Nuclear theory for Majorana (neutrinoless) $\beta\beta$ decay

Javier Menéndez

University of Barcelona
Institute of Cosmos Sciences

International School of Nuclear Physics, 47th Course:
Neutrinos in Cosmology, in Astro-, Particle- and Nuclear Physics

Erice, 21st September 2026

UNIVERSITAT DE BARCELONA
MINISTERIO DE CIENCIA, INNOVACIÓN Y UNIVERSIDADES
Financiado por la Unión Europea NextGenerationEU
Plan de Recuperación, Transformación y Resiliencia
AGENCIA ESTATAL DE INVESTIGACIÓN
ICCUB

Outline

Majorana $\beta\beta$ decay ($0\nu\beta\beta$)

Nuclear matrix elements for light-neutrino-exchange $0\nu\beta\beta$ decay

Majorana $\beta\beta$ decay NMEs with uncertainties

Creation of matter in nuclei: $0\nu\beta\beta$ decay

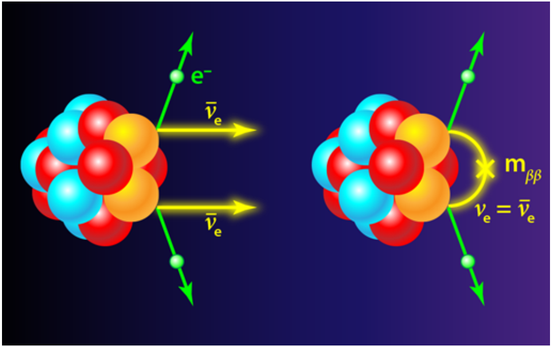
Lepton number conserved in all processes observed:

single β decay,
$\beta\beta$ decay with ν emission...

Neutral massive particles (Majorana ν's) allow lepton number violation:

neutrinoless $\beta\beta$ decay
creates two matter particles (electrons)

Agostini, Benato, Detwiler, JM, Vissani, Rev. Mod. Phys. 95, 025002 (2023)



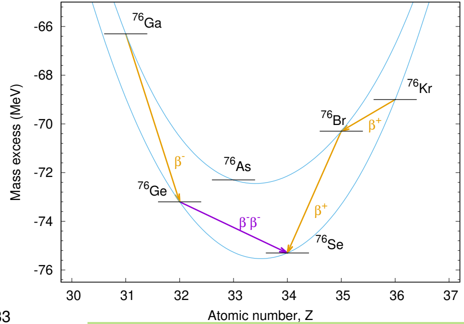
$\beta\beta$ decay

Second order process in the weak interaction

Only observable in nuclei where (much faster) β-decay is forbidden energetically due to nuclear pairing interaction

$$BE(A) = -a_v A + a_s A^{2/3} + a_c \frac{Z(Z-1)}{A^{1/3}} + \frac{(A-2Z)^2}{A} + \begin{cases} -\delta_{pairing} & \text{N,Z even} \\ 0 & \text{A odd} \\ \delta_{pairing} & \text{N,Z odd} \end{cases}$$

or where β-decay is very suppressed by ΔJ angular momentum change



$^{48}Ca \rightarrow {}^{48}Ti$

$^{76}Ge \rightarrow {}^{76}Se$

$^{82}Se \rightarrow {}^{82}Kr$

$^{96}Zr \rightarrow {}^{96}Mo$

$^{100}Mo \rightarrow {}^{100}Ru$

$^{110}Pd \rightarrow {}^{110}Cd$

$^{116}Cd \rightarrow {}^{116}Sn$

$^{124}Sn \rightarrow {}^{124}Te$

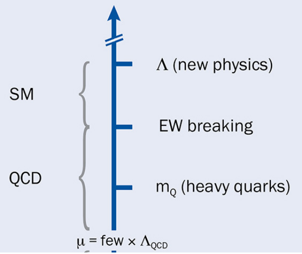
$^{130}Te \rightarrow {}^{130}Xe$

$^{136}Xe \rightarrow {}^{136}Ba$

$^{150}Nd \rightarrow {}^{150}Sm$

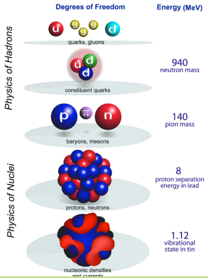
Scales in new-physics searches using nuclei

New physics scale: $\Lambda \gg 250$ GeV



Electroweak scale:

$$v = \left(\sqrt{2}G_F\right)^{-1/2} \sim 250 \text{ GeV}$$



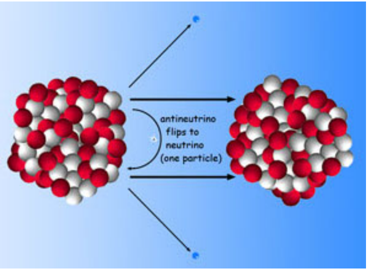
QCD (hadron) scale: $m_N \sim$ GeV

Nuclear scale: $k_F \sim m_\pi \sim 200$ MeV

Nuclear matrix elements for new-physics searches

Neutrinos, many other beyond-standard-model physics studied in experiments using nuclei

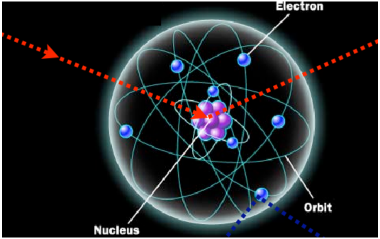
Nuclear structure encoded in nuclear matrix elements ($M^{0\nu\beta\beta}$, $\mathcal{F}_i$) key to plan, fully exploit experiments



$$0\nu\beta\beta\text{: } \left(T_{1/2}^{0\nu\beta\beta}\right)^{-1} \propto g_A^4 \left|M^{0\nu\beta\beta}\right|^2 m_{\beta\beta}^2$$

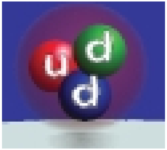
$$\text{Dark matter: } \frac{d\sigma_{\chi\mathcal{N}}}{d\boldsymbol{q}^2} \propto \left|\sum_i c_i\,\zeta_i\,\mathcal{F}_i\right|^2$$

$$\text{CE}\nu\text{NS: } \frac{d\sigma_{\nu\mathcal{N}}}{d\boldsymbol{q}^2} \propto \left|\sum_i c_i\,\zeta_i\,\mathcal{F}_i\right|^2$$



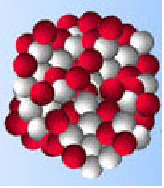
Electric dipole moments...

