The Unitary Fermi Gas as a Neutron Matter Emulator for Nuclear Astrophysics

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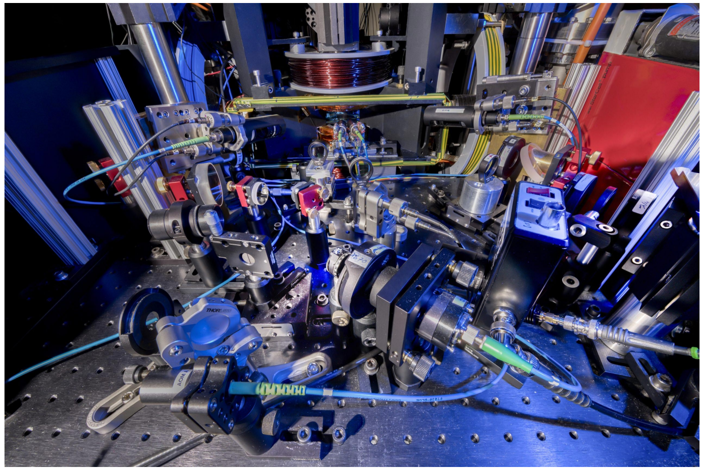
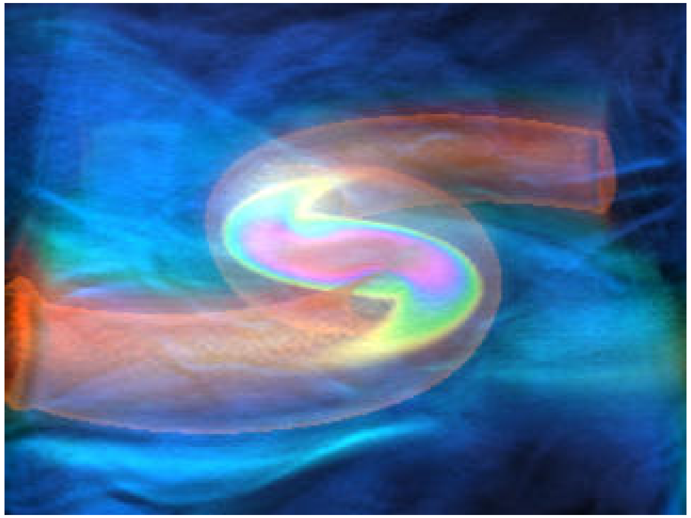




Why study the unitary Fermi gas?

Nuclear forces are quite complicated, and so is nuclear many-body theory (for the equation of state, reaction rates, mean free paths, transport properties, etc.).

The unitary Fermi gas is a very simple and clean model system for strongly correlated quantum many-body systems.

Universality connects atomic and nuclear systems.

Impressive progress in experimental control: Tune interactions, temperature, external fields, linear response, etc.

What do we learn?

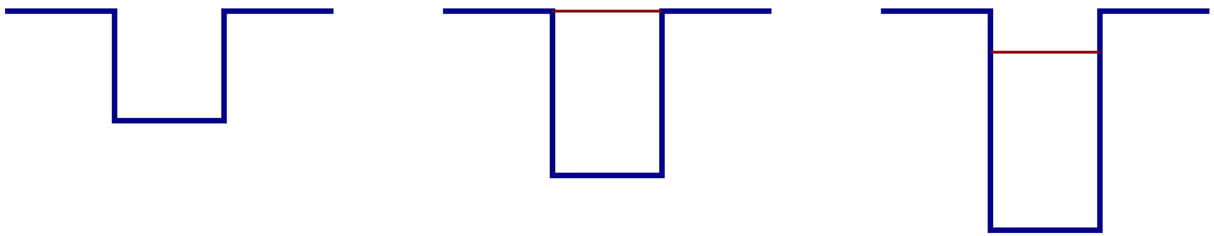
Numerical predictions that directly translate to nuclear systems (e.g. EOS).

Numerical benchmarks for nuclear many-body theory (EOS, response functions, reaction theory).

Improved understanding of few and many-body effects (short range correlations, reaction theory).

Non-relativistic fermions in unitarity limit

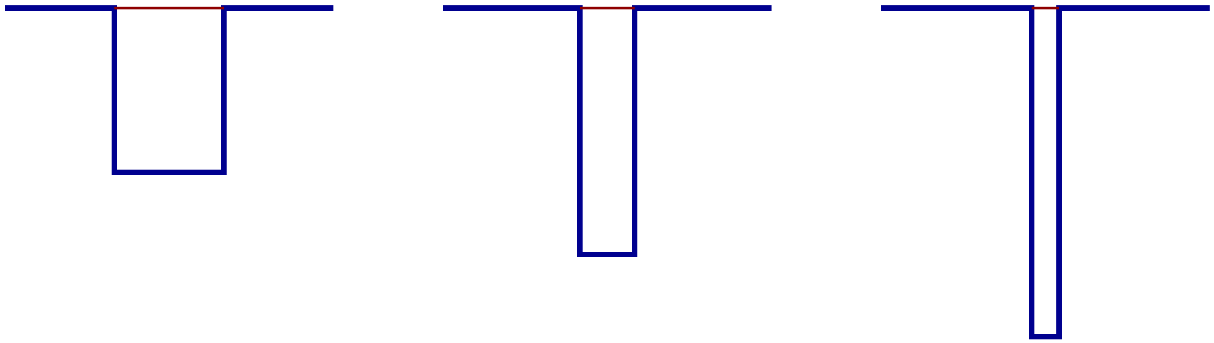
Two body interaction: Consider simple square well potential



$a < 0$ $\qquad$ $a = \infty,\ \epsilon_B = 0$ $\qquad$ $a > 0,\ \epsilon_B > 0$

Non-relativistic fermions in unitarity limit

Now take the range to zero, keeping $\epsilon_B \simeq 0$



$$\mathcal{T} = \frac{1}{ik}$$

The "Unitarity" Limit

Fermi gas at unitarity: Field Theory

Non-relativistic fermions at low momentum

$$\mathcal{L}_{\rm eff} = \psi^\dagger \Big(i\partial_0 + \frac{\nabla^2}{2M} \Big) \psi - \frac{C_0}{2} (\psi^\dagger\psi)^2$$

Unitarity limit: $a \to \infty$ (DR: $C_0 \to \infty$)

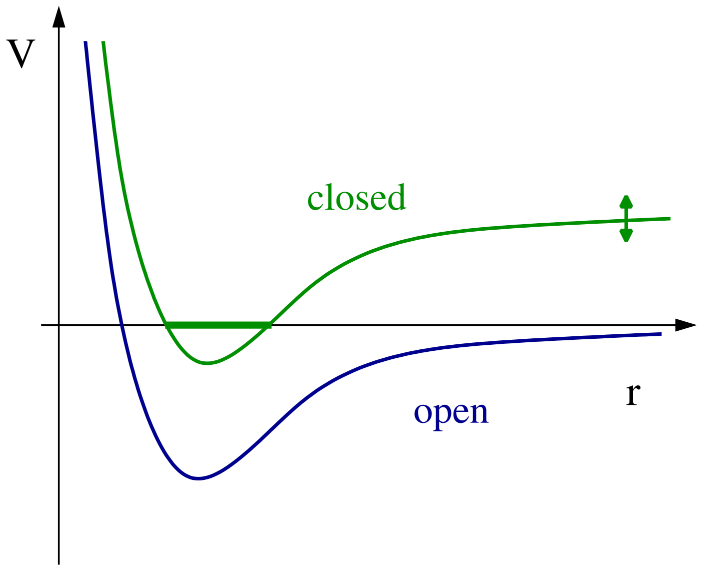
This limit is smooth: HS-trafo, define $\Psi = (\psi_\uparrow, \psi_\downarrow^\dagger)$

