Advancing β-Decay Formalism

a Full QFT Treatment for Tritium Endpoint Precision

Matilde Pitzalis

Università degli studi di Cagliari, INFN sez. Cagliari

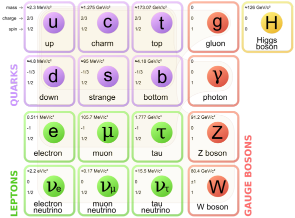
Erice: International school of nuclear physics, 47th course

17/09/2026

Table of Contents

The absolute neutrino mass scale

The first experimental evidence of Beyond the Standard Model Physics: the neutrino mass

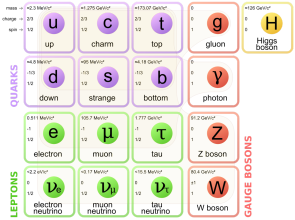


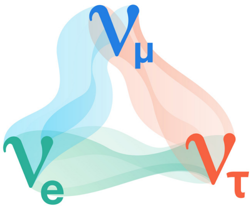
- Massless in the Standard Model
- Experiments proved that neutrinos periodically change flavour during flight

[Fukuda F. et al (Super-Kamiokande Collaboration), Phys. Rev. Lett. 81, 1562(1998); Q.R. Ahmad et al. (SNO Collaboration), Phys. Rev. Lett. 89, 011301 (2002)]

The absolute neutrino mass scale

The first experimental evidence of Beyond the Standard Model Physics: the neutrino mass





- Massless in the Standard Model
- Experiments proved that neutrinos periodically change flavour during flight

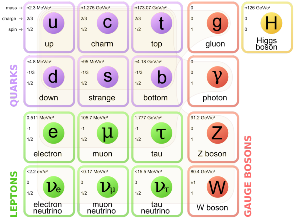
$$P_{\nu_\alpha \to \nu_\beta} \propto \sin^2\left(\frac{\Delta m^2_{ij} L}{4E}\right)$$

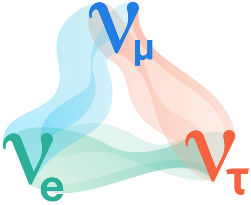
$$\begin{gathered}\text{3 mass eigenstates } \nu_1, \nu_2, \nu_3 \\ \neq \\ \text{3 flavour eigenstates } \nu_e, \nu_\mu, \nu_\tau\end{gathered}$$

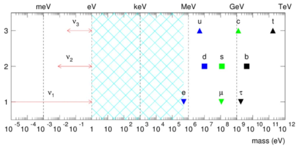
[Fukuda F. et al (Super-Kamiokande Collaboration), Phys. Rev. Lett. 81, 1562(1998); Q.R. Ahmad et al. (SNO Collaboration), Phys. Rev. Lett. 89, 011301 (2002)]

The absolute neutrino mass scale

The first experimental evidence of Beyond the Standard Model Physics: the neutrino mass







[de Gouvea, A. TASI Lectures on Neutrino Physics (2004).]

- Massless in the Standard Model
- Experiments proved that neutrinos periodically change flavour during flight

$$P_{\nu_\alpha \to \nu_\beta} \propto \sin^2\left(\frac{\Delta m^2_{ij} L}{4E}\right)$$

Oscillations only measure squared mass differences, leaving the absolute mass scale completely unknown

3 mass eigenstates ν_1, ν_2, ν_3

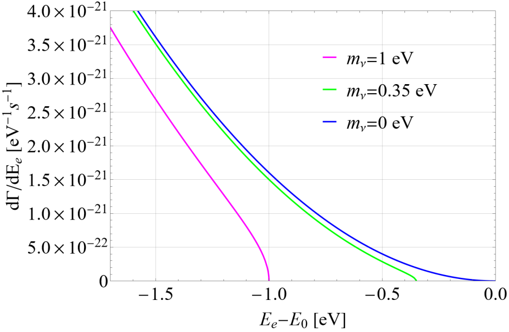
$\neq$

3 flavour eigenstates ν_e, ν_μ, ν_τ

$$m_\nu^2 = \sum_i |U_{ei}|^2 m_i^2$$

[Fukuda F. et al (Super-Kamiokande Collaboration), Phys. Rev. Lett. 81, 1562(1998); Q.R. Ahmad et al. (SNO Collaboration), Phys. Rev. Lett. 89, 011301 (2002)]

Direct measurements of the neutrino mass



$$^3_1\mathrm{T} \longrightarrow {}^3_2\mathrm{He} + e^- + \bar{\nu}_e$$

- End-point energy: $E_0 \sim 18.6$ keV
- Lifetime: $\tau \sim 17.78$ yrs

[E.G. Myers, Phys. Rev. Lett. 114, 013003 (2015); j.J. Simpson, Phys. Rev. C 35, 2 (1987)]

What we measure: The distortion at the end-point is caused by the effect of the non-zero mass of the emitted neutrino on the kinematic phase-space factor of the decay rate.

