FFAG, The working marriage of magnets and dynamics

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Beam dynamics
or
Magnet design?

- Small community.
- "New" concept.
- Unconventional magnets.

→ **Same people!!**

Outline

- FFAG accelerators
  - Definition - History
  - betatron oscillations - chromaticity
- Scaling beam dynamics
  - Circular case
  - Straight case

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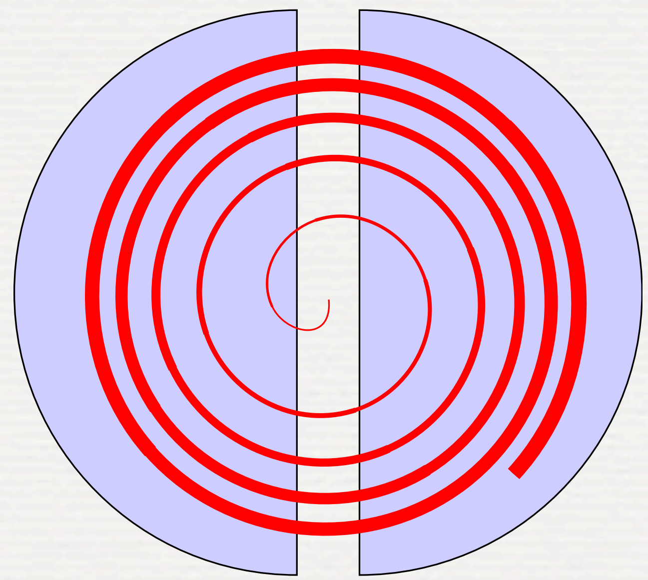
FFAG accelerator

FIXED FIELD ALTERNATING GRADIENT

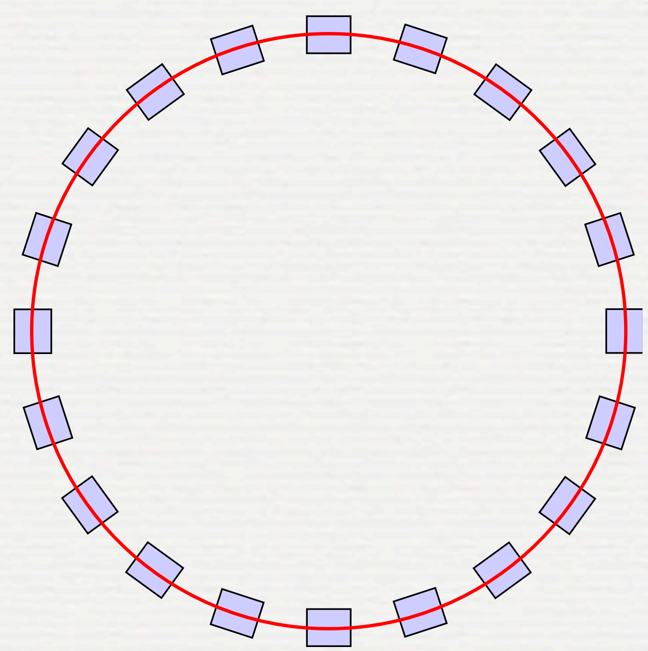
It combines

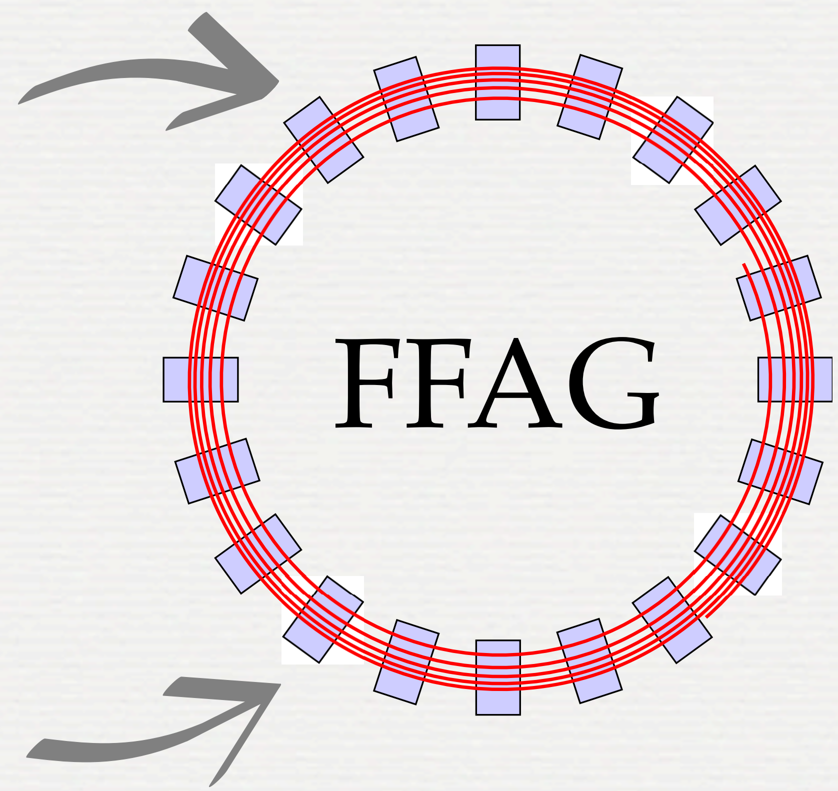
- **a static guide field** like cyclotrons:



AND

- **a strong focusing.** like synchrotrons:



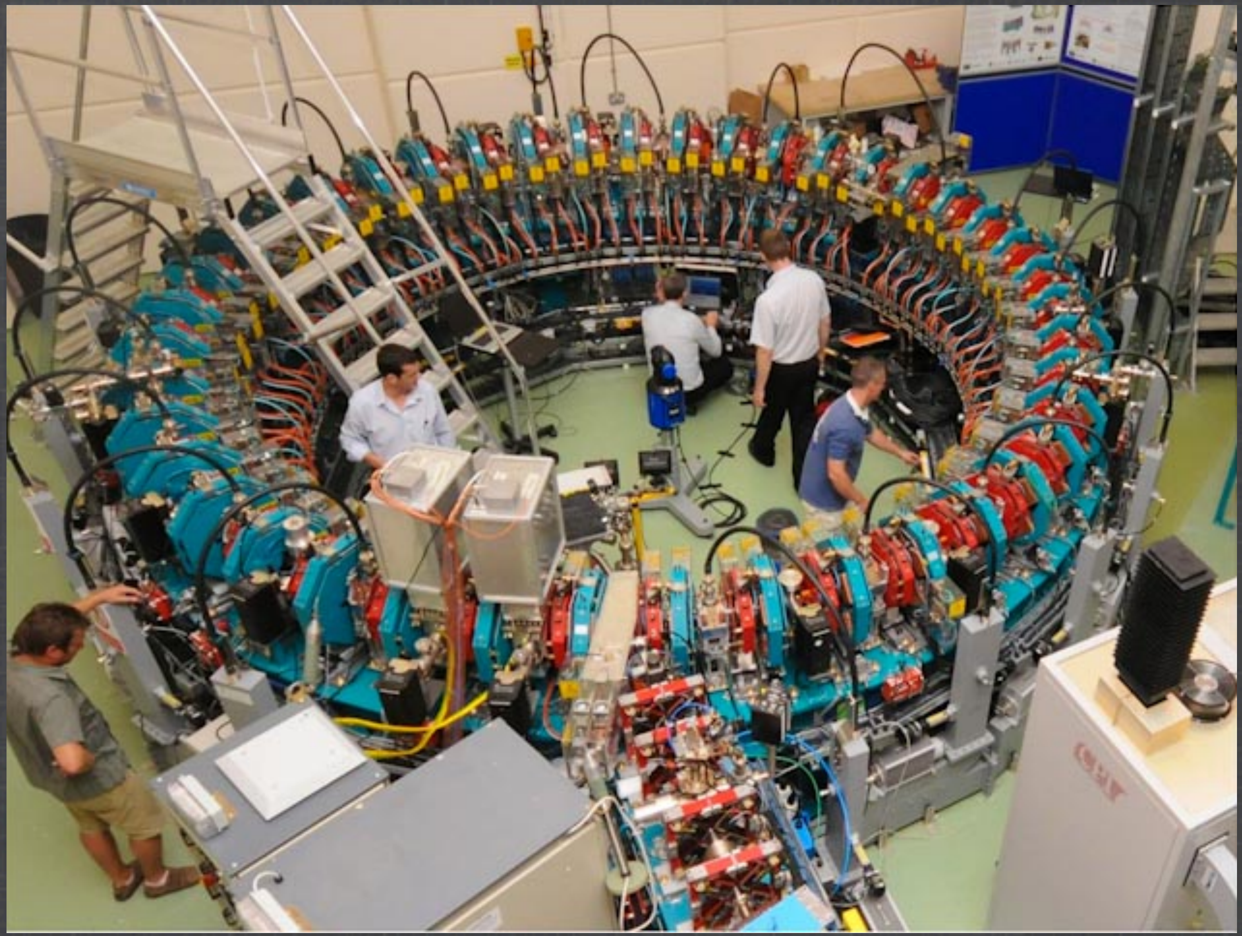


FFAG accelerator

2 types of FFAGs:

- "Scaling" FFAG, non-linear field, constant tune.
- "Non-scaling" FFAG, linear optics, fast resonance crossing

e-model for non-scaling FFAG: EMMA (Daresbury, UK)



FFAG history

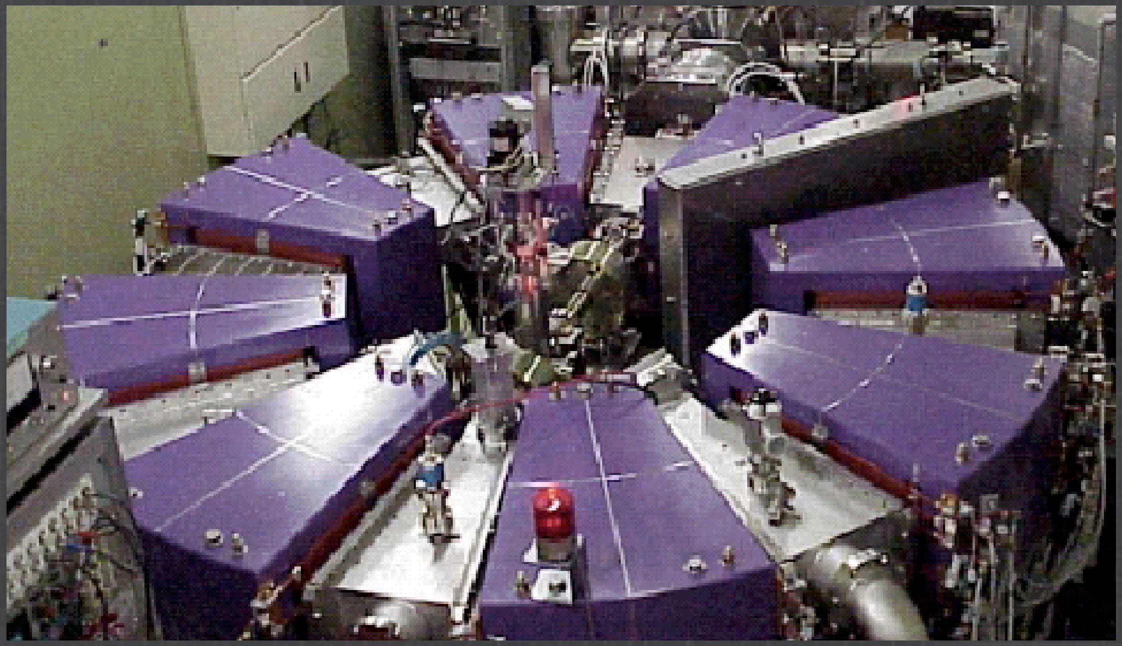
Ohkawa (1953), Kerst & Symon, Kolomenskii.