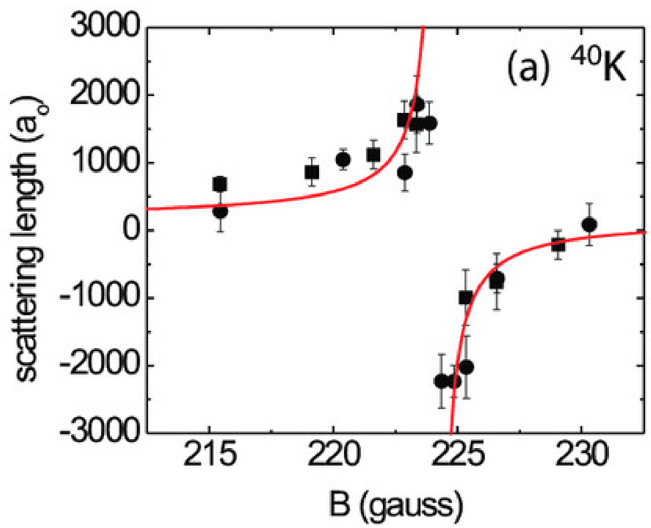
$$\mathcal{L} = \Psi^\dagger \left[i\partial_0 + \sigma_3 \frac{\vec{\nabla}^2}{2m} \right] \Psi + \Big(\Psi^\dagger \sigma_+ \Psi \phi + h.c. \Big) - \frac{1}{C_0} \phi^* \phi \ ,$$

$\phi \sim C_0 \psi_\uparrow \psi_\downarrow$ auxiliary "pair" or "dimer" field.

Experimental realization: Feshbach resonances

Atomic gas with two spin states: "↑" and "↓"

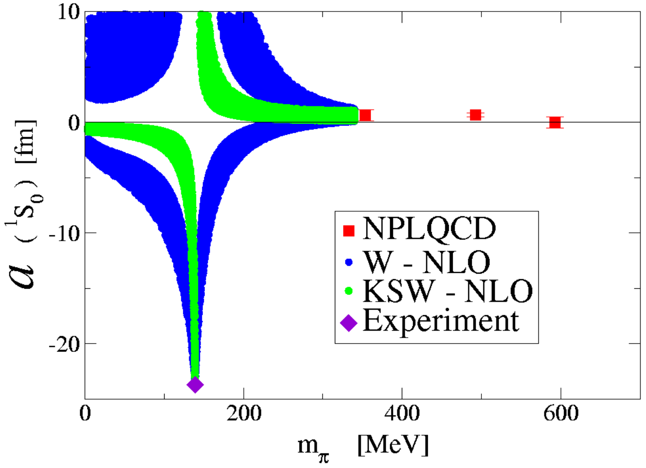




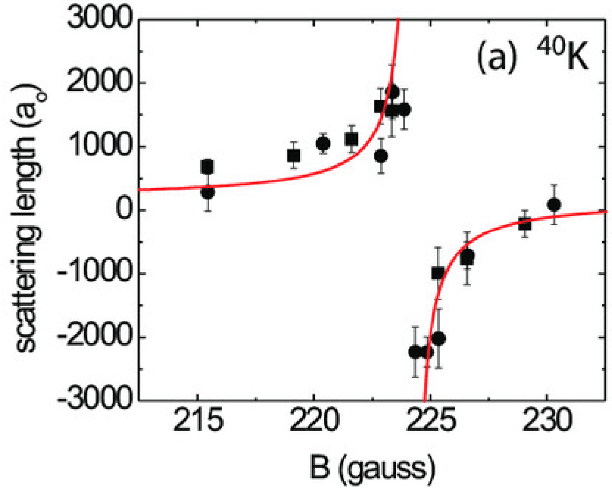
Feshbach resonance $$a(B) = a_0\left(1 + \frac{\Delta}{B - B_0}\right)$$

Universality: From neutrons to atoms

Neutron Matter



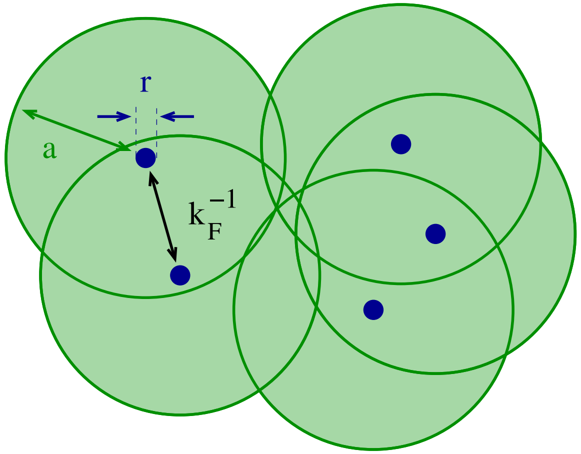
^{40}K Feshbach resonance



What do these systems have in common?

dilute: $r\rho^{1/3} \ll 1$

strongly correlated: $a\rho^{1/3} \gg 1$



Outline

1. Equation of state: From trapped atoms to neutron stars.

2. The contact: From the tail of the momentum distribution to short range correlations in nuclei.

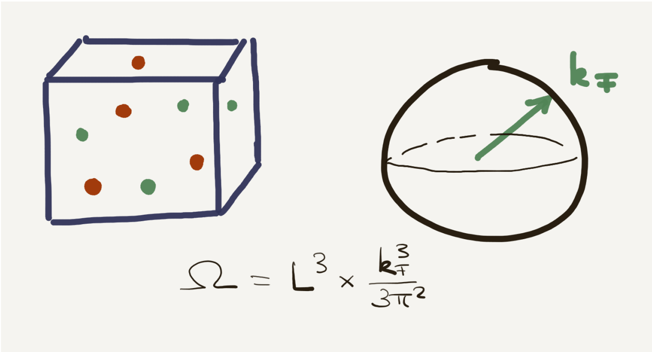
3. Mind the tail: Response functions for ν-scattering.

4. Un-nuclear physics: From trapped few-body systems to the disintegration of halo nuclei.

5. Everything flows: Elliptic flow from traps to heavy ions.

1. Equation of state

Free Fermi gas at zero temperature



$$\frac{E}{N} = \frac{3}{5}\frac{k_F^2}{2m} \qquad \frac{N}{V} = \frac{k_F^3}{3\pi^2}$$

$$\mathcal{E} = E/V \sim (N/V)^{5/3}$$

Unitarity limit ($a \to \infty$, $r \to 0$). No dimensionful parameters.

$$\frac{E}{N} = \xi\,\frac{3}{5}\frac{k_F^2}{2m} \qquad k_F \equiv (3\pi^2 N/V)^{1/3}$$

Prize problem (George Bertsch, 1998): Determine ξ.