Particle, hadron and nuclear physics



Majorana $\beta\beta$ decay
interplay of particle, hadron and nuclear physics:

ν's: interaction with quarks, fundamental bosons
Quarks and gluons: embedded in the nucleon
Nucleons: form complex, many-nucleon nuclei



General $\beta\beta$ half-life:

$$\left(T_{1/2}^{0\nu\beta\beta}\right)^{-1} = G^{0\nu} g_A^4 \left|M^{0\nu\beta\beta}\right|^2 m_{\beta\beta}^2$$

$G^{0\nu}$: kinematics ($e^-, \cdots$)

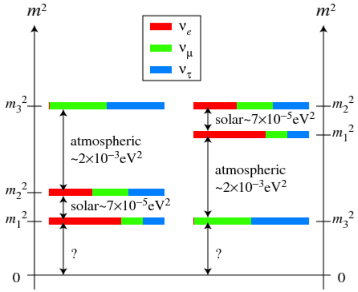
$m_{\beta\beta}$, g_A:
ν masses and mixing matrix elements, particle physics
proportional to g_A, hadron matrix elements, hadron physics

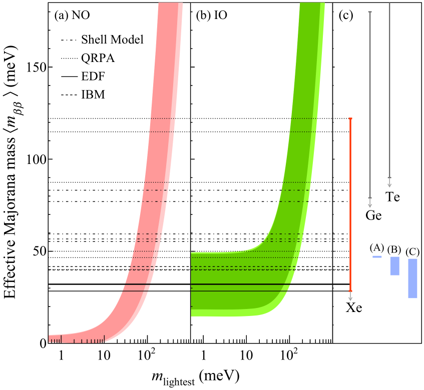
$M^{0\nu\beta\beta}$: nuclear structure physics

Next generation experiments: inverted hierarchy

Decay rate sensitive to
neutrino masses, hierarchy
$m_{\beta\beta} = |\sum U_{ek}^2 m_k|$

$$\left(T_{1/2}^{0\nu\beta\beta}\right)^{-1} = G_{0\nu}\, g_A^4 \left|M^{0\nu\beta\beta}\right|^2 m_{\beta\beta}^2$$

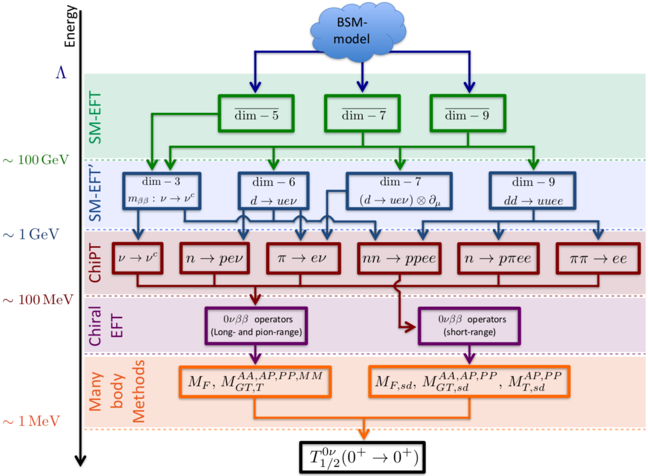




Matrix elements assess
if next-generation experiments
fully cover "inverted hierarchy"

KamLAND-Zen, PRL135 262501(2025)

Effective field theory for $0\nu\beta\beta$ decay



Cirigliano et al. JHEP 12 097 (2018)

$0\nu\beta\beta$ decay half-life

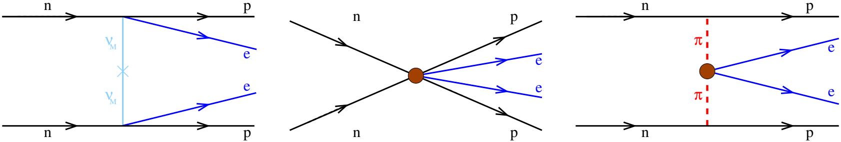
Half-life of $0\nu\beta\beta$ decay sensitive to
$m_{\beta\beta} \sim 1/\Lambda$ (dim-5 operator), new-physics scales $\widetilde{\Lambda}$ (dim-7) or $\widetilde{\Lambda}'$ (dim-9)

$$T_{1/2}^{-1} = G_{01}\, g_A^4 \left(M_{\text{light}}^{0\nu}\right)^2 m_{\beta\beta}^2 + m_N^2 \tilde{G}\, \tilde{g}^4\, \tilde{M}^2 \left(\frac{v}{\widetilde{\Lambda}}\right)^6 + \frac{m_N^4}{v^2} \widetilde{G}'\, \widetilde{g}'^4\, \widetilde{M}'^2 \left(\frac{v}{\widetilde{\Lambda}'}\right)^{10}$$

G_{01}, $\widetilde{G}$, $\widetilde{G}'$: phase-space factors (electrons), very well known

g_A, g_ν^{NN}, $\widetilde{g}$, $\widetilde{g}'$: coupling to hadron(s), experiment or calculate with QCD