- MURA project (1960s): e-model, induction acceleration.

No practical machine for 40 years.

- Complicated magnetic field configuration: 3D design.
- RF cavity: Variable frequency and high gradient

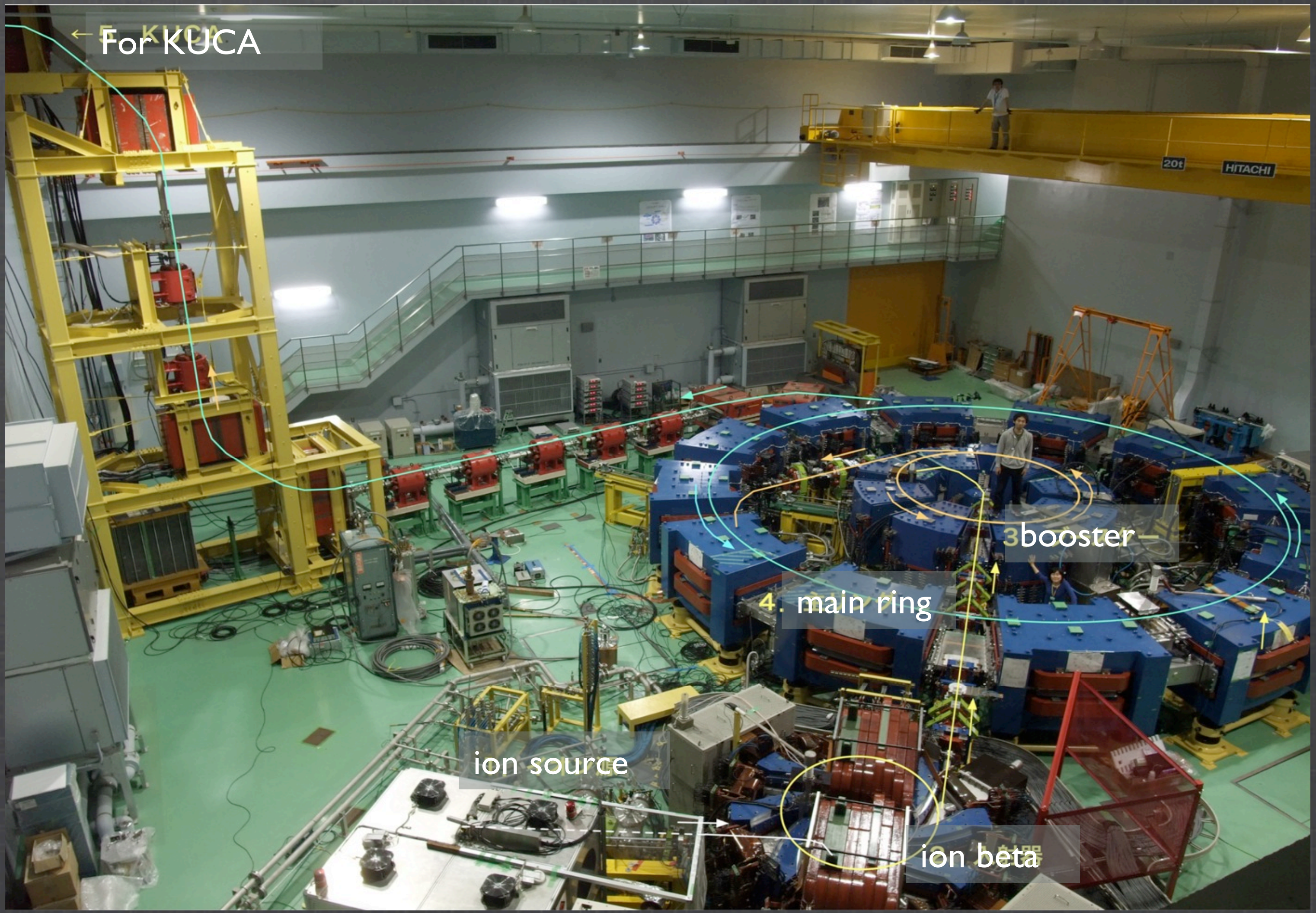
2000: first proton FFAG

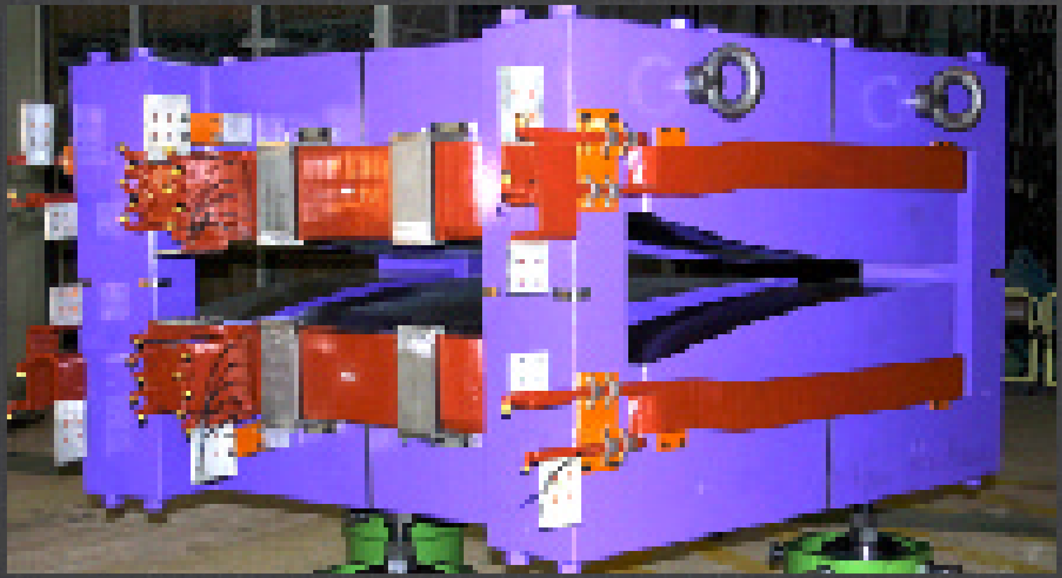


FFAG history (Continued)

2003 Proton FFAG complex at Kyoto University

- Return yoke-free magnets

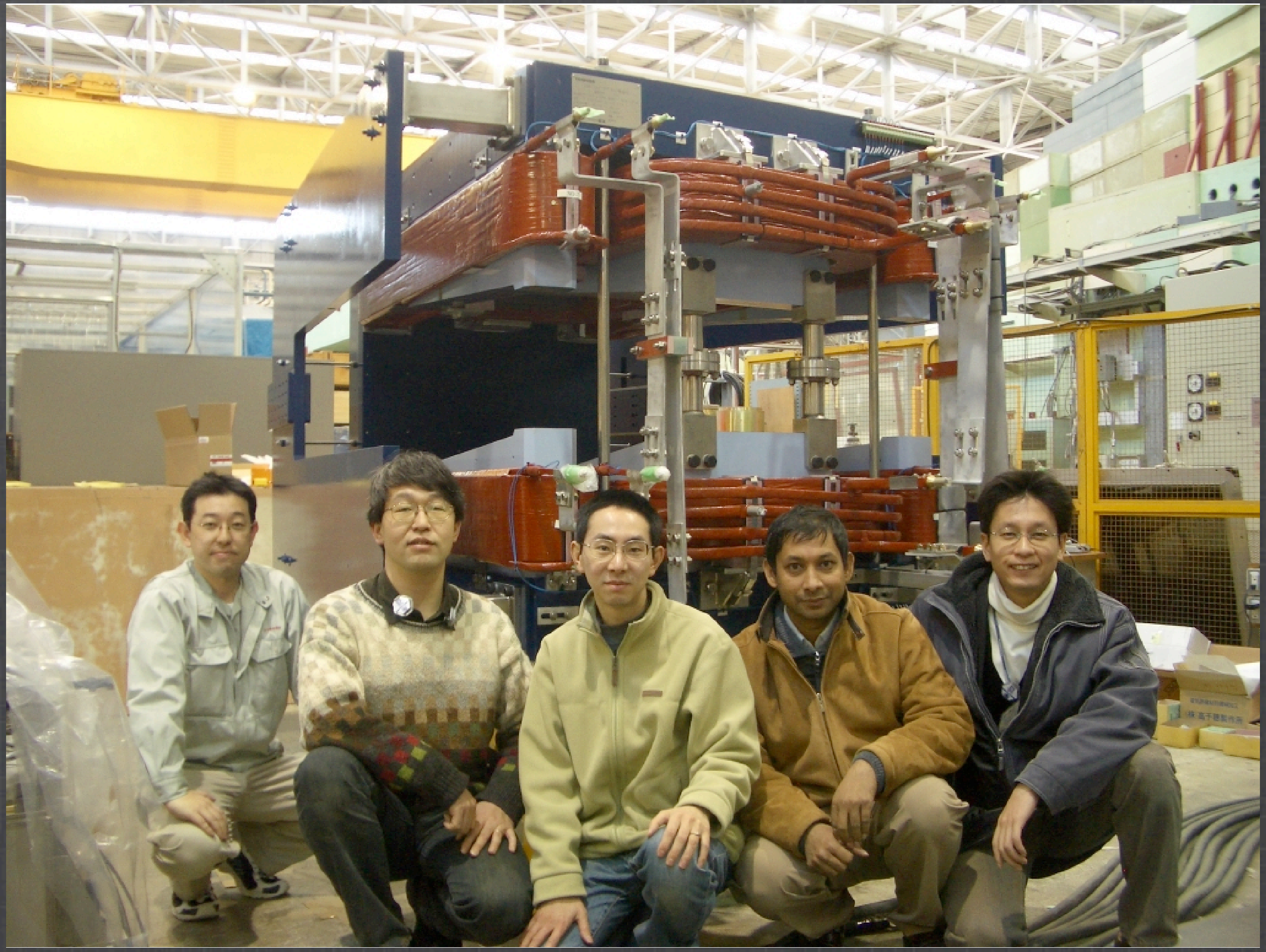




FFAG history (Continued)

Muon accelerator: PRISM

- C-shape magnets
- large aperture
- Challenging dynamics & design



Transverse motion in particle accelerators

Linearized equations of motion:

$$\frac{\partial^2 y}{\partial s^2} + K_y(s) y = 0 \qquad y = x \text{ or } z$$

Periodic case: Hill's equations

→ General solution: $y = \sqrt{\epsilon}\sqrt{\beta(s)}\cos(\nu\phi(s) + \phi_0)$

Betatron oscillations: pseudo-harmonic oscillation of frequency ν (tune) and varying amplitude $\sqrt{\beta(s)}$.