Is $\xi > 0$ (is the system stable)?

How to measure ξ with trapped atoms

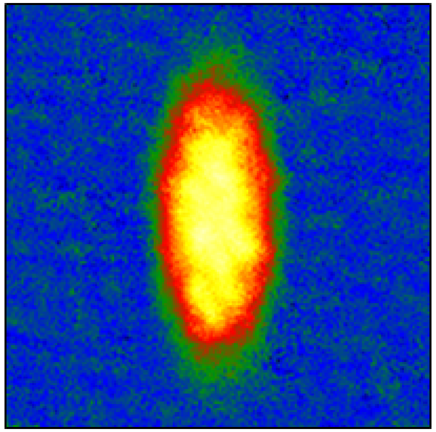
Trapped gas in hydrostatic equilibrium

$$\frac{1}{n}\vec{\nabla}P = -\vec{\nabla}V_{ext} \qquad P = \frac{2}{3}\mathcal{E}$$

Pressure determines size of the cloud ($V_{ext} = \frac{1}{2}m\omega^2 x^2$).

$$r(a=0) = \sqrt{\frac{2E_F}{m\omega^2}} \qquad r(a=\infty) = \xi^{1/4} r(0)$$

Cloud size can be measured with a CCD camera and a ruler

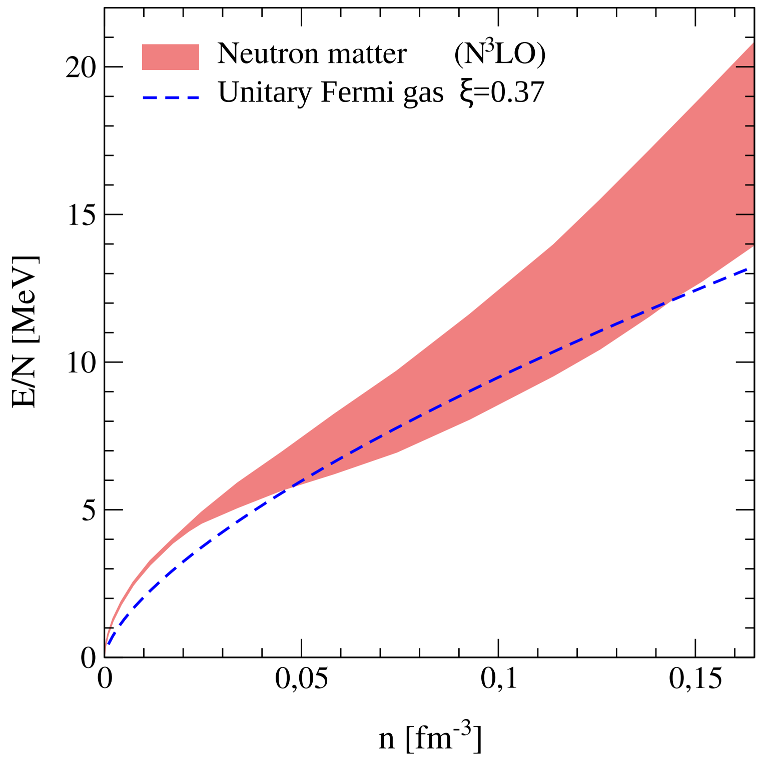


modern value

$$\xi = 0.37(5)$$

(MIT, Sommer et al.)

Neutron matter equation of state



Schaefer, Schwenk

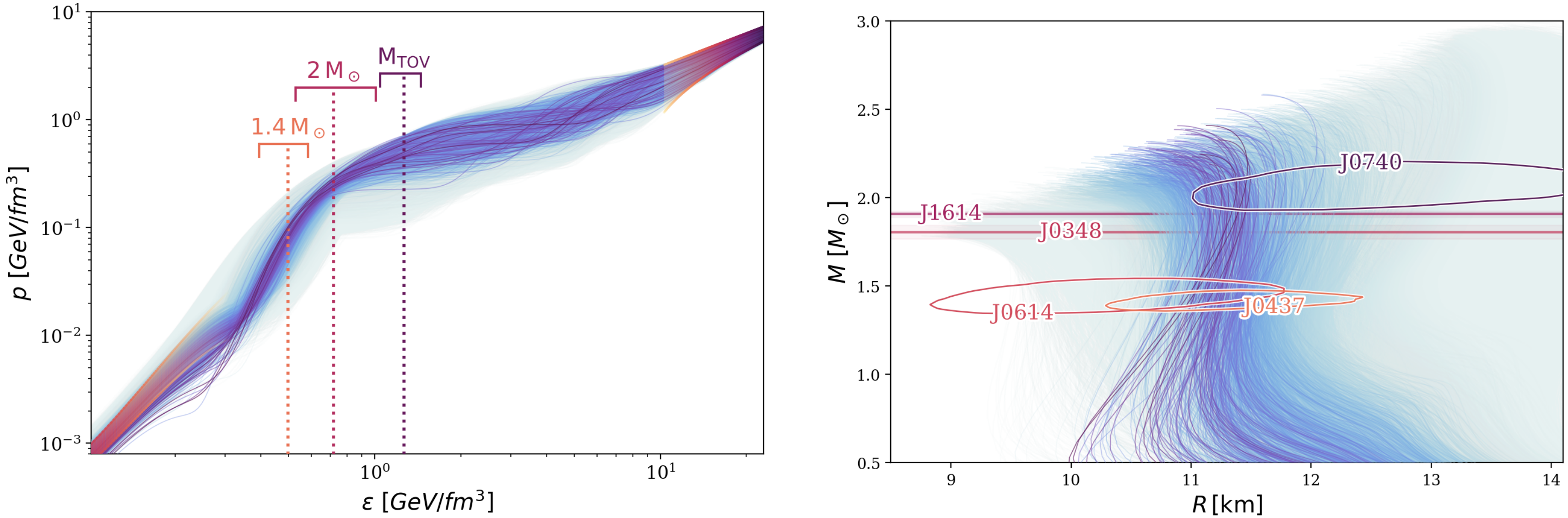
Physik Journal (2014)

$n \lesssim 0.1\,\mathrm{fm}^{-3}$: Unitary gas with a^{-1}, r corrections.

$n \gtrsim 0.1\,\mathrm{fm}^{-3}$: Repulsive 2-body, 3-body forces.

$n \gtrsim 0.2\,\mathrm{fm}^{-3}$: New degrees of freedom.

Neutron Matter EOS and Mass-Radius relation



Use low (nuclear) and high (pQCD) density EOS as priors.
Constrain intermediate densities by observation (NICER etc.).

