Few-nucleon systems and Efimov physics

Fabian Hildenbrand, IAS-4, Forschungszentrum Jülich, Germany

collaborators: S. Elhatisari, H.-W. Hammer, Helen Meyer, U.-G. Meißner, Zhengxue Ren and …

Outline

- Motivation
- Results from NLEFT
- Structure of Three-Body hypernuclei from pionless EFT
  - Universal Correlations

Chiral Nuclear Effective Field Theory

JÜLICH FORSCHUNGSZENTRUM

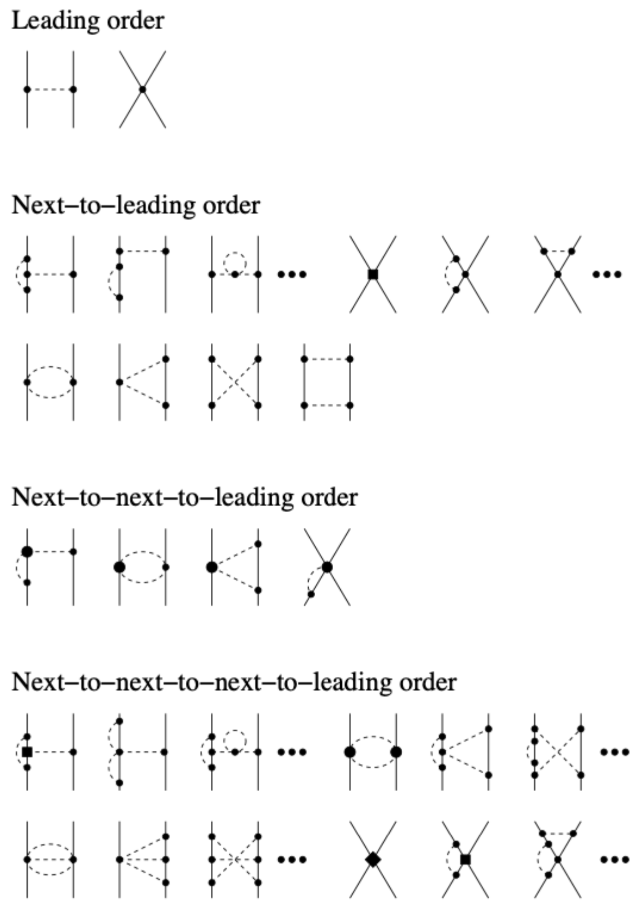
- Pick the correct scale : $\sim 10^{-15}\,\mathrm{m} = 1$ fermi
- Pick the relevant degrees of freedom : protons and neutrons and pions

This is not Lattice QCD, no quarks or gluons!

construct effective field theory

- Low energy chiral effective field theory of QCD
    - No model, systematic improvement is possible due to a hierarchy of forces
    - Systematic error analysis possible
    - Typical nuclear systems are far away from breakdown scale

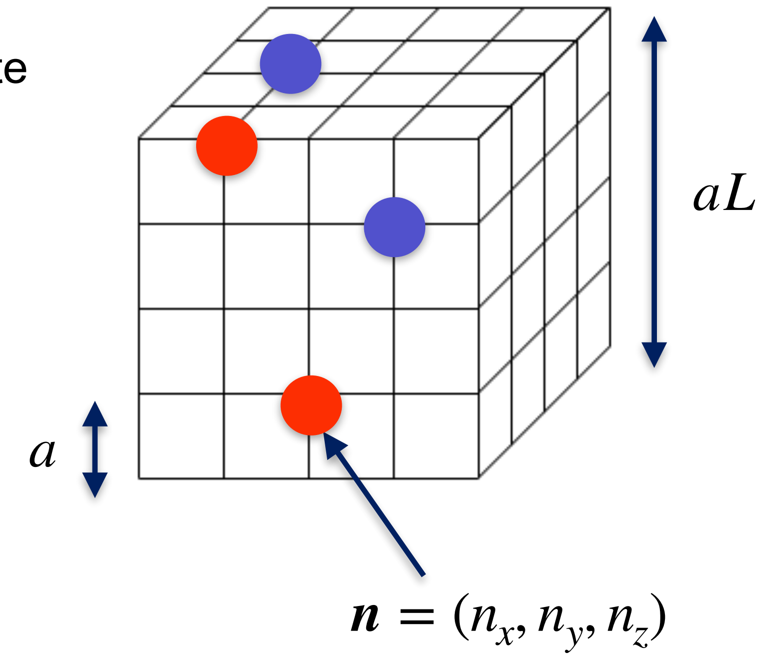
combine with lattice methods



(E.Epelbaum, H.-W.Hammer, U.-G. Meißner)

