FINITE TEMPERATURE FERMIONIC CHARGE AND CURRENT DENSITIES IN CONICAL SPACE WITH A CIRCULAR EDGE

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CONTENT

- Introduction
- Problem setup
- Fermionic mode functions
- Expectation values of the charge and current densities
- Expressions for separate regions of space
- Numerical analysis
- Summary

INTRODUCTION

- 2D models
  - High temperature limits of (3+1)-dimensional field theories
  - Lower dimensional gravity
  - Dirac materials (graphene, topological insulators, Weyl semimetals)

- Boundary conditions in 2D models
  - Condensed matter systems, finite regions, interfaces
  - Topological defects, conical singularity
  - AdS/CFT correspondence

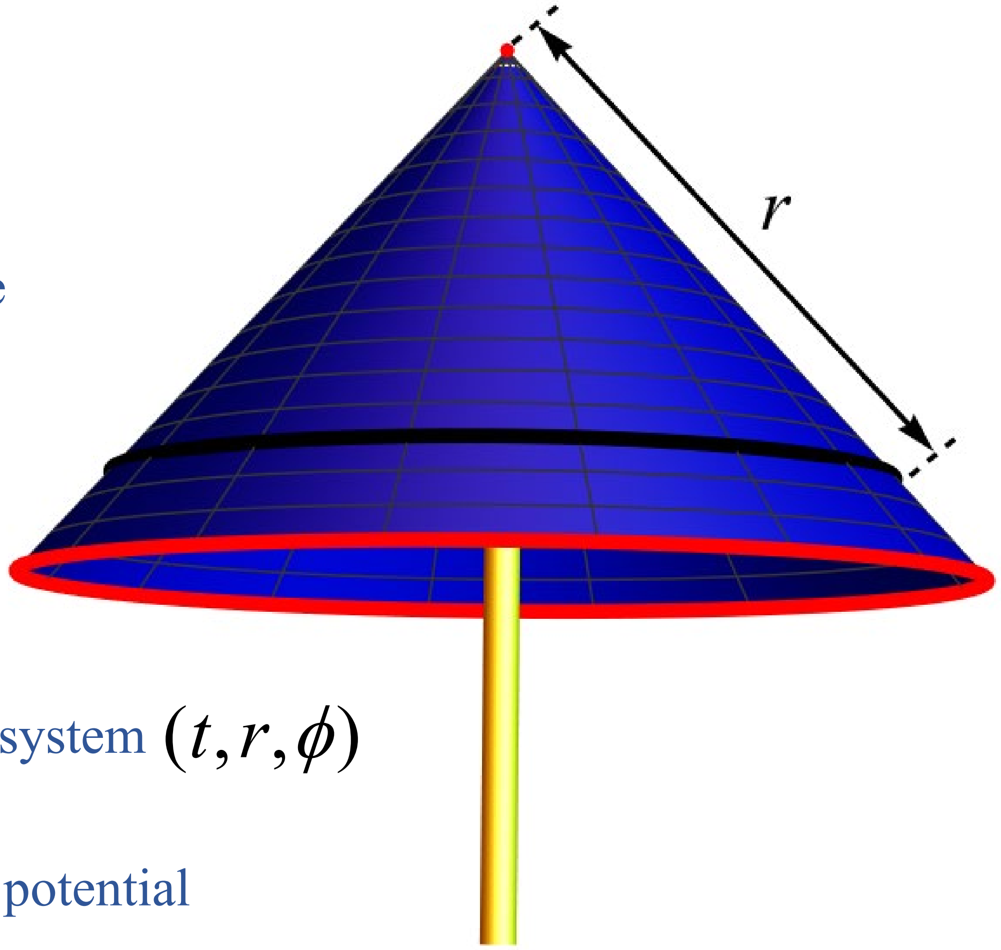
- Finite temperature in 2D physics
  - Practical application (optoelectronics, sensors, and nanotechnology)
  - Affecting electronic, magnetic and optical properties of materials

PROBLEM SETUP: GEOMETRY

- Conical spacetime

$$ds^2 = dt^2 - dr^2 - r^2 d\phi^2$$

$$r \geqslant 0, \quad 0 \leqslant \phi < \phi_0$$



$\phi_0 = 2\pi$ ⇒ (2+1)-dimensional Minkowski spacetime

$\phi_0 < 2\pi$ ⇒ Cone with the apex at $r = 0$

$2\pi - \phi_0$ → Planar angle deficit

- External electromagnetic field

$A_\mu = (0, 0, A)$ → Vector potential in the coordinates system (t, r, ϕ)

$A_\phi = -A / r$ → Physical components of the vector potential

⇕

Infinitely thin magnetic flux $\Phi = -\phi_0 A$ located at $r = 0$

$q \equiv 2\pi / \phi_0$	$\alpha = -e\Phi / 2\pi$	$\alpha = \alpha_0 + n_0, \lvert \alpha_0 \rvert < 1/2, n_0 \in \mathbb{N}$

PROBLEM SETUP: FIELD

- Spinor field (2-component) $\longrightarrow$ $\psi(x)$

Dirac equation $\longrightarrow$ $\boxed{\left(i\gamma^{\mu}D_{\mu}-sm\right)\psi(x)=0}$ $\quad D_{\mu}=\partial_{\mu}+\Gamma_{\mu}+ieA_{\mu}$

e	Charge of the field quantum	
$s=\pm 1$	2 inequivalent irreducible representations of the Clifford algebra in (2+1)-dimensions	
Γ_{μ}	Spin connection	$\Gamma_{\mu}=\frac{1}{4}\gamma^{(a)}\gamma^{(b)}e^{\nu}_{(a)}e_{(b)\nu;\mu}$
$\gamma^{(a)}$	Flat space Dirac matrices	$\gamma^{(0)}=\sigma_3,\gamma^{(1)}=i\sigma_1,\gamma^{(2)}=i\sigma_2$
σ_n	Pauli matrices	
$e^{\mu}_{(a)}$	Basis tetrads $e^{\mu}_{(0)}=(1,0,0),\ e^{\mu}_{(1)}=\left(0,\cos(q\phi),-\frac{\sin(q\phi)}{r}\right),\ e^{\mu}_{(2)}=\left(0,\sin(q\phi),\frac{\cos(q\phi)}{r}\right)$	
γ^{μ}	Dirac matrices $\gamma^{\mu}=e^{\mu}_{(a)}\gamma^{(a)} \Leftrightarrow \gamma^{0}=\begin{pmatrix}1&0\\0&-1\end{pmatrix},\ \gamma^{1}=i\begin{pmatrix}0&e^{-iq\phi}\\e^{iq\phi}&0\end{pmatrix},\ \gamma^{2}=\frac{1}{r}\begin{pmatrix}0&e^{-iq\phi}\\-e^{iq\phi}&0\end{pmatrix}$	

FERMIONIC MODE FUNCTIONS

- Complete set of mode functions $\longrightarrow$ $\left\{\psi_\sigma^{(+)}(x), \psi_\sigma^{(-)}(x)\right\}$
- Circular boundary at $r = a$ (MIT bag boundary condition (BC))

$$\left(1 + i n_\mu \gamma^\mu\right)\psi(x) = 0,\ r = a$$

- Exterior region $r \geqslant a$

Quantum numbers $\longrightarrow$ $(\gamma, j),\ 0 \leqslant \gamma < \infty,\ j = \pm 1/2, \pm 3/2, \ldots$

Radial part expressed in terms of $J_\nu(\gamma r)$ and $Y_\nu(\gamma r)$ $\longrightarrow$

Bessel function

Neumann function

$$\nu = \beta_j, \beta_j + \epsilon_j$$
$$\beta_j = \alpha_j - \epsilon_j / 2$$
$$\alpha_j = q\,|\,j + \alpha\,|$$
$$\epsilon_j = \begin{cases} 1, & j + \alpha > 0 \\ -1, & j + \alpha < 0 \end{cases}$$

- Interior region $r \leqslant a$

Quantum numbers $\longrightarrow$ $(l, j),\ l = 1, 2, \ldots,\ j = \pm 1/2, \pm 3/2, \ldots$

