KU LEUVEN kulak

Electromagnetic Field Simulation for Accelerator Magnets

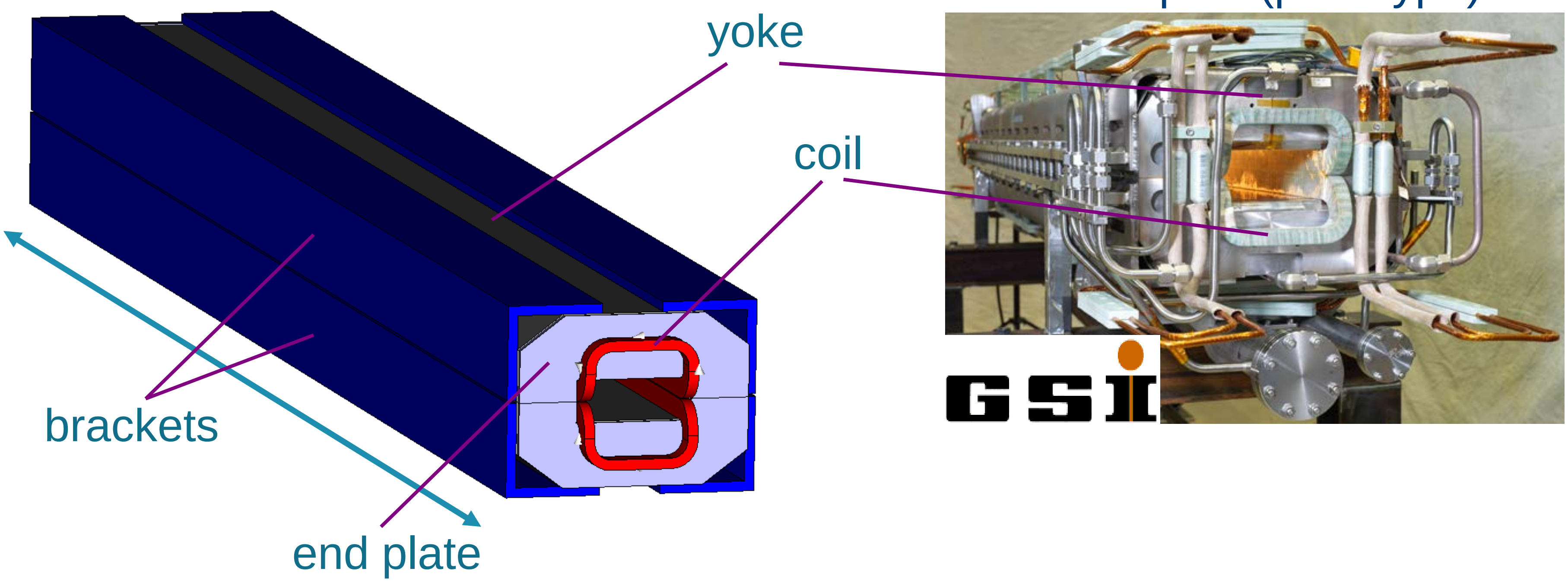
Herbert De Gersem

+ contributions from B. Masschaele, A. Mierau, S. Koch, T. Roggen, U. Römer, S. Schöps, J. Trommler, T. Weiland

Darmstadt, 2nd December 2013

Example: GSI-SIS-100 magnet

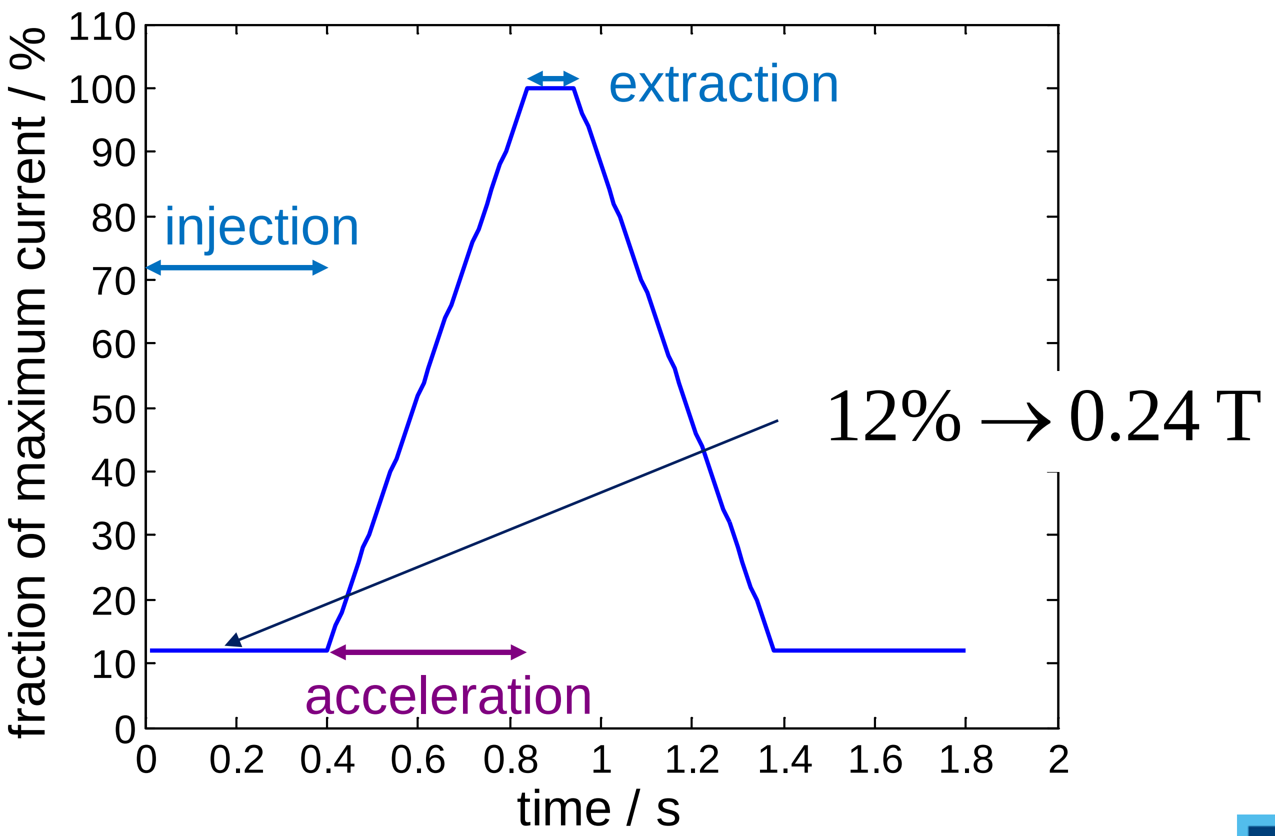
SIS100 dipole (prototype)



length: 3 m

Example: GSI-SIS-100 magnet

excitation profile



Magnetoquasistatic formulation

differential equation: $$\nabla \times \left(\nu \nabla \times \vec{A} \right) + \sigma \frac{\partial \vec{A}}{\partial t} = \vec{J}_{\mathrm{s}}$$

reluctivity

magnetic vector potential

conductivity

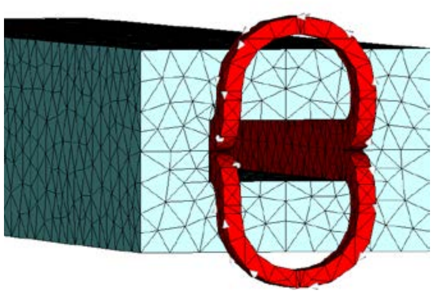
applied current density

Discretisation in space

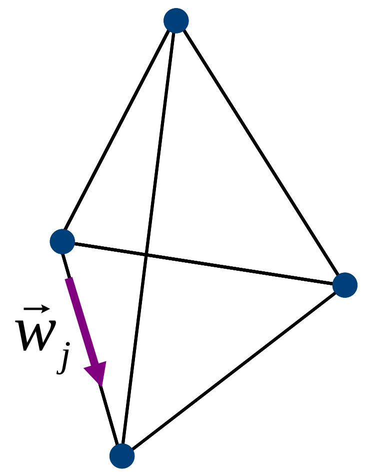
differential equation: $$\nabla\times\left(\nu\nabla\times\vec{A}\right)+\sigma\frac{\partial\vec{A}}{\partial t}=\vec{J}_\mathrm{s}$$

spatial discretisation $$\vec{A}\approx\vec{A}_\mathrm{FE}=\sum_j\widehat{a}_j\vec{w}_j$$

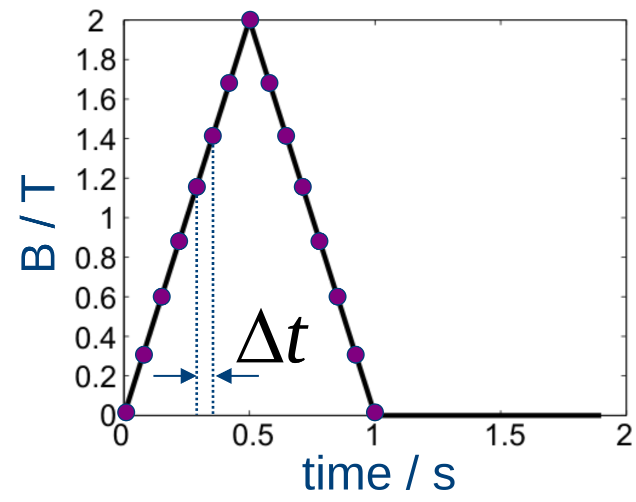
semi-discrete system: $$\mathbf{K}_\nu\widehat{\mathbf{a}}+\mathbf{M}_\sigma\frac{\mathrm{d}\widehat{\mathbf{a}}}{\mathrm{d}t}=\widehat{\mathbf{j}}_\mathrm{s}$$



shape functions:
edge finite elements
(curl-conforming)



Discretisation in time



differential equation: $\nabla\times\left(\nu\nabla\times\right.$

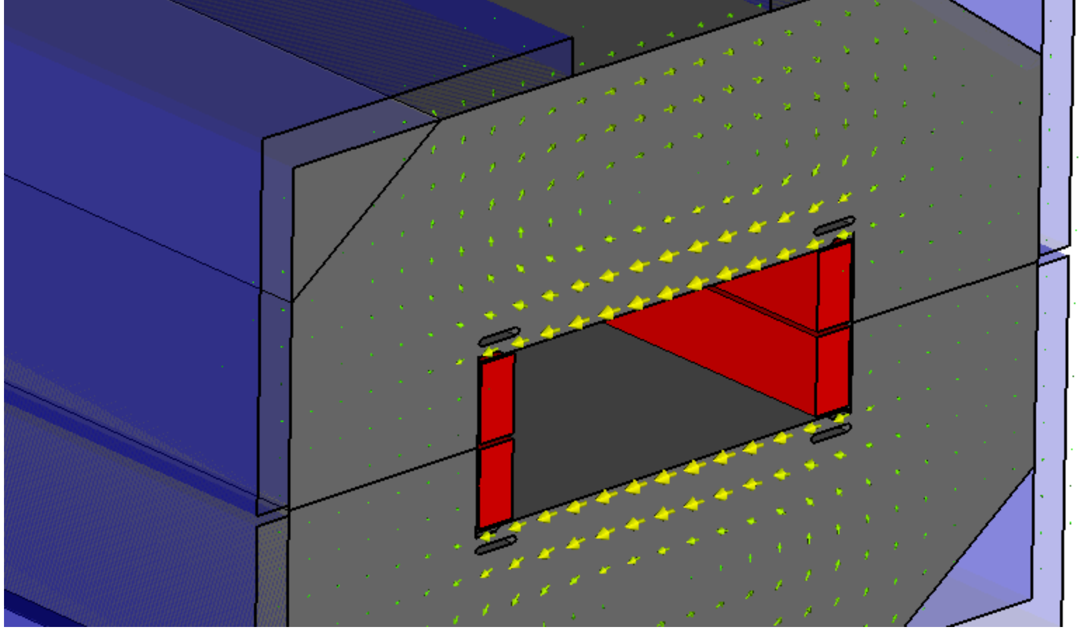
spatial discretisation

semi-discrete system: $$\mathbf{K}_{\nu}\hat{\mathbf{a}}+\mathbf{M}_{\sigma}\frac{\mathrm{d}\hat{\mathbf{a}}}{\mathrm{d}t}=\hat{\hat{\mathbf{j}}}_{\mathrm{s}}$$

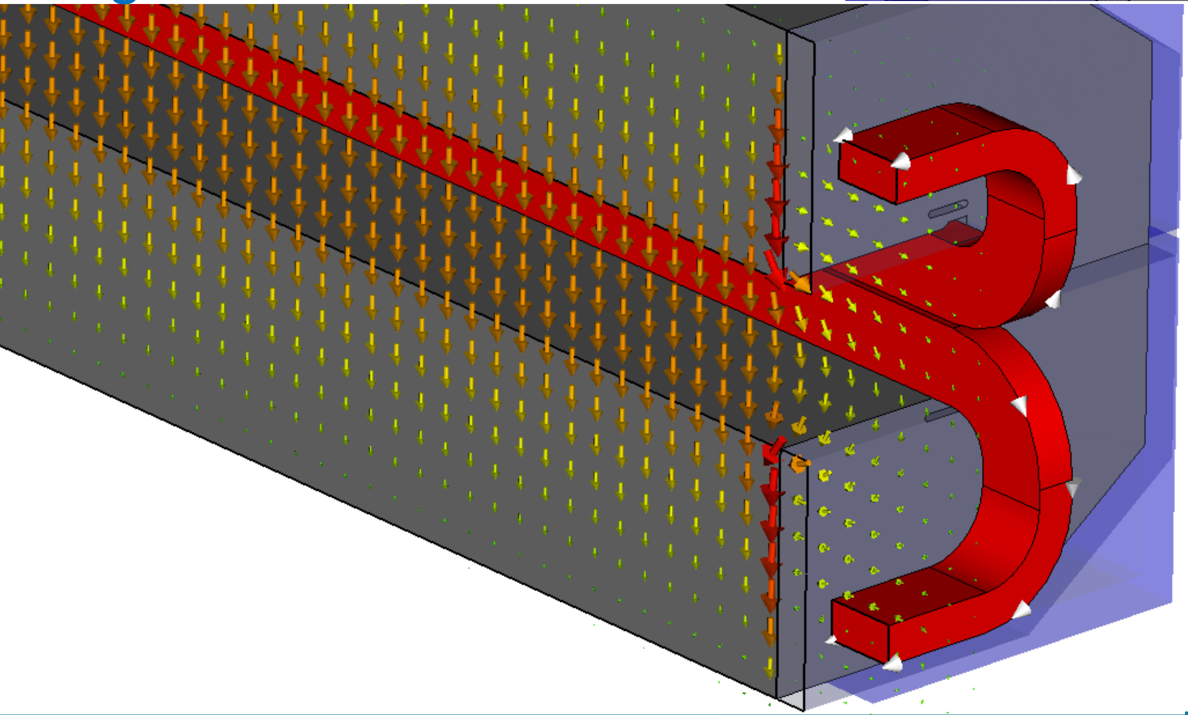
temporal discretisation

discrete system: $$\left(\mathbf{K}_{\nu}+\alpha\mathbf{M}_{\sigma}\right)\hat{\mathbf{a}}_{k+1}=f\left(\hat{\hat{\mathbf{j}}}_{\mathrm{s}},\hat{\mathbf{a}}_{k},\mathbf{K}_{\nu},\mathbf{M}_{\sigma}\right)$$