For a very precise measurement we have to know extremely well the decay spectrum!

Experimental state-of-the-art

KATRIN (Karlsruhe Tritium Neutrino Experiment)

- Molecular tritium source:

$$T_2 \longrightarrow \mathrm{HeT}^+ + e^- + \bar{\nu}_e$$

- Current mass limit:

$$m_\nu < 0.45\ \mathrm{eV} \quad \text{at } 90\%\ \mathrm{CL}$$

[Aker M. et al. (KATRIN Collab.), Science, 388 (2025) 180.]

Experimental state-of-the-art

KATRIN (Karlsruhe Tritium Neutrino Experiment)

- Molecular tritium source:

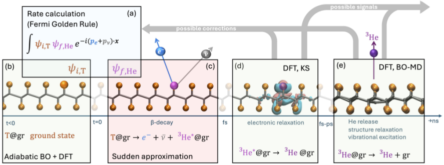
$$T_2 \longrightarrow \mathrm{HeT}^+ + e^- + \bar{\nu}_e$$

- Current mass limit:

$$m_\nu < 0.45\ \mathrm{eV} \quad \text{at } 90\%\ \mathrm{CL}$$

[Aker M. et al. (KATRIN Collab.), Science, 388 (2025) 180.]

PTOLEMY (PonTecorvo Observatory for Light, Early-Universe, Massive-Neutrino Yield)



See F. Pofi's Talk

- Atomic tritium in graphene matrices source
- Aimed target mass sensitivity:

$$\sim 0.05\ \mathrm{eV}$$

- Ultimate goal: detection of the Cosmic Neutrino Background (CνB)

[Baracchini E. et al. (PTOLEMY Collab.), arXiv:1808.01892 (2018); Casale A. et al., Phys. Rev. C, 113 (2026) 054607]

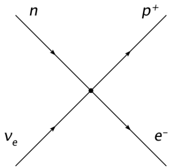
Table of Contents

Fermi's theory of beta decay

The most common way to treat β decay is to employ Fermi's effective, point-like weak-interaction theory [E. Fermi, Z. Phys. 88 (1934)].

$$\Gamma = 2\pi \underbrace{|H_{if}|^2}_{\text{Transition matrix element}} \times \underbrace{\rho(E_0)}_{\text{Final state density}}$$

$${}^{A}_{Z}\mathrm{X} \rightarrow {}^{A}_{Z+1}\mathrm{Y} + \bar{\nu}_e + e^-$$



The phase space factor contains:

- $p_e(E_e + m_e)$ is the kinematic factor of the electron;
- $\epsilon_f = E_0 - E_e$ is the neutrino energy;
- $\sqrt{\epsilon_f^2 - m_\nu^2}$ is the neutrino impulse;

$$\boxed{\frac{d\Gamma}{dE_e} = \frac{G_F^2}{2\pi^3} |H_{\text{if}}|^2 \, p_e(E_e + m_e) \, \epsilon_f \sqrt{\epsilon_f^2 - m_\nu^2} \, \Theta(\epsilon_f - m_\nu)}$$

In natural units, $c = \hbar = 1$. [Values from S. Navas et al (Particle Data Group), Phys.Rev.D 110 (2024) 3, 030001]

The leptonic factorisation approximation

Because the weak interaction is point-like, it is standard practice to factorise the full matrix element into a purely leptonic piece and a purely nuclear piece, assuming

$$|H_{if}|^2 = \left|\langle\Psi_f|\hat{H}|\Psi_i\rangle\right|^2 = \left|\int_{\text{nucl.}} \psi_e^* \phi_f^* \hat{H} \phi_i \psi_\nu \, dV\right|^2$$

$$\Downarrow$$

The leptonic factorisation approximation

Because the weak interaction is point-like, it is standard practice to factorise the full matrix element into a purely leptonic piece and a purely nuclear piece, assuming

$$|H_{if}|^2 = \left|\langle \Psi_f | \hat{H} | \Psi_i \rangle\right|^2 = \left| \int_{\text{nucl.}} \psi_e^* \phi_f^* \hat{H} \phi_i \psi_\nu \, dV \right|^2$$

$$\Downarrow$$

$$|\psi_e(r)|^2 |\psi_\nu(r)|^2$$

The leptonic wave functions are approximately constant across the nuclear volume:

- The neutrino wave function is a plane wave
$$\psi_\nu \approx 1$$
- The electron wave function varies very slowly within the nuclear volume
$$\psi_e(r) \approx \psi_e(0)$$

The leptonic factorisation approximation

Because the weak interaction is point-like, it is standard practice to factorise the full matrix element into a purely leptonic piece and a purely nuclear piece, assuming

$$|H_{if}|^2 = \left|\langle \Psi_f | \hat{H} | \Psi_i \rangle\right|^2 = \left| \int_{\text{nucl.}} \psi_e^* \phi_f^* \hat{H} \phi_i \psi_\nu \, dV \right|^2$$

$$\Downarrow$$

$$|\psi_e(r)|^2 |\psi_\nu(r)|^2$$

The leptonic wave functions are approximately constant across the nuclear volume:

- The neutrino wave function is a plane wave

$$\psi_\nu \approx 1$$

- The electron wave function varies very slowly within the nuclear volume

$$\psi_e(r) \approx \psi_e(0)$$

$$|M_{\text{nucl}}|^2$$

For a superallowed $\frac{1}{2}^+ \longrightarrow \frac{1}{2}^+$ mixed transition, $\langle F \rangle^2 = 1$ and $\langle GT \rangle^2 = 3$, giving

$$\begin{aligned} |M_{\text{nucl}}|^2 &= g_V^2 \langle F \rangle^2 + g_A^2 \langle GT \rangle^2 \\ &\simeq g_V^2 + 3\, g_A^2 \end{aligned}$$

which multiplies the CKM element $|V_{ud}|^2$ associated with the $d \longrightarrow u$ quark transition.

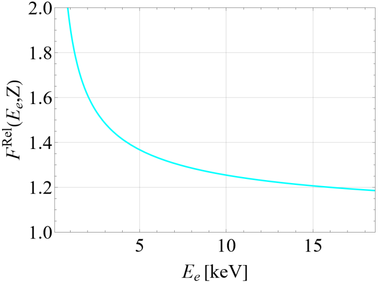
[T-Y. Saito et al, PLB, Vol. 242, Iss. 1, (1990)]

Fermi function

The Fermi function is a correction for the Coulomb distortion of the outgoing electron in the daughter nucleus field.

$$F(Z, E_e) = \frac{|\psi_e(0)_{\mathrm{Coul.}}|^2}{|\psi_e(0)|^2}$$

[E.J.Konopinski and G.E.Uhlenbeck, Phys. Rev. 48, 7 (1935)]



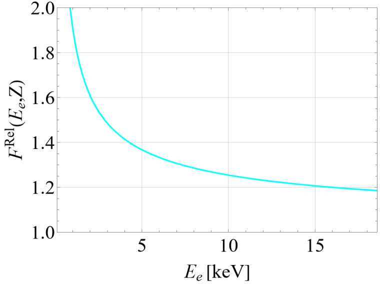
$$\frac{d\Gamma}{dE_e} = \frac{G_F^2}{2\pi^3} |V_{\mathrm{ud}}|^2 |M_{\mathrm{nucl}}|^2 \; F(Z, E_e)\, p_e(E_e + m_e)\, \epsilon_f \sqrt{\epsilon_f^2 - m_\nu^2}\, \Theta(\epsilon_f - m_\nu)$$

Fermi function

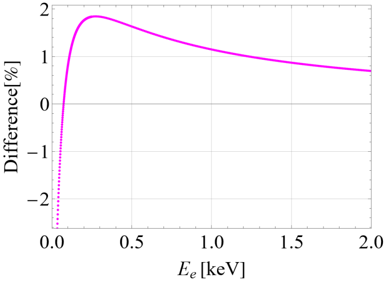
The Fermi function is a correction for the Coulomb distortion of the outgoing electron in the daughter nucleus field.

$$F(Z, E_e) = \frac{|\psi_e(0)_{\text{Coul.}}|^2}{|\psi_e(0)|^2}$$

[E.J.Konopinski and G.E.Uhlenbeck, Phys. Rev. 48, 7 (1935)]



Exact Fermi function: obtained by solving numerically the DHFS equations with the RADIAL package [F. Salvat and J.M. Fernandez-Varea, Computer Physics Communications 240, 195 (2019)].



