

Partial-Wave Analysis of Pion Scattering Reactions

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- Spectroscopy of Baryons.
- **SAID** for Baryon Spectroscopy.
- πN Elastic Scattering.
 - $N(1440)1/2^+$.
- πN inElastic Scattering.
 - $\pi^- p \rightarrow \eta p$.
 - $\pi^- p \rightarrow KY$.
 - $\pi \rightarrow 2\pi$.
- Pion Photoproduction.
- Pion Electroproduction.



Spectroscopy of Baryons



10/6/2013

RRTF Workshop, Darmstadt, Germany, Oct 2013

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A bit of History...

PHYSICAL REVIEW

VOLUME 91, NUMBER 1

JULY 1, 1953

Angular Distribution of Pions Scattered by Hydrogen*

H. L. ANDERSON, E. FERMI, R. MARTIN, AND D. E. NAGLE
Institute for Nuclear Studies, University of Chicago, Chicago, Illinois

(Received March 6, 1953)



The results have been interpreted in terms of phase shift analysis on the assumption that the scattering is mainly due to states of isotopic spins $\frac{3}{2}$ and $\frac{1}{2}$ and angular momenta s_1 , p_1 and p_1 .

PHYSICS REPORTS (Review Section of Physics Letters) 96, Nos. 2 & 3 (1983) 71-204. North-Holland Publishing Company

BARYON SPECTROSCOPY

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Received 29 September 1982

In 1952 Fermi and coworkers (Andersen et al. [1952]) discovered the first baryon resonance – the $\Delta(1238)$. Since then, hundreds of resonances have been identified and nuclear democracy has given way to fundamental quarks. Baryon spectroscopy is now thirty years old and perhaps approaching a mid-life crisis.

For it is inevitable in such a fast-moving field as high energy particle physics, that experiments have moved on beyond the resonance region to higher energies and different priorities. Thus it is probably no exaggeration to say that we now have essentially *all* the experimental data relevant to the low-energy baryon spectrum, that we are *ever* likely to obtain. It is therefore timely to review both the accumulated mass of resonance data, together with the techniques used in its analysis, and also our theoretical framework for understanding the results. The latter is inevitably based on quarks and, by and large, on a

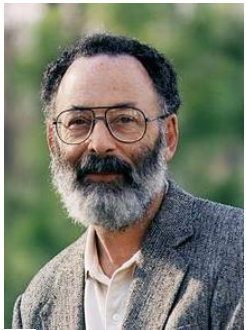


Spectroscopy of Baryons



“It is clear that we still need much more information about the existence and parameters of many baryon states, especially in the $N=2$ mass region, before this question of non-minimal $SU(6) \times O(3)$ super-multiplet can be settled.” **Dick Dalitz, 1976.**

“The first problem is the notion of a resonance is not well defined. The ideal case is a narrow resonance far away from the thresholds, superimposed on slowly varying background. It can be described by a Breit-Wigner formula and is characterized by a pole in the analytic continuation of the partial wave amplitude into the low half of energy plane.” **Gerhard Hoehler, 1987.**



“Why N^* s are important – The *first* is that nucleons are the stuff of which our world is made. My *second* reason is that they are simplest system in which the quintessentially non-Abelian character of QCD is manifest. The *third* reason is that history has taught us that, while relatively simple, Baryons are sufficiently complex to reveal physics hidden from us in the mesons.” **Nathan Isgur, 2000.**



p	$1/2^+$	****	$\Delta(1232)$	$3/2^+$	****	Σ^+	$1/2^+$	****	Ξ^0	$1/2^+$	****	Λ_c^+	$1/2^+$	****
n	$1/2^+$	****	$\Delta(1600)$	$3/2^+$	***	Σ^0	$1/2^+$	****	Ξ^-	$1/2^+$	****	$\Lambda_c(2595)^+$	$1/2^-$	***
$N(1440)$	$1/2^+$	****	$\Delta(1620)$	$1/2^-$	****	Σ^-	$1/2^+$	****	$\Xi(1530)$	$3/2^+$	****	$\Lambda_c(2625)^+$	$3/2^-$	***
$N(1520)$	$3/2^-$	****	$\Delta(1700)$	$3/2^-$	****	$\Sigma(1385)$	$3/2^+$	****	$\Xi(1620)$	$3/2^+$	****	$\Lambda_c(2765)^+$	*	
$N(1535)$	$1/2^-$	****	$\Delta(1750)$	$1/2^+$	*	$\Sigma(1480)$	*		$\Xi(1690)$	$3/2^-$	***	$\Lambda_c(2880)^+$	$5/2^+$	***
$N(1650)$	$1/2^-$	****	$\Delta(1900)$	$1/2^-$	**	$\Sigma(1560)$	**		$\Xi(1820)$	$3/2^-$	***	$\Lambda_c(2940)^+$	****	
$N(1675)$	$5/2^-$	****	$\Delta(1905)$	$5/2^+$	****	$\Sigma(1580)$	$3/2^-$	*	$\Xi(1950)$	$3/2^-$	***	$\Sigma_c(2455)$	$1/2^+$	****
$N(1680)$	$5/2^+$	****	$\Delta(1910)$	$1/2^+$	****	$\Sigma(1620)$	$1/2^-$	**	$\Xi(2030)$	$3/2^+$	****	$\Sigma_c(2520)$	$3/2^+$	****
$N(1685)$	*		$\Delta(1920)$	$3/2^+$	**	$\Sigma(1660)$	$1/2^+$	***	$\Xi(2250)$	$3/2^+$	***	$\Sigma_c(2800)$	****	
$N(1700)$	$3/2^-$	***	$\Delta(1930)$	$5/2^-$	**	$\Sigma(1670)$	$3/2^-$	****	$\Xi(2250)$	**		Ξ_c^+	$1/2^+$	***
$N(1710)$	$1/2^+$	***	$\Delta(1940)$	$3/2^+$	**	$\Sigma(1690)$	$3/2^-$	****	$\Xi(2370)$	**		Ξ^0	$1/2^+$	***
$N(1720)$	$3/2^+$	***	$\Delta(1950)$	$7/2^-$	**	$\Sigma(1750)$	$1/2^-$	****	$\Xi(2500)$	**		Ξ_c^-	$1/2^+$	***
$N(1860)$	$1/2^+$	***	$\Delta(2000)$	$5/2^+$	*	$\Sigma(1770)$	$3/2^+$	****	$\Omega(1370)^-$	$3/2^+$	*	Ξ_c^0	$1/2^+$	****
$N(1870)$	$1/2^-$	***	$\Delta(2100)$	$1/2^-$	**	$\Sigma(1775)$	$5/2^+$	****	$\Omega(1670)^-$	$3/2^+$	*	$\Xi_c(2645)$	$3/2^+$	****
$N(1880)$	$1/2^-$	***	$\Delta(2200)$	$1/2^-$	**	$\Sigma(1840)$	$3/2^+$	*	$\Omega(1700)^-$	$3/2^+$	*	$\Xi_c(2790)$	$1/2^-$	****
$N(1890)$	$1/2^+$	***	$\Delta(2300)$	$9/2^+$	**	$\Sigma(1880)$	$1/2^+$	**	$\Omega(1700)^-$	$3/2^+$	*	$\Xi_c(2815)$	$3/2^-$	****
$N(1900)$	$3/2^+$	***	$\Delta(2350)$	$5/2^-$	*	$\Sigma(1900)$	$1/2^+$	****	$\Omega(1700)^-$	$3/2^+$	*	$\Xi_c(2930)$	*	
$N(1990)$	$7/2^-$	**	$\Delta(2390)$	$7/2^+$	*	$\Sigma(1940)$	$3/2^-$	***	$\Omega(1700)^-$	$3/2^+$	*	$\Xi_c(2980)$	****	
$N(2000)$	$5/2^+$	**	$\Delta(2400)$	$9/2^-$	**	$\Sigma(2000)$	$1/2^-$	*	$\Omega(1700)^-$	$3/2^+$	*	$\Xi_c(3055)$	**	
$N(2040)$	$3/2^+$	*	$\Delta(2420)$	$11/2^+$	****	$\Sigma(2030)$	$7/2^+$	****	$\Omega(1700)^-$	$3/2^+$	*	$\Xi_c(3080)$	****	
$N(2060)$	$5/2^-$	**	$\Delta(2470)$	$13/2^-$	**	$\Sigma(2070)$	$5/2^+$	*	$\Omega(1700)^-$	$3/2^+$	*	$\Xi_c(3123)$	*	
$N(2100)$	$1/2^+$	*	$\Delta(2490)$	$15/2^+$	**	$\Sigma(2080)$	$3/2^+$	**	$\Omega(1700)^-$	$3/2^+$	*	Ω_c^0	$1/2^+$	****
$N(2120)$	$3/2^-$	**				$\Sigma(2100)$	$7/2^-$	*	$\Omega_c^0(2770)^0$	$3/2^+$	****	Ω_c^+	$1/2^+$	****
$N(2190)$	$7/2^-$	****	Λ	$1/2^+$	****	$\Sigma(2250)$	****		Ξ_c^+	*		Ξ_c^0	$1/2^+$	****
$N(2220)$	$9/2^+$	****	$\Lambda(1405)$	$1/2^-$	****	$\Sigma(2455)$	**		Ξ_c^-	*		Λ_b^0	$1/2^+$	****
$N(2250)$	$9/2^-$	****	$\Lambda(1520)$	$3/2^-$	****	$\Sigma(2620)$	**		Ξ_c^0	*		Σ_b^+	$1/2^+$	****
$N(2600)$	$11/2^-$	***	$\Lambda(1600)$	$1/2^+$	***	$\Sigma(3000)$	*		Ξ_c^+	*		Σ_b^0	$3/2^+$	****
$N(2700)$	$13/2^+$	**	$\Lambda(1670)$	$1/2^-$	****	$\Sigma(3170)$	*		Ξ_c^0	*		Ξ_b^+	$1/2^+$	****
			$\Lambda(1690)$	$3/2^-$	****				Ξ_c^+	*		Ξ_b^0	$1/2^+$	****
			$\Lambda(1800)$	$1/2^-$	****				Ξ_c^0	*		Ξ_b^-	$1/2^+$	****
			$\Lambda(1810)$	$1/2^+$	****				Ξ_c^+	*		Ω_b^-	$1/2^+$	****
			$\Lambda(1820)$	$5/2^+$	****									
			$\Lambda(1830)$	$5/2^-$	****									
			$\Lambda(1870)$	$3/2^+$	****									
			$\Lambda(2000)$	*										
			$\Lambda(2020)$	$7/2^+$	*									
			$\Lambda(2100)$	$7/2^-$	****									
			$\Lambda(2110)$	$5/2^+$	****									
			$\Lambda(2325)$	$3/2^-$	*									
			$\Lambda(2350)$	$9/2^+$	****									
			$\Lambda(2585)$	**										

- PDG12 has **112** Baryon Resonances (58 4^* & 3^* of them).
- For example for $SU(6) \times O(3)$, it would be **434** resonances, if all revealed
three 70- and **four 56-** multiplets were filled in.



- There are many **more states** in the QCD inspired models than currently observed.

- A quick check of the PDG Listings reveals that resonance parameters of many established states are not well determined.

There are Many Ways to Study N^*

Prolific source of N^* & Δ^* baryons

Measure many channels with different combinations of quantum numbers.

$\pi N \rightarrow \pi N, \pi\pi N, \dots$

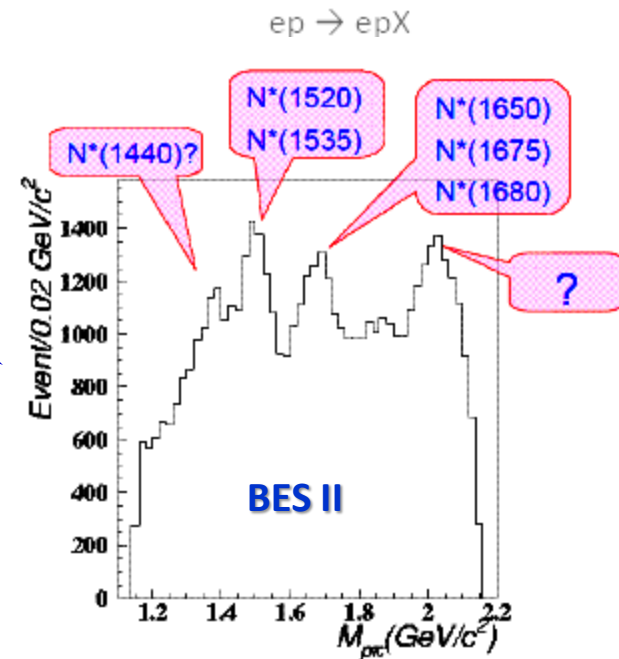
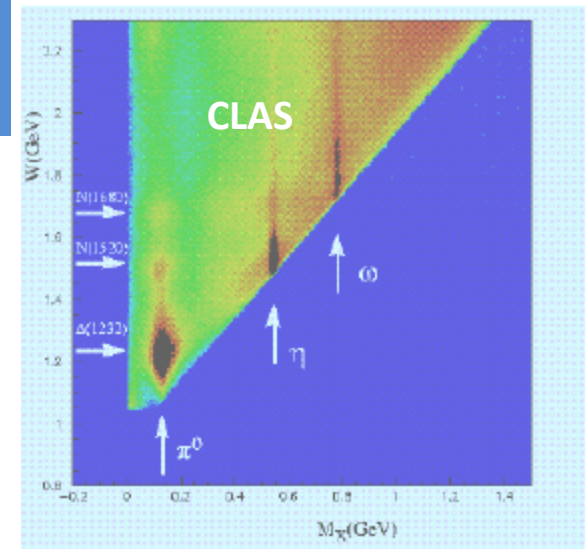
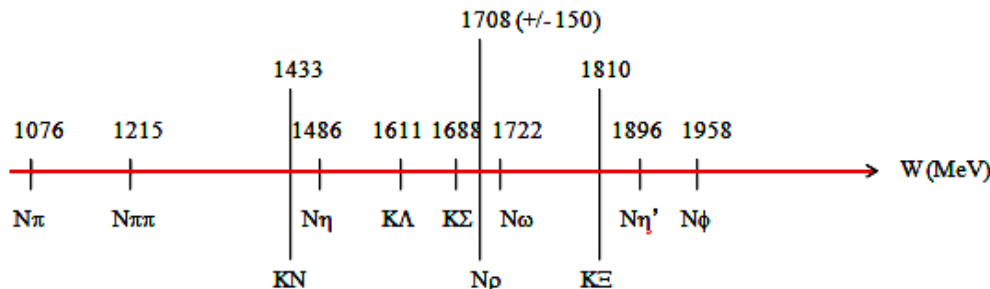
$\gamma N \rightarrow \pi N, \pi\pi N, \dots$

$\gamma^* N \rightarrow \pi N, \pi\pi N, \dots$

$pp \rightarrow pp\pi^0, pp\pi\pi, \dots$

$J/\psi \rightarrow p\bar{p}\pi^0, p\bar{n}\pi^-, \dots$

- Most of **PDG** info comes from these sources and **PWA** is main of them.
- πN elastic scattering is highly constrained.
- Resonance structure is correlated.
- Two-body final state, fewer amplitudes.



PWA for non-Strange Baryons & SAID Database

Originally: PWA arose as the technology to determine amplitude of the reaction via fitting scattering data which is a non-trivial mathematical problem

[*Solution of ill-posed problem*

– Hadamard, Tikhonov, *et al*]

Resonances appeared as a by-product

[Bound states objects with definite quantum numbers, mass, lifetime, *etc*]

That is the strategy of the
GW/VPI πN PWA since 1987



Below 4 GeV

Partial-Wave Analyses at GW

[See Instructions]

31,479

Pion-Nucleon

Pion-Pion-Nucleon

Kaon-Nucleon

Nucleon-Nucleon

Pion Photoproduction

Pion Electroproduction

Kaon Photoproduction

Eta Photoproduction

Eta-Prime Photoproduction

Pion-Deuteron (elastic)

Pion-Deuteron to Proton+Proton

[W = 1320 to 1930 MeV]

241,214 evts

38,162

113,900

6,235

1,914

5,267

27,265

9,086

1,030

6,083

For $\pi \rightarrow 2\pi$, we use log-likelihood while for the rest – least-squares technologies.



SQSD for Baryon Spectroscopy

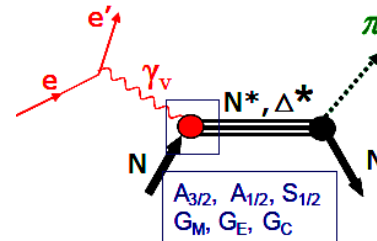


N^* and Δ^* States coupled to πN

[SAID: <http://gwdac.phys.gwu.edu/>]

- GW SAID N^* program consists of $\pi N \rightarrow \pi N \longrightarrow \gamma N \rightarrow \pi N \longrightarrow \gamma^* N \rightarrow \pi N$
As was established by Dick Arndt on 1997.

Assuming dominance of 2-hadronic channels
[πN elastic & $\pi^- p \rightarrow \eta n$], we parameterize
 $\gamma^* N \rightarrow \pi N$ in terms of $\pi N \rightarrow \pi N$ amplitudes.



- One of the most convincing ways to study a non-strange baryon Spectroscopy [a key to our understanding of QCD] is πN PWA.

- Non-strange objects in the PDG Listings come mainly from: Karlsruhe-Helsinki, Carnegie-Mellon-Berkeley, GW, & BnGa now.
- The main source of EM couplings is the GW & BnGa analyses.



CNS DAC Home
▶ CNS DAC [SAID]
CNS Home
TUTORIALS



Partial-Wave Analyses at GW

[See Instructions]

Pion-Nucleon
Pi-Pi-N (under construction)
Kaon-Nucleon
Nucleon-Nucleon
Pion Photoproduction
Pion Electroproduction
Kaon Photoproduction
Eta Photoproduction
Eta-Prime Photoproduction
Pion-Deuteron (elastic)
Pion-Deuteron to Proton+Proton

Analyses From Other Sites

Mainz (MAID – Analyses)
Nijmegen (Nucleon-Nucleon OnLine)
Bonn-Gatchina (PWA)

Contact

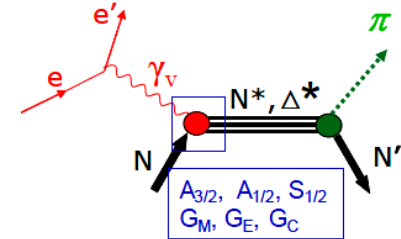
William J. Briscoe
Ron L. Workman
Igor I. Strakovsky

Igor Strakovsky 9



GW N^* Program

- Energy dependent **SM08** and associated **SES & SQS**
- $W = 1080 - 2000 \text{ MeV}$ $Q^2 = 0 - 6 \text{ GeV}^2$
- PWs = 60 [multipoles] $[J < 6]$
- Prms = 171
- **Constraint:** πN + Pion Photo Prod PWAs [no theoretical input]



0.85 World Electro Prod from JLab CLAS

PWA Problems:

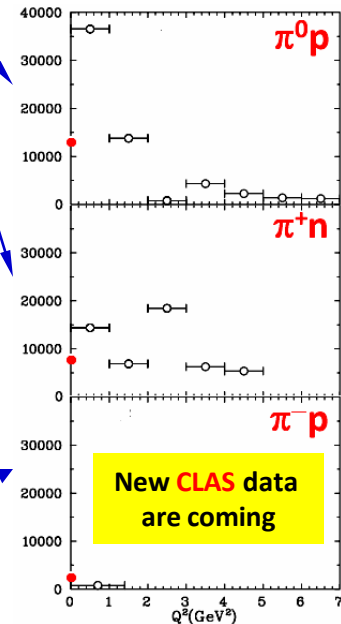
- Additional [18 S] Multipoles
- Q^2 dependence

Database Problems:

- Most of data are **unPolarized** measurements
- There are no $\pi^0 n$ data and very few $\pi^- p$ [no Pol measurements] That does not allow to determine **n-couplings** at $Q^2 > 0$

Reaction	Data	χ^2
$\gamma^* p \rightarrow \pi^0 p$	55,766	81,284
$\gamma^* p \rightarrow \pi^+ n$	51,312	80,004
Redundant	14,772	17,375
Total	121,850	178,663
$\gamma N \rightarrow \pi N$	25,358	53,458
All Photo*	147,208	232,121
$\pi N \rightarrow \pi N$	31,479	57,157
All πN	178,687	289,278
$\gamma^* n \rightarrow \pi^- p$	801	
$\gamma^* n \rightarrow \pi^0 n$	No Data	

Q^2 -Data



πN Elastic Scattering



SAID for $\pi N \rightarrow \pi N$ & $\pi^- p \rightarrow \eta n$

[R. Arndt, W. Briscoe, I.S., R. Workman, Phys Rev C 74, 045205 (2006)]

- Energy dependent **SPO6/WI08** and associated **SES**
- $T = 0 - 2600$ MeV
- **4**-channel Chew-Mandelstam **K-matrix** parameterization
- 3 mapping variables: $g^2/4\pi$, $a[\pi p]$, E_{th}
- PWs = 30 πN {15 [I=1/2] + 15 [I=3/2]} + 4 ηN
- Prms = 99 [I=1/2] + 89 [I=3/2]

[W = 1078 - 2460 MeV]
 [πN , $\pi\Delta$, ρN , ηN]

[I < 9]



• **1st generation** ('57-'79)
 Used by CMB79 and KH84 analyses.
 10k $\pi^\pm p$ each & 1.5k CXS.
 17% data is polarized.