Results



magnetic field

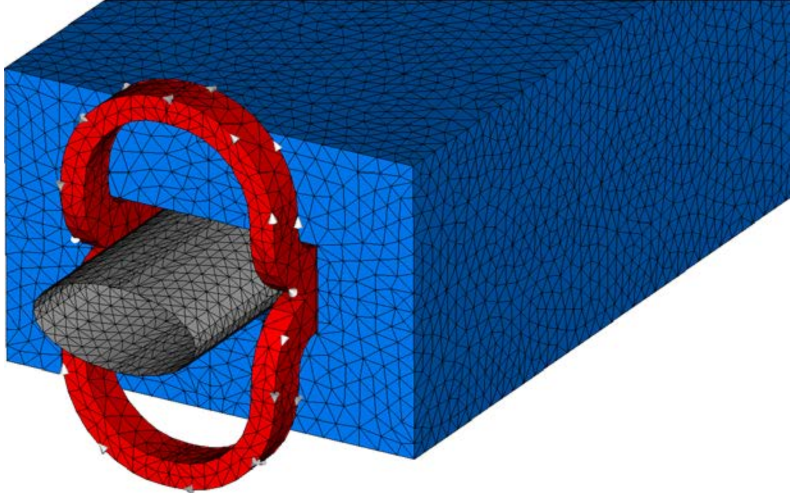


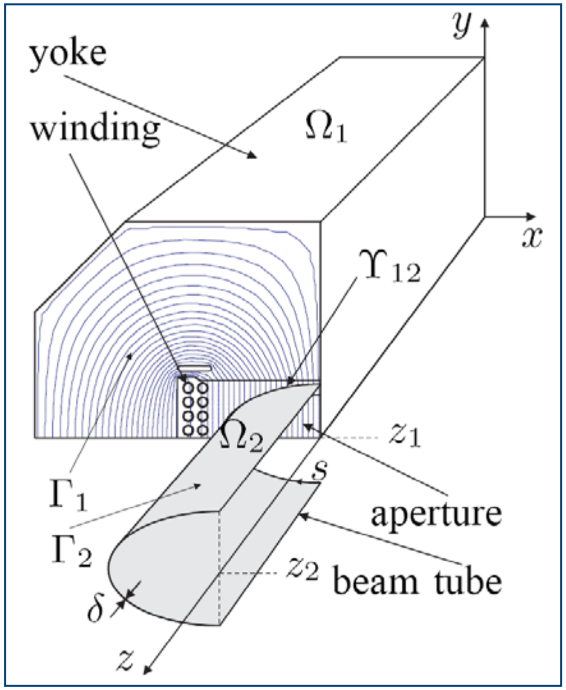
eddy currents
in the end plane

simulation by
CST EMStudio®

Overview

- magnet simulation (standard 3D FE solver)
- challenges
  - o geometrical details
  - o materials
  - o transient effects
  - o high accuracy
- magnet simulation (dedicated 3D FE solver)
- hybrid models
  - hybrid discretisation
  - domain decomposition
- conclusions





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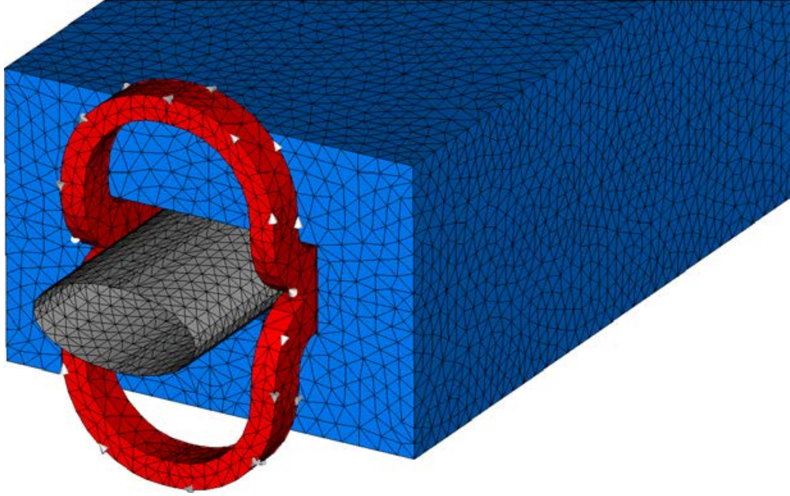
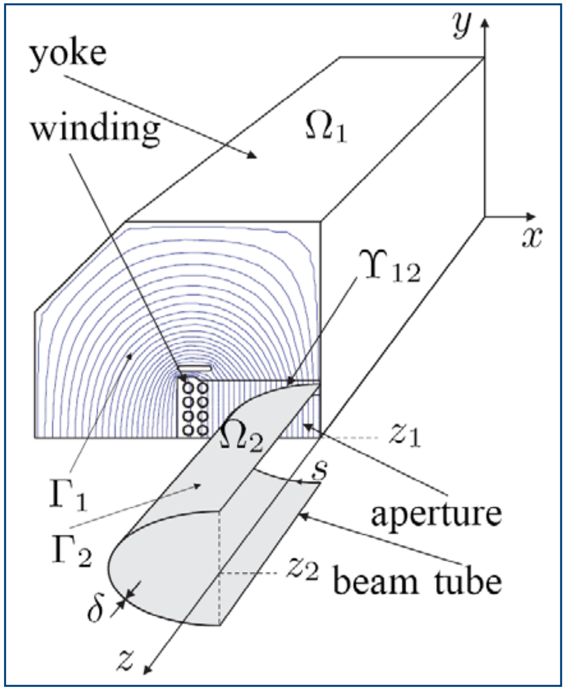




Challenge 1: Detailed geometry

yoke

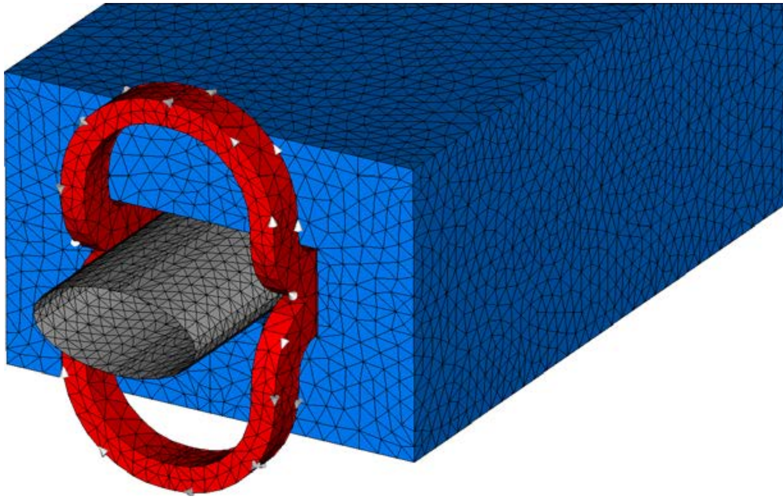
- length (meter)
  vs. lamination thickness (mm)
- shimming, holes

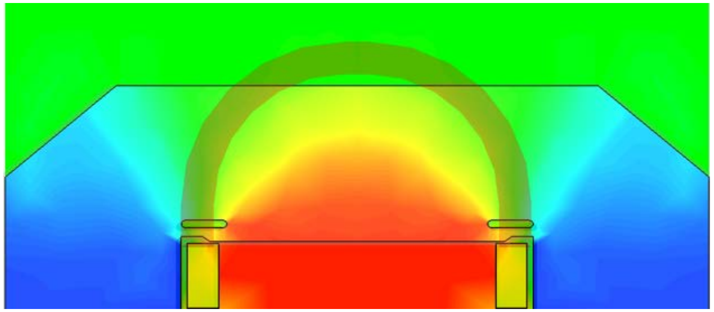
beam tube

- < 1mm thick

end-winding parts

- determine the eddy currents
  in the end plates





Challenge 2: Materials

yoke iron:

- anisotropic (rolling & transverse direction)

$$\overline{\overline{\nu}}(\vec{B}) = R^T \begin{bmatrix} \nu_{\text{rol}} & & \\ & \nu_{\text{trans}} & \\ & & \nu_{\text{trans}} \end{bmatrix} R$$

ν_{rol} reluctivity in the rolling direction

ν_{trans} reluctivity in the transversal direction

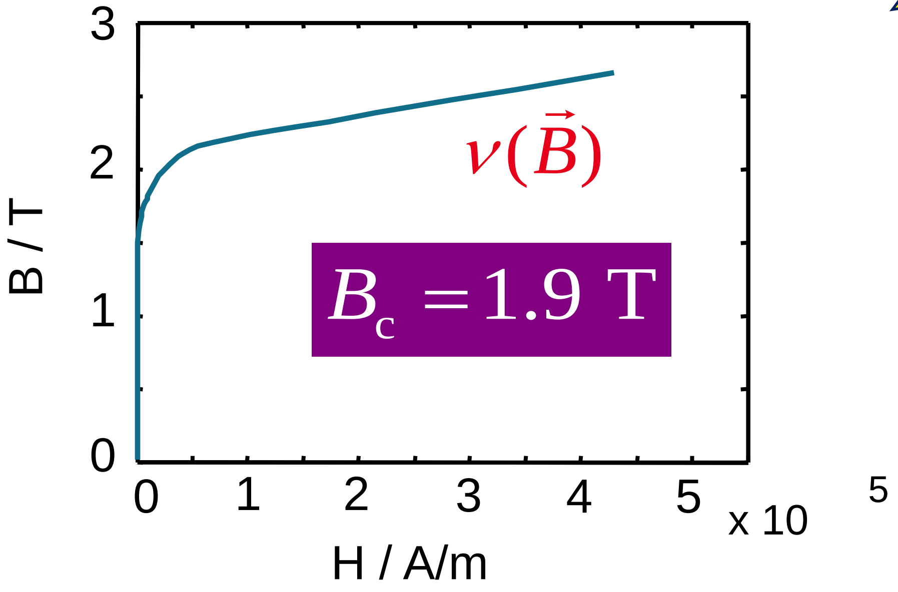
R local rotation matrix

Challenge 2: Materials

yoke iron:

- anisotropic (rolling & transverse direction)
- nonlinear (saturation)

Newton method



Challenge 2: Materials

yoke iron:

- anisotropic (rolling & transverse direction)
- nonlinear (saturation)
- hysteretic (remanent field)

Jiles-Atherton model

Preisach model

estimation of losses by Steinmetz-Bertotti

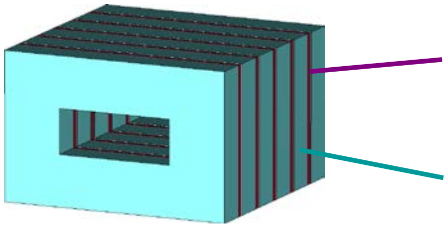
Challenge 2: Materials

yoke iron:

- anisotropic (rolling & transverse direction)
- nonlinear (saturation)
- hysteretic (remanent field)
- composite (lamination)

stacking factor

$$\gamma_{\mathrm{st}} \approx 0.95 \leq\sim 1$$



(simple) homogenisation along lamination direction

$$\frac{1}{\nu_{xy}} = \frac{\gamma_{\mathrm{st}}}{\nu_{\mathrm{Fe}}} + \frac{1-\gamma_{\mathrm{st}}}{\nu_0}$$

perpendicular to laminates

$$\nu_z = \gamma_{\mathrm{st}}\nu_{\mathrm{Fe}} + \left(1-\gamma_{\mathrm{st}}\right)\nu_0$$

Challenge 2: Materials

yoke iron:

- anisotropic (rolling & transverse direction)
- nonlinear (saturation)
- hysteretic (remanent field)
- composite (lamination)
- variability

stochastics, sensitivity

see presentation of Sebastian Schöps

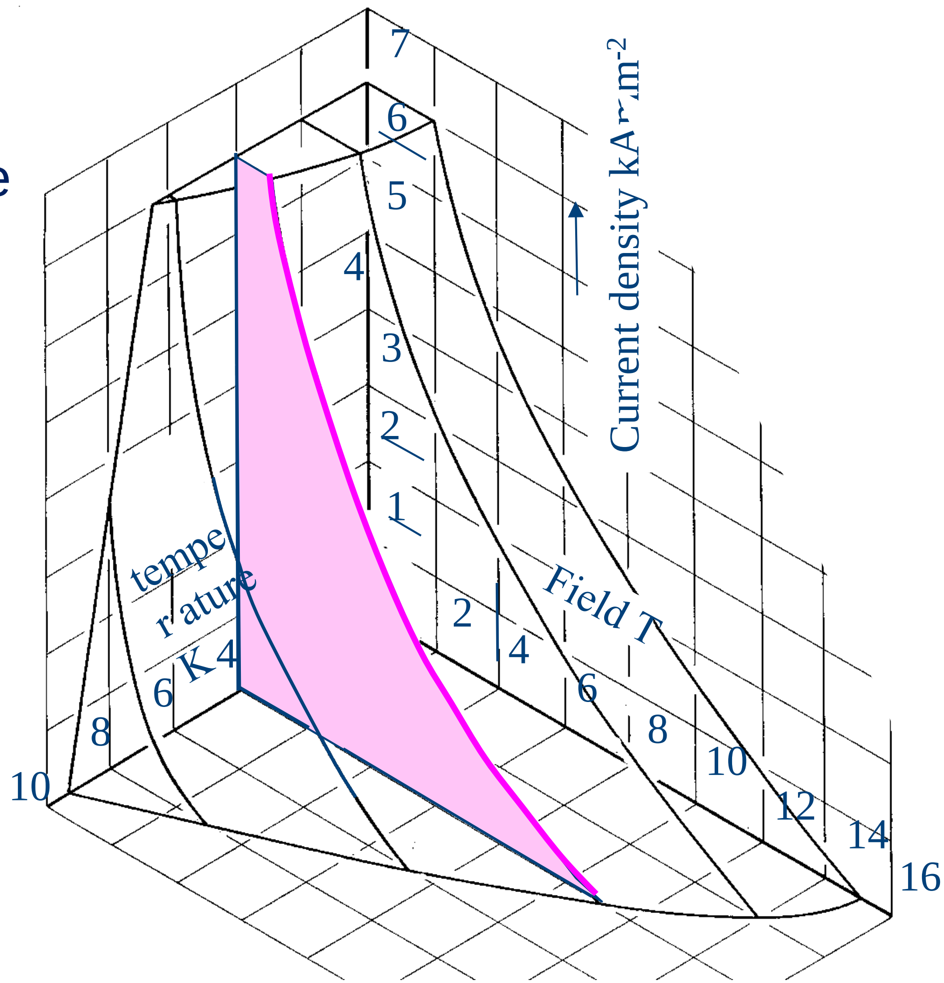
Challenge 2: Materials

yoke iron:

- anisotropic (rolling & transverse
- nonlinear (saturation)
- hysteretic (remanent field)
- composite (lamination)
- variability

superconductor:

- critical current
- temperature
- magnetic field



Challenge 3: Transient phenomena

lamination

- hysteresis + remanence

Jiles-Atherton model

Preisach model

estimation of the remanence
(based on data from material vendor)

Challenge 3: Transient phenomena

lamination

- hysteresis + remanence
- eddy currents

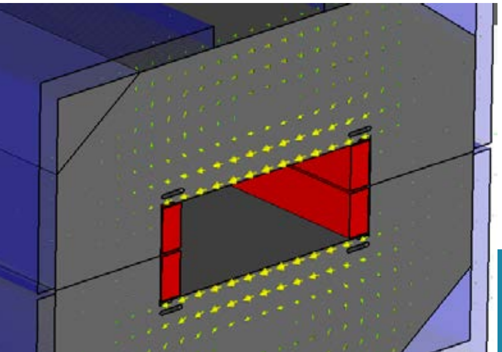
$$\nabla \times \left(\nu \nabla \times \vec{A} \right) + \sigma \frac{\partial \vec{A}}{\partial t} = \vec{J}_{\mathrm{s}}$$

eddy current term

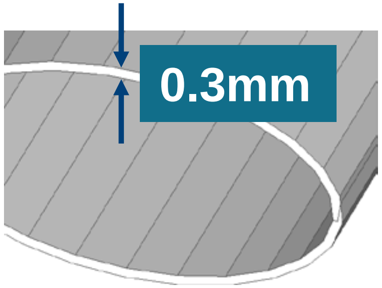
+ (simple) homogenisation $\sigma_{xy} = \gamma_{\mathrm{st}} \sigma_{\mathrm{Fe}}$