Betatron resonances

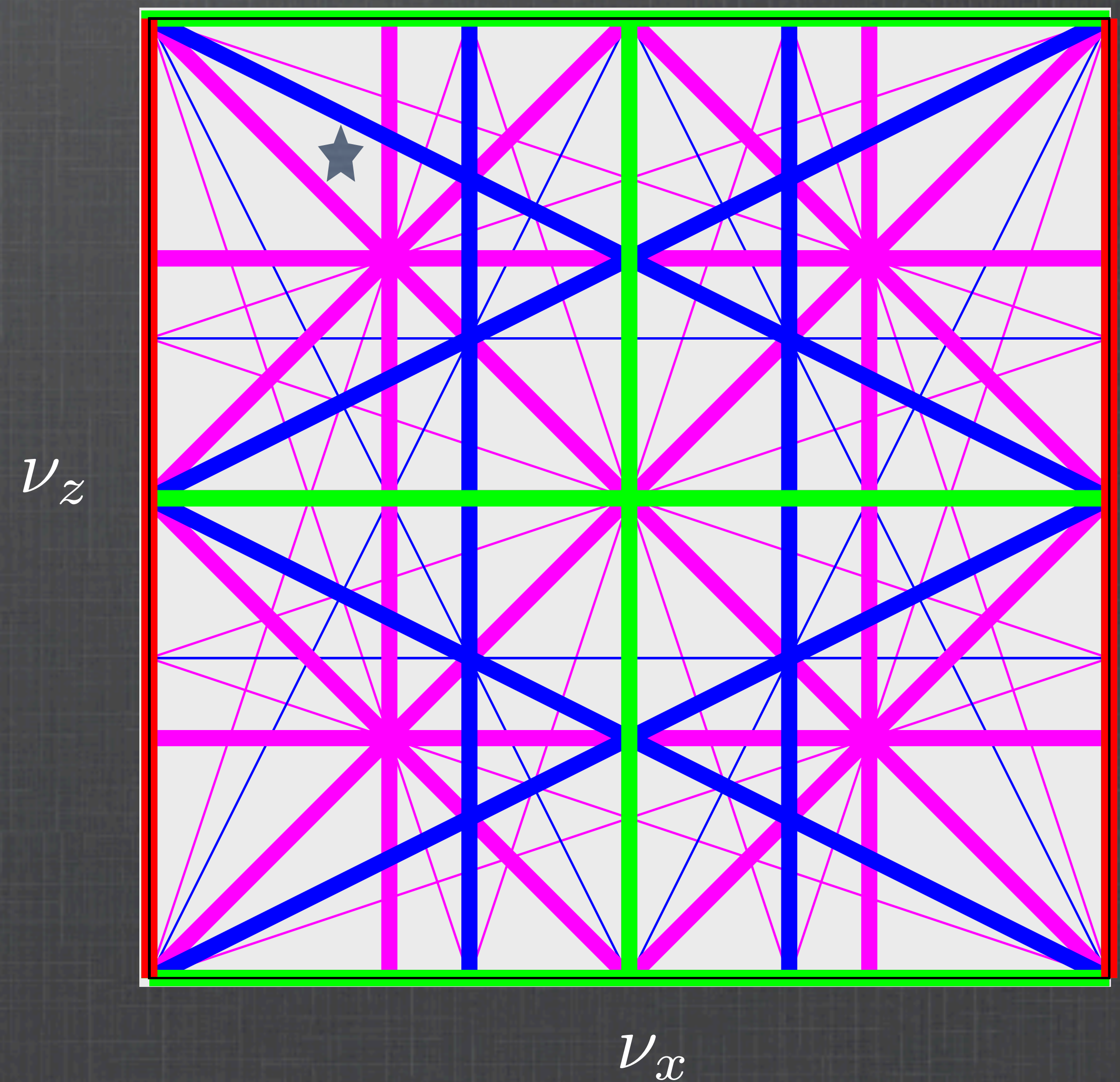
Non-linear components are considered as perturbations of the linear equations of motion.

Resonance conditions:

$$m_x\nu_x + m_z\nu_z = q,$$

$$(m_x, m_z, q) \in \mathbb{N}^3$$

Working point (ν_x, ν_z) positioned in the tune diagram.



Chromaticity: Variation of tune with respect to particle energy.

→ **<u>Zero-chromaticity</u>**: Invariance of both horizontal and vertical tune with respect to energy.

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Invariance of the betatron oscillations

keep independent of momentum the transverse linearized equations of motion.

➡ zero-chromatic system for any momentum range.

Circular case

Linearized equations of motion for a momentum *p*:

$$\begin{cases} \dfrac{d^2x}{d\Theta^2} + \dfrac{R^2}{\rho^2}(1-n)x = 0, \\ \dfrac{d^2z}{d\Theta^2} + \dfrac{R^2}{\rho^2}nz = 0. \end{cases}$$

(x, s, z): curvilinear coordinates.

New system of coordinates (x, Θ, z)

$\Theta = s/R$ with $R = \dfrac{1}{2\pi}\oint ds$

n: field index

ρ : curvature radius

Independent of momentum *p:*

$$\begin{cases} \left(\dfrac{\partial(R/\rho)}{\partial p}\right)_\Theta = 0, \\ \left(\dfrac{\partial n}{\partial p}\right)_\Theta = 0. \end{cases}$$

$\left(\frac{\partial(R/\rho)}{\partial p}\right)_\Theta = 0$ ➡ Similarity of the reference trajectories.

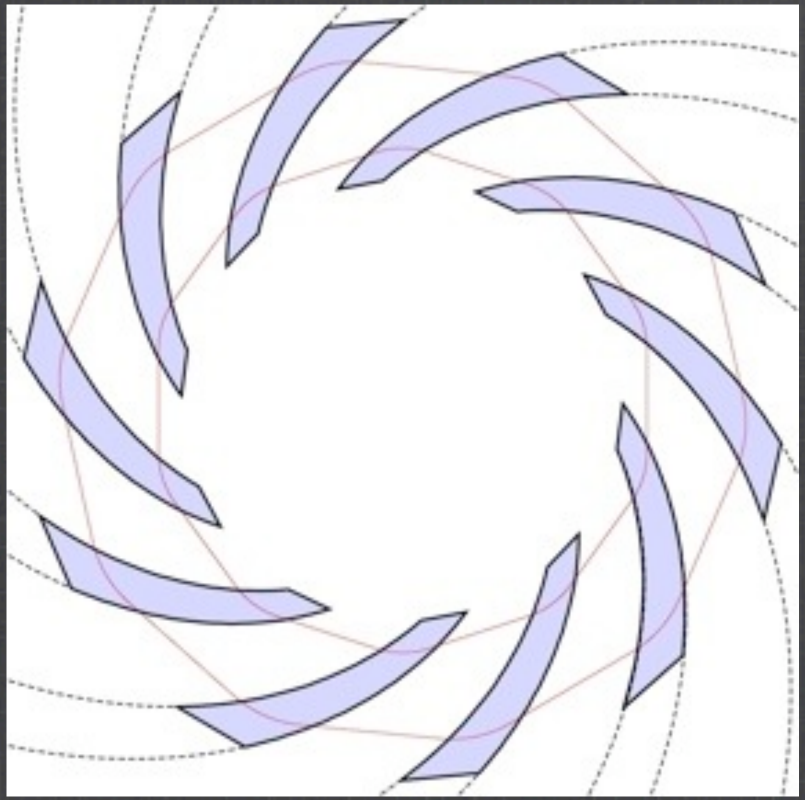
$\left(\frac{\partial n}{\partial p}\right)_\Theta = 0$ ➡ Invariance of the focusing strength.

Circular case

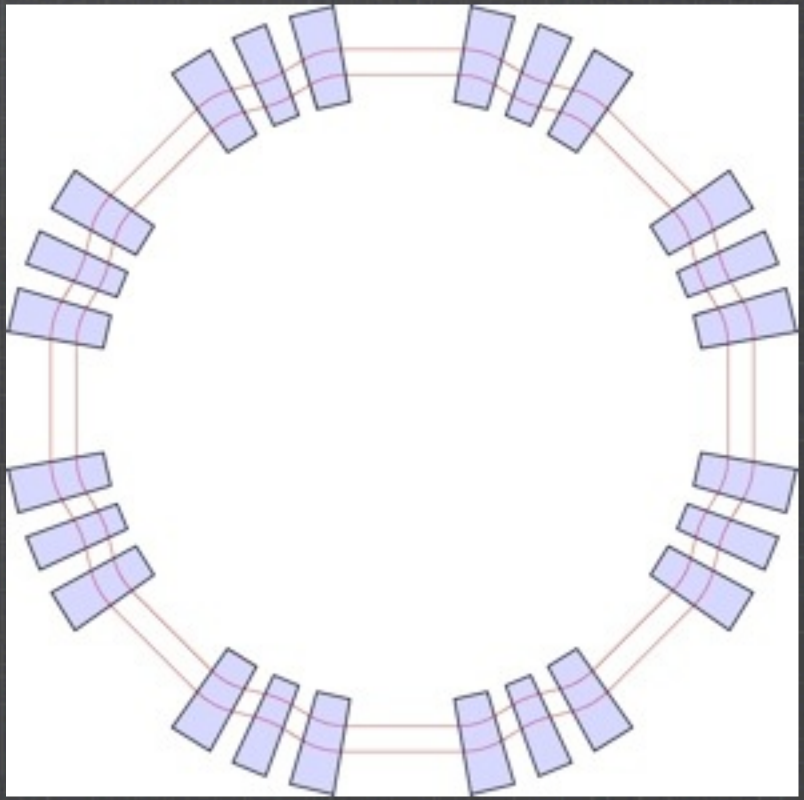
Invariance of the betatron oscillations ⟷ similarity of the closed orbits and invariance of the field index