• **2nd generation** ('80-'06)
 SAID fits:
 13k $\pi^\pm p$ each, 3k CXS & 0.3k $\pi^- p \rightarrow \eta n$
 25% data is polarized.
 Meson Factories [LAMPF, TRIUMF, & PSI] are the main source of new measurements.
 There is no discrimination against data

• **3rd generation** (07'+)
 New data may come from
J-PARC, HADES, EPECUR, etc

Reaction	Data	χ^2
$\pi^+ p \rightarrow \pi^+ p$	13,354	27,136
$\pi^- p \rightarrow \pi^- p$	11,978	22,632
$\pi^- p \rightarrow \pi^0 n$	3,115	6,068
$\pi^- p \rightarrow \eta n$	257	650
DR constraint	2,775	671
Total	31,479	57,157



[0 - 2600 MeV] \rightarrow 10 data/MeV

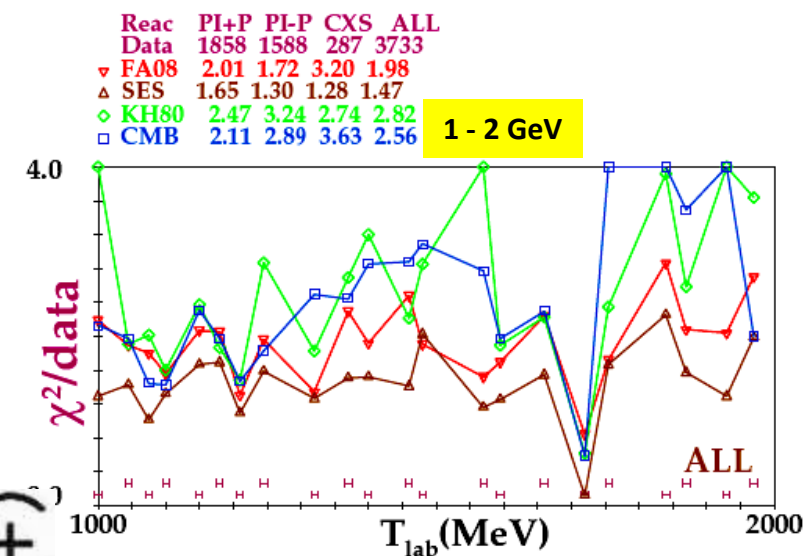
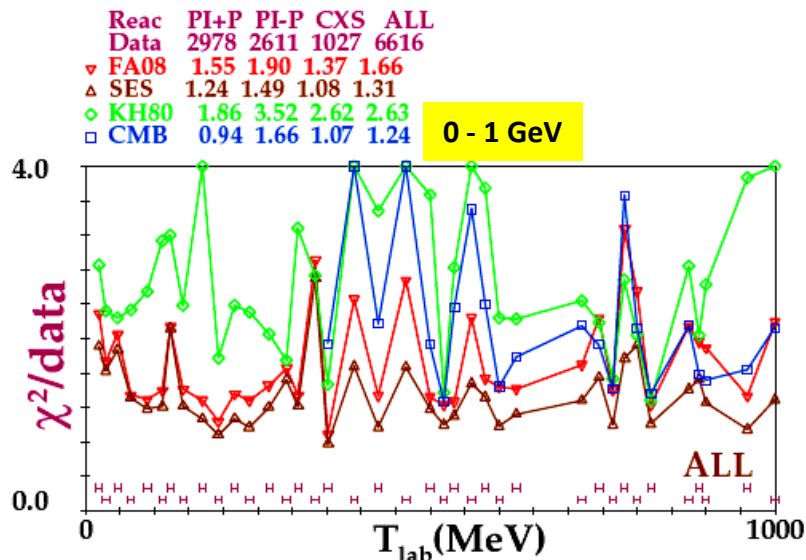
[550 - 800 MeV] \rightarrow 1 data/MeV

27 σ^{tot} & 37 P data
 above 800 MeV \rightarrow 0.03 data/MeV

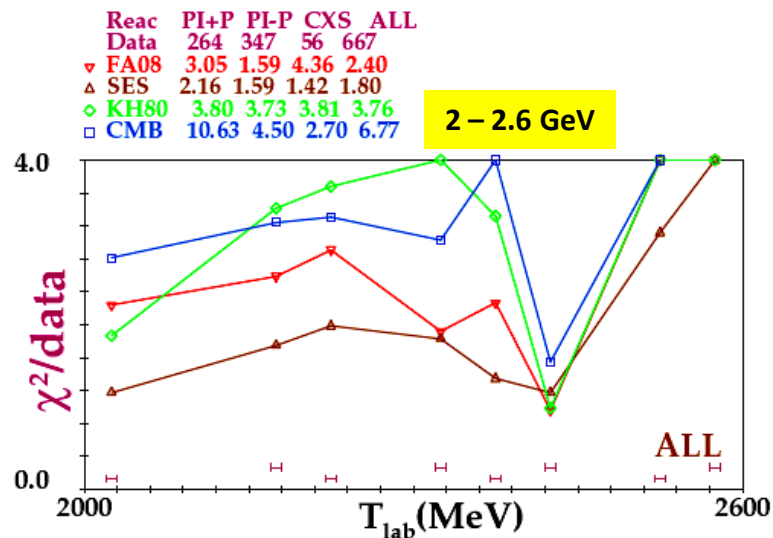
DRs have been derived from the *first principles*.

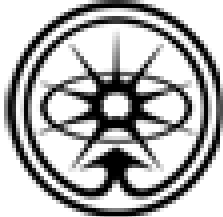


Fit to πN Elastic Scattering Data



- Some of structures in **35-year** old solutions.
- [KH and CMB] are still considered as resonances.
- $W > 1.5$ GeV is less constrained by data.





New Observables

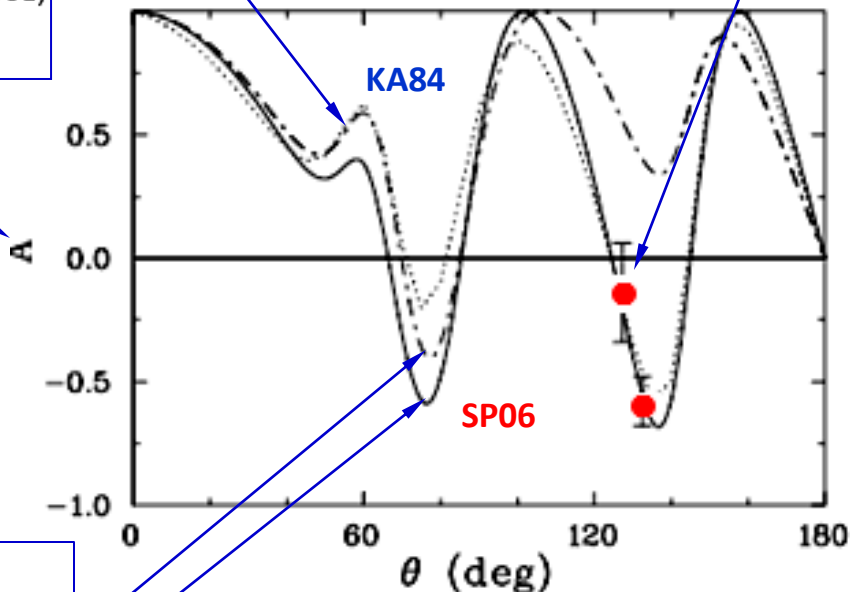
[R. Arndt, W. Briscoe, IS, R. Workman, Phys Rev C 74, 045205 (2006)]

Some **Old** solutions may be not able to reproduce **New** measurements

πN scattering data:

$d\sigma/d\Omega$ (unpolarized)
P (polarized target or recoil nucleon)
R and A (polarized target and recoil measured)

Not Independent: $P^2 + R^2 + A^2 = 1$



$\pi^+ p$: 1300 MeV

Data:

ITEP: $\pi^+ p \rightarrow \pi^+ p$ @ 1300 MeV
[I. Alekseev *et al* Phys Lett B 351, 585 (1995)]

PWA:

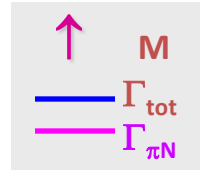
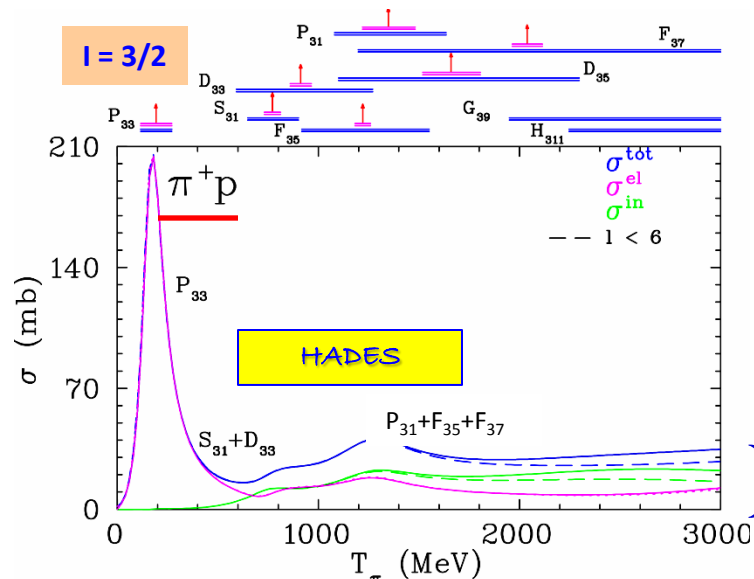
KA84: Karlsruhe-Helsinki fit, 1984
KB84: KH Barrelet corrected solution, 1997
SP06: GW fit, 2006

Polarized measurements is an important part of a hadron program



Where is a Resonance ?

[R. Arndt, W. Briscoe, IS, R. Workman, Phys Rev C 74, 045205 (2006)]



Here you are...

To determine Resonance parameters with SAID:

- Fit data to get $T_{\pi\pi}$
- Fit $T_{\pi\pi}$ over narrow W range
 \rightarrow Pole position
- Then fit DATA over same W range
 \rightarrow BW parameters

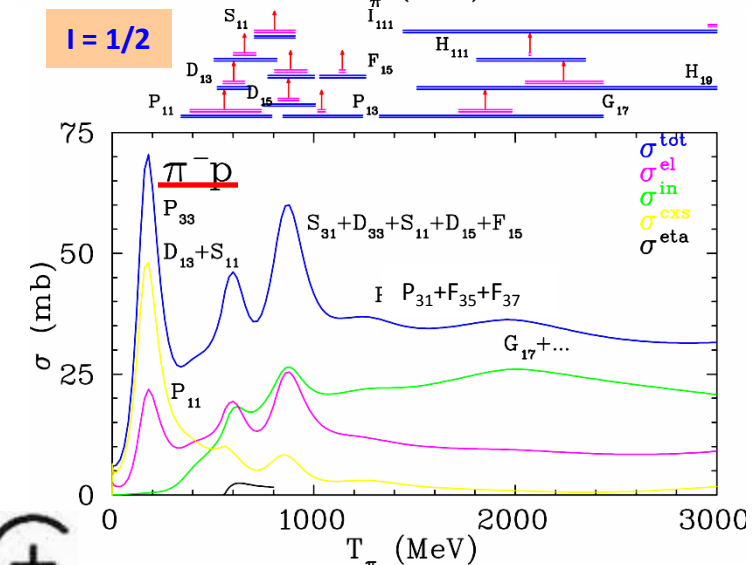
Main techniques:

- Pole on complex energy plane
- BW fit (data, SES, or global)

Assertions:

- Argand plot, $\text{Im}(\text{Re})$
- Speed plot, $\text{Sp}(W) = |dT/dW|$
- Time-delay, $\tau = d\delta/dW$
- Crossover energy, $\text{Re}A = 0$
- etc

- Below $W = 1900$ MeV,
 The highest wave contribution is small.



GW DAC Search for \mathcal{N}^* and Δ^*

- We are considering a resonance as a **Pole** in the complex plane which is not far away from the physical axis.

- Applied directly to the data via **BW** + **Bckgr**

- Assume: $S \rightarrow S_R S_B$

$$S_R = 1 + 2iT_R$$

$$T_R = (\Gamma_e/2) / [W_R - W - i(\Gamma_e/2 + \Gamma_I/2)]$$

$$\Gamma = \Gamma_e + \Gamma_I \quad \Gamma_e = \rho_e \Gamma R \quad \Gamma_I = \rho_i \Gamma (1 - R)$$

$$T_B = K_B(1 - iK_B)^{-1} \quad K_B = a + b(W - W_R) + c$$

- Map $\chi^2[W_R, \Gamma]$ while searching all other **PW** parameters
Look for **significant** improvement

- Subjective variables are

- Energy binning
- Strength of constraints
- Which **PW** to be searched

- Standard PWA

- Tends (by construction) to miss narrow Resonances with $\Gamma < 30$ MeV
- Reveals only wide Resonances, but not too wide [$\Gamma < 500$ MeV]
and possessing not too small BR [$BR > 0.04$]

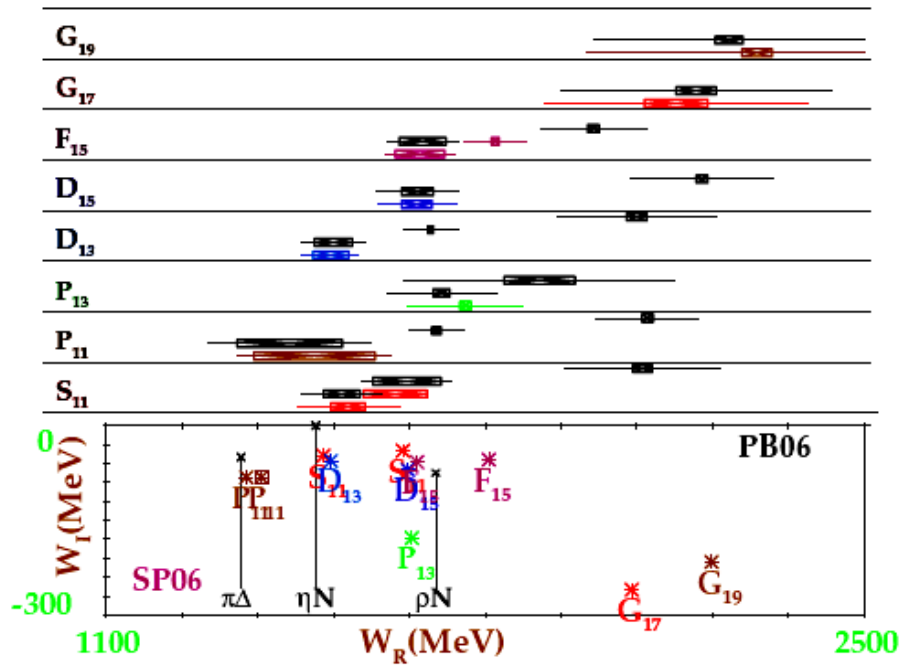
- Modified PWA

- Allows to put a resonance by hands with subsequent refitting the data
Then the search will allow to see how reliable/tolerable it is

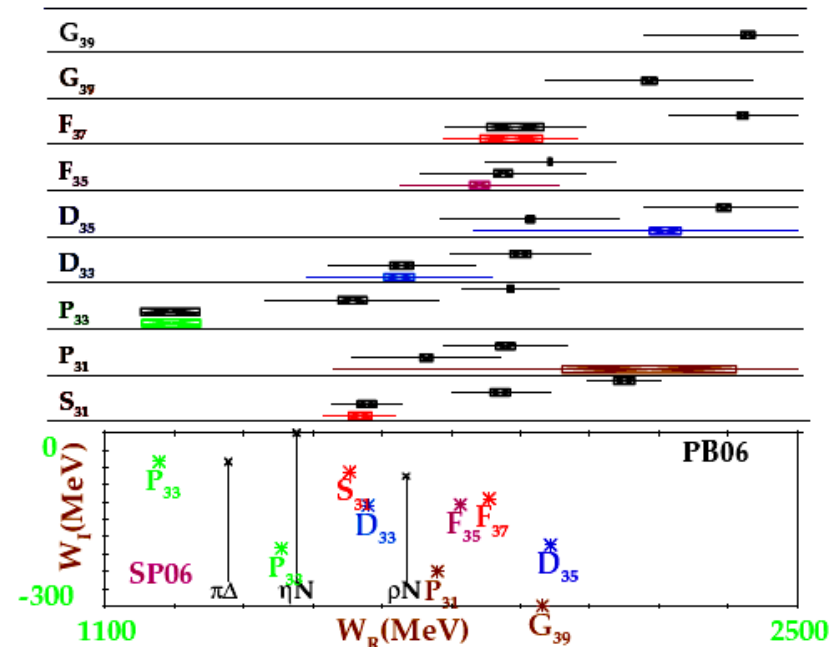


Complex Energy Plane vs BW fits

$l = 1/2$



$l = 3/2$



• There are shifts between Pole and BW prms.

$$M = \text{Re}W_p$$

$$\Gamma = 2 * \text{Im}W_p$$

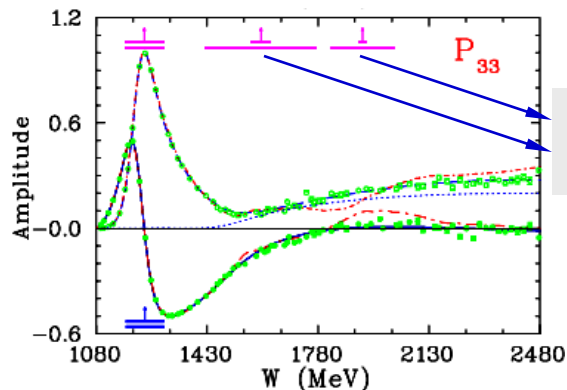
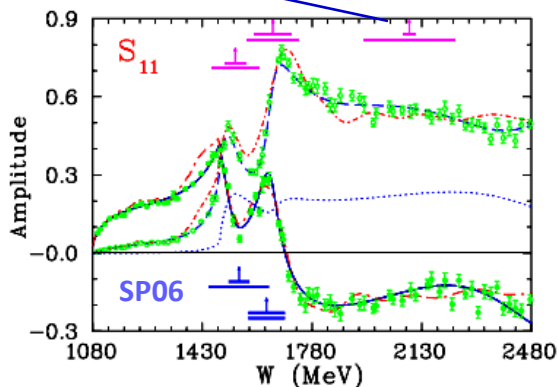


Partial Waves [$\mathcal{L}_{(2I)(2J)}$]

[R. Arndt, W. Briscoe, IS, R. Workman, Phys Rev C 74, 045205 (2006)]

Overall: the difference between **KA'84** and **GW'08** is rather small but... resonances may be essentially different

N(2090)*

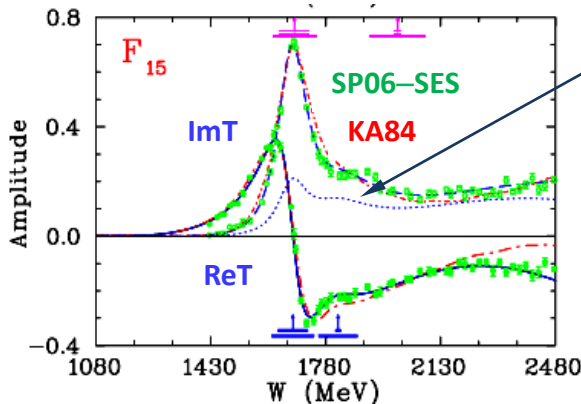
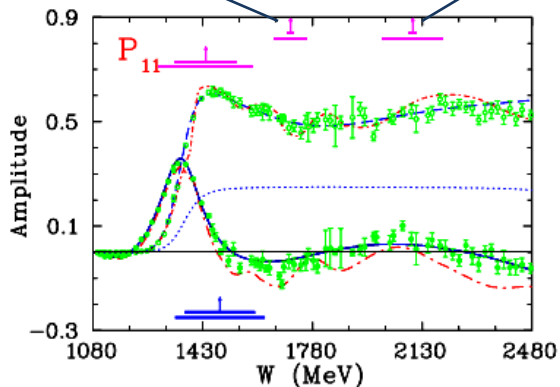


$\Delta(1920)***$
 $\Delta(1600)***$

1964: 1.2k data below 700 MeV
2012: 32k data below 2600 MeV

N(1710)***

N(2100)*



$\text{Im}T \cdot T^* \geq 0$ [unitarity boundary]

There are several SESs which one can possibly consider as samples of complete experiments.

Pion PhotoProduction complete experiments are under the consideration now.

PDG12 [J. Beringer *et al* [PDG] J Phys D 86, 010001 (2012)]

KA84 [R. Koch, Z Phys C 29, 597 (1985)]



$\mathcal{O}(1440)1/2^+$



Courtesy of Donna Arndt, 2010



$N(1440)^{****}$ – What is Known

[J. Beringer *et al* / [PDG] Phys Rev D 86, 010001 (2012)]



Two-faced Janus
Roman God of Gates & Doors

• **Dick Arndt:** “This is one of mysterious Resonances”

PDG	PWA-Pole	Ref	Re(MeV)	$-2\text{Im}(\text{MeV})$	
	BnGa12		1370 ± 4	190 ± 7	
	SAID-SP06		1359	164	1 st Riemann sheet
			1388	166	2 nd Riemann sheet
	KH93		1385	164	
	CMU80		1375 ± 30	180 ± 40	

PDG	PWA-BW	Ref	Mass(MeV)	Width(MeV)	BR
	BnGa12		1430 ± 8	365 ± 35	0.62 ± 0.03
	SAID-SP06		1485 ± 1.2	284 ± 18	0.787 ± 0.016
	KSU92		1462 ± 10	391 ± 34	0.69 ± 0.03
	CMU80		1440 ± 30	340 ± 70	0.68 ± 0.04
	KH79		1410 ± 12	135 ± 10	0.51 ± 0.05



$$f(E) = \frac{k}{(E^2 - M^2)^2 + M^2\Gamma^2}$$

$M = \text{Re}W_p$
 $\Gamma = 2\text{Im}W_p$



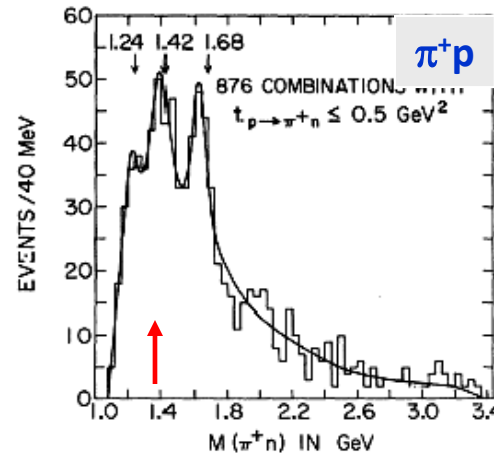
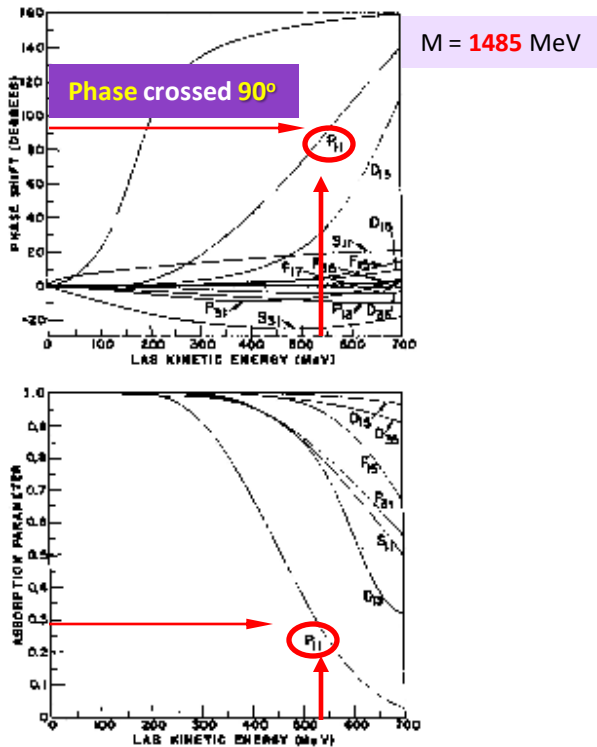
Discovery and First Direct Measurements of $N(1440)$

- Just after the $N(1440)$ discovery via πN PWA. [L.D. Roper, *Phys Rev Lett* **12**, 340 (1964)]

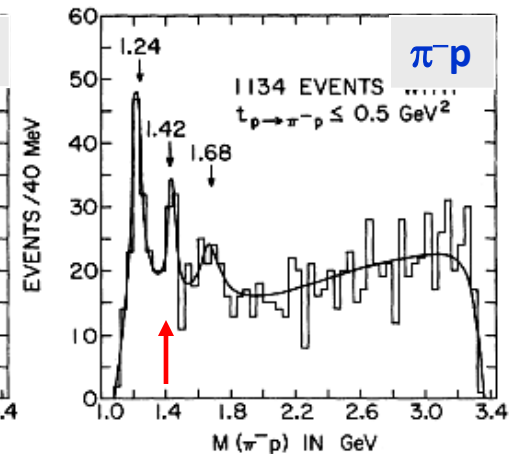
- Several direct searches got a signal. [R.B. Bell et al *Phys Rev Lett* **20**, 164 (1968)]

BROOKHAVEN
NATIONAL LABORATORY

- BNL-LHBC: $\pi^\pm p$ at 6 GeV/c



$M = 1405 \pm 30$ MeV $\Gamma = 100$ MeV
Significance $[N_s/\sqrt{(N_b+N_s)}] = 3.1 \sigma$



$M = 1436 \pm 20$ MeV $\Gamma = 50$ MeV
Significance $[N_s/\sqrt{(N_b+N_s)}] = 2.8 \sigma$

Courtesy of Cole Smith, June 2005

- Both BNL masses are less than $M = 1485$ MeV determined originally via πN PWA and by $SP06$ BW



Complex Energy Plane for P_{11}

[R. Arndt, W. Briscoe, IS, R. Workman, Phys Rev C 74, 045205 (2006)]

• **BW:** $W_R = 1485.0 \pm 1.2$ MeV
 $\Gamma = 248 \pm 18$ MeV

Branch-points:

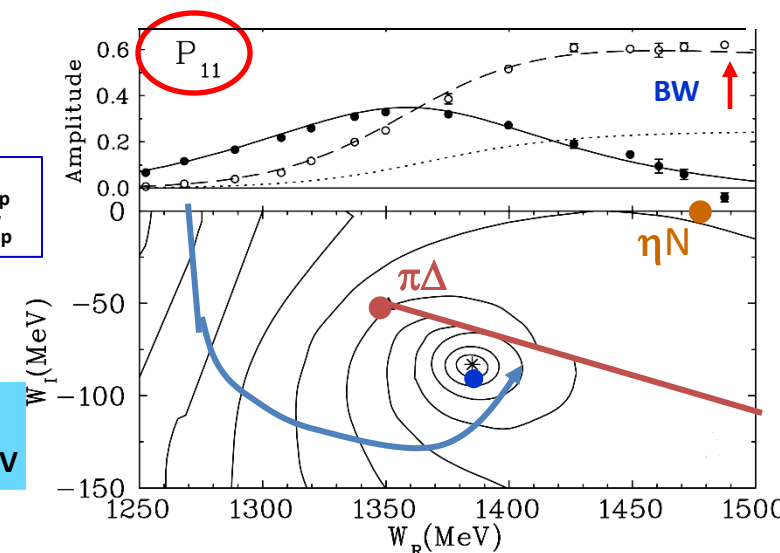
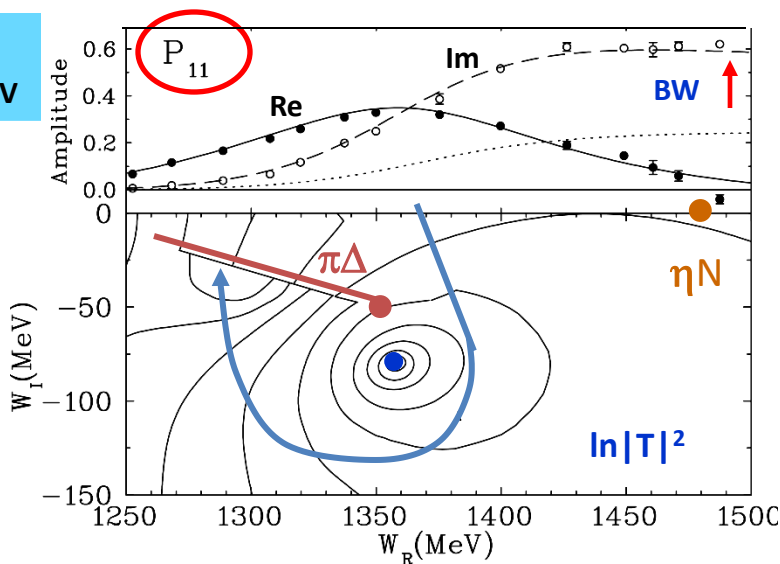
- $\pi\Delta$ thr [$W = 1350 - i50$ MeV]
- ηN thr [$W = 1487 - i0$ MeV]
- $\pi\Delta$ Branch Cut is two-body and has 2 Riemann sheets

- **Sheet 1** is the sheet reached most directly the **real axis**
- **Sheet 2** is behind the $\pi\Delta$ Branch Cut

- **N(1440)** is a Resonance which manifests itself via **2 Poles** at **2 different Riemann sheets** (with respect to the $\pi\Delta$ cut)

- Due to nearby $\pi\Delta$ Branch Point, both **poles** are not far from physical region

- Simple BW is not adequate to such a complex structure
[2 Poles & 2 Branch-Points $\pi\Delta$ & ηN]



• **1st Riemann sheet**
Pole 1: $W_p = 1359 - i82$ MeV

• There is a **shift** between Pole positions at **two sheets**, due to a non-zero jump on the $\pi\Delta$ -cut

$$M = \text{Re}W_p$$

$$\Gamma = 2 * \text{Im}W_p$$

• **2nd Riemann sheet**
Pole 2: $W_p = 1388 - i83$ MeV



Two Pole Observation

PHYSICAL REVIEW D

VOLUME 32, NUMBER 5

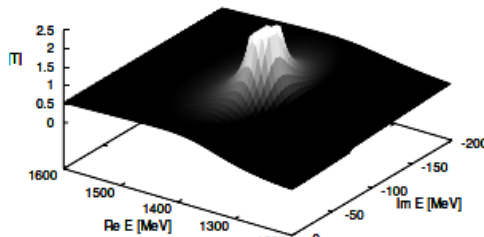
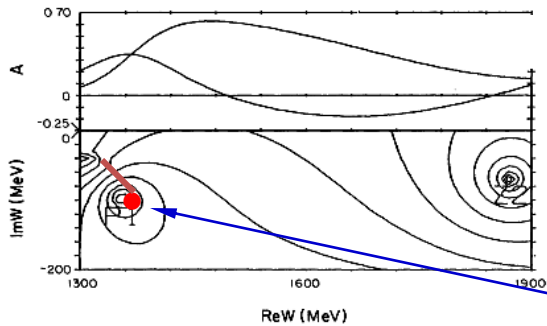
1 SEPTEMBER 1985

Pion-nucleon partial-wave analysis to 1100 MeV

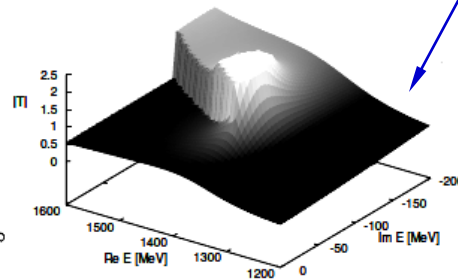
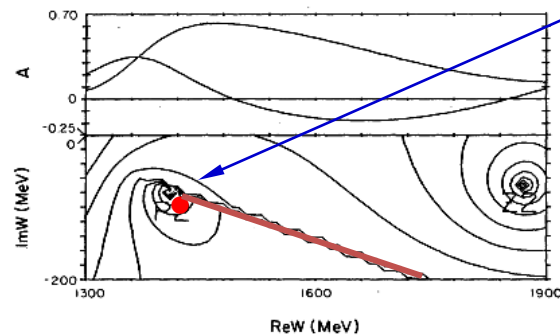
Richard A. Arndt, John M. Ford,* and L. David Roper

Department of Physics, Virginia Polytechnic Institute and State University, Blacksburg, Virginia

(Received 24 January 1985)



Pole 1: $W = 1359 - i100$ MeV
Pole 2: $W = 1410 - i80$ MeV



• [R.E. Cutkosky & S. Wang, Phys Rev D 42, 235 (1990)]

Pole 1: $W = 1370 - i114$ MeV
Pole 2: $W = 1360 - i120$ MeV

• **SAID PWA:**

[R. Arndt *et al*

Chinese Phys C 33, 1063 (2009)]

Pole 1: $W_p = 1359 - i82$ MeV

Pole 2: $W_p = 1388 - i83$ MeV

• **Juelich Model:**

[M. Doering *et al* Nucl Phys A829, 17C (2009)]

Pole 1: $W = 1387 - i73$ MeV

Pole 2: $W = 1387 - i71$ MeV

• **JLab ANL-Osaka (EBAC) Model:**

[H. Kamano *et al* Phys Rev C 81, 065207 (2010)]

Pole 1: $W = 1357 - i76$ MeV

Pole 2: $W = 1364 - i105$ MeV



10/6/2013

RRTF Workshop, Darmstadt, Germany, Oct 2013

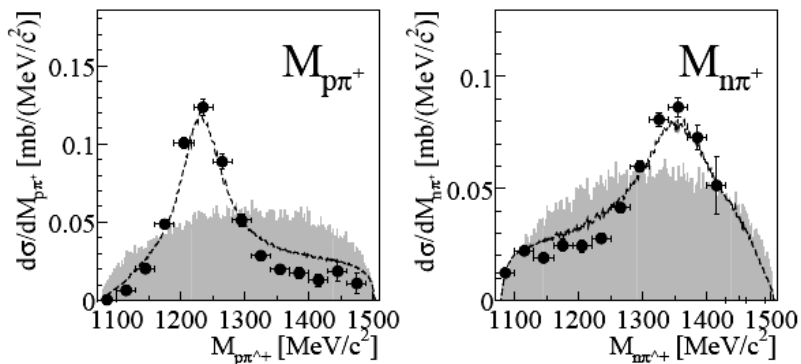
Igor Strakovsky 23



Direct Measurements of $N(1440)$: Hadronic Probes

- **CELSIUS-WASA: $pp \rightarrow n p \pi^+$**

[T. Skorodko *et al* Eur Phys J A **61**, 168 (2009)]

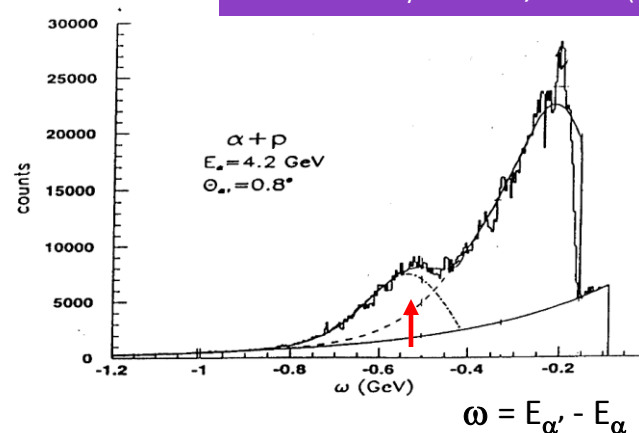


M=1360 MeV
 $\Gamma = 150$ MeV

- Looks similar to Pole at 1st sheet in GW πN

- **SATURNE II: $\alpha p \rightarrow \alpha' X$**

[H.P. Morsch and P. Zupranski, Phys Rev C **61**, 024002 (2000)]



M=1390±20 MeV
 $\Gamma = 190±30$ MeV

- Looks similar to Pole at 2nd sheet in GW πN

[S. Hirenzaki *et al* Phys. Rev. C **53**, 277 (1996)]

M=1430 MeV
 $\Gamma = 300$ MeV

• All Masses in Direct Measurements are smaller than BW and close to GW πN Pole positions.

- **SATURNE II: $\alpha p \rightarrow \alpha' p \pi \pi$**

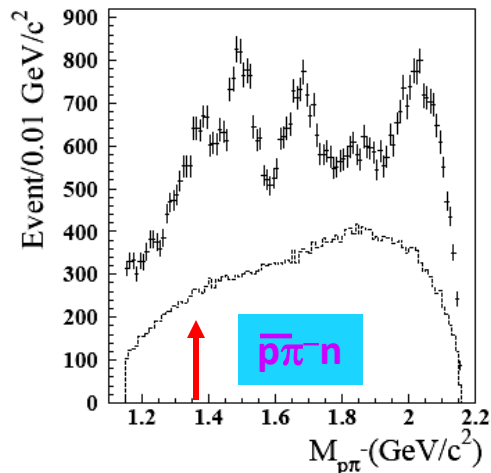
[G.D. Alkhasov *et al* Phys Rev C **78**, 025205 (2008)]

Difficulties in $N(1440)$ description do not allow to make a conclusive treatment.



Direct Measurements of $N(1440)$: EM Probes

- Relative contributions of various singularities may be different in different processes



• **BES II: $e^+e^- \rightarrow J/\psi \rightarrow p\bar{\pi}^-n + p\pi^+n$**

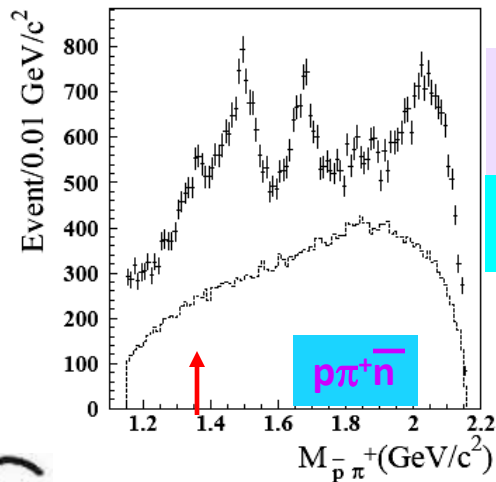
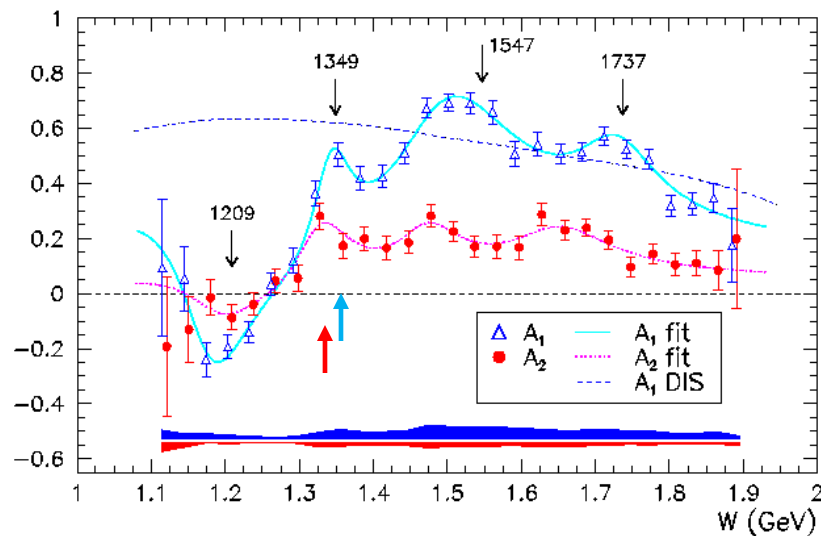
[M. Ablikim *et al* (BES Collaboration)
Phys Rev Lett **97**, 062001 (2006)]

• **JLab-RSS: $ep \rightarrow e'X$**

[F.R. Wesselmann *et al*
Phys Rev Lett **98**, 132003 (2007)]

Virtual Photon Asymmetries:

$$A_1 = \frac{1}{(E-E')} \left((E-E'\cos\theta)A_{||} - \frac{E'\sin\theta}{\cos\phi}A_{\perp} \right) \quad A_2 = \frac{\sqrt{Q^2}}{2E} \left(A_{||} - \frac{E-E'\cos\theta}{E'\sin\theta\cos\phi}A_{\perp} \right)$$



PWA: $J^P=1/2^+$

$M=1358 \pm 6 \pm 16$ MeV
 $\Gamma=179 \pm 26 \pm 50$ MeV

• Looks similar to Pole at 1st sheet in GW πN

$$M = \text{Re}W_p \\ \Gamma = 2*\text{Im}W_p$$

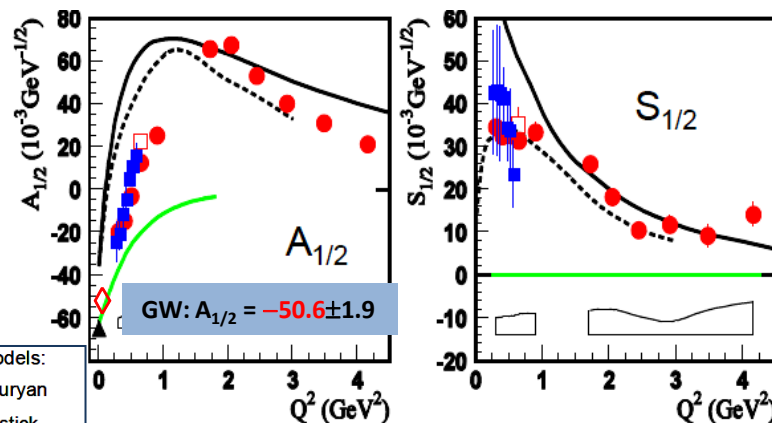
• Evidence for **two poles** ?

$M=1338 \pm 10$ MeV $M=1346 \pm 5$ MeV
 $\Gamma=65 \pm 26$ MeV $\Gamma=71 \pm 35$ MeV



$N(1440)$ Puzzle for Jlab CLAS12

- Most of analyses of $N(1440)$ are based on its BW parameterization, which assumes that the Res is related to an isolated Pole
- However, the latest GW PWA for the elastic πN scattering gives evidence that $N(1440)$ corresponds to a more complicated case of several nearby singularities in the amplitude
- Then, the BW description is only an efficient one for $N(1440)$, which could be different in different processes
- Some inelastic data indirectly support this point:
they give the $N(1440)$ BW mass and width essentially different from the PDG BW values



Light front models:
 — I. Aznauryan
 - - - S. Capstick
 — hybrid $P_{11}(1440)$

- The analysis of the recent CLAS Single and Double π^+ Electro Prod data [$W = 1.15 - 1.69$ GeV, $Q^2 = 1.7 - 4.5$ GeV²] allows to extract helicities for $\gamma^* p \rightarrow N(1440)P_{11}$ transition
 [1 π : I.G. Aznauryan *et al* Phys Rev C 80, 055203 (2009)
 2 π : V. Mokeev, PC 2010]
- Model predictions allow to conclude that $N(1440)$ is a first radial excitation of $3q$ ground state

- Since Q^2 -dependences for contributions of different singularities may be different, the set of several singularities might provide the $N(1440)$ BW mass and width depending on the Q^2

• This problem can be studied in future measurements with JLab CLAS12



J. Beringer *et al* (PDG) Phys Rev D **86**, 010001 (2012)

- More than half of states have poor evidence.
- Most of states need more work to do.
- Most of QCD models predict more states than observed.
- **Where are missing resonances?**

GW SAID Contribution

Status as seen **I = 1/2**

Particle	J^P	Status									
		overall	πN	γN	$N\eta$	$N\sigma$	$N\omega$	ΔK	ΣK	$N\rho$	$\Delta\pi$
N	$1/2^+$	****									
N(1440)	$1/2^+$	****	****	****		***			*	***	
N(1520)	$3/2^-$	****	****	****		***			***	***	
N(1535)	$1/2^-$	****	****	****		****			**	*	
N(1650)	$1/2^-$	****	****	***		***	**		**	***	
N(1675)	$5/2^-$	****	****	***		*			*	***	
N(1680)	$5/2^+$	****	****	****	*	**			***	***	
N(1685)	?	*									
N(1700)	$3/2^-$	***	***	**		*	*		*	***	
N(1710)	$1/2^+$	***	***	***		***	**		*	**	
N(1720)	$3/2^+$	****	****	***		***	**	**	**	*	
N(1860)	$5/2^+$	**	**			*	*		*	*	
N(1875)	$3/2^-$	***	*	***		**	***	**		***	
N(1880)	$1/2^+$	**	*	*		**	*				
N(1895)	$1/2^-$	**	*	**		**	*				
N(1900)	$3/2^+$	***	**	***		**	***	**	*	**	
N(1990)	$7/2^+$	**	**	**		*	*				
N(2000)	$5/2^+$	**	*	**		**	*	*	**		
N(2040)	$3/2^+$	*									
N(2060)	$5/2^-$	**	**	**		*	**				
N(2100)	$1/2^+$	*									
N(2150)	$3/2^-$	**	**	**		**	**		**	**	
N(2190)	$7/2^-$	****	****	***		26 N*	**	*			
N(2220)	$9/2^+$	****	****	****		11 ****	**				
N(2250)	$9/2^-$	****	****	****		5 ***	**				
N(2600)	$11/2^-$	***	***	***		7 **	**				
N(2700)	$13/2^+$	**	**	**		3 *	**				

I = 3/2

Particle	J^P	Status										Status as seen in —	
		overall	πN	γN	$N\eta$	$N\sigma$	$N\omega$	ΔK	ΣK	$N\rho$	$\Delta\pi$		
$\Delta(1232)$	$3/2^+$	****	****	****	F								
$\Delta(1600)$	$3/2^+$	***	***	***	o					*	***		
$\Delta(1620)$	$1/2^-$	****	****	***	r					***	***		
$\Delta(1700)$	$3/2^-$	****	****	****	b					**	***		
$\Delta(1750)$	$1/2^+$	*	*		i								
$\Delta(1900)$	$1/2^-$	**	**	**	d				**	**	**		
$\Delta(1905)$	$5/2^+$	****	****	****	d				***	**	**		
$\Delta(1910)$	$1/2^+$	****	****	**	e			*	*	**			
$\Delta(1920)$	$3/2^+$	***	***	**	n			***	***	**			
$\Delta(1930)$	$5/2^-$	***	***	**									
$\Delta(1940)$	$3/2^-$	**	*	**	F				(seen in $\Delta\eta$)	***	*	***	
$\Delta(1950)$	$7/2^+$	****	****	****	o				***	*	***		
$\Delta(2000)$	$5/2^+$	**	**	**	r					**	**		
$\Delta(2150)$	$1/2^-$	*	*		b								
$\Delta(2200)$	$7/2^-$	*	*		i								
$\Delta(2300)$	$9/2^+$	**	**	**	d								
$\Delta(2350)$	$5/2^-$	*	*		d								
$\Delta(2390)$	$7/2^+$	*	*		n								
$\Delta(2400)$	$9/2^-$	**	**	7 ****									
$\Delta(2420)$	$11/2^+$	****	****	3 ***									
$\Delta(2750)$	$13/2^-$	**	**	7 **									
$\Delta(2950)$	$15/2^+$	**	**	5 *									

26 N*
11 ****
5 ***
7 **
3 *

22 Δ^*
7 ****
3 ***
7 **
5 *

BnGa Additional States

The latest GWU analysis (ARNDT 06) finds no evidence for this resonance.