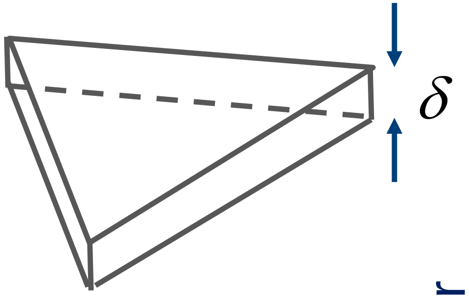
$$\sigma_z = 0$$

or + multi-scale model (hand-shaking)



Challenge 3: Transient phenomena





lamination

- hysteresis + remanence
- eddy currents

beam tube

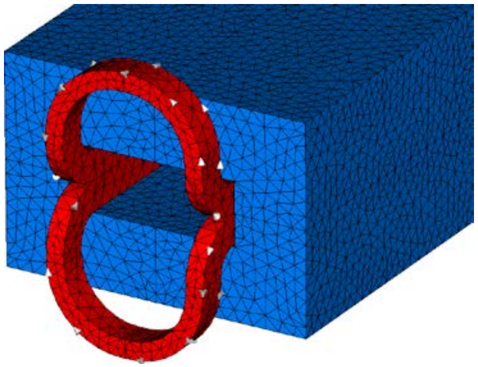
- eddy currents

shell elements

additional matrix contributions $\mathbf{K}_\delta$ and $\mathbf{M}_\delta$

assembling into system matrix by $\mathbf{Q}$

$$\mathbf{K}_\nu + \sigma \mathbf{M}_\sigma + \mathbf{Q}^{\mathrm{T}}\left(\mathbf{K}_\delta + \alpha \mathbf{M}_\delta\right)\mathbf{Q} = \mathbf{K}_{\mathrm{full}} + \alpha \mathbf{M}_{\mathrm{full}}$$



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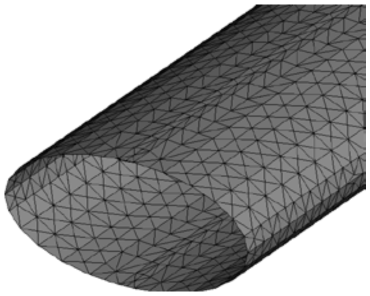

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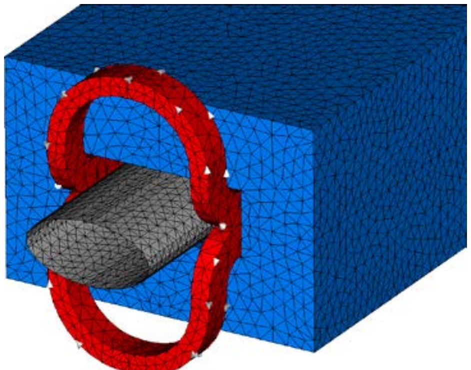


+ Jens Trommler

Challenge 3: Transient phenomena

lamination

- hysteresis + remanence
- eddy currents

beam tube

- eddy currents

superconductor

- persistent currents

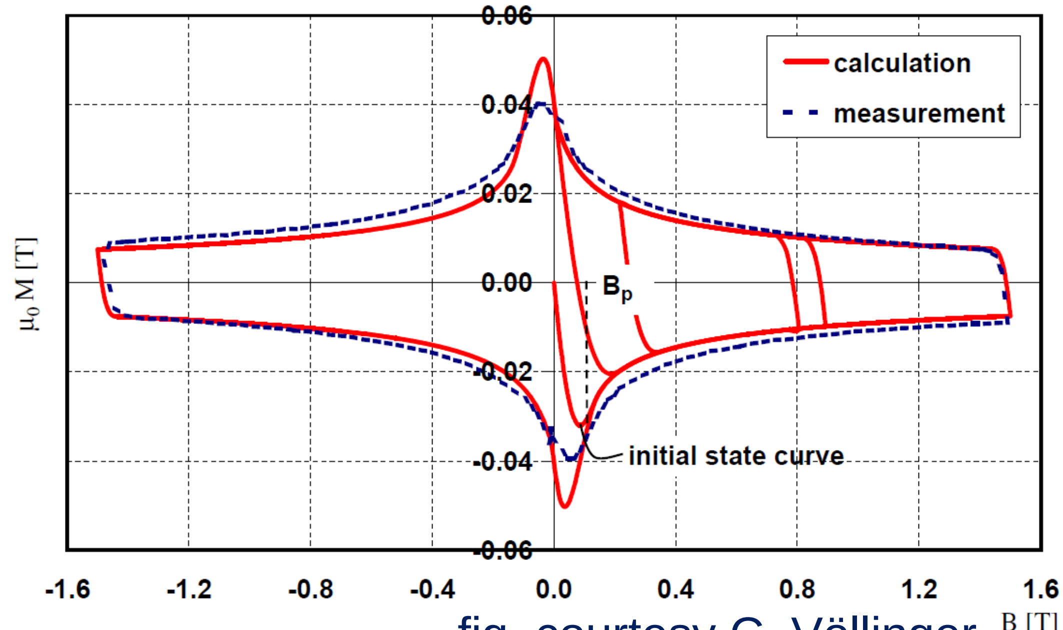


fig. courtesy C. Völlinger

Bean model → magnetisation (Christine Völlinger) implemented in ROXIE

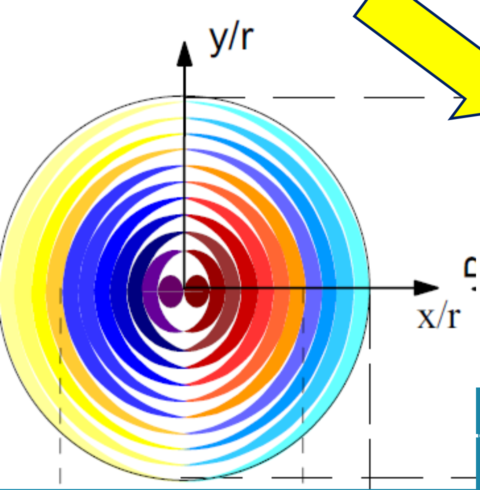
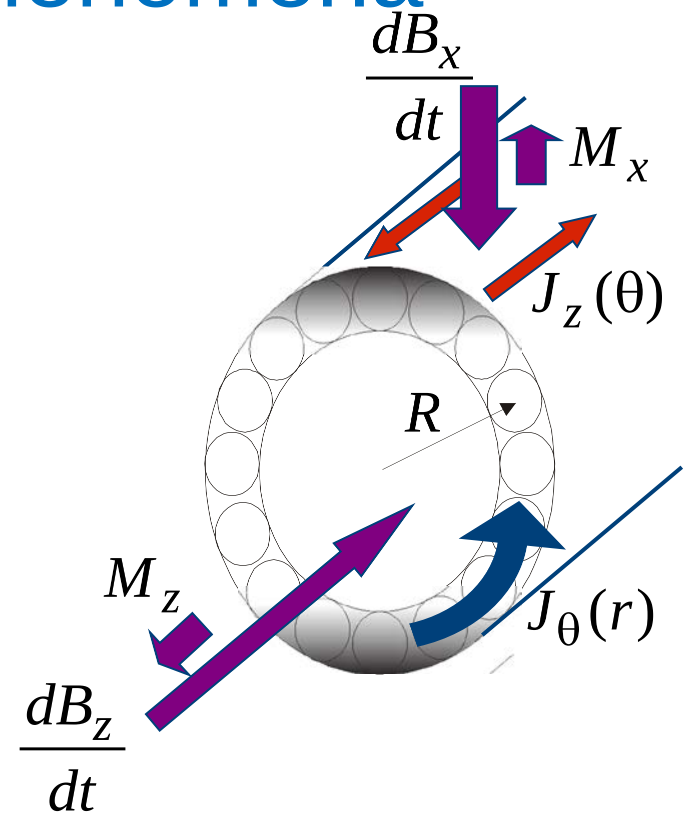


fig. courtesy C. Völlinger

Challenge 3: Transient phenomena



lamination

- hysteresis + remanence
- eddy currents

beam tube

- eddy currents

superconductor

- persistent currents
- coupling currents
- cable eddy currents

$$\nabla\times\left(\nu\nabla\times\vec{A}\right)+\sigma\frac{\partial\vec{A}}{\partial t}+\nabla\times\left(\nu_0\overline{\overline{\tau}}_{\mathrm{cb}}\nabla\times\frac{\partial\vec{A}}{\partial t}\right)=\vec{J}_{\mathrm{s}}$$

additional magnetisation

Challenge 4: High accuracy requirements

losses

- dimensioning of the cooling system
- hot spots
- quench

aperture field

- multipoles during injection, ramping and extraction
- + influence of eddy currents

huge models

parallelisation, multi-core computers

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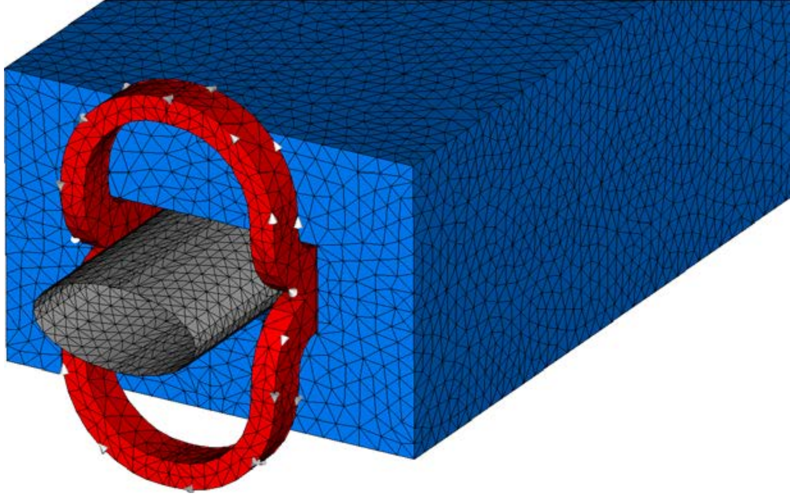
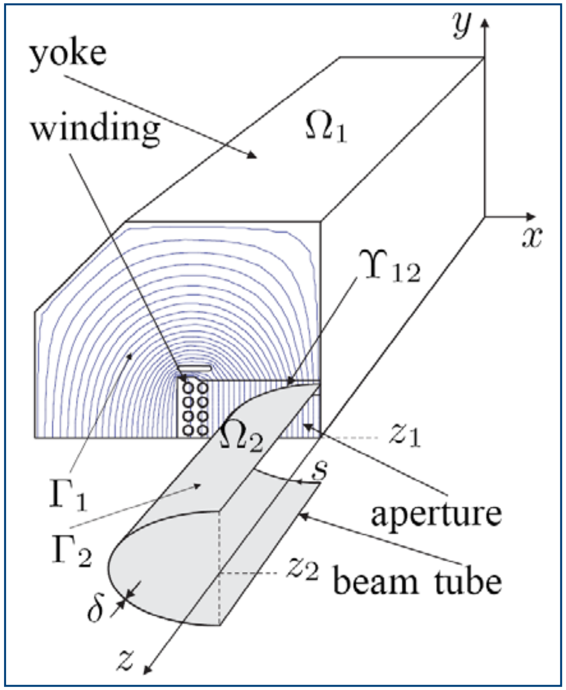
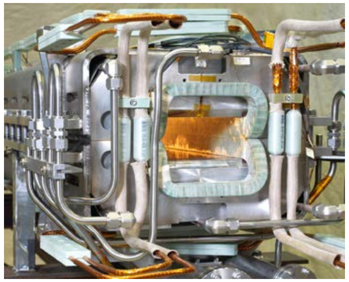
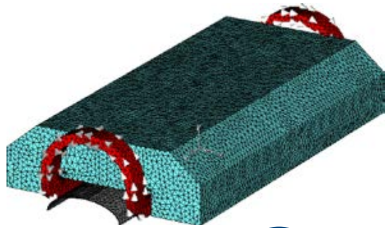




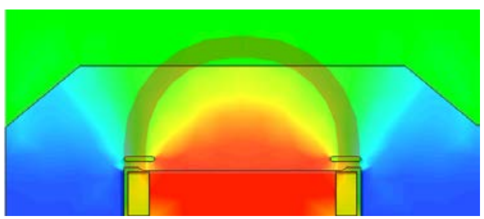
Dedicated Simulation Tool

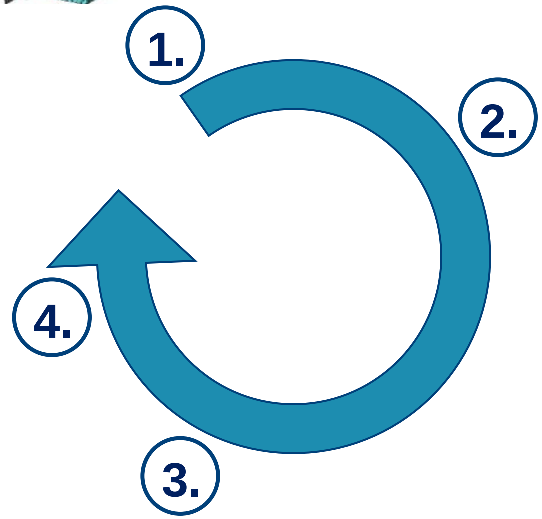




CST Studio Suite®
- CAD modelling
- meshing

- visualisation





Matlab
- postprocessing
- visualisation

FEMSTER, LLNL

TRILINOS, Sandia Labs

own software
- FE assembly (higher order FEs)
- transient solver
- nonlinear materials
- system solver

