PID Likelihoods for the Barrel DIRC

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Outline

- Maximum Likelihood mean of a Gaussian Distribution
- Toy Monte carlo studies and effect of background on true Gaussian distribution
- Reconstructed θ_c and Time in Barrel DIRC
- Track based Maximum Likelihood and PID
- Summary

Maximum likelihood mean of the Gaussian distribution

Probability

$$f(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\ \sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right),$$

PDF

$$f(x_1, \ldots, x_n \mid \mu, \sigma^2) = \prod_{i=1}^{n} f(x_i \mid \mu, \sigma^2) = \left(\frac{1}{2\pi\sigma^2}\right)^{n/2} \exp\left(-\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{2\sigma^2}\right),$$

Minimize
For
mean

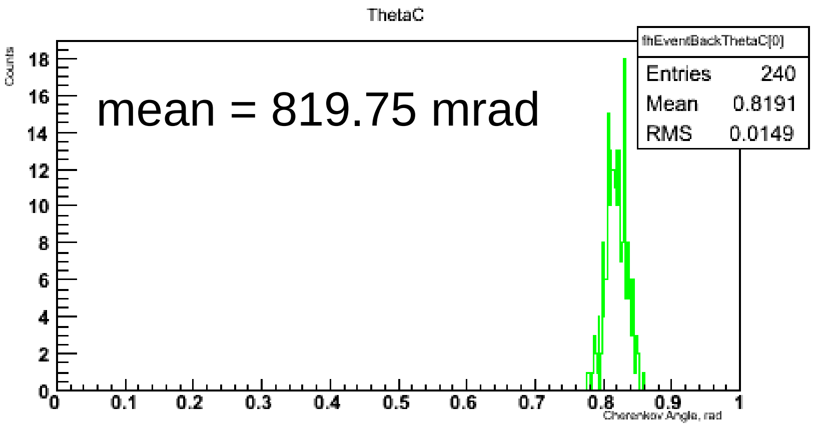
$$0 = \frac{\partial}{\partial\mu} \log\left(\left(\frac{1}{2\pi\sigma^2}\right)^{n/2} \exp\left(-\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2 + n(\bar{x}-\mu)^2}{2\sigma^2}\right)\right)$$

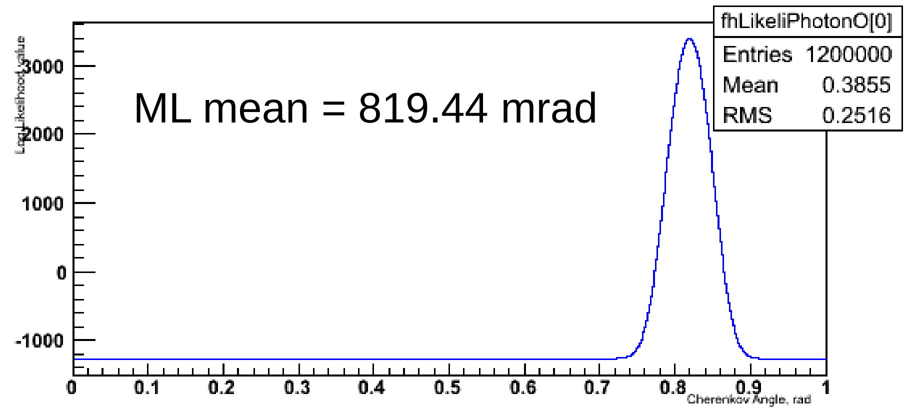
$$= \frac{\partial}{\partial\mu}\left(\log\left(\frac{1}{2\pi\sigma^2}\right)^{n/2} - \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2 + n(\bar{x}-\mu)^2}{2\sigma^2}\right)$$

$$= 0 - \frac{-2n(\bar{x}-\mu)}{2\sigma^2}$$

$$\hat{\mu} = \bar{x} = \sum_{i=1}^{n} x_i/n.$$

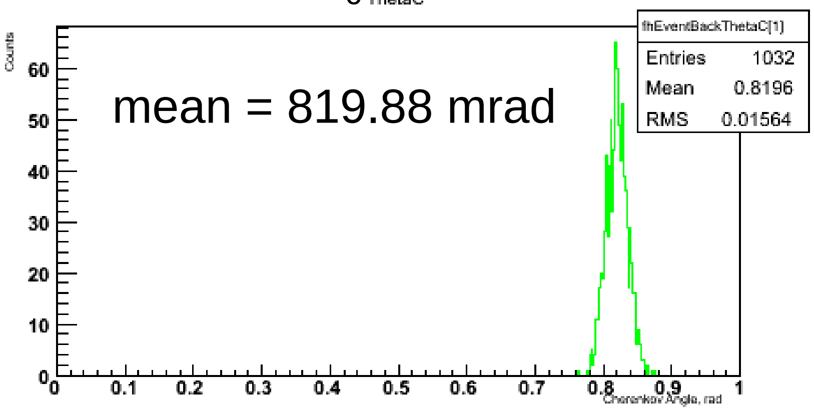
Typical Gaussian events and the Maximum Likelihood mean