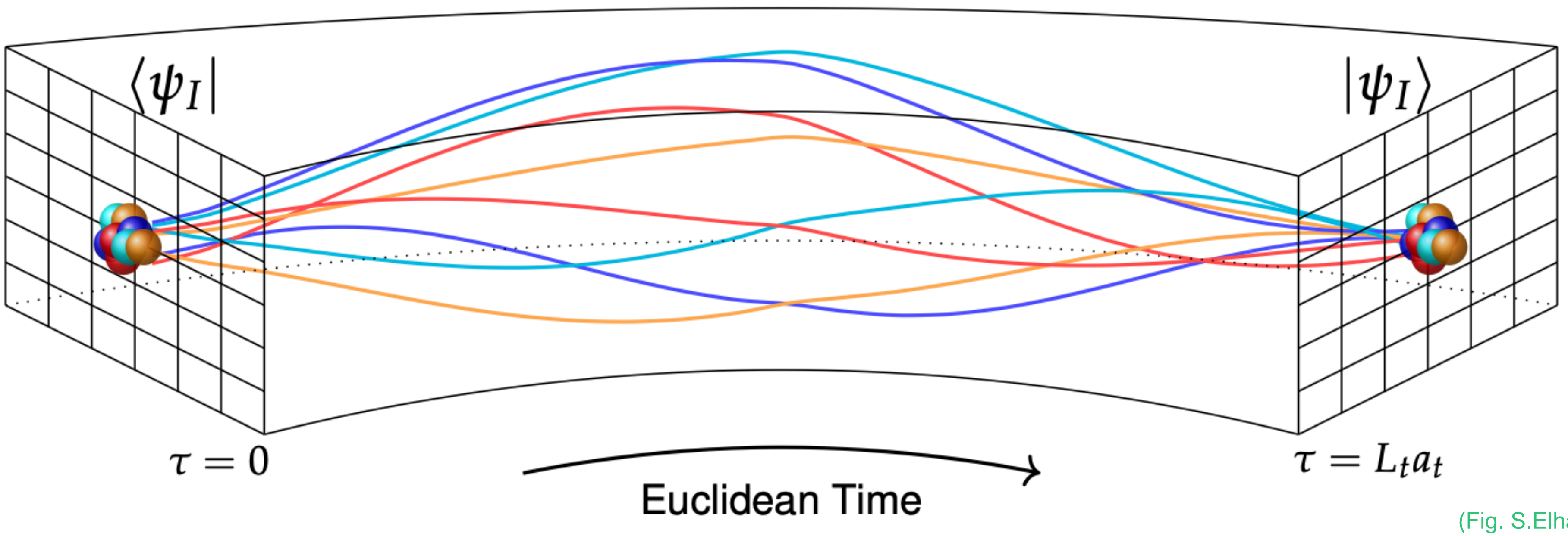
Method: Lattice Monte Carlo

- Lattice $\Rightarrow$ cubic volume of size $(La)^3$ with discrete lattice site



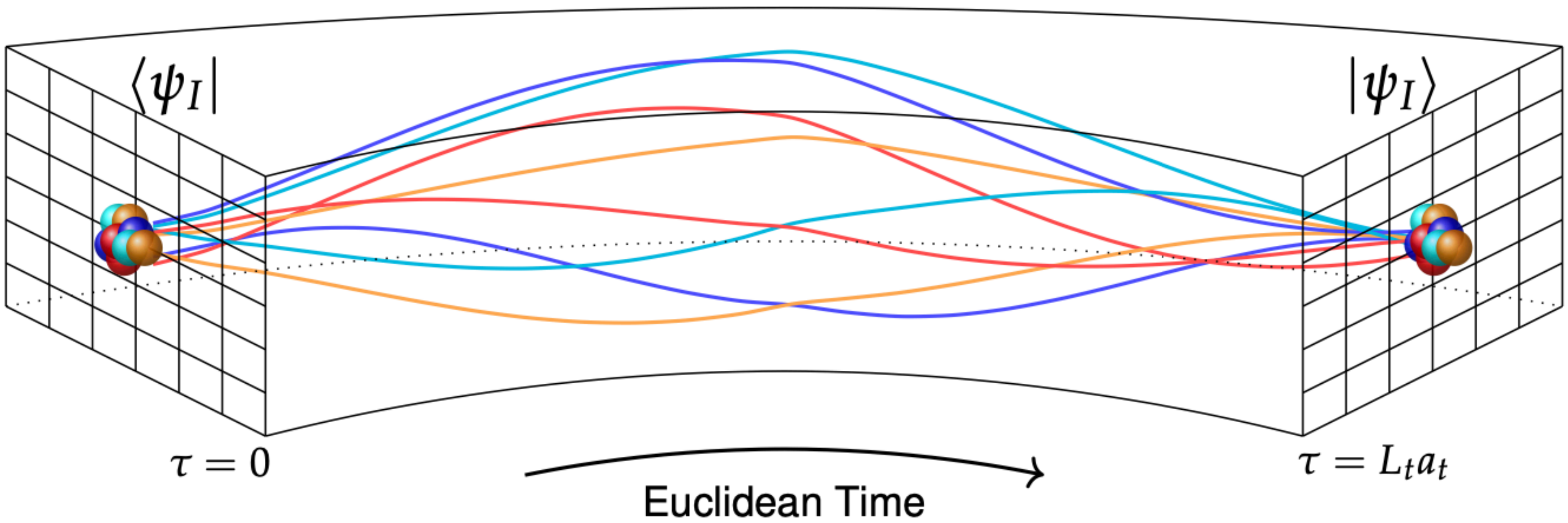
- Discretized chiral potentials, contact interactions one-pion exchange, Coulomb (Epelbaum et al.)

- Do Euclidean time evolution and extract i.e. energies as transient energy $E = -\dfrac{d}{d\tau} \ln(Z(\tau))$



(Fig. S.Elhatisari)

Method: Lattice Monte Carlo II



(Fig. S.Elhatisari)

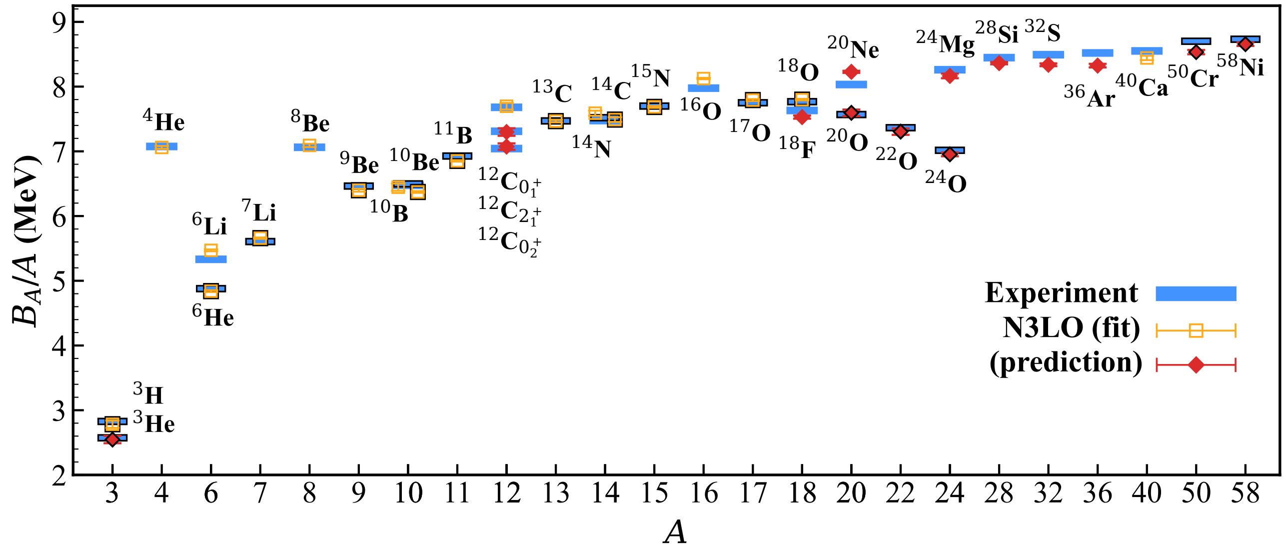
- Auxiliary fields to handle many particles efficiently:
- Idea: replace interactions between nucleons with interaction of a nucleon with an auxiliary field

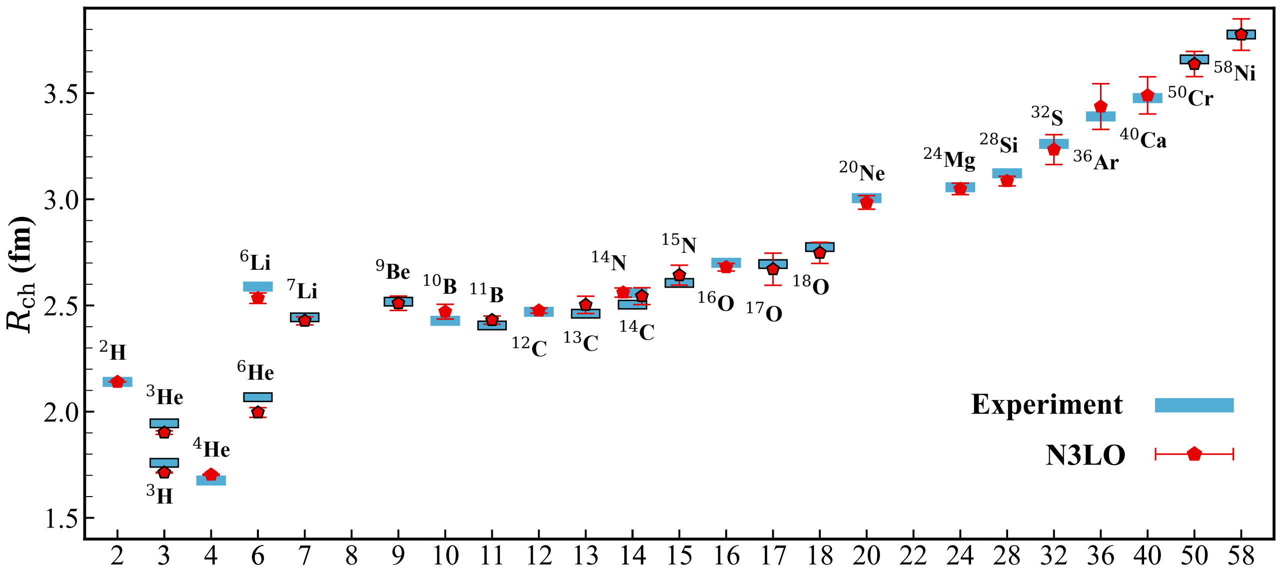
$$\exp\left(-\frac{C}{2}(N^\dagger N)^2\right) = \sqrt{\frac{1}{2}} \int \mathrm{d}A \exp\left[-\frac{A^2}{2} + \sqrt{C}A(N^\dagger N)\right]$$

Since nucleons only interact with an auxiliary field⇒ perfect for parallel computing