$M_{\text{long}}^{0\nu}$, $M_{\text{short}}^{0\nu}$, $\widetilde{M}$, $\widetilde{M}'$: nuclear matrix elements, many-body challenge



$0\nu\beta\beta$ decay mediated by sterile neutrinos

- Dependence of the NME on m_ν:

$$H^X(r) = \frac{2}{\pi}\frac{R}{g_A^2}\int_0^\infty f^X(pr)\frac{h^X(p^2)}{\left(\sqrt{p^2+m_\nu^2}\right)\left(\sqrt{p^2+m_\nu^2}+\langle E^m\rangle-\frac{1}{2}(E_i-E_f)\right)}p^2\,dp$$

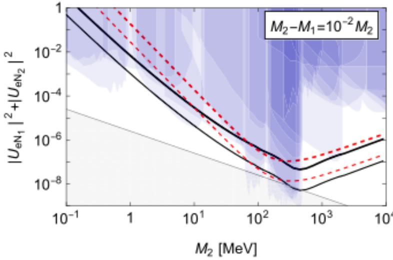
- Dependence of two-body coupling $g_\nu^{\rm NN}$ on m_ν
- $m_\nu \gtrapprox 1$ GeV, integrate ν's out at the quark level, match to lepton-number-violating operator

Cancellation: $\sum_{i=1}^{n+3} U_{ei}^2 m_i = (M_\nu)^*_{ee} = 0$

Blennow et al. JHEP 07 096 (2010)

Drives subleading (N^{2}LO) NME
ultrasoft ν's: low-momentum $p \ll m_\pi$
dominant because of milder scaling

Dekens et al. PRC 108 045501 (2023)
Dekens et al. JHEP 09 201 (2024)



$0\nu\beta\beta$ mediated by new-physics heavy particles

Stantard Model extensions
trigger $0\nu\beta\beta$ decay (heavy ν, M_R...)

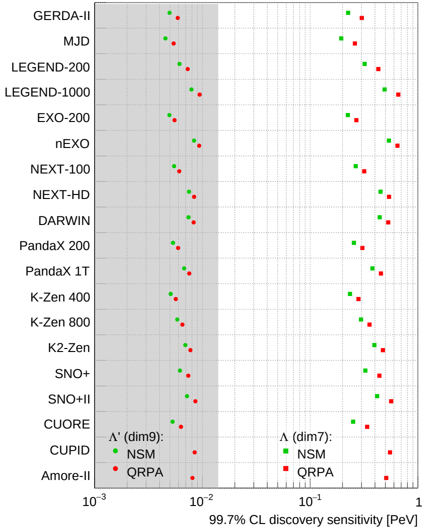
Phase-space,
hadronic/nuclear matrix elements,
known or calculated

Effective field theory

Cirigliano et al JHEP 12 097 (2018)

dimension-7 ($\sim 1/\Lambda^3$),
dimension-9 ($\sim 1/\Lambda^5$) operators

constrained by current searches
$\Lambda \gtrsim 250$ TeV (dim-7)
$\Lambda \gtrsim 5$ TeV (dim-9)



Outline

Majorana $\beta\beta$ decay ($0\nu\beta\beta$)

Nuclear matrix elements for light-neutrino-exchange $0\nu\beta\beta$ decay

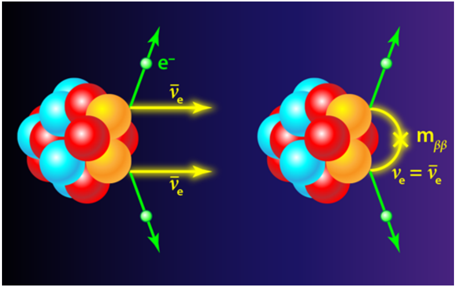
Majorana $\beta\beta$ decay NMEs with uncertainties

Nuclear matrix elements

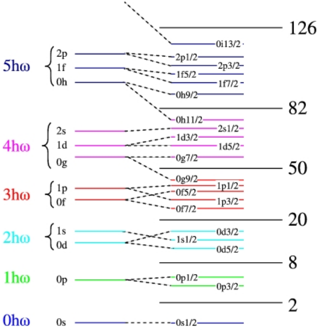
Nuclear matrix elements needed in low-energy new-physics searches

$$\langle\, \text{Final}\,|\mathcal{L}_{\text{leptons}-\text{nucleons}}|\,\text{Initial}\,\rangle = \langle\, \text{Final}\,|\int dx\, j^{\mu}(x) J_{\mu}(x)\,|\,\text{Initial}\,\rangle$$

- Nuclear structure calculation of the initial and final states:
  Shell model, QRPA, IBM,
  Energy-density functional
  Ab initio many-body theory
  QMC, Coupled-cluster, IMSRG...
- Lepton-nucleus interaction:
  Hadron current in nucleus:
  phenomenological,
  effective theory of QCD



Nuclear shell model



Only keep essential degrees of freedom

- High-energy orbitals: always empty
- Valence space: where many-body problem is solved
- Inert core: always filled

$$H|\Psi\rangle = E|\Psi\rangle \;\rightarrow H_{eff}|\Psi\rangle_{eff} = E|\Psi\rangle_{eff}$$

$$|\Psi\rangle_{eff} = \sum_\alpha c_\alpha |\phi_\alpha\rangle\,, \quad |\phi_\alpha\rangle = a^+_{i1}a^+_{i2}...a^+_{iA}|0\rangle$$

Shell model diagonalization:

$\sim 10^{12}$ Slater dets. Caurier et al. RMP77 (2005)

$\gtrsim 10^{30}$ Slater dets. variational

Otsuka, Shimizu, Tsunoda: PScr. 92 063001 (2017)
Dao, Nowacki: PRC 105, 054314 (2022)
Bally, Rodríguez: EPJA 60 62 (2024)