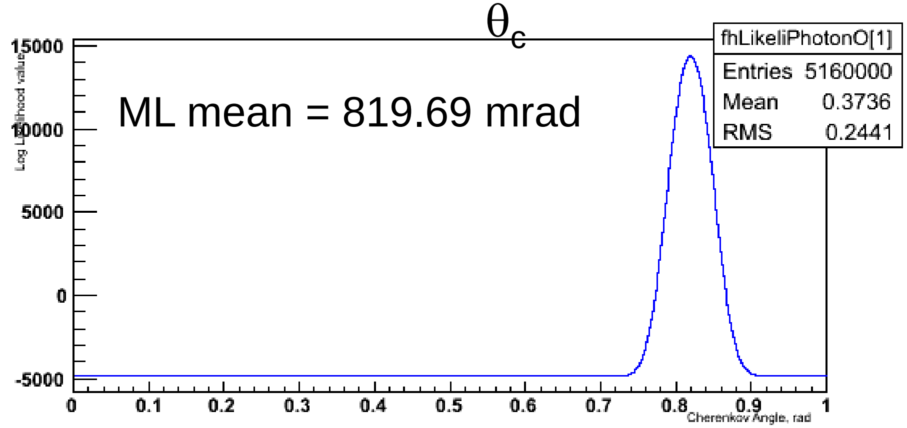




θ_c

θ_c





θ_c

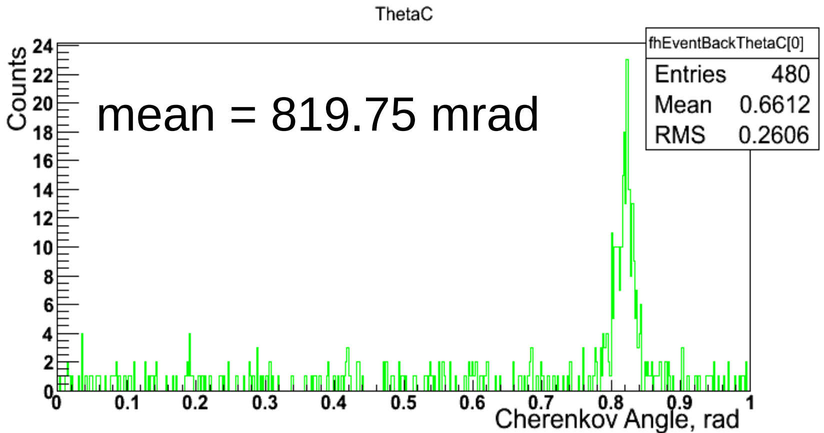
θ_c

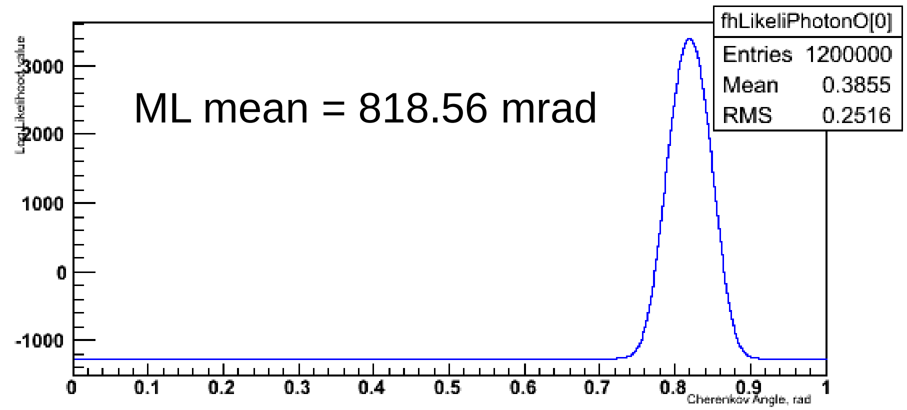
Event

Log Likelihood value

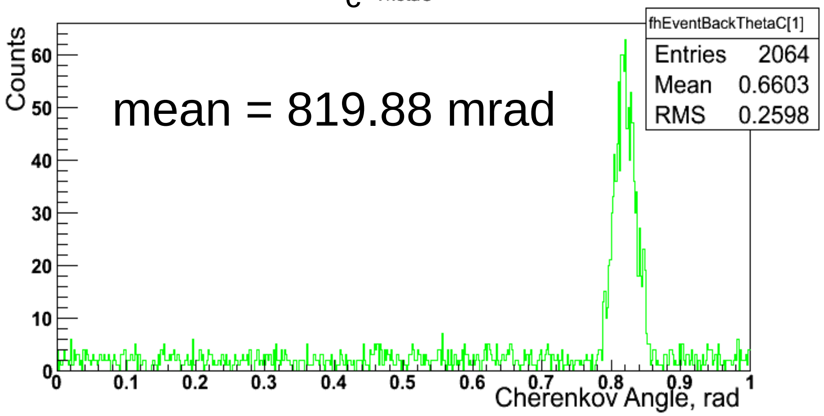
The Maximum likelihood value is obtained with zero cross over at the maximum

Gaussian + background and the Maximum Likelihood mean

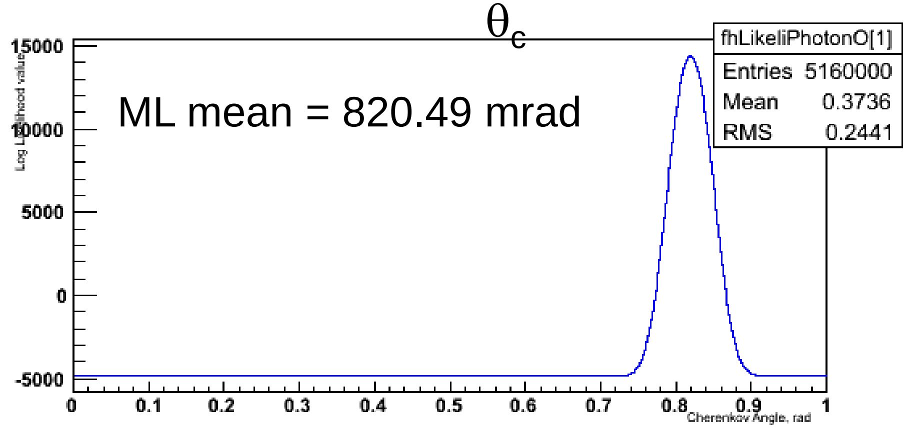




θ_c



θ_c



θ_c θ_c

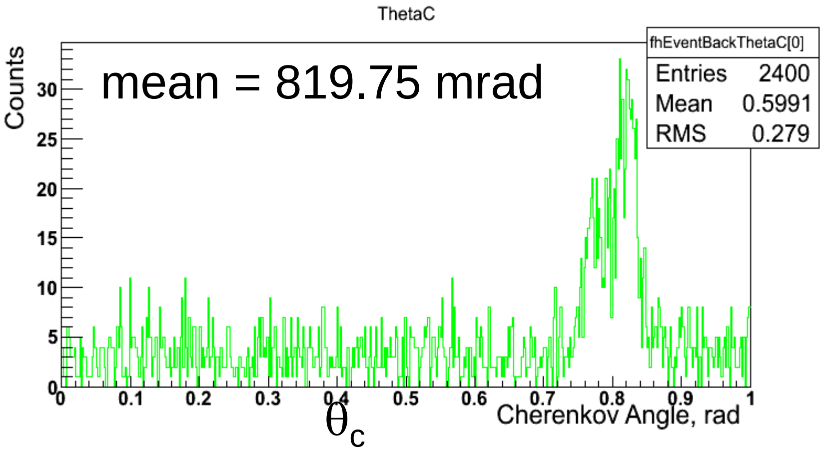
Event

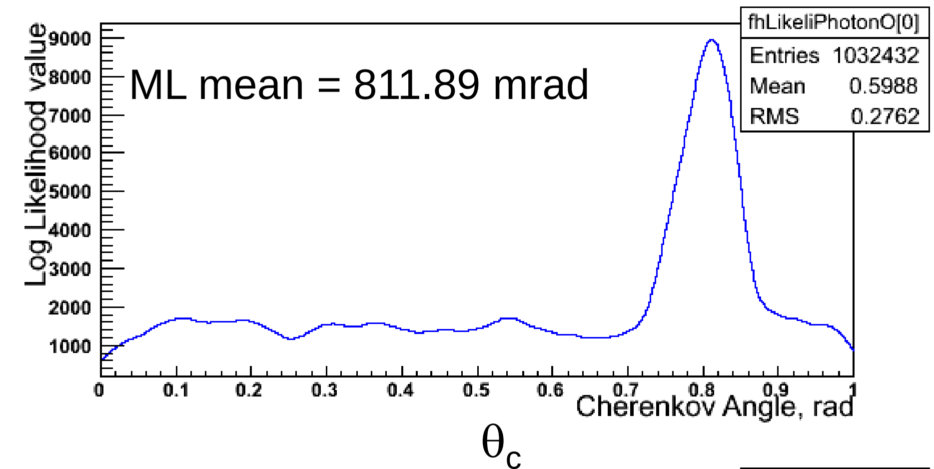
Log Likelihood value

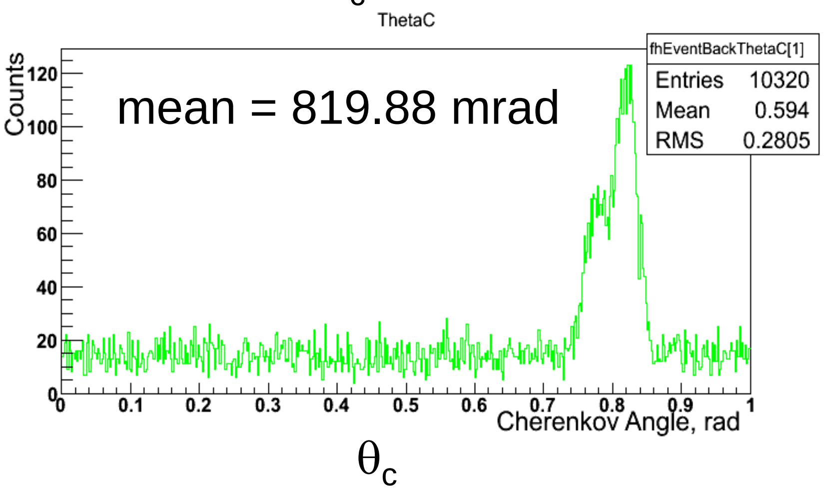
The Maximum likelihood value is obtained with zero cross over at the maximum

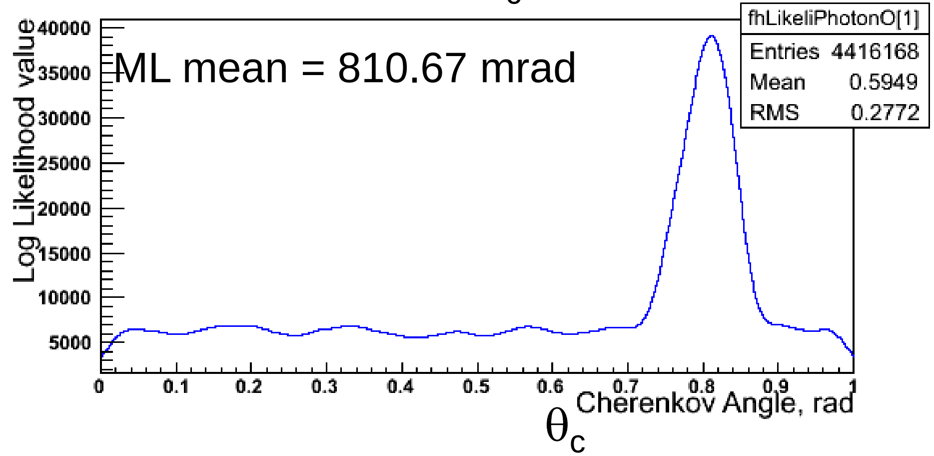
Probability $$f(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\ \sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right), \qquad + \chi$$