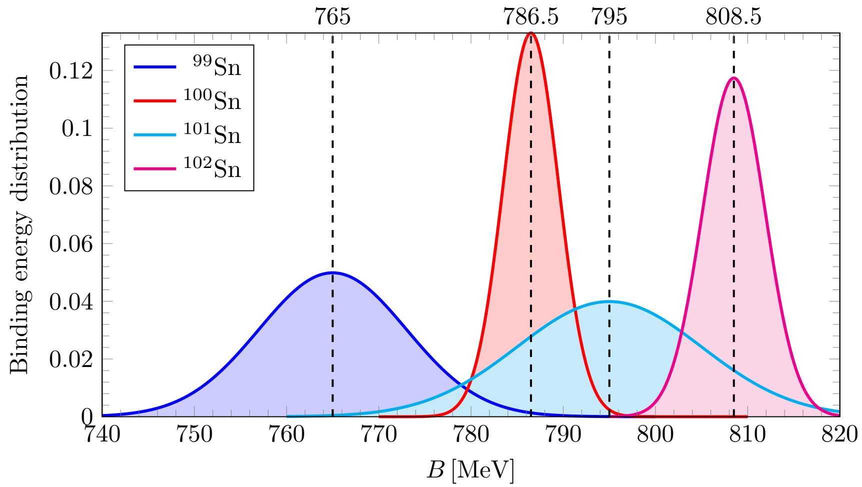
Very successful results in the nuclear sector

(S.Elhatisari,…,FH et al.)





- Ground and exited states are well reproduced over a large excerpt of the nuclear chart
- No fitting to charge radii, everything is a prediction!
- Utilizes Wavefunction matching
- Recent calculations in the $A \sim 100$ region



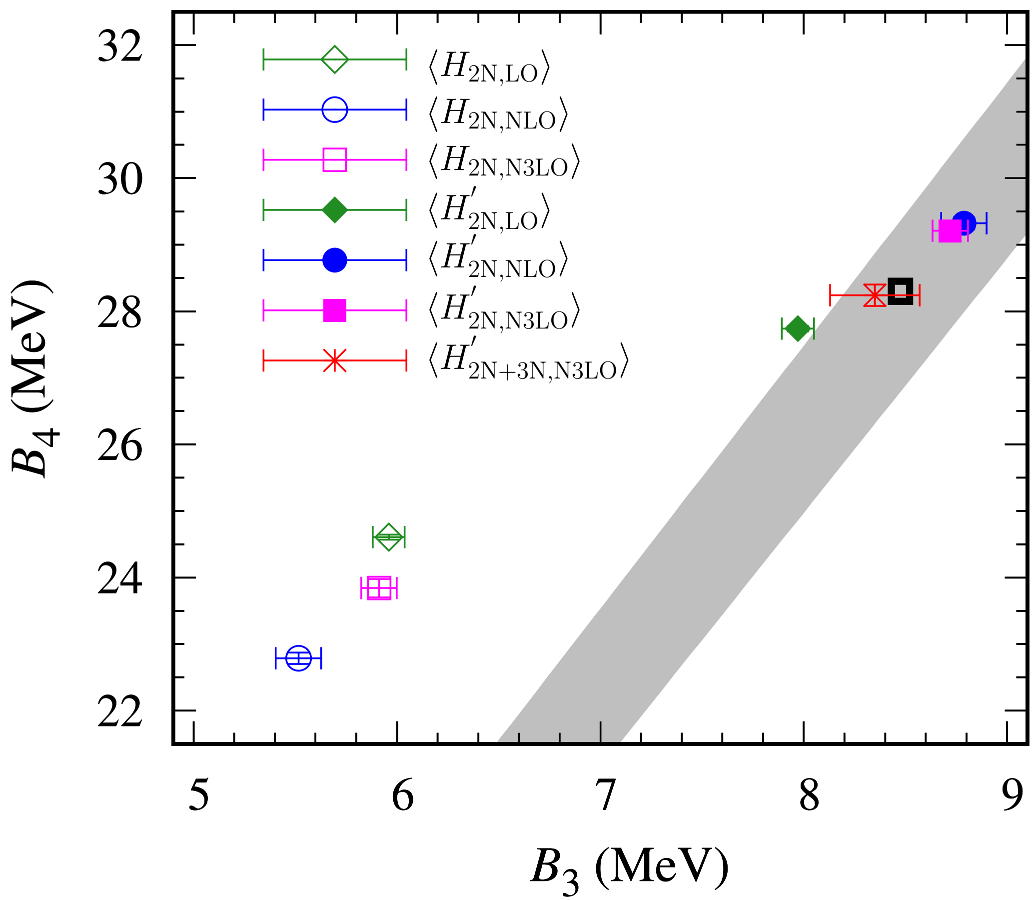
(FH et al.)

Few-nucleon systems are great to test interactions

- Monte-Carlo Simulations feature much smaller statistical uncertainties.
- Some systems can be solved numerically exact.

Tjon Band for nuclear systems

- Correlation between three- and four-body binding energy.
- All realistic nuclear potentials must fall within this band
- WFM interaction fulfils this requirement.

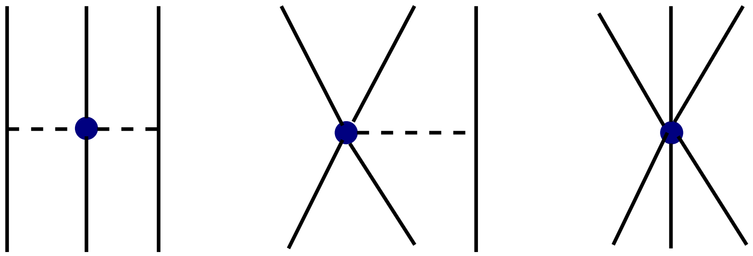


(S.Elhatisari,…,FH et al.)

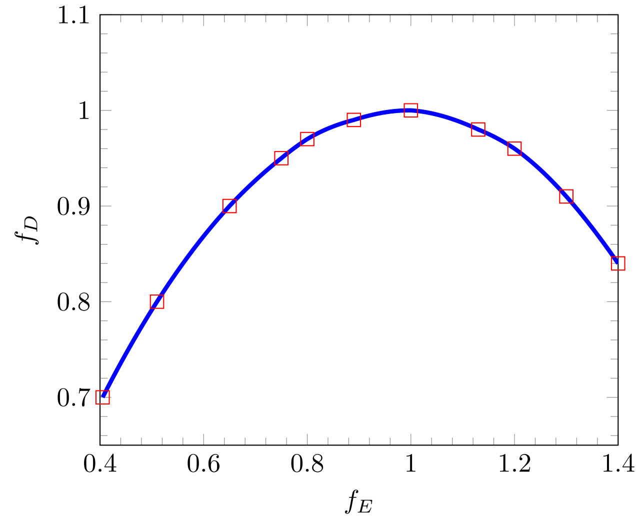
Triton-beta-decay

(Elhatisari, FH, Meißner. 2024)

- Typical input to three-body forces in other approaches
- Exact full non-perturbative (two-body and three-body) GPU Lanczos calculation