Geometrical field index: $k = \frac{R}{\bar{B}}\frac{d\bar{B}}{dR}$

$$\boxed{B(r,\theta) = B_0 \left(\frac{r}{r_0}\right)^k \cdot \mathcal{F}(\theta - \tan\zeta \ln\frac{r}{r_0})}$$

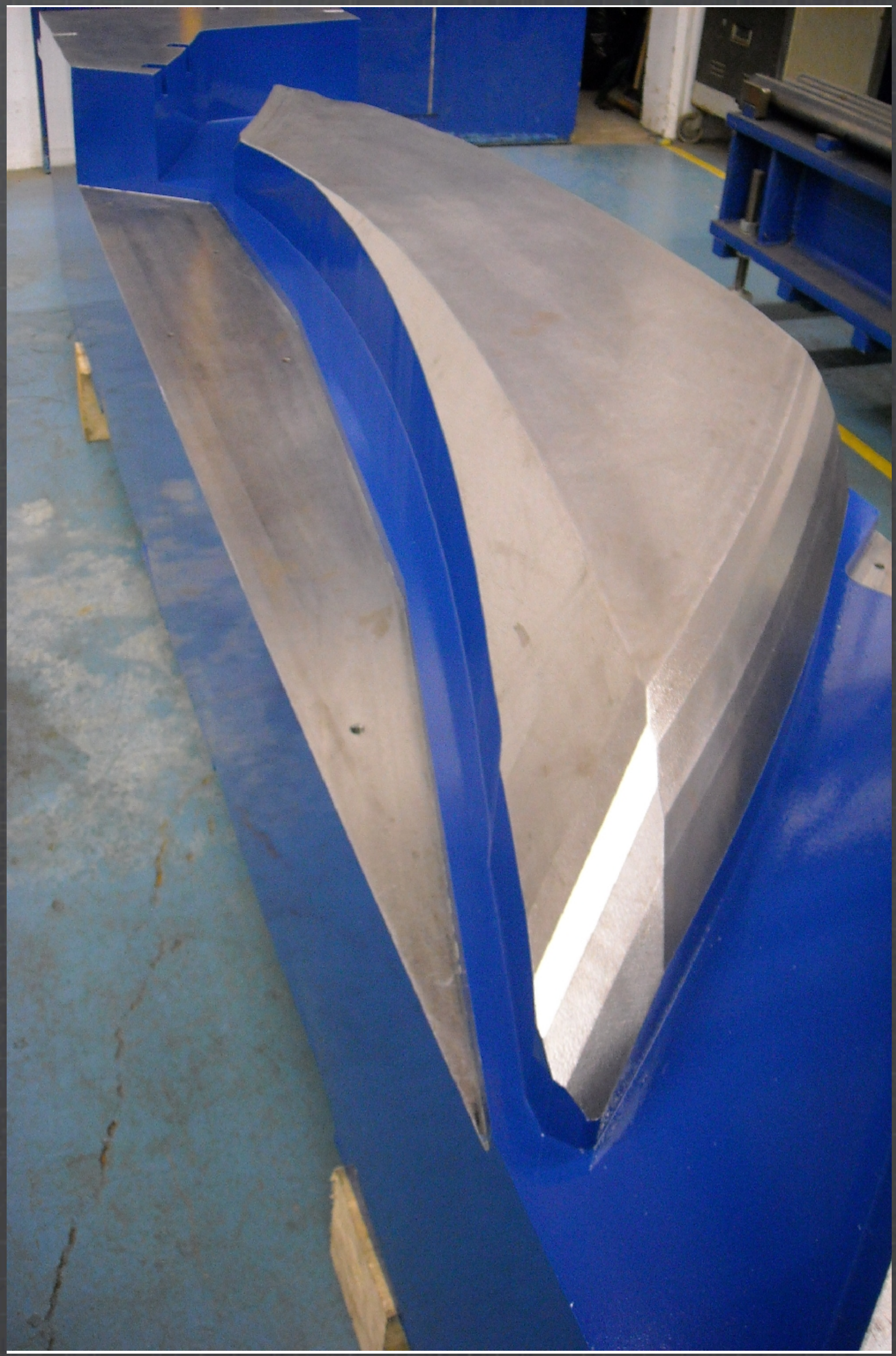


Spiral sector: $\zeta = const.$



Radial sector: $\zeta = 0$

Spiral case: RACCAM



Magnet built by SIGMAPHI

Straight case

Linearized equations of motion for a momentum *p*:

$$\begin{cases} \dfrac{d^2x}{ds^2} + \dfrac{1-n}{\rho^2}x = 0, \\ \dfrac{d^2z}{ds^2} + \dfrac{n}{\rho^2}z = 0. \end{cases}$$

(x, s, z): curvilinear coordinates

n: field index

ρ : curvature radius

Independent of momentum *p:*

$$\begin{cases} \left(\dfrac{\partial \rho}{\partial p}\right)_s = 0, \\ \left(\dfrac{\partial n}{\partial p}\right)_s = 0. \end{cases}$$

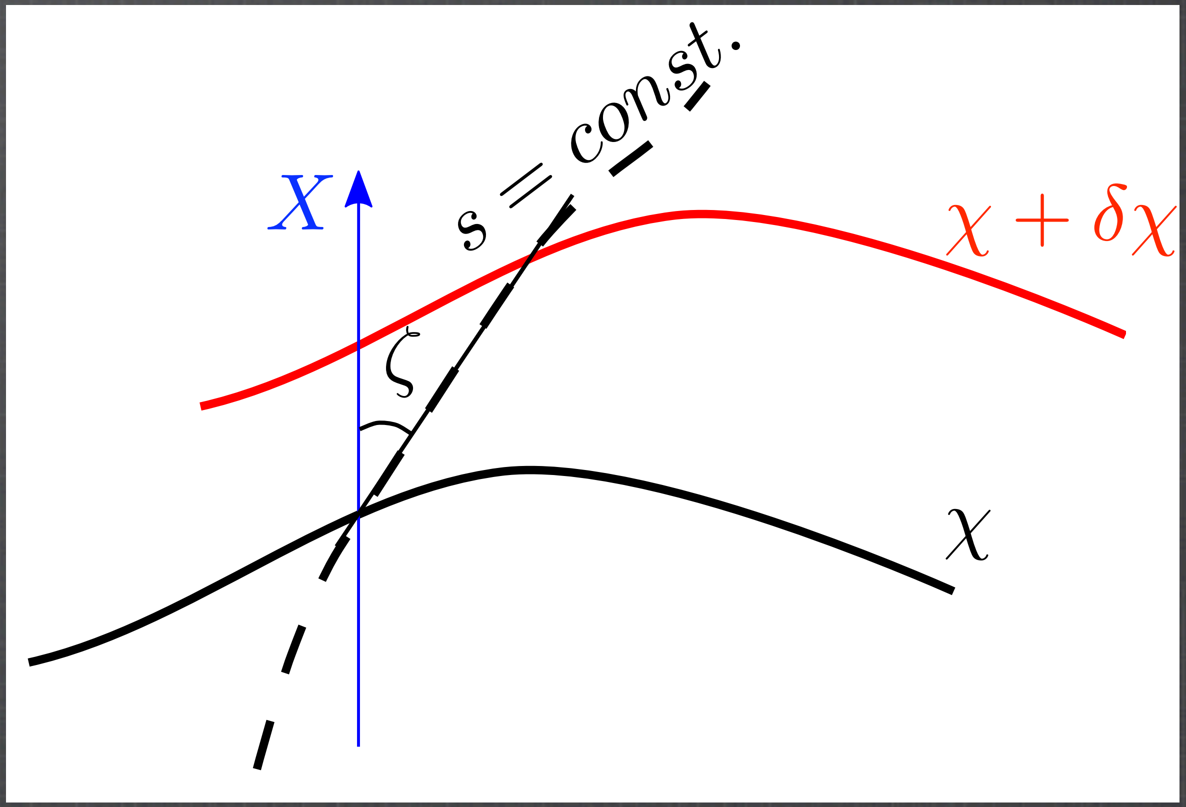
$\Rightarrow$ Similarity of the reference trajectories

$\Rightarrow$ Invariance of the focusing strength

Change of coordinates: introduction of average abscissa χ.

Straight case

Introduction of normalized field gradient: $m = \frac{1}{B}\frac{dB}{d\chi}$



Invariance of the focusing strength gives condition on *m*:

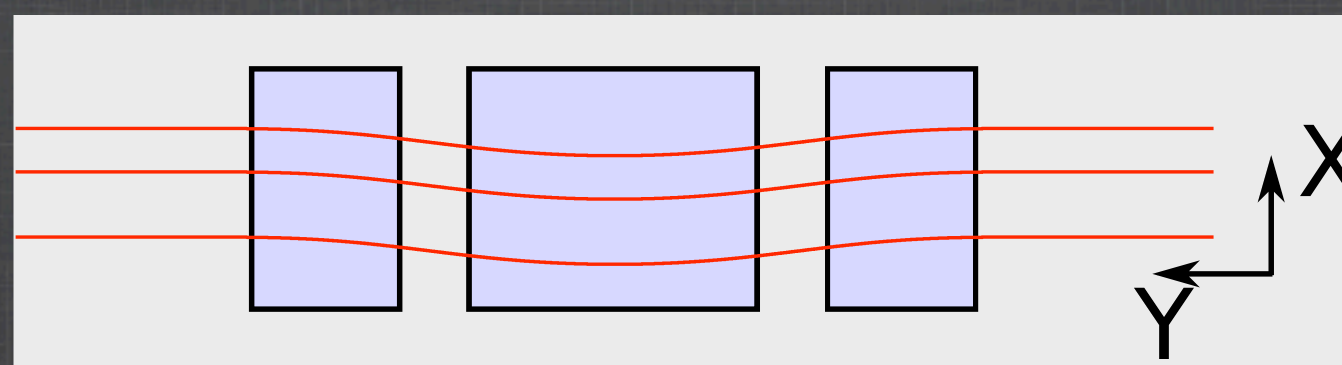
$m = m_1 + m_2 \tan\zeta(\chi)$

$$B(\chi, s) = B_0 e^{\left[m_1(\chi - \chi_0) + m_2 \int_{\chi_0}^{\chi} \tan\zeta(\chi) d\chi\right]} \mathcal{F}(s)$$