Gaussian + background and the Maximum Likelihood mean









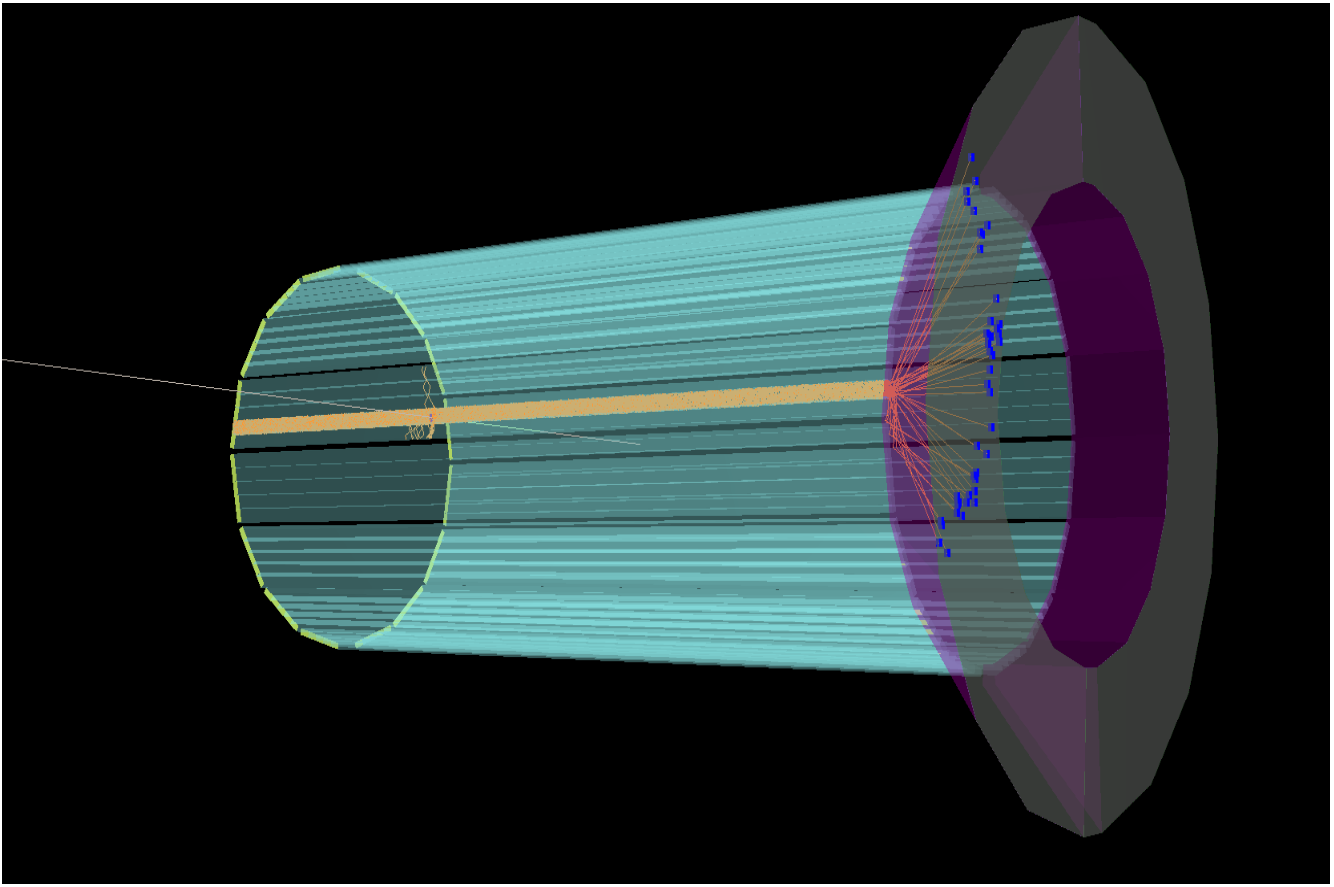
Event

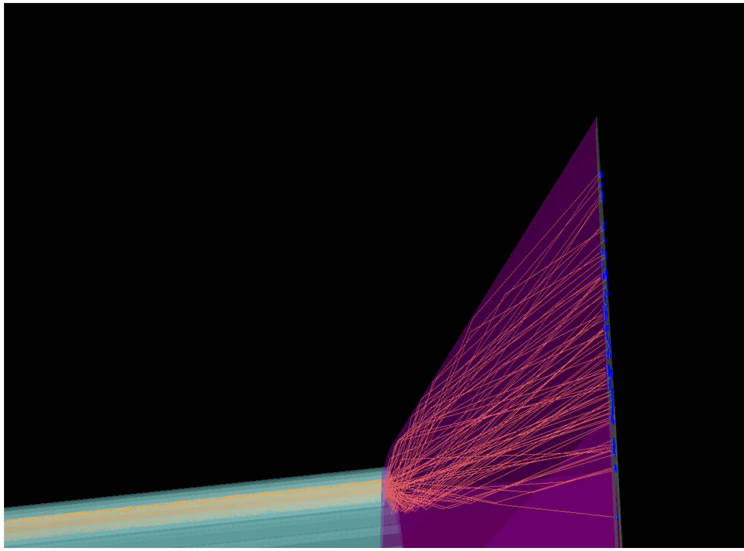
Log Likelihood value

The Maximum likelihood value is obtained with zero cross over at the maximum

Probability $$f(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\ \sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right), \quad + X$$

Reconstruction of Cherenkov angle (θ_c)





Different reflections in expansion volume

2.5 meter bars and 30 cm Expansion volume

Ambiguities (different possibilities of reflections in expansion volume and bar)