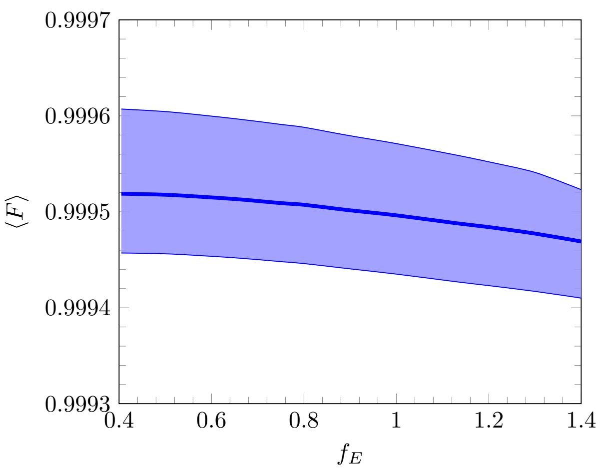
Curve of constant binding energy

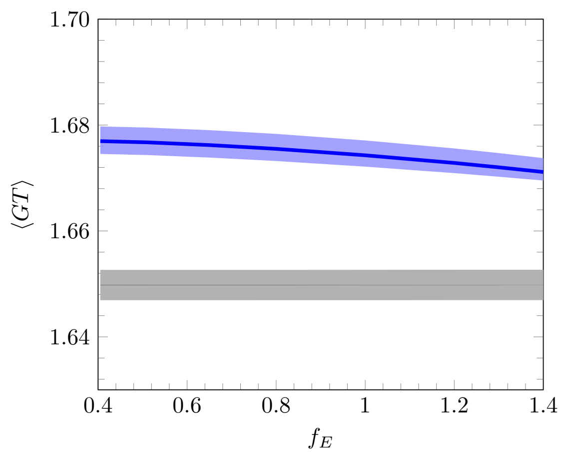


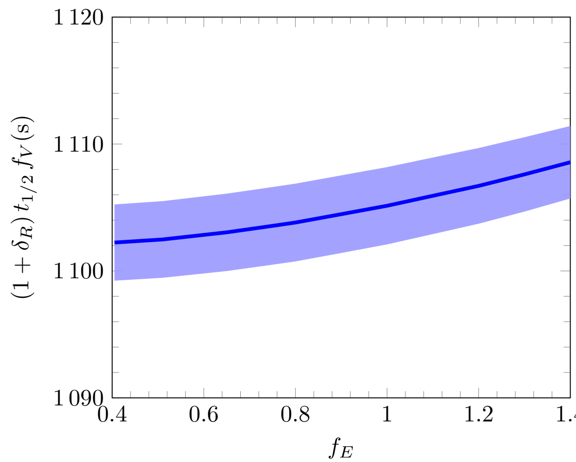
$$\langle F \rangle = \sum_{n=1}^{3} \langle {}^3\mathrm{He} \| \tau_{n,+} \| {}^3\mathrm{H} \rangle = 0.9998$$

$$\langle GT \rangle = \sum_{n=1}^{3} \langle {}^3\mathrm{He} \,||\, \sigma_n \tau_{n,+} \,||\, {}^3\mathrm{H} \rangle = 1.6497(23)$$

$$(1+\delta_R)\, t_{1/2}\, f_V = 1132.1(25)s$$





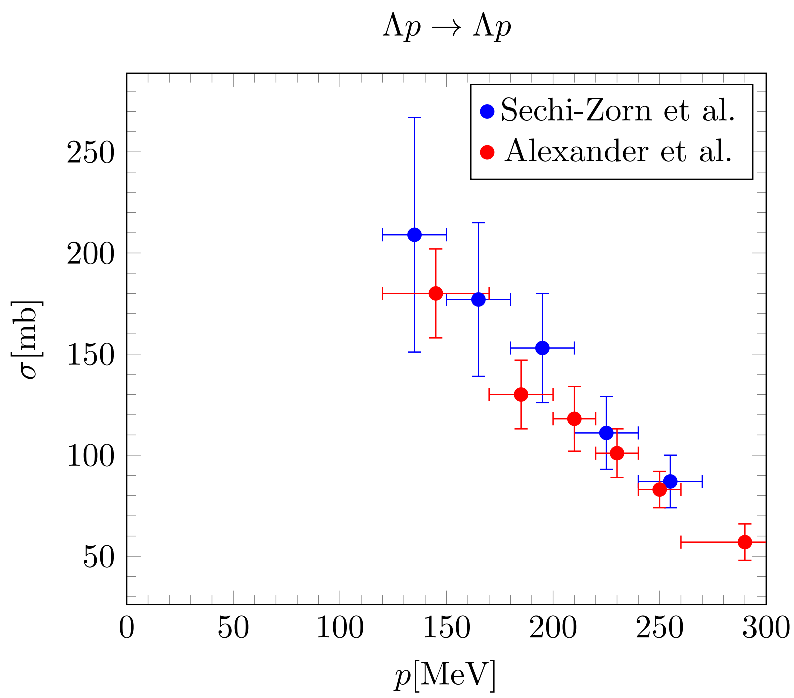


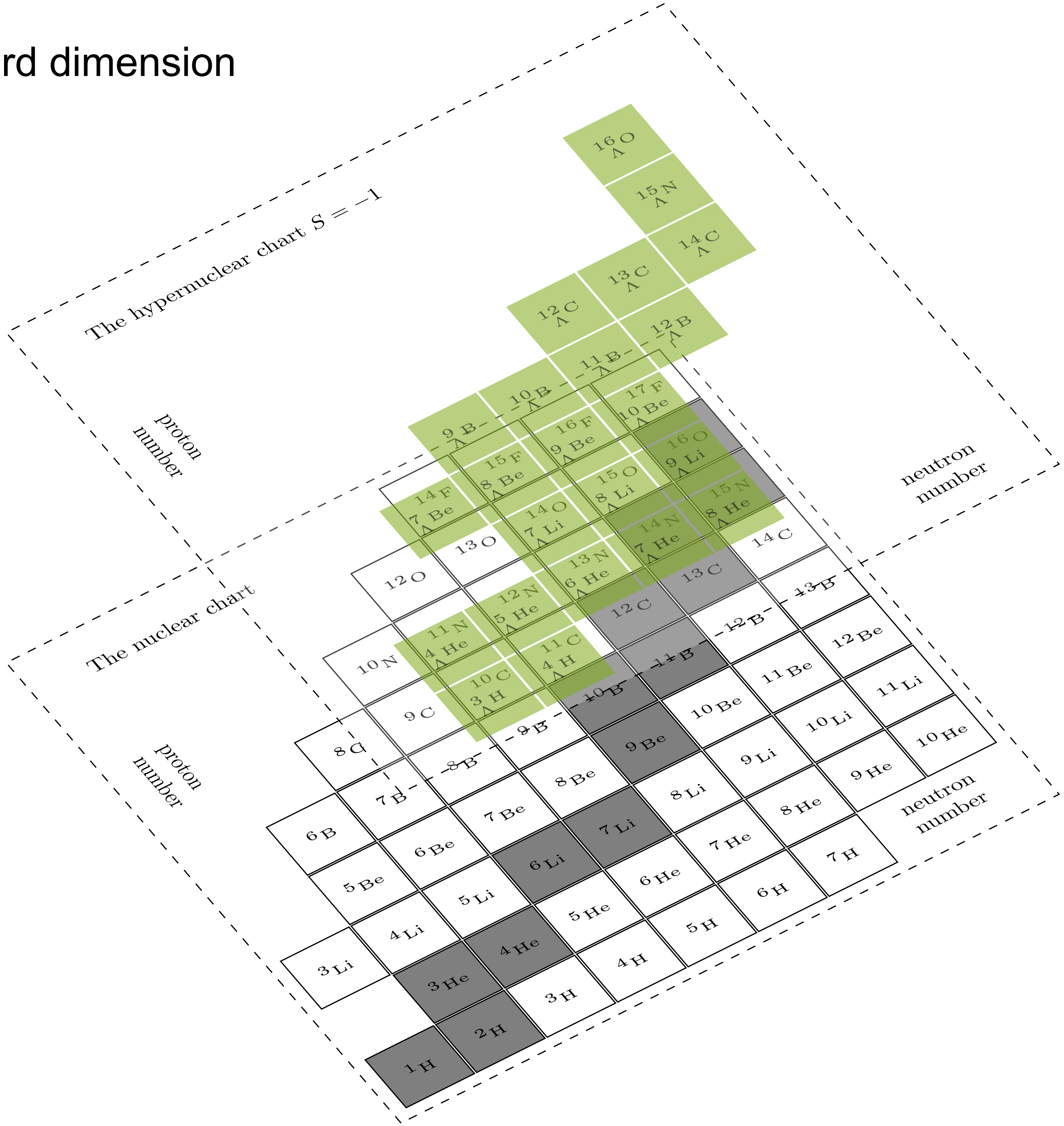
Hypernuclear physics in a nutshell

- Strangeness extents the nuclear chart to a third dimension

- Unique opportunity to study the strong force without the Pauli principle

- Typical approach from nuclear physics does not work since two-body data is sparse