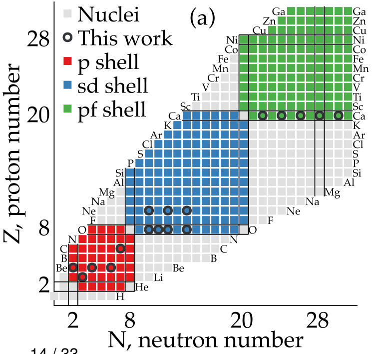
H_{eff} includes effects of

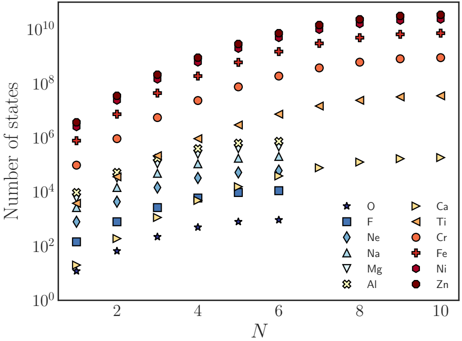
- inert core
- high-energy orbitals

Shell-model scaling

Even for medium-mass nuclei
shell-model calculations become demanding toward the mid-shell

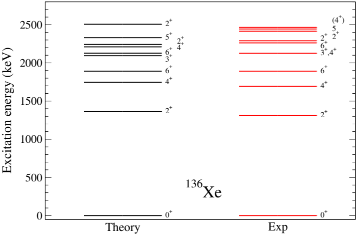
with number of Slater determinants (many-body basis states)
reaching $\sim 10^{10}$ for relatively simple nuclei

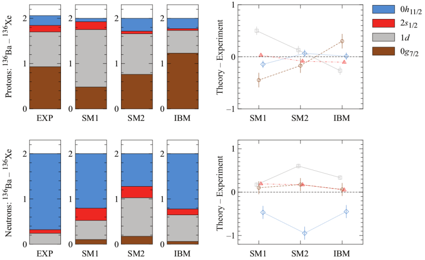


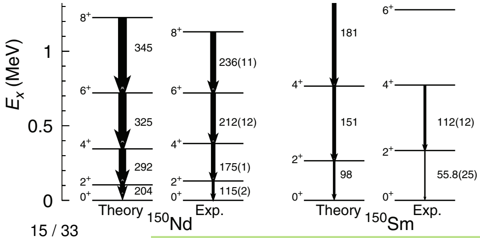


Tests of nuclear structure

Spectroscopy well described: masses, spectra, transitions, knockout...







Schiffer et al. PRL100 112501(2009)

Kay et al. PRC79 021301(2009)

...

Szwec et al., PRC94 054314 (2016)

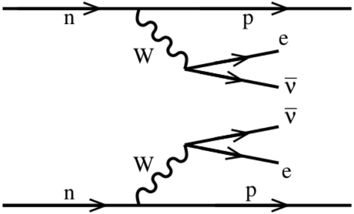
Rodríguez et al. PRL105 252503 (2010)

...

Vietze et al. PRD91 043520 (2015)

Two-neutrino $\beta\beta$ decay, 2νECEC

2$\nu\beta\beta$ decay same initial, final states
similar operator structure ($\sigma\tau$) as 0$\nu\beta\beta$
Comparison of predicted 2$\nu\beta\beta$ decay vs data



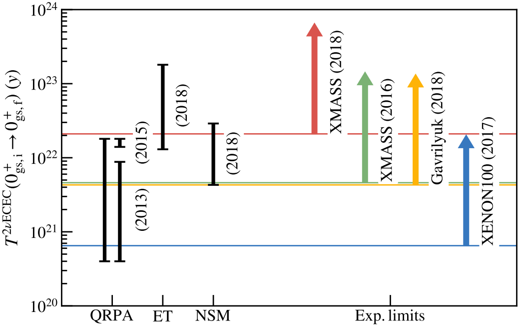
$$M^{2\nu\beta\beta} = \sum_k \frac{\langle 0_f^+ | \sum_n \sigma_n \tau_n^- | 1_k^+ \rangle \langle 1_k^+ | \sum_m \sigma_m \tau_m^- | 0_i^+ \rangle}{E_k - (M_i + M_f)/2}$$

$2\nu\beta\beta$ decay, 2νECEC of ^{124}Xe

Two-neutrino $\beta\beta$ predicted for ^{48}Ca before measurement

Caurier, Poves, Zuker, PLB 252 13(1990)

Recent predictions for 2νECEC ^{124}Xe half-life:
shell model error bar largely dominated by “quenching” uncertainty



Suhonen
JPG 40 075102 (2013)

Pirinen, Suhonen
PRC 91, 054309 (2015)

Coello Pérez, JM, Schwenk
PLB 797 134885 (2019)

Shell model, QRPA and Effective theory (ET) predictions suggest experimental detection close to XMASS 2018 limit

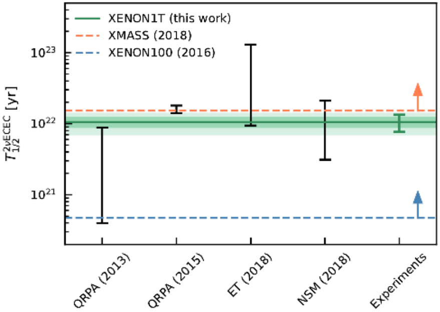
$2\nu\beta\beta$ decay, 2νECEC of ^{124}Xe

Two-neutrino $\beta\beta$ predicted for ^{48}Ca before measurement

Caurier, Poves, Zuker, PLB 252 13(1990)

Recent predictions for 2νECEC ^{124}Xe half-life:

shell model error bar largely dominated by “quenching” uncertainty



Suhonen
JPG 40 075102 (2013)

Pirinen, Suhonen
PRC 91, 054309 (2015)

Coello Pérez, JM, Schwenk

PLB 797 134885 (2019)

XENON1T
Nature 568 532 (2019)

PRC106, 024328 (2022)

Shell model, QRPA, Effective theory (ET)
good agreement with XENON1T measurement!