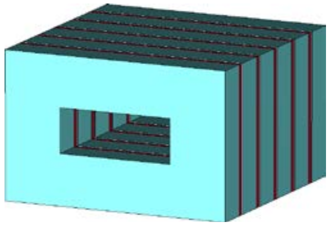
parallelisation

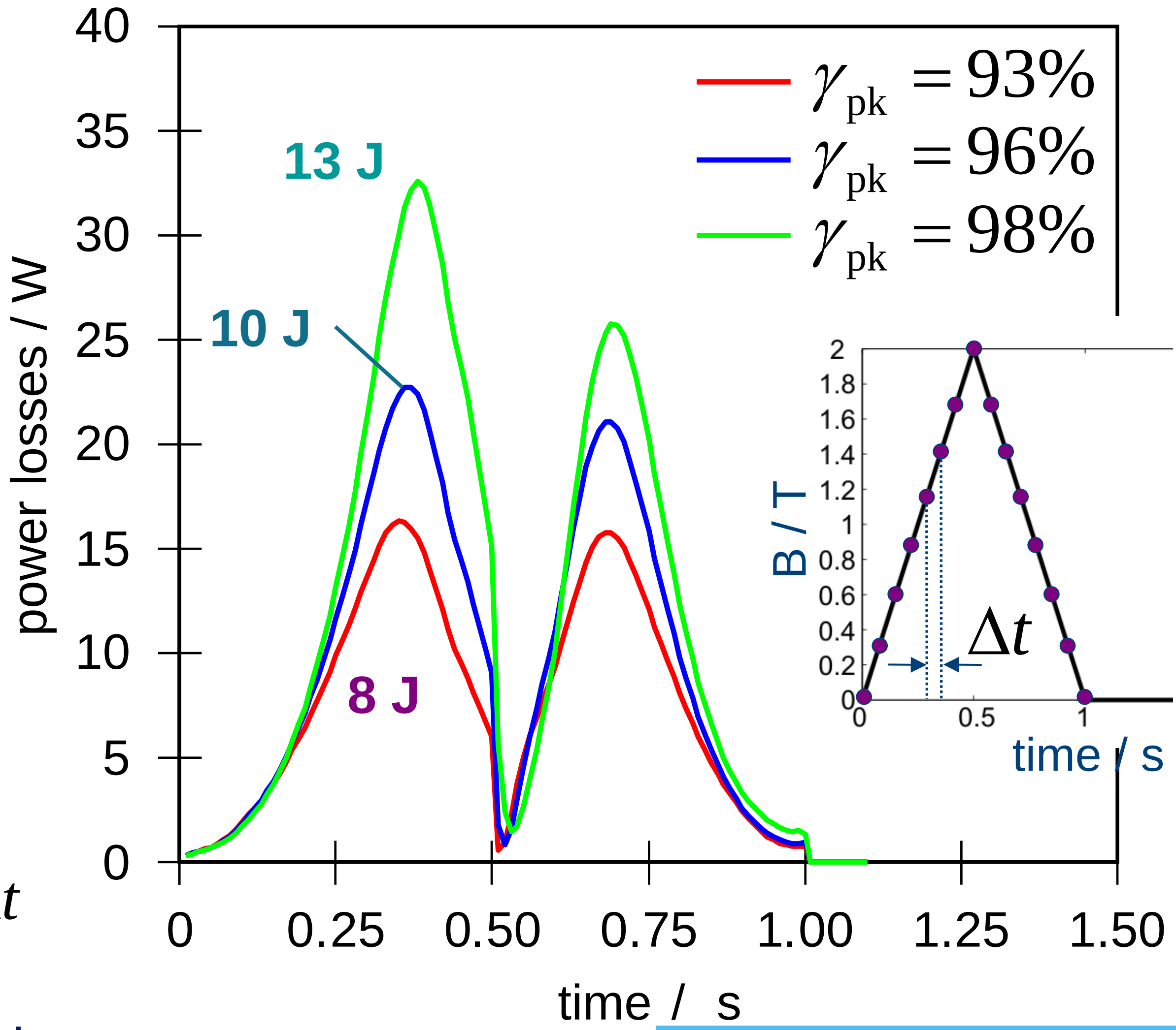
+ Stephan Koch, Jens Trommler

Results: Eddy-Current Losses

eddy-current losses over one cycle

for different stacking factors γ_{pk}





loss energy: $W = \int_0^T P \mathrm{d}t$

+ Stephan Koch, Jens Trommler

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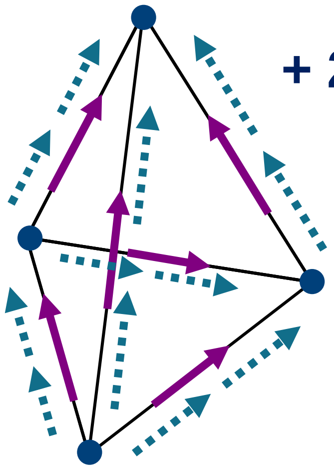
Results: Loss Energy

- discretization:
  - increase number of elements
  - increase order of approximation

$$\vec{A} \approx \vec{A}_{\mathrm{FE}} = \sum_j a_j \vec{w}_j^{\mathrm{tv}}$$

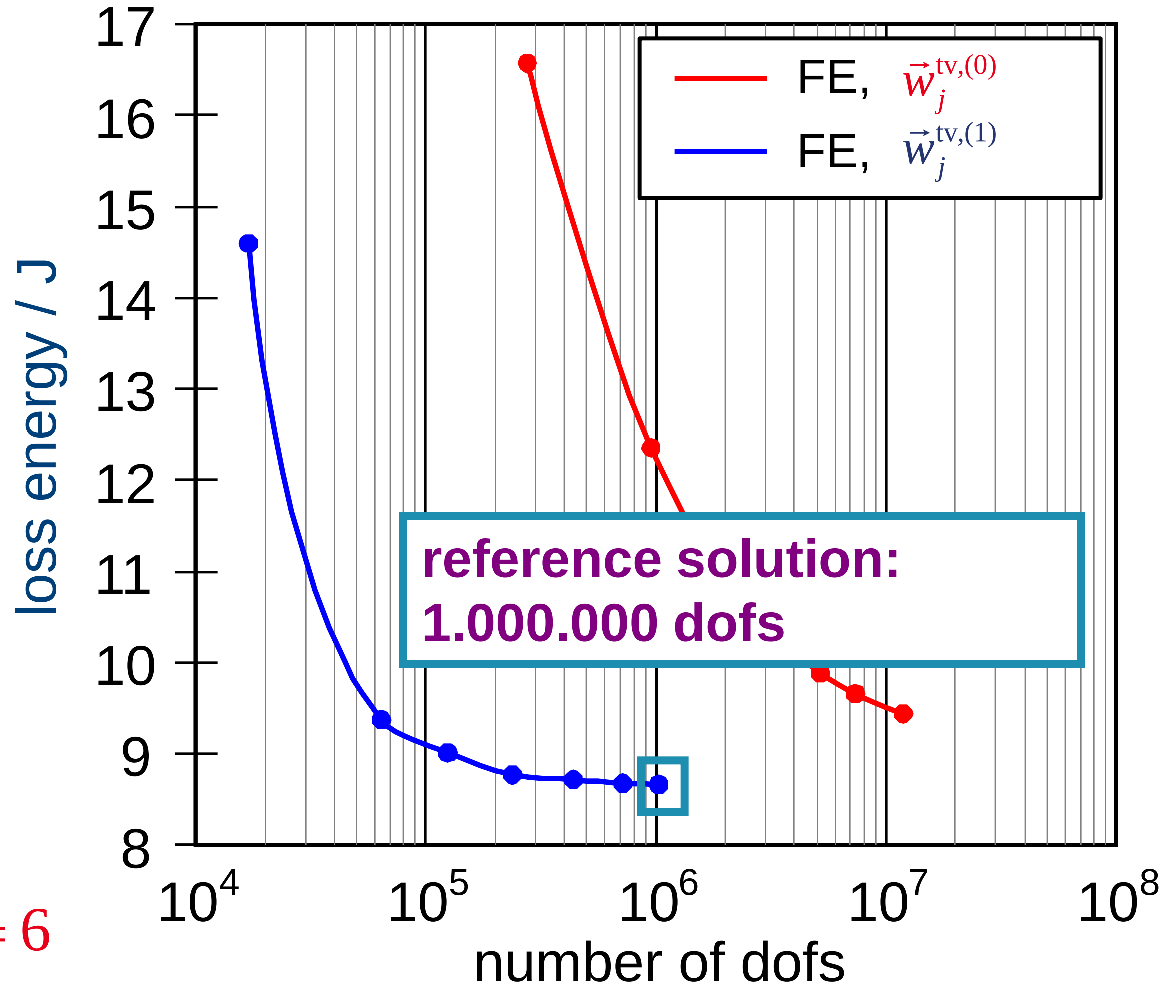
degrees of freedom:



+ 2 per face

$\vec{w}_j^{\mathrm{tv},(0)}$ $n_{\mathrm{dof}} = 6$

$\vec{w}_j^{\mathrm{tv},(1)}$ $n_{\mathrm{dof}} = 20$



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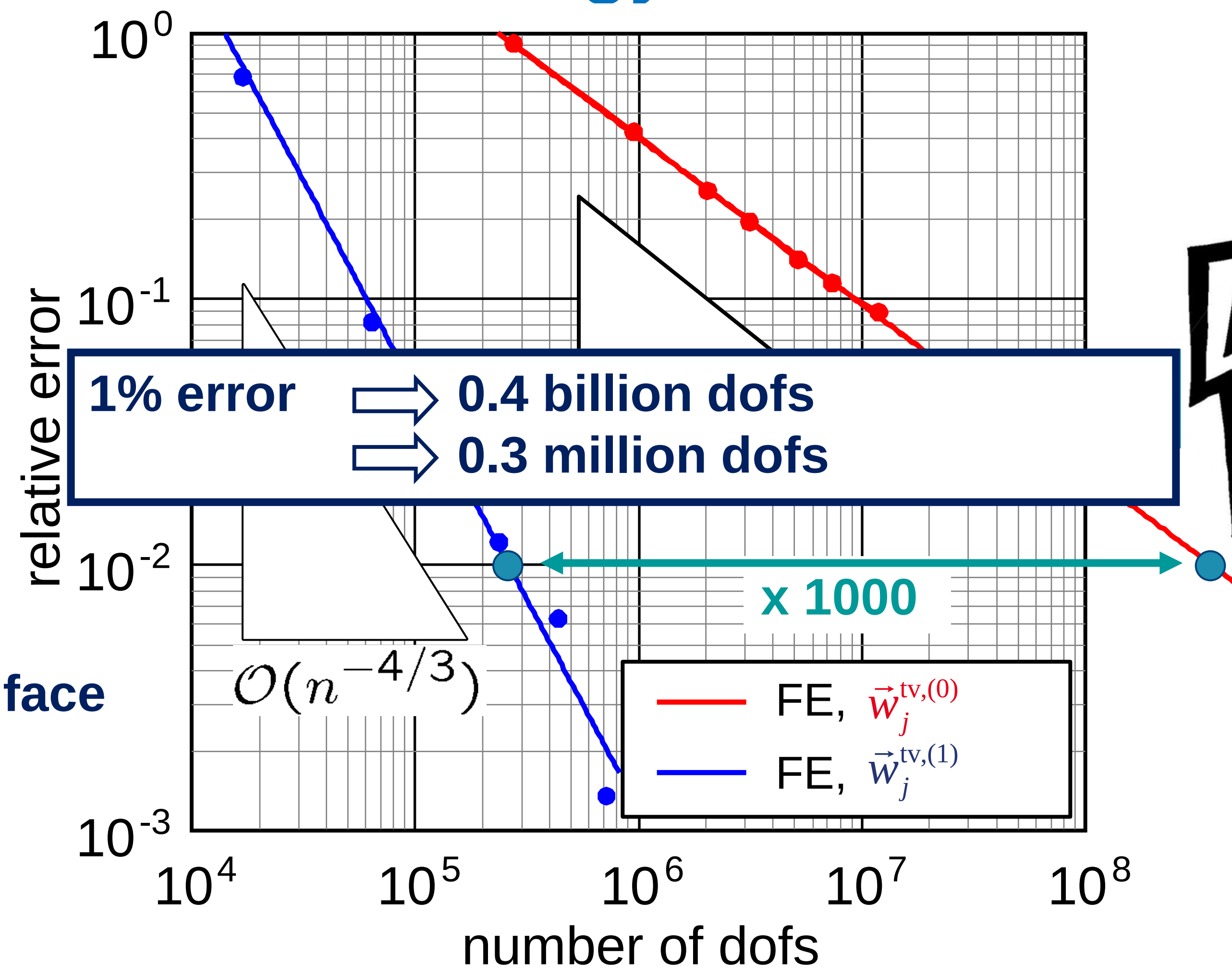
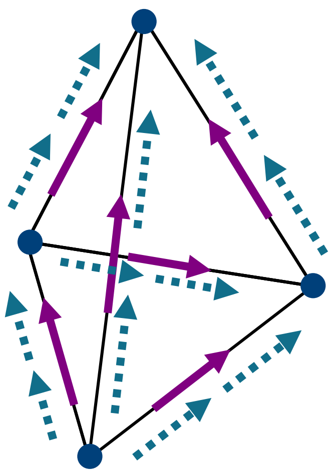
Convergence: Loss Energy

- **relative error with respect to reference solution**



1% error ⇨ 0.4 billion dofs
⇨ 0.3 million dofs

degrees of freedom:



+ 2 per face

$\vec{w}_j^{\mathrm{tv},(0)}$

$\vec{w}_j^{\mathrm{tv},(1)}$

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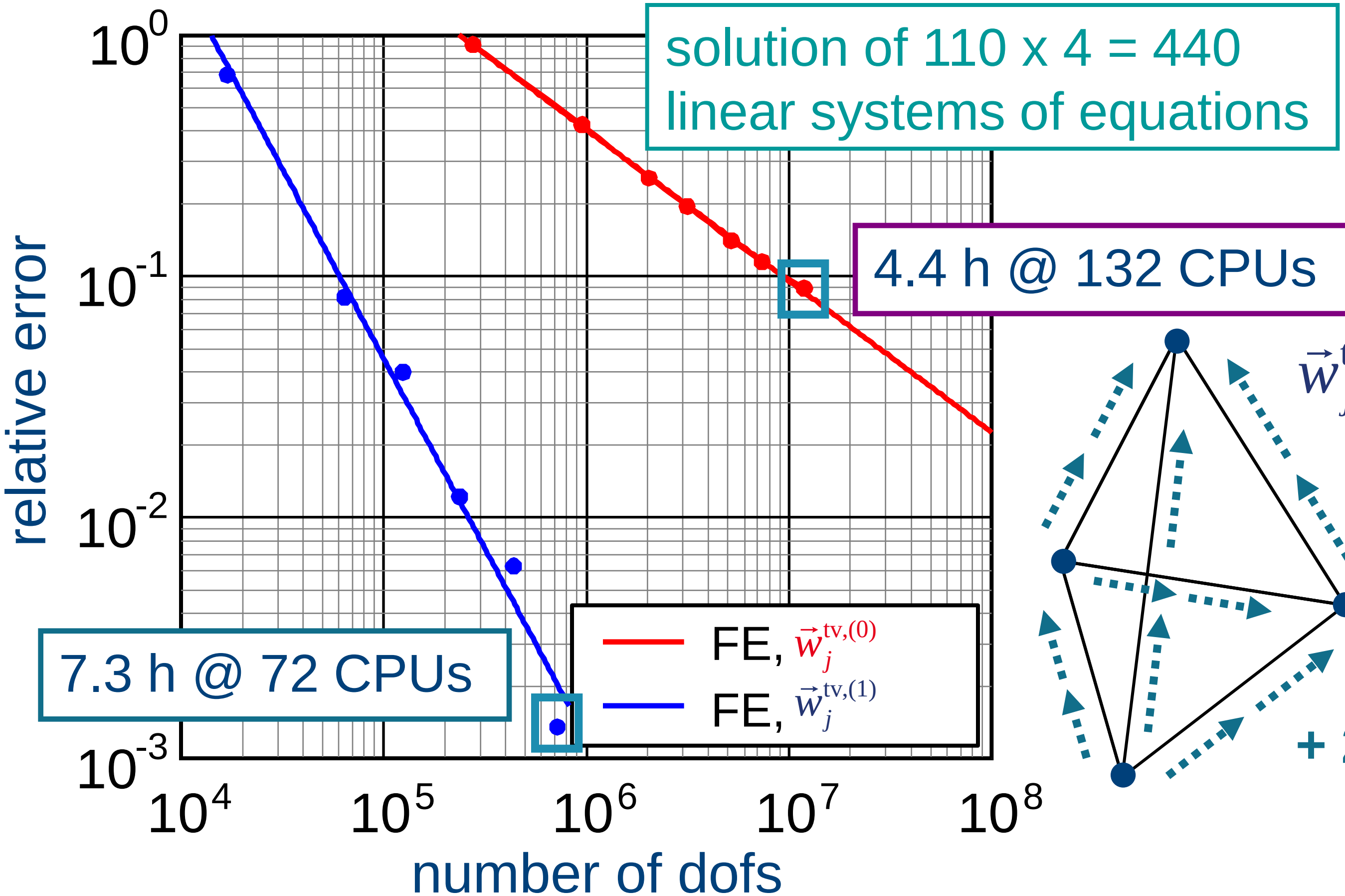
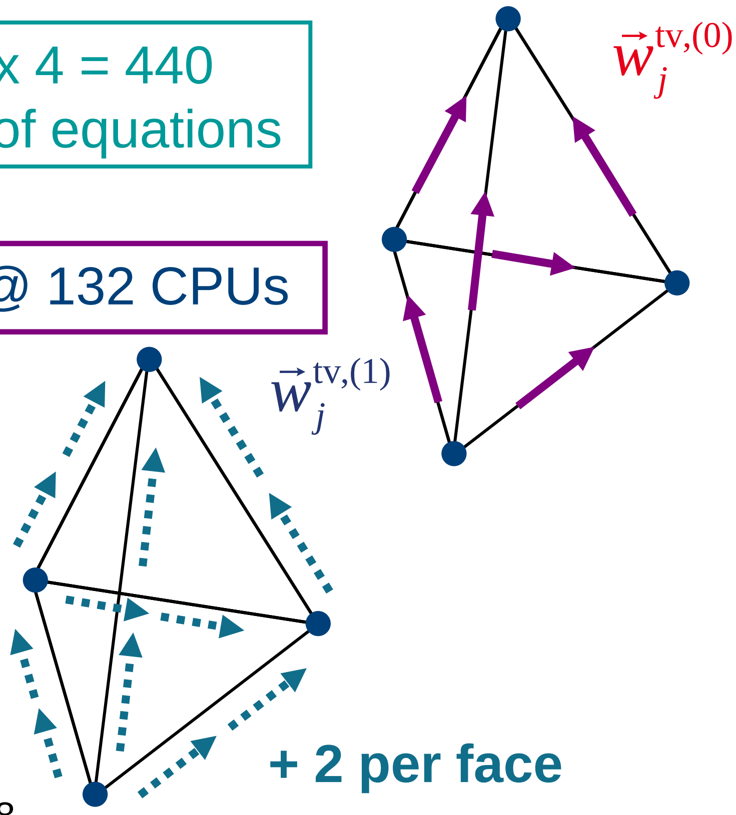
Comparison: Shape Functions





