}$ is usually referred to as the pole mass of the resonance **S**.



A plot of the multi-valued imaginary part of the complex logarithm function, which shows the branches.
As a complex number **z** goes around the origin, the imaginary part of the logarithm goes **up** or **down**.
This makes the origin a *branch point* of the function.

$$z = re^{i\theta}$$
$$\ln z = \ln r + i\theta$$