Fast simulation tools for Raman amplification

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September 30, 2013

Abstract

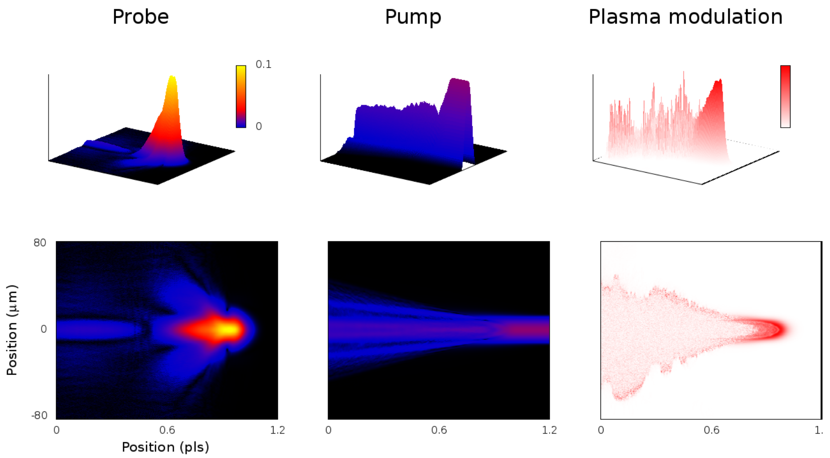
Raman amplification in plasma is a possible source of ultra-short, ultra-intense laser pulses.

Multidimensional effects can play a significant role.

BUT conventional simulation techniques are computationally intensive.

Develop a fast, multidimensional simulation model for Raman.

Abstract


Probe
Pump
Plasma modulation
0.1
0
80
0
-80
Position (µm)
0
0.6
1.2
0
0.6
1.2
0
0.6
1.
Position (pls)

Outline

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Ultra-intense laser pulses have many interesting applications

Have to be careful you don't damage your amplifier or optics

Motivation

Use plasma as a gain medium

- No damage threshold
- Inherent compression mechanisms
- Scalable frequency

Motivation

Use plasma as a gain medium

Make current laser systems smaller, cheaper

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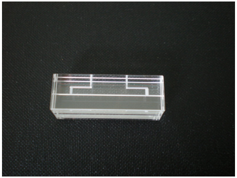
Optic from the Vulcan PetaWatt laser

Motivation

Use plasma as a gain medium

Make current laser systems smaller, cheaper



Plasma capillary from Strathclyde

Motivation

Use plasma as a gain medium

Make current laser systems smaller, cheaper

Motivation

Use plasma as a gain medium

Make current laser systems smaller, cheaper

OR

Motivation

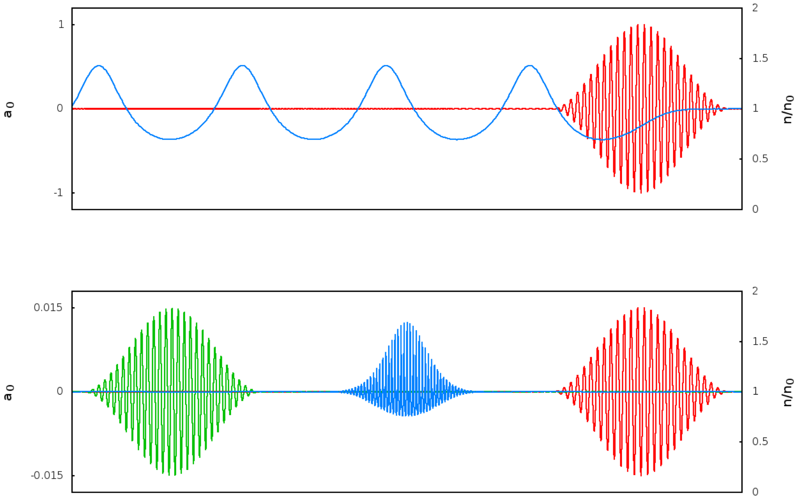
Use plasma as a gain medium

Make current laser systems smaller, cheaper

OR

Do things which are not possible with conventional technologies

Plasma waves



Two counterpropagating laser pulses of different frequency create a beatwave

and drive a plasma wave through the ponderomotive force.