BC $\Rightarrow$ $\bar{J}_{\beta_j}^{(\pm)}(\gamma a) = 0$ $\Rightarrow$ Eigenvalues $\longrightarrow$ $\gamma a = \gamma_{j,l}^{(\pm)}$

Ref: $\longrightarrow$ A.A. Saharian, E.R. Bezerra de Mello, A.A. Saharyan, Phys. Rev. **D 100**, 105014 (2019)

EXPECTATION VALUES OF THE CHARGE AND CURRENT DENSITIES

$$\boxed{\left\langle j^{\nu}\right\rangle = e\,\mathrm{tr}\left[\hat{\rho}\bar{\psi}\gamma^{\nu}\psi\right],\ \nu=0,1,2}$$

$\left\langle j^{\nu}\right\rangle$	Ensemble average
$\bar{\psi}$	Dirac adjoint $\quad \bar{\psi}=\psi^{\dagger}\gamma^{0}$
$\hat{\rho}$	Density matrix $\quad \hat{\rho}=\frac{1}{Z}e^{-\beta(\hat{H}-\mu'\hat{Q})},\ \beta=\frac{1}{T}$
T	Temperature
μ'	Chemical potential
Z	Partition function $\quad Z=\mathrm{tr}\left[e^{-\beta\left(\hat{H}-\mu'\hat{Q}\right)}\right]$

- Expansion of the field operator

$$\psi=\sum_{\sigma}\left[a_{\sigma}\psi_{\sigma}^{(+)}+b_{\sigma}^{+}\psi_{\sigma}^{(-)}\right]$$

$$\bar{\psi}=\sum_{\sigma}\left[a_{\sigma}^{+}\bar{\psi}_{\sigma}^{(+)}+b_{\sigma}\bar{\psi}_{\sigma}^{(-)}\right]$$

- Anticommutation relations

$$\left\{a_{\sigma},a_{\sigma'}^{+}\right\}=\left\{b_{\sigma},b_{\sigma'}^{+}\right\}=\delta_{\sigma\sigma'}$$

$$\left\{a_{\sigma},a_{\sigma'}\right\}=\left\{b_{\sigma},b_{\sigma'}\right\}=$$

$$=\left\{a_{\sigma},b_{\sigma'}^{+}\right\}=\left\{b_{\sigma},a_{\sigma'}^{+}\right\}=0$$

DECOMPOSITIONS OF EXPECTATION VALUES

Vacuum / Temperature

$$\langle j^\nu \rangle = \langle j^\nu \rangle_{\text{vac}} + \underbrace{\langle j^\nu \rangle_{T+} + \langle j^\nu \rangle_{T-}}_{\langle j^\nu \rangle_T}$$

- Vacuum expectation values

$$\langle j^\nu \rangle_{\text{vac}} = -\frac{e}{2}\sum_{\sigma}\sum_{\lambda=\pm} \lambda \bar{\psi}_\sigma^{(\lambda)}(x)\gamma^\nu \psi_\sigma^{(\lambda)}(x)$$

- Contributions from particles and antiparticles

$$\langle j^\nu \rangle_{T\pm} = \pm e \sum_{\sigma} \frac{\bar{\psi}_\sigma^{(\pm)}(x)\gamma^\nu \psi_\sigma^{(\pm)}(x)}{e^{\beta(E_\sigma \mp \mu)}+1}, \quad \mu = e\mu'$$

$\pm E_\sigma \longrightarrow$ Energies corresponding to the modes $\psi_\sigma^{(\pm)}(x)$

Boundary-free / Boundary-induced

$$\langle j^\nu \rangle = \langle j^\nu \rangle^{(0)} + \langle j^\nu \rangle^{(b)}$$

$$\Downarrow$$

$$\langle j^\nu \rangle^{(0)} = \langle j^\nu \rangle^{(0)}_{\text{vac}} + \underbrace{\langle j^\nu \rangle^{(0)}_{T+} + \langle j^\nu \rangle^{(0)}_{T-}}_{\langle j^\nu \rangle^{(0)}_T}$$

$$\langle j^\nu \rangle^{(b)} = \langle j^\nu \rangle^{(b)}_{\text{vac}} + \underbrace{\langle j^\nu \rangle^{(b)}_{T+} + \langle j^\nu \rangle^{(b)}_{T-}}_{\langle j^\nu \rangle^{(b)}_T}$$

$$\langle j^\nu \rangle_{\text{vac}} = \langle j^\nu \rangle^{(0)}_{\text{vac}} + \langle j^\nu \rangle^{(b)}_{\text{vac}}$$

$$\langle j^\nu \rangle_{T\pm} = \langle j^\nu \rangle^{(0)}_{T\pm} + \langle j^\nu \rangle^{(b)}_{T\pm}$$

EXPECTATION VALUES: VACUUM STATE

$$\boxed{\left\langle j^{\nu}\right\rangle_{\text{vac}}=\left\langle j^{\nu}\right\rangle_{\text{vac}}^{(0)}+\left\langle j^{\nu}\right\rangle_{\text{vac}}^{(b)}}$$

- Boundary-free VEVs

$$\left\langle j^{\nu}\right\rangle_{\text{vac}}^{(0)}=-\frac{e}{2\pi r}\left\{\sum_{l=1}^{[q/2]}{}'(-1)^{l}\sin(2\pi l\alpha_0)f_{\nu}\left(2mr\sin(\pi l/q)\right)-\frac{q}{\pi}\int_0^{\infty}dy\frac{f_{\nu}\left(2mr\cosh y\right)f(q,\alpha_0,y)}{\cosh(2qy)-\cos(q\pi)}\right\}$$

- Boundary-induced VEVs (Exterior region)

$$\left\langle j^{\nu}\right\rangle_{\text{vac}}^{(\text{b})}=\frac{e}{\pi\phi_0}\sum_j\int_m^{\infty}dx\,x\,\text{Re}\left[\frac{\bar{I}_{\beta_j}(ax)}{\bar{K}_{\beta_j}(ax)}\frac{U_{\nu,\beta_j}^{K}(rx)}{\sqrt{x^2-m^2}}\right]$$

- Boundary-induced VEVs (Interior region)

$$\left\langle j^{\nu}\right\rangle_{\text{vac}}^{(\text{b})}=\frac{e}{\pi\phi_0}\sum_j\int_m^{\infty}dx\,x\,\text{Re}\left[\frac{\bar{K}_{\beta_j}(ax)}{\bar{I}_{\beta_j}(ax)}\frac{U_{\nu,\beta_j}^{I}(rx)}{\sqrt{x^2-m^2}}\right]$$

$$f_0(z)=sme^{-z},\qquad f_2(z)=2m^2e^{-z}(1+z)/z^2$$

$$f(q,\alpha_0,y)=\sum_{\chi=\pm1}\chi\cos\left[q\pi(1/2-\chi\alpha_0)\right]\cosh\left[q(1+2\chi\alpha_0)y\right]$$

$[w]\longrightarrow$ Integer part of w

Modified Bessel functions

$$\bar{F}_{\beta_j}(u)=uF'_{\beta_j}(u)-\left[\epsilon_j\beta_j-\delta^{(\text{J})}\left(sm_a+i\sqrt{u^2-m_a^2}\right)\right]F_{\beta_j}(u);\ F=I,K$$

$$U_{0,\beta_j}^{F}(rx)=\sum_{\chi=\pm1}\left(sm+\chi i\sqrt{x^2-m^2}\right)F_{\alpha_j-\chi\epsilon_j/2}^{2}(rx)$$

$$U_{2,\beta_j}^{F}(rx)=-\delta_F\frac{2x}{r}F_{\beta_j}(rx)F_{\beta_j+\epsilon_j}(rx)$$

$$F=I,K$$

Ref: (Vacuum, 1 boundary) $\longrightarrow$

E. R. Bezerra de Mello, V. Bezerra, A.A. Saharian, V.M. Bardeghyan, Phys. Rev. **D 82**, 085033 (2010)

EXPECTATION VALUES: BOUNDARY-FREE

$$\boxed{\left\langle j^{\nu}\right\rangle^{(0)} = \left\langle j^{\nu}\right\rangle^{(0)}_{\text{vac}} + \left\langle j^{\nu}\right\rangle^{(0)}_{T+} + \left\langle j^{\nu}\right\rangle^{(0)}_{T-}}$$

where

$$\left\langle j^{0}\right\rangle^{(0)}_{T\pm} = \frac{\pm e}{2\phi_0}\sum_j \int_0^\infty d\gamma \frac{\gamma / E}{e^{\beta(E\mp\mu)}+1}\sum_{\chi=\pm 1}(E \pm \chi s m) J^2_{\alpha_j - \chi\epsilon_j/2}(\gamma r)$$

$$\left\langle j^{2}\right\rangle^{(0)}_{T\pm} = \frac{e}{\phi_0 r}\sum_j \int_0^\infty d\gamma \frac{\epsilon_j \gamma^2 / E}{e^{\beta(E\mp\mu)}+1} J_{\beta_j}(\gamma r) J_{\beta_j+\epsilon_j}(\gamma r)$$