Most probable reflection possibilities in expansion volume are

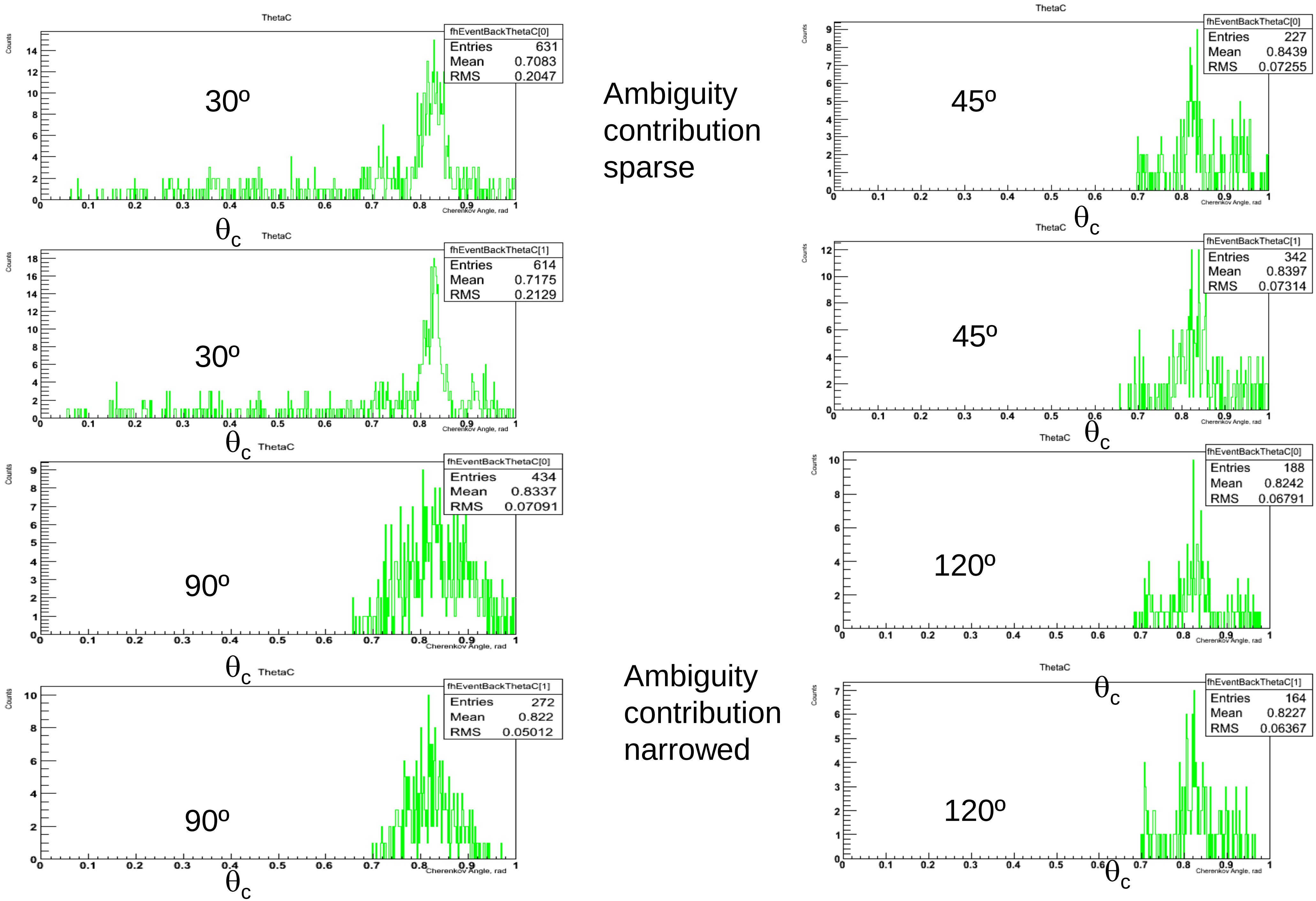
Direct,

Bottom,

UP and

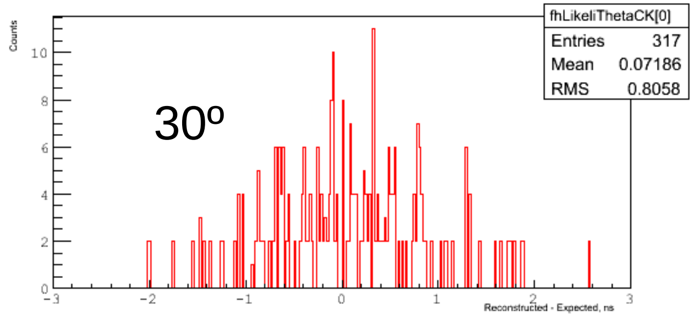
Bottom + UP

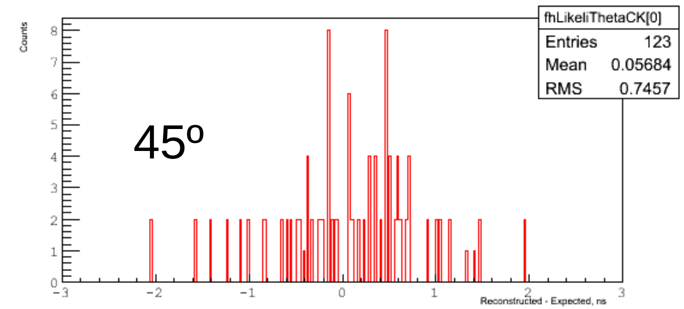
Reconstructed Cherenkov angle (θ_c) at different polar angles

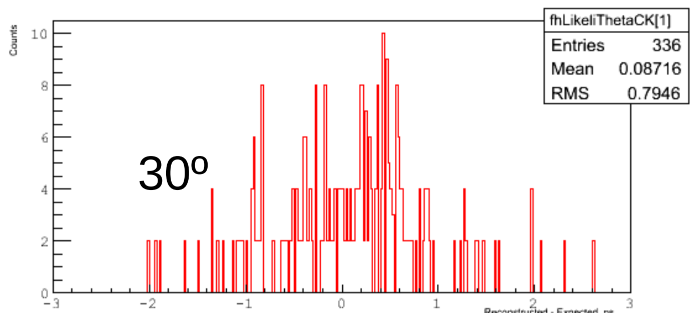


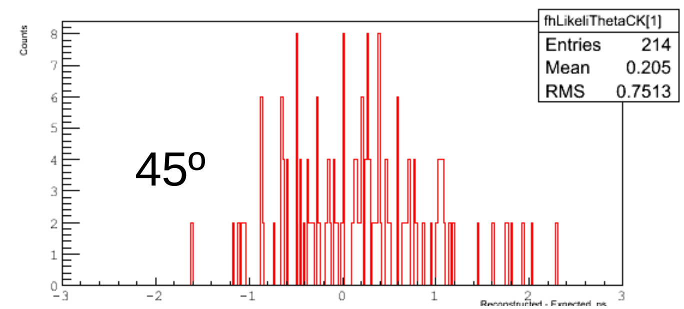
Effect of Chromatic Dispersion on photon time