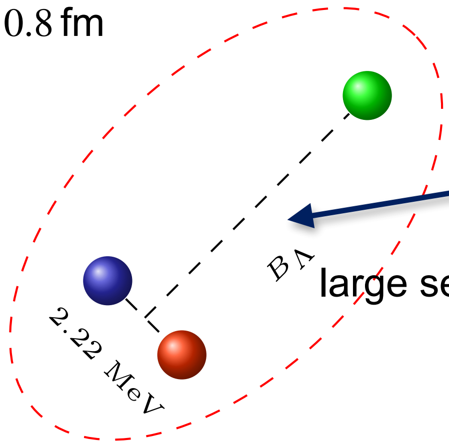




- Gateway : Three-Body Systems

Construction of a first Lattice ΛN interaction

$\sqrt{\langle r^2_{\Lambda d} \rangle} \approx 10.8$ fm



large separation

Emulsion:

$B_\Lambda = 0.130 \pm 0.050$ MeV Juric 1973

Heavy Ion:

$B_\Lambda = 0.406 \pm 0.120$ MeV Star 2020

$B_\Lambda = 0.102 \pm 0.063$ MeV Alice 2023

World Average:

$B_\Lambda = 0.105 \pm 0.026$ MeV Mainz 2025

Tuesday morning

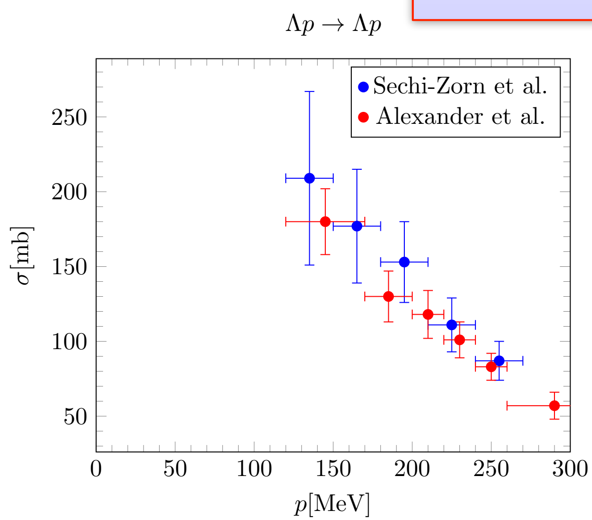
Shallow S-Wave State

$$J^P = \frac{1}{2}^+$$

Distinguishable

$$I = 0 \Rightarrow \frac{1}{\sqrt{2}}(pn - np)\Lambda$$

Combine 2-Body data with hypertriton in exact calculation



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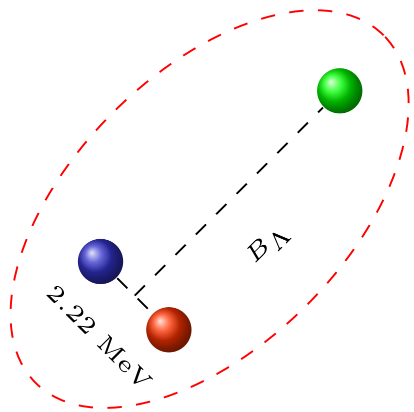
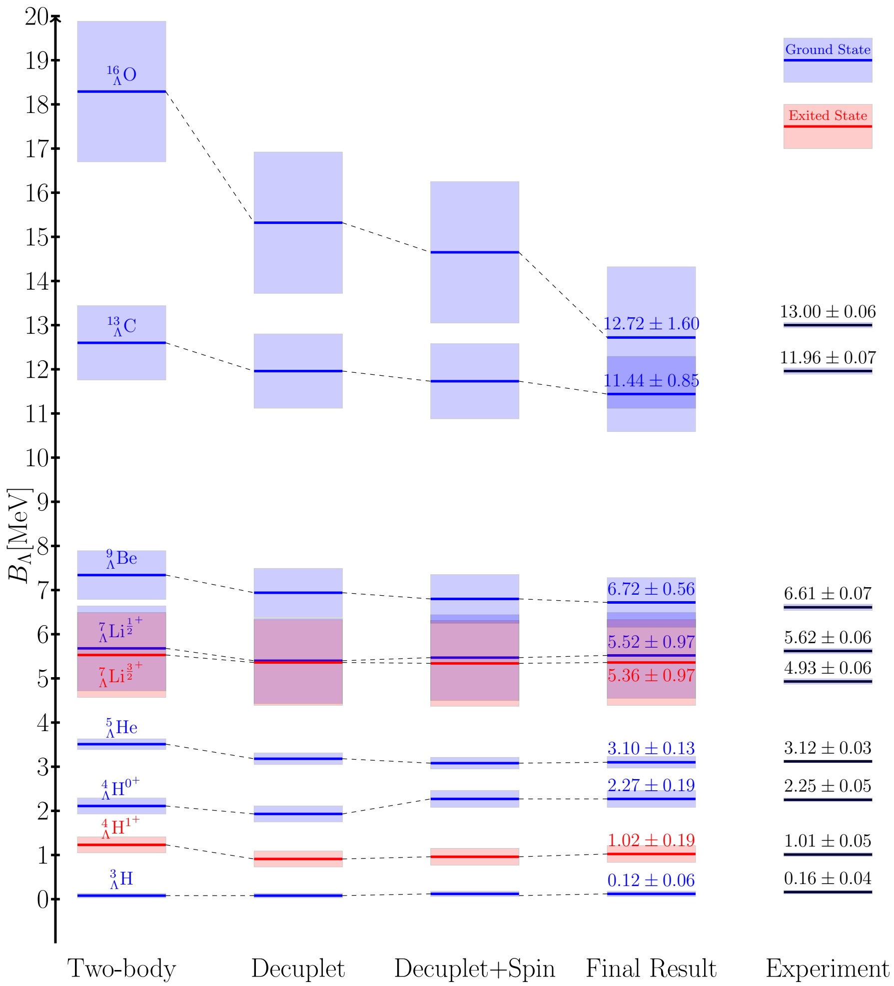
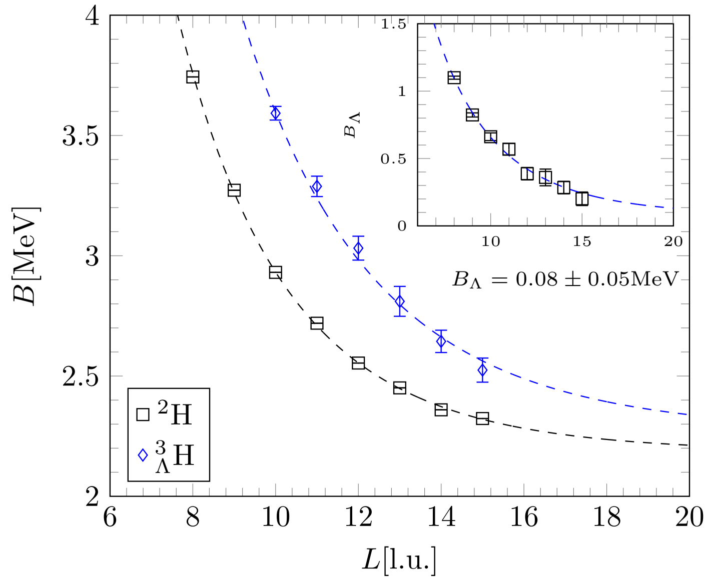


Results for hypernuclei (FH et al)





$$V_{ct}^{\Lambda NN} = C_1(1 - \boldsymbol{\sigma}_2 \cdot \boldsymbol{\sigma}_3)(3 + \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3)$$
$$+ C_2 \boldsymbol{\sigma}_1 \cdot (\boldsymbol{\sigma}_2 + \boldsymbol{\sigma}_3)(1 - \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3)$$
$$+ C_3(3 + \boldsymbol{\sigma}_2 \cdot \boldsymbol{\sigma}_3)(1 - \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3)$$