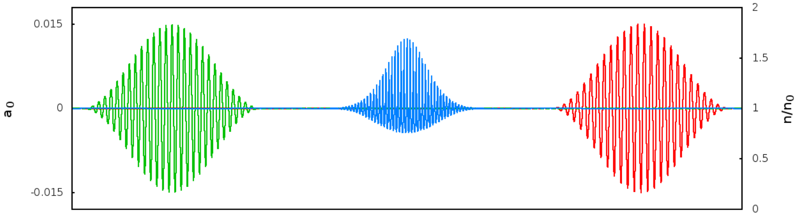
The resulting density perturbation scatters the higher frequency pump into the lower frequency probe.

Many physical processes relevant to Raman amplification require a particle description

- wavebreaking
- particle trapping
- Landau damping

Particle in Cell

Raman amplification in plasma



Wavelength of excited plasma wave is short ($\sim \lambda_{laser}/2$)

and high amplitude waves lead to anharmonic structure.

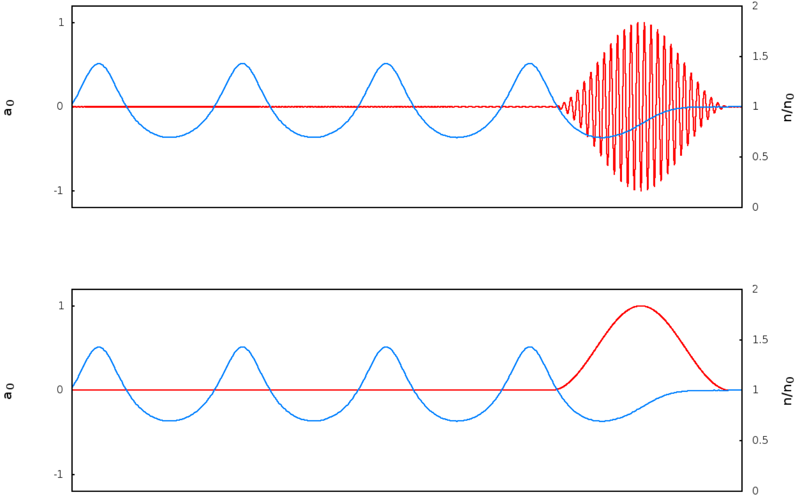
Requires a high resolution - large computational overheads

Envelope models

We look to existing techniques used to reduce the computational overhead of simulations of laser plasma interactions

Remove fast laser oscillations by switching to laser envelopes

Envelope model - wakefield



Mora and Antonsen, Phys. Plasmas **4** 217 (1997)

Cowan *et al*, J. Comp. Phys. **230** 61 (2011)

Envelope model derivation - in one slide

Laser pulse evolution calculated from the wave equation:

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$$\left(\partial_t^2 - c^2\nabla^2\right)\vec{a} = \frac{e}{mc\varepsilon_0}J_\perp$$

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where $\vec{a}$ is the reduced vector potential.

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Transform into a co-moving frame, $\xi = z/c - t$, $\tau = t$

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Make the quasistatic approximation

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$$= \chi\vec{a}$$

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Make the quasistatic approximation
where χ is the (renormalised) plasma susceptibility.

Laser pulse evolution calculated from the wave equation:

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Introduce an envelope representation of the laser field

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$\vec{a} = a\,\mathrm{e}^{-i\omega\xi}\,\vec{u}$

Envelope model derivation - in one slide

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\end{aligned}
$$

Neglect the second derivative in τ (evolves slowly in co-moving frame).

Envelope model derivation - in one slide

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\left(\partial_\tau^2 - 2\partial_\tau\partial_\xi + 2i\omega\partial_\tau - c^2\nabla_\perp^2\right) a &= \chi a \\
\left(-2\partial_\tau\partial_\xi + 2i\omega\partial_\tau - c^2\nabla_\perp^2\right) a &= \chi a
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Neglect the second derivative in τ (evolves slowly in co-moving frame).
Suppresses backscatter and reflection.

