

Three-nucleon forces and shell structure of neutron-rich Ca isotopes

Javier Menéndez

Institut für Kernphysik (TU Darmstadt) and ExtreMe Matter Institute (EMMI)

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**Theoretical Approach:
NN+3N forces in Shell Model**

**Neutron rich Ca isotopes:
Shell evolution $N = 32$ shell closure**

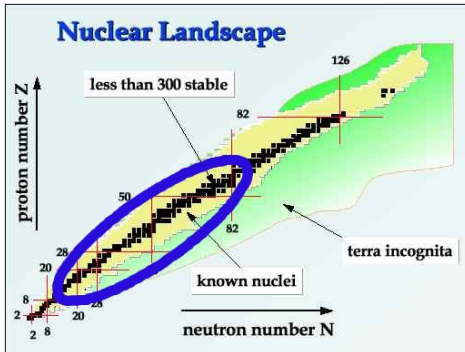
Proton rich $N=8$ and $N=20$ isotopes

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Nuclear Structure approach



Big variety of nuclei in the nuclear chart, $A \sim 2 \dots 300$

Systematic *ab initio* calculations only possible in the lightest nuclei

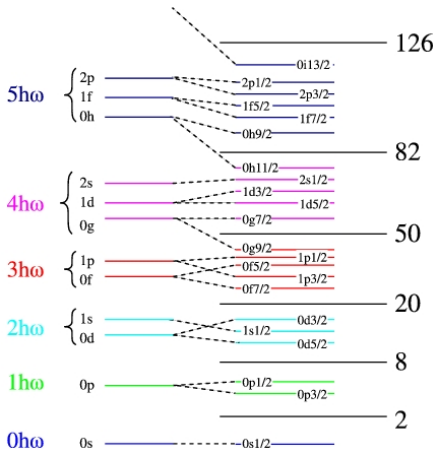
Poses a hard many-body problem: design approximate methods suited for different regions

Interacting Shell Model:

Solve the problem choosing the (more) relevant degrees of freedom

Use realistic nucleon-nucleon (NN) and three-nucleon (3N) interactions

The Interacting Shell Model



Chose as basis states that of the 3D Harmonic Oscillator

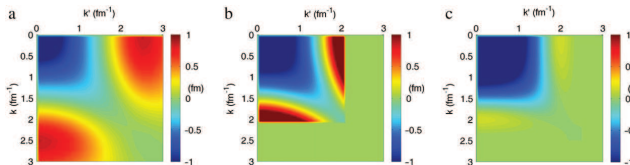
To keep the problem feasible, the configuration space is separated into

- Outer orbits: orbits that are always empty
- Valence space: the space in which we explicitly solve the problem
- Inner core: orbits that are always filled

$$\text{Dim} \sim \binom{(\rho+1)(\rho+2)_\nu}{N} \binom{(\rho+1)(\rho+2)_\pi}{Z}$$

Many Body Perturbation Theory

Better convergence through V_{lowk} transformation

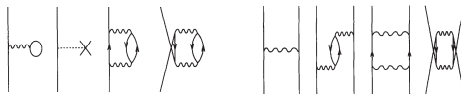


Many-body Perturbation Theory
up to third order to build
an effective Shell Model interaction
in a valence space

$$H|\Psi\rangle = E|\Psi\rangle \rightarrow H_{eff}|\Psi\rangle_{eff} = E|\Psi\rangle_{eff}$$

Single Particle Energies
(SPEs)

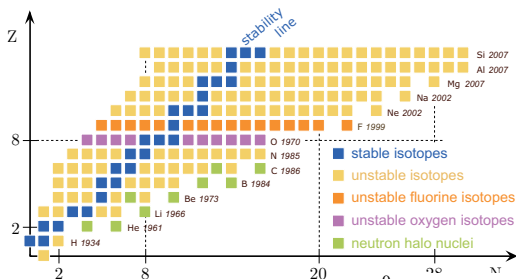
Two-Body Matrix Elements
(TBMEs)