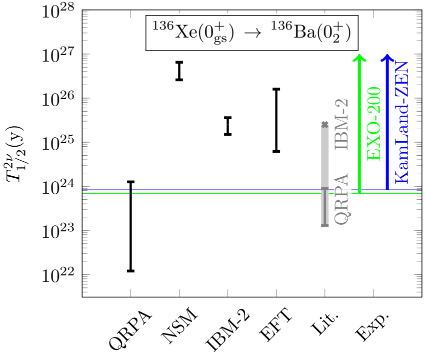
$2\nu\beta\beta$ decay of ^{136}Xe to ^{136}Ba 0_2^+

Current experiments sensitive to two-neutrino $\beta\beta$ of ^{136}Xe to ^{136}Ba 0_2^+

EXO-200: Chin. Phys. C 47 (2023)

KamLAND-Zen: Nucl. Phys. A 946 171 (2016)

PANDA-X: JHEP 05 089 (2025)



Nuclear shell model
QRPA, EFT and IBM
very different predictions!

Barea et al.
PRC 91 034304 (2015)

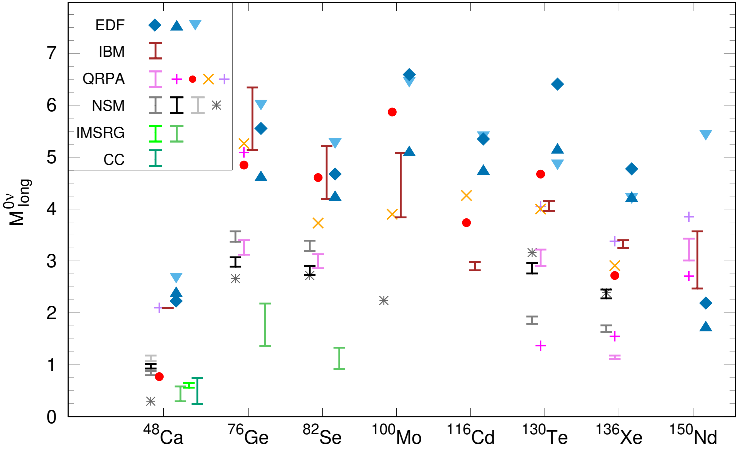
Pirinen, Suhonen
PRC 91, 054309 (2015)

Jokiniemi, Romeo, Brase, Kotila et al.
PLB 838 137689 (2023)

Very good test of theoretical calculations!

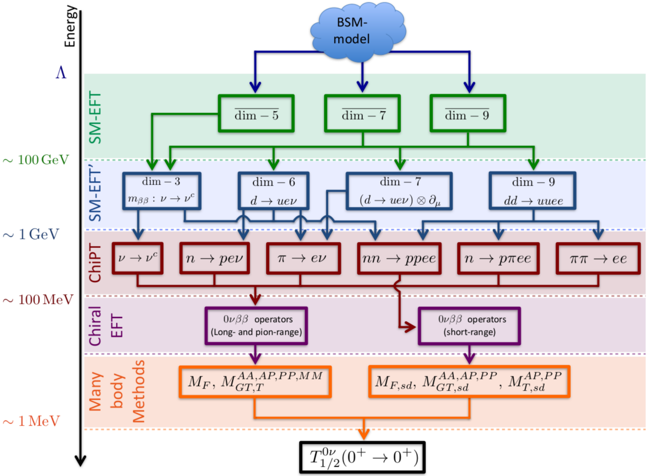
$0\nu\beta\beta$ decay nuclear matrix elements

Large difference in nuclear matrix element calculations



Agostini, Benato, Detwiler, JM, Vissani, Rev. Mod. Phys. 95, 025002 (2023)

Effective field theory for $0\nu\beta\beta$ decay



Cirigliano et al. JHEP 12 097 (2018)

Light-neutrino exchange: contact operator

Short-range operator contributes to light-neutrino exchange
for RG invariance of two-nucleon decay amplitude: high-energy ν's

$$T_{1/2}^{-1} = G_{01}\, g_A^4 \left(M_{\rm long}^{0\nu} + M_{\rm short}^{0\nu}\right)^2 m_{\beta\beta}^2,$$

Cirigliano et al. PRL120 202001(2018)

Unknown value (and sign) of the hadronic coupling $g_\nu^{\rm NN}$!

Lattice QCD calculations could obtain value of $g_\nu^{\rm NN}$

Davoudi, Kadam, Phys. Rev. Lett. 126, 152003 (2021), PRD105 094502('22)

match $nn \to pp + ee$ amplitude calculated with dispersion QCD methods

Cirigliano et al. PRL126 172002 (2021), JHEP 05 289 (2021)

relativistic χEFT $nn \to pp + ee$ amplitude calculation

Yang et al. PRL134, 242502 (2025)

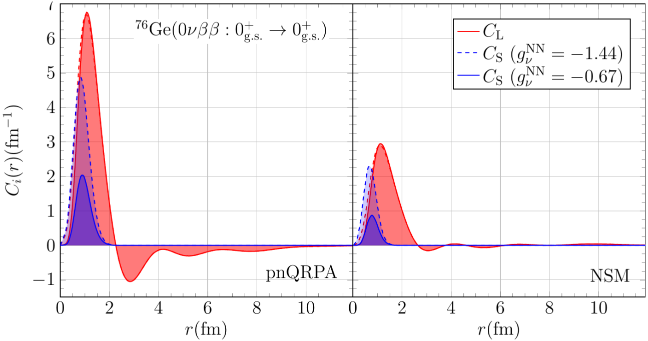
charge-independence breaking of nuclear Hamiltonians