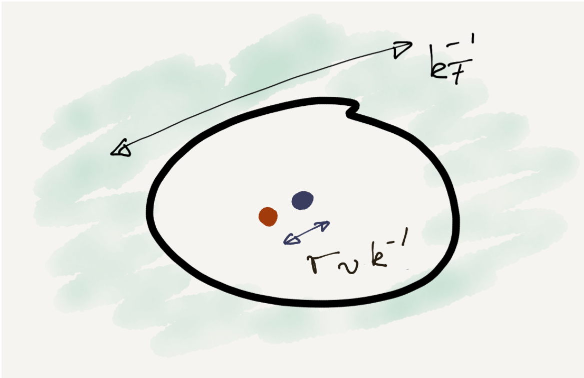
Gorda et al. 2512.18044.

2. Short range correlations and the "Contact"

Consider short distance structure of unitary gas

$$\langle n_\uparrow(R+r/2)n_\downarrow(R-r/2)\rangle \simeq \frac{1}{16\pi^2}\frac{\mathcal{C}}{r^2}$$

$\mathcal{C}$: *Tan's contact density*



Related object: Momentum distribution

$$n_\sigma(k) = \int d^3R \int d^3r\, e^{-ikr}\langle \psi_\sigma^\dagger(R-r/2)\psi_\sigma(R+r/2)\rangle$$

The large momentum (short distance) tail of the distribution is

$$n_\sigma(k) = \frac{C}{k^4} \qquad\qquad C = \int d^3r\, \mathcal{C}(r) \quad \textit{Contact}$$

Shina Tan, cond-mat/0505200.

Short range correlations, continued

The contact is related to the the pair density

$$\mathcal{C} = \langle m^2 \phi^\dagger \phi \rangle \qquad \phi \sim C_0 \psi_\uparrow \psi_\downarrow$$

Many universal relations. Example: Thermodynamics

$$\left.\frac{dE}{da^{-1}}\right|_s = -\frac{h^2 \mathcal{C}}{4\pi m}$$

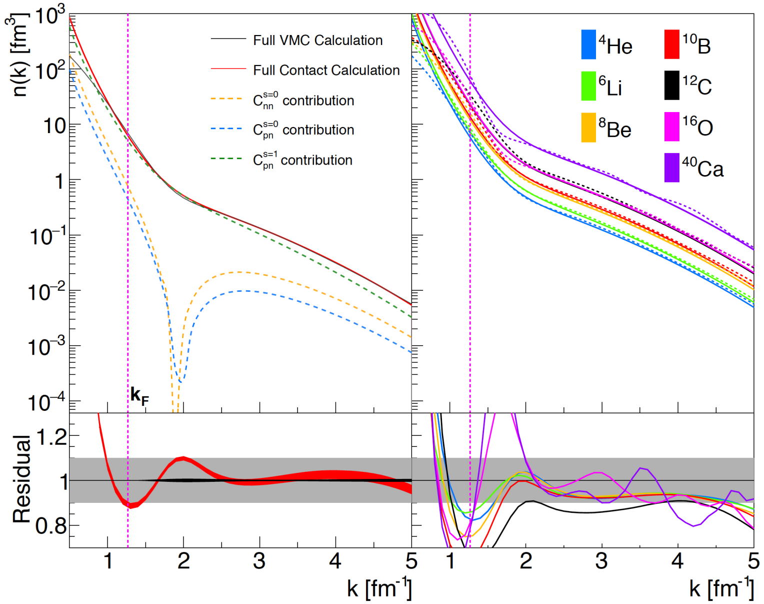
At high T the RHS can be computed using the Virial expansion

$$\mathcal{C} = \frac{8\pi^2 n^2}{mT}$$

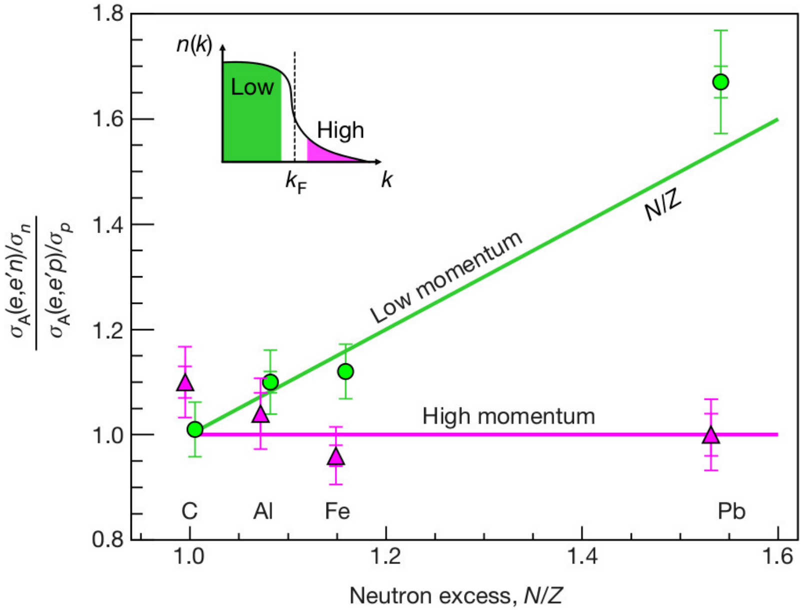
Momentum distribution $n_\sigma \sim C/k^4$ fixed by EOS.

For $T = 0$ have $\mathcal{C} = \frac{2\zeta}{5\pi}(3\pi^2 n)^{4/3}$ with $\zeta \simeq 0.95$.

Short range correlations and the contact in nuclei



Momentum distribution in nuclei (theory)
Weiss et al. 1612.00923



Pair density in nuclei
CLAS collaboration
Nature (2018)

3. Structure Functions

Structure functions govern neutrino NC/CC current interactions

$$\frac{d^2\sigma_{NC}}{d\cos\theta d\omega} = G_F^2\left[c_V f_V(\theta) S(\omega,q) + c_A f_A(\theta) S_\sigma(\omega,q)\right]$$

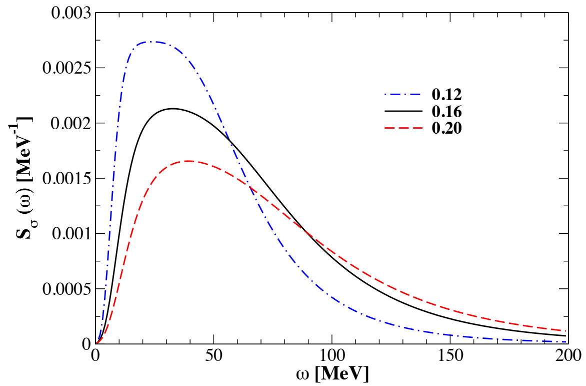
$$\frac{d^2\sigma_{CC}}{d\cos\theta dE_e} = G_F^2\left[f_V(\theta) S_\tau(\omega,q) + g_A f_A(\theta) S_{\tau\sigma}(\omega,q)\right]$$

$$S_O = \int dt\, dx\, e^{i\omega t - iqx}\langle \rho_O(x,t)\rho_O(0,0)\rangle\,, \quad \rho_A = \psi^\dagger O \psi,$$