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Envelope model derivation - in one slide

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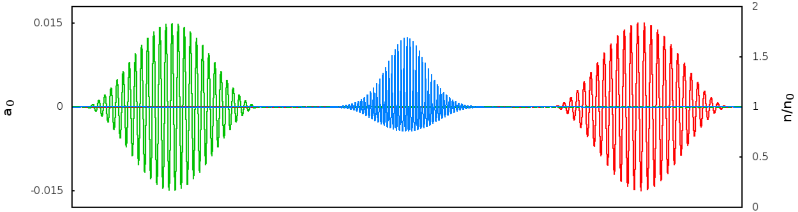
Envelope model - beatwave



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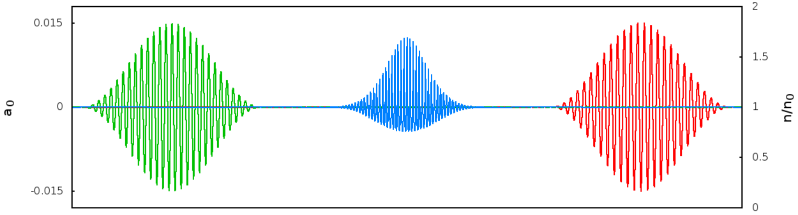

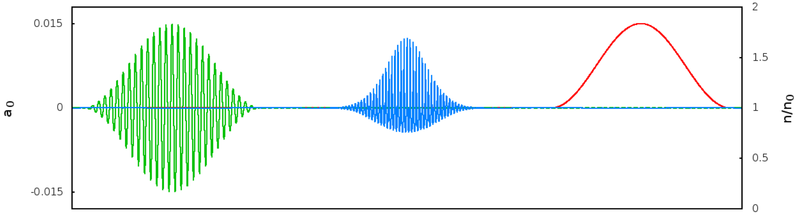

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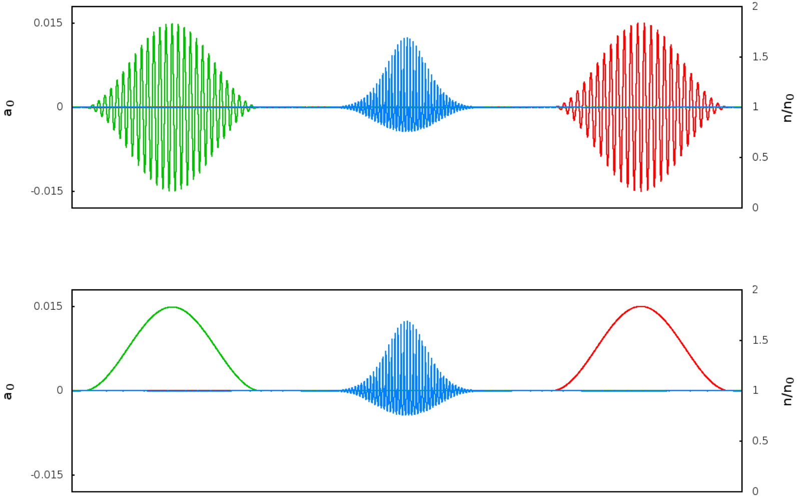


Separation of scales

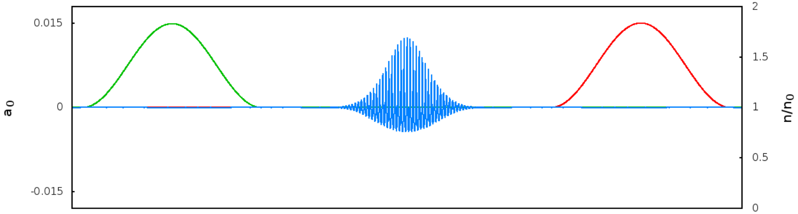
Laser envelopes vary slowly in space and time

Excited plasma wave varies slowly in time, but rapidly in space.

Solution - treat plasma wave as electrostatic.

Hur *et al*, Phys. Plasmas **11** 5204 (2004)

Envelope model - beatwave



Envelope model derivation - beatwave

Envelope solver for a single pulse:

$$\left(-2\partial_\tau\partial_\xi + 2i\omega\partial_\tau - c^2\nabla_\perp^2\right) a = \chi a$$

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Envelope solver for two pulses: $\vec{a} = a\,\mathrm{e}^{-i\omega_a\xi}\,\vec{u} + b\,\mathrm{e}^{i\omega_b(\xi+2\tau)}\,\vec{u}$