Cirigliano et al. PRC100, 055504 (2019)

Long and short-range NME in heavy nuclei

Relatively stable contribution of new term M_S/M_L
among different nuclei used in $0\nu\beta\beta$ experiments:

20% – 50% impact of short-range NME in shell model
30% – 70% impact of short-range NME in QRPA



Jokiniemi, Soriano, JM, Phys. Lett. B 823 136720 (2021)

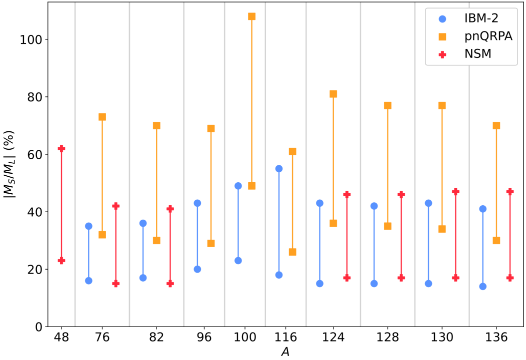
Short-range NME: method comparison

Relatively consistent contribution of M_S/M_L among many-body methods:

20% – 50% in shell model
30% – 70% in QRPA
Jokiniemi, Soriano, JM
Phys. Lett. B 823 136720 (2021)

20% – 50% in IBM
Kauppinen and Kotila
PRC 112, 034329 (2025) (2025)

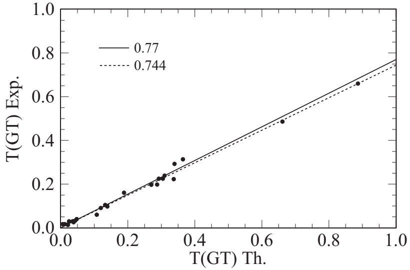
43% in IM-GCM for ^{48}Ca
Wirth et al.
PRL127 242502 (2021)



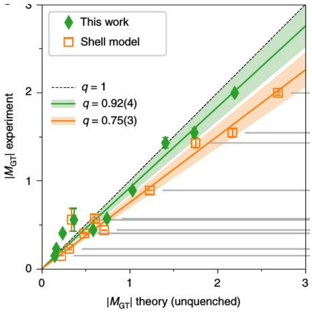
Uncertainty dominated by coupling g_ν^{NN}

β–decay Gamow-Teller transitions: “quenching”

β decays (e^- capture): nuclear shell model vs ab initio



Martinez-Pinedo et al. PRC53 2602(1996)



Gysbers et al. Nature Phys. 15 428 (2019)

$$\langle F|\sum_i [g_A\,\boldsymbol{\sigma}_i\tau_i^-]^{\rm eff}\,|I\rangle\,,\quad [\boldsymbol{\sigma}_i\tau]^{\rm eff}\approx 0.7\boldsymbol{\sigma}_i\tau$$

Shell model: $\boldsymbol{\sigma}_i\tau$ “quenching”

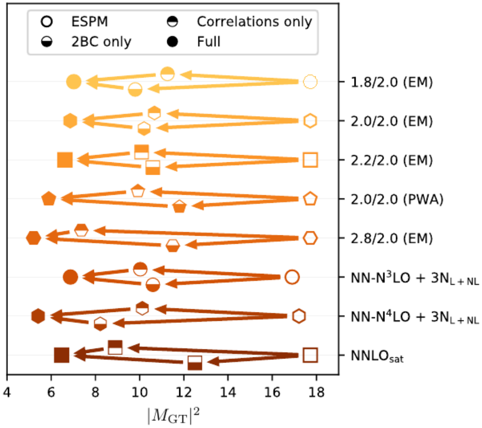
quenching: effects not in model

Ab initio calculations including meson-exchange currents and additional nuclear correlations

do not need “quenching”

Origin of β decay "quenching"

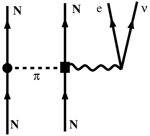
Which are main effects missing in conventional β-decay calculations?
Test case: GT decay of ^{100}Sn



Relatively similar and complementary impact of

- nuclear correlations
- meson-exchange currents

Gysbers et al.
Nature Phys. 15 428 (2019)

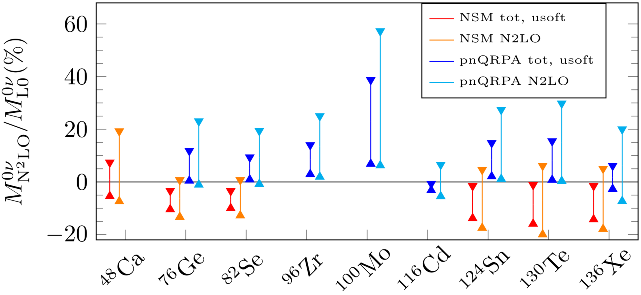


N^2LO $0\nu\beta\beta$ nuclear matrix elements

Overall $\sim (10-20)\%$ N^2LO NME contributions, but larger in few cases: negative in shell model, while positive in QRPA

Agree with beyond-closure-approximation effects (χEFT prediction!)

Castillo, Jokiniemi, Soriano, JM, PLB 860 139181 (2025)



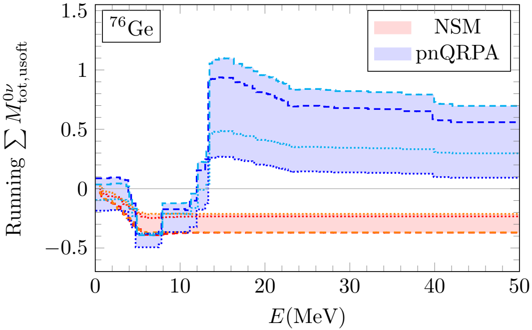
Uncertainties dominated by scale dependence and short-range terms

Ultrasoft N^{2}LO $0\nu\beta\beta$ nuclear matrix element

Physics of the running sum directly relevant
for the ultrasoft neutrinoless $\beta\beta$ nuclear matrix element as well!