Status of Non-strange Resonances Twenty Years Ago – Rosenfeld Tables

L. Montanet *et al* (PDG) Phys Rev D **50**, 1173 (1994)



$I = 1/2$

$I = 3/2$

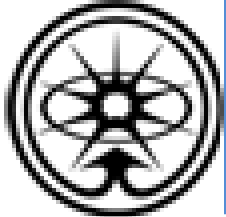
Particle	$L_{2I,2J}$	Overall status	Status as seen in							
			$N\pi$	$N\eta$	AK	ΣK	$\Delta\pi$	$N\rho$	$N\gamma$	
$N(939)$	P_{11}	****								
$N(1440)$	P_{11}	****	****	*			***	*	***	
$N(1520)$	D_{13}	****	****	*			****	****	****	
$N(1535)$	S_{11}	****	****	****			*	**	***	
$N(1650)$	S_{11}	****	****	*	***	**	***	**	***	
$N(1675)$	D_{15}	****	****	*	*		****	*	****	
$N(1680)$	F_{15}	****	****				****	****	****	
$N(1700)$	D_{13}	***	***	*	**	*	**	*	**	
$N(1710)$	P_{11}	***	***	**	**	*	**	*	***	
$N(1720)$	P_{13}	****	****	*	**	*	*	**	**	
$N(1900)$	P_{13}	**	**				*			
$N(1990)$	F_{17}	**	**	*	*	*			*	
$N(2000)$	F_{15}	**	**	*	*	*	*	**		
$N(2080)$	D_{13}	**	**	*	*				*	
$N(2090)$	S_{11}	*	*							
$N(2100)$	P_{11}	*	*							
$N(2190)$	G_{17}	****	****	*	*	*	*	*	*	
$N(2200)$	D_{15}	**	**	*	*					
$N(2220)$	H_{19}	****	****	*						
$N(2250)$	G_{19}	****	****	*						
$N(2600)$	I_{111}	***	***							
$N(2700)$	K_{113}	**	**							

22 N^*
11 ****
3 ***
6 **
2 *

Particle	$L_{2I,2J}$	Overall status	Status as seen in							
			$N\pi$	$N\eta$	AK	ΣK	$\Delta\pi$	$N\rho$	$N\gamma$	
$\Delta(1232)$	P_{33}	****	****	F					****	
$\Delta(1600)$	P_{33}	***	***	o			***	*	**	
$\Delta(1620)$	S_{31}	****	****	r			****	****	***	
$\Delta(1700)$	D_{33}	****	****	b	*		***	**	***	
$\Delta(1750)$	P_{31}	*	*	i						
$\Delta(1900)$	S_{31}	***	***	d	*	*	*	**	*	
$\Delta(1905)$	F_{35}	****	****	d	*	*	**	**	***	
$\Delta(1910)$	P_{31}	****	****	e	*	*	*	*	*	
$\Delta(1920)$	P_{33}	***	***	n	*	*	**	*	*	
$\Delta(1930)$	D_{35}	***	***		*				**	
$\Delta(1940)$	D_{33}	*	*	F						
$\Delta(1950)$	F_{37}	****	****	o	*	****	*	****		
$\Delta(2000)$	F_{35}	**	**	r				**		
$\Delta(2150)$	S_{31}	*	*	b						
$\Delta(2200)$	G_{37}	*	*	i						
$\Delta(2300)$	H_{39}	**	**	d						
$\Delta(2350)$	D_{35}	*	*	d						
$\Delta(2390)$	F_{37}	*	*	e						
$\Delta(2400)$	G_{39}	**	**							
$\Delta(2420)$	H_{311}	****	****						*	
$\Delta(2750)$	I_{313}	**	**							
$\Delta(2950)$	K_{315}	**	**							

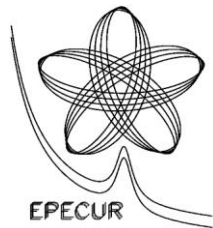
22 Δ^*
7 ****
4 ***
5 **
6 *





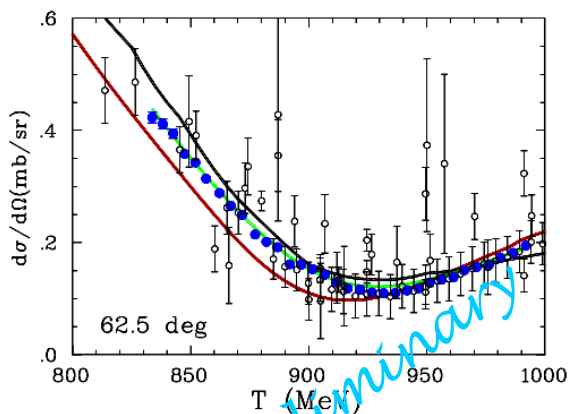
ITEP for $\pi^+ p \rightarrow \pi^+ p$ & $\pi^- p \rightarrow \kappa^0 \Lambda$

[I.G. Alekseev *et al.* arXiv: 1204.6433]



• **Precise cross section measurements:**

- $\pi p \rightarrow \pi p$: $d\sigma/d\Omega$ – **0.5%** statistical precision and **1 MeV** momentum step
- $\pi^- p \rightarrow \kappa^0 \Lambda$: σ_{REAC} – **15 %** statistical precision and the same mom step

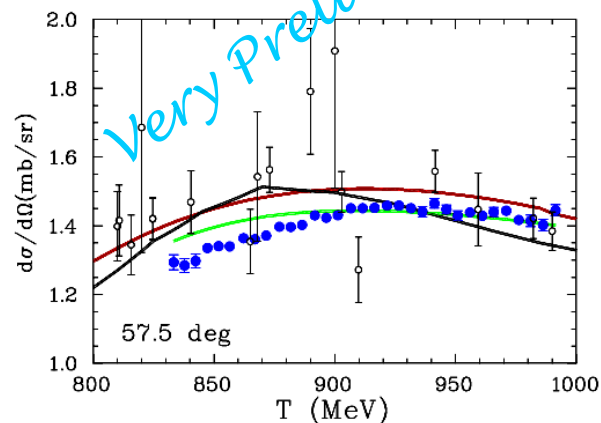
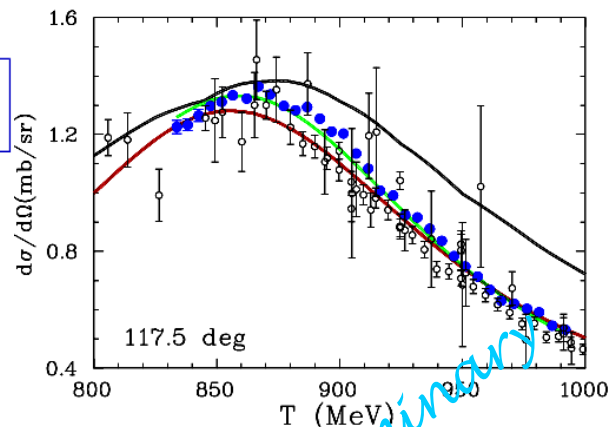


○ EPECUR
 $T_{\pi} = 834\text{--}992$ MeV
 $\theta = 48\text{--}117$ deg

○ Previous [available in Hoehler's time]:
 ± 2.5 deg or
 ± 2 MeV

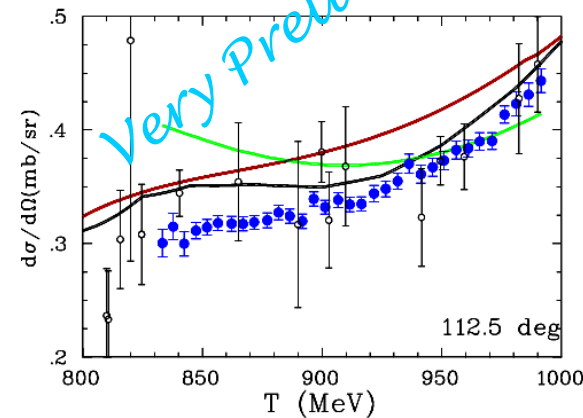
$$\pi p \rightarrow \pi p$$

Wi08 KA84
 $\chi^2/d.o. = 6090/495$ $36394/495$



$$\pi^+ p \rightarrow \pi^+ p$$

Wi08 KA84
 $\chi^2/d.o. = 4748/528$ $2387/528$
11980/528

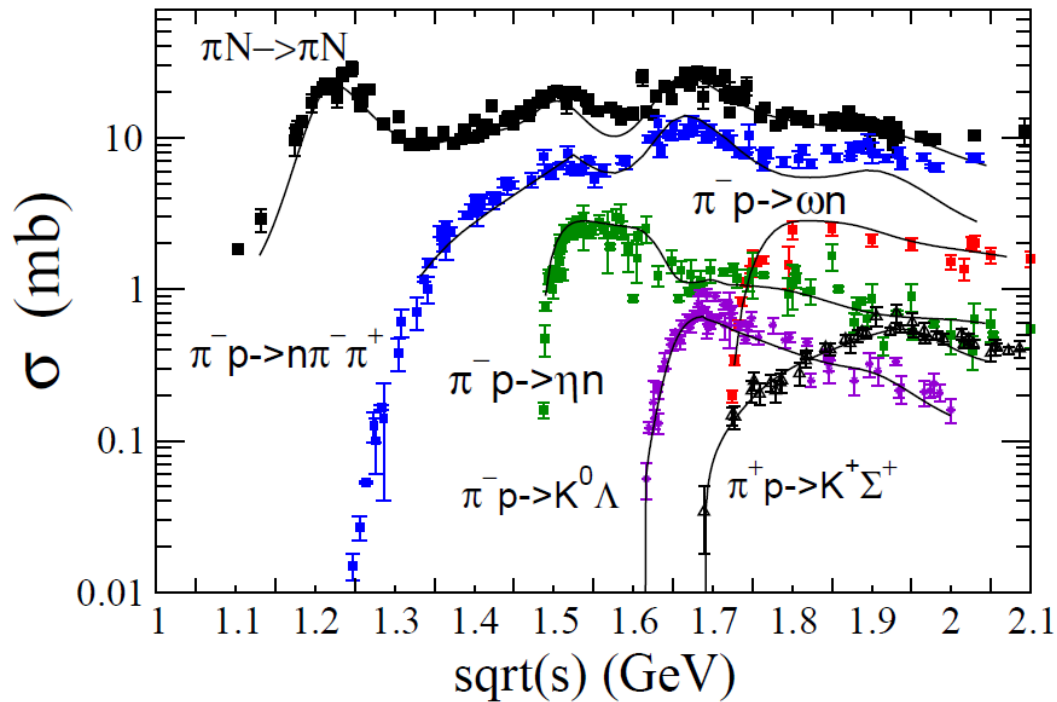


— GW-Wi08
— KH-KA84
— Gridnev12

} EPECUR data are out of the fit
 } EPECUR data are in the fit No norm



πN in Elastic Scattering

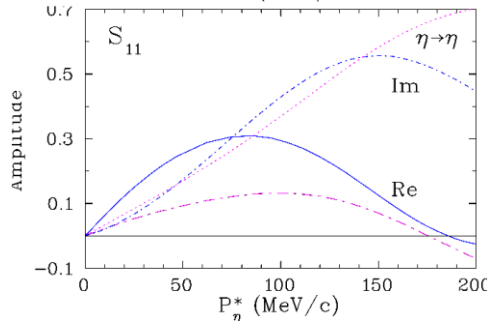
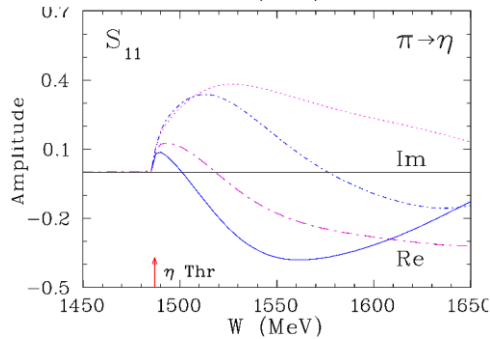
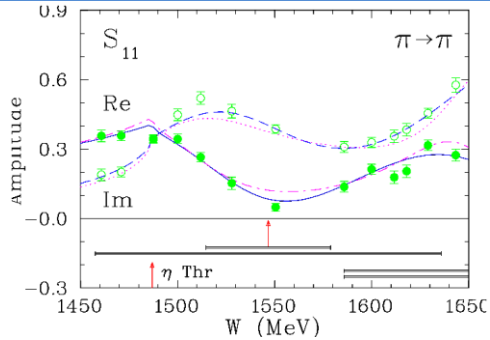


Courtesy of Vitaly Shklyar, Spring 2013

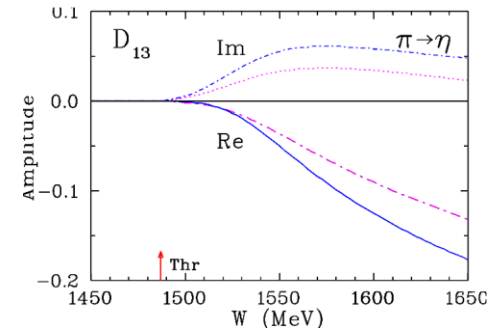
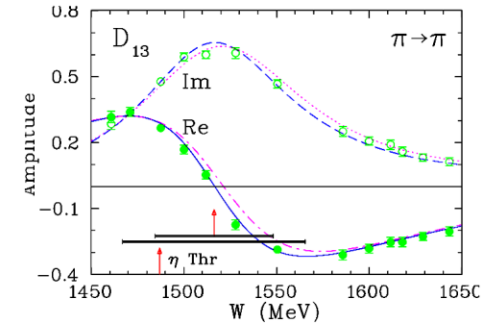


Coupled Channel Fit for S_{11} & D_{13} : $\eta\eta$ BRs

[R. Arndt, W. Briscoe, IS, R. Workman, A. Gridnev, Phys Rev C 72, 045202 (2005)]



• Limited energy range limits possibilities to determine resonance parameters



$N(1535)S_{11}$: $\Gamma_{\eta} > \Gamma_{\pi}$

$N(1520)D_{13}$: $\Gamma_{\eta}/\Gamma = 0.0008 - 0.0016$

D_{13} [Mainz (γ, η): $\Gamma_{\eta}/\Gamma = 0.0008 \pm 0.0001$

D_{13} [Giessen, multi-ch]: $\Gamma_{\eta}/\Gamma = 0.0023 \pm 0.0004$

[G. Penner and U. Mosel, Phys Rev C 66, 055211 (2002)
L. Tiator et al Phys Rev C 60, 035210 (1999)]

— No E913/914 BNL thr data

— Added E913/914 BNL thr data

[S. Prakhov et al Phys Rev C 72, 015203 (2005)]

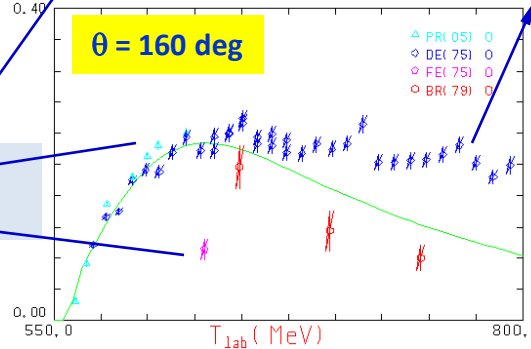
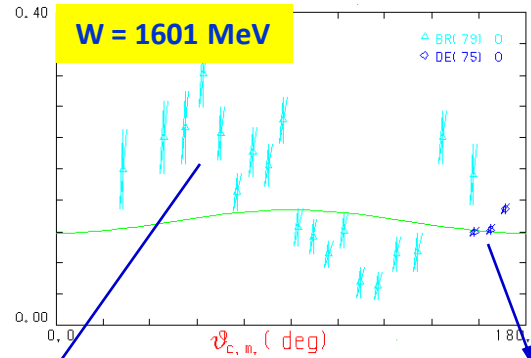
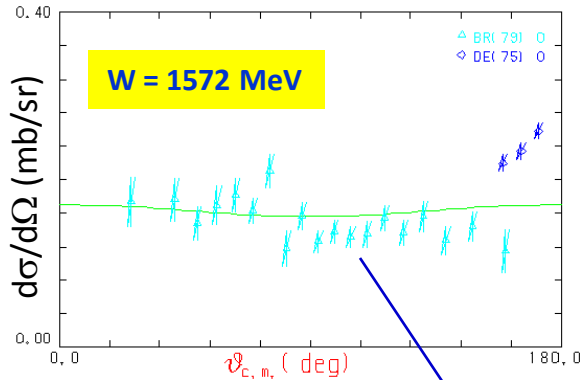


$\pi^- p \rightarrow \eta n$ Puzzle

[R. Arndt, W. Briscoe, IS, R. Workman, Phys Rev C **74**, 045205 (2006)]

• Several groups evaluated $\pi^- p \rightarrow \eta n$ data

- Cutkosky *et al* Phys Rev D **20**, 2804 (1979)
- Koch & Pietarinen, Nucl Phys **A336**, 331 (1980)
- Wighman *et al* Phys Rev D **38**, 3365 (1988)
- Clajus & Nefkens, πN News Lett **7**, 76 (1992)
- Arndt *et al* Phys Rev C **74**, 045205 (2006)



[Debenham *et al*, Phys Rev D **12**, 2545 (1975)]

[Brown *et al*, Nucl Phys **B153**, 89 (1979)]

[Feltesse *et al*, Nucl Phys **B93**, 242 (1975)]



was the 7 GeV proton synchrotron operating in the Rutherford Appleton Laboratory in UK between 1964 and 1978

- Most of **Nimrod** data do not satisfy requirements [systematics (**10%** or more), momentum err (up to **50 MeV/c**), and so on]
- For that reason, **SAID** is not able to use them in $\pi^- p \rightarrow \pi^- p$, $\pi^0 n$, & ηn PWAs



$\pi^- p \rightarrow \eta n$ Puzzle

[R. Arndt, W. Briscoe, IS, R. Workman, Phys Rev C 74, 045205 (2006)]
 [R. Arndt, W. Briscoe, IS, R. Workman, A. Gridnev, Phys Rev C 72, 045202 (2005)]

$N(1535) 1/2^-$

$I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ Status: ****

$\Gamma(N\pi)/\Gamma_{total}$

VALUE
0.35 to 0.55 OUR ESTIMATE
0.355 ± 0.002
0.394 ± 0.009
0.51 ± 0.05
0.50 ± 0.10
0.38 ± 0.04

DOCUMENT ID	TECN	COMMENT
ARNDT 06	DPWA	$\pi N \rightarrow \pi N, \eta N$
GREEN 97	DPWA	$\pi N \rightarrow \pi N, \eta N$
MANLEY 92	IPWA	$\pi N \rightarrow \pi N \& N\pi\pi$
CUTKOSKY 80	IPWA	$\pi N \rightarrow \pi N$
HOEHLER 79	IPWA	$\pi N \rightarrow \pi N$

$N(1650) 1/2^-$

$I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ Status: ****

$\Gamma(N\pi)/\Gamma_{total}$

VALUE (%)	DOCUMENT ID	TECN	COMMENT
50 to 90 (≈ 70) OUR ESTIMATE			
51 ± 4	ANISOVICH 12A	DPWA	Multichannel
100	ARNDT 06	DPWA	$\pi N \rightarrow \pi N, \eta N$
73.5 ± 1.1	GREEN 97	DPWA	$\pi N \rightarrow \pi N, \eta N$
65 ± 10	CUTKOSKY 80	IPWA	$\pi N \rightarrow \pi N$
61 ± 4	HOEHLER 79	IPWA	$\pi N \rightarrow \pi N$

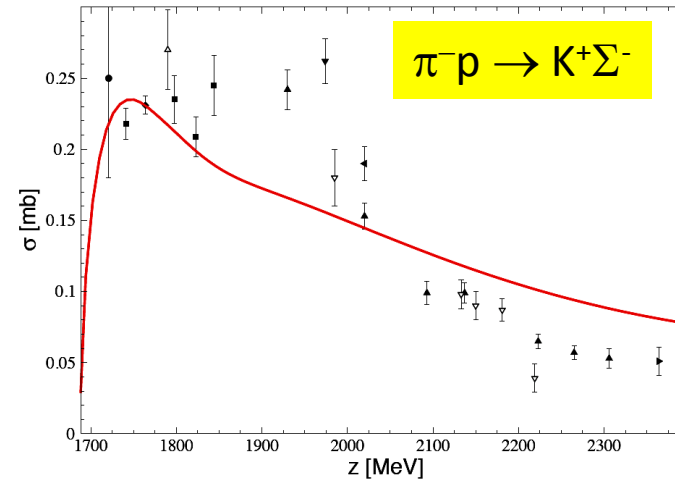
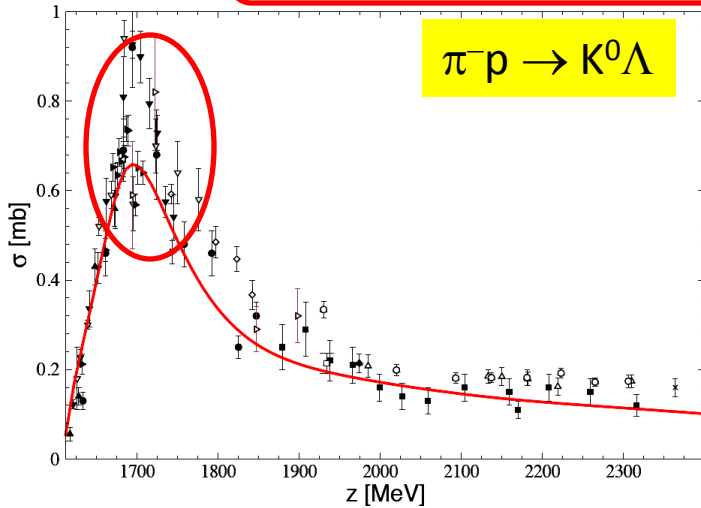
$\Gamma(N\eta)/\Gamma_{total}$

VALUE (%)	DOCUMENT ID	TECN	COMMENT
5 to 15 OUR ESTIMATE			
18 ± 4	ANISOVICH 12A	DPWA	Multichannel
1.0 ± 0.6	PENNER 02C	DPWA	Multichannel
6 ± 1	VRANA 00	DPWA	Multichannel

- Why there is a difference between two $\frac{1}{2}^-$
- There is no room for ηn in SAID PWA

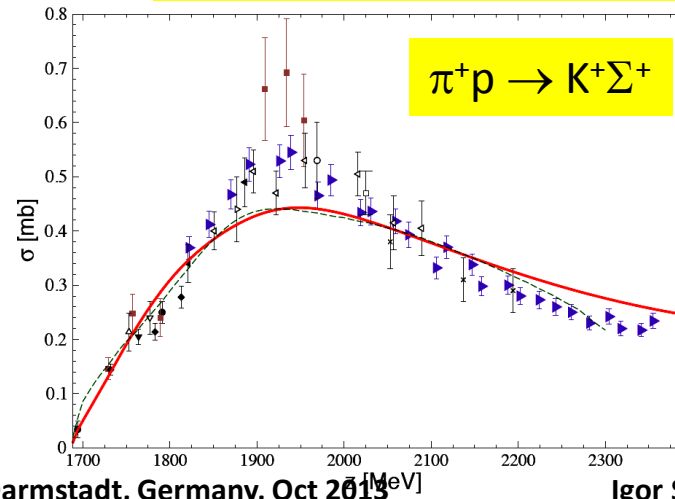
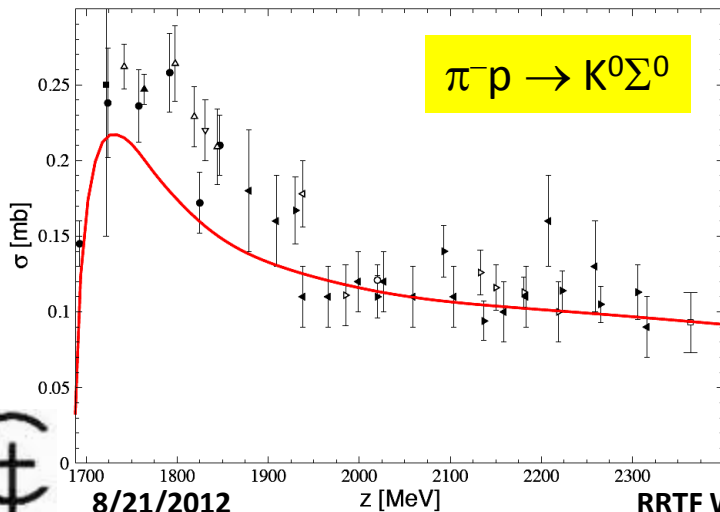
$\pi N \rightarrow KY$ Puzzle

- The evaluation for reactions with KY , $\eta'N$, ωN , and ϕN , final states are **not possible** now because of small databases.