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$$\left(-2\partial_\tau\partial_\xi + 2i\omega_b\left(\partial_\tau - 2\partial_\xi\right) - c^2\nabla_\perp^2\right) b = \chi b + \chi\,\mathrm{e}^{-i((\omega_b+\omega_a)\xi+2\omega_b)}\,a$$

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Introduce new slowly-varying quantities,
$\tilde{\chi} = \langle\chi\rangle$, $\tilde{\psi} = \left\langle \mathrm{e}^{i((\omega_b+\omega_a)\xi+2\omega_b)}\,\chi\right\rangle$

Envelope model derivation - beatwave

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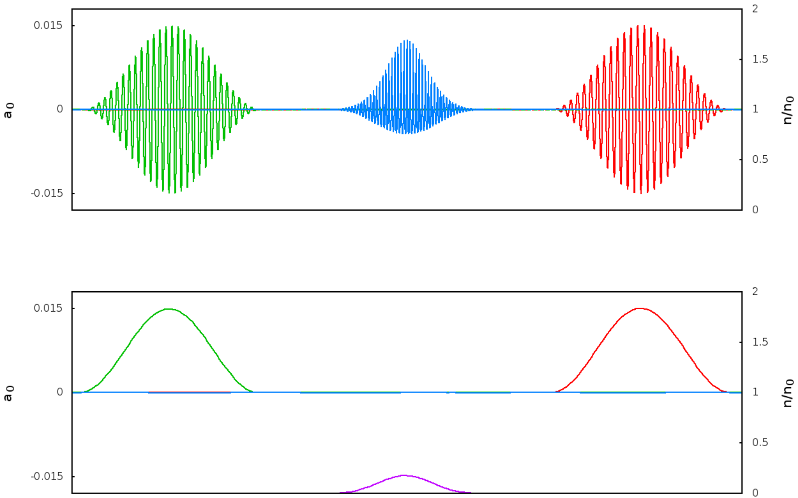
Envelope model - beatwave



Model Overview

Envelope laser solver using averaged values

Electrostatic solver using a fine grid

Both updated using same long timestep - lower computational overhead

Results

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Experimental results by Ren *et al*:

Nature Physics 3, 732 - 736 (2007)

Intensity gain of 10.000 times
1.3×10^{12} to 1.3×10^{16} $\mathrm{W\,cm^{-2}}$

Energy gain of 200 times
16 μJ to 3.2 mJ

Simulation

Use similar parameters:

Probe amplitude $= 0.001$
Pump amplitude $= 0.01$
Probe diameter $= 55\ \mu$m
Probe diameter $= 55\ \mu$m
Probe FWHM $= 500$ ps
Plasma density $= 1.3\times10^{19}\ \ \mathrm{cm}^{-3}$
Plasma length $= 2$ mm

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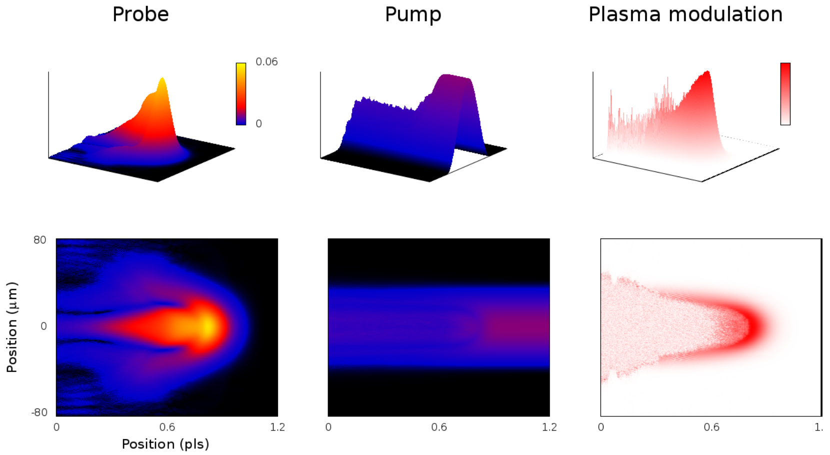
Simulation - homogeneous plasma



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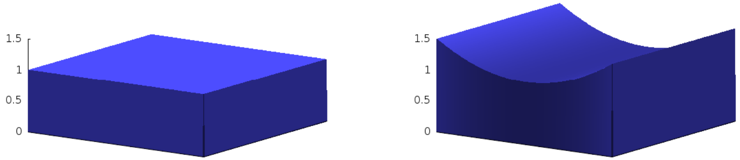
Possible to improve gain?