+ S. Koch, J. Trommler

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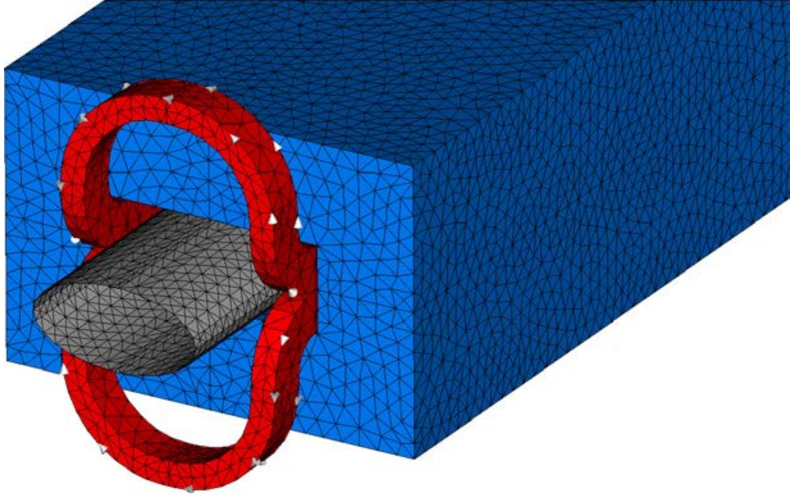
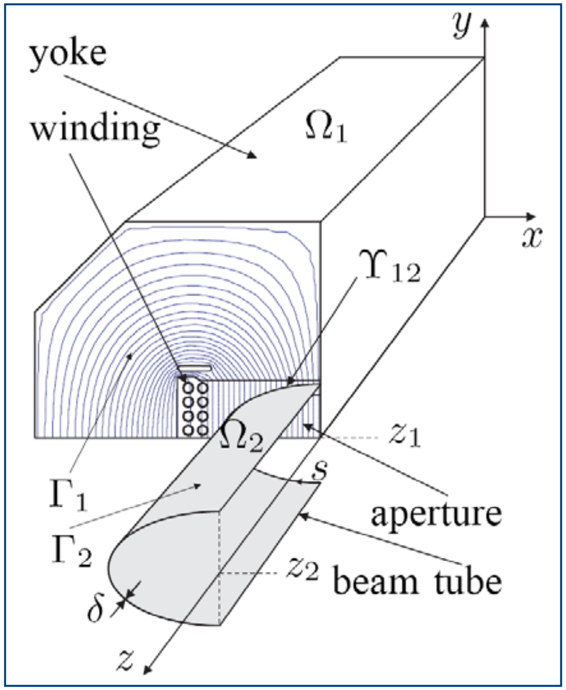
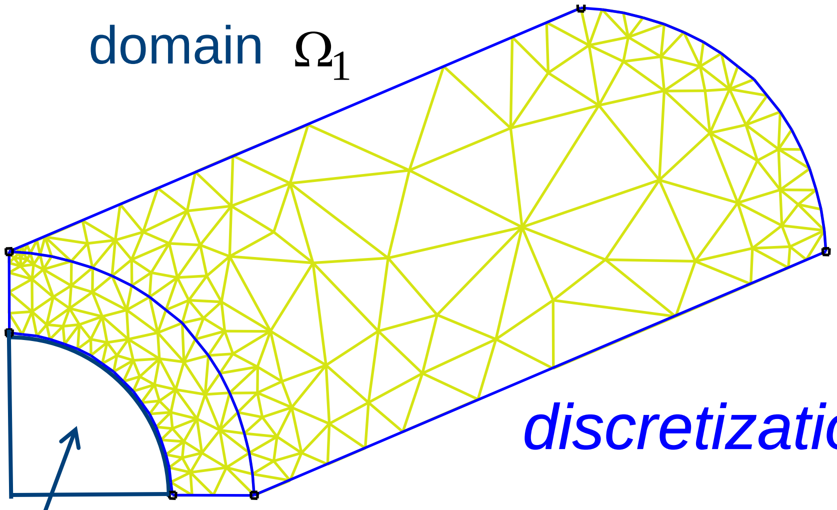
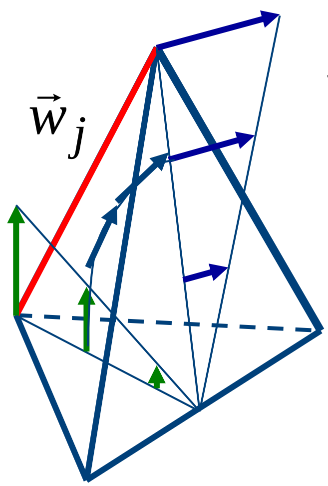




Hybrid formulation



$$\begin{cases} \vec{A} = \sum_j u_j \vec{w}_j & \text{in } \Omega_1 \\ \psi = \sum_q v_q N_q & \text{in } \Omega_2 \end{cases}$$



magnetoquasistatic formulation

$$\nabla \times \left(\nu \nabla \times \vec{A} \right) + \sigma \frac{\partial \vec{A}}{\partial t} = \vec{J}_s \quad \text{in } \Omega_1$$

$$-\nabla \cdot \left(\mu \nabla \psi \right) = 0 \quad \text{in } \Omega_2$$

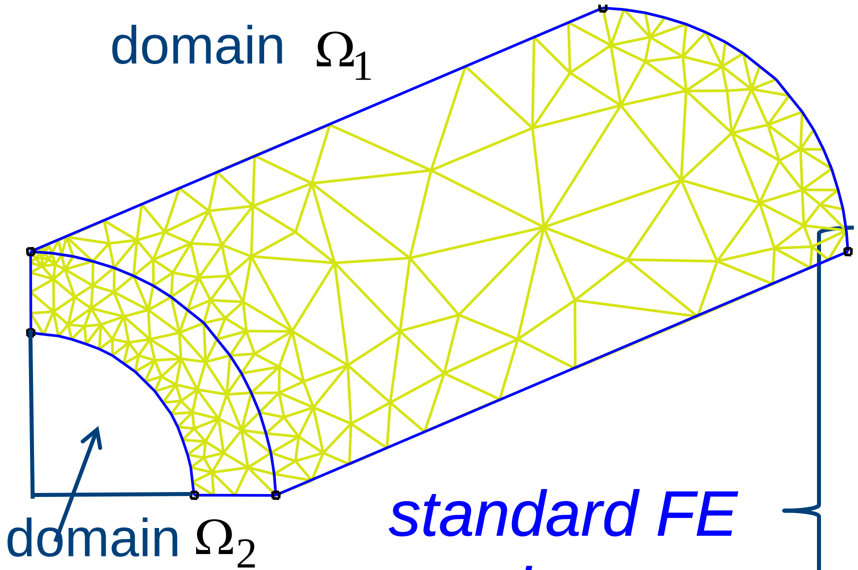
+ continuity requirements

$$\begin{cases} \vec{B}_1 = \nabla \times \vec{A} & \text{in } \Omega_1 \\ \vec{H}_2 = -\nabla \psi & \text{in } \Omega_2 \end{cases}$$

system of equations

$$\begin{bmatrix} \mathbf{K} + \mathbf{M} \frac{d}{dt} & \mathbf{B}^T \\ \mathbf{B} \frac{d}{dt} & -\mathbf{G} \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \mathbf{v} \end{bmatrix} = \begin{bmatrix} \mathbf{f} \\ 0 \end{bmatrix}$$

Hybrid formulation



system of equations

$$\begin{bmatrix} \mathbf{K} + \mathbf{M}\dfrac{d}{dt} & \mathbf{B} \\ \mathbf{B}^T & -\mathbf{G} \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \mathbf{v} \end{bmatrix} = \begin{bmatrix} \mathbf{f} \\ 0 \end{bmatrix}$$

standard FE matrices

$$\mathbf{K}_{ij} = \int_{\Omega_1} \left(\nu \nabla \times \vec{w}_j\right) \cdot \left(\nabla \times \vec{w}_i\right) d\Omega$$

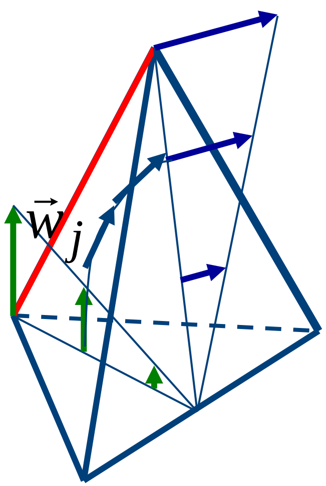
$$\mathbf{M}_{ij} = \int_{\Omega_1} \sigma \vec{w}_j \cdot \vec{w}_i \, d\Omega$$

$$\mathbf{f}_i = \int_{\Omega_1} \vec{J}_s \cdot \vec{w}_i \, d\Omega$$

$$\mathbf{G}_{pq} = \int_{\Omega_2} \mu \nabla N_q \cdot \nabla N_p \, d\Omega$$

coupling block

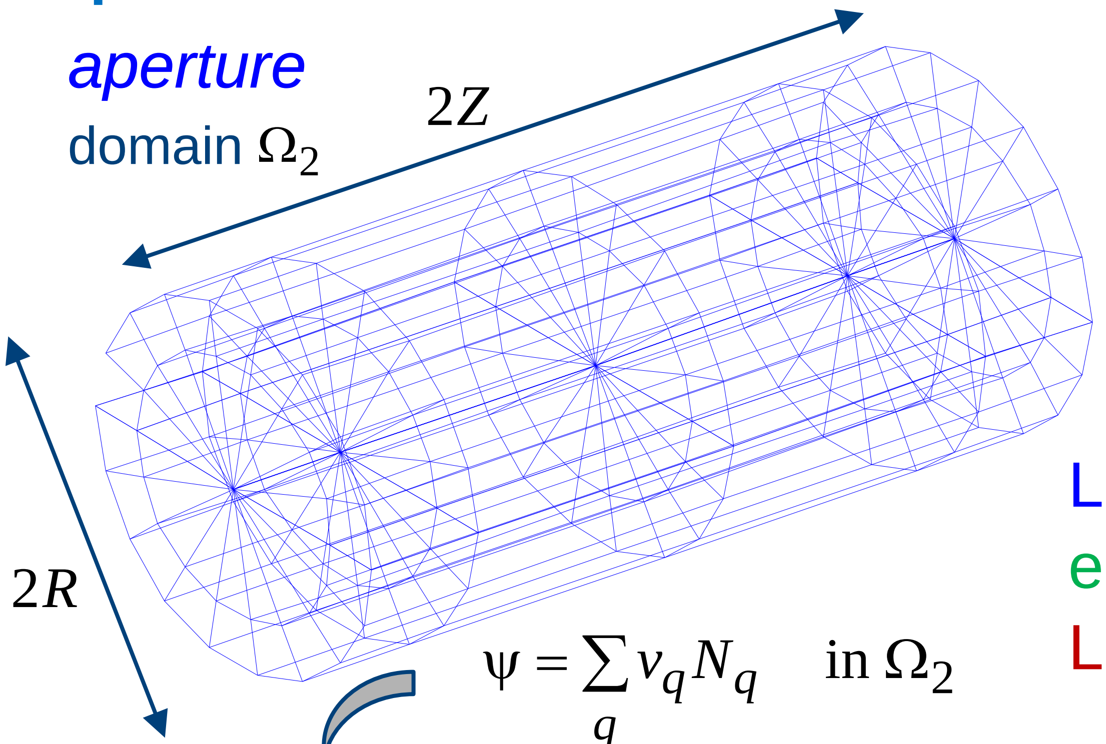
$$\mathbf{B}_{iq} = -\int_{\Omega_1 \cap \Omega_2} \left(\nabla N_q \times \vec{w}_i\right) \cdot d\vec{S}$$

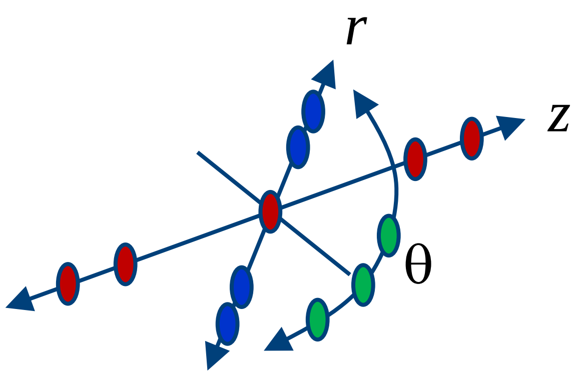


Spectral discretisation

aperture

domain Ω_2





Legendre distribution in *r*

equidistant distribution in *θ*

Legendre distribution in *z*

$$\psi = \sum_q v_q N_q \quad \text{in } \Omega_2$$

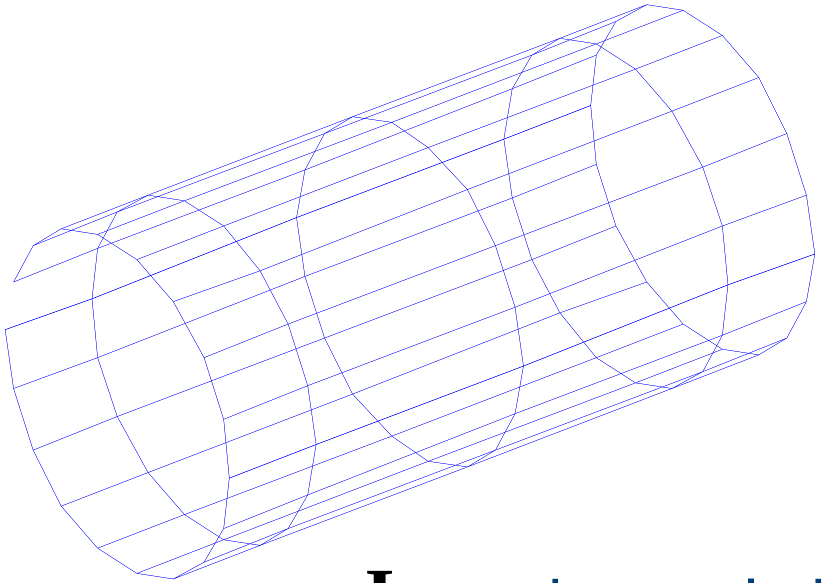
$$N_q(r,\theta,z) = N_{q_1,q_2,\lambda}(r,\theta,z) = P_{q_1}\left(\frac{r}{R}\right) e^{-j\lambda_q \theta} P_{q_2}\left(\frac{z}{Z}\right)$$

$$\mathbf{G} = \begin{bmatrix} \mathbf{D}_r^T & \mathbf{D}_\theta^T & \mathbf{D}_z^T \end{bmatrix} \mu_0 \begin{bmatrix} \mathbf{D}_r & \mathbf{D}_\theta & \mathbf{D}_z \end{bmatrix}^T$$