Need response at fairly large frequency, e.g. energy loss rate

$$n + n \to n + n + \nu\bar{\nu}$$

$$Q \sim G_F^2 \int d\omega\, \omega^6 e^{-\omega/T} S_\sigma(\omega)$$



Shen et al., 1205.6499

Short range correlations

High frequency behavior (at fixed $y = \omega/q^2$) controlled by OPE

$$A(x,t)B(0,0) = \sum_i |x|^{\Delta_i - \Delta_A - \Delta_B} f_i(|x^2|/t) O_i(0)$$

Single particle response (RF spectroscopy)

$$I(\omega) = -\frac{1}{\pi} Im G(\omega) = \frac{\langle \phi^\dagger \phi \rangle}{4\pi^2} \frac{1}{\omega^{3/2}}$$

Structure factor (density)

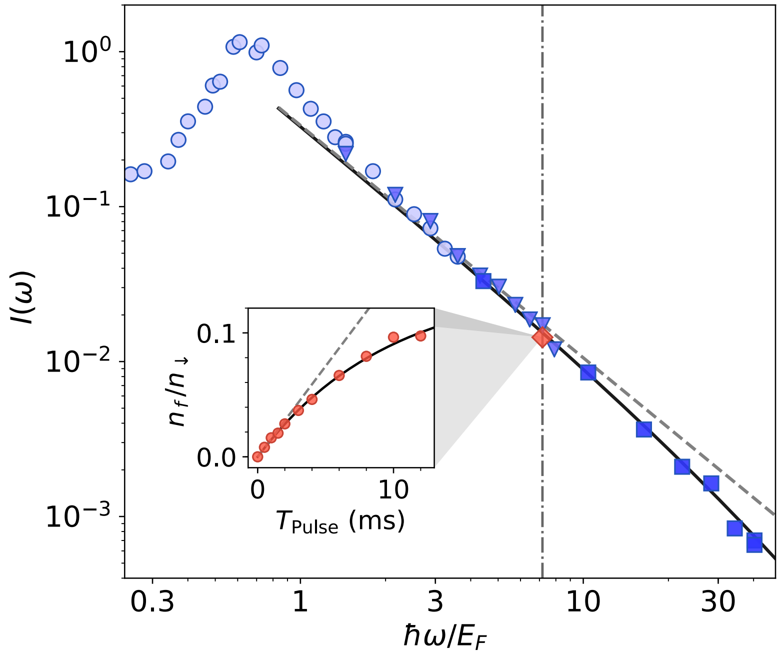
$$S(\omega, q) = \frac{4\langle \phi^\dagger \phi \rangle}{45\pi^2 m} \frac{q^4}{\omega^{7/2}}$$

Structure factor (spin density)

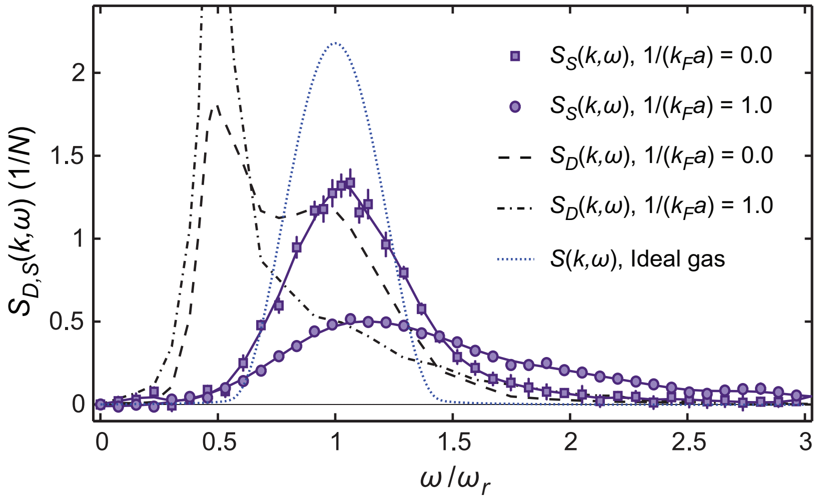
$$S_\sigma(\omega, q) = \frac{\langle \phi^\dagger \phi \rangle}{3\pi^2 m^{1/2}} \frac{q^2}{\omega^{5/2}}$$

Son, Thompson, 1002.0922. Hu, Liu, 1101.1134

Measurements of the response in cold gases



RF spectroscopy



Spin response

Mukherjee et al. (MIT), 2019. Vale et al. (Swinburne), 2012.

4. Conformal symmetry and Un-nuclear physics

Unitary Fermi gas is invariant under scale

$$x \to sx, \quad t \to s^2 t \qquad [D,H] = 2iH \qquad D = \int dx\, x \cdot \jmath$$

and conformal transformations

$$x \to \frac{x}{1+ct}, \quad t \to \frac{t}{1+ct} \qquad [C,H] = iD \qquad C = \int dx\, \frac{x^2}{2}\, n$$

Constrains correlation functions, e.g. pair propagator

$$G_\Phi(\omega, p) = \frac{1}{\sqrt{p^2/(4m) - \omega}}$$

Conformal symmetry: State operator correspondence

Generalized to operators $\mathcal{U}_n(t,x)$ with higher mass $M = nm$

$$i\langle T\mathcal{U}_n^\dagger \mathcal{U}_n\rangle_{\omega,p} = \left(\frac{p^2}{2M} - \omega\right)^{\Delta_n - 5/2} \qquad \textit{conformal dimension } \Delta_n$$

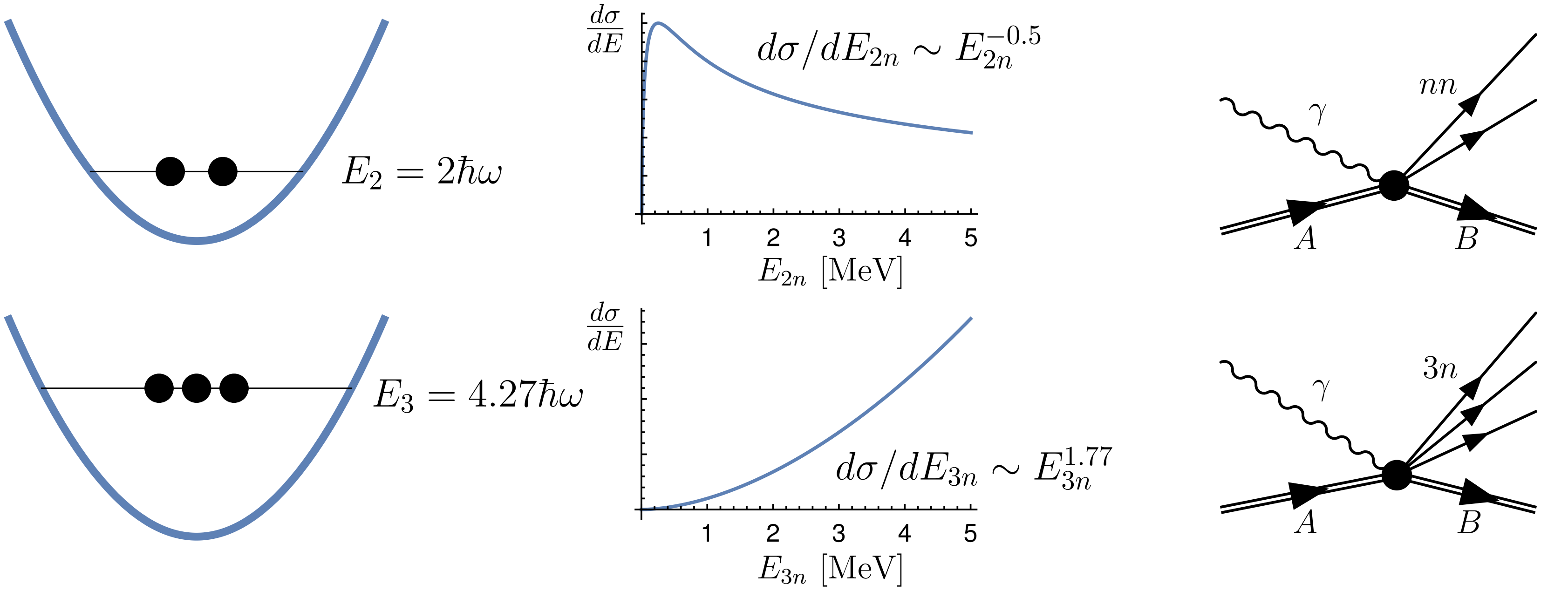
Δ_n related to ground state in harmonic potential