Ref:
(Boundary-free, Finite temp.) → S. Bellucci, E.R. Bezerra de Mello, E. Bragança, A.A. Saharian, Eur. Phys. J. **C 76**, 359 (2016)

EXPECTATION VALUES: BOUNDARY-INDUCED

$$\left\langle j^{\nu}\right\rangle = \left\langle j^{\nu}\right\rangle^{(0)} + \left\langle j^{\nu}\right\rangle^{(b)}$$

- Exterior region

$$\left\langle j^{0}\right\rangle^{(\mathrm{b})} = \frac{2Te}{\phi_0}\sum_j\sum_{n=0}^{\infty}\mathrm{Re}\left\{\frac{\bar{I}_{\beta_j}(u_n a)}{\bar{K}_{\beta_j}(u_n a)}\sum_{\chi=\pm1}\left[sm+\chi\mu+\chi i\pi(2n+1)T\right]K^2_{\alpha_j-\chi\epsilon_j/2}(u_n r)\right\}$$

$$\left\langle j^{2}\right\rangle^{(\mathrm{b})} = \frac{2Te}{\phi_0}\sum_j\sum_{n=0}^{\infty}\mathrm{Re}\left[\frac{\bar{I}_{\beta_j}(u_n a)}{\bar{K}_{\beta_j}(u_n a)}U^{K}_{2,\beta_j}(u_n r)\right]$$

- Interior region

$$\left\langle j^{0}\right\rangle^{(\mathrm{b})} = \frac{2eT}{\phi_0}\sum_j\sum_{n=0}^{\infty}\mathrm{Re}\left[\frac{\bar{K}_{\beta_j}(u_n a)}{\bar{I}_{\beta_j}(u_n a)}\sum_{\chi=\pm1}\left[sm+\chi\mu+\chi i\pi(2n+1)T\right]I^2_{\alpha_j-\chi\epsilon_j/2}(u_n r)\right]$$

$$\left\langle j^{2}\right\rangle^{(\mathrm{b})} = \frac{2eT}{\phi_0}\sum_j\sum_{n=0}^{\infty}\mathrm{Re}\left[\frac{\bar{K}_{\beta_j}(u_n a)}{\bar{I}_{\beta_j}(u_n a)}U^{I}_{2,\beta_j}(u_n r)\right]$$

Odd functions under simultaneous replacements

$$\alpha \to -\alpha,\ \mu \to -\mu$$

$$\bar{F}_{\beta_j}(u) = uF'_{\beta_j}(u) - \left[\epsilon_j\beta_j - \delta^{(\mathrm{J})}\left(sm_a + i\sqrt{u^2-m_a^2}\right)\right]F_{\beta_j}(u);\ F = I, K$$

$$u_n = \left\{\left[\pi(2n+1)T - i\mu\right]^2 + m^2\right\}^{1/2}$$

MASSLESS FIELD WITH ZERO CHEMICAL POTENTIAL

$\boxed{m = 0 \text{ and } \mu = 0}$

- Exterior region

$$\left\langle j^0 \right\rangle^{(b)} = \frac{2Te}{a\phi_0} \sum_j \sum_{n=0}^{\infty} \frac{K^2_{\beta_j+\epsilon_j}(zr/a) - K^2_{\beta_j}(zr/a)}{K^2_{\beta_j+\epsilon_j}(z) + K^2_{\beta_j}(z)} \Bigg|_{z=(2n+1)\pi aT}$$

$$\left\langle j^2 \right\rangle^{(\mathrm{b})} = \frac{4Te}{\phi_0} \sum_j \sum_{n=0}^{\infty} \sum_{\chi=\pm 1} \frac{\chi I_{\alpha_j-\chi\epsilon_j/2}(z) K_{\alpha_j-\chi\epsilon_j/2}(z)}{K^2_{\beta_j+\epsilon_j}(z) + K^2_{\beta_j}(z)} U^K_{2,\beta_j}(zr/a) \Bigg|_{z=(2n+1)\pi aT}$$

- Interior region

$$\left\langle j^0 \right\rangle^{(b)} = \frac{2eT}{a\phi_0} \sum_j \sum_{n=0}^{\infty} \frac{I^2_{\beta_j+\epsilon_j}(zr/a) - I^2_{\beta_j}(zr/a)}{I^2_{\beta_j+\epsilon_j}(z) + I^2_{\beta_j}(z)} \Bigg|_{z=(2n+1)\pi aT}$$

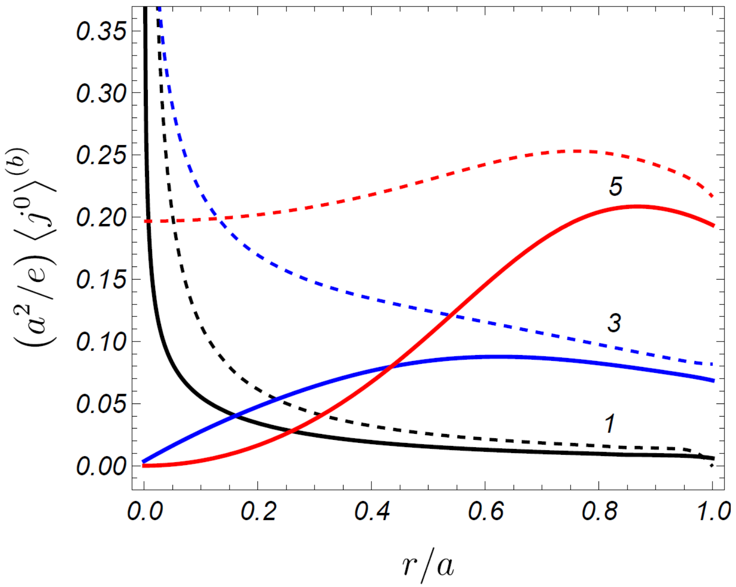
$$\left\langle j^2 \right\rangle^{(\mathrm{b})} = \frac{2eT}{\phi_0} \sum_j \sum_{n=0}^{\infty} \sum_{\chi=\pm 1} \frac{\chi I_{\alpha_j-\chi\epsilon_j/2}(z) K_{\alpha_j-\chi\epsilon_j/2}(z)}{I^2_{\beta_j}(z) + I^2_{\beta_j+\epsilon_j}(z)} U^I_{2,\beta_j}(zr/a) \Bigg|_{z=(2n+1)\pi aT}$$

$$\Downarrow$$

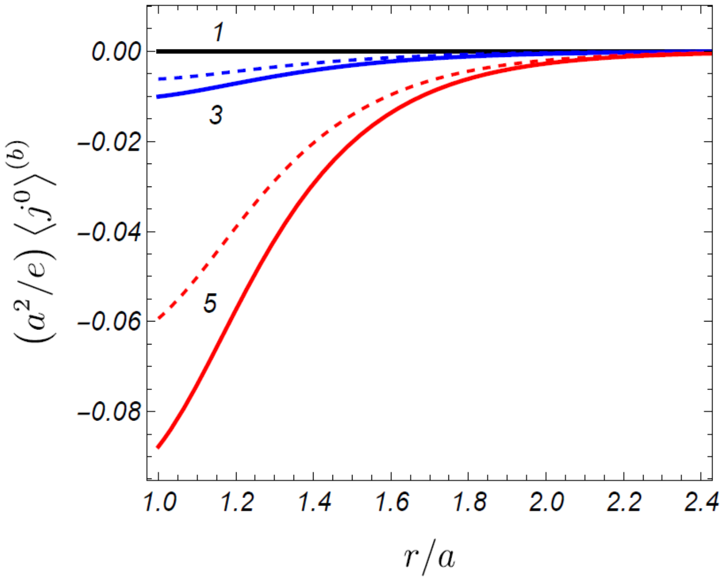
Expectation values do not depend on the parameter s