Cirigliano et al. PRC 97 065501 (2018)

Castillo, Jokiniemi, Soriano, JM, PLB 860 139181 (2025)



Determines sign
of this contribution:

negative in shell model
positive in QRPA

Is it possible to measure high-enegy GT strength in heavy $\beta\beta$ emitters?

Is it possible to better measure GT transitions for the β^+ leg?

Outline

Majorana $\beta\beta$ decay ($0\nu\beta\beta$)

Nuclear matrix elements for light-neutrino-exchange $0\nu\beta\beta$ decay

Majorana $\beta\beta$ decay NMEs with uncertainties

Correlation analysis: NMEs vs nuclear properties

Use correlation of $0\nu\beta\beta$ with observables to calibrate NME predictions:

Two-neutrino $\beta\beta$ decay

Horoi et al. PRC106 054302(2022), 107 045501(2023)

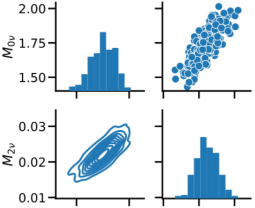
Jokiniemi et al. PRC107 044305 (2023)

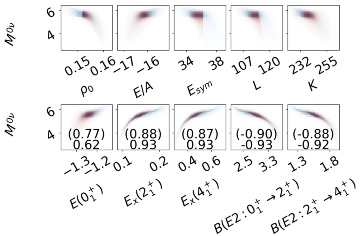
Observables related to nuclear deformation

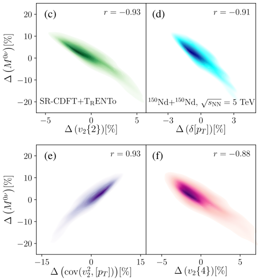
Zhang et al. PRC112 L021302 (2025), Li et al. PRL135 022301 (2025)

Pairing-channel phase-shift at rel. high energy:

Belley et al. PRC 113 014319 (2026)







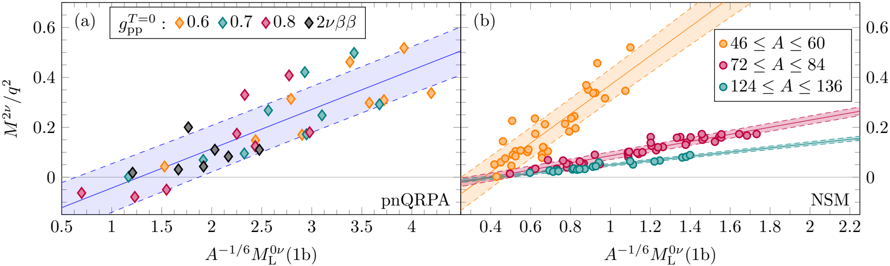
Correlation of $0\nu\beta\beta$ decay and $2\nu\beta\beta$ decay

Good correlation between 2ν and 0ν modes of $\beta\beta$ decay in nuclear shell model (systematic calculations of different nuclei) and QRPA calculations (decays of $\beta\beta$ emitters with different g_{pp} values)

Similar but not common correlation, depends on mass for shell model

$0\nu\beta\beta - 2\nu\beta\beta$ correlation also observed in ^{48}Ca, ^{136}Xe

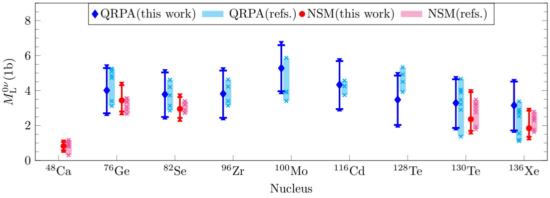
Horoi et al. PRC106 054302 (2022), PRC107 045501 (2023)

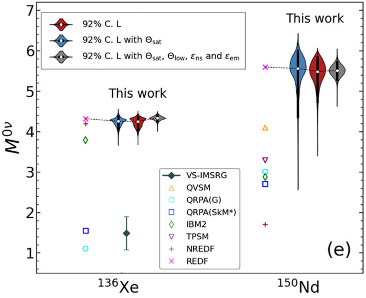


Jokiniemi, Romeo, Soriano, JM, PRC 107 044305 (2023)

$0\nu\beta\beta$ NMEs with theoretical uncertainties

NMEs calibrated with data based on correlations
of $0\nu\beta\beta$ and other observables





NMEs consistent with previous results,
but also take into account model uncertainties

Jokiniemi et al. PRC107 044305 (2023)

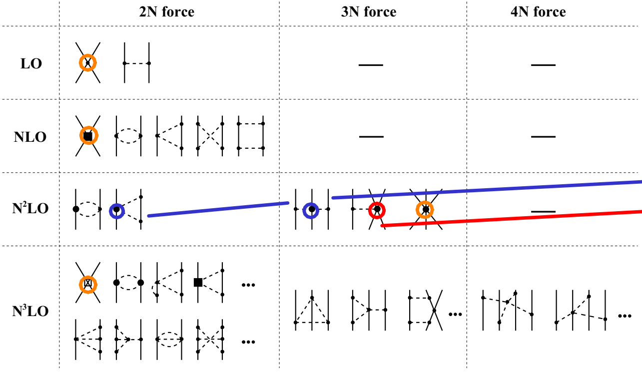
Zhang et al. PRC112 L021302 (2025)

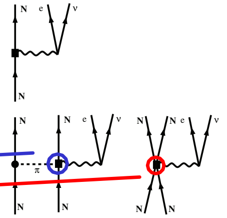
Chiral effective field theory

Chiral EFT: low energy approach to QCD, nuclear structure energies

Approximate chiral symmetry: pion exchanges, contact interactions

Systematic expansion: nuclear forces and electroweak currents





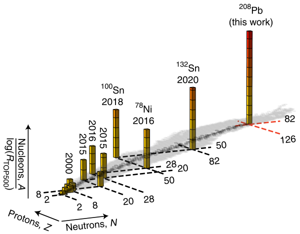
Park, Krebs, Baroni...