+FIT : Dehler, Weiland (1994)

in 2D : HDG, Clemens, Weiland (2003)

Coupling blocks



interface $\Omega_1 \cap \Omega_2$

$$\mathbf{B}_{iq} = -\int_{\Omega_1 \cap \Omega_2} \left(\nabla N_q \times \vec{w}_i \right) \cdot d\vec{S}$$

$$\mathbf{B}_{iq} = -\mathbf{I}_{\mathrm{col}}^{T} \mathbf{F}_{\mathrm{2d}}^{-1} \begin{bmatrix} \hat{\mathbf{D}}_{\theta} \\ \hat{\mathbf{D}}_{z} \end{bmatrix} \hat{\mathbf{Q}}$$

$\mathbf{I}_{\mathrm{col}}$ interpolation matrix

$\mathbf{F}_{\mathrm{2d}}$ 2D FFT

$\hat{\mathbf{D}}_{\theta}$ difference matrix restricted to the interface

$\hat{\mathbf{Q}}$ selection of points at the interface

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System solution (1)

1. full FE model used for reference (symmetric, definite)

$$\mathbf{A}_{\mathrm{ref}}\mathbf{u}_{\mathrm{ref}} = \mathbf{f}_{\mathrm{ref}}$$

⇨ Conjugate Gradients (CG)

Incomplete Cholesky (IC) preconditioner

2. coupled system (symmetric, indefinite)

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B}^T & -\mathbf{G} \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \mathbf{v} \end{bmatrix} = \begin{bmatrix} \mathbf{f} \\ 0 \end{bmatrix}$$

⇨ Minimal Residual (MINRES)

2a. block(IC,exact) 2b. additive Schwarz

$$\begin{bmatrix} \tilde{\mathbf{A}}^{-1} & 0 \\ 0 & \mathbf{G}^{-1} \end{bmatrix} \qquad \begin{bmatrix} \tilde{\mathbf{A}}^{-1} + \mathbf{B}^H\mathbf{G}\mathbf{B} & 0 \\ 0 & \mathbf{G}^{-1} \end{bmatrix}$$

3. Schur complement system (symmetric, definite)

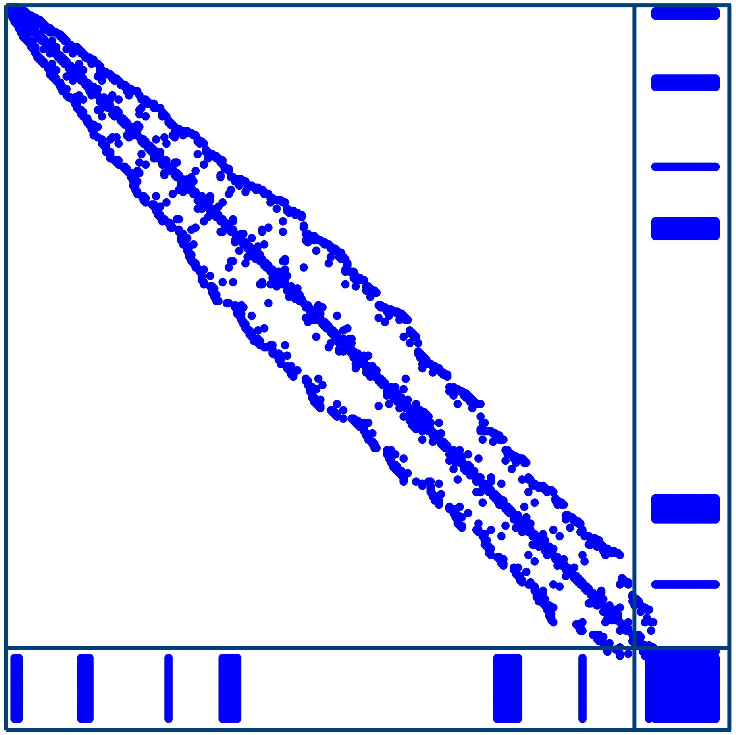
$$\left(\mathbf{A} + \mathbf{B}\mathbf{G}^{-1}\mathbf{B}^H\right)\mathbf{u} = \mathbf{f}$$

⇨ Conjugate Gradients (CG) 3a. IC $\tilde{\mathbf{A}}^{-1}$

3b. additive Schwarz $\tilde{\mathbf{A}}^{-1} + \mathbf{B}^H\mathbf{G}\mathbf{B}$

System solution (2)

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B}^T & -\mathbf{G} \end{bmatrix} =$$



(+) $\mathbf{B}$ and $\mathbf{G}$ applied by (partially) dense algebraic matrices

(*) $\mathbf{B}$ and $\mathbf{G}$ carried out „on the fly“

selection by index sets
interpolation by sparse matrices
2D Fast Fourier Transforms

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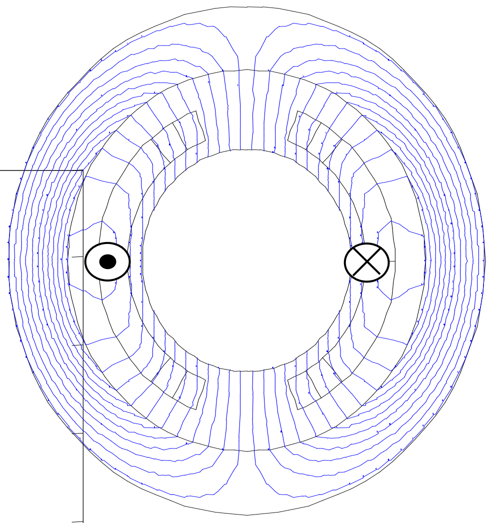
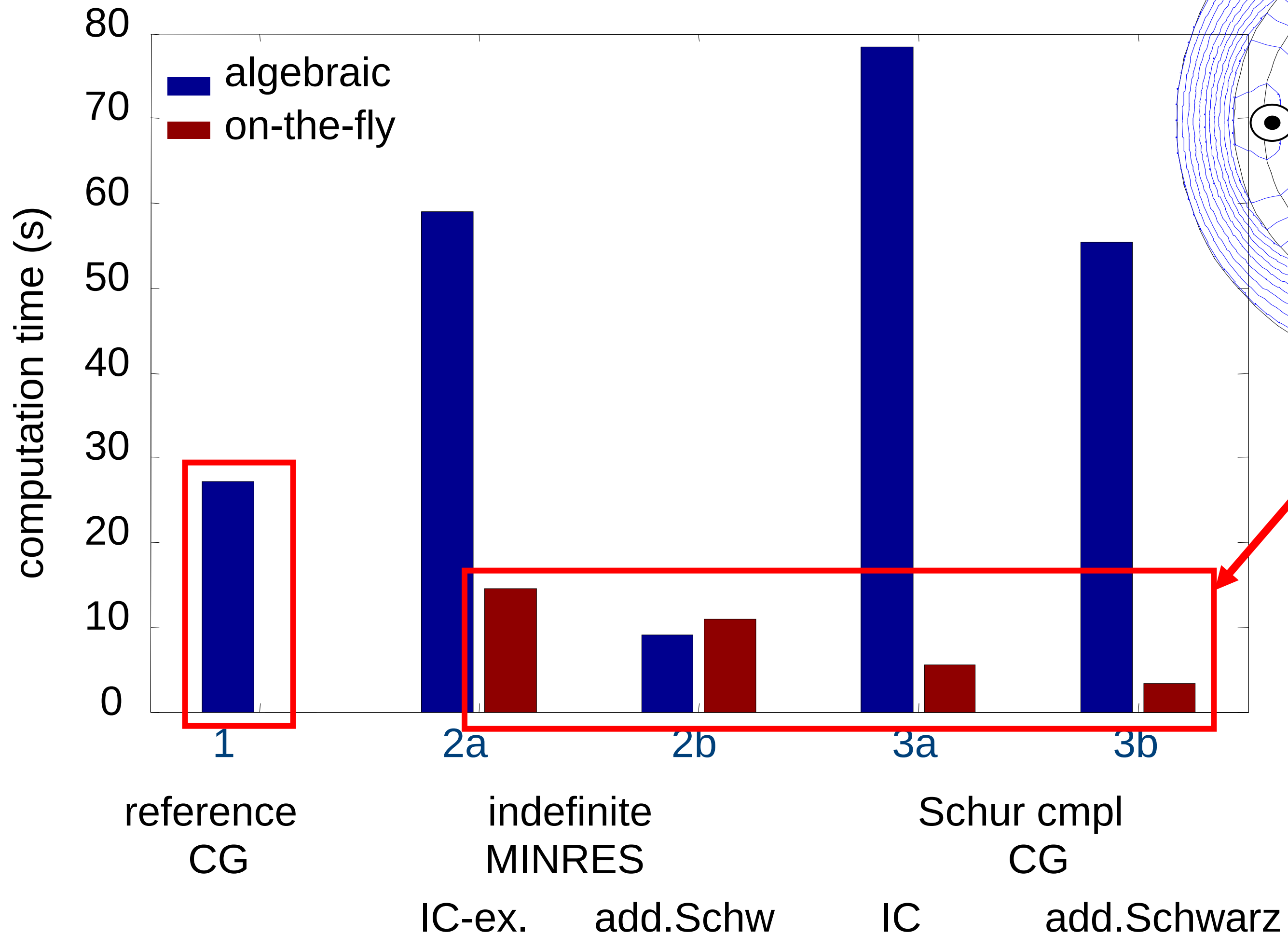
Numerical tests

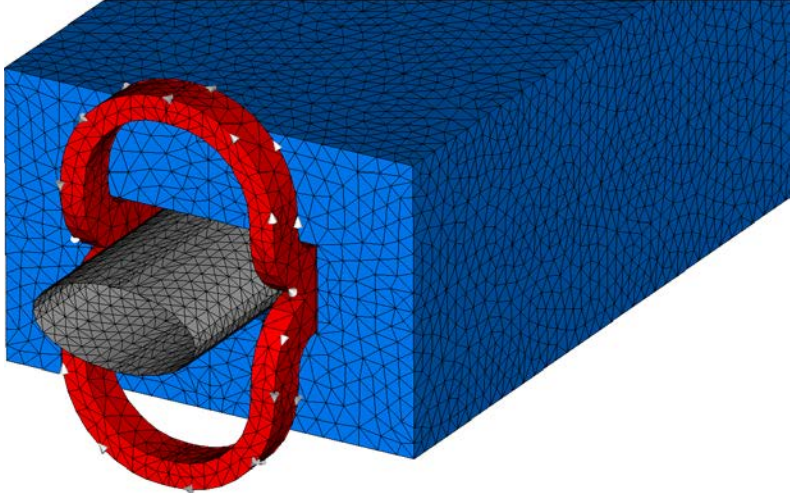
test model



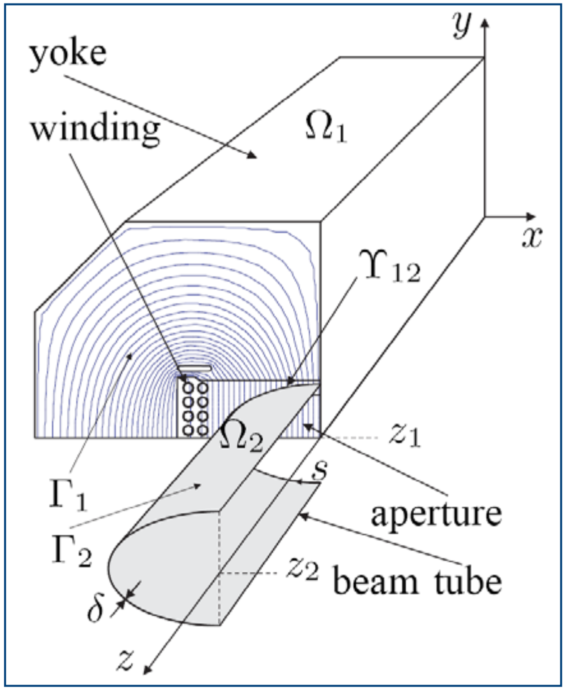


significantly faster than reference

Overview



- magnet simulation (standard 3D FE solver)
- challenges
  - o geometrical details
  - o materials
  - o transient effects
  - o high accuracy
- magnet simulation (dedicated 3D FE solver)
- hybrid models
  - hybrid discretisation
  - domain decomposition
- conclusions



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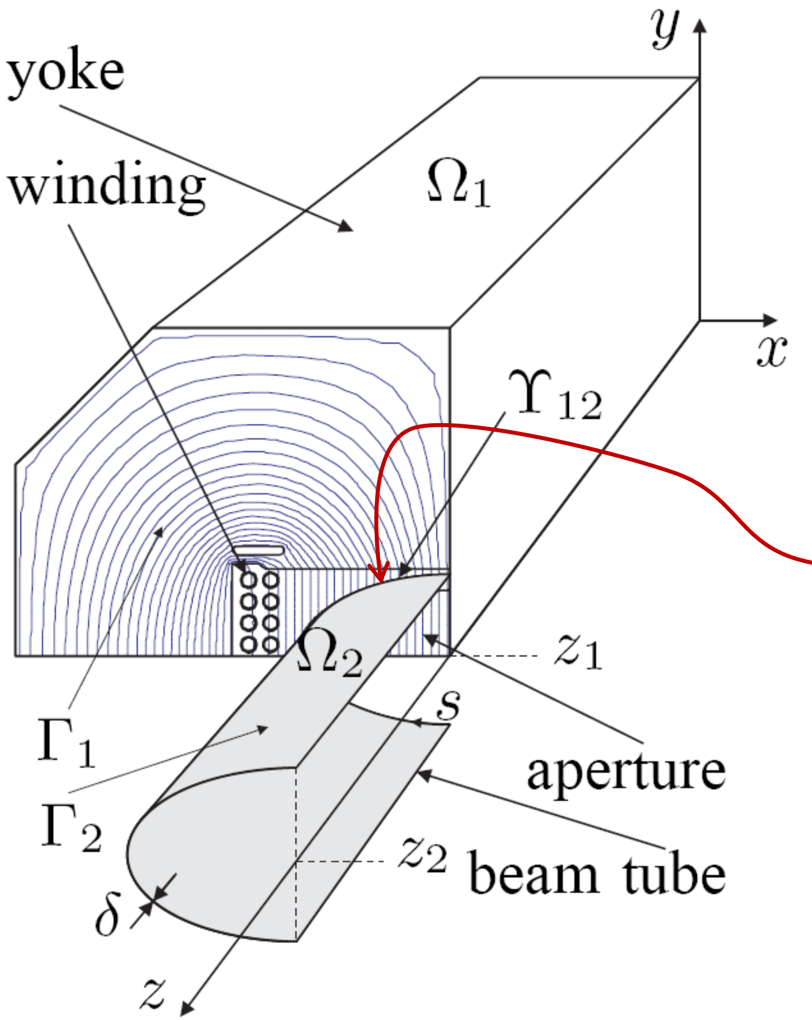
Beam-tube end model



standard 2D model

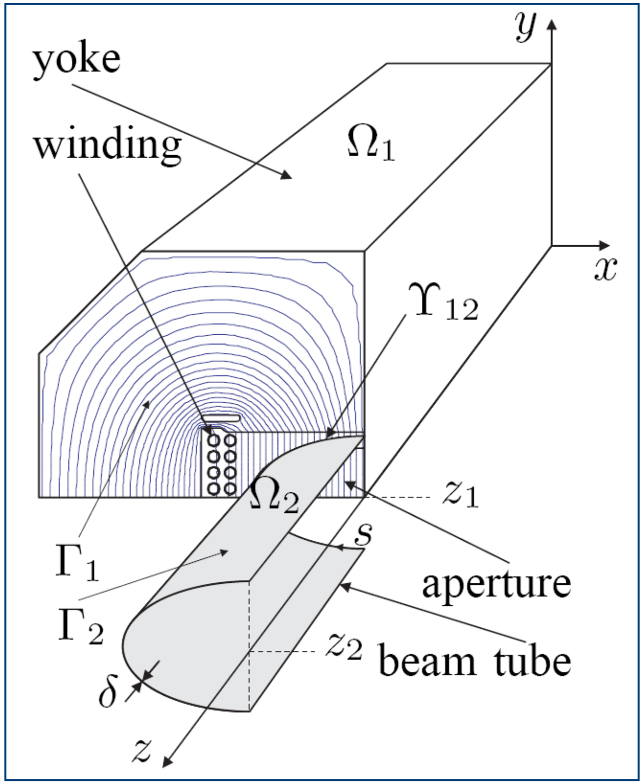
cross-section Γ_1
of volume Ω_1

common interface Υ_{12}

additional 2D model
for the beam-tube end

cross-section Γ_2
of volume Ω_2

Coupled formulation



magnetoquasistatic formulation

$$\nabla\times\left(\nu\nabla\times\vec{A}\right)+\sigma\frac{\partial\vec{A}}{\partial t}+\sigma\nabla\varphi=\vec{J}_s \quad \text{in } \Omega_1$$

$$-\nabla\cdot\left(\sigma\frac{\partial\vec{A}}{\partial t}\right)-\nabla\cdot\left(\sigma\nabla\varphi\right)=0 \quad \text{in } \Omega_2$$

