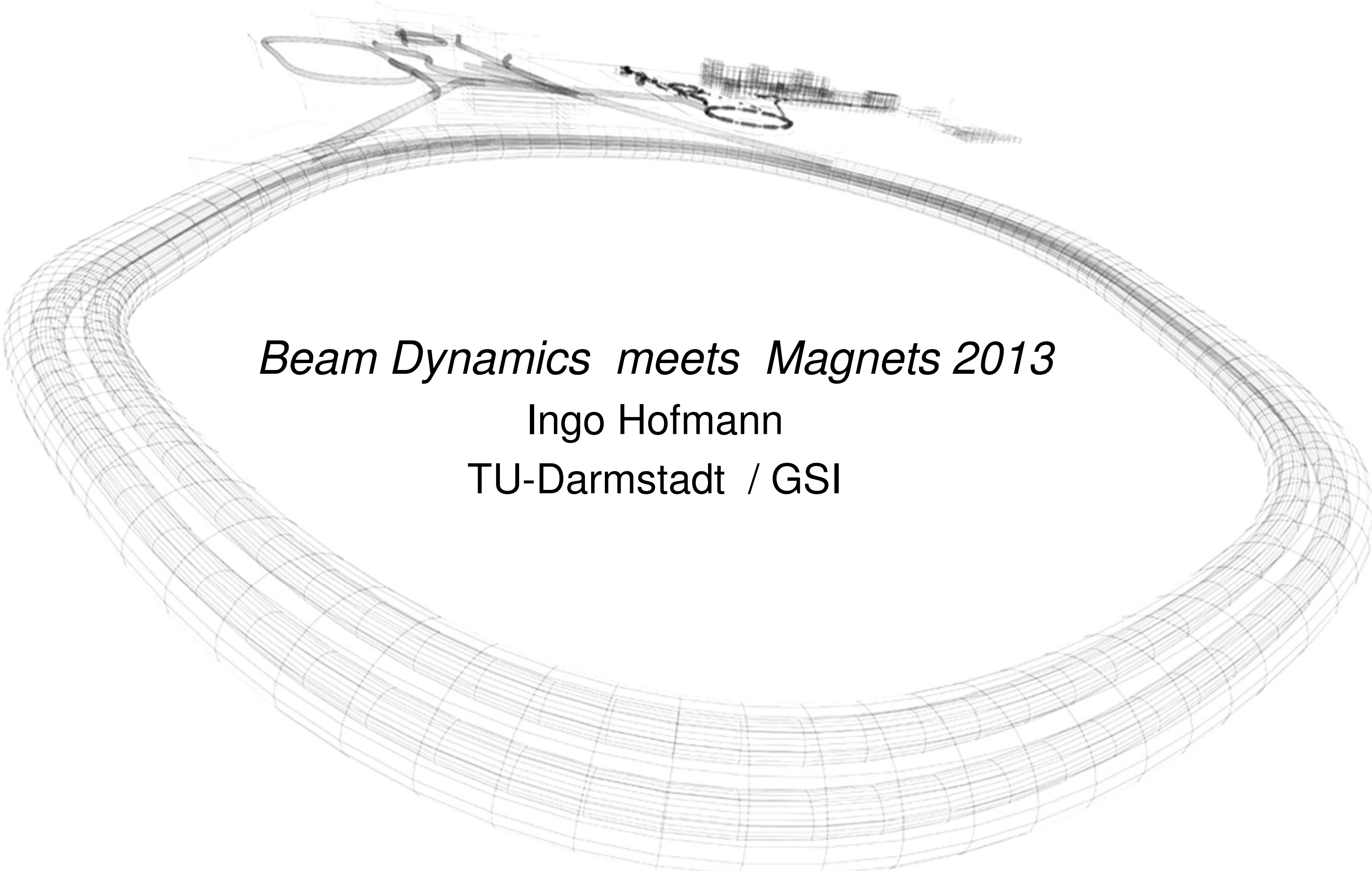
Comparison of quadrupolar & solenoidal focusing and "order" effects



Beam Dynamics meets Magnets 2013

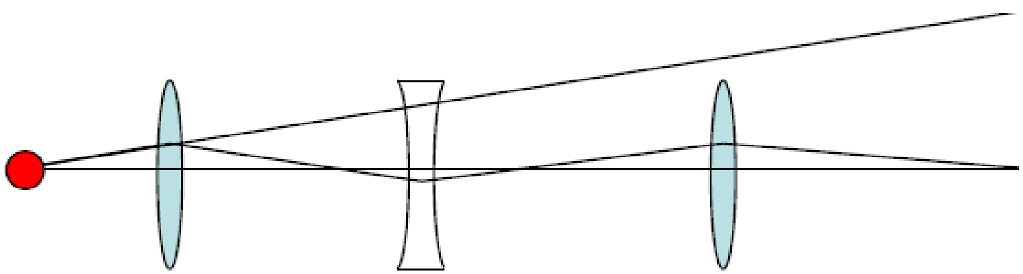
Ingo Hofmann
TU-Darmstadt / GSI

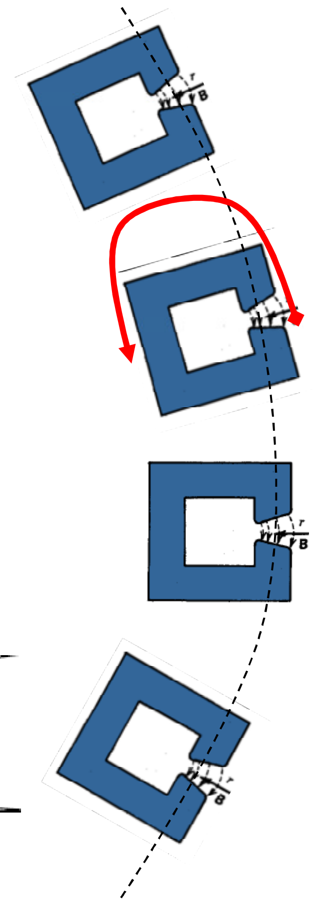
1. Introduction
2. Solenoids and quadrupoles – comparative evaluation (single magnets – no lattices)
3. Examples - applications (non-standard - novel)
4. Higher order effects
5. Concluding remarks