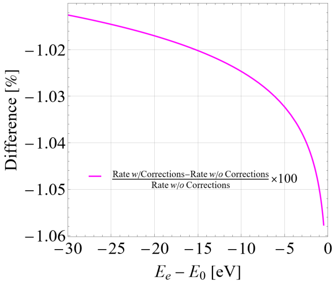
$$\frac{d\Gamma}{dE_e} = \frac{G_F^2}{2\pi^3} |V_{\text{ud}}|^2 |M_{\text{nucl}}|^2 \, F(Z, E_e) \, p_e(E_e + m_e) \, \epsilon_f \sqrt{\epsilon_f^2 - m_\nu^2} \, \Theta(\epsilon_f - m_\nu)$$

Corrective Factors

$$\frac{d\Gamma}{dE_e} = \frac{G_F^2 m_e^5 c^4}{2\pi^3 \hbar^7} |V_{\mathrm{ud}}|^2 |M_{\mathrm{nucl}}|^2 \; F(Z, E_e)\, p_e E_e \cdot SLCI \sum_f GRQ \cdot P_f \epsilon_f \sqrt{\epsilon_f^2 - m_\nu^2 c^4}\, \Theta(\epsilon_f - m_\nu c^2)$$

- Radiative Corrections G
- Screening Effects S
- Recoil Effects R
- Finite Structure of the Nucleus L and C
- Recoiling Coulomb Field Q
- Orbital-Electron Interactions I

[L. Hayen et al, Rev. Mod. Phys. 90, 015008 (2018); Kleesiek M. et al., Eur. Phys. J. C, 79 (2019) 204]



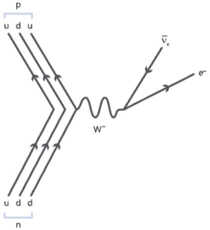
PRELIMINARY!

Radiative corrections are the most impactful near the endpoint, causing a depletion of the spectrum while all other corrections are nearly constant in the last 30 eV, so they only cause a shift.

A new approach

$$\langle e^-(p_e)\bar{\nu}_e(p_\nu) f|H_{\rm eff}|i\rangle = \frac{G_F V_{ud}}{\sqrt{2}} \int d\boldsymbol{x}\, \bar{\psi}^{(-)}_{e^-,p_e,s_e}(\boldsymbol{x})\, \gamma^\mu (1-\gamma_5)\, v_{s_\nu}(\boldsymbol{p}_\nu)\, e^{-i\boldsymbol{p}_\nu\cdot\boldsymbol{x}}\, \langle f|J_\mu(\boldsymbol{x})|i\rangle$$

- We introduce a fully non-factorised formulation within the effective weak-interaction theory.
- This approach retains the complete spatial dependence of the leptonic wave functions throughout the interaction.
- The decay rate is evaluated through direct spatial integration of the solutions of the Dirac equation alongside the nuclear current.



$$\Gamma = \frac{(G_F V_{ud})^2}{\pi^2} \int_{m_e}^{E_0} dE_e\, p_e E_e (E_0 - E_e)^2 \sum_{J,L,\kappa_e,\kappa_\nu} \frac{1}{2J_i+1} \left|\langle f\|\Xi_{JL}(\kappa_e,\kappa_\nu)\|i\rangle\right|^2$$

[Horiuchi W. et al., Prog. Theor. Exp. Phys., 2021 (2021) 103D03; A. Ravlic et al, Phys. Rev. C 111, 064323 (2025); Cadeddu M. et al., Phys. Rev. Lett. (accepted, 2026), arXiv:2512.20560v3]

Going beyond the factorisation scheme

The transition matrix element for β^- decay is expressed as a spatial convolution of the leptonic and hadronic currents, preserving the V–A structure:

$$\Xi_{JLM}(\kappa_e, \kappa_\nu) = S_{\kappa_e} \int dr$$

Time-like Vector	$\times\Big\{-\rho_V^0(r)\big(G_{\kappa_e}(r)g_{\kappa_\nu}(r)S_{0JJ}(\kappa_e,\kappa_\nu) + F_{\kappa_e}(r)f_{\kappa_\nu}(r)S_{0JJ}(-\kappa_e,-\kappa_\nu)\big)$
Space-like Vector	$+\rho_V^i(r)\big(G_{\kappa_e}(r)f_{\kappa_\nu}(r)S_{1LJ}(\kappa_e,-\kappa_\nu) - F_{\kappa_e}(r)g_{\kappa_\nu}(r)S_{1LJ}(-\kappa_e,\kappa_\nu)\big)$
Time-like Axial	$+\rho_A^0(r)\big(G_{\kappa_e}(r)f_{\kappa_\nu}(r)S_{0JJ}(\kappa_e,-\kappa_\nu) - F_{\kappa_e}(r)g_{\kappa_\nu}(r)S_{0JJ}(-\kappa_e,\kappa_\nu)\big)$
Space-like Axial	$-\rho_A^i(r)\big(G_{\kappa_e}(r)g_{\kappa_\nu}(r)S_{1LJ}(\kappa_e,\kappa_\nu) + F_{\kappa_e}(r)f_{\kappa_\nu}(r)S_{1LJ}(-\kappa_e,-\kappa_\nu)\big)\Big\}$