Full diagonalizations using codes ANTOINE and NATHAN

Caurier et al. RMP77 427(2005) and **compare to experiment**

3N forces: Oxygen dripline



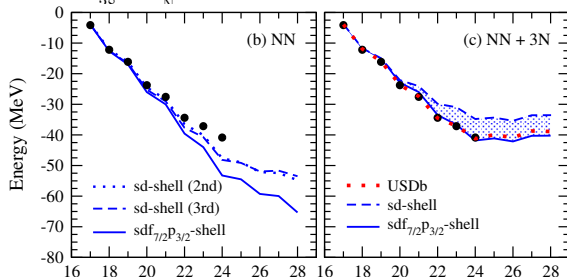
Chiral NN+3N forces to describe O dripline at ^{24}O

Otsuka et al.

PRL105 032501 (2010)

3N forces provide
repulsion
missing in NN-only forces

3N forces crucial also for
reliable description of
spectra






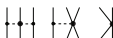

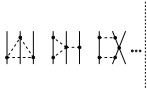
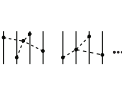
Forces and Currents in Chiral EFT

Chiral EFT: low energy approach to QCD for nuclear structure energies

Approximate chiral symmetry of QCD: pions pseudo-Goldstone bosons

Short-range couplings are fitted to experiment once

Systematic expansion: nuclear forces and electroweak currents

	2N force	3N force	4N force
LO		—	—
NLO		—	—
N ² LO			—
N ³ LO			

NN forces up to N³LO

3N forces up to N²LO

NN fitted to:

- NN scattering data

3N fitted to:

- ³H Binding Energy
- ⁴He radius

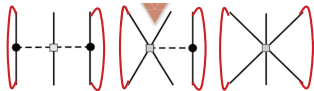
3N Forces

Treatment of 3N forces:

normal-ordered 2B: 2 valence, 1 core particle
 \Rightarrow (effective) Two-body Matrix Elements (TBME)

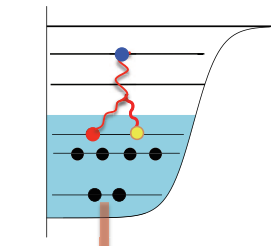
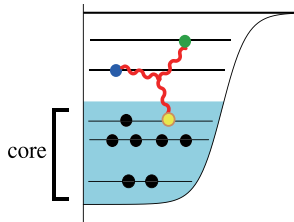


normal-ordered 1B: 1 valence, 2 core particles
 \Rightarrow (effective) Single particle energies (SPE)



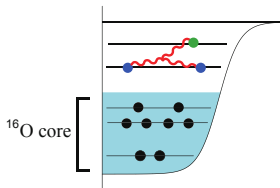
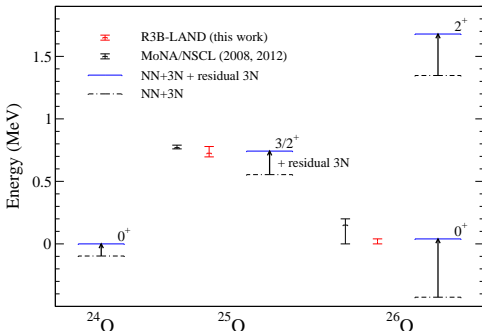
residual 3B:

\Rightarrow Estimated to be suppressed by $N_{valence}/N_{core}$



Residual 3N Forces

In the most neutron-rich oxygen isotopes,
3N forces between 3 valence neutrons
can give a relevant contribution



Residual 3N contributions are repulsive

They are small compared to normal-ordered 3N force, but increase with N

Very good agreement with resonances in ^{25}O and ^{26}O

Caesar, Simonis et al
PRC88 034313 (2013)