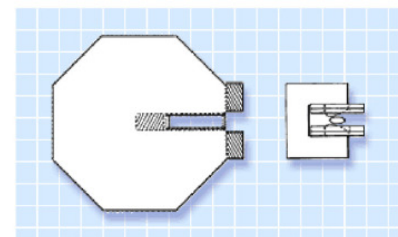
Alternating gradient (strong) focusing – key invention to enable high energy physics

Issue appeared with increasing energies

- **Cosmotron 3 GeV "weak focusing" accelerator at BNL** - 2000 t steel
  - extrapolated that 200.000 t steel (=20 x Eiffel tower) would be needed for 30 GeV
  - SU: Synchrocyclotron 1 GeV 10.000 t steel
- **1952 Livingston had the idea to revert some magnets, Courant did the work and Snyder the theory!**
- **birth of strong focusing era**







Size comparison between the Cosmotron's weak-focusing magnet (L) and the AGS alternating gradient focusing magnets

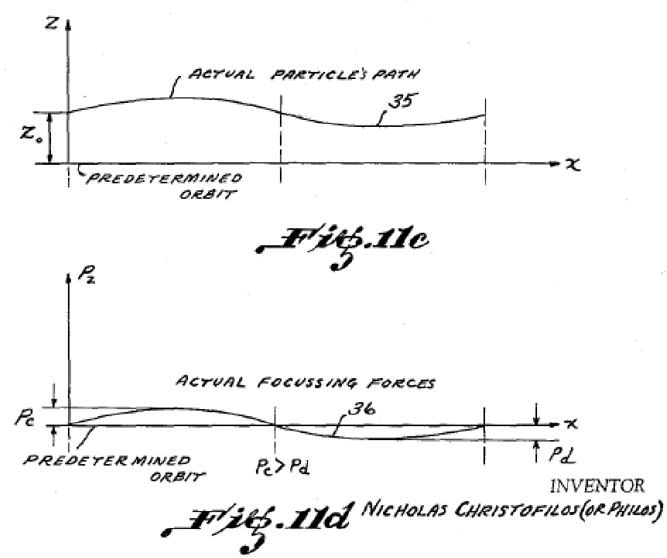
later separated: dipoles plus alternating quadrupoles

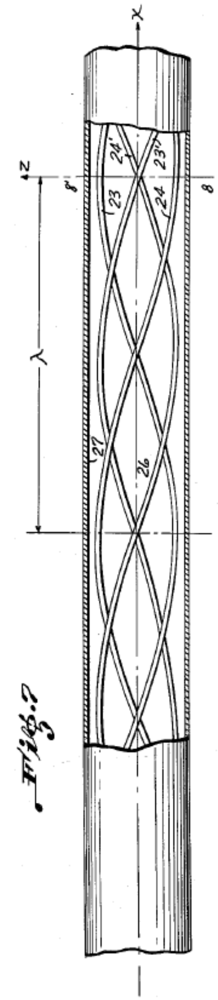
Invented 2 years earlier by Nicholas Christophilos

- **invention** (1950) by Nicholas Christophilos (US Patent – no publication)
- unknown to Courant and co
  - first published in 1952: Courant-Snyder-theory (variables) → real modeling
  - thereafter attributed in a 1 page PRL the invention to Christophilos

Christofilos

- demonstrated in 1954 at Cornell **1.3 GeV electron accelerator**
- in 1956 invention of **FFAG** (K. Symon, D. Kerst, T. Ohkawa) at MURA → electrons to 50 MeV
- **applied (>1959) to ~ 30 GeV CERN-PS and BNL-AGS and U70 in Serpukhov**





What can be learnt?

- ✓ Revolutionary inventions necessary from time to time
- ✓ Challenge: theorists ←→ experimenters

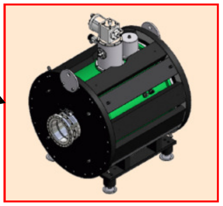
Focusing options – differences in order and fringe (end) fields

first order lenses

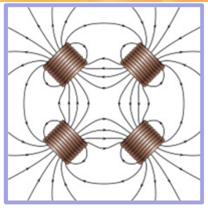
Maxwell's equations!
no 2d or 3d focusing in 1st order or vacuum

second order lenses

- **Solenoids:** focusing but in "second order"



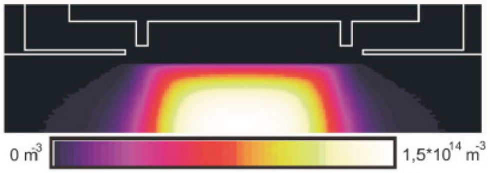
- **Quadrupoles (F & D)**



beam not in vacuum:

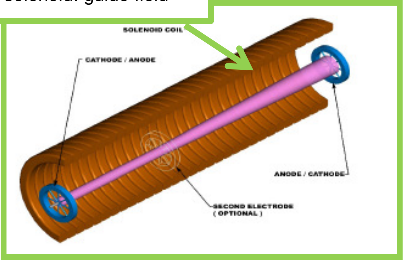
- **Gabor lens**: high voltage

J. Pozimski, O. Meusel, 2005   FAIR LEBT 2013 (optional)

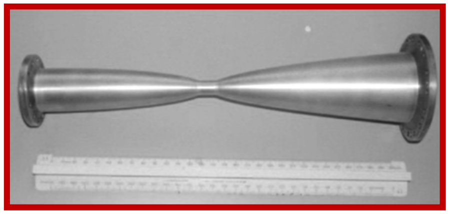


- **Plasma lens / z-Pinch**: pulsed discharge

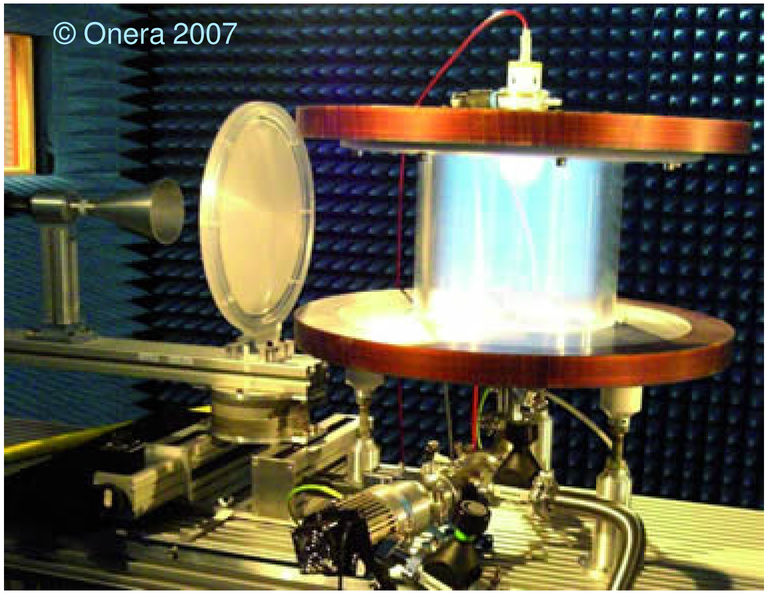
solenoid: guide field



- **Magnetic horn**

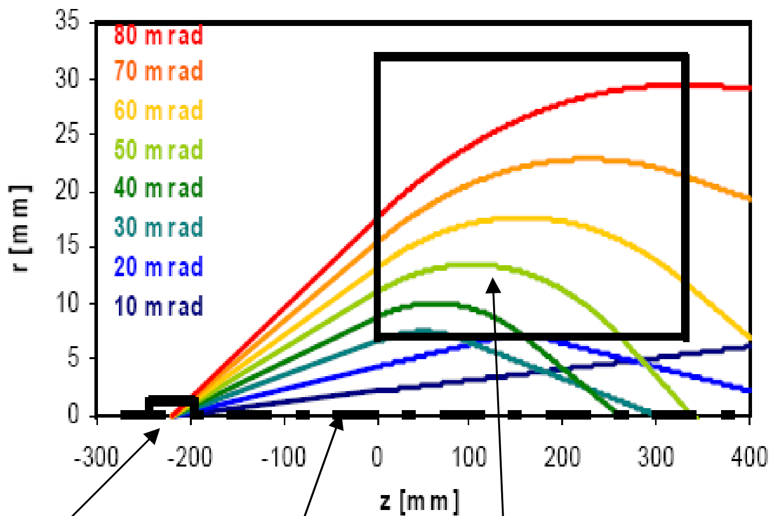


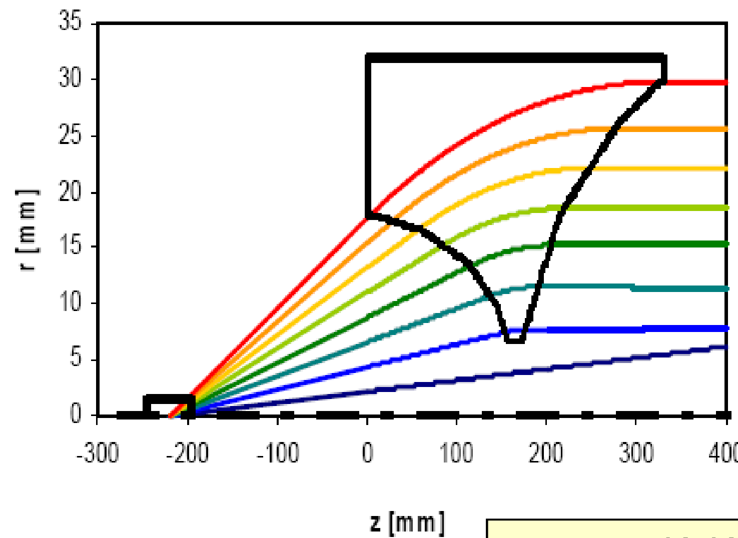
Plasma lens: also includes a coil magnet



Onera (F): Experimental assembly for the study of the deviation of an electromagnetic wave by a plasma lens. Right part: the containment chamber of the plasma between the two Helmholtz coils.

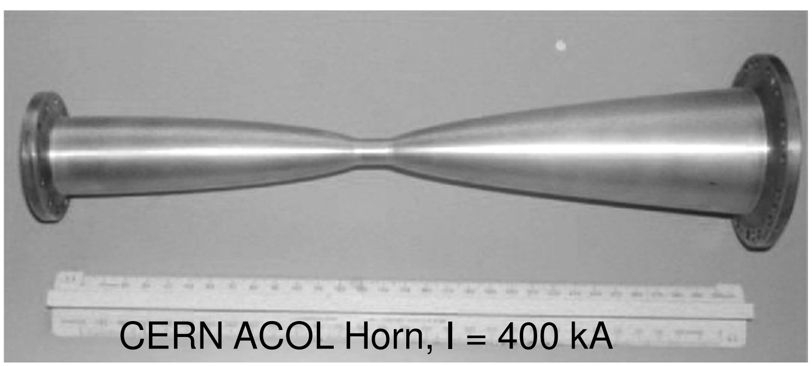
Magnetic Horn: can be designed directly achromatic by tailoring conductor (to be used for pbar at FAIR)





courtesy K. Knie, GSI

target symmetry axis horn (magnetic field area)