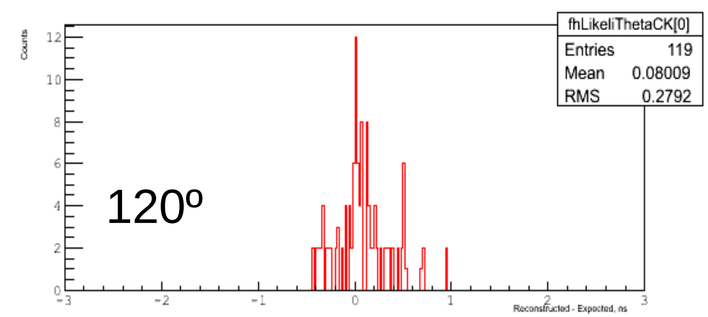
Longer path

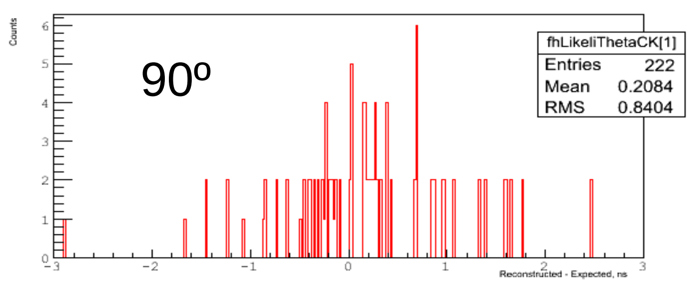


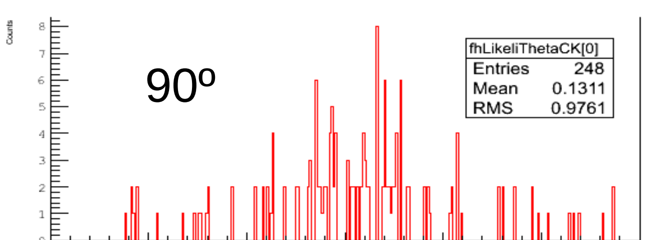


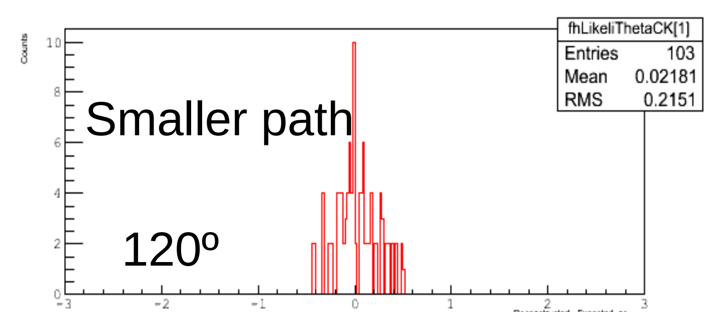








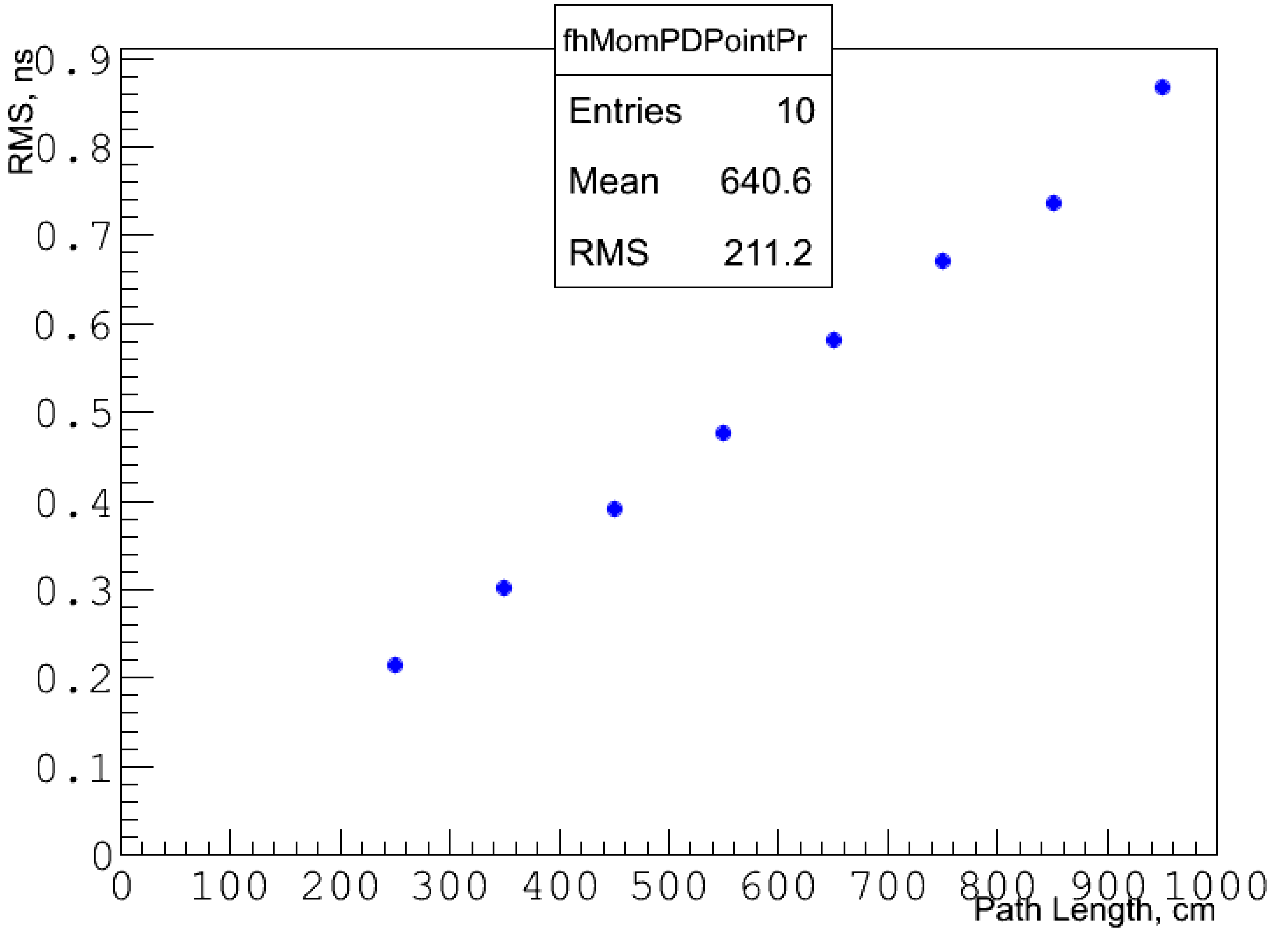




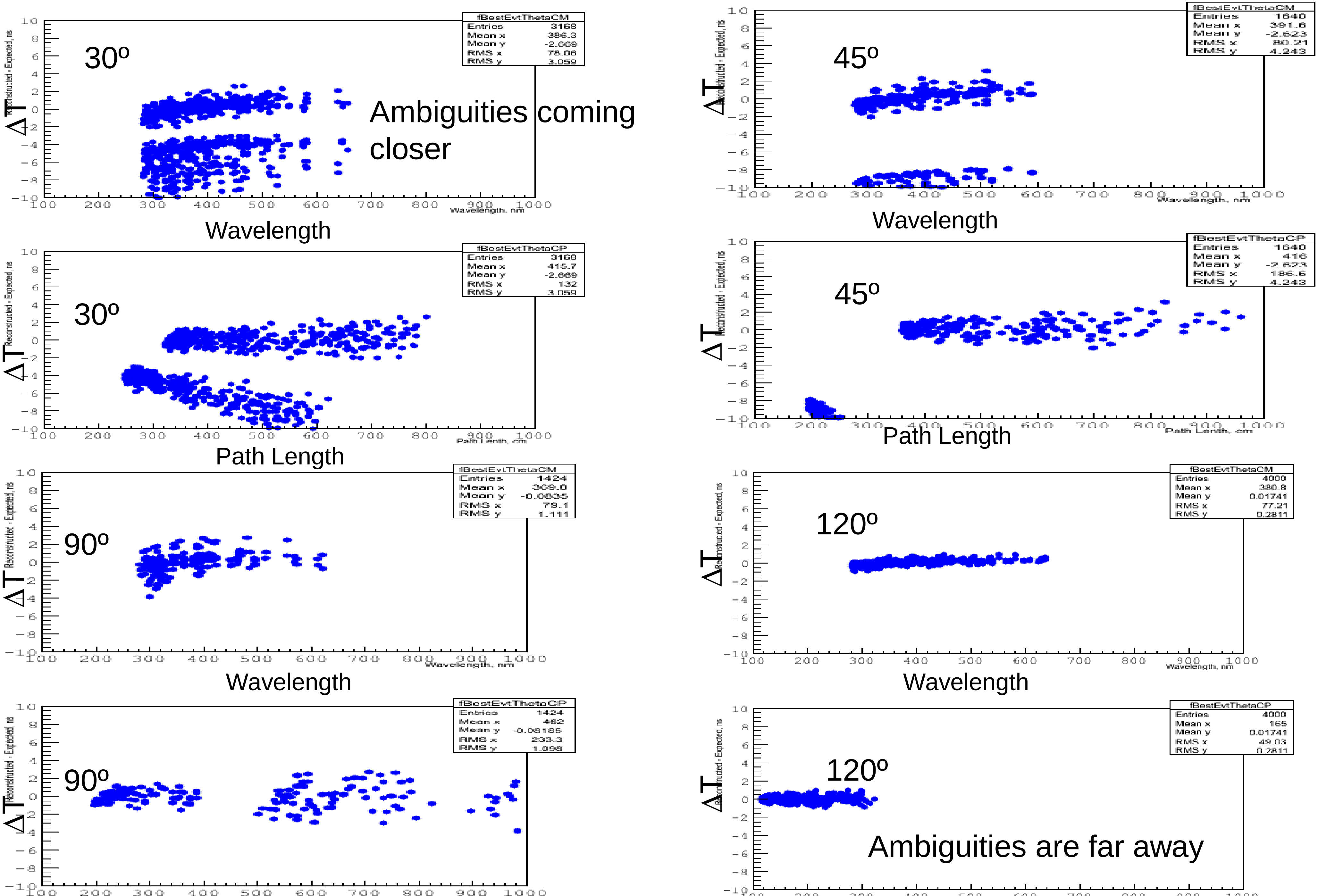
ΔT (reconstructed - expected)

ΔT (reconstructed - expected)