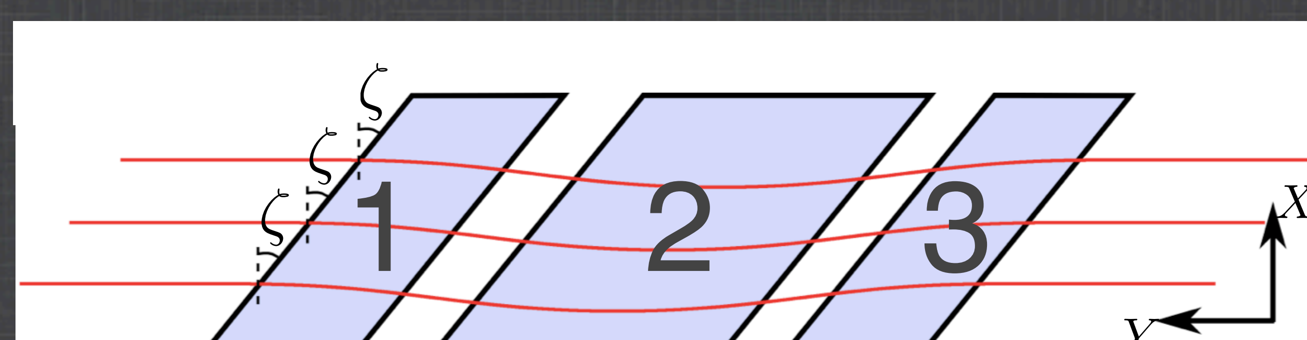
Straight case

$\zeta = const.$ ⟷ $m = const.$

$$B(X,Y) = B_0 e^{m(X-X_0)} \mathcal{F}\left(Y - (X - X_0)\tan\zeta\right)$$



Rectangular case: $\zeta = 0$

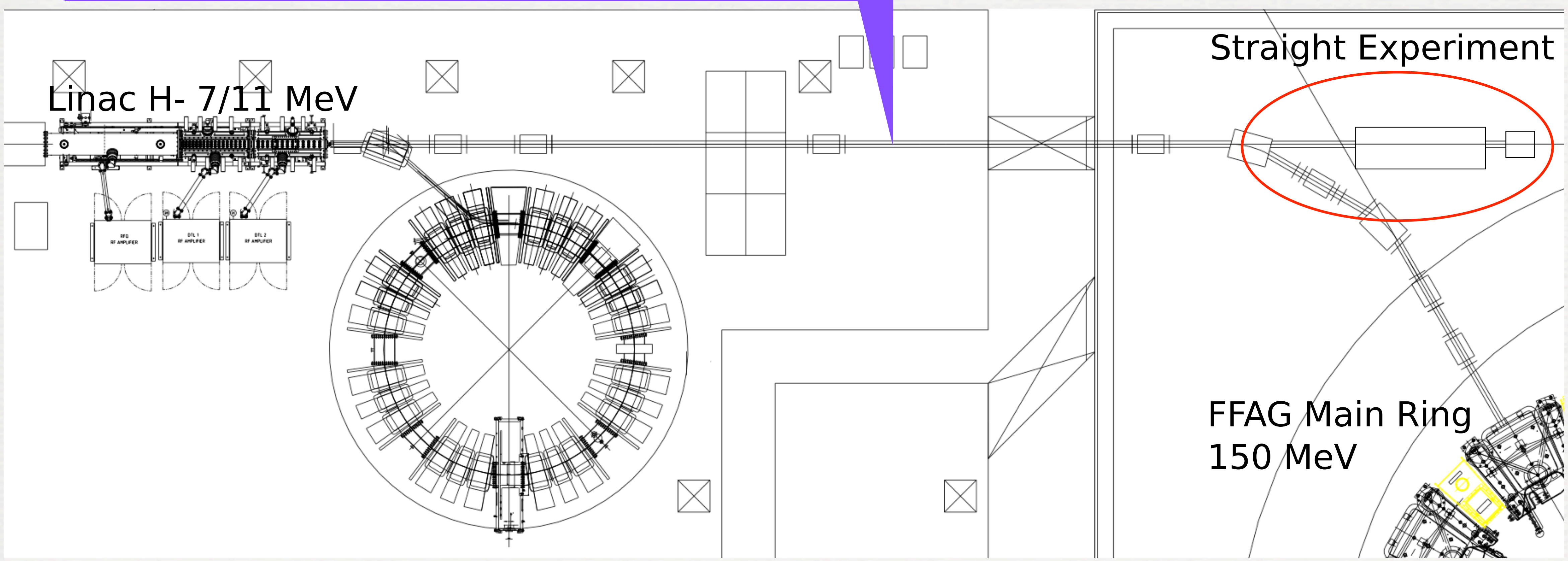


Tilted straight case: $\zeta = const.$

Straight experiment

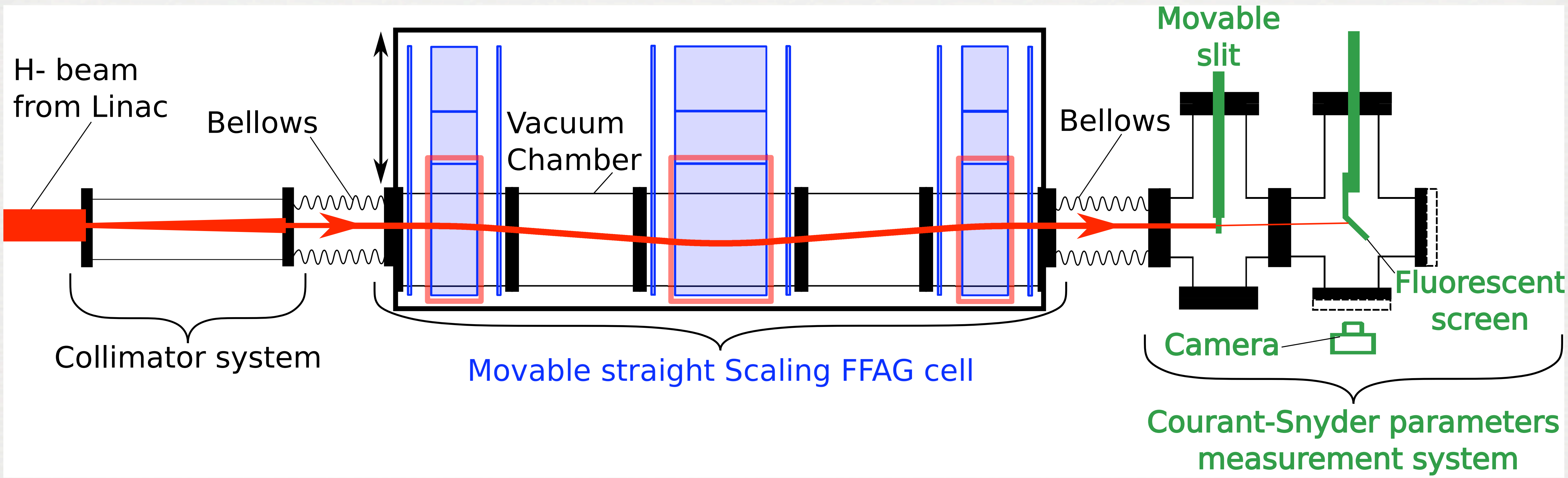
➡Design and manufacturing of a straight scaling cell prototype, and measure of the horizontal phase advance for 2 different energies.

H^- linac injection beam line



Use of 2 energies: 7 MeV and 11 MeV.

Layout of the experiment



Straight Scaling FFAG cell design

- C-shape Magnets to have easy access to the pole.
- Cell able to move horizontally to match the different reference trajectories.
- Rectangular magnets.

Type	FDF
m-value	11 m^{-1}
Total length	4.68 m
Length of F magnet	15 cm
Length of D magnet	30 cm
Max. B Field (D magnet)	0.3 T
Max. B Field (F magnet)	0.2 T
Horizontal phase advance	87.7 deg.
Vertical phase advance	106.2 deg.

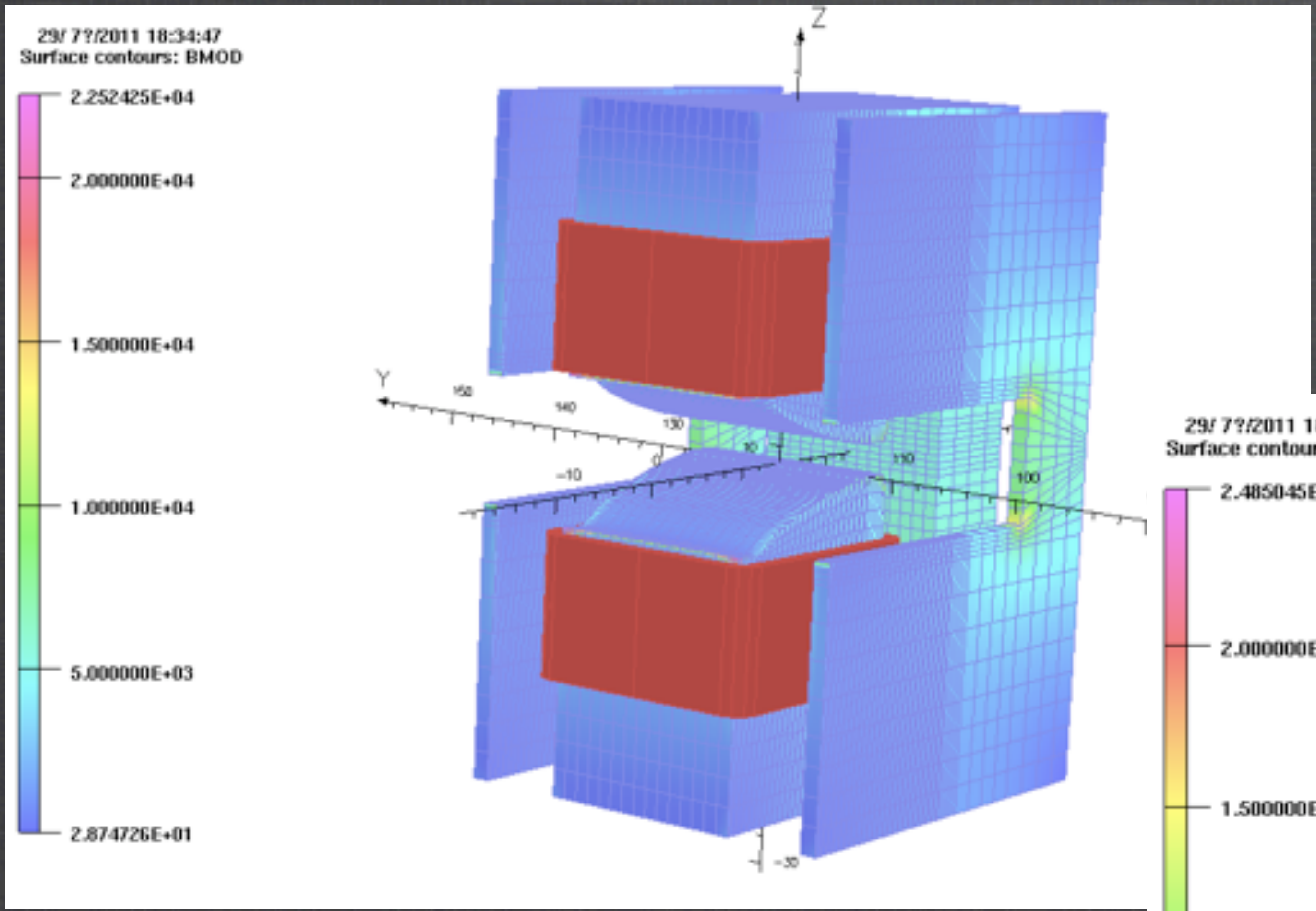
- Coils:
  - Max 3500 A.T/coil.
  - 18 turns x 4 layers = 72 turns of 5 mm x 2 mm cross section wire.