Going beyond the factorisation scheme

The transition matrix element for β^- decay is expressed as a spatial convolution of the leptonic and hadronic currents, preserving the V–A structure:

$$\Xi_{JLM}(\kappa_e, \kappa_\nu) = S_{\kappa_e} \int dr$$

$$\begin{aligned}
&\text{Time-like Vector} && \times\Big\{-\rho_V^0(r)\big(G_{\kappa_e}(r) g_{\kappa_\nu}(r) S_{0JJ}(\kappa_e, \kappa_\nu) + F_{\kappa_e}(r) f_{\kappa_\nu}(r) S_{0JJ}(-\kappa_e, -\kappa_\nu)\big) \\
&\text{Space-like Vector} && \quad + \rho_V^i(r)\big(G_{\kappa_e}(r) f_{\kappa_\nu}(r) S_{1LJ}(\kappa_e, -\kappa_\nu) - F_{\kappa_e}(r) g_{\kappa_\nu}(r) S_{1LJ}(-\kappa_e, \kappa_\nu)\big) \\
&\text{Time-like Axial} && \quad + \rho_A^0(r)\big(G_{\kappa_e}(r) f_{\kappa_\nu}(r) S_{0JJ}(\kappa_e, -\kappa_\nu) - F_{\kappa_e}(r) g_{\kappa_\nu}(r) S_{0JJ}(-\kappa_e, \kappa_\nu)\big) \\
&\text{Space-like Axial} && \quad - \rho_A^i(r)\big(G_{\kappa_e}(r) g_{\kappa_\nu}(r) S_{1LJ}(\kappa_e, \kappa_\nu) + F_{\kappa_e}(r) f_{\kappa_\nu}(r) S_{1LJ}(-\kappa_e, -\kappa_\nu)\big)\Big\}
\end{aligned}$$

- $G_{\kappa_e}(r), F_{\kappa_e}(r)$ Electron wave functions, exactly computed with RADIAL package
- $g_{\kappa_\nu}(r), f_{\kappa_\nu}(r)$ Neutrino wave functions, spherical Bessel functions of order 0 and 1
- The nuclear states are described via nuclear transition densities

Going beyond the factorisation scheme

The transition matrix element for β^- decay is expressed as a spatial convolution of the leptonic and hadronic currents, preserving the V–A structure:

$$\Xi_{JLM}(\kappa_e,\kappa_\nu) = S_{\kappa_e}\int dr$$

Time-like Vector
Space-like Vector
Time-like Axial
Space-like Axial

$$\begin{aligned}
\times\Big\{ & -\rho_V^0(r)\big(G_{\kappa_e}(r)g_{\kappa_\nu}(r)S_{0JJ}(\kappa_e,\kappa_\nu) + F_{\kappa_e}(r)f_{\kappa_\nu}(r)S_{0JJ}(-\kappa_e,-\kappa_\nu)\big) \\
& + \rho_V^i(r)\big(G_{\kappa_e}(r)f_{\kappa_\nu}(r)S_{1LJ}(\kappa_e,-\kappa_\nu) - F_{\kappa_e}(r)g_{\kappa_\nu}(r)S_{1LJ}(-\kappa_e,\kappa_\nu)\big) \\
& + \rho_A^0(r)\big(G_{\kappa_e}(r)f_{\kappa_\nu}(r)S_{0JJ}(\kappa_e,-\kappa_\nu) - F_{\kappa_e}(r)g_{\kappa_\nu}(r)S_{0JJ}(-\kappa_e,\kappa_\nu)\big) \\
& - \rho_A^i(r)\big(G_{\kappa_e}(r)g_{\kappa_\nu}(r)S_{1LJ}(\kappa_e,\kappa_\nu) + F_{\kappa_e}(r)f_{\kappa_\nu}(r)S_{1LJ}(-\kappa_e,-\kappa_\nu)\big)\Big\}
\end{aligned}$$