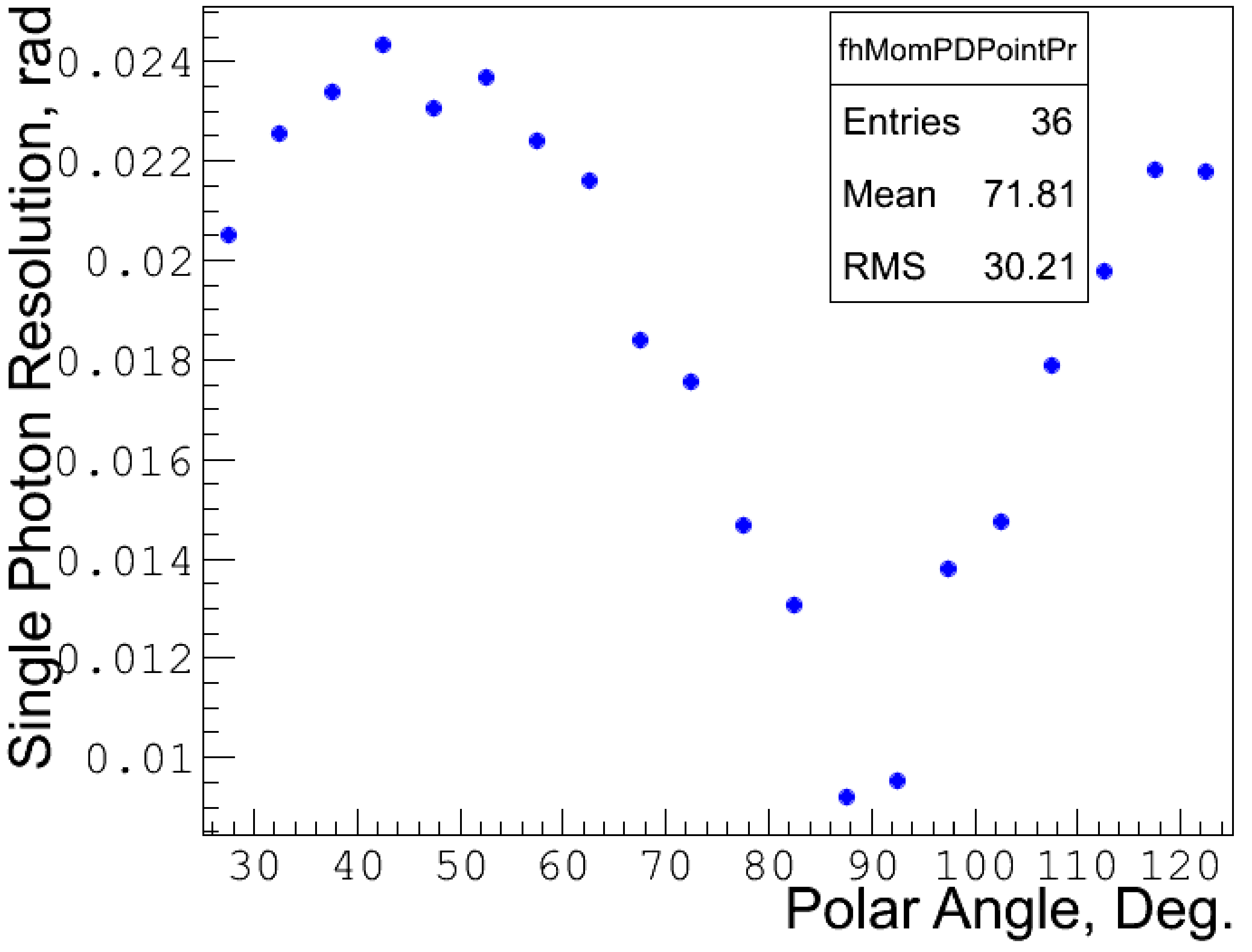
RMS of ΔT (T_{rec} - T_{exp}) Photon Path length



Right vs wrong Ambiguity time at different polar angles



Single Photon Resolution with charge particle polar angle



Log Likelihood function and selection criteria

First approach: To get the track theta independent of Particle Hypothesis

Track θ

$$LogL = \Sigma_i^{N_{amb}} Log(\frac{1}{\sqrt{2\pi\sigma_s^2}} exp\frac{-(\Theta_c - \Theta_i)^2}{2\sigma_s^2} + \frac{\Theta_c}{\Theta_m^2}) \quad (1)$$

Where, σ_s = 10-20 mrad is the single photon resolution.

Hypothesis test

$$LogL_{\pi,k,p} = Log(exp\frac{-(\Theta_{tr} - \Theta_{\pi,k,p})^2}{2\sigma_{tr}^2}) \quad (2)$$

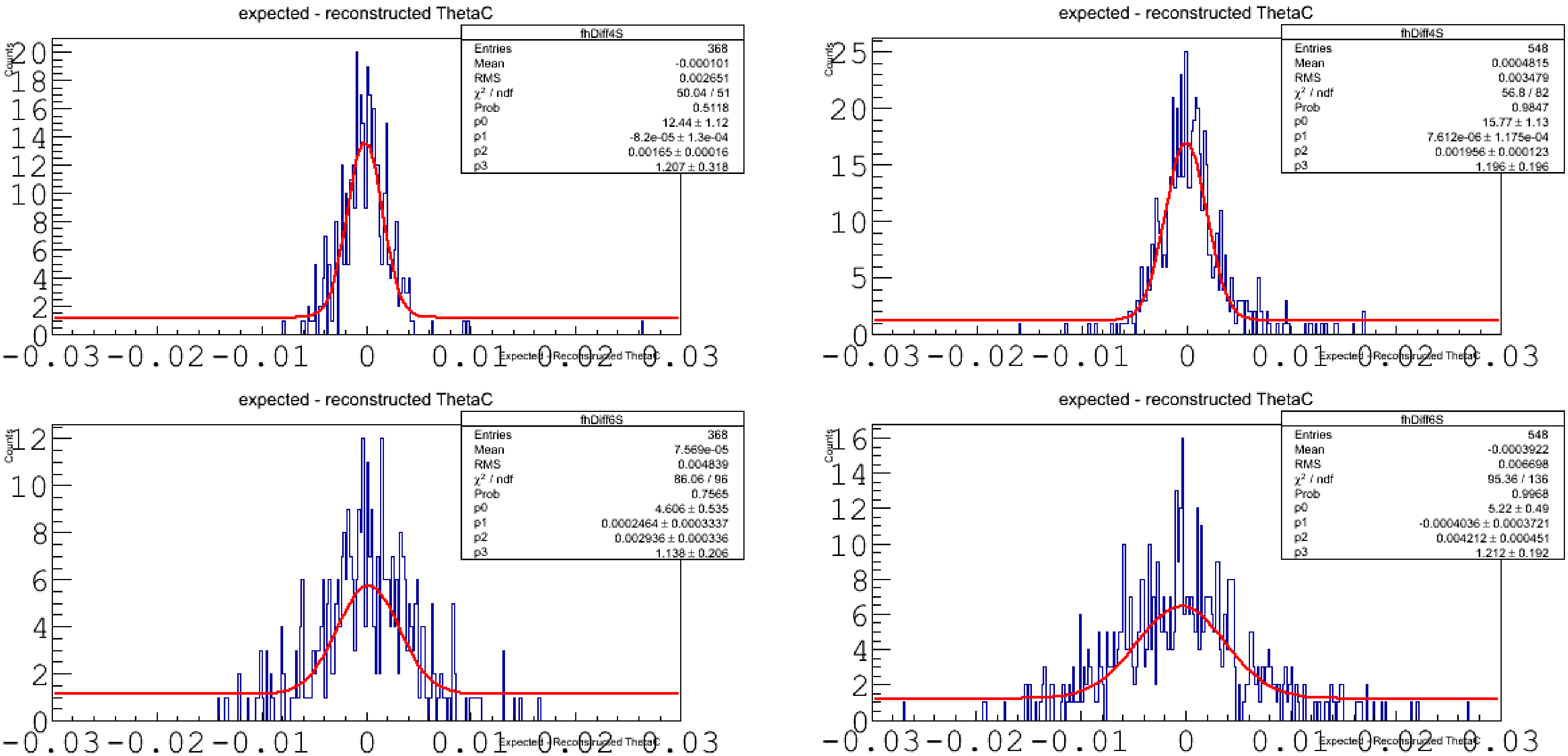
Where, σ_{tr} = 1.-3.0 mrad is the track resolution and
Θ_{tr} is the Cherenkov angle of the track based on Maximum Likelihood
$\Theta_{\pi,k,p}$ is the expected Cherenkov angle.