RADIAL DEPENDENCE

Interior



Exterior



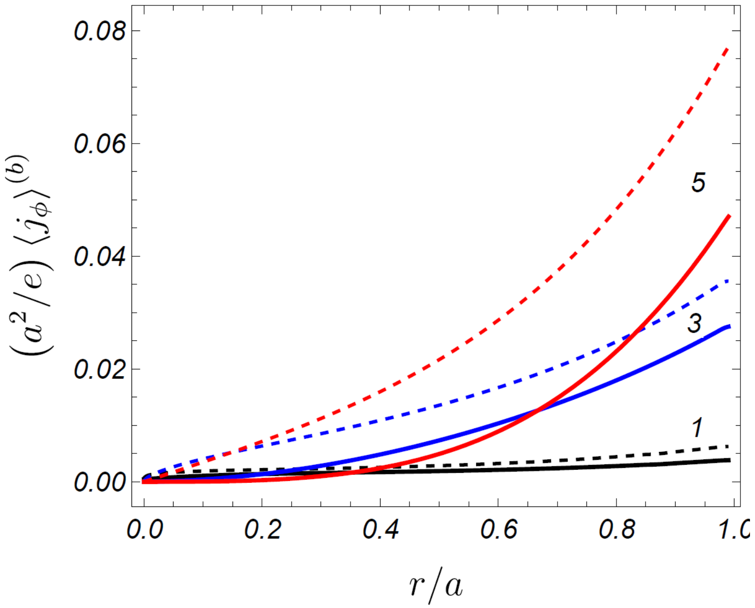
$m=0$ and $\mu=0$

Charge density

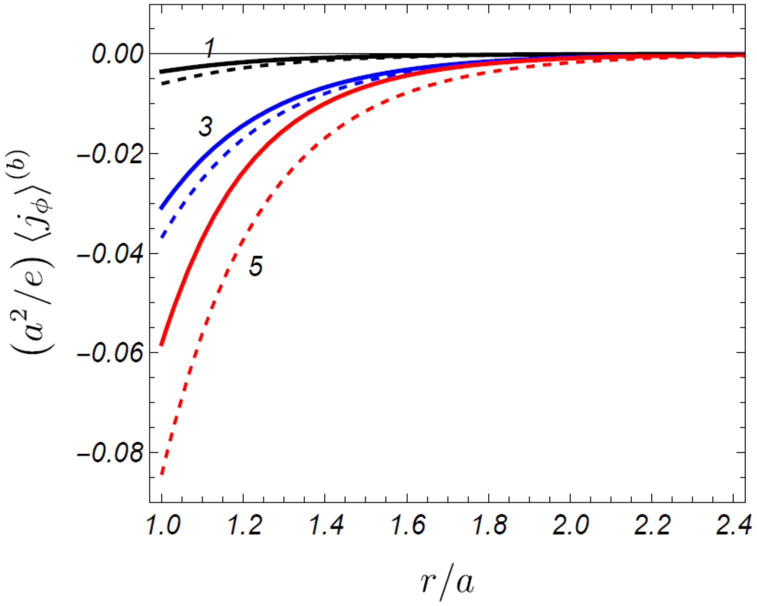
Full / Dashed

$q=1,3,5$

$Ta=0.5,\ \alpha_0=0.2/0.4$



Current density



DEPENDENCE ON TEMPERATURE

$r/a = 0.5$ (Interior)

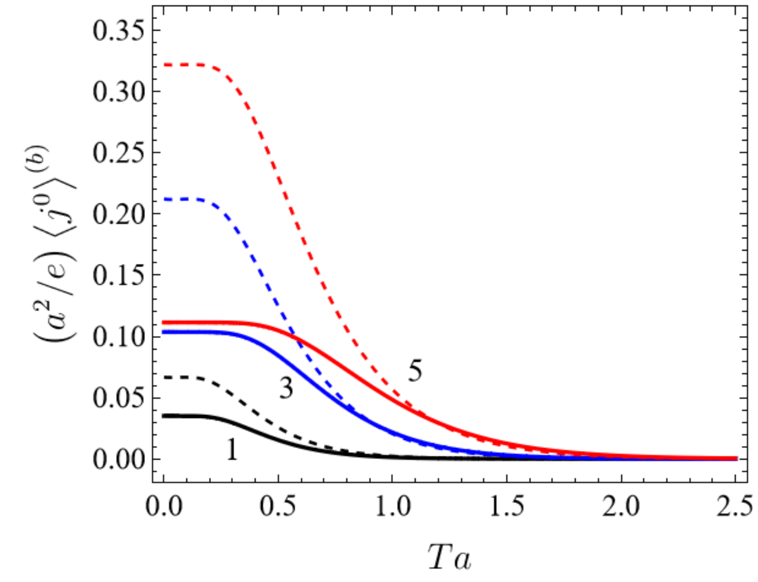
$m = 0$ and $\mu = 0$

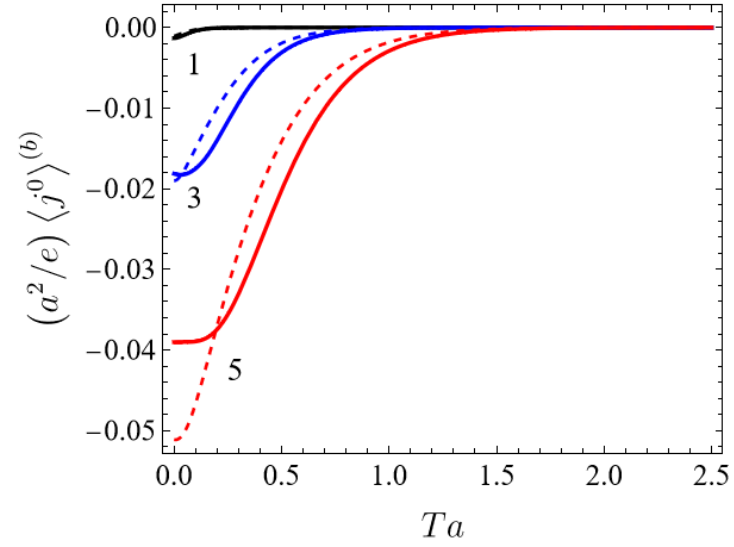
$r/a = 1.5$ (Exterior)