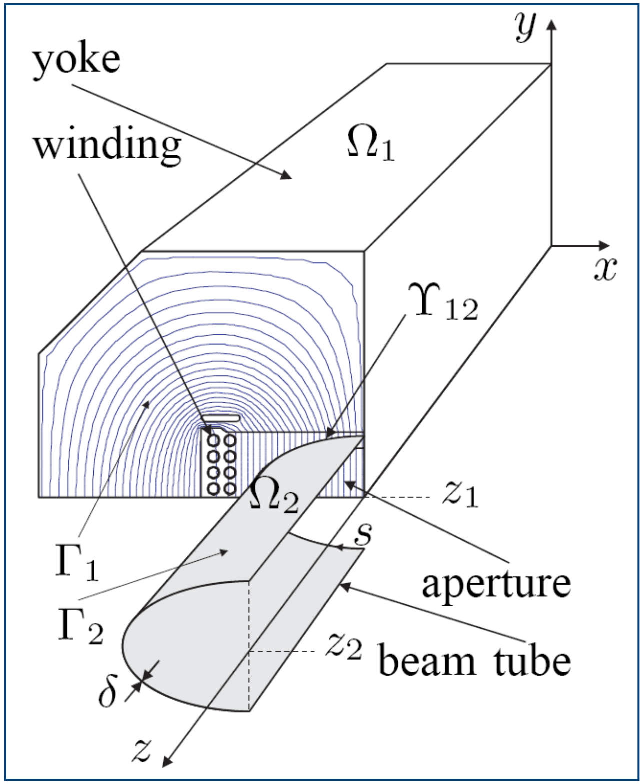
FE shape functions

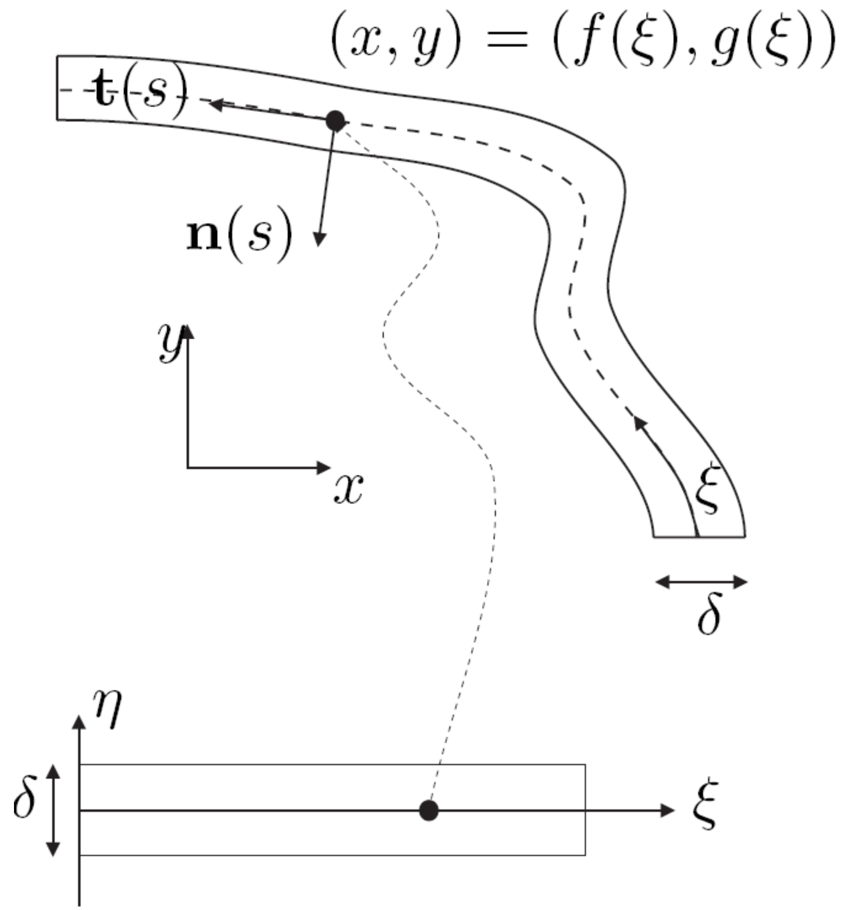
$$\begin{cases}\vec{w}_j=\frac{1}{z_1}N_j(x,y)\,\vec{e}_z & \text{in } \Omega_1\\ P_{\tilde{q}}=M_{\tilde{q}}(s,z_1)\frac{z}{z_1} & \text{in } \Omega_1\\ P_q=M_q(s,z) & \text{in } \Omega_2\end{cases}$$

system of equations

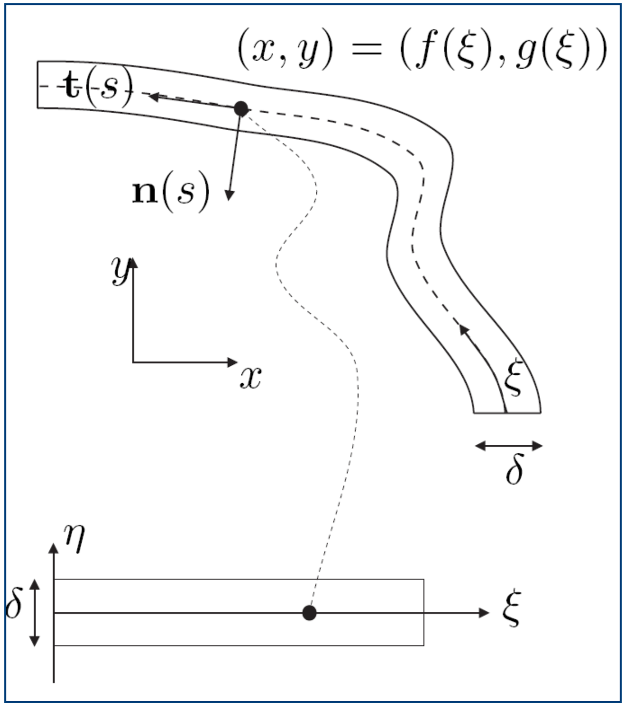
$$\begin{bmatrix}\mathbf{K}+\mathbf{M}\frac{d}{dt} & \mathbf{B}^T\\ \mathbf{B}\frac{d}{dt} & \mathbf{G}\end{bmatrix}\begin{bmatrix}\mathbf{u}\\ \mathbf{v}\end{bmatrix}=\begin{bmatrix}\mathbf{f}\\ 0\end{bmatrix}$$

Arbitrary tube cross-section





Beam-tube end model



parametrization of Υ_{12}:

$$(x, y) = \big(f(\xi), g(\xi) \big)$$

coordinate transformation

$$\begin{cases} x = f(\xi) - g'(\xi)\eta \\ y = g(\xi) + f'(\xi)\eta \\ z = z \end{cases}$$

Laplace-Beltrami

$$-\frac{1}{1-k(\xi)\eta}\frac{\partial}{\partial\xi}\left(\frac{\sigma}{1-k(\xi)\eta}\frac{\partial\varphi}{\partial\xi}\right) - \frac{\partial}{\partial\eta}\left(\sigma\frac{\partial\varphi}{\partial\eta}\right) - \frac{\partial}{\partial z}\left(\sigma\frac{\partial\varphi}{\partial z}\right) = 0$$

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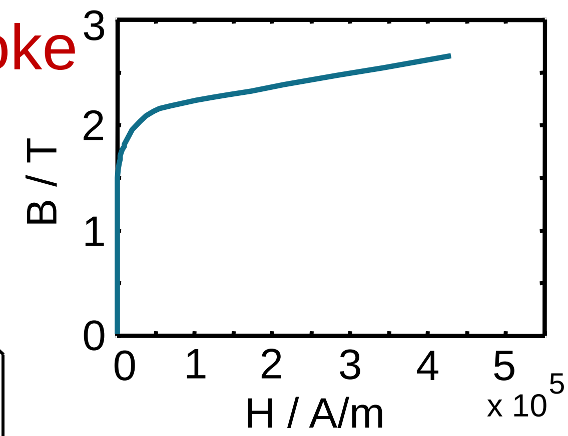
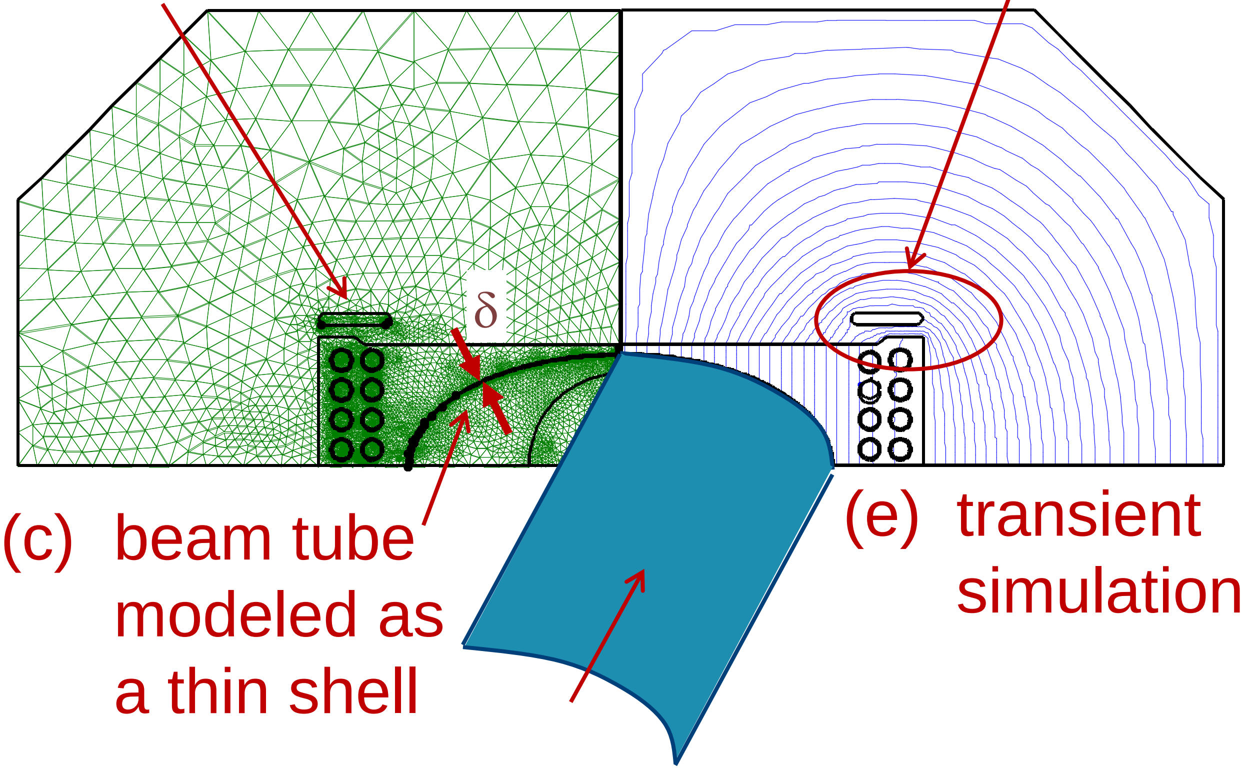
Example: GSI-SIS-100 magnet