→ ~5 A/mm^2 → Indirect water cooling system.

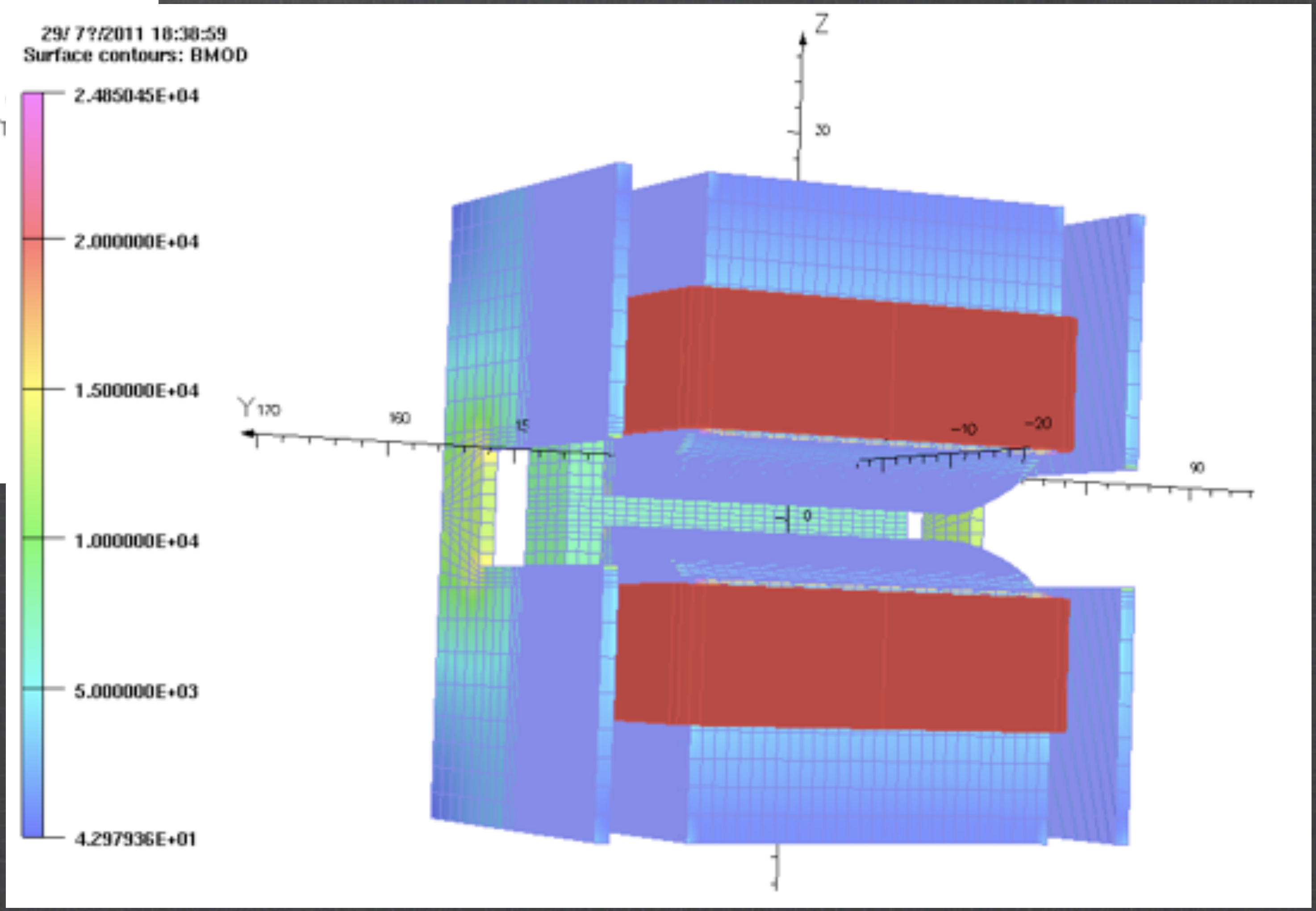
- Power supply per magnet (D): 100 A, 30 V.
- Whole system power consumption: ~1 kW.

Magnet design

Pole shape configured with POISSON, then TOSCA.



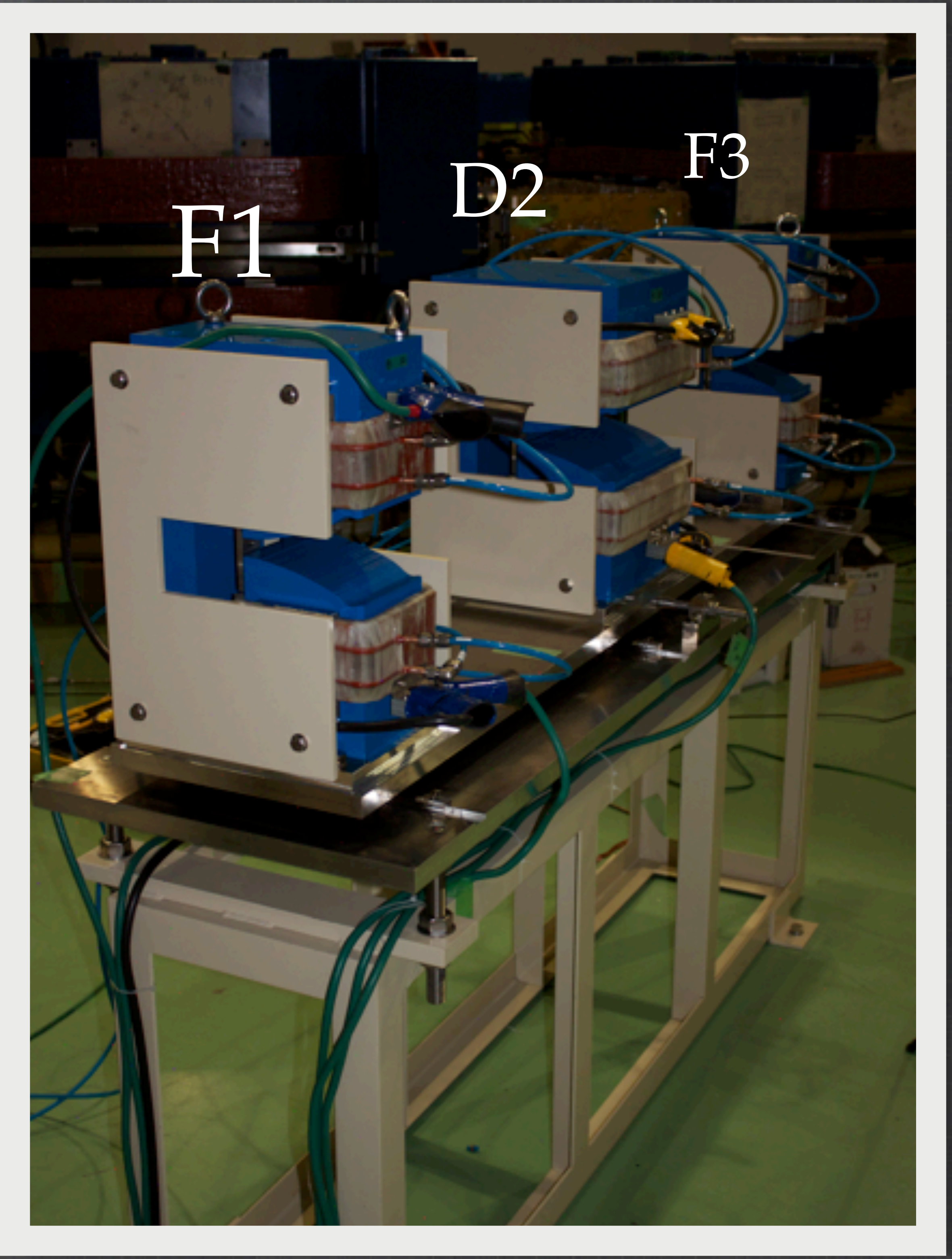
Magnetic field in D magnet (30 cm long). TOSCA model.