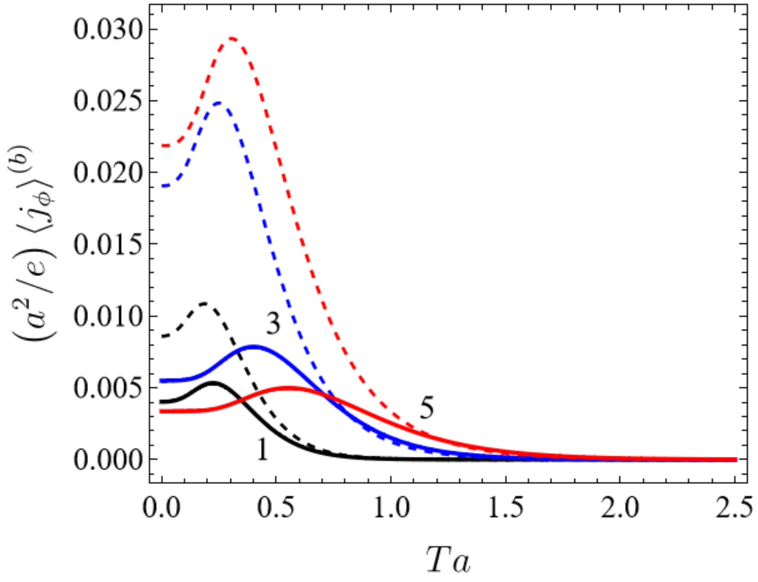
Full / Dashed

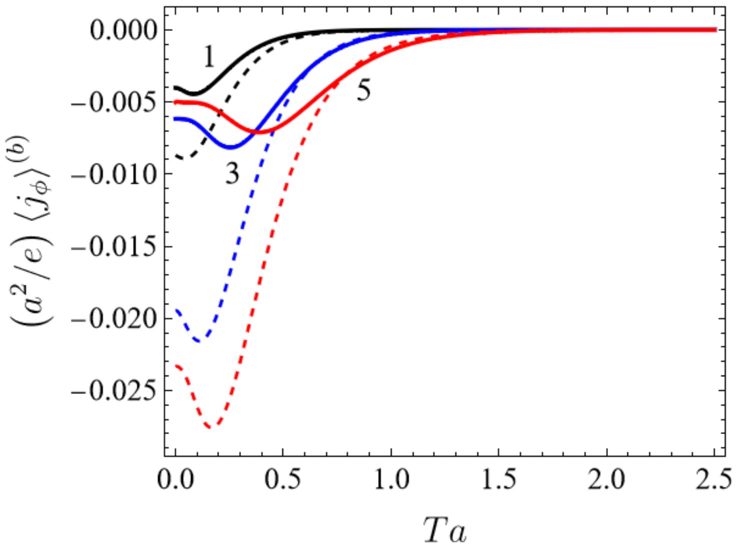
$\alpha_0 = 0.2 / 0.4$

$q = 1, 3, 5$



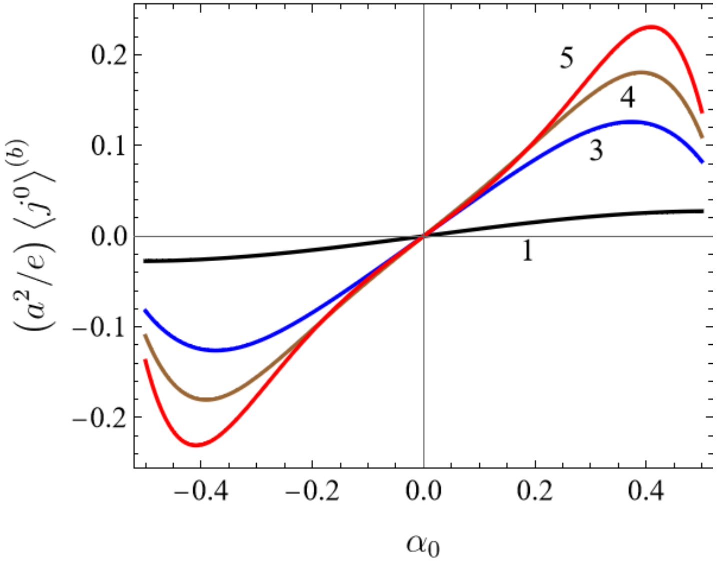






DEPENDENCE ON PARAMETERS α_0, q

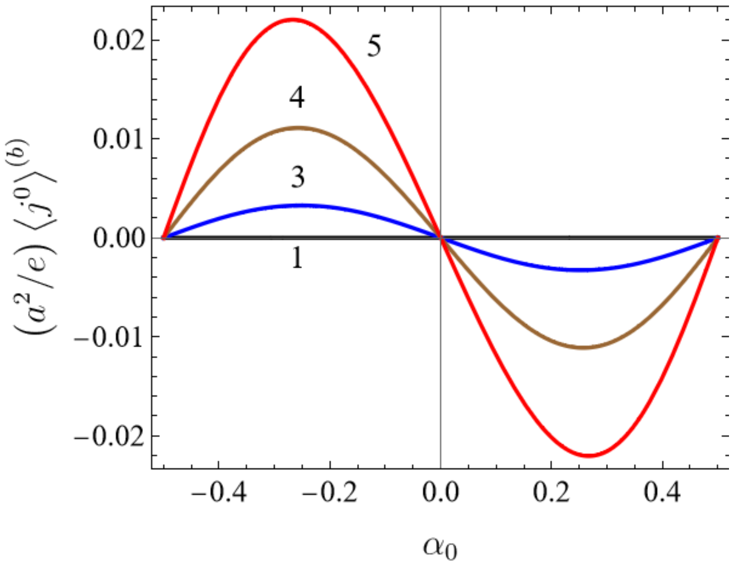
$r/a = 0.5$ (Interior)

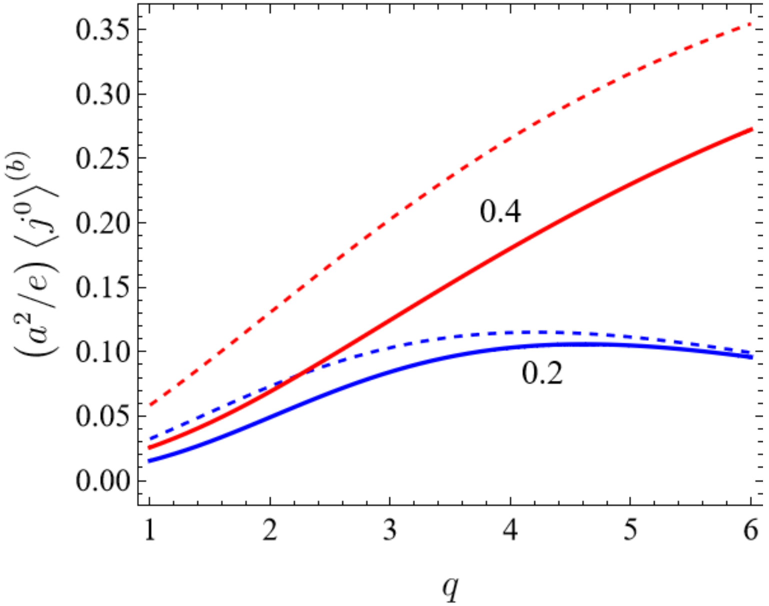


$m = 0$ and $\mu = 0$

$Ta = 0.5, \ \ q = 1,3,4,5$

$r/a = 1.5$ (Exterior)