Introduce plasma channel

Vieux *et al*, New Journal of Physics **13** (2011)

Pai *et al*, Phys. Rev. Lett. **101** 065005 (2008)



density at boundary 50% higher than on axis

density at beam waist 5.6% higher than on axis

Simulation - plasma channel

Use similar parameters:

Probe amplitude = 0.001
Pump amplitude = 0.0172
Probe diameter = 55 μm
Probe diameter = 19 μm
Probe FWHM = 500 ps
Plasma density = 1.3×10^{19} cm^{-3} (on axis)
Plasma length = 2 mm

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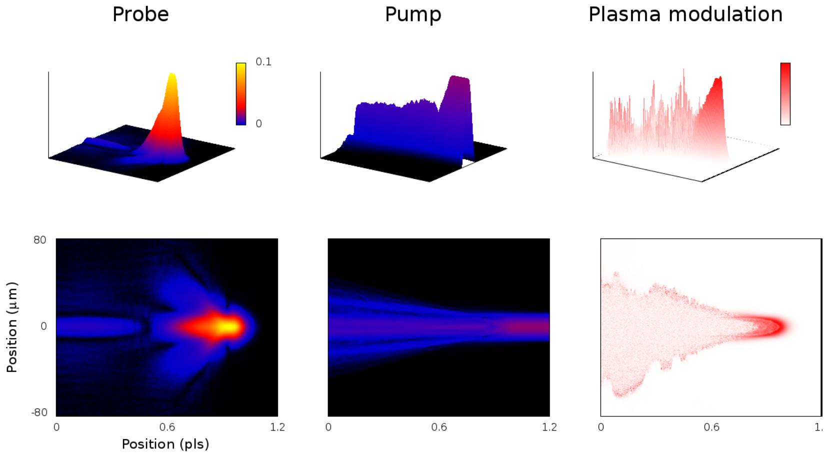
Movie


Probe
Pump
Plasma modulation
0.1
0
80
0
-80
Position (µm)
0
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0
0.6
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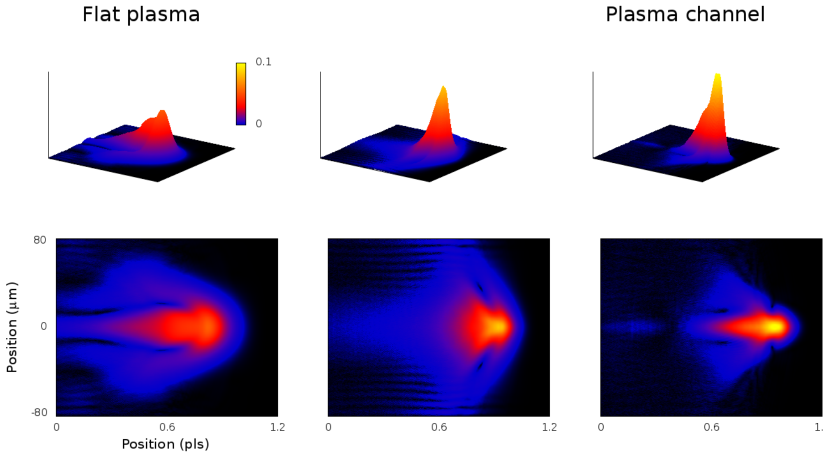
0
0.6
1.

Simulation results

Introduction of a plasma channel gives

- more focused pulse
- higher peak intensity

Movie



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Introduction of a plasma channel gives

- more focused pulse
- higher peak intensity

Contributions from both

- the use of a matched pump
- and refractive guiding of probe

Conclusions

Conclusions

- Development of fast simulation model for Raman amplification
- Allows simulation of multidimensional effects, refraction, dispersion
- Much faster than conventional methods (Yee-PIC/Vlasov)
  - Parameter scans
  - Long interaction lengths
  - Large diameter beams

Conclusions

Initial simulations run for realistic parameters

Introduction of a plasma channel gives higher intensity

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Conclusions

We have a code. It works.

If you have a laser and you'd like to do some Raman

get in touch!

-FIN-