- $G_{\kappa_e}(r), F_{\kappa_e}(r)$ Electron wave functions, exactly computed with RADIAL package
- $g_{\kappa_\nu}(r), f_{\kappa_\nu}(r)$ Neutrino wave functions, spherical Bessel functions of order 0 and 1
- The nuclear states are described via nuclear transition densities

Going beyond the factorisation scheme

The transition matrix element for β^- decay is expressed as a spatial convolution of the leptonic and hadronic currents, preserving the V–A structure:

$$\Xi_{JLM}(\kappa_e, \kappa_\nu) = S_{\kappa_e} \int dr$$

$$\begin{array}{ll}
\text{Time-like Vector} & \times\Big\{-\rho_V^0(r)\big(G_{\kappa_e}(r)g_{\kappa_\nu}(r)S_{0JJ}(\kappa_e,\kappa_\nu) + F_{\kappa_e}(r)f_{\kappa_\nu}(r)S_{0JJ}(-\kappa_e,-\kappa_\nu)\big) \\
\text{Space-like Vector} & \quad + \rho_V^i(r)\big(G_{\kappa_e}(r)f_{\kappa_\nu}(r)S_{1LJ}(\kappa_e,-\kappa_\nu) - F_{\kappa_e}(r)g_{\kappa_\nu}(r)S_{1LJ}(-\kappa_e,\kappa_\nu)\big) \\
\text{Time-like Axial} & \quad + \rho_A^0(r)\big(G_{\kappa_e}(r)f_{\kappa_\nu}(r)S_{0JJ}(\kappa_e,-\kappa_\nu) - F_{\kappa_e}(r)g_{\kappa_\nu}(r)S_{0JJ}(-\kappa_e,\kappa_\nu)\big) \\
\text{Space-like Axial} & \quad - \rho_A^i(r)\big(G_{\kappa_e}(r)g_{\kappa_\nu}(r)S_{1LJ}(\kappa_e,\kappa_\nu) + F_{\kappa_e}(r)f_{\kappa_\nu}(r)S_{1LJ}(-\kappa_e,-\kappa_\nu)\big)\Big\}
\end{array}$$

- $G_{\kappa_e}(r), F_{\kappa_e}(r)$ Electron wave functions, exactly computed with RADIAL package
- $g_{\kappa_\nu}(r), f_{\kappa_\nu}(r)$ Neutrino wave functions, spherical Bessel functions of order 0 and 1
- The nuclear states are described via nuclear transition densities

$$\rho_{TD}(r) = \phi_f^*(r)\hat{O}\phi_i(r)$$

From the factorised to the non-factorised scheme

Standard (factorised)

- Leptonic wave functions assumed **constant** across the nuclear volume:

$$\psi_\nu \approx 1, \qquad \psi_e(r) \approx \psi_e(0)$$

- Matrix element **factorises**:

$$|H_{if}|^2 \longrightarrow \underbrace{|\psi_e(0)|^2 |\psi_\nu(0)|^2}_{\text{leptonic}} \times \underbrace{|M_{\text{nucl}}|^2}_{\text{nuclear}}$$

- Nuclear structure enters only via **reduced matrix elements**:

$$|M_{\text{nucl}}|^2 = g_V^2 \langle F \rangle^2 + g_A^2 \langle GT \rangle^2$$

- Coulomb distortion added **separately**, as a correction factor $F(Z, E_e)$

From the factorised to the non-factorised scheme

Standard (factorised)

- Leptonic wave functions assumed **constant** across the nuclear volume:

$$\psi_\nu \approx 1, \qquad \psi_e(r) \approx \psi_e(0)$$

- Matrix element **factorises**:

$$|H_{if}|^2 \longrightarrow \underbrace{|\psi_e(0)|^2|\psi_\nu(0)|^2}_{\text{leptonic}} \times \underbrace{|M_{\text{nucl}}|^2}_{\text{nuclear}}$$

- Nuclear structure enters only via **reduced matrix elements**:

$$|M_{\text{nucl}}|^2 = g_V^2\langle F\rangle^2 + g_A^2\langle GT\rangle^2$$

- Coulomb distortion added **separately**, as a correction factor $F(Z, E_e)$

$\Longrightarrow$

Our approach (non-factorised)

- **Full spatial dependence** of the leptonic wave functions retained throughout the nuclear volume
- Matrix element is a **spatial convolution** of leptonic and hadronic currents:

$$\Xi_{JLM} = S_{\kappa_e} \int dr\ \{\ldots\}$$

- Nuclear structure enters via the full **nuclear transition densities**:

$$\rho_V(r), \quad \rho_A(r)$$

- Coulomb distortion **built in exactly**, via the full DHFS electron wave functions