CERN ACOL Horn, I = 400 kA

- \+ large energy and angle acceptance!
- \- mostly suitable for GeV particles
- \- technically demanding (pulsed)

Solenoids vs. quadrupoles

"solenoids are good at low energy" / injectors / for round beams – where is the transition?
searching for collector optics for laser accelerated protons

I.Hofmann, PRST-AB 2013

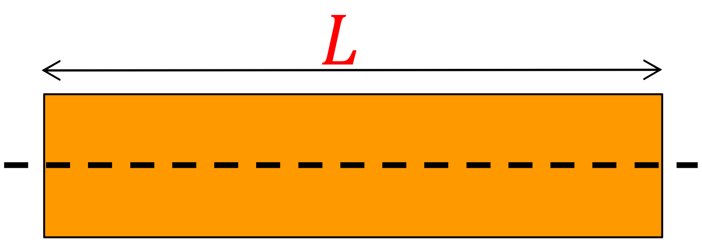
Compare systems using thin lens approximation *(see Martin Reiser's book p. 102ff)*:

- same overall length of lens system
- same maximum B
- point-to-parallel focusing ←

Solenoid:

$1/f_s = k^2 L$

$k^2 = [qB/(2m\gamma v)]^2$



Single quadrupole:

$1/f_1 = \pm\kappa l$

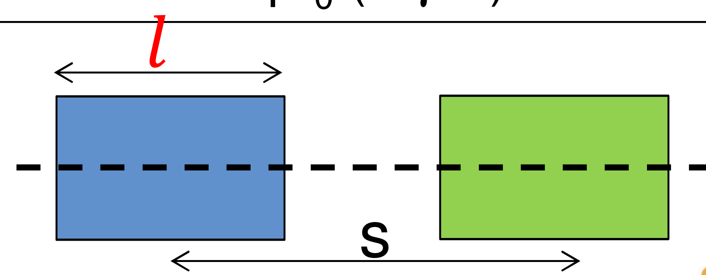
Doublet (s: distance of centers)

$1/F_d = 1/f_1 + 1/f_2 - s/(f_1 f_2)$

$f_1 = f$ and $f_2 = -f$

$1/F_d = \kappa^2 l^2 s$ (s matters!)

$\kappa = qB_0/(m\gamma a v)$



Quantify focusing enhancement doublet versus solenoid (not only relevant criterion)

Equivalent systems: $B_{sol}=B_{pole\text{-}tip}$ & same overall length

$$T_d \equiv \frac{1/F_d}{1/f_s} = \frac{4l^2 s}{a^2 L_{sol}}$$

gap between quads =length:

$s=2l$ & $L=3l$

doublet enhancement factor T_d:

$$T_d = \frac{8}{27}\left(\frac{L}{a}\right)^2$$

for same focal length require:

$$B_{sol} = \sqrt{T_d}\, B_d$$

→ Doublet strength superior for $L > 2a$

Purely geometrical issue!