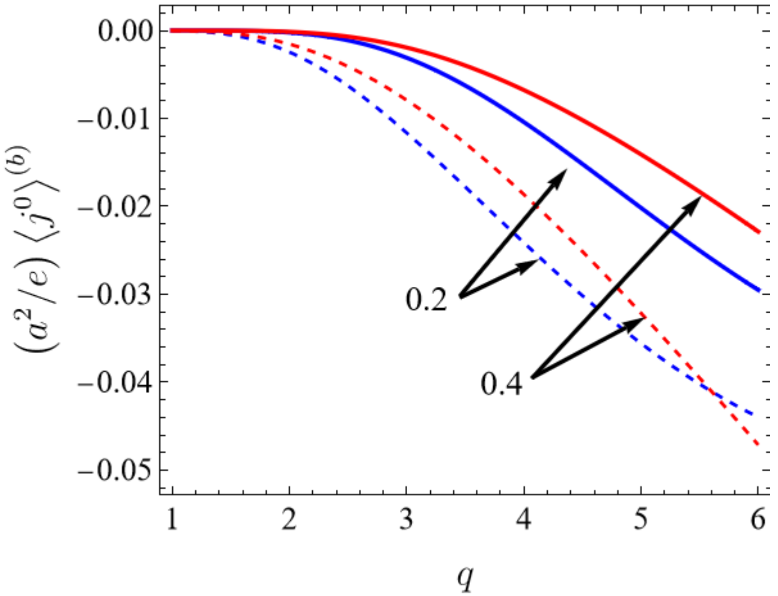


$m = 0$ and $\mu = 0$

Full / Dashed

$\Updownarrow$

$\alpha_0 = 0.2, 0.4, \ \ Ta = 0.5/0.25$

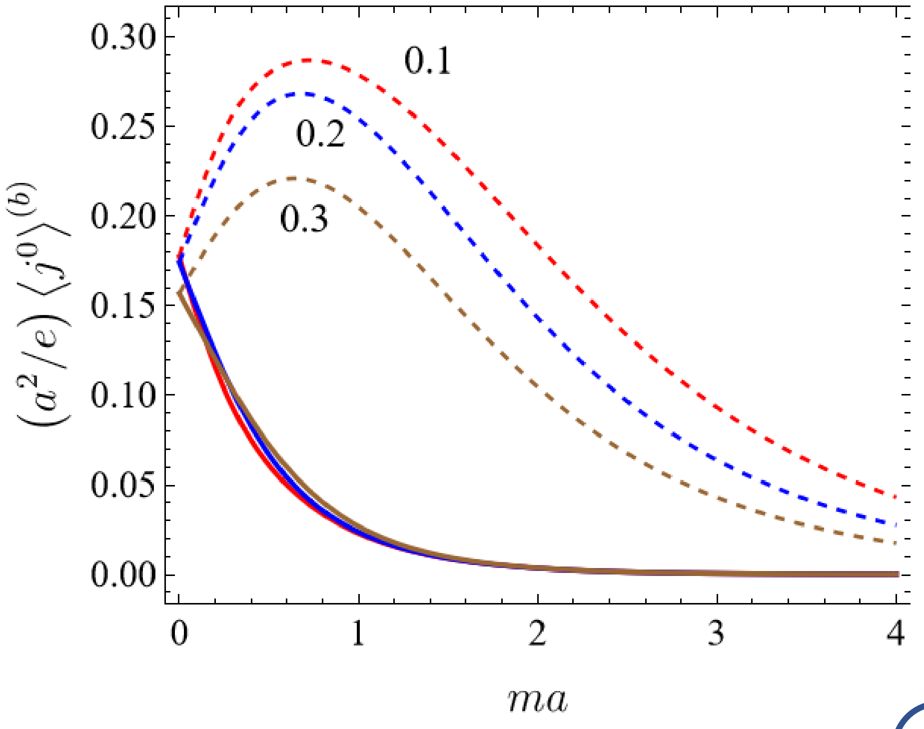


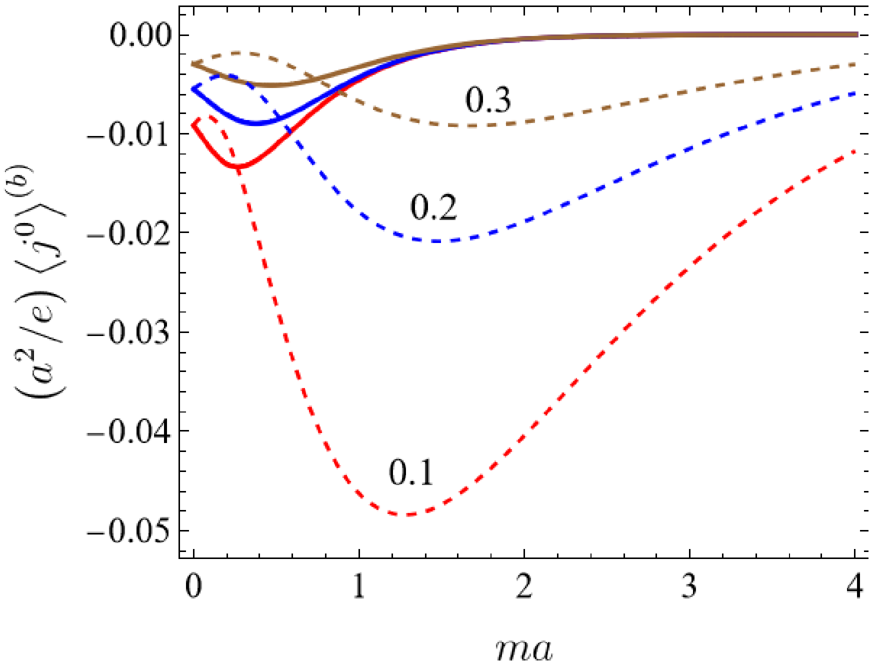
DEPENDENCE ON MASS

$r/a = 0.5$ (Interior)

$\mu = 0$

$r/a = 1.5$ (Exterior)





$q = 2.5, \alpha_0 = 0.4,$

$Ta = 0.1, 0.2, 0.3,$ $s = +1/-1$

Full / Dashed

BOUNDARY CONDITION: GENERAL FORM

- Boundary condition

$$\left(1+\eta i n_\mu \gamma^\mu\right)\psi(x)=0,\, r=a,\, \eta=\pm 1$$

Special case $\eta=+1 \longrightarrow$ MIT bag BC

- Expectation values

$$\langle j^\nu \rangle^{(\mathrm{b})}_{(s,\mu,\eta)} = (-1)^{1-\nu/2} \langle j^\nu \rangle^{(\mathrm{b})}_{(-s,-\mu,-\eta)}$$

$$\langle j^\nu \rangle^{(0)}_{(s,\mu)} = (-1)^{1-\nu/2} \langle j^\nu \rangle^{(0)}_{(-s,-\mu)}$$

$\Downarrow$

The same for the total expectation values $\langle j^\nu \rangle_{(s,\mu,\eta)}$

PARITY AND TIME-REVERSAL INVARIANT MODELS

- ∃ 2 inequivalent irreducible representations of the Clifford algebra

2 sets of γ-matrices ⟶ $\gamma^{\mu}_{(s)} = (\gamma^0, \gamma^1, \gamma^2_{(s)}),$

$$\gamma^2_{(s)} = -is\gamma^0\gamma^1 / r,\ s = \pm 1$$

$s = +1$ ⟷ Present problem

- If ∄ magnetic field ➡ ∃ fermionic models which are invariant under the P- and T-reversal transformations

Lagrangian density ⟶ $L = \sum_{s=\pm 1} \bar{\psi}_{(s)}(i\gamma^{\mu}_{(s)} D_{\mu} - m)\psi_{(s)}$

- The corresponding problem can be mapped to the present problem

PARITY AND TIME-REVERSAL INVARIANT MODELS

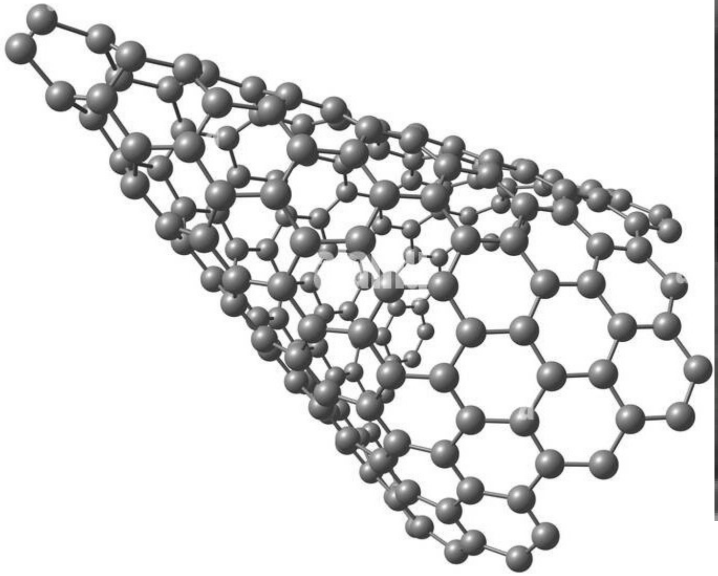
Field	$\psi_{(s)}$	$\psi'_{(s)} = \gamma^0 \gamma^{(1-s)/2} \psi_{(s)}$
Boundary conditions	$\left(1+\eta_{(s)} i n_\mu \gamma^\mu_{(s)}\right)\psi_{(s)} = 0,\ r=a$	$\left(1+s\eta_{(s)} i n_\mu \gamma^\mu\right)\psi'_{(s)} = 0,\ r=a$
Field equations	$\left(i\gamma^\mu_{(s)} D_\mu - m\right)\psi_{(s)}(x) = 0$	$\left(i\gamma^\mu D_\mu - sm\right)\psi'_{(s)}(x) = 0$
Current densities	$j^\nu_{(s)} = e\bar{\psi}_{(s)}\gamma^\nu_{(s)}\psi_{(s)}$	$j'^\nu_{(s)} = e\bar{\psi}'_{(s)}\gamma^\nu_{(s)}\psi'_{(s)}$