Second approach: Maximize the Particle Hypothesis

$$LogL_{\pi,k,p} = \Sigma_i^{N_{amb}} Log(\frac{1}{\sqrt{2\pi\sigma_s^2}} exp\frac{-(\Theta_c - \Theta_{\pi,k,p})^2}{2\sigma_s^2} + \frac{\Theta_c}{\Theta_m^2}) \quad (3)$$

Selection is based of the Separation power
Log likelihood difference = -0.5, -2.0, -4.5 for 1σ, 2σ, 3σ separation, respectively

Track θ Resolution



Expected – reconstructed θ

Expected – reconstructed θ

P > 3GeV/C

P < 3GeV/C

Particle Selection criteria

First approach: To get the track theta independent of Particle Hypothesis

$$LogL = \Sigma_i^{N_{amb}} Log(\frac{1}{\sqrt{2\pi\sigma_s^2}} exp\frac{-(\Theta_c - \Theta_i)^2}{2\sigma_s^2} + \frac{\Theta_c}{\Theta_m^2}) \quad (1)$$

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$$LogL_{\pi,k,p} = Log(exp\frac{-(\Theta_{tr} - \Theta_{\pi,k,p})^2}{2\sigma_{tr}^2}) \quad (2)$$

Where, σ_{tr} = 1.-3.0 mrad is the track resolution and
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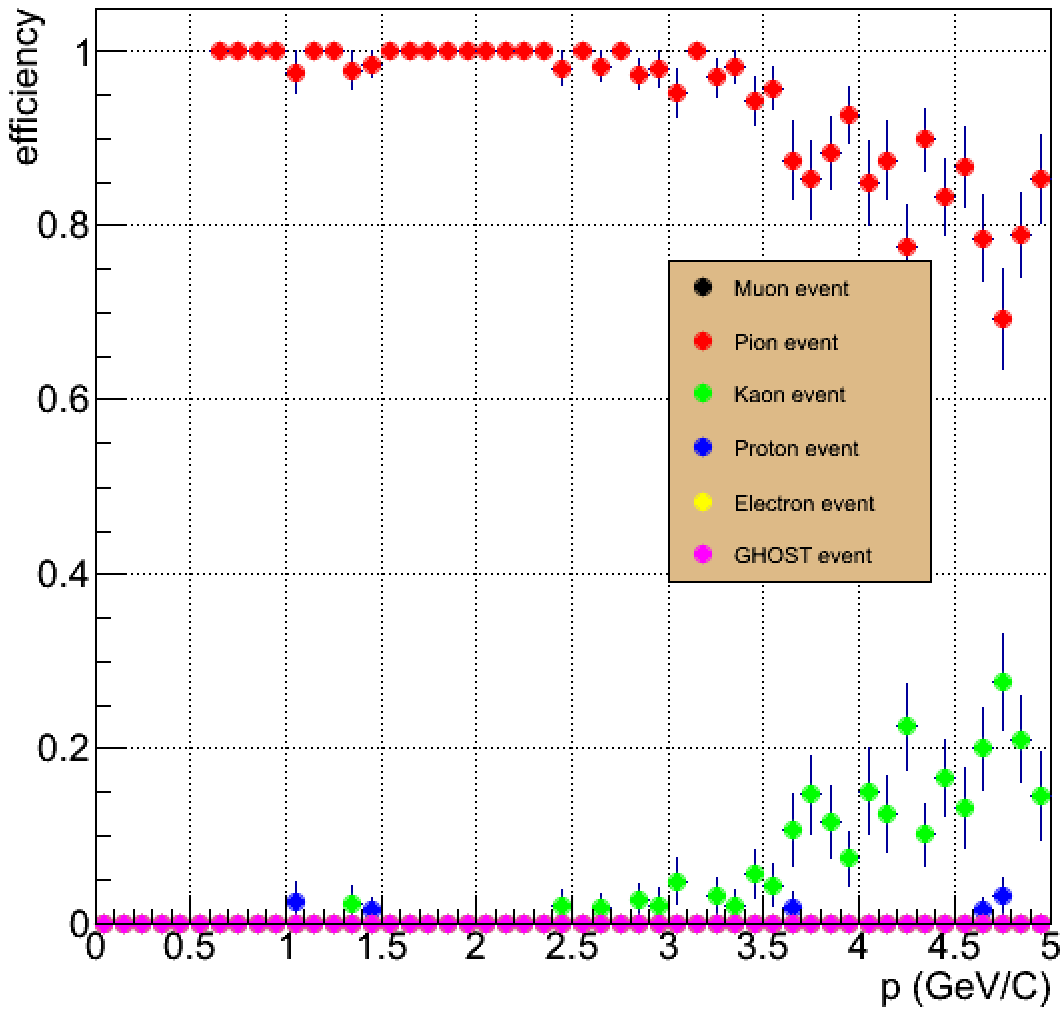
Selection is based on the Separation power
Log likelihood difference = -0.5, -2.0, -4.5,… for 1σ, 2σ, 3σ,... separation, respectively

Initial Momentum is a priory information required to calculate the expected θ_c

PID efficiency of Pion



Single Photon resolution σ = 17 mrad

Track resolution = 3 mrad

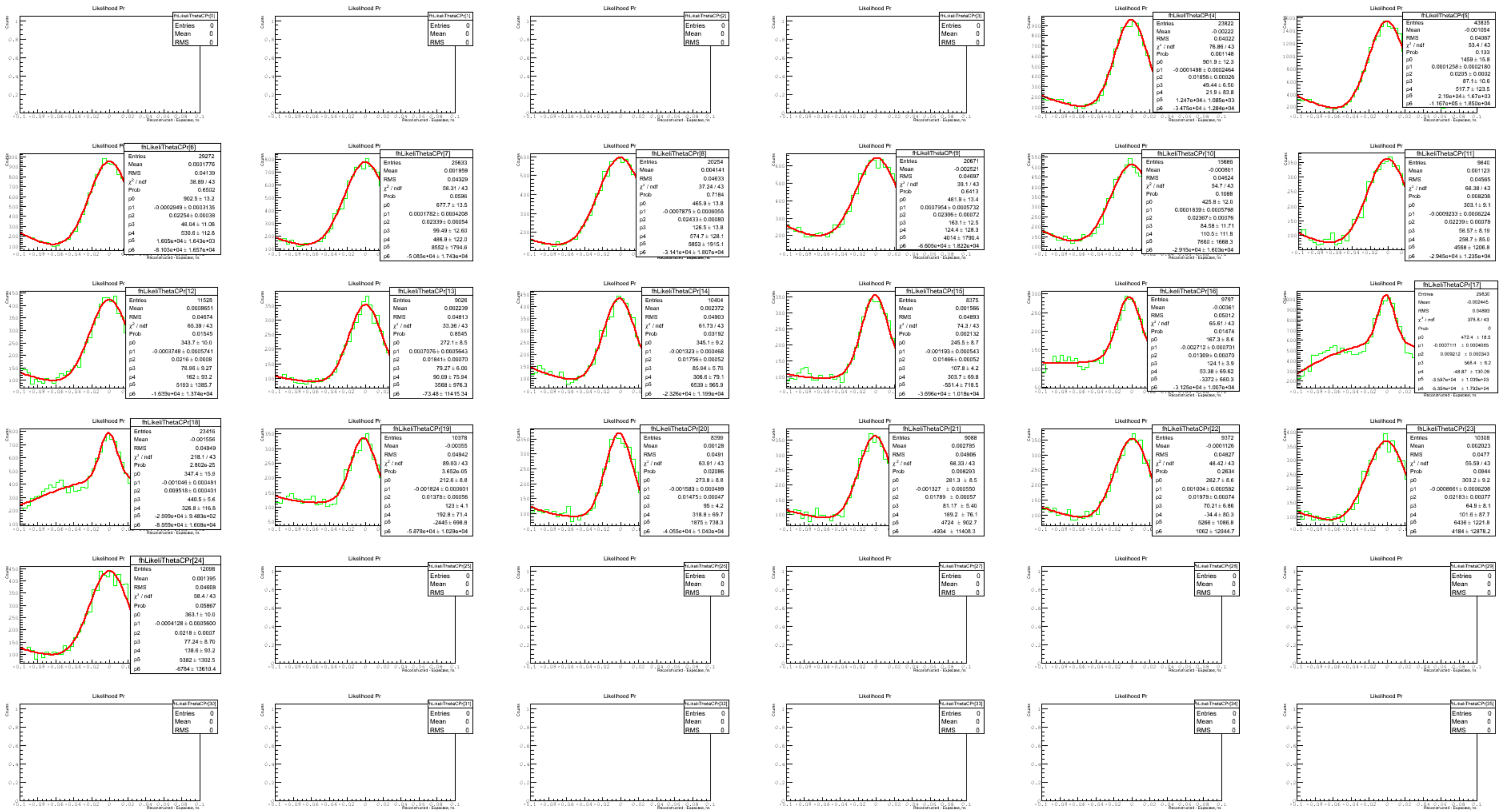
$$\text{efficiency} = \frac{\text{correct identified particles}}{\text{Total no. of particles}}$$