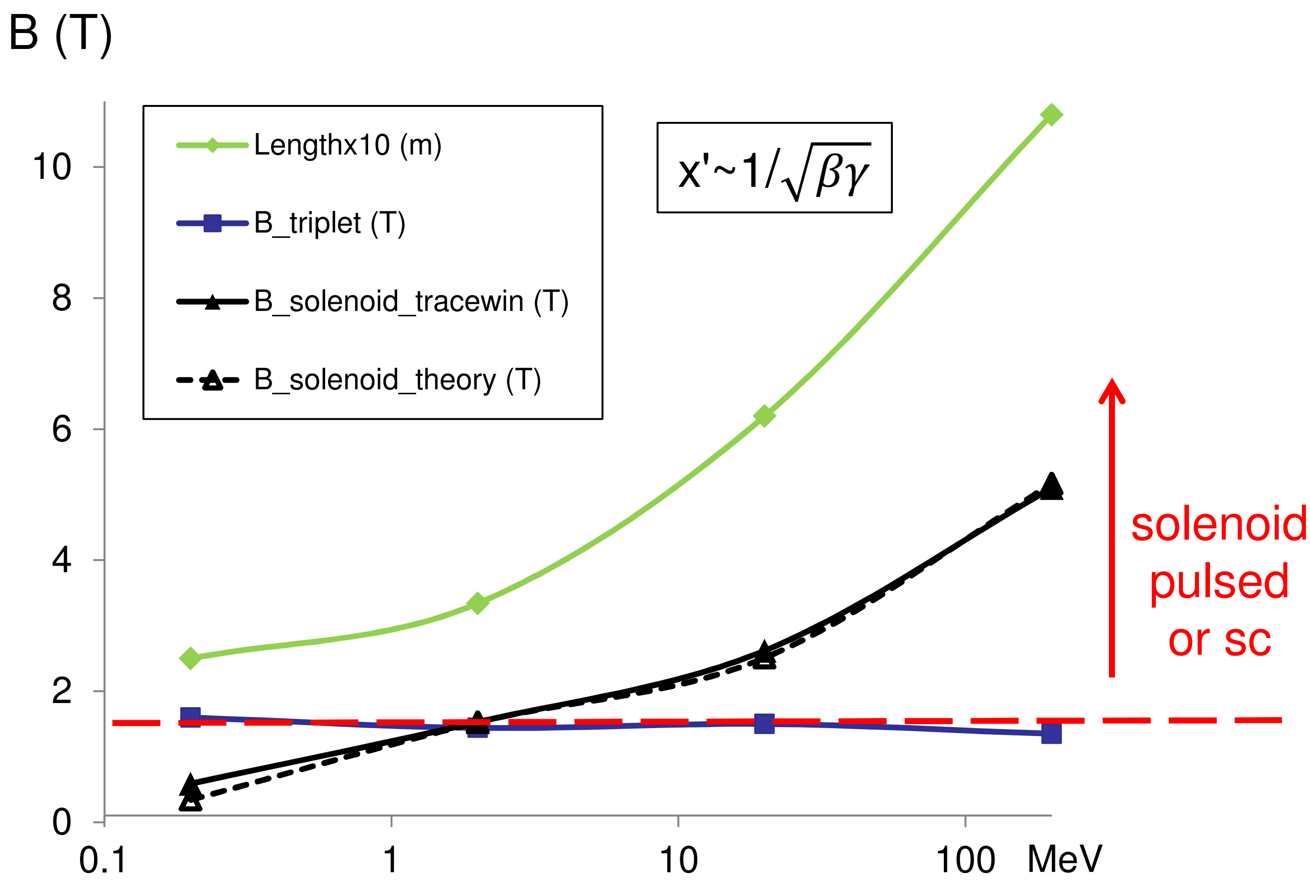
Application to solenoid - triplet

for triplet
replace 8 by 4

$$T_d = \frac{8}{27}\left(\frac{L}{a}\right)^2$$

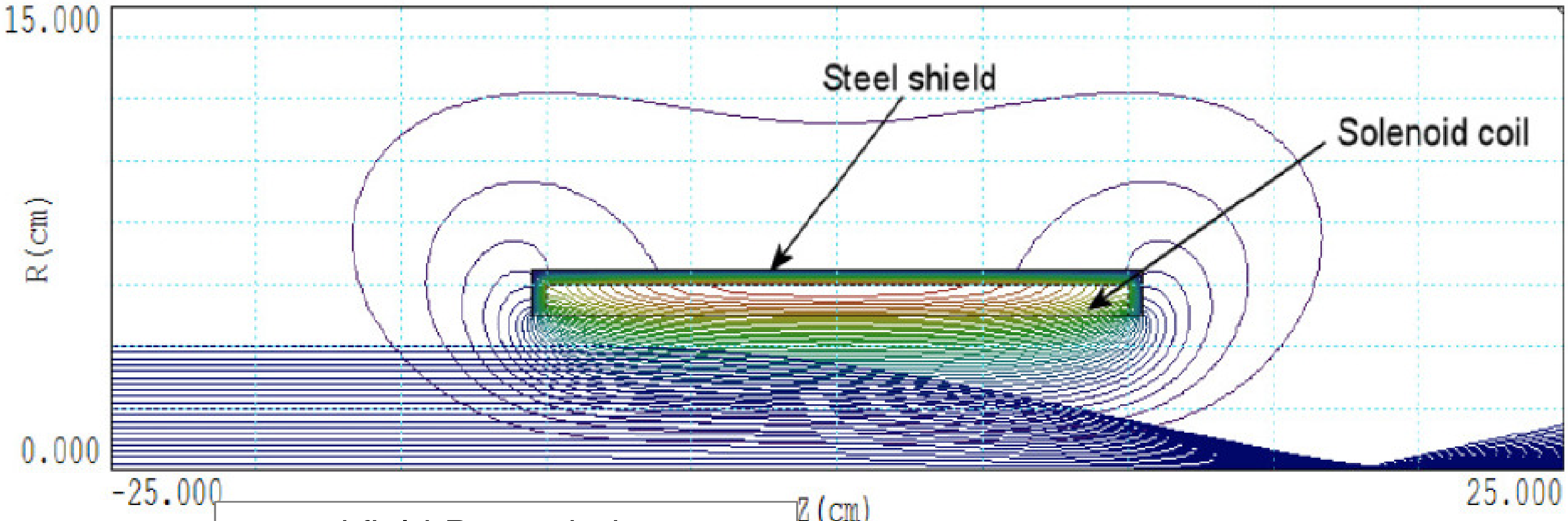
$$\rightarrow$$

$$T_{tr} = \frac{4}{27}\left(\frac{L}{a}\right)^2$$



→ excellent confirmation of thin lens formulae

Why is it so?



end field-B_r needed to get
rotation $v_\theta \sim v_z B_r$
→ r-focusing force ~ v_θ B_z
focusing =
end field x core effect

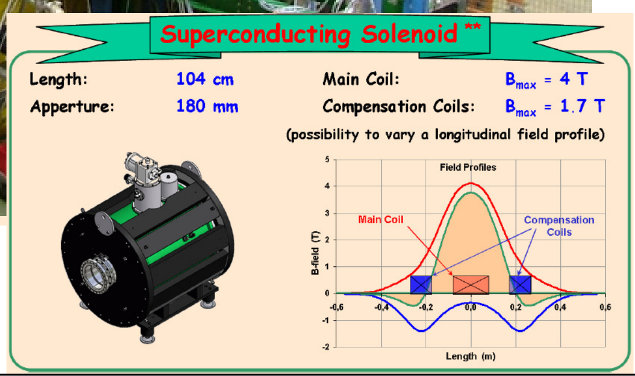
Applications with solenoids and/or quads as option:

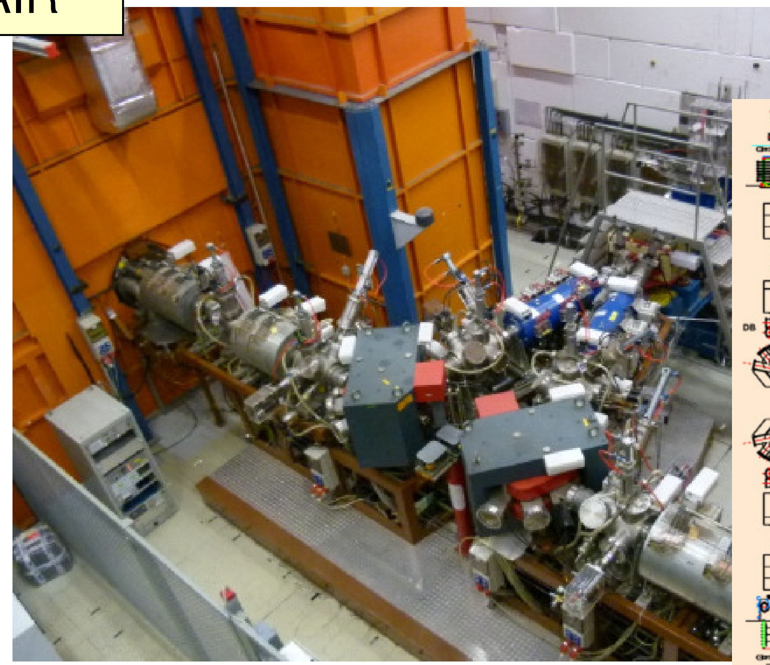
- solenoids hardly seen in high energy physics –