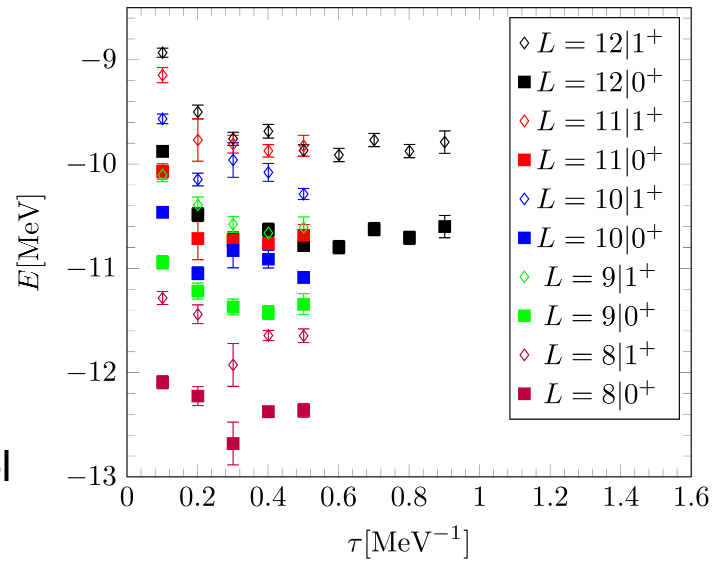
Proper extrapolation:

$$B_\Lambda(^3_\Lambda\text{H}) = 0.08 \pm 0.05 \text{ MeV}$$

Experiment

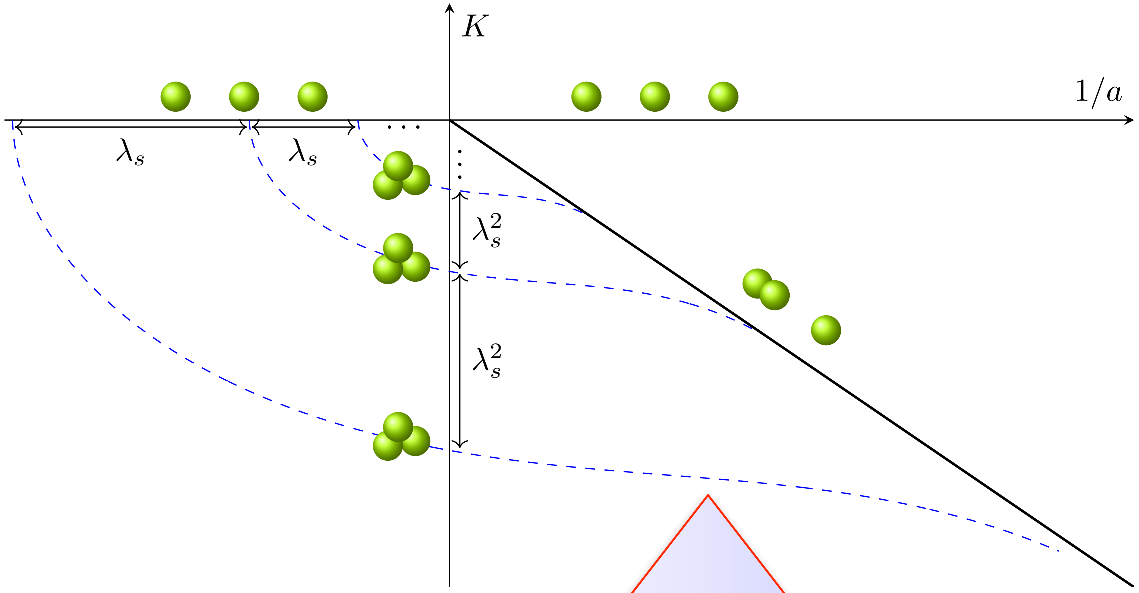
0.105 ± 0.026 MeV

- Hypertriton has large Λ separation of ~ 11 fm



Finite Box under control

Efimov physics for hypernuclear systems



Physics Determined by a and Λ_*

Universal Relations Between Observables

B_Λ and $\langle r^2 \rangle$

B_Λ and τ

B_Λ and $a_{\Lambda d}$

Pionless effective field theory
Controllable Uncertainties
Systematic Improvement

Shallow S-Wave State

$J^P = \frac{1}{2}^+$

Distinguishable

2 Isospin Channels

Large Scattering Length

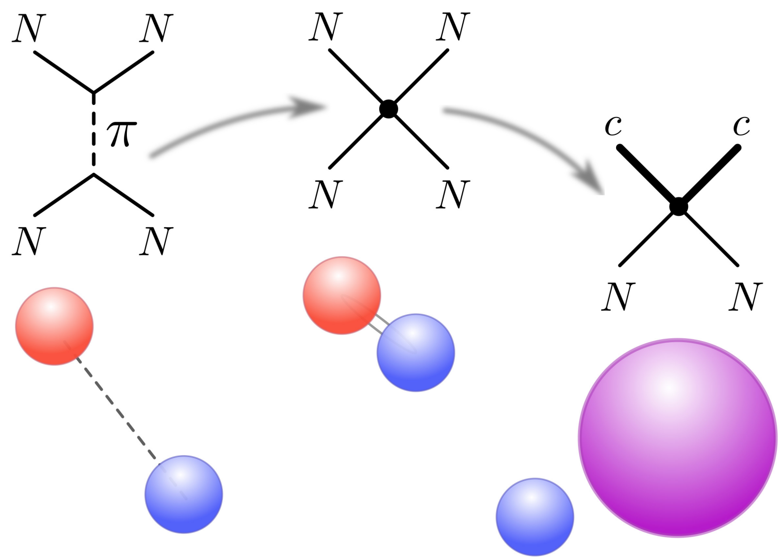
Why pionless Effective Field Theory(EFT)?

What is pionless EFT (in a nutshell)?

Simplifies a fundamental theory to it essential parts

Focus on the relevant degrees of freedom

Offer a systematic way to improve the theory



Picture: FB Physik TU Darmstadt

Integrate out heavy particles out of the theory

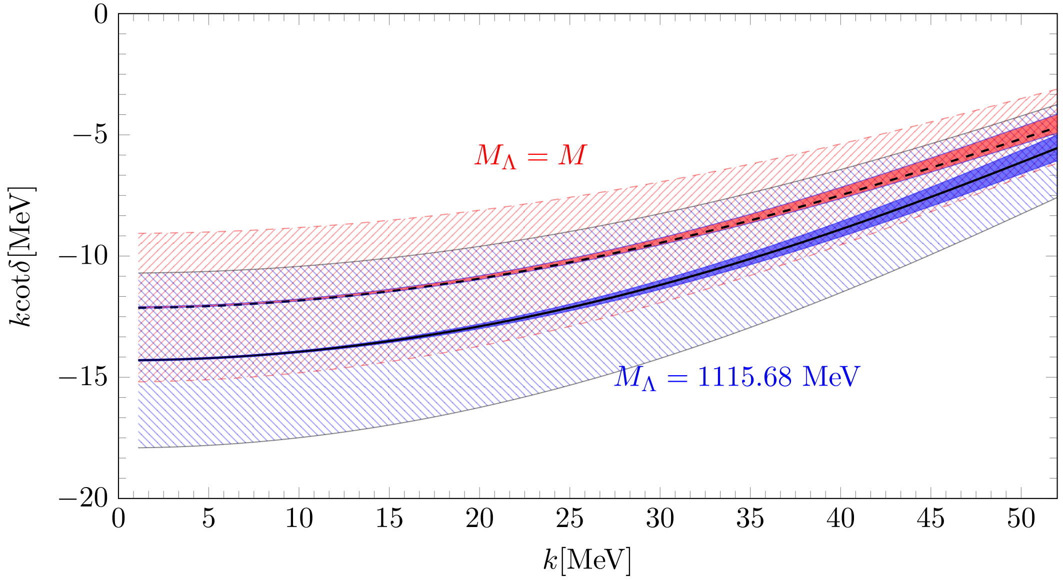
$$\frac{g^2}{m_\pi^2 - q^2} \approx \frac{g^2}{m_\pi^2} + \frac{g^2 q^2}{m_\pi^4} \qquad \xrightarrow{\gamma \sim q \ll m_\pi}$$