From the factorised to the non-factorised scheme

Standard (factorised)

- Leptonic wave functions assumed **constant** across the nuclear volume:

$$\psi_\nu \approx 1, \qquad \psi_e(r) \approx \psi_e(0)$$

- Matrix element **factorises**:

$$|H_{if}|^2 \longrightarrow \underbrace{|\psi_e(0)|^2|\psi_\nu(0)|^2}_{\text{leptonic}} \times \underbrace{|M_{\text{nucl}}|^2}_{\text{nuclear}}$$

- Nuclear structure enters only via **reduced matrix elements**:

$$|M_{\text{nucl}}|^2 = g_V^2\langle F\rangle^2 + g_A^2\langle GT\rangle^2$$

- Coulomb distortion added **separately**, as a correction factor $F(Z, E_e)$

$\Longrightarrow$

Our approach (non-factorised)

- **Full spatial dependence** of the leptonic wave functions retained throughout the nuclear volume
- Matrix element is a **spatial convolution** of leptonic and hadronic currents:

$$\Xi_{JLM} = S_{\kappa_e} \int dr\ \{\dots\}$$

- Nuclear structure enters via the full **nuclear transition densities**:

$$\rho_V(r), \quad \rho_A(r)$$

- Coulomb distortion **built in exactly**, via the full DHFS electron wave functions

Key change: the point-like approximation $\psi_e\psi_\nu \to \psi_e(0)\psi_\nu(0)$ is dropped; the full spatial overlap between leptonic and nuclear wave functions is retained and directly integrated.

Proof-of-concept

The same non-factorised approach, applied to neutrino capture on ^{71}Ga, already reduced the predicted cross-section enough to significantly mitigate the long-standing Gallium Anomaly, without invoking new physics [Cadeddu M. et al., Phys. Rev. Lett. (accepted, 2026), arXiv:2512.20560v3].

- Realistic transition densities
- Exact electron wave functions
- Dynamical interplay between the nuclear transition density and the exact leptonic wave functions

ACCEPTED PAPER

Possible solution to the gallium anomaly moving beyond the leptonic wave-function factorization

M. Cadeddu, N. Cargioli, F. Dordei, L. Ferro, C. Giunti, and M. Pitzalis

Phys. Rev. Lett. - **Accepted** 31 July, 2026

DOI: https://doi.org/10.1103/5pvf-mcr1

Export Citation

Abstract

For over thirty years, a $\sim 20\%$ deficit, now exceeding 5σ, has persisted between measured and predicted neutrino capture rates on ^{71}Ga, as observed in radioactive source experiments (namely GALLEX, SAGE, and more recently BEST) using ^{51}Cr and ^{37}Ar. This long-standing discrepancy, referred to as the gallium anomaly, has posed a significant challenge to our understanding of both experimental methods and theoretical predictions. In this work, we revisit the theoretical calculation of the neutrino capture cross-section by moving beyond the standard treatment of the leptonic wave functions, revealing limitations in the commonly used factorization approach based on the detailed balance principle. Incorporating phenomenologically constrained Gamow-Teller transition densities, able to correctly reproduce the precisely measured half-life of ^{71}Ge, we find that the revised cross-section can be significantly reduced, potentially resolving the gallium anomaly without invoking new physics.

Proof-of-concept

The same non-factorised approach, applied to neutrino capture on ^{71}Ga, already reduced the predicted cross-section enough to significantly mitigate the long-standing Gallium Anomaly, without invoking new physics [Cadeddu M. et al., Phys. Rev. Lett. (accepted, 2026), arXiv:2512.20560v3].

- Realistic transition densities
- Exact electron wave functions
- Dynamical interplay between the nuclear transition density and the exact leptonic wave functions

ACCEPTED PAPER

Possible solution to the gallium anomaly moving beyond the leptonic wave-function factorization

M. Cadeddu, N. Cargioli, F. Dordei, L. Ferro, C. Giunti, and M. Pitzalis

Phys. Rev. Lett. - **Accepted** 31 July, 2026

DOI: https://doi.org/10.1103/5pvf-mcr1

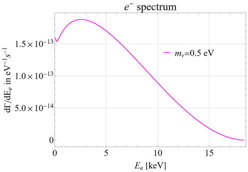
Export Citation

Abstract

For over thirty years, a $\sim 20\%$ deficit, now exceeding 5σ, has persisted between measured and predicted neutrino capture rates on ^{71}Ga, as observed in radioactive source experiments (namely GALLEX, SAGE, and more recently BEST) using ^{51}Cr and ^{37}Ar. This long-standing discrepancy, referred to as the gallium anomaly, has posed a significant challenge to our understanding of both experimental methods and theoretical predictions. In this work, we revisit the theoretical calculation of the neutrino capture cross-section by moving beyond the standard treatment of the leptonic wave functions, revealing limitations in the commonly used factorization approach based on the detailed balance principle. Incorporating phenomenologically constrained Gamow-Teller transition densities, able to correctly reproduce the precisely measured half-life of ^{71}Ge, we find that the revised cross-section can be significantly reduced, potentially resolving the gallium anomaly without invoking new physics.