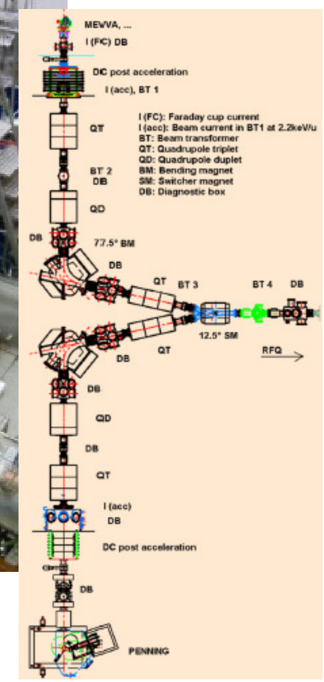
1. In low energy beam transport of linacs: options for high-current U^{4+} for FAIR

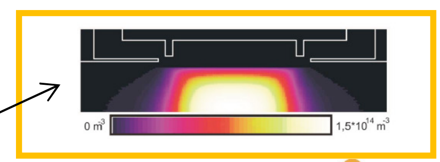
source: R. Hollinger, GSI

30 mA U^{4+} needed for FAIR



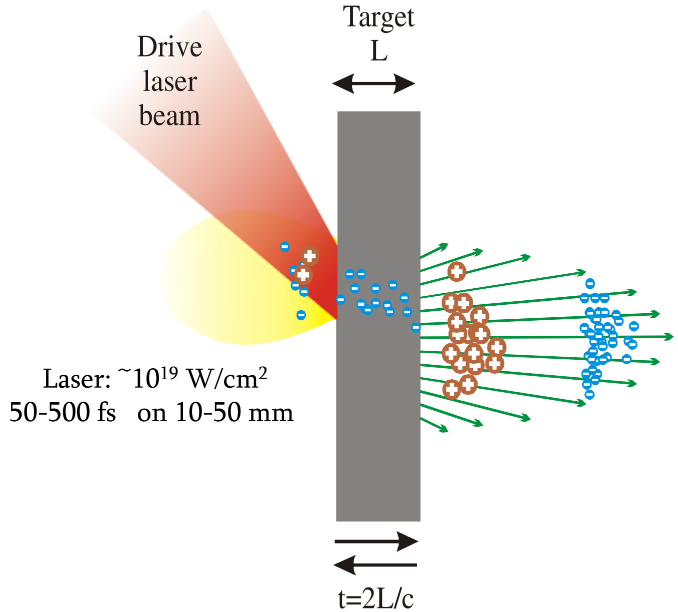




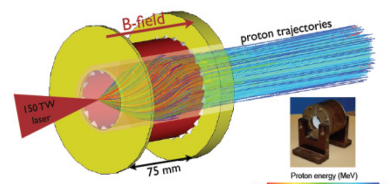


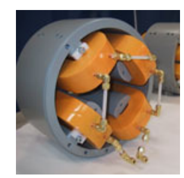
1. Solenoid efficient, **but space charge compensation a problem**
2. **Triplet seems to support higher** compensation degree
3. Gabor "space charge" lens considered as another option (IAP Frankfurt)

2. Collection of secondary particles
   - here example of laser accelerated protons
   - collaboration GSI (PHELIX) – TU Darmstadt etc.



protons (10-70 MeV):
- large divergence ~ 20°
- large energy spread ΔE/E ~ 50%
- challenge to collection





Possible applications (if > 70 MeV):
- therapy
- imaging
- neutrons

Higher order effects: 2nd order chromatic applied to collection & focusing of laser protons

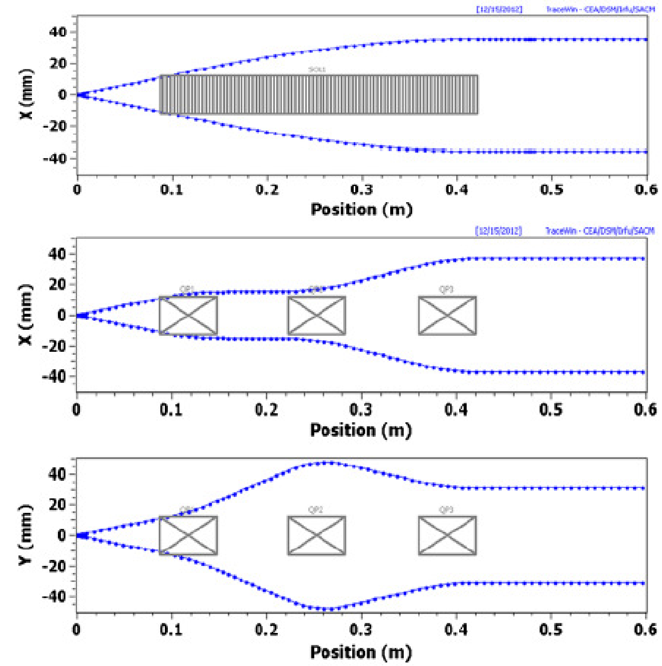
$$\alpha_{x,y} = \frac{\delta f_{x,y}/f_{x,y}}{\delta E/E}$$