No explict $\Lambda \Leftrightarrow \Sigma$
But Three-Body-Force

$^3S_1(NN) + \Lambda$ (Hypertriton)
$^1S_0(NN) + \Lambda$ (Λnn)

$^3S_1(\Lambda N) + N$ (Both)
$^1S_0(\Lambda N) + N$ (Both)

The Phillips line for the Hypertriton



Use chiral EFT inputs for ΛN interaction

Phase shift are however independent of details of the interaction

→ Shallowness of the hypertriton

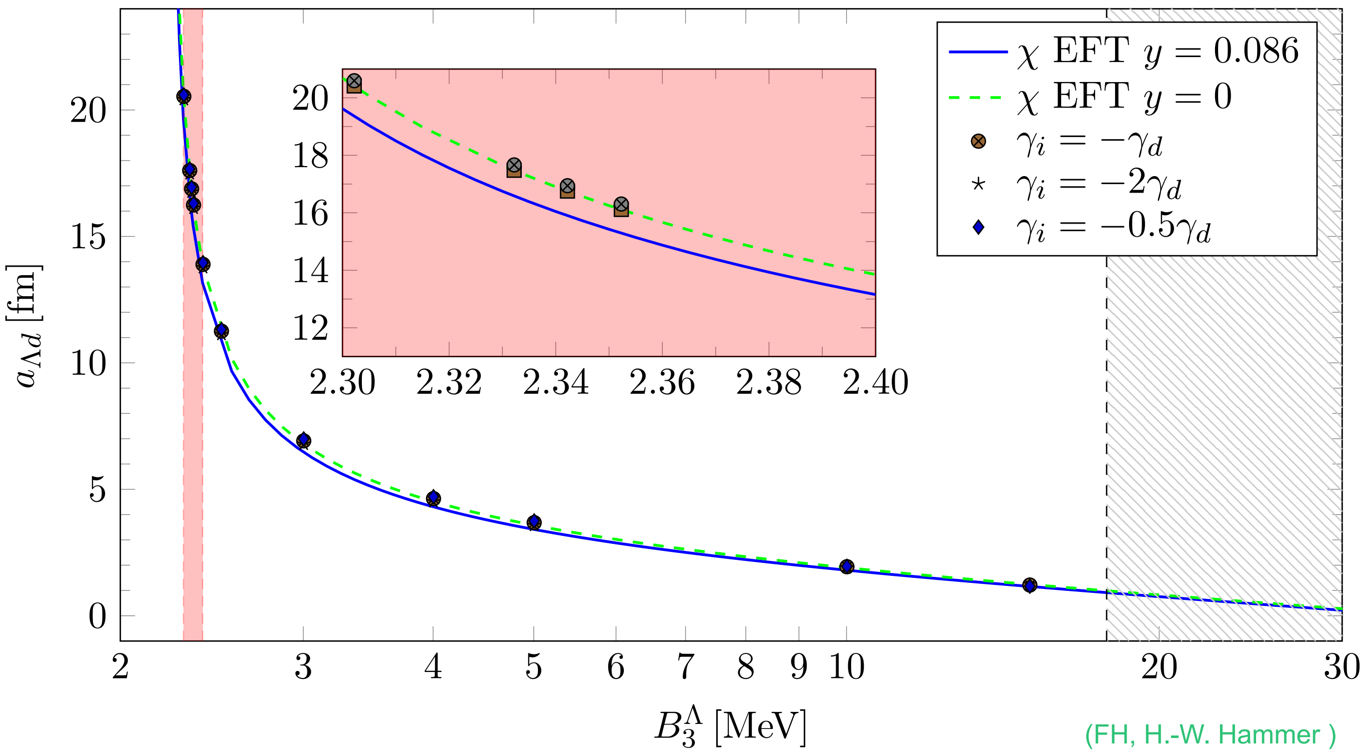
$a_{\Lambda d} = 15.4^{+4.3}_{-2.3}$ fm $\quad r_{\Lambda d} \approx 1.3$ fm

Strong dependence on B_Λ

Independent of the Λ pole position

Universal relation :

$B_\Lambda \Leftrightarrow a_{\Lambda d}$



(FH, H.-W. Hammer)

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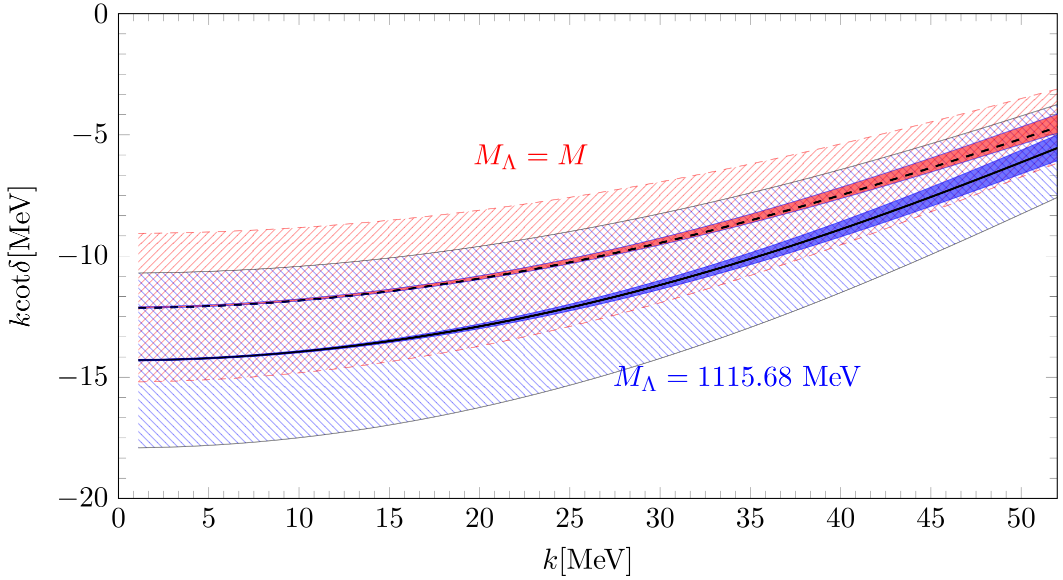