What's next:

We apply the fully non-factorised formalism to the superallowed transition $^3_1\mathrm{T} \longrightarrow {}^3_2\mathrm{He} + e^- + \bar{\nu}_e$ computing the decay rate through direct spatial integration of the leptonic wave functions and nuclear transition densities.



Thank you!

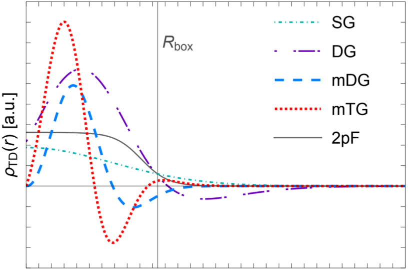
BACKUP

The Gallium Anomaly

The IBD amplitude depends on the full **weak transition density**

$$\rho_{\mathrm{TD}}(\mathbf{r}) = \Psi^*_{^{71}\mathrm{Ge}}(\mathbf{r})\, \hat{H}_{\mathrm{GT}}\, \Psi_{^{71}\mathrm{Ga}}(\mathbf{r})$$

integrated directly against the exact DHFS electron radial functions $g_\kappa(r)$, $f_\kappa(r)$ and the neutrino spherical Bessel functions j_0, j_1, without invoking the detailed-balance principle and leptonic factorisation at $r = 0$.



Since $\rho_{\mathrm{TD}}(r)$ is not known from first principles, several phenomenological shapes are fit to reproduce simultaneously:

- the $^{71}\mathrm{Ga}(\nu_e, e^-)^{71}\mathrm{Ge}$ cross-sections from $^{51}\mathrm{Cr}/^{37}\mathrm{Ar}$ sources
- the precisely measured $^{71}\mathrm{Ge}$ EC half-life, $t^{\mathrm{exp}}_{1/2} = 11.465(3)$ d

- **Single-lobe** shapes (SG) cannot resolve the anomaly
- Shapes with a **node / sign change** (DG-type) **do** resolve it, within $\sim 20\%$ cross-section reduction
- **2pF** (transition density ≡ nuclear charge density) is a conceptually unjustified benchmark and does not solve the anomaly

To solve the anomaly we need both realistic non-factorised lepton wave functions *and* a spatially extended, sign-changing transition density.

The Corrective factors

Beyond the Fermi function $F_{\rm rel}(Z,E)$, seven additional multiplicative correction factors enter the differential spectrum [L. Hayen et al, Rev. Mod. Phys. 90, 015008 (2018); Kleesiek M. et al., Eur. Phys. J. C, 79 (2019) 204]:

$$\left(\frac{d\Gamma}{dE}\right)_C = \frac{G_F^2|V_{ud}|^2}{2\pi^3}\left(g_V^2+3g_A^2\right)F_{\rm rel}(Z,E)\,p\,(E+m_e)\cdot SLCI\sum_f GRQ\cdot P_f\,\epsilon_f\sqrt{\epsilon_f^2-m_\nu^2}\,\Theta(\epsilon_f-m_\nu)$$

- G **Radiative corrections**: electromagnetic effects from virtual and real photons
- S **Screening**: corrects $F(Z,E)$ for screening of the Coulomb field by the 1s-orbital electrons left behind by the parent molecule
- R **Recoil effects**: relativistic 3-body phase space, weak magnetism, $V-A$ interference, $\mathcal{O}(1/M)$ with $M = m_{^3\mathrm{He}}$
- L **Finite nuclear extension**: the daughter Coulomb field does not scale as $1/r^2$ inside the finite nuclear radius
- C **Finite nuclear extension**: convolution of the electron/neutrino wave functions with the nucleonic wave function over the nuclear volume
- Q **Recoiling Coulomb field**: the electron propagates in the field of a recoiling, not stationary, charge
- I **Orbital-electron exchange**: quantum interaction between the departing β-electron and the 1s-orbital electrons

G, R, Q depend on the endpoint energy and are summed over final states f; S, L, C, I do not and factor outside the sum.