$\alpha_{x,y} = 1.9$ solenoid

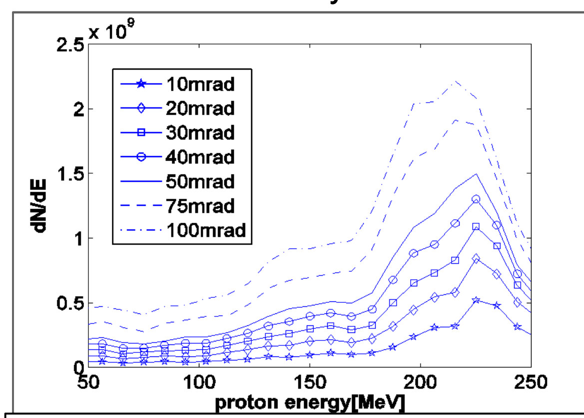
$\alpha_{x,y} = 1.8 / 3.9$ triplet

$\alpha_x * \alpha_y$ = same for <u>equivalent</u> solenoid or triplet

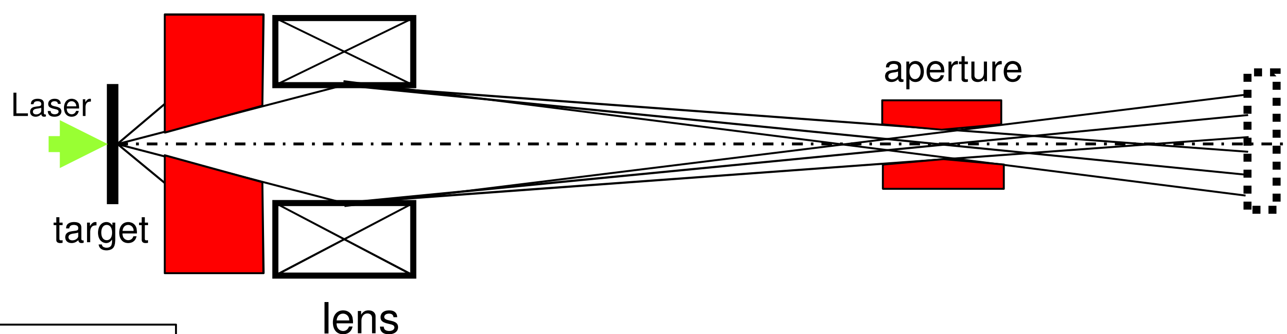
in spite of much smoother focusing of solenoid



theory:



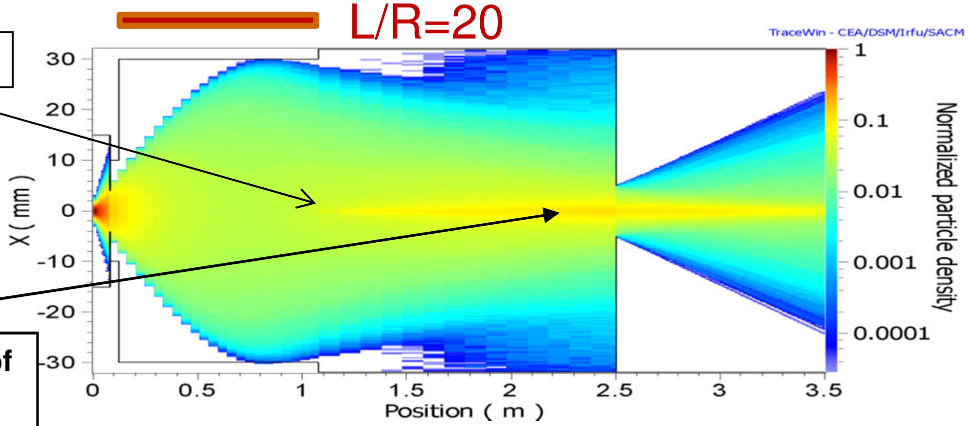
I. Hofmann J. Meyer-ter-Vehn, X. Yan, H. AlOmari, NIM 2012



Tracking: chromatic (2nd order) aberration + spherical (3rd order)

long vs. short solenoid

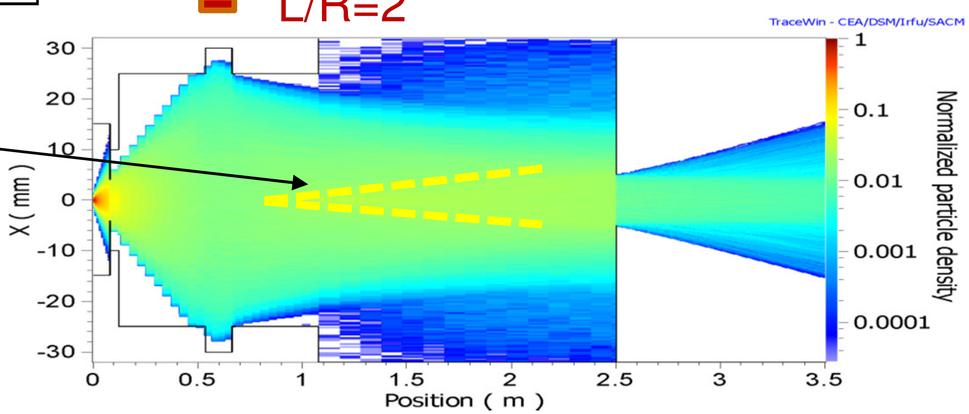
L/R=20



Lowest energy focus (50 MeV)

Chromatic focal line = „image" of small source emittance
- allows selection of energy by an aperture