Curves:
Jülich DCC
model, 2012

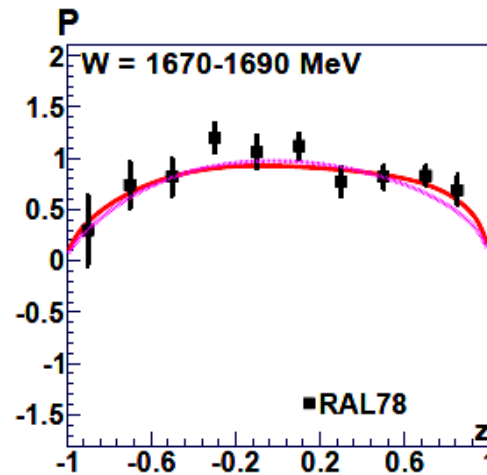
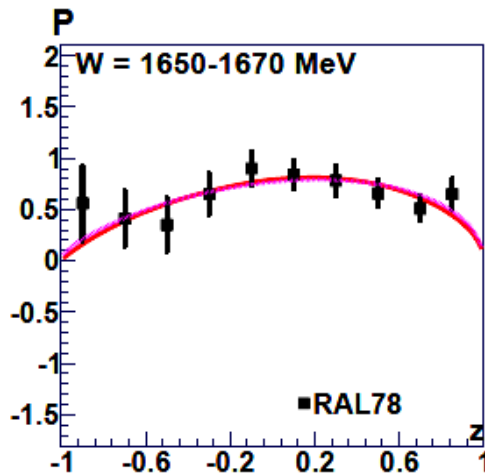
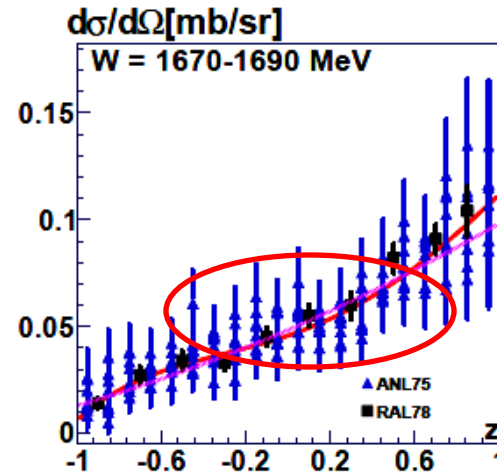
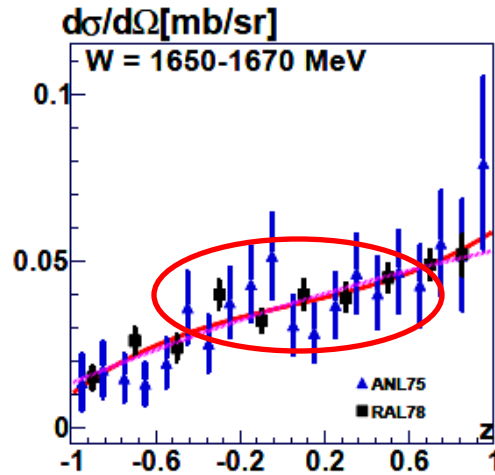
Courtesy of Kanzo Nakayama, GW EIC Workshop, April 2012



$\pi N \rightarrow K \Lambda$

nimrod
The 7GeV proton synchrotron

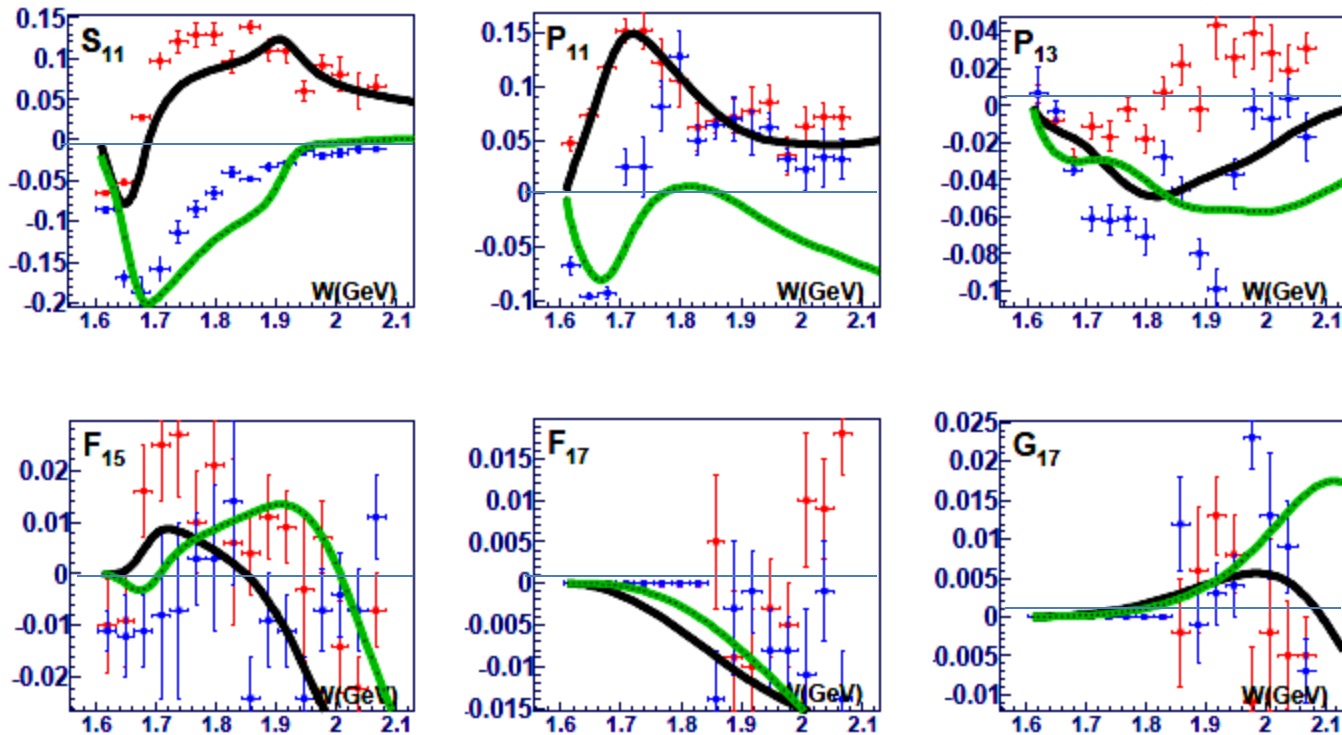
Argonne
NATIONAL LABORATORY



Courtesy of Eberhard Klempt, PWA7 Workshop, Sept 2013



$\pi N \rightarrow \mathcal{K} \Lambda$ Amplitudes



- Points with error bars: **SES** of Shresta and Manley. **Curves:** [BnGa2011](#).
- Shresta and Manley start from a **model-dependant fit** and select the solution of the SES which is closest to the energy-dependent fit.
- They first freeze the S_{11} wave, then S_{11} and P_{11} , to the energy dependent solution.

Courtesy of Eberhard Klempt, PWA7 Workshop, Sept 2013



$\pi \rightarrow 2\pi$

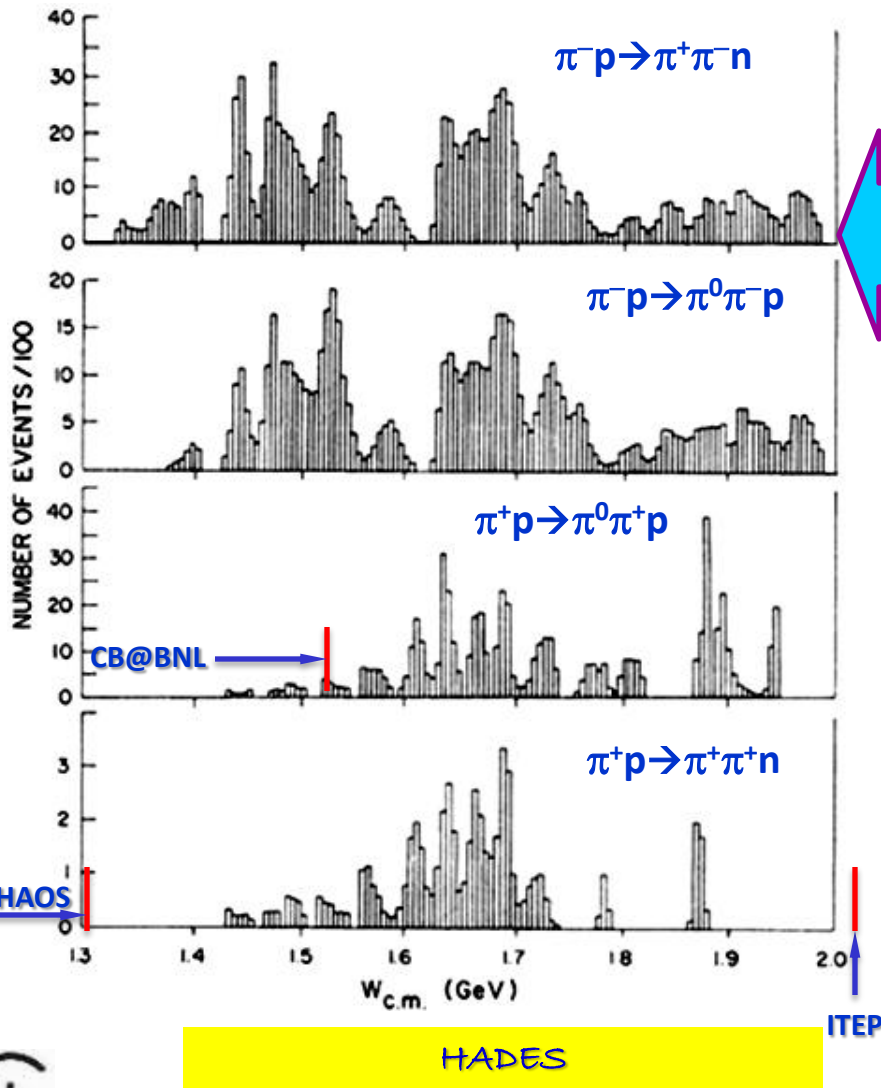


$\pi^- p \rightarrow \pi^+ \pi^- n$
 $\pi^- p \rightarrow \pi^0 \pi^0 n$
 $\pi^- p \rightarrow \pi^- \pi^0 p$
 $\pi^+ p \rightarrow \pi^+ \pi^0 p$
 $\pi^+ p \rightarrow \pi^+ \pi^+ n$



Our knowledge of $\pi\Delta$, ρN , and other quasi-two-body $\pi\pi N$ channels comes mainly from **isobar-model** analyses of the $\pi N \rightarrow \pi\pi N$ data.


Previous $\pi N \rightarrow \pi \pi N$ Measurements





- **241,214 Bubble Chamber** events for $\pi N \rightarrow \pi \pi N$ have been analyzed in **Isobar-model PWA** at $W = 1320$ to **1930 MeV**.

[D.M. Manley, **R. Arndt**, Y. Goradia, V. Teplitz, Phys Rev D **30**, 904 (1984)]

- **Recent post-Bubble Chamber** measurements:

- **349,611** events for $\pi^- p \rightarrow \pi^0 \pi^0 n$ from **CB@BNL** at $W = 1213$ to **1527 MeV**.
 [S. Prakhov *et al* Phys Rev C **69**, 045202 (2004)]

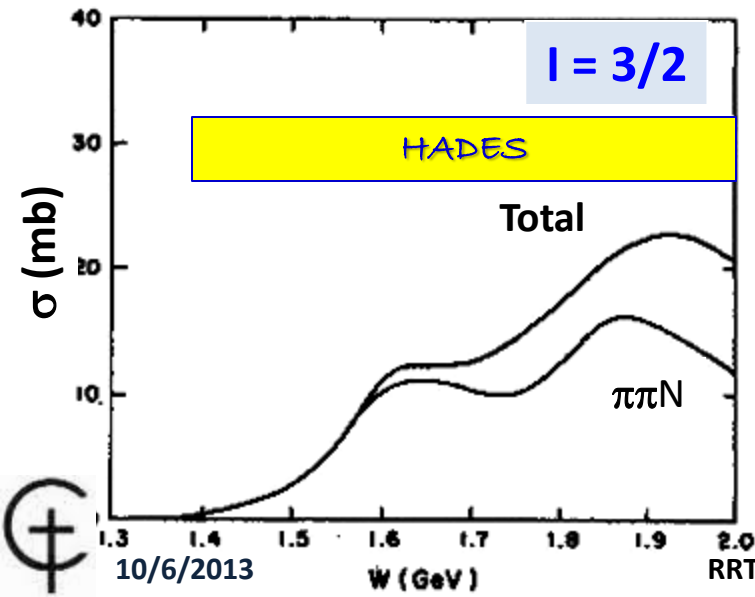
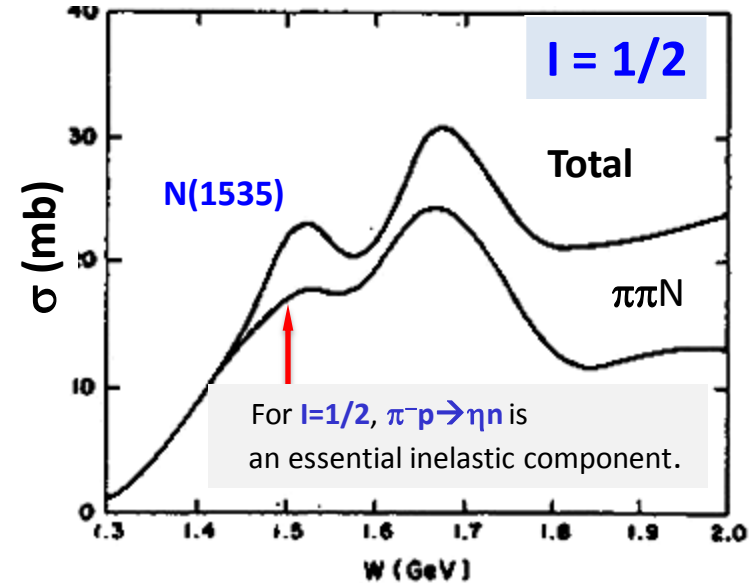
- **20,000** events for $\pi^+ p \rightarrow \pi^+ \pi^+ n$ from **TRIUMF CHAOS@TRIUMF** at $W = 1257$ to **1302 MeV**. [M. Kermani *et al* PRC **58**, 3431 (98)]


- **40,000** events for $\pi^- p \rightarrow \pi^- \pi^+ n$ from **ITEP** at $W = 2060$ MeV.
 [I. Alekseev *et al* Phys At Nucl **61**, 174 (1998)]

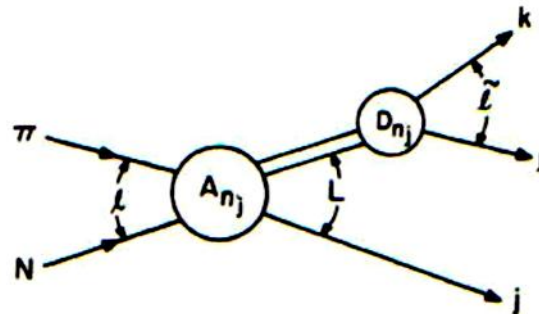


$\pi N \rightarrow \pi \pi N$ in Isobar Model

[D.M. Manley, R. Arndt, Y. Goradia, V. Teplitz, Phys Rev D 30, 904 (1984)]



- $\pi N \rightarrow \pi \pi N$ is the dominant inelastic reaction in πN scattering above 1300 MeV, $\sigma_{\text{inel}} \sim \sigma(\pi \pi N)$
- Drawbacks** – analysis of 3-body final states is complicated (many partial waves are involved).



The **total amplitude** for a given charge channel can be written as a coherent sum over all **isobars** and **partial waves**.

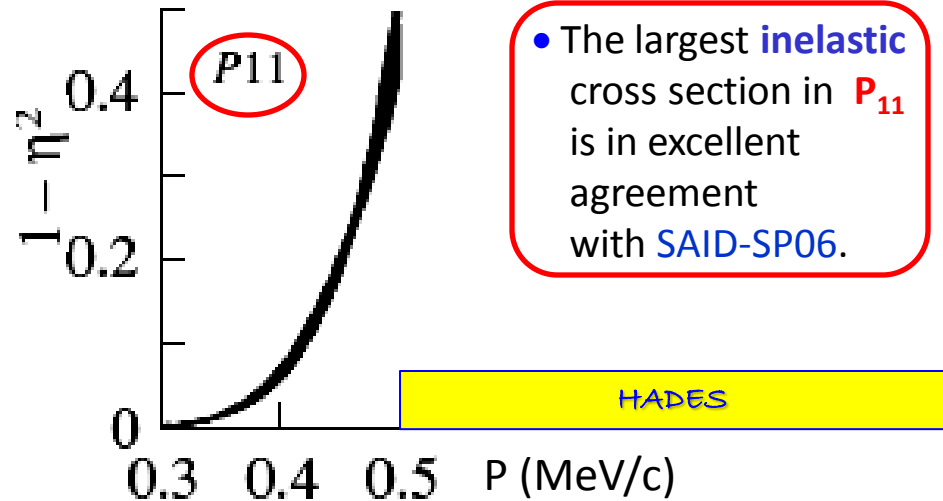
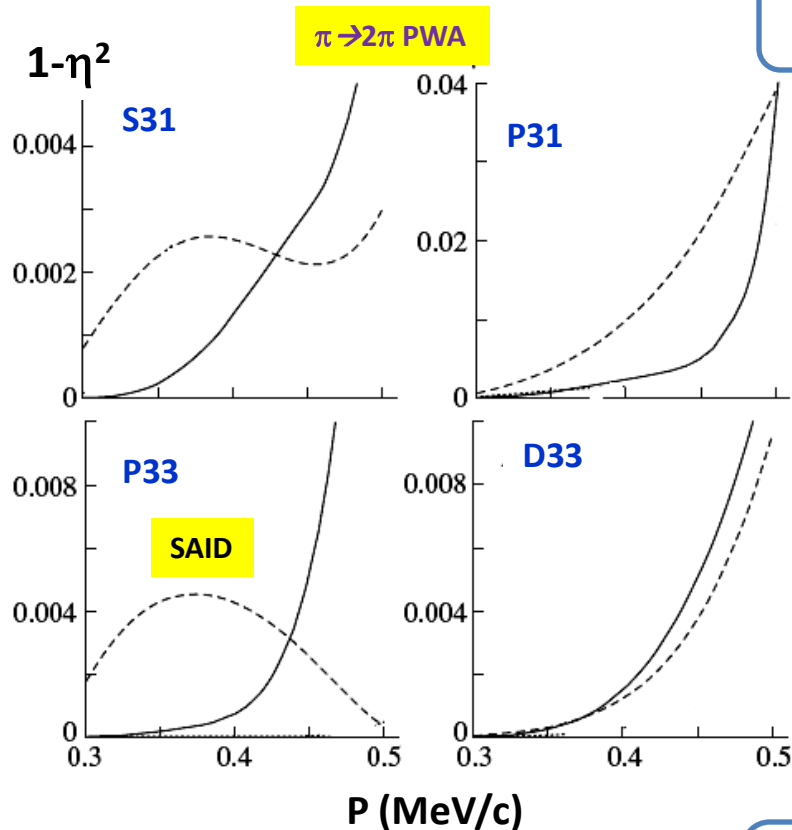
- Many of the 3- and 4-star resonances have large decay branching ratios to $\pi \pi N$ channels.
- There remains a strong need for detailed new measurements in all charge channels!



$\pi N \rightarrow \pi \pi N$ in Isobar Model at low Energies

[V. Kozhevnikov & S. Sherman, Phys Atom Nucl 71, 1860 (2008)]

- Unfortunately, it is hard to merge $\pi N \rightarrow \pi N$ and $\pi N \rightarrow \pi \pi N$ databases to make a joint PWA.



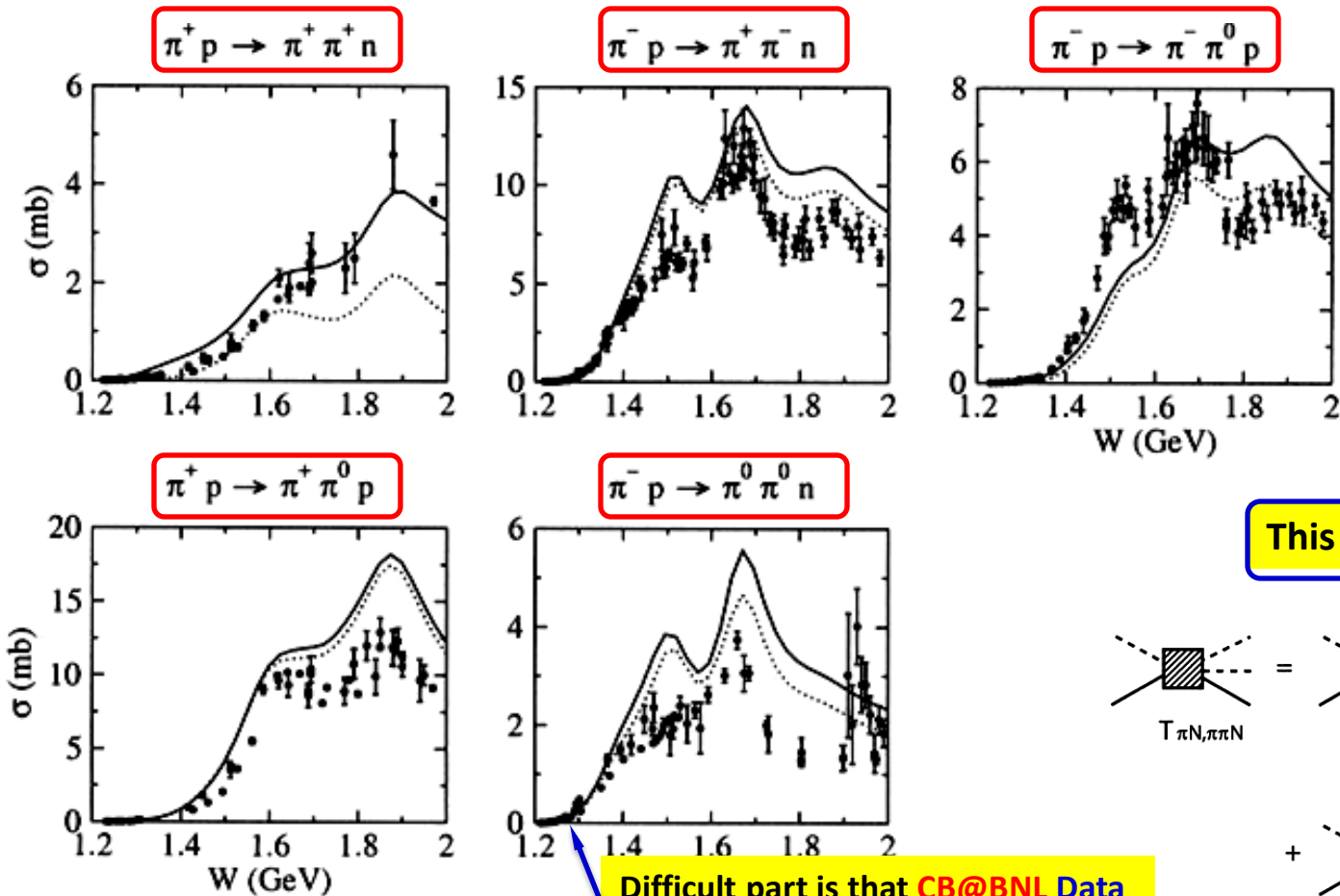
- The largest inelastic cross section in P_{11} is in excellent agreement with SAID-SP06.

- A complete analysis of $\gamma N \rightarrow \pi \pi N$ ideally would require fitting all data obtained with both pion and photon beams.