Summary

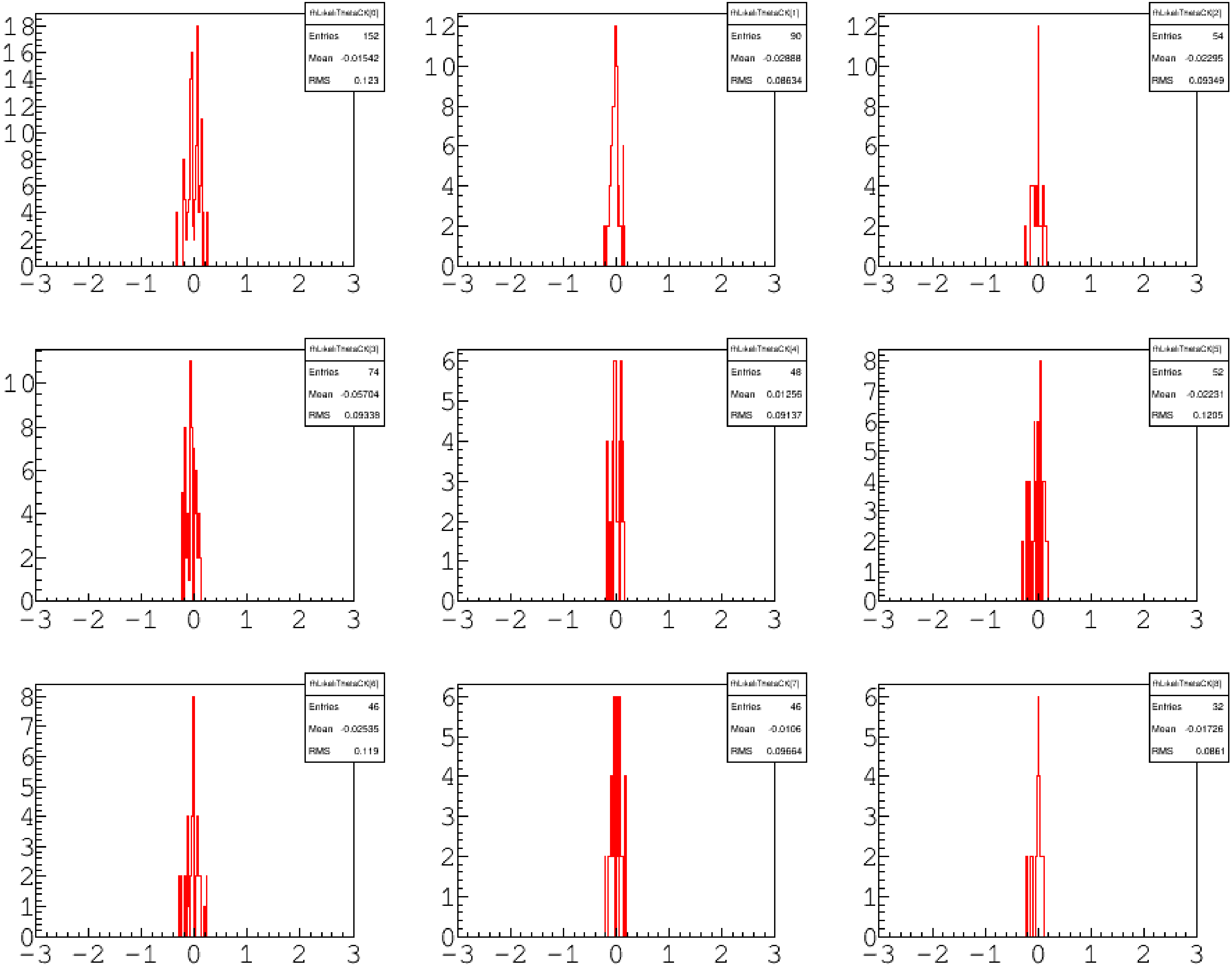
- Track based Maximum Likelihood method is developed and study has been carried out for tank type expansion volume.
- Further study for different geometries is continued.

Thanks for Attention!

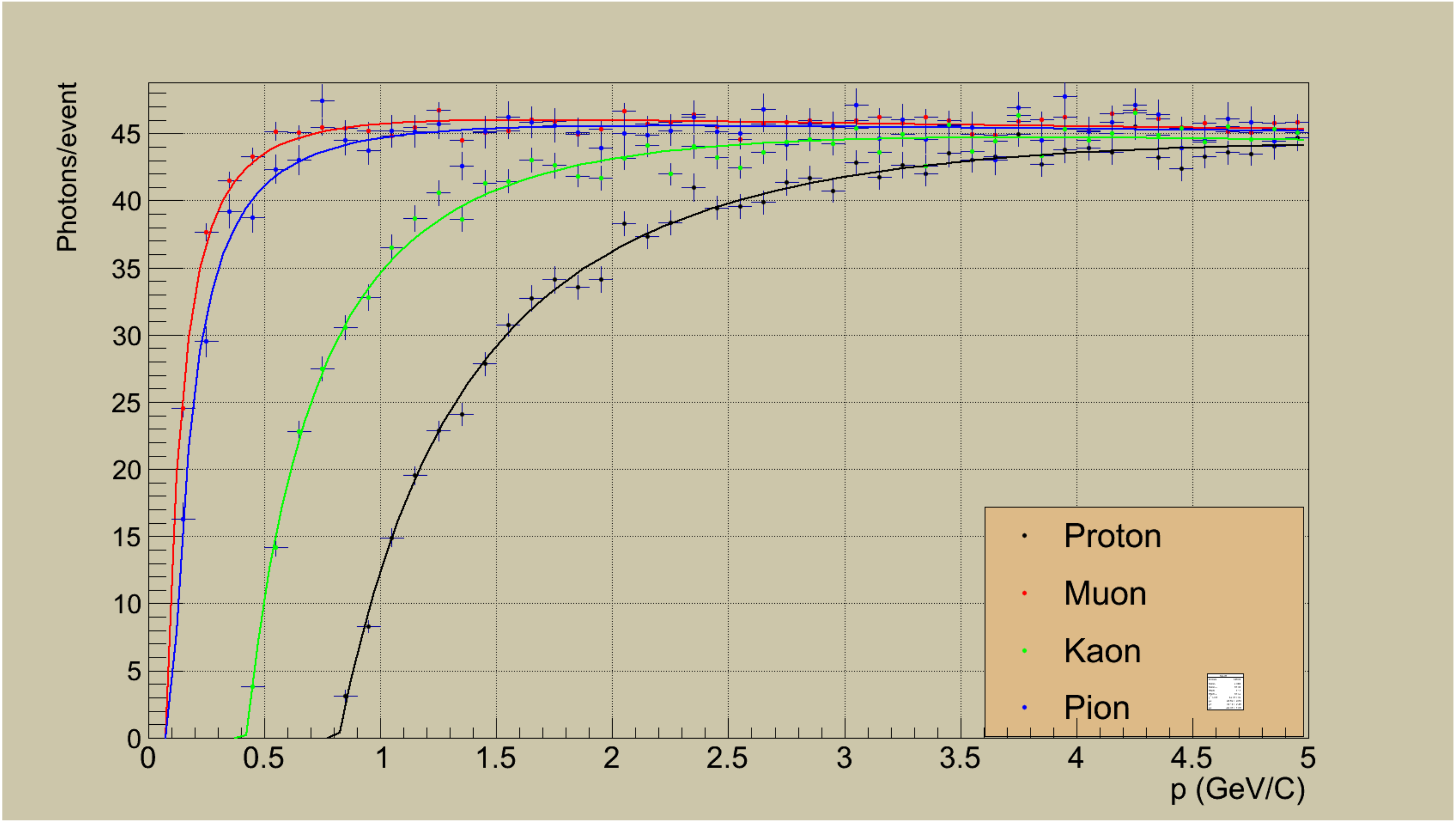
Backup slides



Single Photon Resolution with polar angle



Entries 152
Mean -0.01542
RMS 0.123
Entries 90
Mean -0.02888
RMS 0.08634
Entries 54
Mean -0.02295
RMS 0.09349
Entries 74
Mean -0.05704
RMS 0.09339
Entries 48
Mean 0.01256
RMS 0.09137
Entries 52
Mean -0.02231
RMS 0.1205
Entries 46
Mean -0.02535
RMS 0.119
Entries 46
Mean -0.0106
RMS 0.09664
Entries 32
Mean -0.01726
RMS 0.0861



Photons/event
p (GeV/C)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
0
5
10
15
20
25
30
35
40
45
Proton
Muon
Kaon
Pion