$$E_n = \Delta_n \hbar\omega \qquad \textit{state} - \textit{operator correspondence}$$

E.g single free particle: $\Delta_1 = 3/2$

Hammer, Son, 2103.12610. Baym, Schaefer, 2109.06924.

Un-nuclear physics: Nuclear reactions



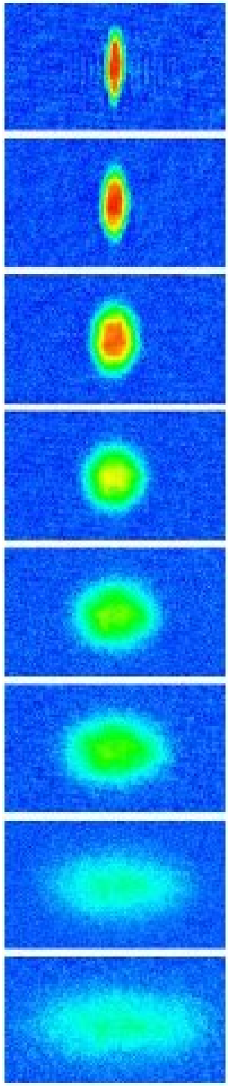
$$x + A \to B + Nn$$

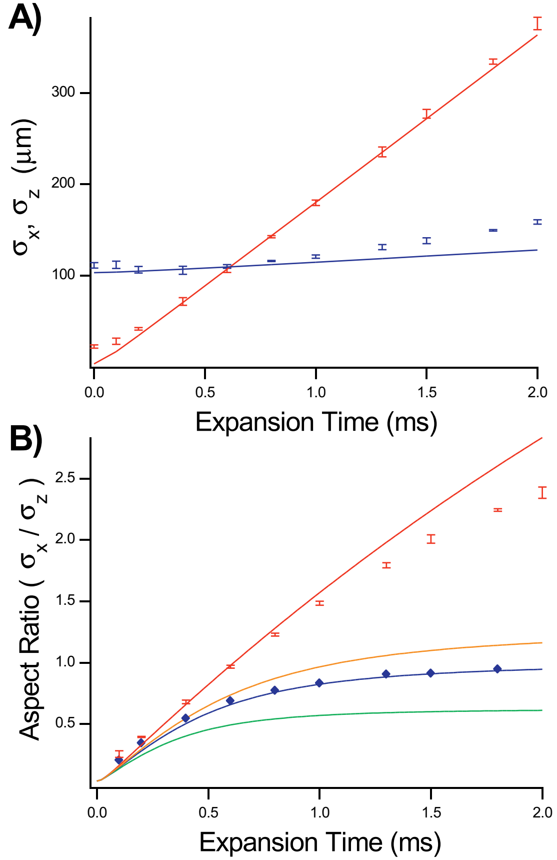
$$\frac{d\sigma}{dE} \sim E^{\Delta_n - 5/2}$$

$$[(ma^2)^{-1} \sim 0.5\, MeV] < E < [(mr^2)^{-1} \sim 5\, MeV]$$

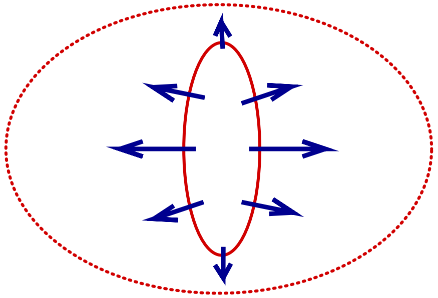
Microscopic calculations: Backert et al., 2601.19408

5. Elliptic flow in the unitary Fermi gas





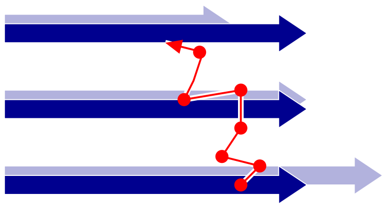
Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy



O'Hara et al. (2002)

Shear viscosity: Theory

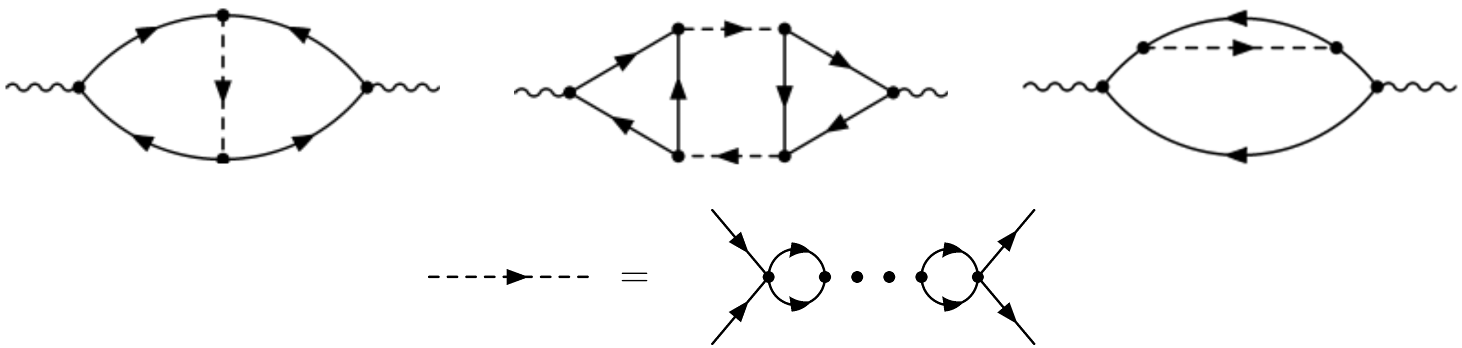
Kinetic theory: Momentum transport by diffusion of atoms