Short-range couplings fitted to experiment once

Weinberg, van Kolck, Kaplan, Savage, Wise, Meißner, Epelbaum...

Ab initio predictions for nuclear neutron radius

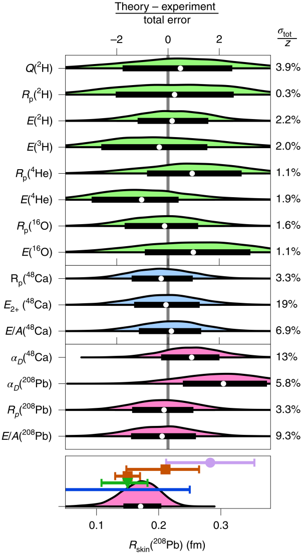
Remarkable progress ab initio calculations of (relatively uncorrelated) heavy nuclei, ^{208}Pb



Determine ^{208}Pb neutron skin using Bayesian approach sampling of 10^9 (parameters of) nuclear Hamiltonians

Hu, Jiang, Miyagi et al.

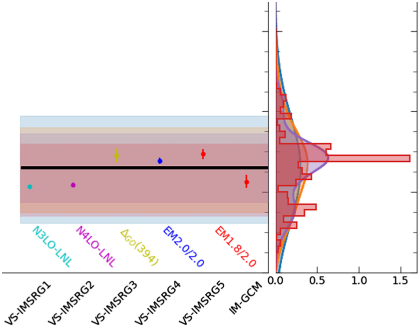
Nature Phys. 18 1196 (2022)



UNIVERSITAT DE BARCELONA

Ab initio NME with theoretical uncertainty

Similar study to obtain density of NME values for ^{76}Ge

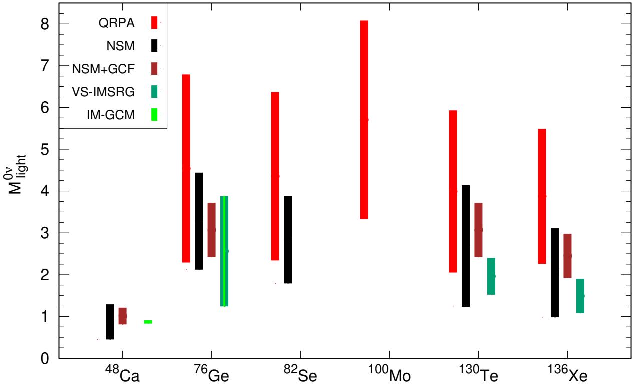


Belley, Yao et al. PRL132 182502 (2024)

Theoretical uncertainty dominated by nuclear Hamiltonian
and difference in NME results
obtained with two independent many-body methods

$0\nu\beta\beta$ decay total (long- and short-range) NMEs

Not-so-large difference in nuclear matrix element calculations!



Wirth et al.
PRL127 242502 (2021)

Belley et al.
arXiv:2307:15156

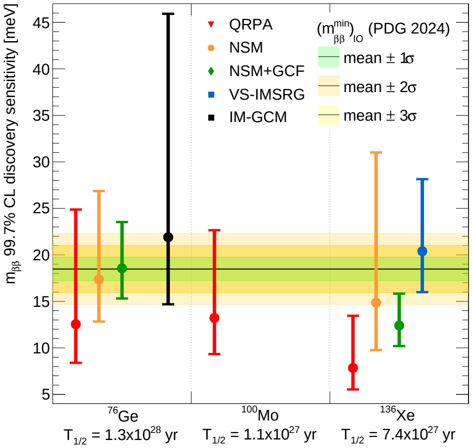
Belley et al.
PRL132 182502 (2024)

Jokiniemi et al.
PRC 107 044305 (2023)

Weiss et al.
PRC106 065501 (2022)

Gómez-Cadenas, Martín-Albo, JM, Mezzeto, Monrabal, Sorel, Riv. Nuovo Cim. 46, 619 (2023)

Uncertainty in physics reach of $0\nu\beta\beta$ experiments



Agostini, Benato, Detwiler, JM, Vissani

Phys. Rev. C 104 L042501 (2021), updated

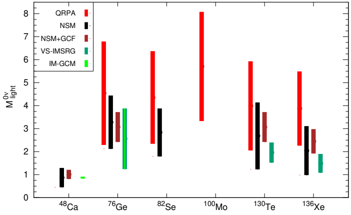
Nuclear matrix element theoretical uncertainty critical to anticipate $m_{\beta\beta}$ sensitivity of future experiments

Current uncertainty in $m_{\beta\beta}$ prevents to foresee if next-generation experiments will fully cover parameter space of “inverted” neutrino mass hierarchy

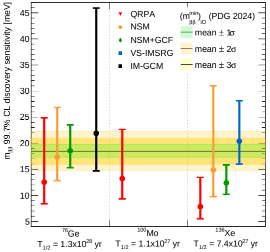
Uncertainty needs to be reduced!

Summary

Calculations of $0\nu\beta\beta$ NMEs
challenge nuclear many-body methods,
reach and exploitation of $0\nu\beta\beta$ searches
demand reliable NMEs
reduce present theoretical uncertainties



Nuclear calculations must describe well
known nuclear spectroscopy
and include physics of "quenching"
and new short-range NME
Calibration with data ($2\nu\beta\beta$, deformation...)
inform theoretical uncertainty of NMEs



Current $0\nu\beta\beta$ NMEs sizeable errorbars
and reasonable overlap of predictions
from different many-body methods

Thank you very much for your attention!