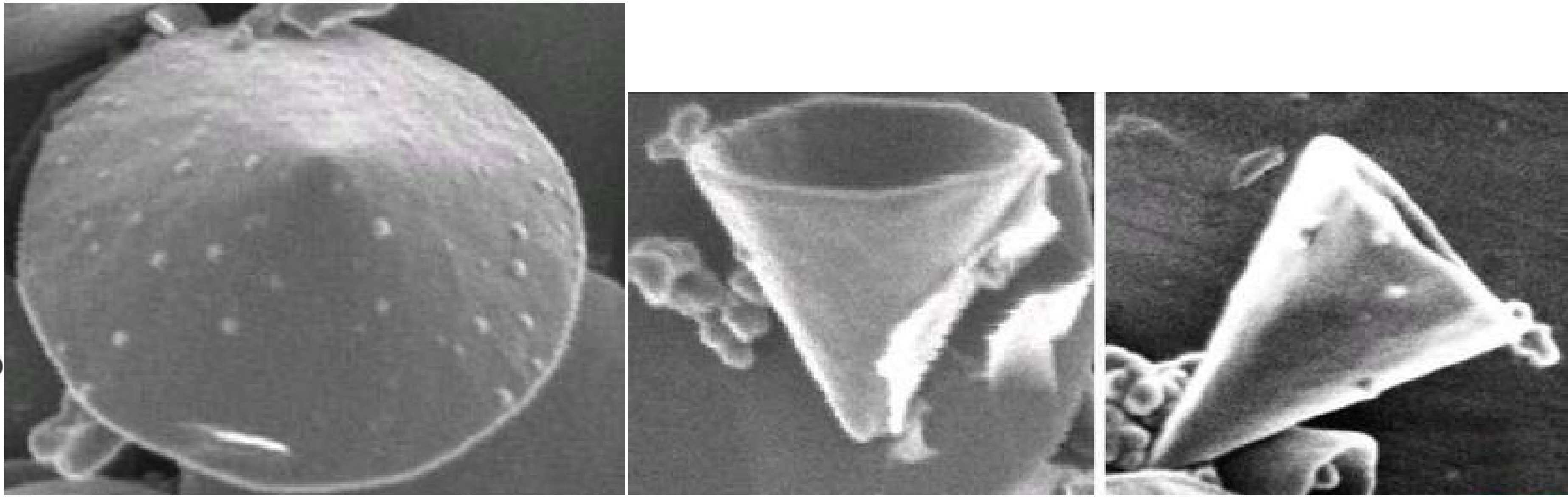
$$\langle j^\nu_{(s)} \rangle = s^{\nu/2} \langle j'^\nu_{(s)} \rangle,\ \nu = 0,2$$

Total expectation values

$$\boxed{\langle J^\nu \rangle = \sum_{s=\pm 1} s\eta_{(s)}^{1-\nu/2} \left\langle j^\nu \right\rangle_{(\eta_{(s)}, s\eta_{(s)}\mu, +1)}}$$

APPLICATIONS IN GRAPHITIC CONES





SEM images of free-standing hollow carbon nanocones produced by pyrolysis of heavy oil

Planar angle deficit $\longrightarrow$ $$2\pi - \phi_0 = \pi n_c / 3, \quad n_c = 1,2,\ldots,5$$

APPLICATIONS IN GRAPHITIC CONES

- Dirac model describing the long wavelength properties of the electronic subsystem of graphene

$s = \pm 1$ $\longrightarrow$ Points $\mathbf{K}_+$ and $\mathbf{K}_-$ of the graphene Brillouin zone

Spinors $\longrightarrow$ $\psi_{(s)} = \begin{pmatrix} \psi_{s,AS} \\ \psi_{s,BS} \end{pmatrix}$

$S = \pm 1$ $\longrightarrow$ Additional degree of freedom related to spin

$\psi_{s,AS}, \psi_{s,BS}$ $\longrightarrow$ Electron wave functions on the A and B sites of the graphene lattice

- Dirac equation for π-electrons

4-component spinors $\longrightarrow$ $\Psi_S = \begin{pmatrix} \psi_{(+1)} \\ \psi_{(-1)} \end{pmatrix}$

4x4 Dirac matrices $\longrightarrow$ $\gamma^{\mu}_{(4)} = \sigma_{\mathrm{P3}} \otimes \gamma^{\mu}$ $\qquad \sigma_{\mathrm{P3}} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Spin connection $\longrightarrow$ $\Gamma^{(4)}_{\mu} = I \otimes \Gamma_{\mu}$ $\qquad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$c \to v_{\mathrm{F}} \approx 7.9 \times 10^{7}$ cm/sec

c $\longrightarrow$ Speed of light

v_{F} $\longrightarrow$ Fermi velocity of electrons

SUMMARY

- We have considered the combined effects of finite temperature and circular boundary on the expectation values of the charge and current densities in a (2+1)-dimensional conical spacetime with an arbitrary value of the planar angle deficit
- In two-dimensional spaces there exist two inequivalent representations of the Clifford algebra and we have presented the investigation for both the fields realizing those representations
- The expectation values of the charge and current densities in both the exterior and interior regions are decomposed into boundary-free and boundary-induced contributions
- Boundary-induced contributions are periodic functions of the magnetic flux with the period equal to the flux quantum
- Boundary-induced contributions are odd functions under the simultaneous reflections $\alpha \to -\alpha,\ \mu \to -\mu$
- At large distances from the boundary the charge density is dominated by the Minkowskian part which does not depend on radial coordinate, whereas the current density decays exponentially

THANK YOU

EXPECTATION VALUES: EXTERIOR REGION

- Thermal part

$$\langle j^0 \rangle_{T\pm} = \frac{\pm e}{2\phi_0} \sum_j \int_0^\infty d\gamma \frac{\gamma}{E} \frac{\sum_{\chi=\pm1} (E \pm \chi s m) g^{(\pm)2}_{\beta_j, \alpha_j - \chi\epsilon_j/2}(\gamma a, \gamma r)}{\left[e^{\beta(E \mp \mu)} + 1 \right] \left[\bar{J}^{(\pm)2}_{\beta_j}(\gamma a) + \bar{Y}^{(\pm)2}_{\beta_j}(\gamma a) \right]}$$

  - Boundary-free thermal part (particles and antiparticles) $\longrightarrow$ $\langle j^0 \rangle^{(0)}_{T\pm}$ $\Rightarrow$ $\dfrac{g^{(\pm)2}_{\beta_j,\nu}(x,y)}{\bar{J}^{(\pm)2}_{\beta_j}(x) + \bar{Y}^{(\pm)2}_{\beta_j}(x)} \to J^2_\nu(y); \nu = \beta_j, \beta_j + \epsilon_j$
  - Boundary-induced part (particles and antiparticles) $\longrightarrow$ $\langle j^0 \rangle^{(b)}_{T\pm} = \langle j^0 \rangle_{T\pm} - \langle j^0 \rangle^{(0)}_{T\pm}$
  - Boundary-induced thermal part $\longrightarrow$ $\langle j^0 \rangle^{(b)}_{T} = \langle j^0 \rangle^{(b)}_{T+} + \langle j^0 \rangle^{(b)}_{T-}$
  - Boundary-induced part $\longrightarrow$ $\langle j^0 \rangle^{(b)} = \langle j^0 \rangle^{(b)}_{\mathrm{vac}} + \langle j^0 \rangle^{(b)}_{T}$

- Total expectation values $\longrightarrow$ $\langle j^\nu \rangle = \langle j^\nu \rangle^{(0)} + \langle j^\nu \rangle^{(b)}$

EXPECTATION VALUES: INTERIOR REGION

- Thermal part

$$\langle j^{\nu}\rangle_{T\lambda} = \frac{\lambda e}{2\phi_0 a}\sum_j \sum_{l=1}^{\infty} \frac{T_{\beta_j}^{(\lambda)}\left(\gamma_{j,l}^{(\lambda)}\right) g^{(\nu)}\left(\gamma_{j,l}^{(\lambda)}/a\right)}{e^{\beta(E-\lambda\mu)}+1}$$

$g^{(\nu)}\left(\gamma_{j,l}^{(\pm)}/a\right)$ are expressed in terms of $J_{\nu}\left(\gamma_{j,l}^{(\pm)}r/a\right)$

$$E_{j,l}^{(\pm)} = \sqrt{\gamma_{j,l}^{(\pm)2}/a^2 + m^2}$$

- Summation formula ⟶ Generalized Abel-Plana type formula

Ref:

A.A. Saharian, The Generalized Abel-Plana Formula with Applications to Bessel Functions and Casimir Effect (2008) (**arXiv:0708.1187**)

A.A. Saharian, E.R. Bezerra de Mello, *J. Phys. A: Math. Gen.* **37**, 3543 (2004)

- Total expectation values

$$\langle j^{\nu}\rangle_{T\pm}^{(b)} \xrightarrow[\langle j^{\nu}\rangle_{T+}^{(b)}+\langle j^{\nu}\rangle_{T-}^{(b)}]{} \langle j^{\nu}\rangle_{T}^{(b)} \xrightarrow[\text{adding }\langle j^{\nu}\rangle_{\text{vac}}^{(b)}]{} \langle j^{\nu}\rangle^{(b)} \xrightarrow[\text{adding }\langle j^{\nu}\rangle^{(0)}]{} \langle j^{\nu}\rangle$$

RADIAL DEPENDENCE

- Large distances from the boundary

$$\begin{cases} m \neq 0 \\ \mu \neq 0 \end{cases} \text{ and } \begin{cases} u_n r \gg 1 \\ T \gtrsim m, |\mu| \end{cases}$$

$$\left\langle j^{\nu} \right\rangle^{(b)} \propto e^{-2u_0 r}$$

$$\begin{cases} m = 0 \\ \mu = 0 \end{cases} \text{ and } \quad Tr \gg 1$$

$$\left\langle j^{0} \right\rangle^{(b)}, \left\langle j^{2} \right\rangle^{(b)} \propto e^{-2\pi Tr} / r^2$$

$$\langle j^0 \rangle^{(0)} \approx \langle j^0 \rangle^{(\mathrm{M})}$$ does not depend on r

$$\langle j^2 \rangle^{(0)} \approx \langle j^2 \rangle^{(0)}_{\mathrm{t}} \propto \begin{cases} e^{-2ru_0}, & \phi_0 > \pi \\ e^{-2ru_0 \sin(\phi_0/2)}, & \phi_0 < \pi \end{cases}$$

$$\left\langle j^{\nu} \right\rangle^{(0)}_{\mathrm{vac}} \propto \begin{cases} e^{-2mr}, & \phi_0 > \pi \\ e^{-2mr \sin(\phi_0/2)}, & \phi_0 < \pi \end{cases}$$

$$\left\langle j^{0} \right\rangle^{(\mathrm{b})}_{\mathrm{vac}}, r\left\langle j^{2} \right\rangle^{(\mathrm{b})}_{\mathrm{vac}} \propto e^{-2mr} / r^{3/2}$$

$$\boxed{q \equiv 2\pi / \phi_0}$$

$$\left\langle j^{0} \right\rangle^{(0)}_{\mathrm{vac}} = 0$$

$$\left\langle j^{2} \right\rangle^{(0)}_{\mathrm{vac}} \propto 1/r^2$$

$$\left\langle j^{0} \right\rangle^{(\mathrm{b})}_{\mathrm{vac}} \propto 1/r^{2\rho+2}$$

$$\left\langle j^{2} \right\rangle^{(\mathrm{b})}_{\mathrm{vac}} \propto \begin{cases} 1/r^{2\rho+4}, & \rho > 1/2 \\ 1/r^{4\rho+3}, & \rho < 1/2 \end{cases}$$

$$\boxed{\rho = q\left(1/2 - |\alpha_0|\right)}$$

RADIAL DEPENDENCE

- Near the cone apex

$$\begin{cases} m \neq 0 \\ \mu \neq 0 \end{cases} \text{and } u_n r \ll 1$$

Contribution of the irregular mode

$$\langle j^\nu \rangle^{(b)} \propto r^{q-2q|\alpha_0|-1} \Rightarrow \begin{cases} 2|\alpha_0| > 1-1/q \longrightarrow \langle j^0 \rangle^{(b)} & \text{diverges on the apex} \\ 2|\alpha_0| < 1-1/q \longrightarrow \langle j^0 \rangle^{(b)} & \text{vanishes on the apex} \\ \langle j_\phi \rangle^{(b)} & \text{vanishes on the apex} \end{cases}$$

$2|\alpha_0| > 1-1/q \longrightarrow$ Expectation values in the boundary-free geometry are dominated by the vacuum parts $\longrightarrow \begin{cases} \langle j^0 \rangle^{(0)} \propto r^{q(1-2|\alpha_0|)-1} \\ \langle j_\phi \rangle^{(0)} \propto 1/r^2 \end{cases}$

- On the boundary

Approaching from exterior $\longrightarrow$

$$\langle j^0 \rangle_{T\pm} = -\langle j_\phi \rangle_{T\pm} = \frac{4e}{\pi^2 \phi_0} \sum_j \int_0^\infty d\gamma \frac{\gamma(sm/E \pm 1)}{e^{\beta(E \mp \mu)}+1} \frac{1}{\bar{J}_{\beta_j}^{(\pm)2}(\gamma a) + \bar{Y}_{\beta_j}^{(\pm)2}(\gamma a)}$$

Approaching from interior $\longrightarrow$

$$\langle j^0 \rangle_{T\lambda} = \langle j_\phi \rangle_{T\lambda} = \frac{\lambda e}{\phi_0 a^2} \sum_j \sum_{l=1}^\infty \frac{u T_{\beta_j}^{(\lambda)}\left(\gamma_{j,l}^{(\lambda)}\right) J_{\beta_j}^2\left(\gamma_{j,l}^{(\lambda)}\right)}{e^{\beta(E-\lambda\mu)}+1} \left(1 + \frac{\lambda s m}{E}\right)$$

DEPENDENCE ON TEMPERATURE

- High temperature

$$T \gg m, |\mu|, 1/(r-a) \Rightarrow \begin{cases} \langle j^0 \rangle^{(b)} \propto e^{-2\pi T(r-a)} \\ q>2 \longrightarrow \langle j^0 \rangle_{\mathrm{t}}^{(0)} \propto e^{-2\pi Tr\sin(\pi/q)} \\ q<2 \longrightarrow \langle j^0 \rangle_{\mathrm{t}}^{(0)} \propto e^{-2\pi Tr} \\ \langle j^0 \rangle_{\mathrm{M}}^{(0)} \approx e\mu T\ln 2/\pi \end{cases} \Rightarrow$$

Total $\langle j^0 \rangle$ is dominated by the Minkowskian part

- Zero temperature limit

1) $|\mu|<m \longrightarrow \lim_{T\to 0}\langle j^\nu \rangle = \langle j^\nu \rangle_{\mathrm{vac}}$

2) $|\mu|>m$

Exterior $\longrightarrow$ $$\lim_{T\to 0}\langle j^0 \rangle = \langle j^0 \rangle_{\mathrm{vac}} + \frac{\lambda e}{2\phi_0}\sum_j \int_0^{\sqrt{\mu^2-m^2}} d\gamma \frac{\gamma}{E}\frac{(E+\lambda sm)g^{(\lambda)2}_{\beta_j,\beta_j}(\gamma a,\gamma r)+(E-\lambda sm)g^{(\lambda)2}_{\beta_j,\beta_j+\epsilon_j}(\gamma a,\gamma r)}{\bar{J}^{(\lambda)2}_{\beta_j}(\gamma a)+\bar{Y}^{(\lambda)2}_{\beta_j}(\gamma a)}$$

Interior $\longrightarrow$ $$\lim_{T\to 0}\langle j^0 \rangle = \langle j^0 \rangle_{\mathrm{vac}} + \frac{\lambda e}{2\phi_0 a^2}\sum_j \sum_{l=1}^{l_m}{}' T^{(\lambda)}_{\beta_j}\left(\gamma^{(\lambda)}_{j,l}\right) g^{(0)}\left(\gamma^{(\lambda)}_{j,l}\right)$$

$$\gamma^{(\lambda)}_{j,l_m} \leqslant a\sqrt{\mu^2-m^2} < \gamma^{(\lambda)}_{j,l_m+1}$$

Prime means that the last term $l=l_m$ must be taken with additional factor ½ in case of $\gamma^{(\lambda)}_{j,l_m} = a\sqrt{\mu^2-m^2}$