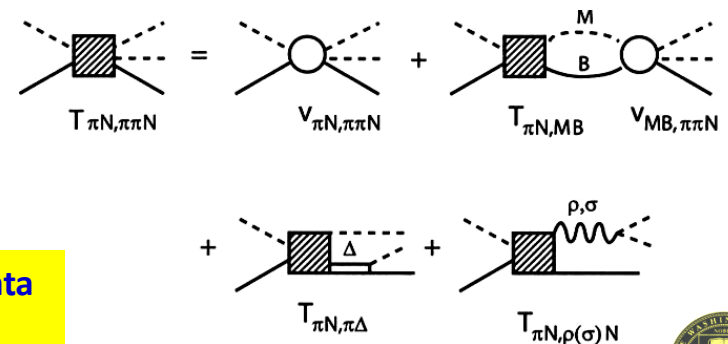


ANL-OSAKA (EBAC) Dynamical Coupled-Channels Study of $\pi N \rightarrow \pi\pi N$ Reactions

[H. Kamano, et al Phys Rev C 79, 025206 (2009)]



This approach may work



Difficult part is that **CB@BNL Data** require acceptance corrections



Resonance $\rightarrow N\rho$ Branching Ratios

	GiBUU12	UrQMD09	KSU12	KSU92	BnGa12	CLAS12	PDG12	
$N(1520)3/2^-$	21	15	20.9(7)	21(4)	10(3)	12.7(4.3)	20(5)	D13
$N(1720)3/2^+$	87	73	1.4(5)	87(5)	10(13)	47.5(21.5)	77.5(7.5)	P13
$\Delta(1620)1/2^-$	29	5	26(2)	25(6)	12(9)	37(12)	16(9)	S31
$\Delta(1905)5/2^+$	87	80	<6	86(3)	42(8)		>60	F35

Partial courtesy of Piotr Salabura, Sept 2013

CLAS12: V. Mokeev *et al*, Phys Rev C **86**, 035203 (2012); V. Mokeev, PC
BnGa12: A.V, Anisovich *et al*, Eur Phys J A **48**, 15 (2012)
GiBUU12: J. Weil *et al*, Eur Phys J A **48**, 111 (2012); J. Weil, PC
KSU92: D.M. Manley and E.M. Saleski, Phys Rev D **45**, 055203 (1992)
KSU12: M. Shrestha and D.M. Manley, Phys Rev D **86**, 055203 (2012)
PDG12: J. Beringer *et al* [RPP] Phys Rev D **86**, 010001 (2012)
UrQMD09: K. Schmidt *et al*, Phys Rev C **79**, 4002 (2009)



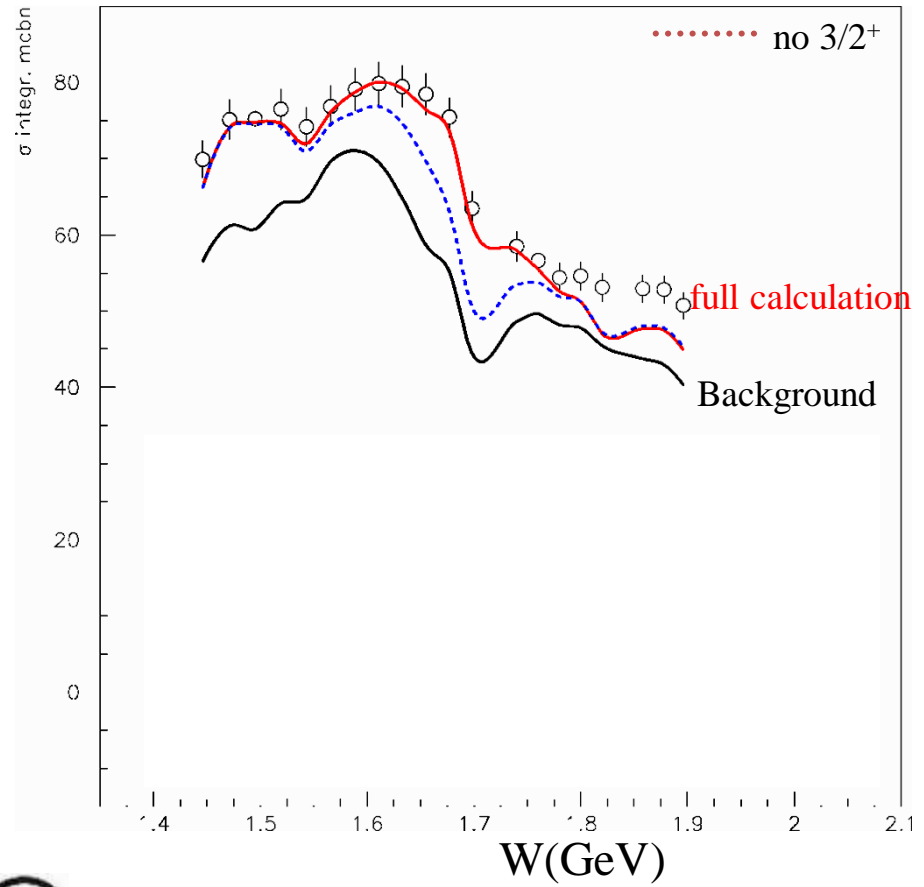
CLAS N^* candidate at 1720 MeV in $p\pi^+\pi^-$?

Courtesy of Victor Moiseev, Sept 2013

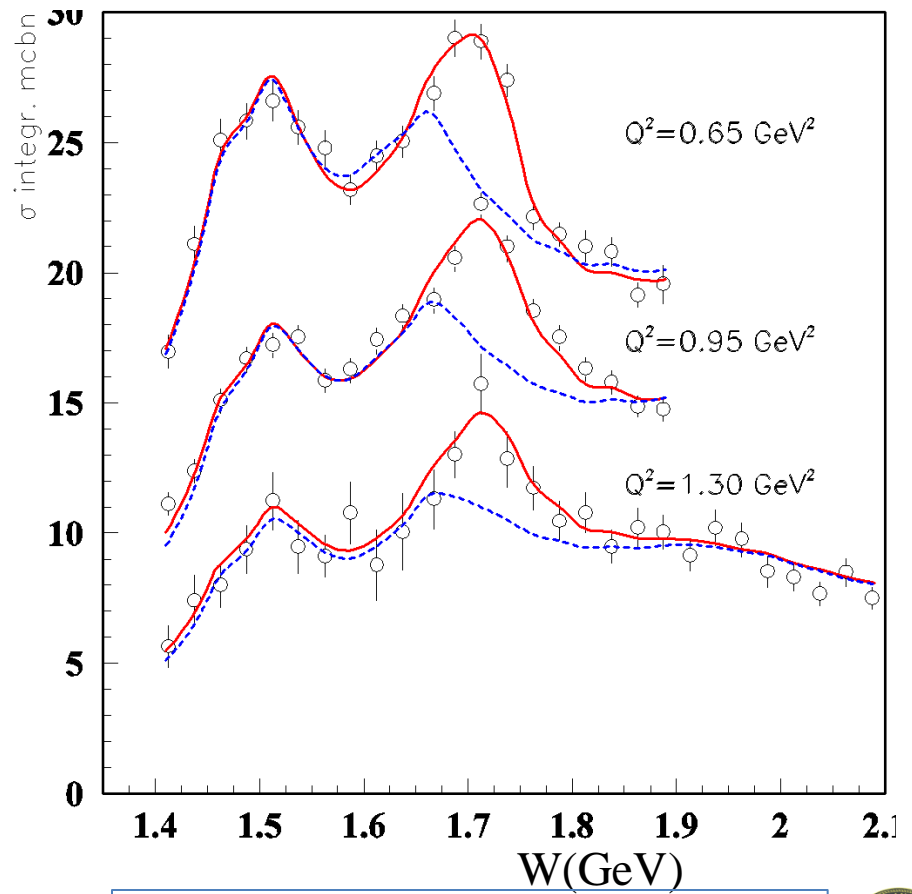


..... no $3/2^+$ (1720)
 — full

Pion Photoproduction



Pion Electroproduction



[M. Ripani et al, Phys.Rev.Lett. 91, 022002 (2003)]

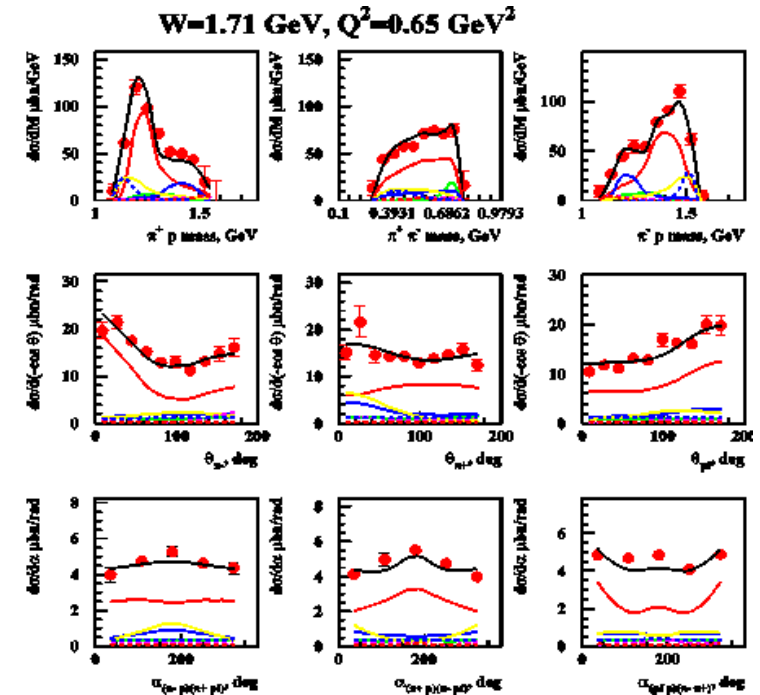


Origin of the structure at $W \sim 1.7$ GeV for the first time

observed in the CLAS $N\pi\pi$ data

- $P_{13}(1720)$ state with hadronic decays fit to the CLAS data $2.94 < \chi^2/dp < 3.15$

	M (GeV)	Γ_{tot} (MeV)	BR($\pi\Delta$) (%)	BR($\rho\rho$) (%)
$P_{13}(1720)$ CLAS	1.728 ± 0.005	133 ± 19	66 ± 26	16 ± 11
$P_{13}(1720)$ PDG	1.70 -1.75	150-400	comp. with 0.	70-85



Courtesy of Victor Moiseev, Sept 2013

- $3/2^+(1725)$ candidate state, hadronic parameters of others N^* 's are within PDG uncertainties $2.78 < \chi^2/dp < 2.9$

	M (GeV)	Γ_{tot} (MeV)	BR($\pi\Delta$) (%)	BR($\rho\rho$) (%)
$3/2^+(1725)$	1.725 ± 0.004	80 ± 6.0	48 ± 10	7.7 ± 2.2
$P_{13}(1720)$	1.747 ± 0.004	161 ± 31	comp. with 0.	60-100

Precise data on $\pi\Delta$ & $\rho\rho$ hadronic decays of $P_{13}(1720)$ is critical in order to understand origin of the structure at $W \sim 1.7$ GeV in $N\pi\pi$ electroproduction cross sections. This information can be obtained in the studies of $\pi N \rightarrow \pi\pi N$ reactions at **J-PARC** and **HADES**...



Pion Photoproduction



Single *Pion* Photoproduction

- An accurate evaluation of the **EM** couplings $N^*(\Delta^*) \rightarrow \gamma N$ from **meson photoproduction** data remains a paramount task in **hadron** physics.
- Only with good data on both the **proton** and **neutron targets**, one can hope to disentangle the **isoscalar** & **isovector EM** couplings of various N^* & Δ^* resonances,
K.M. Watson, Phys Rev **95**, 228 (1954); R.L. Walker, Phys Rev **182**, 1729 (1969)
as well as the **isospin** properties of non-resonant **background amplitudes**.
- The lack of the $\gamma n \rightarrow \pi^- p$ & $\gamma n \rightarrow \pi^0 n$ data does not allow us to be as confident about the determination of **neutron** couplings relative to those of the **proton**.



- The **radiative decay** width of **neutral baryons** may be extracted from π^- & π^0 photoproduction off the **neutron**, which involves a **bound neutron target** and needs the use of **model-dependent nuclear (FSI) corrections**.

A.B. Migdal, JETP **1**, 2 (1955); K.M. Watson, Phys Rev **95**, 228 (1954)



Where We Are Now

- Some of the N^* baryons [$N(1675)D_{15}$, for instance] have *stronger* EM coupling to the **neutron** than to the **proton** but parameters are very uncertain.

$$N(1675) 5/2^- \quad I(J^P) = \frac{1}{2}(\frac{5}{2}^-) \quad \text{Status: } **** \quad D_{15}$$

PDG12:	$N(1675)5/2^- \rightarrow p\gamma$, helicity-1/2 ampl, $A_{1/2}$: $+0.019 \pm 0.008$
PDG12:	$N(1675)5/2^- \rightarrow n\gamma$, helicity-1/2 ampl, $A_{1/2}$: -0.043 ± 0.012
SAID13	$N(1675)5/2^- \rightarrow p\gamma$, helicity-3/2 ampl, $A_{3/2}$: $+0.016 \pm 0.001$
SAID13	$N(1675)5/2^- \rightarrow n\gamma$, helicity-3/2 ampl, $A_{3/2}$: -0.058 ± 0.002

- PDG estimates for the $A_{1/2}$ & $A_{3/2}$ decay amplitudes of the $N(1720)P_{13}$ state are consistent with **zero**, while the recent SAID determination gives small but non-vanishing values.

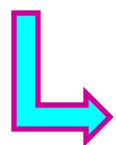
$$N(1720) 3/2^+ \quad I(J^P) = \frac{1}{2}(\frac{3}{2}^+) \quad \text{Status: } **** \quad P_{13}$$

	PDG12	SAID13
PDG12:	$N(1720)3/2^+ \rightarrow p\gamma$, helicity-1/2 ampl, $A_{1/2}$: -0.01 to 0.11	$+0.095 \pm 0.002$
SAID13:	$N(1720)3/2^+ \rightarrow p\gamma$, helicity-3/2 ampl, $A_{3/2}$: -0.019 ± 0.020	-0.048 ± 0.002

- Other unresolved issues relate to the second P_{11} , $N(1710)P_{11}$, that are not seen in the recent πN PWA, contrary to other PWAs used by the PDG12.

$$N(1710) 1/2^+ \quad I(J^P) = \frac{1}{2}(\frac{1}{2}^+) \quad \text{Status: } *** \quad P_{11}$$

The latest GWU analysis (ARNDT 06) finds no evidence for this resonance.

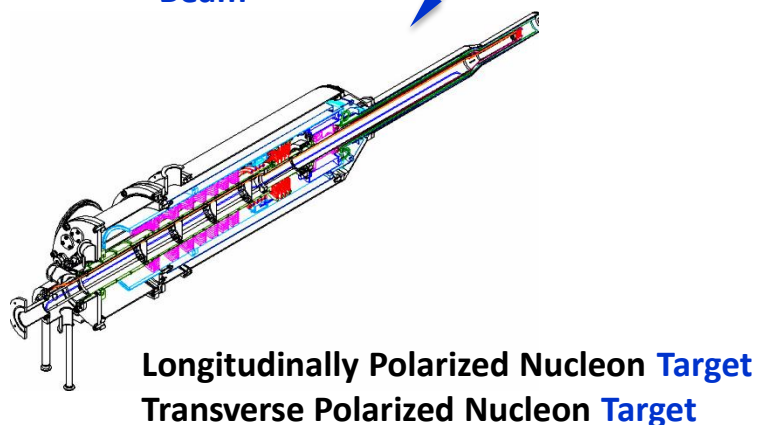
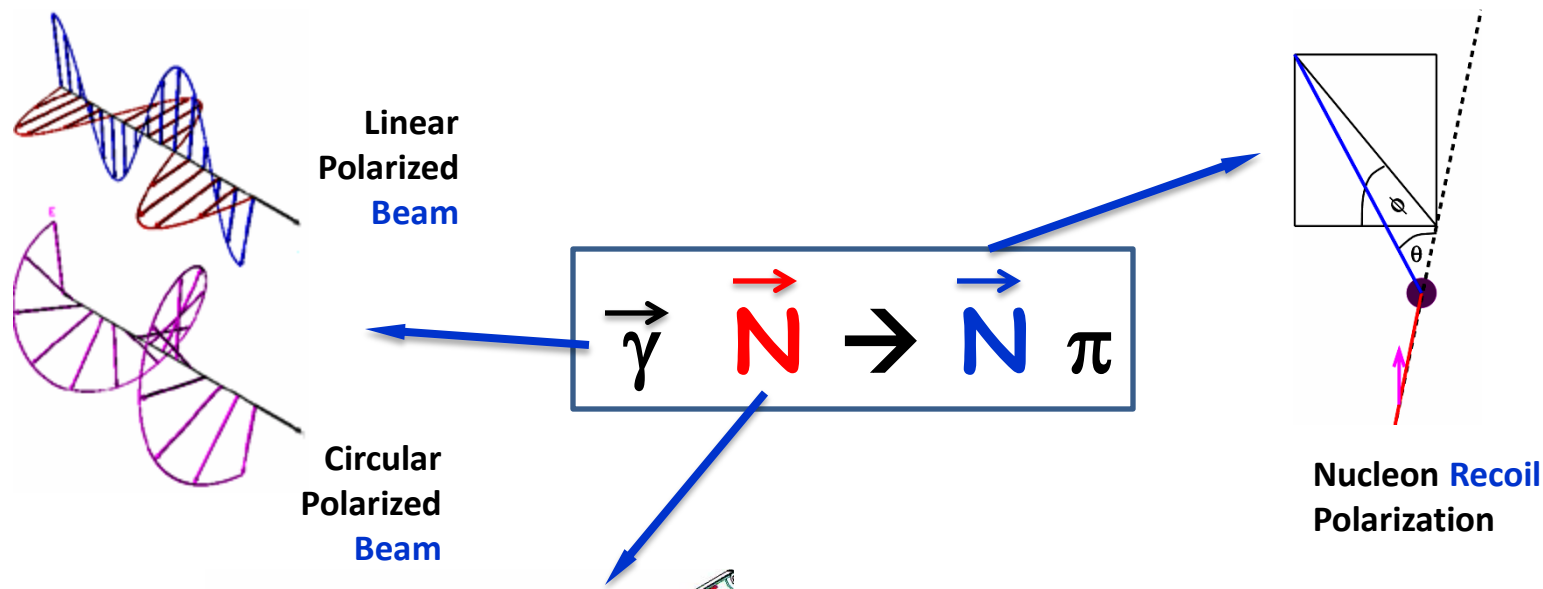


R. Arndt, Ya. Azimov, M. Polyakov, IS, R. Workman, Phys Rev C 69, 035208 (2004)

$N(1680) 1/2^+$



Complete Experiment in Pion Photo Production

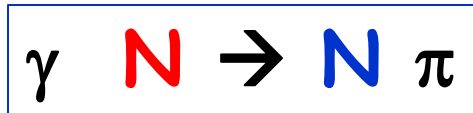


There are several πN SESs which one can possibly consider as samples of complete experiments.

Pion PhotoProduction complete experiments are under the consideration now.



Direct Amplitude Reconstruction in Pion Photo Production



spin: $1 \quad \frac{1}{2} \rightarrow \frac{1}{2} \quad 0$

helicities: $2 \times 2 \times 2 / 2 = 4$

parity conservation 

- In **particle physics**, **helicity** is the projection of the spin \vec{S} onto the direction of momentum, \hat{p} :

$$h = \vec{J} \cdot \hat{p} = \vec{L} \cdot \hat{p} + \vec{S} \cdot \hat{p} = \vec{S} \cdot \hat{p}$$

$$\hat{p} = \frac{\vec{p}}{|\vec{p}|}$$

Therefore, there are **4** independent invariant amplitudes

- In order to determine the pion photo production amplitude, one has to carry out **7** independent measurements at fixed **(W, t)** or **(E_γ, θ)**

8 Ambiguities in the partial-wave analysis of pseudoscalar-meson photoproduction
 Greg Keaton and Ron Workman
 Department of Physics, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061
 (Received 19 April 1996)

This extra observable is necessary to eliminate a **sign ambiguity**.



SAID for Pion Photoproduction

[W. Chen *et al*, Phys Rev C 86, 015206 (2012)]

- Energy dependent **GB12** and associated **SES**
- $E = 145 - 2700$ MeV [W = 1080 - 2460 MeV]
- PWs = 60 [E & M multipoles] [J < 6]
- Prms = 210
- Constraint: $M = (\text{Born} + A)(1+iT_{\pi N}) + BT_{\pi N} + (C+iD)(\text{Im}T_{\pi N} - |T_{\pi N}|^2)$

Born [no free parameters to fit]

πN -PWA [no theoretical input]

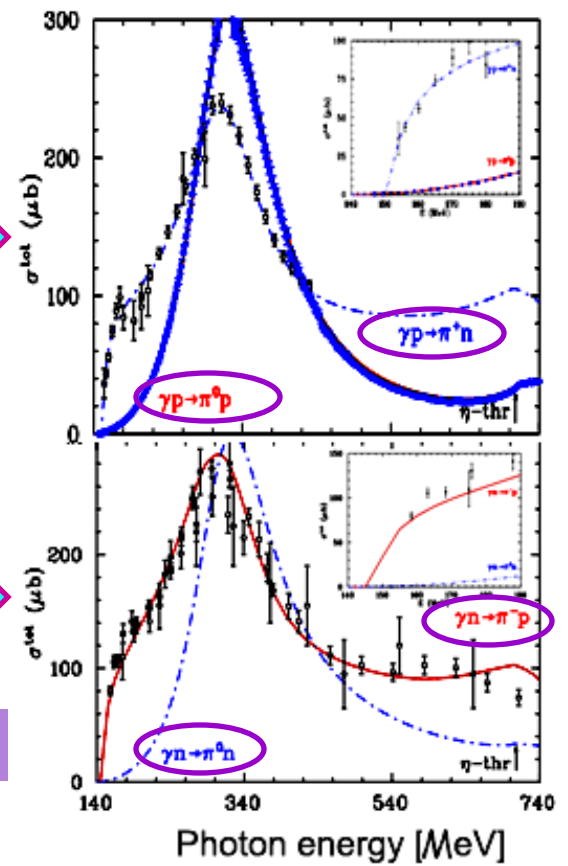
Reaction	Data (Dpol)	χ^2
$\gamma p \rightarrow \pi^0 p$	14,612 (3 %)	32,449
$\gamma p \rightarrow \pi^+ n$	8,510 (5 %)	16,520
$\gamma n \rightarrow \pi^+ p$	3,058 (0 %)	6,396
$\gamma n \rightarrow \pi^0 n$	364 (0 %)	1,201
Total	26,554	56,566

23,122 data

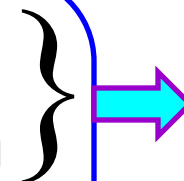
3,422 data

Much less known, 15%

- **1st generation** – ('60-'90)
10k data [85% bremsstrahlung data.]
30% data is polarized.
[limited coverage, broad energy binning.]
- **2nd generation** – ('90-'10) → SAID fits.
25k data [60% tagged data.]
30% data is polarized.
Dearth of neutron data.
- **3rd generation** – ('10+)
New data will come from **JLab**, **CB@MAMI-C**,
SPring-8, **CB-ELSA**, **MAX-lab**, & **LNS**.