(b) adaptive mesh refinement

(a) saturated yoke

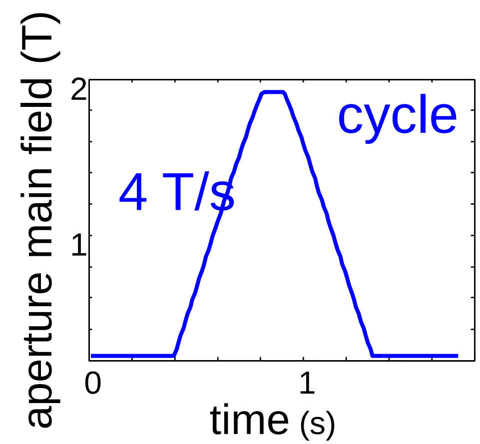




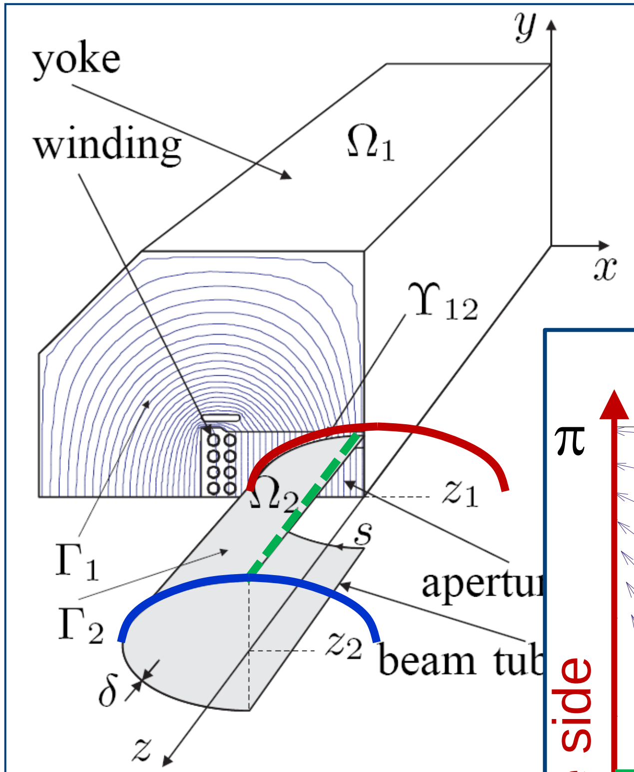
(c) beam tube modeled as a thin shell

(d) additional 2D beam-tube end model

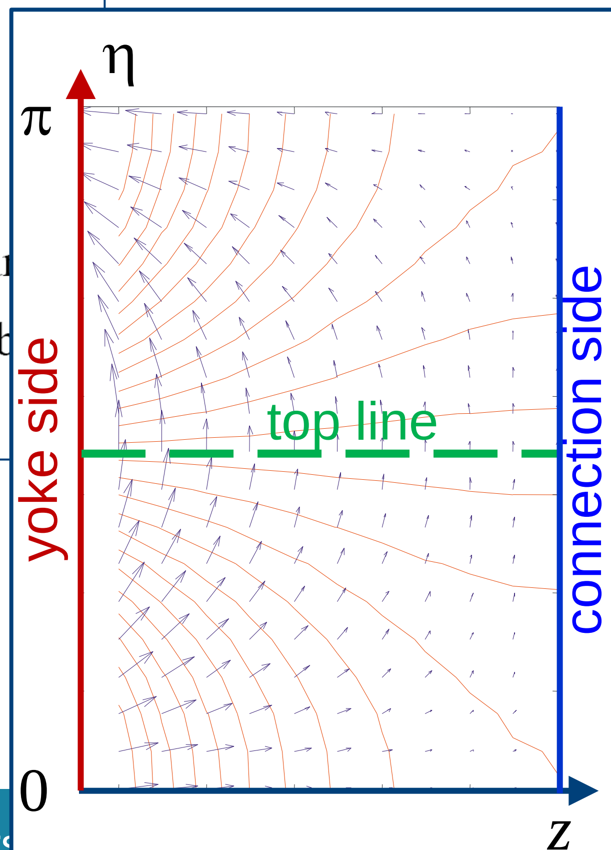
(e) transient simulation



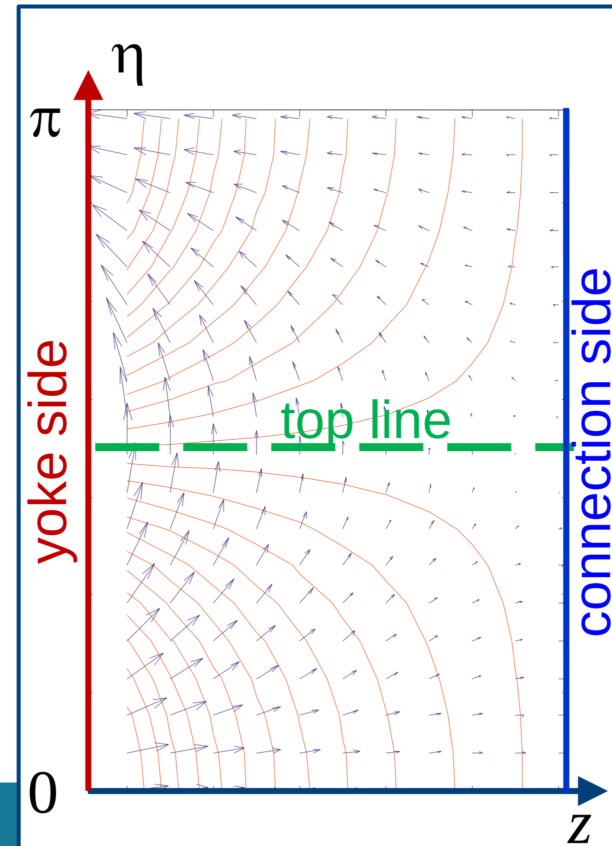
Eddy-current closing paths



connected
beam tube



disconnected
beam tube

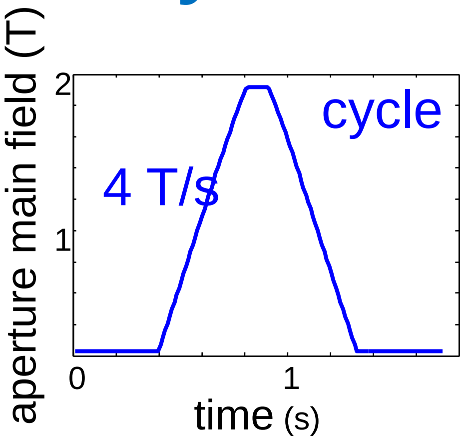


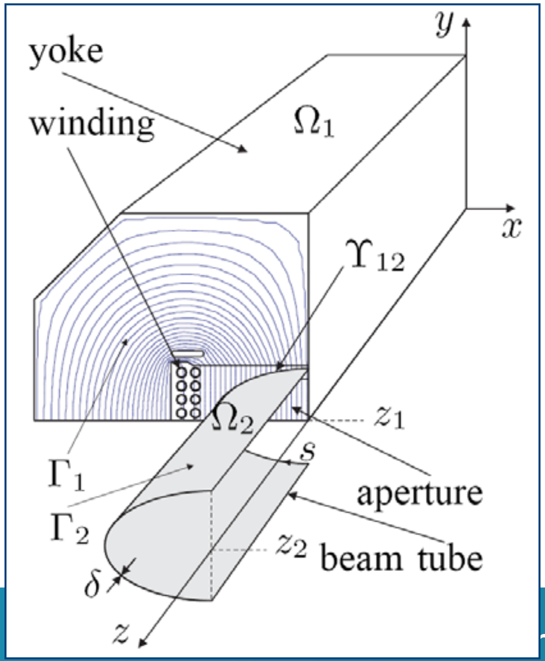
yoke side

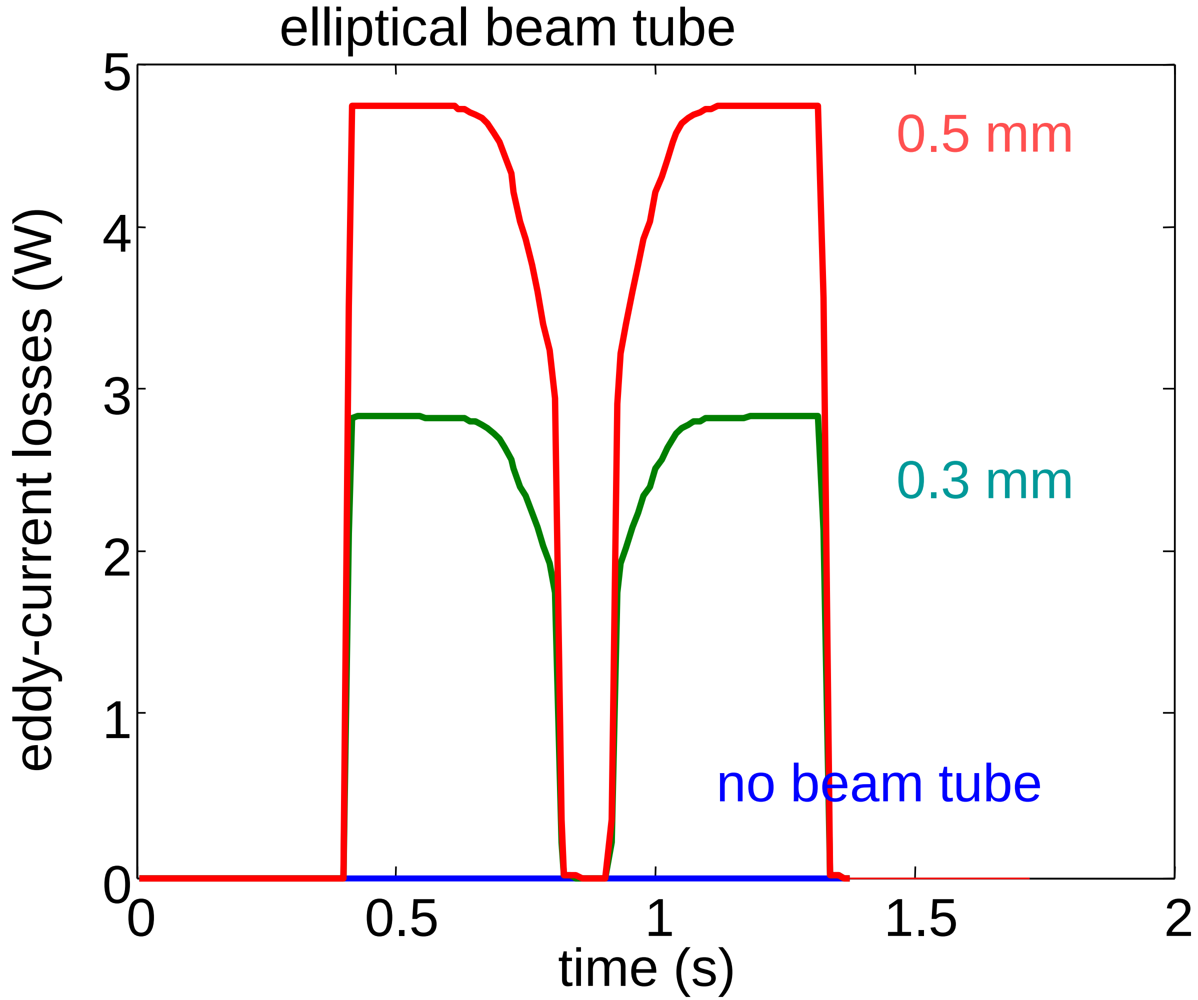
top line

connection side

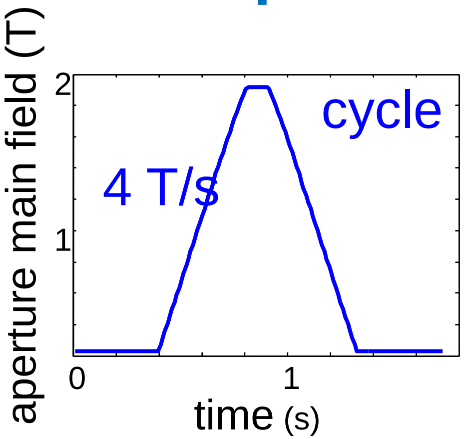
Eddy-current losses

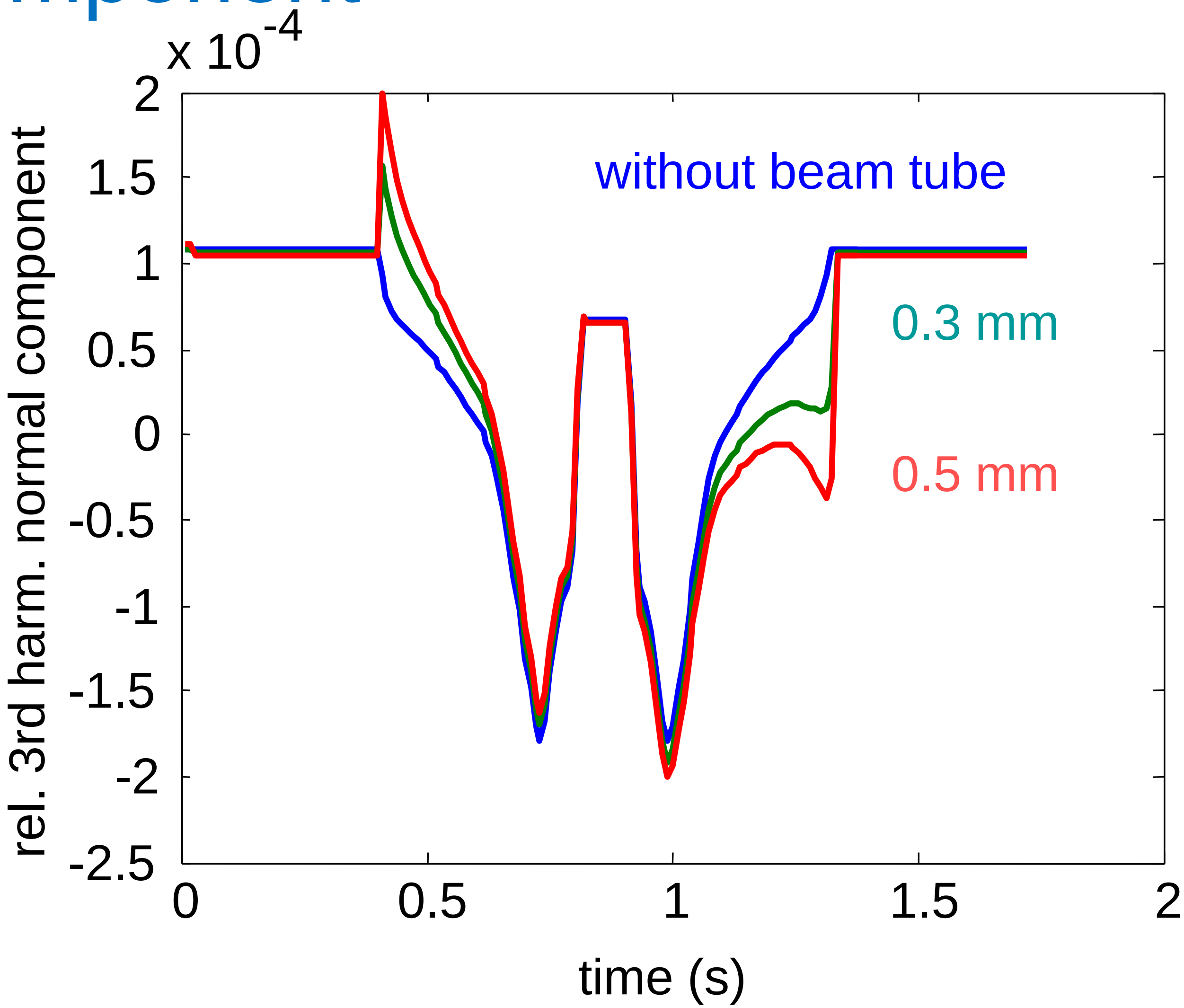


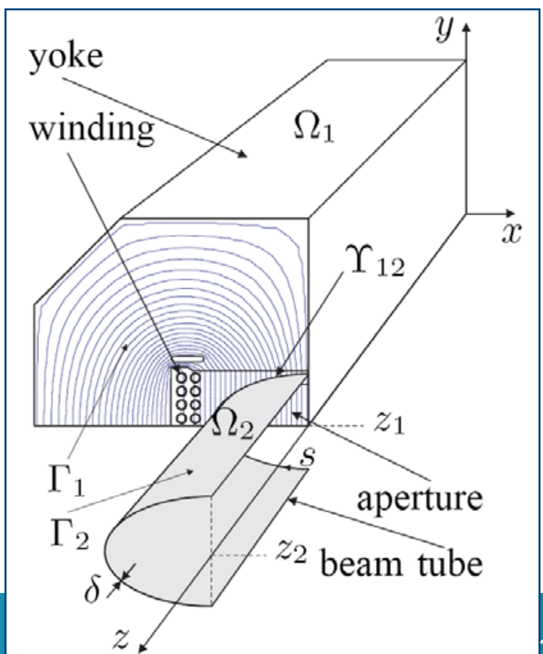




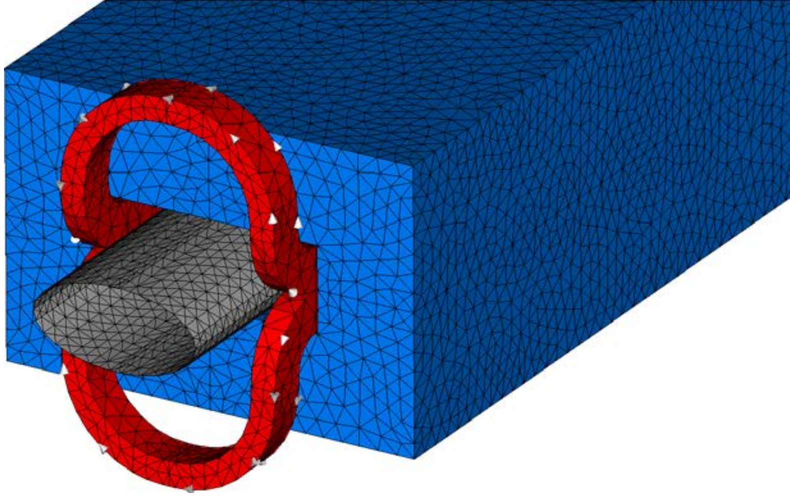
Sextupole component







Conclusions



- nonlinear 3D transient magnetic simulation feasible with of-the-shell software
- challenges remain
  and are problem specific
  - o geometrical details
  - o materials
  - o transient effects
  - o high accuracy
- dedicated methods and software