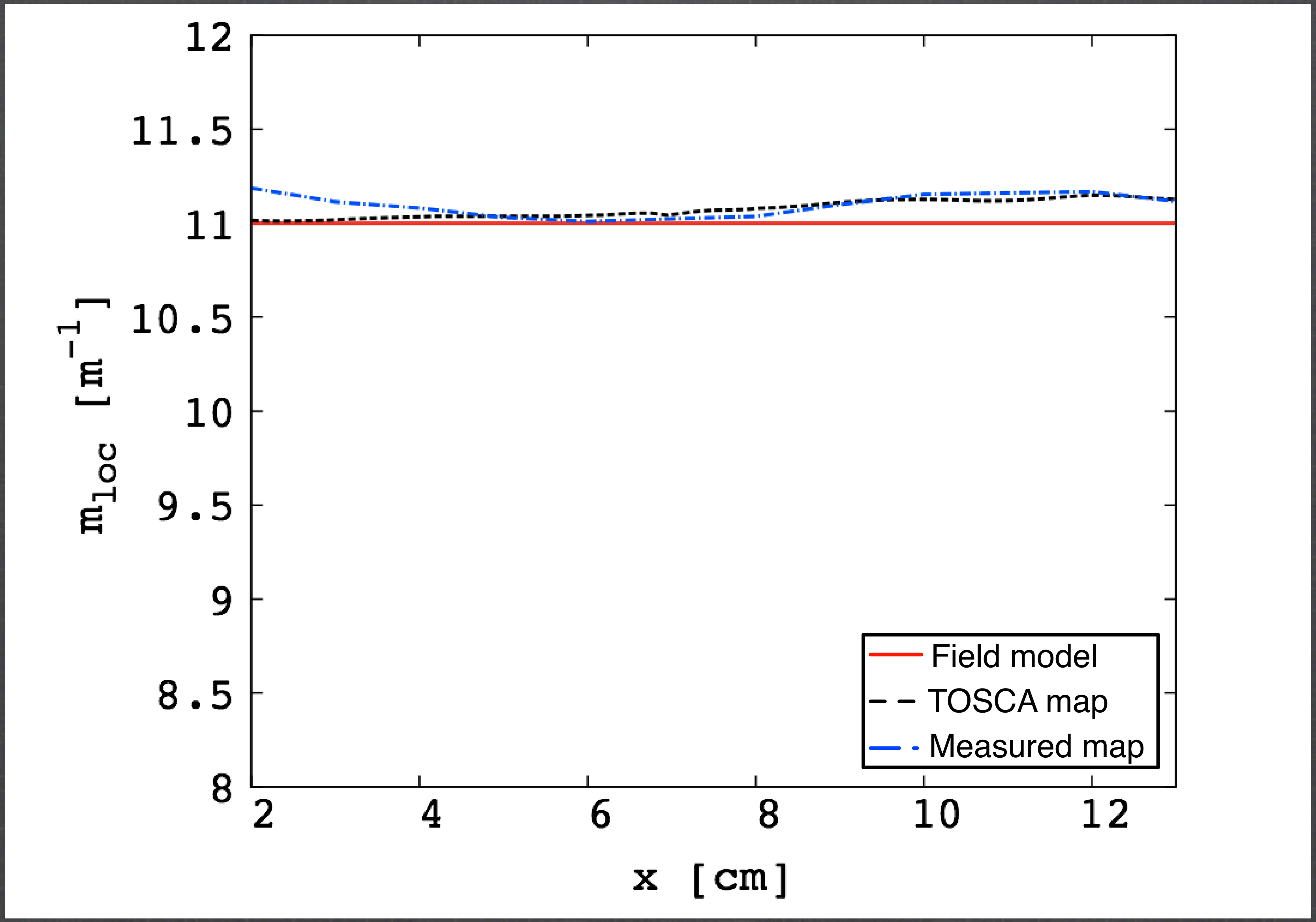
Magnetic field in F magnet (15 cm long). TOSCA model.

FIELD MEASUREMENT

Measured field map created



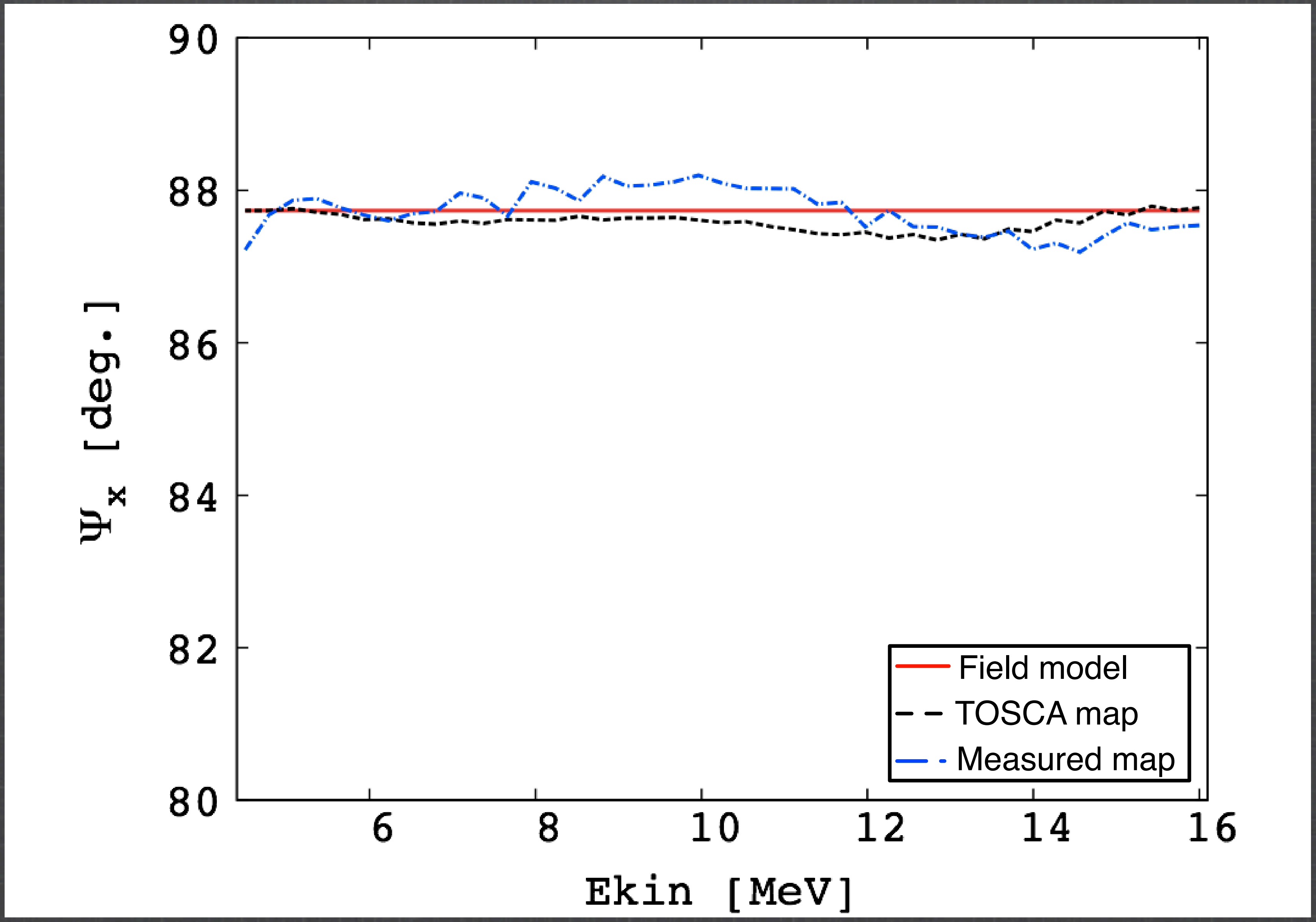
Comparison TOSCA-Measure



Local m-value vs horizontal abscissa with field model (plain red), in TOSCA field map (black dashed) and in measured field map (mixed blue).

→ **Good agreement** (difference < 1%)

Particle tracking



Horizontal phase advances vs kinetic energy with field model (plain red), in TOSCA field map (black dashed) and in measured field map (mixed blue).

Experimental results

$$\tan\psi = -\alpha_1 - \frac{\beta_1 x_1'}{x_1}$$

	$\bar{x_1}$ (mm)	$\bar{x_1'}$ (mrad)	$\bar{\beta_1}$ (m)	$\bar{\alpha_1}$	$\psi_{exp.}$ (deg)	ψ_{TOSCA} (deg)
11 MeV	**2.0**	**-2.4**	**17.7**	**-1.5**	**87.5** $\pm$ 3.3	**87.5**
7 MeV	**1.8**	**-2.1**	**11.7**	**-1.0**	**86.1** $\pm$ 9.6	**87.6**

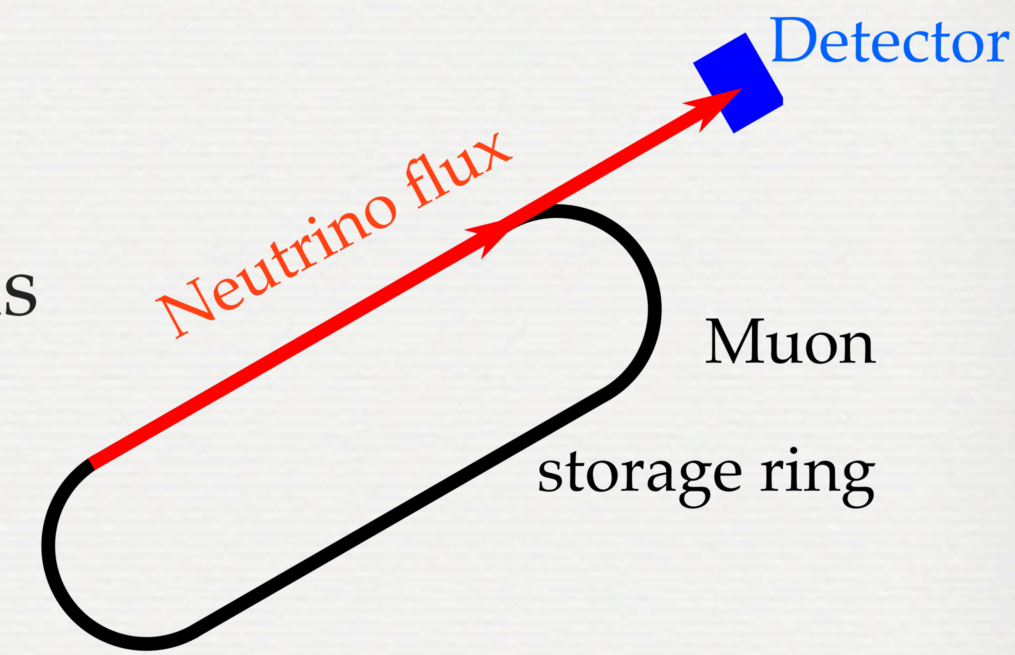
ψ_{exp}(11 MeV)=87.5 deg

ψ_{exp}(7 MeV)=86.1 deg

Straight scaling law clarified.

J.-B. Lagrange *et al*, “Straight scaling FFAG beam line”,
Nucl. Instr. Meth. A, vol. 691, pp. 55–63, 2012.

nuSTORM



Neutrinos from STORed Muons (nuSTORM) with a muon storage ring is investigated for neutrino experiments (neutrino mixing matrix, sterile neutrinos).