- DU13:** included recent **CLAS** $\pi^0 p$ & $\pi^+ n$ Σ [M. Dugger *et al*, arXiv:1308.4028[nucl-ex]
- GB12/GZ12:** included recent **CLAS** $\pi^- p$ $d\sigma/d\Omega$ [W. Chen *et al*, Phys Rev C **86**, 015206 (2012)]
- CM12:** **CM** parameterization for $T_{\pi N}$ [R. Workman *et al*, Phys Rev C **86**, 015202 (2012)]
- SN11/SK11:** included recent **GRAAL** $\pi^- p$ & $\pi^0 n$ Σ
LEPS $\pi^0 p$ $d\sigma/d\Omega$ [R. Workman *et al*, Phys Rev C **85**, 025201 (2012)]
- $$M = (\text{Born} + A)(1 + iT_{\pi N}) + BT_{\pi N} + (C + iD)(\text{Im}T_{\pi N} - |T_{\pi N}|^2)$$
- SP09:** included recent **CLAS** $\pi^+ n$ $d\sigma/d\Omega$ [M. Dugger *et al*, Phys Rev C **79**, 065206 (2009)]
- $$M = (\text{Born} + \alpha_R)(1 + iT_{\pi N}) + \alpha_R T_{\pi N} + \text{higher terms}$$



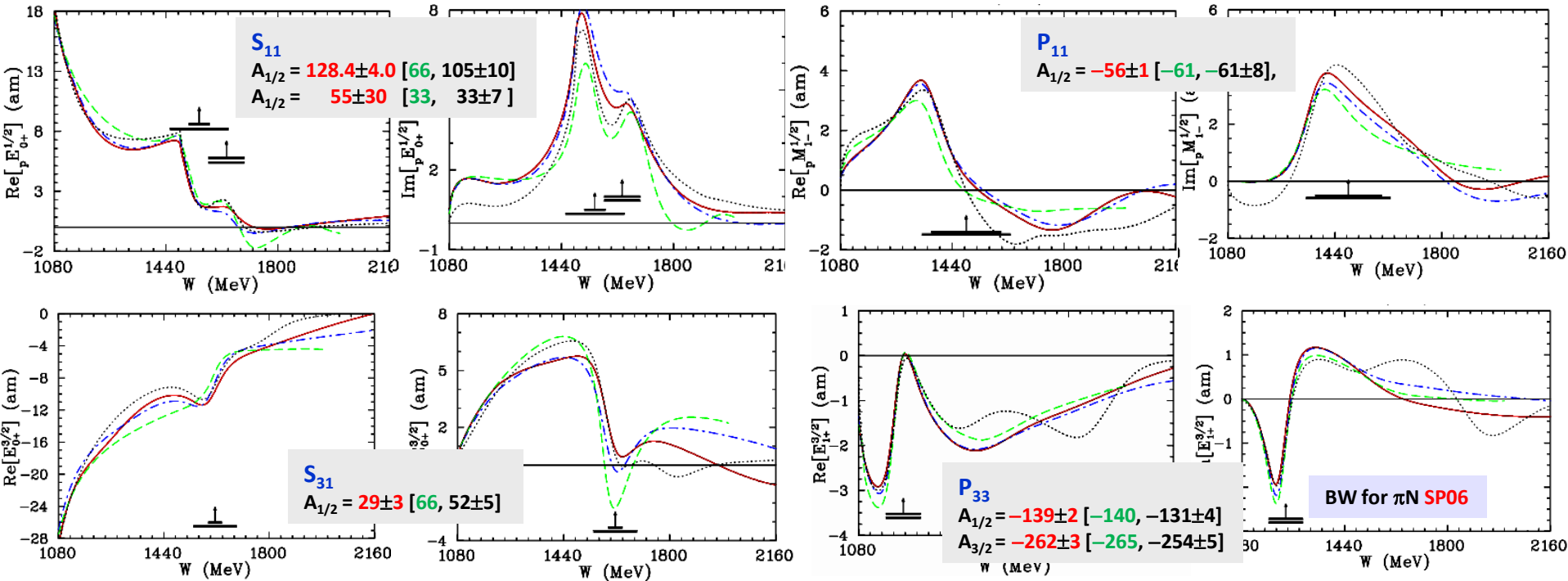
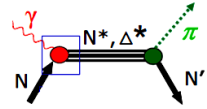
Solution	Energy Limit (MeV)	χ^2/N_{Data}	N_{Data}
DU13	2700	2.23	27,265
GB12	2700	2.09	26,179
CM12	2700	2.01	25,814
SN11	2700	2.08	25,553
SP09	2700	2.05	24,912
FA06	3000	2.18	25,524
SM02	2000	2.01	17,571
SM95	2000	2.37	13,415

- The overall **SAID** χ^2 has remained **stable** against the growing database, which has increased by a **factor of 2** since **1995**.
- Most of this increase coming from **photon-tagging** facilities.

Proton Multipoles from DU13 & CM12

[R. Workman *et al*, Phys Rev C 86, 015202 (2012); M. Dugger *et al*, arXiv:1308.4028 [nucl-ex]

- Overall: the difference between MAID07 or BnGa and SAID DU13 is rather small but... Resonances may be essentially different.



- Significant changes have occurred at high energies.
- Comparisons to earlier SAID fits and fit from the Mainz & BnGa groups show that the new DU13 & CM12 solutions is much more satisfactory at higher energies.

SAID DU13
 SAID CM12
 MAID07
 BnGa13

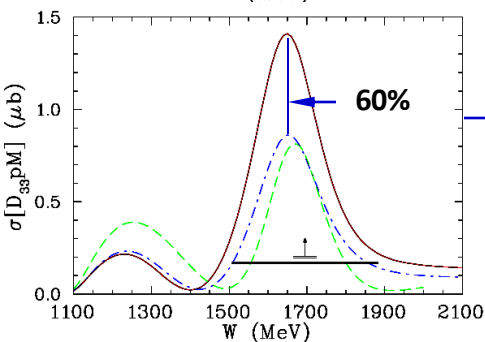
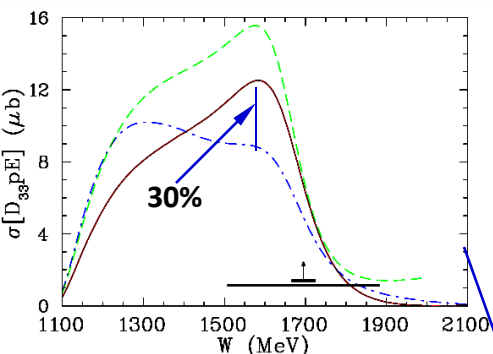
MAID07: D. Drechsel *et al*, Eur Phys J A 34, 69 (2007)
 BnGa: A. Anisovich *et al*, Eur Phys J A 48, 15 (2012)



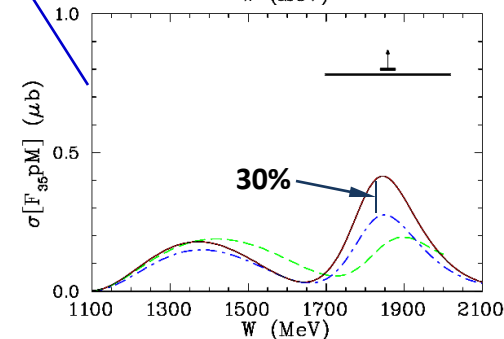
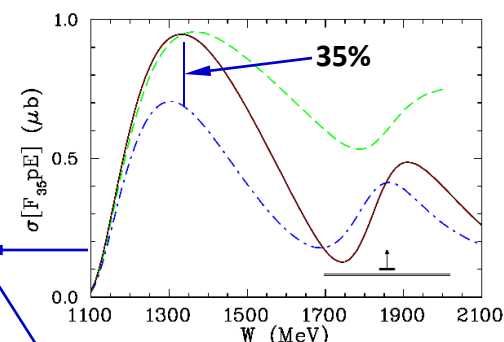
CLAS Σ Data Impact for Proton $S = 0$ & $I = 1/2$ Couplings

[M. Dugger *et al*, arXiv:1308.4028 [nucl-ex]]

• The largest change is found for the $\Delta(1700)3/2^-$ and $\Delta(1905)5/2^+$ states, for which the various analyses disagree significantly in terms of photo-decay amplitudes.



Δ^*	Solution	$A_{1/2}$ ($\text{GeV}^{1/2} \times 10^{-3}$)	$A_{3/2}$ ($\text{GeV}^{1/2} \times 10^{-3}$)
$\Delta(1700)3/2^-$	CM12	105 ± 5	92 ± 4
	DU13	132 ± 5	108 ± 5
	BnGa	160 ± 20	165 ± 20
	MD07	226	210
	PDG12	104 ± 15	85 ± 22
$\Delta(1905)5/2^+$	CM12	19 ± 2	-38 ± 4
	DU13	20 ± 2	-49 ± 5
	BnGa	25 ± 5	-50 ± 4
	MD07	18	-28
	PDG12	26 ± 11	-45 ± 20



Relativized Quark Model:
S. Capstick, Phys Rev D **46**, 2864 (1992)

SAID DU12
SAID CM12
MAID07

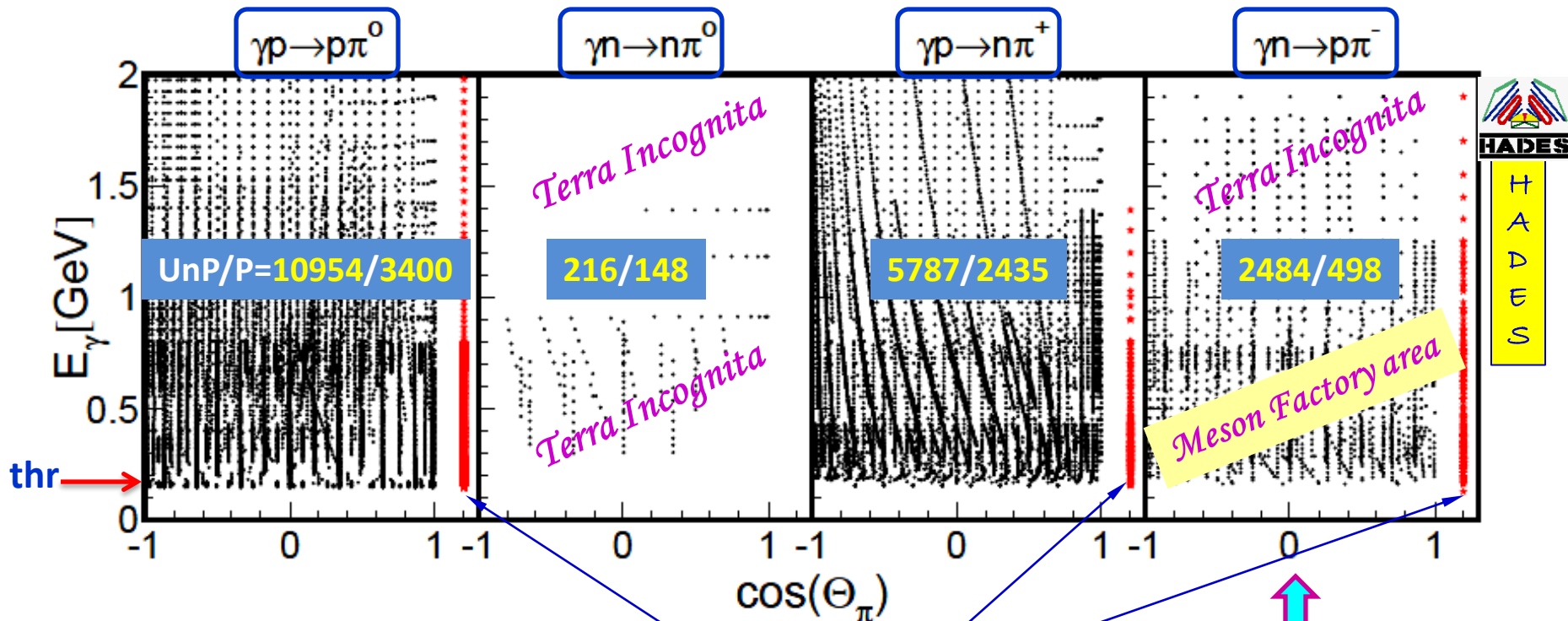


$\gamma n \rightarrow \pi^- p$ Experiment

- The existing $\gamma n \rightarrow \pi^- p$ database contains mainly differential cross sections (**17%** of which are from **polarized** measurements.)
- Many of these are old **bremsstrahlung** measurements with limited **angular** ($\theta = 40 - 140^\circ$) coverage and large **energy** binning ($E_\gamma = 100 - 200$ MeV.) In several cases, the **systematic** uncertainties have not been given.
- At lower energies ($E_\gamma < 700$ MeV,) there are data sets for the inverse π^- photoproduction reaction: $\pi^- p \rightarrow \gamma n$.
This process is free from complications associated with a **deuteron target**.
eg, CB@BNL: A. Shafi, *et al*, PRC70, 035204 (2004)
- However, the disadvantage of using $\pi^- p \rightarrow \gamma n$ is the **large background** because of the **5 to 500** times larger cross section for $\pi^- p \rightarrow \pi^0 n \rightarrow \gamma \gamma n$.



Future exp activity will fill empty spots specifically for n-target.

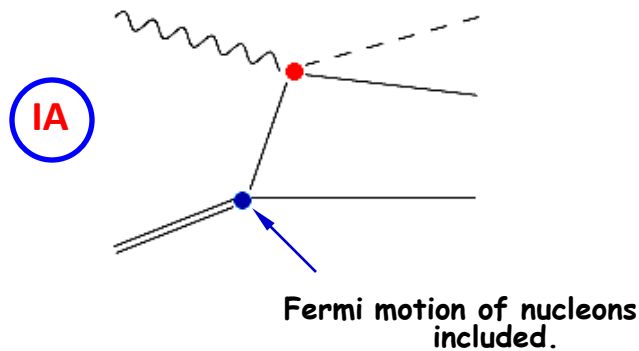


• The existing $\gamma n \rightarrow \pi^- p$ data contains mainly differential cross sections (17% of which are from polarized measurements.)

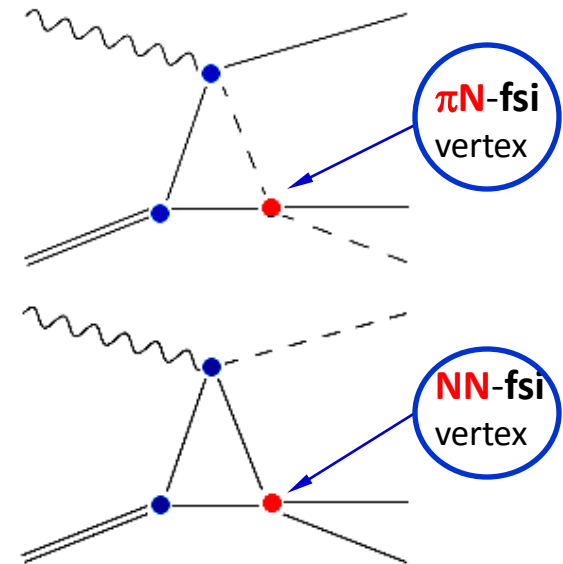
FSI and $\gamma d \rightarrow \pi^- p p \Rightarrow \gamma n \rightarrow \pi^- p$

[V. Tarasov, A. Kudryavtsev, W. Briscoe, H. Gao, & IS, Phys Rev C 84, 035203 (2011)]

- FSI plays a critical role in the state-of-the-art analysis of $\gamma n \rightarrow \pi N$ data.
- For $\gamma n \rightarrow \pi^- p$ the effect: 5% - 60%. It depends on (E, θ) .



Input: SAID $\gamma N \rightarrow \pi N$, $\pi N \rightarrow \pi N$, $NN \rightarrow NN$ amplitudes for 3 leading terms.
DWF: Bonn Potential.

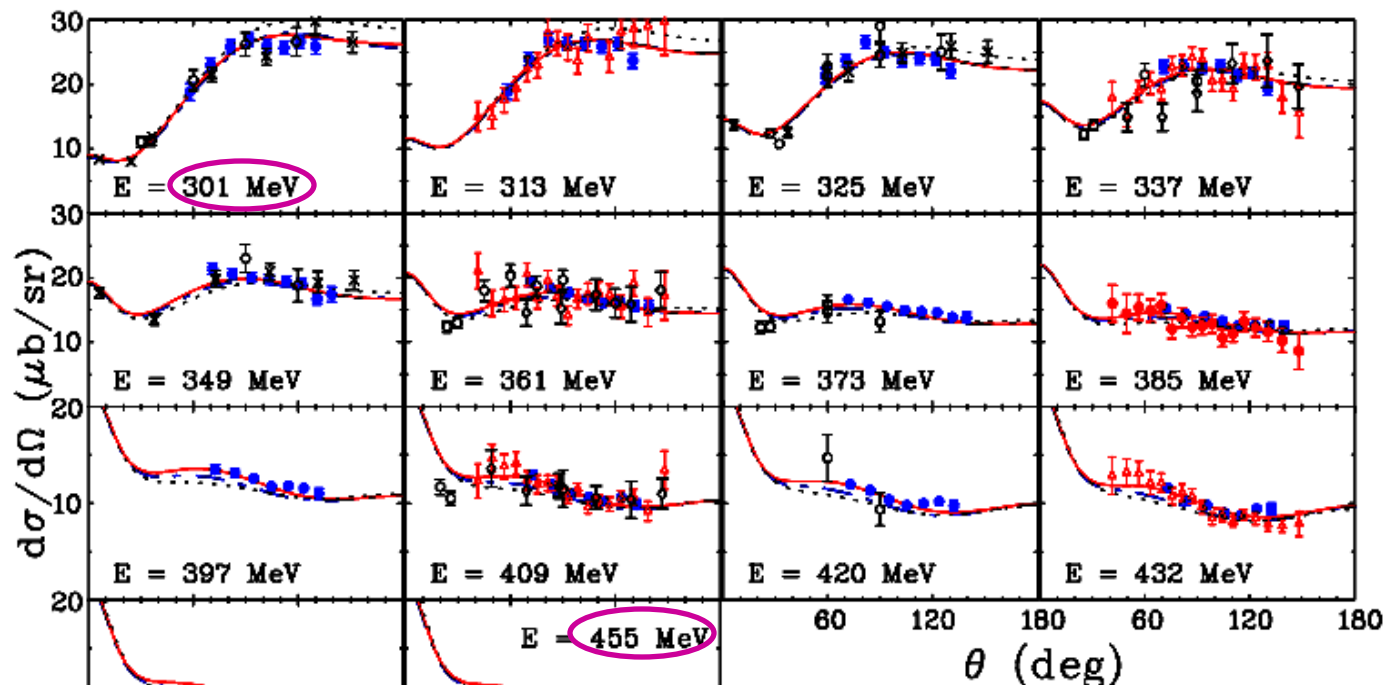


$$R = (d\sigma/d\Omega_{\pi p}) / (d\sigma^{IA}/d\Omega_{\pi p})$$



$$\frac{d\sigma}{d\Omega}(\gamma n) = R^{-1} \frac{d\sigma}{d\Omega}(\gamma d)$$

- MAMI-B data for $\gamma n \rightarrow \pi^- p$ (including FSI corrections) and previous hadronic data for $\pi^- p \rightarrow n \gamma$ appear to agree well.



SAID-PE12
SAID-SN11
MAID07

Data:

- – MAMI-B for $\gamma n \rightarrow \pi^- p$
- △ – CB@BNL for $\pi^- p \rightarrow n \gamma$

A. Shafi, et al, Phys Rev C 70, 035204 (2004)

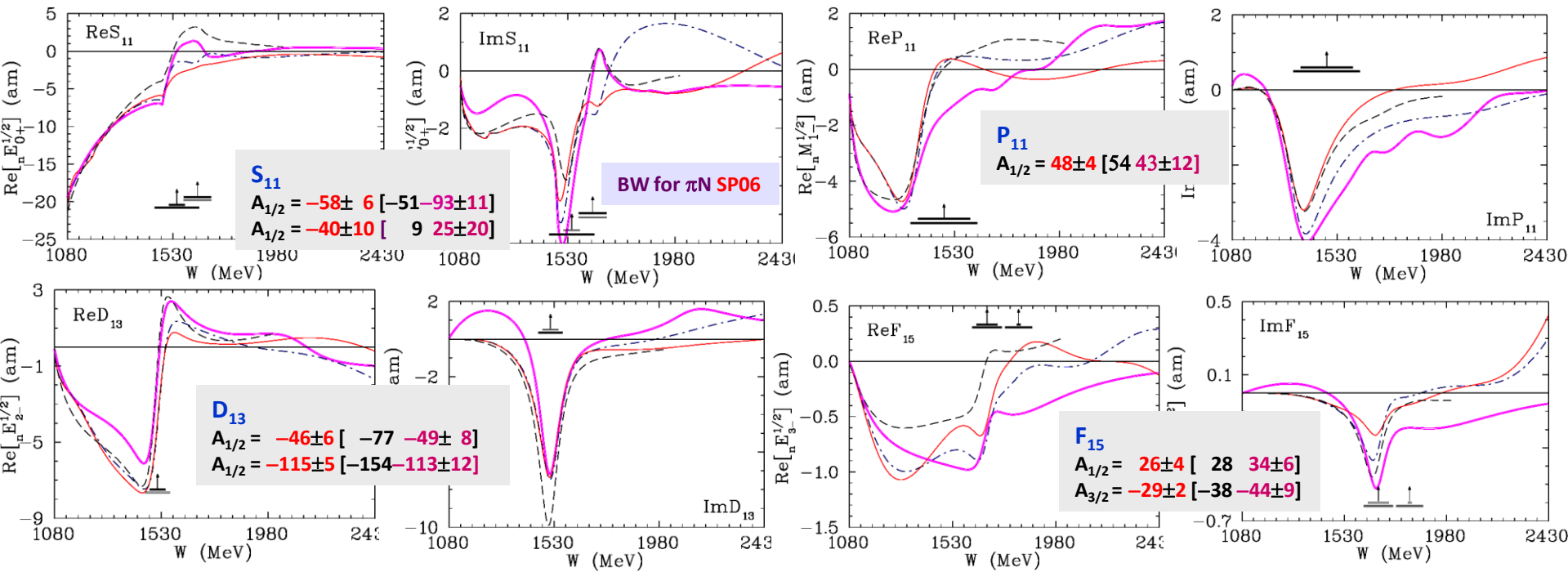
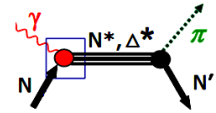
- o – TRIUMF, CERN, LBL, LAMPF for $\pi^- p \rightarrow n \gamma$



Neutron Multipoles from SAID GB12 & SN11

[W. Chen *et al*, Phys Rev C 86, 015206 (2012); R. Workman *et al*, Phys Rev C 85, 025201 (2012)]

- Overall: the difference between MAID07 with BnGa13 and SAID GB12 is rather small but... Resonances may be essentially different.



- Significant changes have occurred at high energies.
- Comparisons to earlier SAID and fit from the Mainz and BnGa groups show that the new GB12 solution is much more satisfactory at higher energies.

SAID GB12
 SAID SN11
 MAID07
 BnGa13

MAID07: D. Drechsel, *et al*, Eur Phys J A 34, 69 (2007)
 BnGa13: A. Anisovich *et al*, Eur Phys J A 49, 67 (2013)



CLAS Data Impact for Neutron $S = 0$ & $I = 1/2$ Couplings

[W. Chen *et al*, Phys Rev C 86, 015206 (2012)]

- **BnGa13** and **SAID GB12** used the same (almost) data to fit them while **BnGa13** has several new **Ad Hoc** resonances.

Resonance	πN SAID	$nA_{1/2}$	Resonance	πN SAID	$nA_{1/2}$	$nA_{3/2}$
$N(1535)1/2^-$ -63 S_{11}	$W_R=1547$ MeV $\Gamma=188$ MeV $\Gamma_{\pi N}/\Gamma=0.36$	-58 ± 6 -60 ± 3 -93 ± 11 -46 ± 27	$N(1520)3/2^-$ -38 -114 D_{13}	$W_R=1515$ MeV $\Gamma=104$ MeV $\Gamma_{\pi N}/\Gamma=0.63$	-46 ± 6 -47 ± 2 -49 ± 8 -59 ± 9 -139 ± 11	-115 ± 5 -125 ± 2 -113 ± 12 -139 ± 11
$N(1650)1/2^-$ -35 S_{11}	$W_R=1635$ MeV $\Gamma=115$ MeV $\Gamma_{\pi N}/\Gamma=1.00$	-40 ± 10 -26 ± 8 25 ± 20 -15 ± 21	$N(1675)5/2^-$ -35 -51 D_{15}	$W_R=1674$ MeV $\Gamma=147$ MeV $\Gamma_{\pi N}/\Gamma=0.39$	-58 ± 2 -42 ± 2 -60 ± 7 -43 ± 12 -58 ± 13	-80 ± 5 -60 ± 2 -88 ± 10 -58 ± 13
$N(1440)1/2^+$ -6 P_{11}	$W_R=1485$ MeV $\Gamma=284$ MeV $\Gamma_{\pi N}/\Gamma=0.79$	48 ± 4 45 ± 15 43 ± 12 40 ± 10	$N(1680)5/2^+$ 19 -23 F_{15}	$W_R=1680$ MeV $\Gamma=128$ MeV $\Gamma_{\pi N}/\Gamma=0.70$	26 ± 4 50 ± 4 34 ± 6 29 ± 10	-29 ± 2 -47 ± 2 -44 ± 9 -33 ± 9

GB12
SN11
BnGa13
PDG

S. Capstick, Phys Rev D 46, 2864 (1992).

- New **GB12** $nA_{1/2}$ & $nA_{3/2}$ couplings shown sometimes a significant deviation from our previous SAID determination (**SN11**) and **PDG12** average values, e.g., for $N(1650)1/2^-$, $N(1675)5/2^-$, and $N(1680)5/2^+$.
- Fresh **BnGa13** has some difference vs. **GB12**, **PDG12**, and the **relativized quark model**, e.g., for $N(1650)1/2^-$, $N(1650)1/2^-$, and $N(1680)5/2^+$.