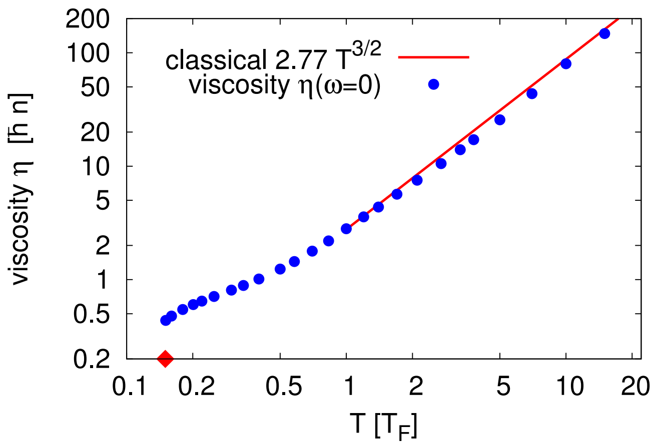
$$\eta = \frac{15}{32\sqrt{\pi}}(mT)^{3/2} \quad (T \gtrsim T_F)$$

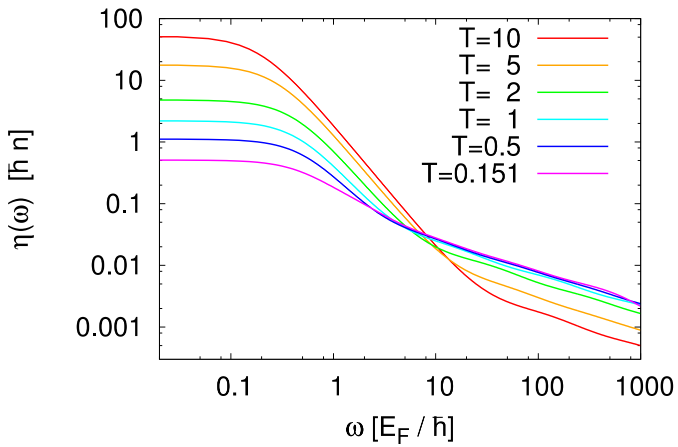
QFT: Diagrammatic content of Boltzmann equation. Kubo formula with Maki-Thompson + Azlamov-Larkin + Self-energy

$$\eta = -\lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}\int dt d^3x\, e^{-i(\omega t - kx)}\,\Theta(t)\langle[\Pi_{xy}(0),\Pi_{xy}(t,x)]\rangle$$



Can be used to extrapolate kinetic theory to $T \sim T_F$



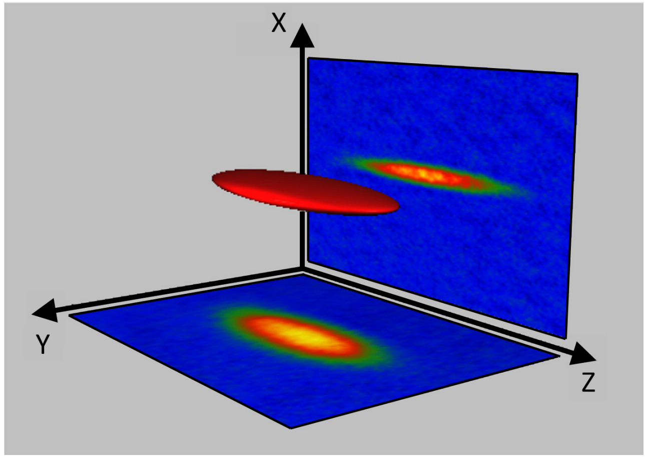


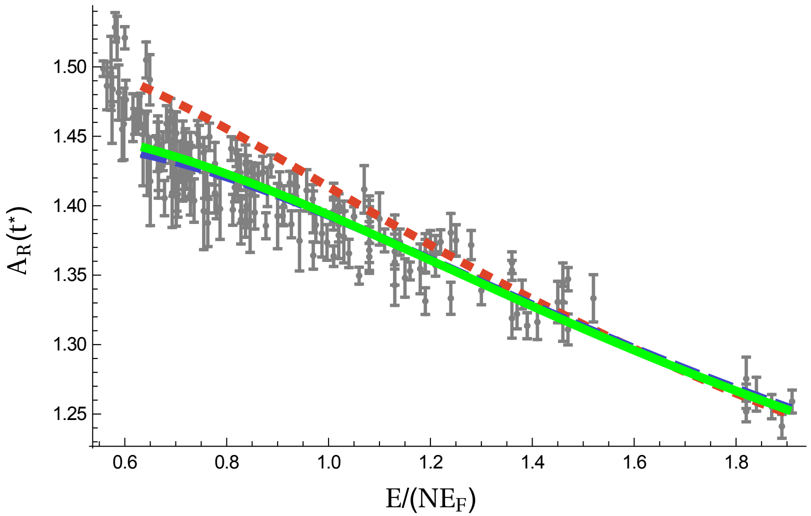
$$\eta(T \sim T_c) \sim \hbar n$$

Drude peak, universal tail.

Enss, Zwerger (2011), see also Levin (2014)

Shear viscosity: Phenomenology



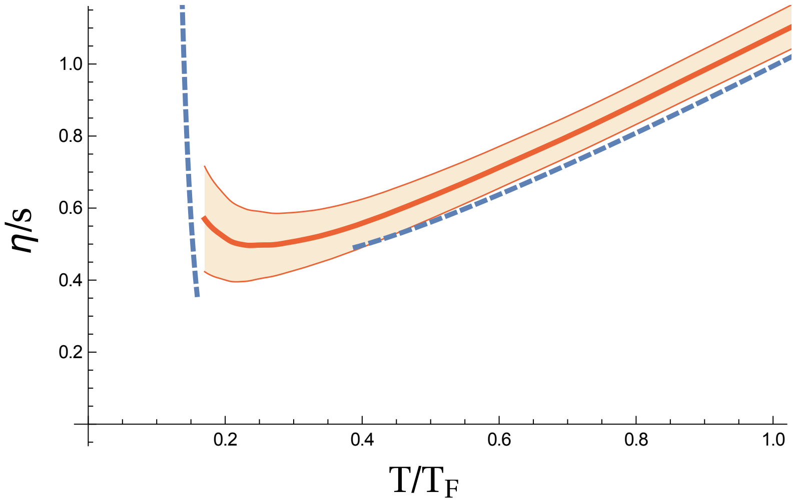


$A_R = \sigma_x/\sigma_y$ as function of total energy. Data: Joseph et al (2016). $E/(NE_F) \sim 0.6$ is the superfluid transition.