Muons decay in neutrinos in the storage ring

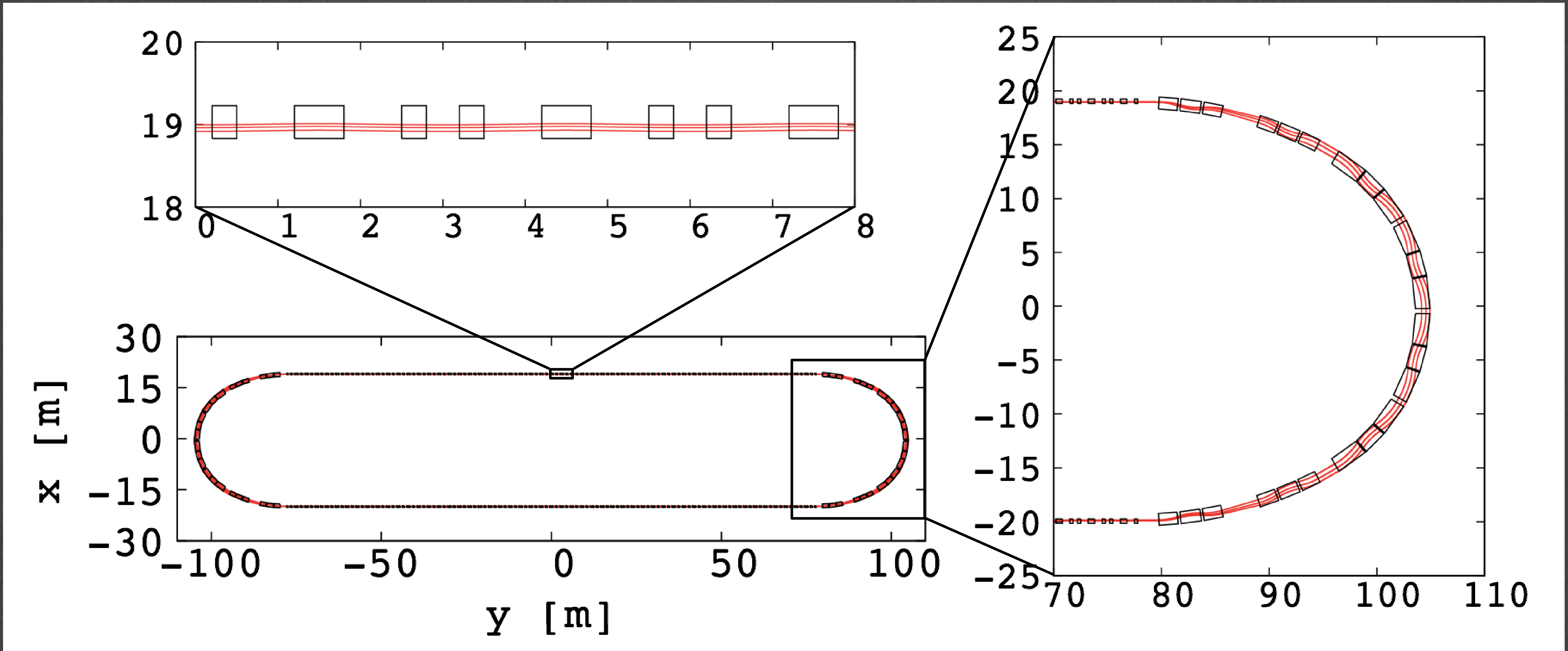
→ Racetrack to collect the maximum decayed neutrinos.

Conventional racetrack storage ring has small longitudinal acceptance.

Dramatically reduces the brightness at the detector.

→ Racetrack FFAG design

FFAG decay ring for nuSTORM



Large transverse acceptance (1000π mm.mrad)

Large momentum acceptance (±16%, up to ±25%)

Summary

- FFAGs have complicated magnetic field configuration, that require a strong collaboration between beam dynamics and magnet designs.
- Usually beam dynamics and magnet design are studied by the same people.
- You are welcome to join us for exciting challenges!

Thank you for your attention

back-up slides

Experimental measurement

$$\begin{pmatrix} x_1 \\ x'_1 \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{\beta_1}{\beta_0}} \cos\psi & a_{12} \\ \frac{-\alpha_1 \cos\psi - \sin\psi}{\sqrt{\beta_1 \beta_0}} & a_{22} \end{pmatrix} \cdot \begin{pmatrix} x_0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \tan\psi = -\alpha_1 - \frac{\beta_1 x'_1}{x_1}$$

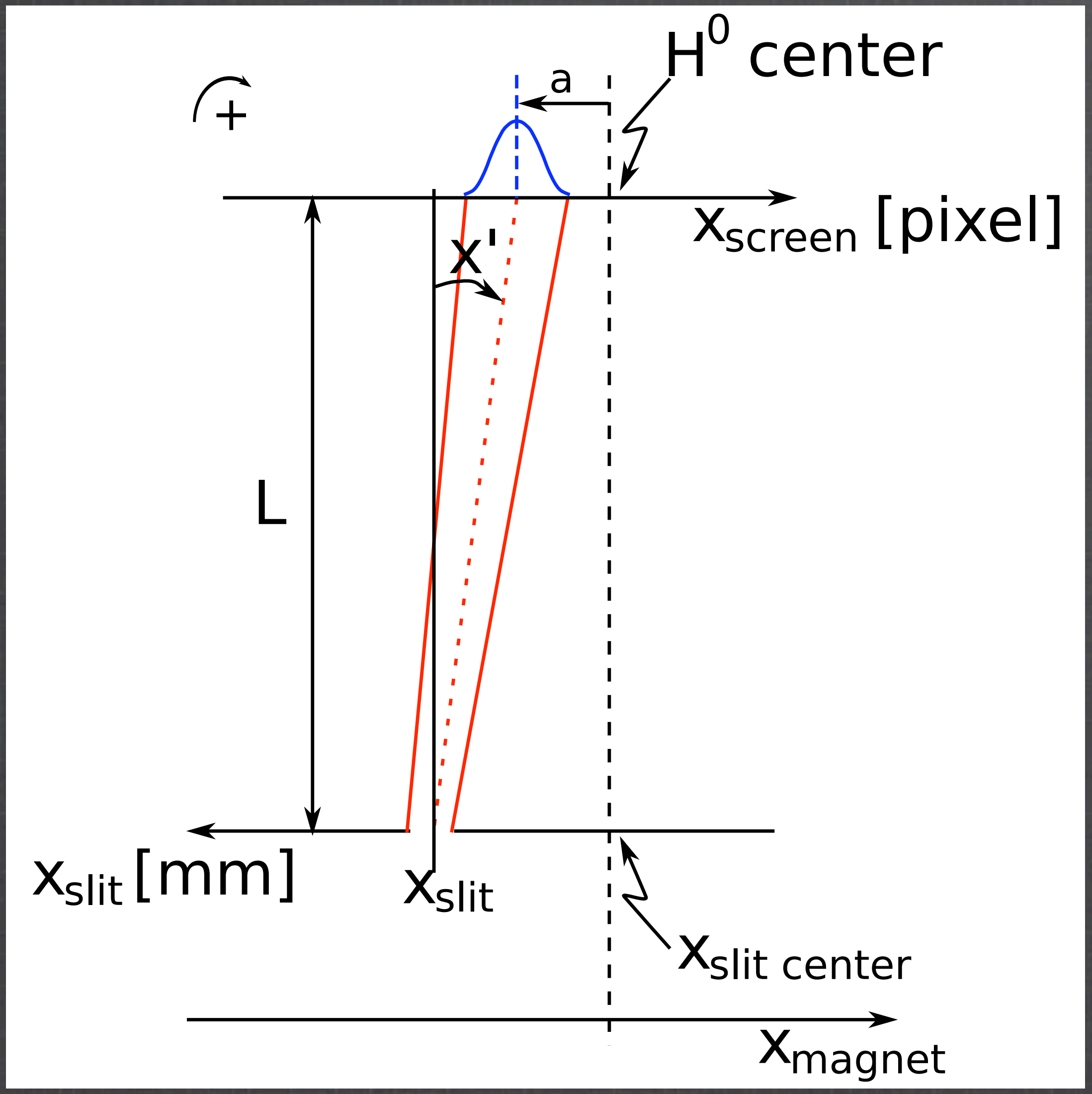
Exit parameters to measure: x_1, x'_1, β_1 and α_1.

For each energy, the beam is launched 3 times:

- on the reference trajectory,
- -10 mm off the reference trajectory,
- +10 mm off the reference trajectory.

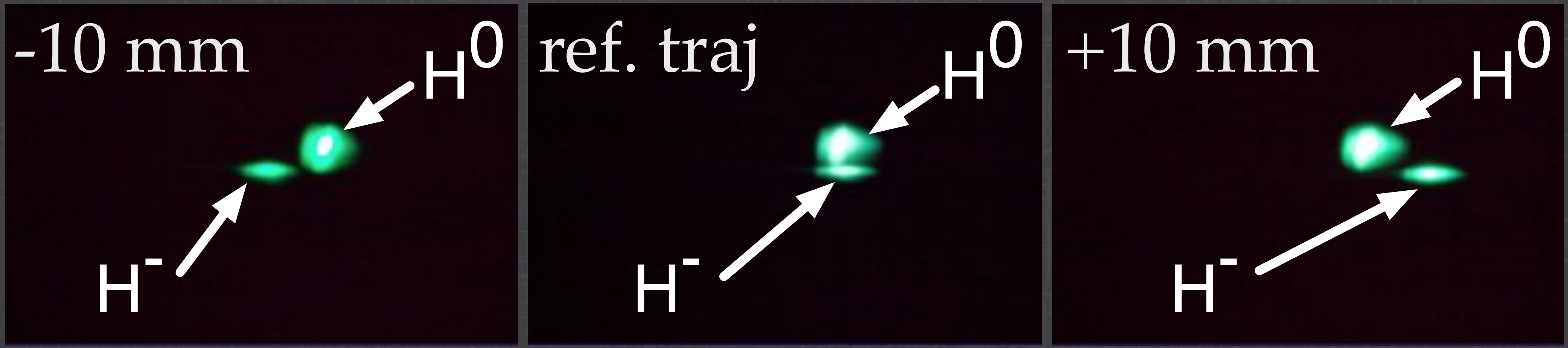
$$\Rightarrow \quad x_1 = \frac{x_{+10} - x_{-10}}{2}, \qquad x'_1 = \frac{x'_{+10} - x'_{-10}}{2}$$

Experimental measurement



angle measurement scheme.

position and beta measurement from pictures without slit.



Experimental measurement

α_1 measurement from

- the slope of the line x' vs. x_{slit}: $slope = -\left(\frac{\alpha}{\beta}\right)_{slit}$
- the beta value

→ Drift transfer matrix tracking to obtain α_1.