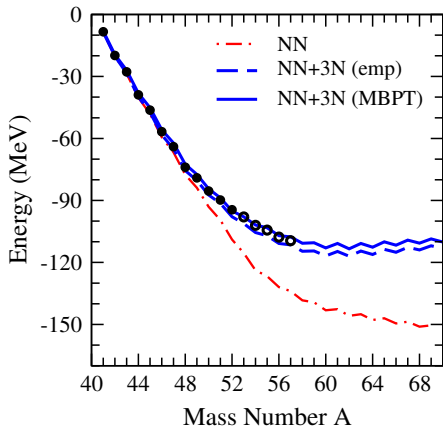
Theoretical Approach:
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**Neutron rich Ca isotopes:
Shell evolution $N = 32$ shell closure**

Proton rich $N=8$ and $N=20$ isotopes

Ca isotopes: Masses

Ca isotopes: explore nuclear shell evolution $N = 20, 28, 32?, 34?$



Ca with respect to ^{40}Ca core

3N forces repulsive contribution,
chiral NN-only forces too attractive

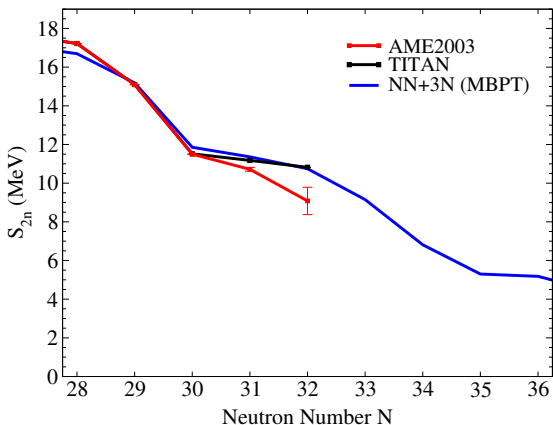
Flat behaviour towards ^{60}Ca does not
allow clear prediction of the dripline

Results sensitive to SPEs, especially
more neutron-rich systems,
MBPT (calculated from NN+3N forces)
Empirical (from GXPF1 interaction)
Estimate of the uncertainty

Two-Neutron separation energies

Ca isotopes (on top of ^{40}Ca core)

Compare $S_{2n} = -[B(N, Z) - B(N - 2, Z)]$ with experiment



Precision measurements
with TITAN changed AME
2003 ~ 1.74 MeV in ^{52}Ca

More flat behaviour in
 ^{50}Ca – ^{52}Ca

3N forces needed in
theoretical calculation

$pf_{9/2}$ valence space

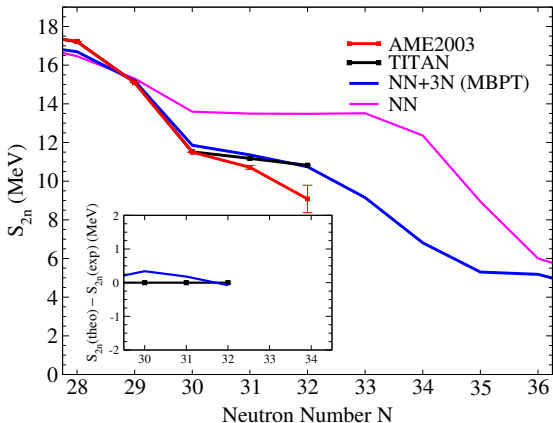
Gallant et al.

PRL 109 032506 (2012)

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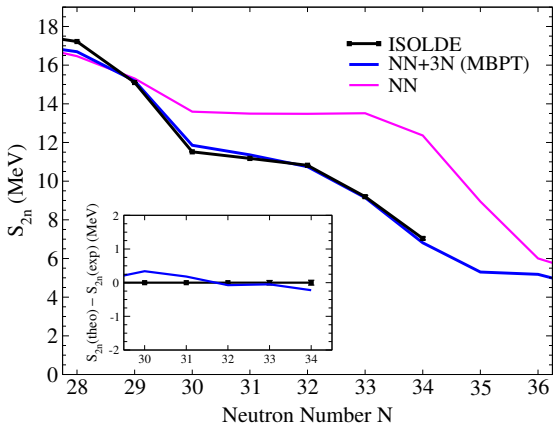
Gallant et al.