Grey, Blue, Green: LO, NLO, NNLO fit.

$$\eta = \eta_0 (mT)^{3/2} \left\{ 1 + \eta_2 n\lambda^3 + \eta_3 (n\lambda^3)^2 + \ldots \right\}$$

Reconstruct η/s (normal fluid)



$T_c \sim 0.17 T_F$. Kinetic theory at low and high T (blue dashed)

Validate method at high temperature:

$$\eta|_{T \gg T_c} = (0.265 \pm 0.02)(mT)^{3/2}$$

$$\eta_0(th) = \frac{15}{32\sqrt{\pi}} = 0.269$$

Phenomenology: Two component model works well, $\eta \sim \eta_0 T^{3/2} + \eta_1 n$

$$\eta/s|_{T_c} = 0.56 \pm 0.20$$

Final thoughts

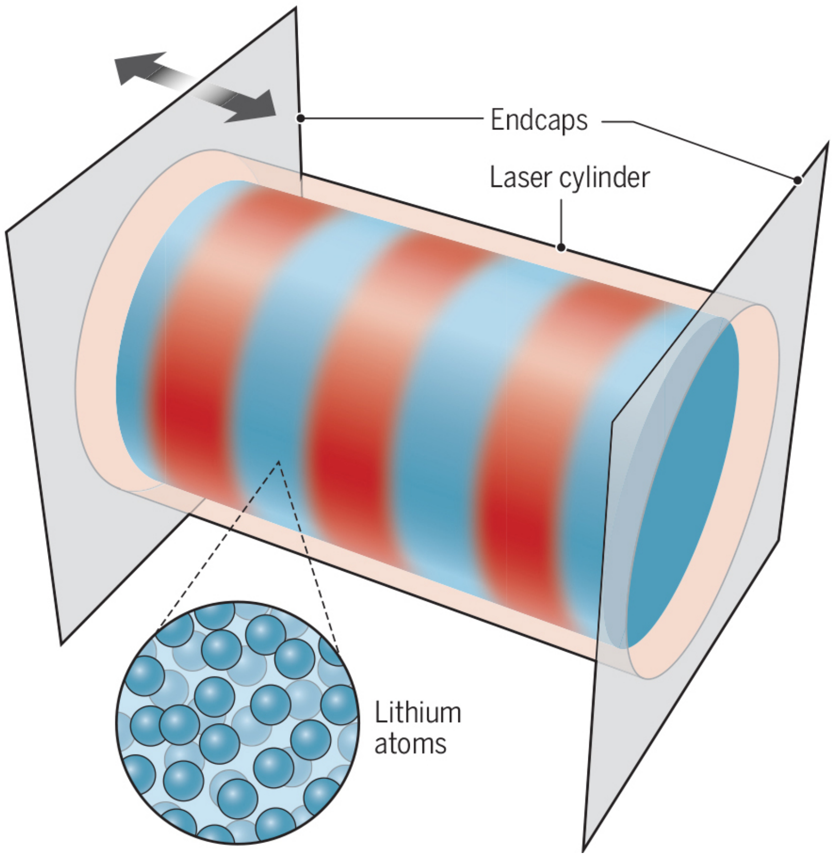
The unitary Fermi gas has become a paradigm for strongly correlated quantum liquids.

Universality relates the cold atomic gas to dilute neutron matter. Important for understanding analytic aspects, and as a benchmark for quantum Monte Carlo calculations.

Range of ideas continues to expand: From thermodynamics to transport, short range correlations and un-nuclear reactions.

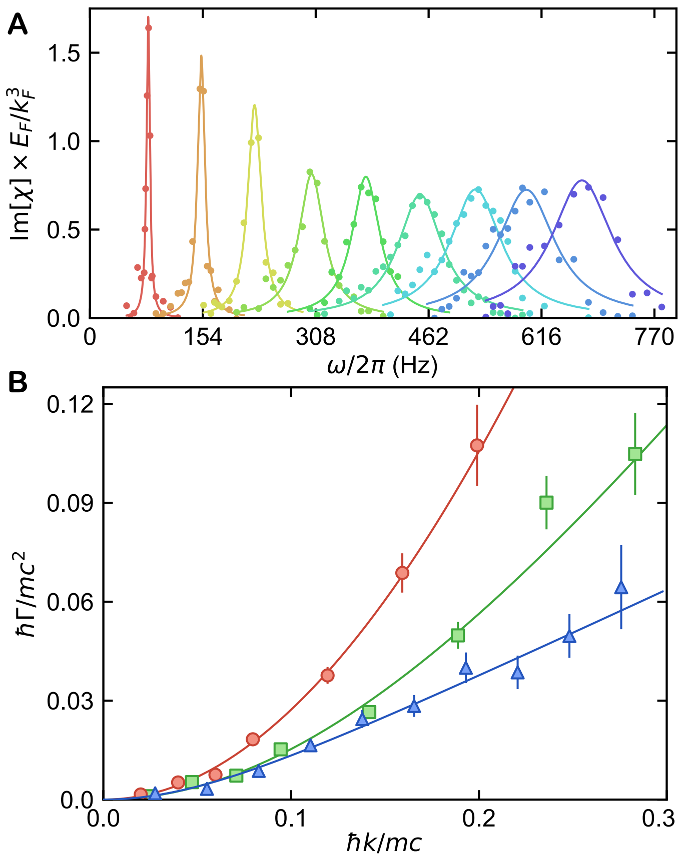
Continued role for ultracold gases as quantum simulators, not just universal gate based quantum computers.

Sound attenuation (MIT)



Sound attenuation (MIT)

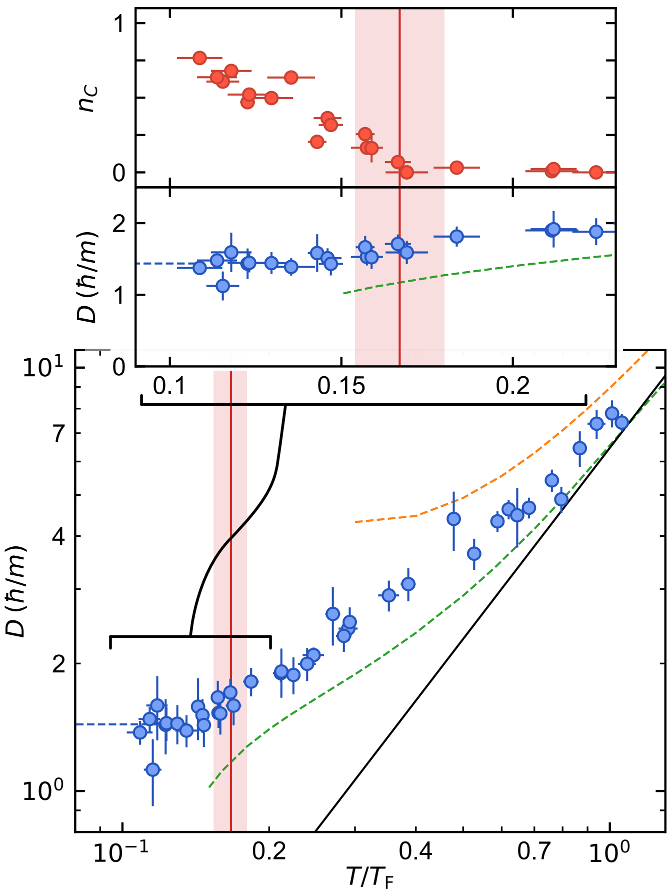
Spectral response $\rho_k(\omega)$.



Damping rate $\Gamma(k)$

$(T/T_F = 0.36, 0.21, 0.13)$.

Sound diffusivity $D_s(T)$



$$D_s = \frac{4\eta}{3\rho} + \frac{4\kappa T}{15P}.$$

Patel et al., Science (2021)