BnGa13: A. Anisovich *et al*, EPJA 49, 67 (2013)



J. Beringer *et al* (PDG) Phys Rev D **86**, 010001 (2012)

- More than half of states have poor evidence.
- Most of states need more work to do.
- Most of QCD models predict more states than observed.
- **Where are missing resonances?**

GW SAID Contribution

Status as seen **I = 1/2**

Particle	J^P	Status									
		overall	πN	γN	$N\eta$	$N\sigma$	$N\omega$	ΔK	ΣK	$N\rho$	$\Delta\pi$
N	1/2 ⁺	****									
N(1440)	1/2 ⁺	****	****	****		***			*	***	
N(1520)	3/2 ⁻	****	****	****		***			***	***	
N(1535)	1/2 ⁻	****	****	****		****			**	*	
N(1650)	1/2 ⁻	****	****	***		***	**		**	***	
N(1675)	5/2 ⁻	****	****	***		*			*	***	
N(1680)	5/2 ⁺	****	****	****	*	**			***	***	
N(1685)	? [?]	*									
N(1700)	3/2 ⁻	***	***	**		*	*		*	***	
N(1710)	1/2 ⁺	***	***	***		***	**		*	**	
N(1720)	3/2 ⁺	****	****	***		***	**	**	**	*	
N(1860)	5/2 ⁺	**	**			*	*		*	*	
N(1875)	3/2 ⁻	***	*	***		**	***	**		***	
N(1880)	1/2 ⁺	**	*	*		**	*				
N(1895)	1/2 ⁻	**	*	**		**	*				
N(1900)	3/2 ⁺	***	**	***		**	***	**	*	**	
N(1990)	7/2 ⁺	**	**	**		*	*				
N(2000)	5/2 ⁺	**	*	**		**	*	**			
N(2040)	3/2 ⁺	*									
N(2060)	5/2 ⁻	**	**	**		*	**				
N(2100)	1/2 ⁺	*									
N(2150)	3/2 ⁻	**	**	**		**	**		**	**	
N(2190)	7/2 ⁻	****	****	***		26 N*	**	*			
N(2220)	9/2 ⁺	****	****	****		11 ****	**	*			
N(2250)	9/2 ⁻	****	****	****		5 ***	**	*			
N(2600)	11/2 ⁻	***	***	***		7 **	**	*			
N(2700)	13/2 ⁺	**	**	**		3 *	**	*			

I = 3/2

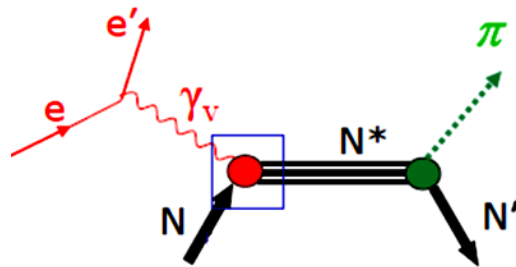
Particle	J^P	Status										Status as seen in —	
		overall	πN	γN	$N\eta$	$N\sigma$	$N\omega$	ΔK	ΣK	$N\rho$	$\Delta\pi$		
Δ (1232)	3/2 ⁺	****	****	****	F								
Δ (1600)	3/2 ⁺	***	***	***	o					*	***		
Δ (1620)	1/2 ⁻	****	****	***	r					***	***		
Δ (1700)	3/2 ⁻	****	****	****	b					**	***		
Δ (1750)	1/2 ⁺	*	*		i								
Δ (1900)	1/2 ⁻	**	**	**	d				**	**	**		
Δ (1905)	5/2 ⁺	****	****	****	d				***	**	**		
Δ (1910)	1/2 ⁺	****	****	**	e			*	*	**			
Δ (1920)	3/2 ⁺	***	***	**	n			***	***	**			
Δ (1930)	5/2 ⁻	***	***	**									
Δ (1940)	3/2 ⁻	**	*	**	F				(seen in $\Delta\eta$)	***	*	***	
Δ (1950)	7/2 ⁺	****	****	****	o				***	*	***		
Δ (2000)	5/2 ⁺	**	**	**	r					**	**		
Δ (2150)	1/2 ⁻	*	*		b								
Δ (2200)	7/2 ⁻	*	*		i								
Δ (2300)	9/2 ⁺	**	**	**	d								
Δ (2350)	5/2 ⁻	*	*		d								
Δ (2390)	7/2 ⁺	*	*		n								
Δ (2400)	9/2 ⁻	**	**	7 ****									
Δ (2420)	11/2 ⁺	****	****	3 ***									
Δ (2750)	13/2 ⁻	**	**	7 **									
Δ (2950)	15/2 ⁺	**	**	5 *									

BnGa Additional States

The latest GWU analysis (ARNDT 06) finds no evidence for this resonance.

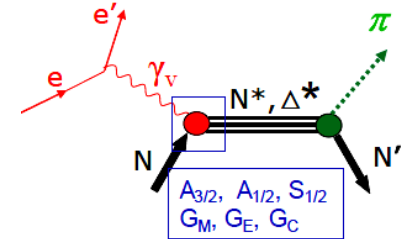


Pion Electroproduction



GW \mathcal{N}^* Program

- Energy dependent **SM08** and associated **SES & SQS**
- $W = 1080 - 2000 \text{ MeV}$ $Q^2 = 0 - 6 \text{ GeV}^2$
- PWs = 60 [multipoles] $[J < 6]$
- Prms = 171
- **Constraint:** πN + Pion Photo Prod PWAs [no theoretical input]



0.85 World Electro Prod from JLab CLAS

PWA Problems:

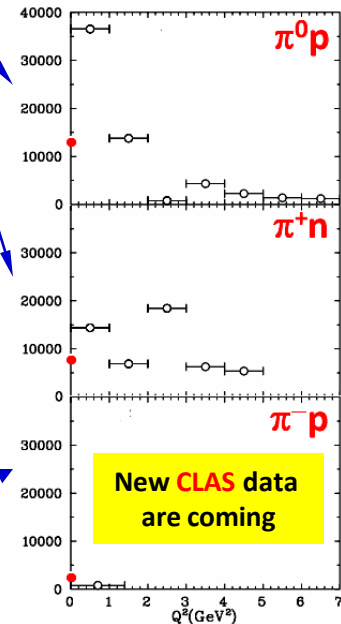
- Additional [18 S] Multipoles
- Q^2 dependence

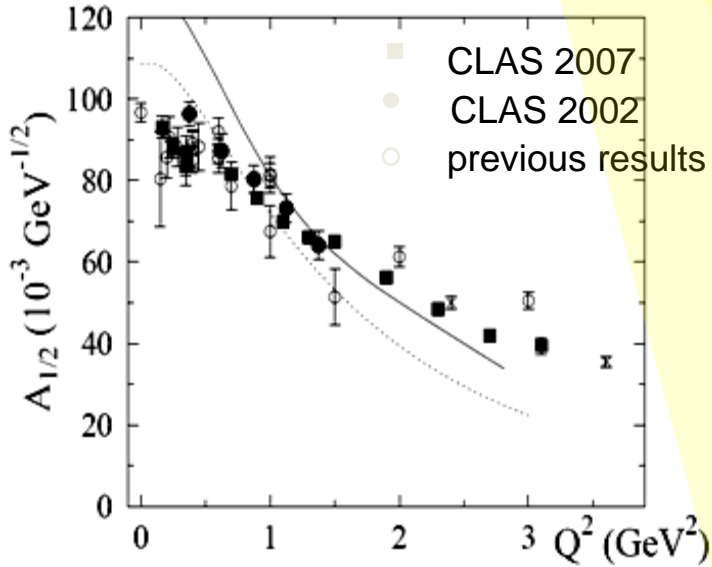
Database Problems:

- Most of data are **unPolarized** measurements
- There are no $\pi^0 n$ data and very few $\pi^- p$ [no Pol measurements] That does not allow to determine **n-couplings** at $Q^2 > 0$

Reaction	Data	χ^2
$\gamma^* p \rightarrow \pi^0 p$	55,766	81,284
$\gamma^* p \rightarrow \pi^+ n$	51,312	80,004
Redundant	14,772	17,375
Total	121,850	178,663
$\gamma N \rightarrow \pi N$	25,358	53,458
All Photo*	147,208	232,121
$\pi N \rightarrow \pi N$	31,479	57,157
All πN	178,687	289,278
$\gamma^* n \rightarrow \pi^- p$	801	
$\gamma^* n \rightarrow \pi^0 n$	No Data	

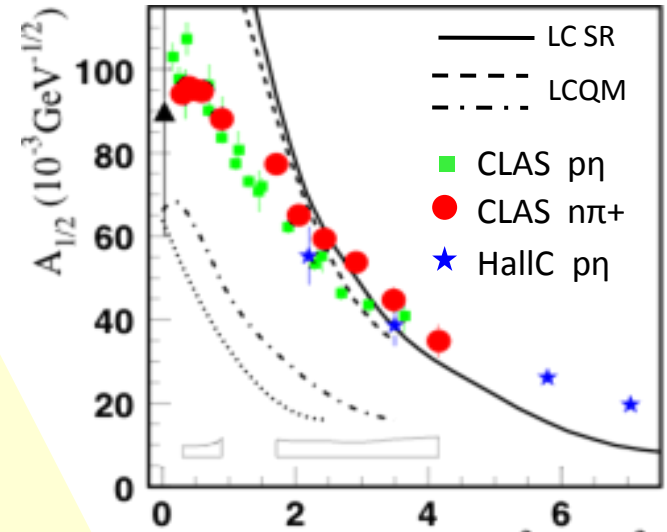
Q^2 -Data



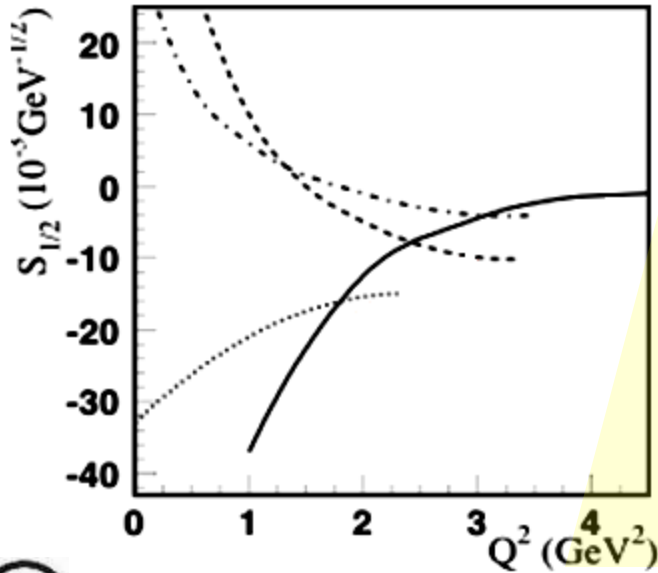


Analysis of $p\eta$
 assumes $S_{1/2}=0$

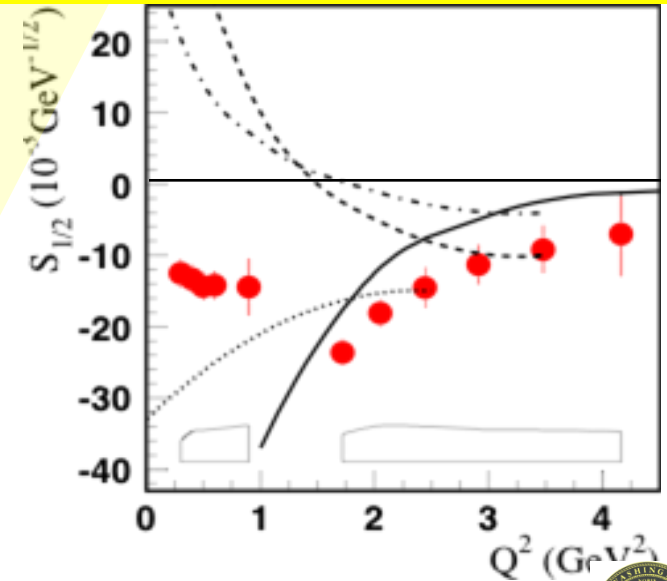
Branching ratios
 $\beta_{N\pi} = \beta_{N\eta} = 0.45$



Courtesy of Kijun Park, QCD2010 @ Montpellier France June -July, 2010



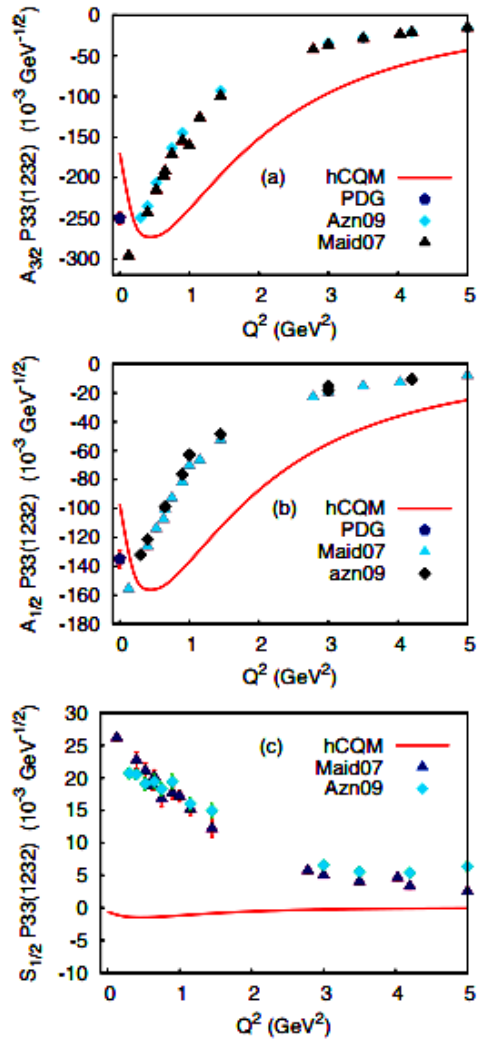
- $A_{1/2}(Q^2)$ from $N\pi$ and $p\eta$ are consistent
- First extraction of $S_{1/2}(Q^2)$ amplitude.



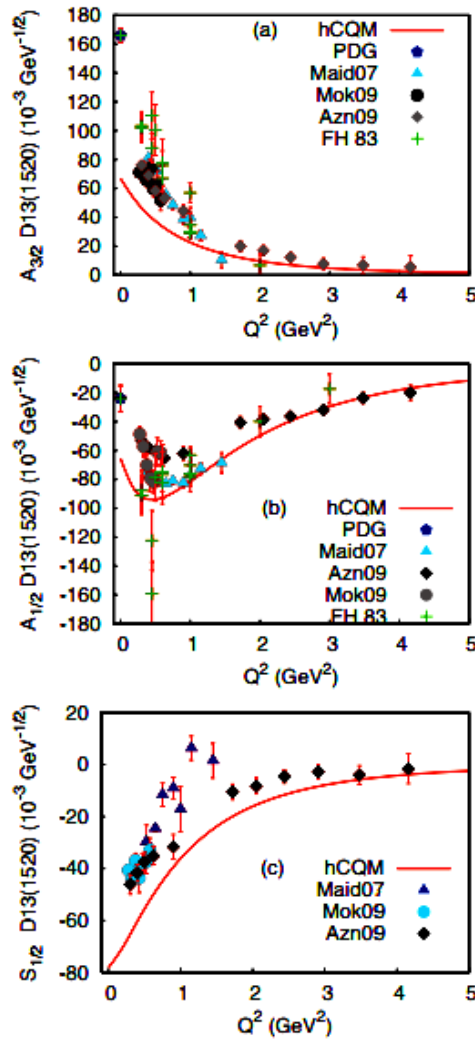
Constituent Quark Model Predictions

[E. Santopinto and M. Giannini, Phys Rev C 86, 065202 (2012)]

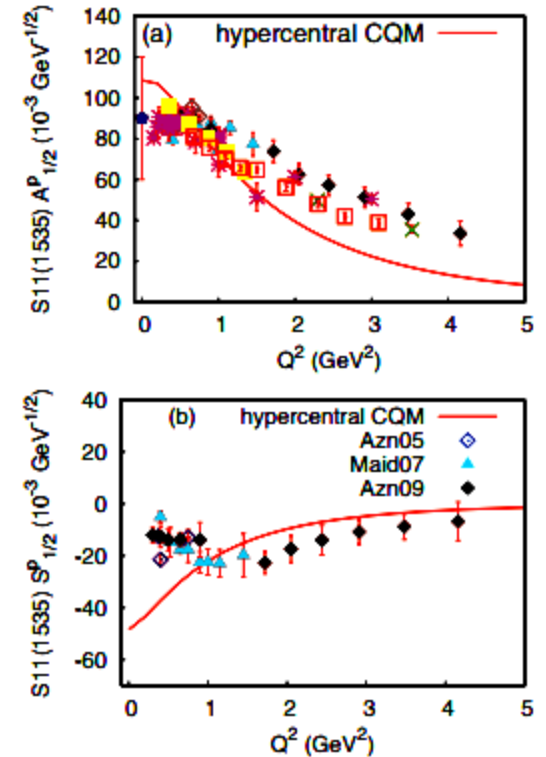
$\Delta(1232)3/2^+$



$N(1520)3/2^-$



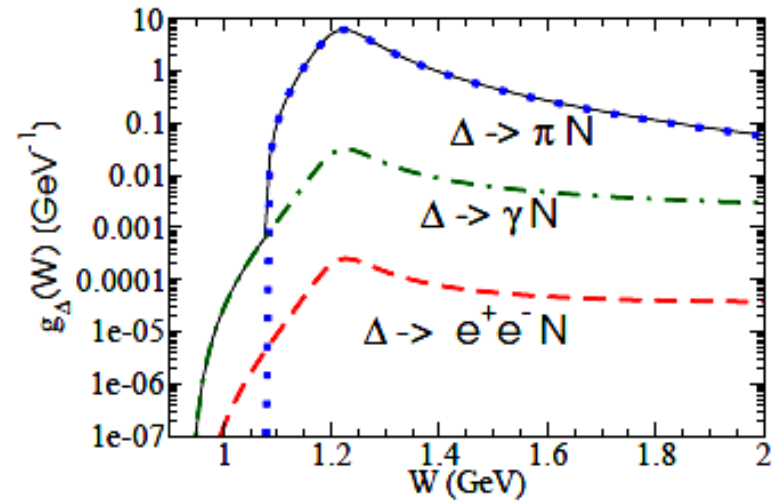
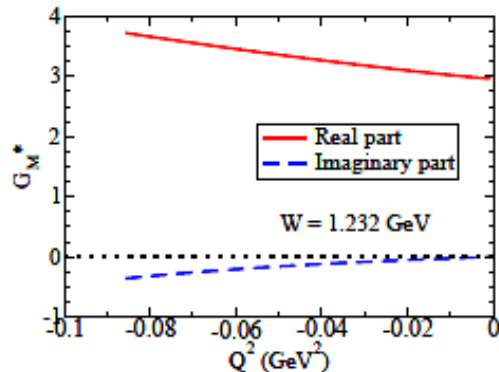
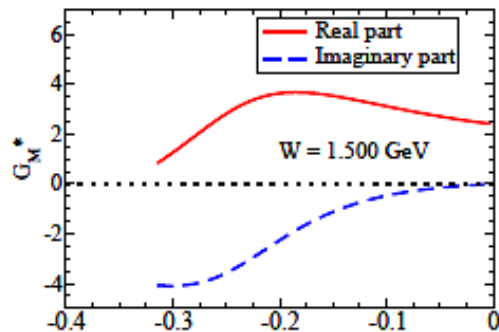
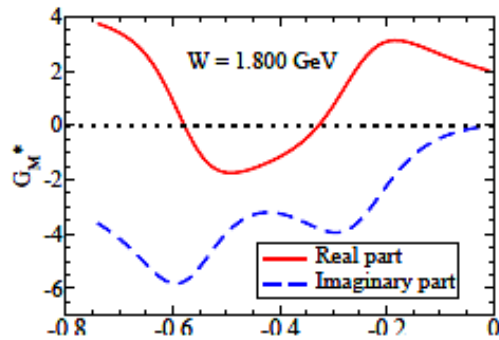
$N(1535)1/2^-$



Timelike $\gamma^* N \rightarrow \Delta$ form factors and Δ Dalitz decay

[G. Ramalho and M.T. Pena, Phys Rev D 85, 113014 (2012)]

$\Delta(1232)3/2^+$



$$g_{\Delta}(W) = A \frac{W^2 \Gamma_{tot}(W)}{(W^2 - M_{\Delta}^2)^2 + W^2 [\Gamma_{tot}(W)]^2}$$

$$\Gamma_{tot}(W) = \Gamma_{\pi N}(W) + \Gamma_{\gamma N}(W) + \Gamma_{e^+e^- N}(W)$$



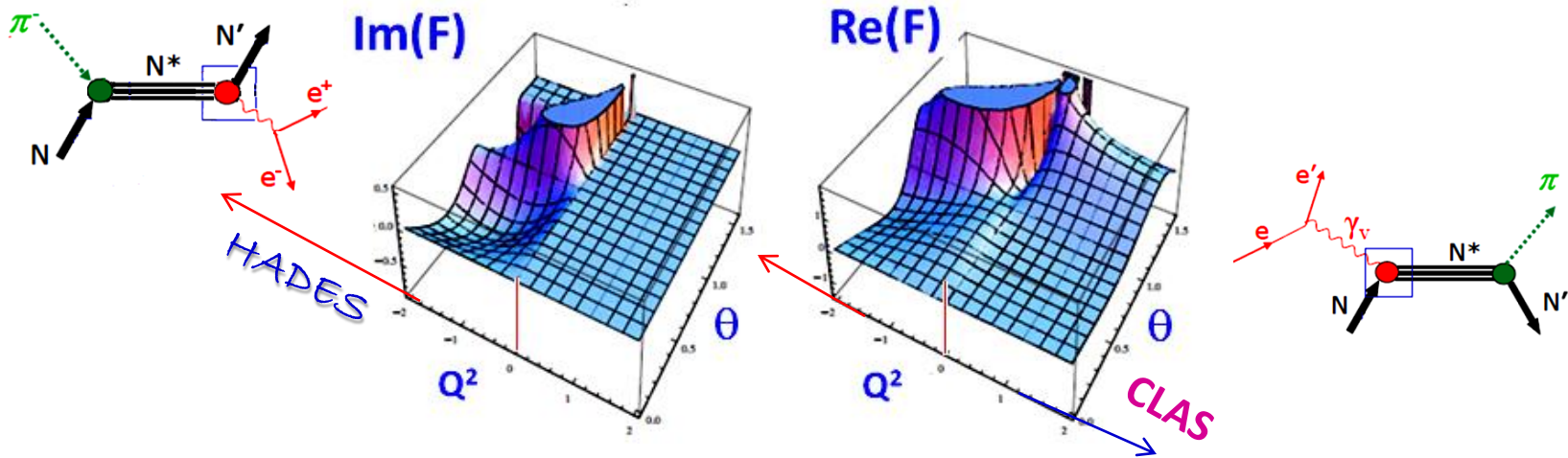
Inverse Pion Electroproduction (IPE)

- IPE is the only process which allows the determination of **EM nucleon** & pion **formfactors** in the intervals

$$0 < k^2 < 4 M^2 = 3.53 \text{ GeV}^2$$

$$0 < k^2 < 4 m_\pi = 0.08 \text{ GeV}^2$$

which are kinematically unattainable from e^+e^- initial states.



- IPE $\pi^-p \rightarrow e^+e^-n$ measurements will significantly complement the current electroproduction $\gamma^*N \rightarrow \pi N$ study for the evolution of baryon properties with increasing momentum transfer by investigation of the case for the *time-like virtual photon*.



Phenomenology for Baryon Resonances