PRL 109 032506 (2012)

^{54}Ca and $N = 32$ shell closure

Ca isotopes (on top of ^{40}Ca core)

Compare $S_{2n} = -[B(N, Z) - B(N - 2, Z)]$ with experiment



Very recently $^{53,54}\text{Ca}$
measured at ISOLDE

Excellent agreement
between calculation and
experiment

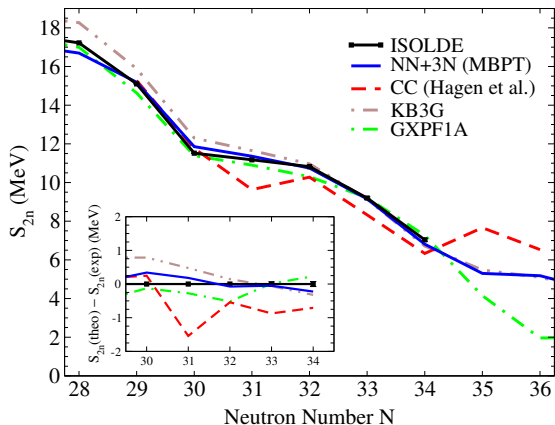
S_{2n} evolution:
 ^{52}Ca – ^{54}Ca as ^{48}Ca – ^{50}Ca :
 $N = 32$ shell closure

Wienholtz et al.
Nature 498 346 (2013)

Two-neutron separation energies

Compare $S_{2n} = -[B(N, Z) - B(N - 2, Z)]$ with experiment

S_{2n} also calculated by other approaches:



Phenomenological interactions also reproduce quite well experiment (input about masses/gaps into the interactions)
Differ markedly beyond ^{54}Ca

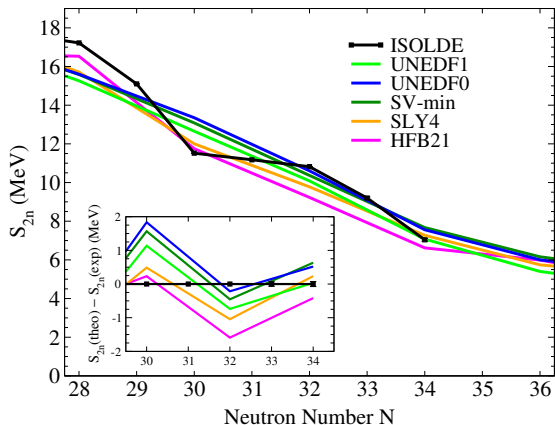
Coupled-Cluster calculations
Hagen et al. PRL109 032502 (2012) very good agreement for even isotopes

Wienholtz et al.
Nature 498 346 (2013)

Two-neutron separation energies

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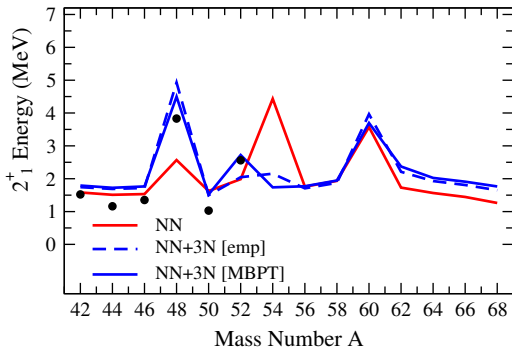
Modern Energy Density Functionals reproduce correctly overall trends and neutron/proton driplines have more difficulties in describing shell closures
Erl er et al. Nature486 509(2012)

Wienholtz et al.
Nature 498 346 (2013)

Shell closures and 2_1^+ energies

2_1^+ energies characterise shell closures of the neutron rich calcium isotopes

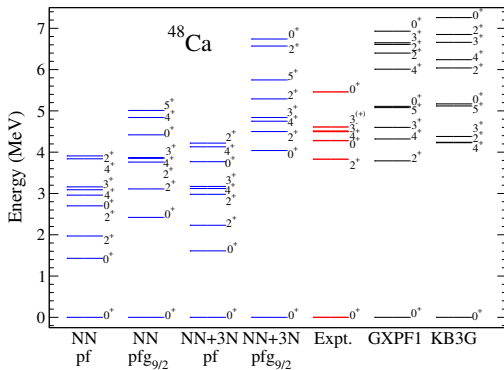
Holt, JM, Schwenk,
JPG40 075105 (2013)