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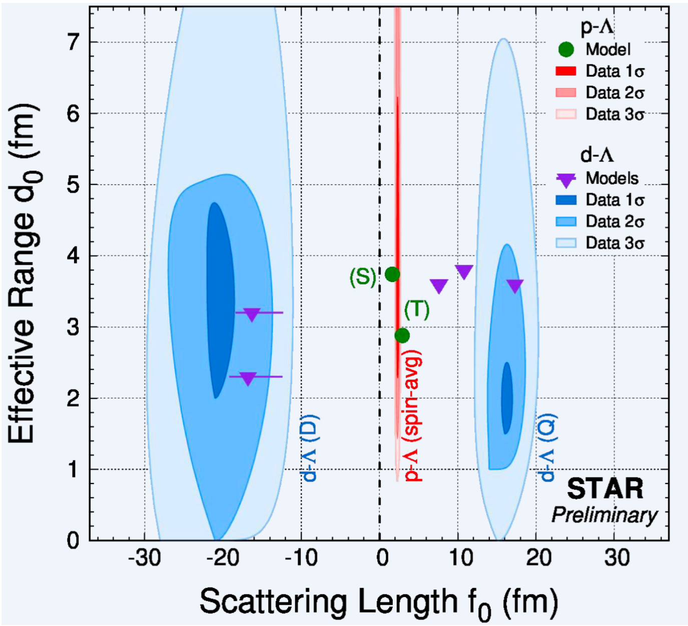
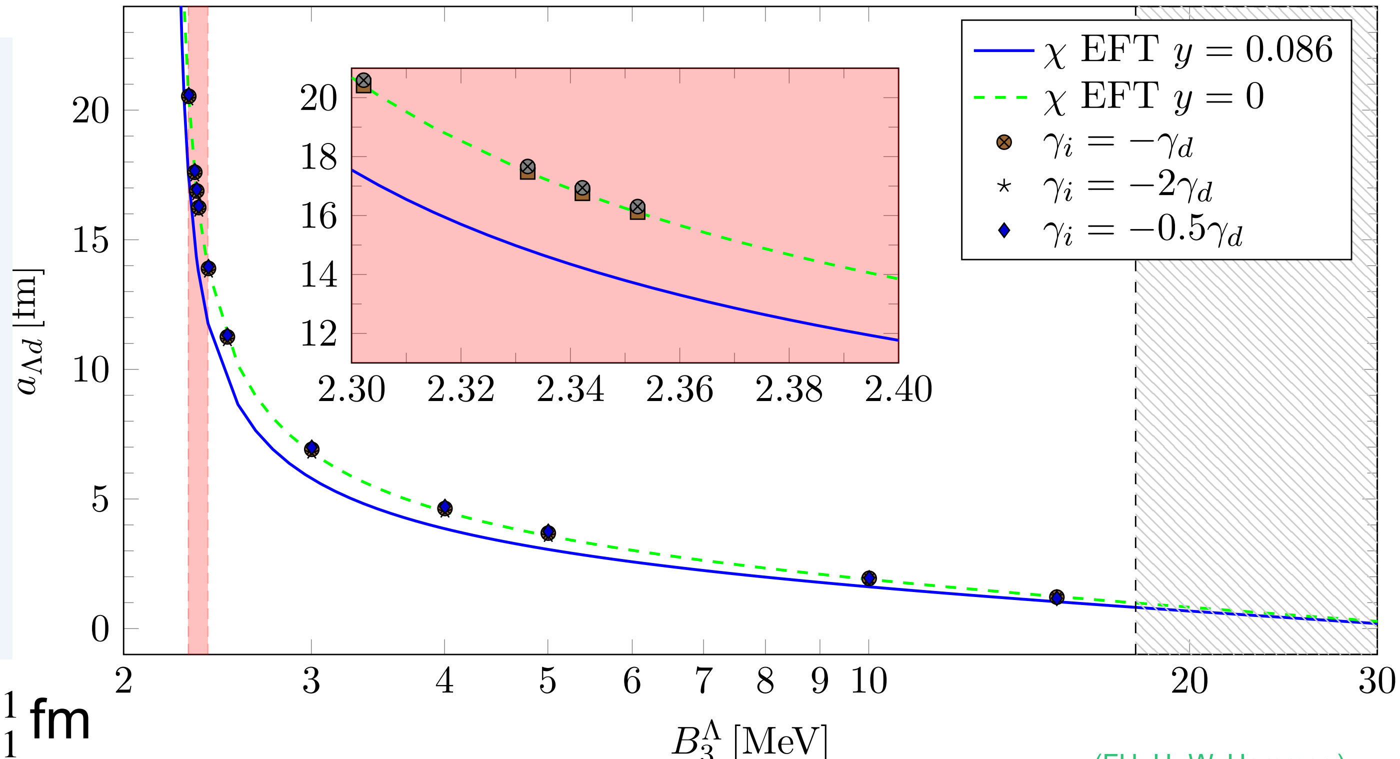
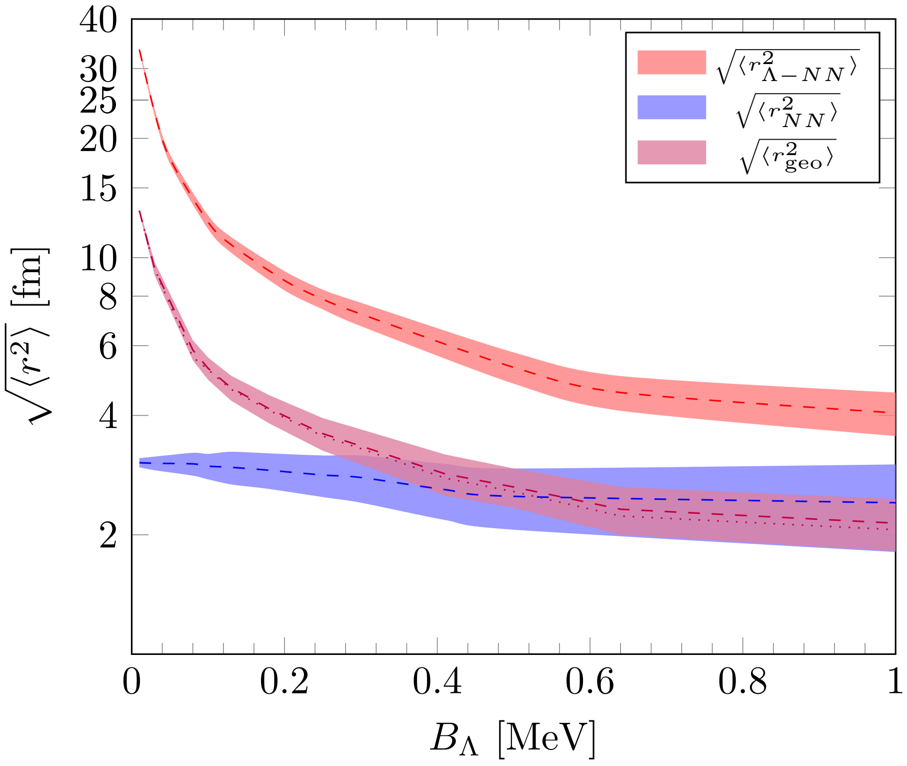


$a_{\Lambda d} = 16^{+2}_{-1}$ fm $\quad$ $r_{\Lambda d} = 2^{+1}_{-1}$ fm



(FH, H.-W. Hammer)

Matter radii for the Hypertriton



Expectation from two-body calculation

$$B_2 = \frac{1}{2\mu a^2} \quad \text{and} \quad \langle r^2 \rangle = \frac{a^2}{2}$$

$$\sqrt{\langle r^2_{NN} \rangle} \approx 3.04\,\text{fm}$$

$$\sqrt{\langle r^2_{\Lambda d} \rangle} \approx 10.34\,\text{fm}$$

$\longrightarrow$

Universal relation between $\langle r^2 \rangle \Leftrightarrow B_\Lambda$

$\sqrt{\langle r^2_{\Lambda-NN'} \rangle}$[fm]	$\sqrt{\langle r^2_{N'-\Lambda N} \rangle}$[fm]	$\sqrt{\langle r^2_{N-N'\Lambda} \rangle}$[fm]	$\sqrt{\langle r^2_{NN'} \rangle}$[fm]	$\sqrt{\langle r^2_{geo} \rangle}$[fm]
10.79	3.96	4.02	2.96	4.66
$+3.04/-1.53$ $+0.03/-0.02$	$+0.40/-0.25$ $+0.03/-0.03$	$+0.41/-0.25$ $+0.03/-0.03$	$+0.06/-0.05$ $+0.03/-0.04$	$+1.19/-0.54$ $+0.01/-0.01$

Insensitive to details of Of the ΛN Interaction

Summary

- Briefly introduced Nuclear lattice effective field theory
- Presented results for charge as well as binding energies for normal nuclear matter
- Discussed the importance of few nucleon systems for nuclear as well as hypernuclear systems

- Discussed the Efimov effect at the example of the hypertriton
- Showed universal relations between binding energy and scattering length
- Discussed the importance of few nucleon systems for nuclear as well as hypernuclear systems