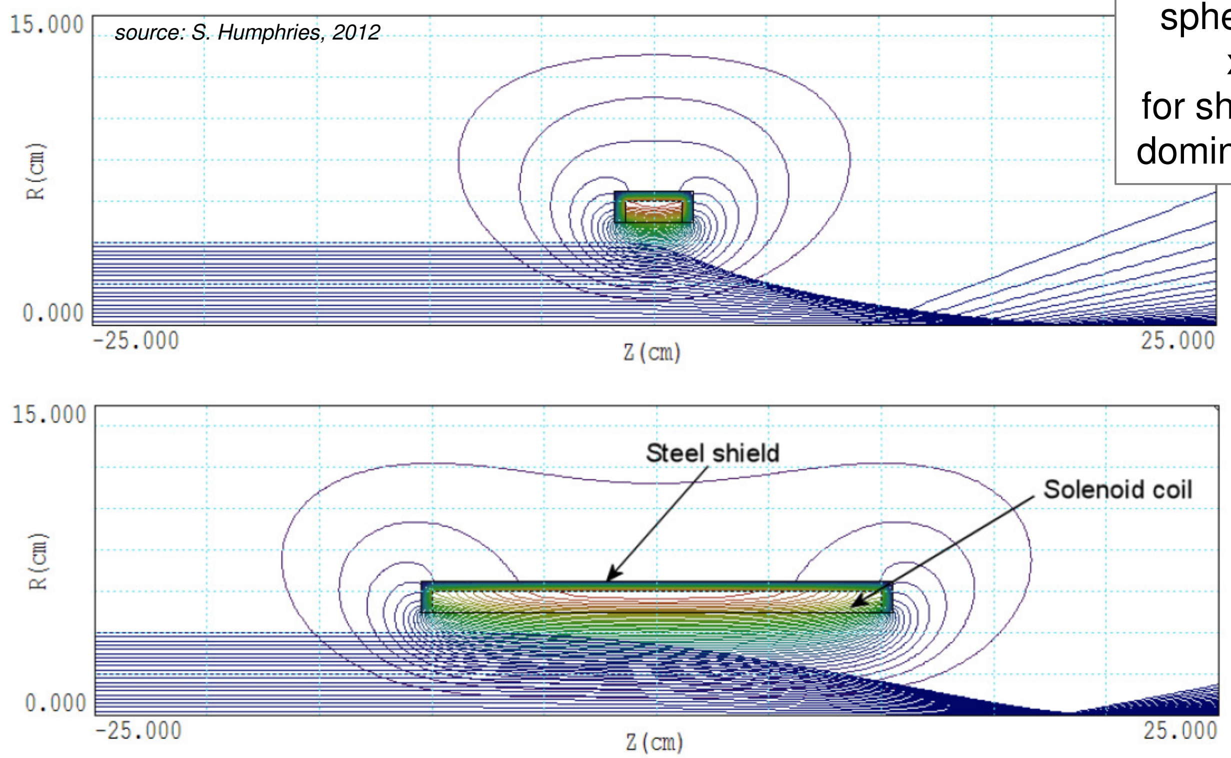
L/R=2



Spherical aberration dominating over chromatic
- cone width increasing with distance
- → equivalent to effective emittance growth (relatively independent of energy)
- destroys energy selection

usually emittance growth from spherical aberration ~ % level (3rd order)

End effects:



spherical aberration
$x' \sim x + \alpha x^3$
for short lens focusing
dominated by end field

End fields source of nonlinearity (core field linear) →
long solenoid: less spherical aberrations, but also less efficient !

Long solenoid not the only one low spherical aberration case

Analytical model: *B. Biswas, Rev. Sci. Instr., 2013*

Examples:

1. very long (in air) solenoid:
   $L=20R_i$ and $R_o/R_i=1.3$ → n=15
   and $C_{3rd}=0.0014\ cm^{-2}$
2. very thin pan cake solenoid:
   $L=0.3R_i$ and $R_o/R_i=20$ → n=1.5
   and $C_{3rd}=0.00145\ cm^{-2}$
3. same spherical aberration!

fractional reduction of focal length
(3rd and 5th order):

$$\Delta r' = -\frac{r}{f_0}\left[1 + C_1 r^2 + C_2 r^4\right],$$

$$B_z(z) = B_o\left(\frac{1}{1+(z/a)^n}\right)$$

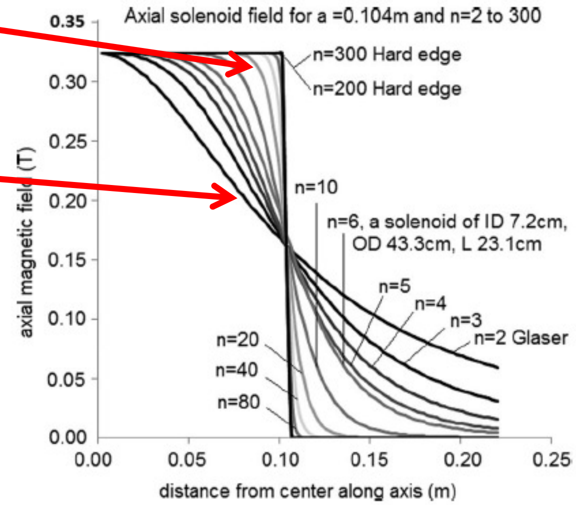


FIG. 1. The solenoid model Eq. (1) field on axis for various order n and $a = 0.104$ m showing soft edge to hard edge transformation for $n > 80$.

$$C_1 = \frac{1}{2}\frac{\int_{-\infty}^{+\infty}\{B'(z)\}^2 dz}{\int_{-\infty}^{+\infty} B^2(z)dz},\quad C_2 = \frac{5}{64}\frac{\int_{-\infty}^{+\infty}\{B''(z)\}^2 dz}{\int_{-\infty}^{+\infty} B^2(z)dz}.$$

Concluding remarks

- Focusing magnets – many options (applications)
- domain of solenoids in low energy
- beam dynamics > 2^{nd} order determined by end fields
- higher order depends much on geometry and design