- Correct closure at $N = 28$ when 3N forces are included
Holt et al. JPG39 085111(2012)
- 3N forces enhance closure at $N = 32$
- 3N forces reduce strong closure at $N = 34$ (1.7-2.2 MeV)
Measured at 2.1 MeV Steppenbeck et al. Nature, in press

^{48}Ca spectrum

Challenge: Doubly-closed nucleus ^{48}Ca



Spectra too compressed
with NN forces only or pf space

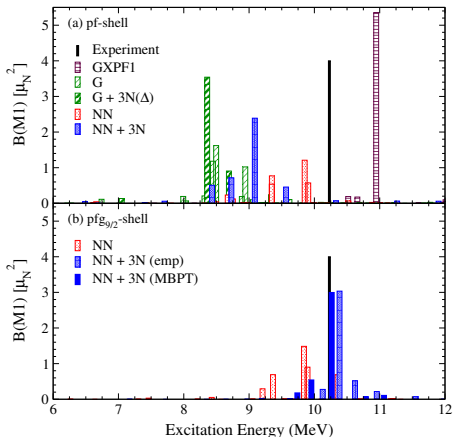
2_1^+ state only \sim appropriate energy
in $pf_{g_{9/2}}$ NN+3N calculation

0_1^+ state too low (1st excited state)
especially compared to
phenomenological interactions

Importance of 3N forces

Importance of including $g_{9/2}$ orbit

B(M1) Transition in ^{48}Ca



B(M1) strength in ^{48}Ca too fragmented
in pf space
Phenomenological calculations
reproduce experimental concentration

In the extended $pfg_{9/2}$ space NN
forces also fragmented strength

NN+3N calculation in $pfg_{9/2}$ very good
agreement with experiment

B(E2) Transition Strengths

Isotope	Transition	KB3G	GXPF1A	MBPT	EXP.
^{46}Ca	$2^+ \rightarrow 0^+$	9.2	9.2	13.3	25.4 ± 4.5
					36.4 ± 2.6
^{46}Ca	$6^+ \rightarrow 4^+$	3.6	3.6	4.8	5.38 ± 0.29
^{47}Ca	$3/2^- \rightarrow 7/2^-$	0.84	3.6	1.0	4.0 ± 0.2
^{48}Ca	$2^+ \rightarrow 0^+$	11.5	11.9	10.3	19 ± 6.4
^{49}Ca	$7/2^- \rightarrow 3/2^-$	0.41	4.0	0.22	0.53 ± 0.21
^{50}Ca	$2^+ \rightarrow 0^+$	8.9	9.1	11.2	7.4 ± 0.2

B(E2)s in reasonable agreement with experiment
(order of magnitude)

Similar quality as phenomenological interactions (very close to KB3G)

^{46}Ca : *sd* degrees of freedom?

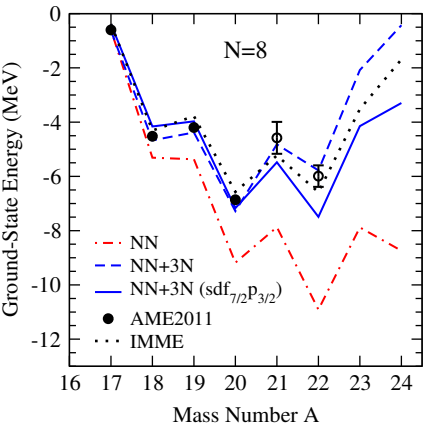
Outline

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Proton dripline at $N = 8$



Holt, JM, Schwenk PRL110 022502 (2013)

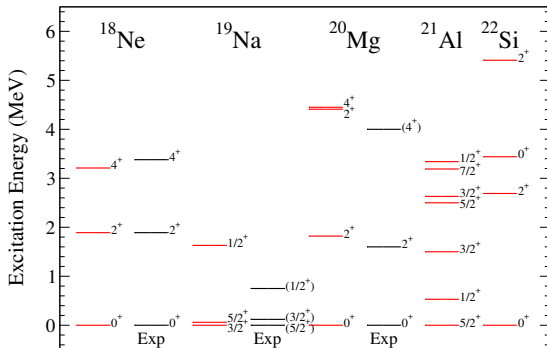
Theory complements/improves
mass extrapolations and
isomeric mass-multiplet formula (IMME)
 $E(A, T, T_z) = E(A, T, -T_z) + 2b(A, T)T_z$

NN forces oberbind
3N forces essential to describe masses
and the predict the proton dripline

Proton dripline not certain
predicted either in ^{20}Mg or ^{22}Si :
 $S_{2p} = -0.12$ (Theory) / $+0.01$ (IMME)
Measurement needed!

Calculations in standard
and extended spaces

Spectra of $N = 8$ isotones



Holt, JM, Schwenk PRL110 022502 (2013)

Including NN+3N forces
good agreement with
known spectra

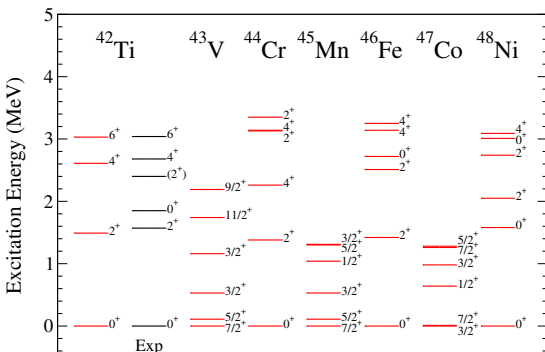
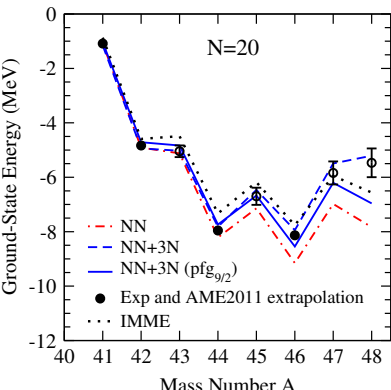
Prediction of 2^+ , 4^+ doublet
close to previously
unpublished 4^+ state in ^{20}Mg
(I. Mukha)

Prediction of ^{21}Al and ^{22}Si
spectra

In ^{22}Si calculations point to a sub-shell closure (analogous to ^{22}O)

More experimental information greatly appreciated!

Masses and spectra of $N = 20$ isotones



Holt, JM, Schwenk PRL110 022502 (2013)

Dripline robustly predicted at ⁴⁶Fe

Good description of ⁴⁸Ni: $S_{2p} = -1.02$ (Th) vs $-1.28(6)$ (Exp) Pomorski (2012)

Summary and Outlook

Shell Model calculation based on chiral EFT (NN+3N forces) and MBPT gives good agreement with experimental masses, two-neutron separation energies, pairing gaps and excitation spectra for oxygen, calcium isotopes and proton-rich $N=8,20$ isotones:

- Oxygen dripline, unbound $^{25,26}\text{O}$ reproduced with residual 3N forces
- Predicted neutron rich Ca S_{2n} 's with NN+3N forces agree with recent measurements of $^{51,52}\text{Ca}$ (TRIUMF) and $^{53,54}\text{Ca}$ (ISOLTRAP)
- Shell structure: prominent closure at $N = 32$
- Ca spectroscopy: spectra, electromagnetic strengths
- Dripline and spectra of proton-rich $N = 8, 20$ isotones predicted

Outlook:

Heavier isotope and isotone chains: include $T=0$ (pn) TBME

Explore uncertainties in the theoretical calculation

Collaborators



J. D. Holt, A. Schwenk, J. Simonis



R³B Collaboration
(C. Caesar, T. Aumann...)



TITAN Collaboration
(A. Gallant, J. Dilling...)



ISOLTRAP Collaboration
(F. Wienholtz, K. Blaum...)