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ELECTRON ACCELERATION IN THE REGIME OF STOCHASTIC HEATING WITHIN A PS-DURATION LASER PULSE

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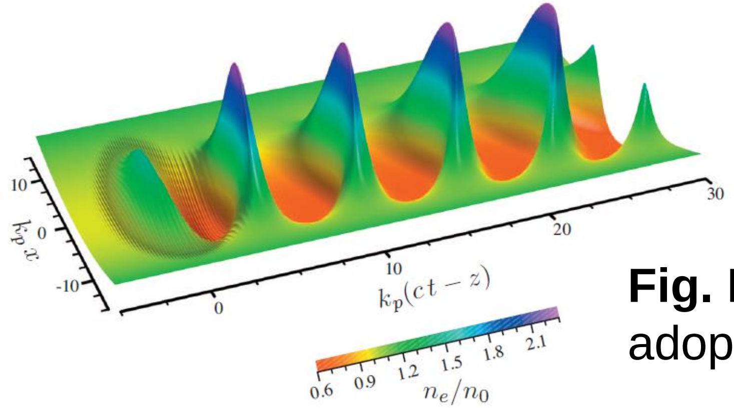
Outline

1) Introduction
2) Motivation
3) Analytical model: electron dynamic in regular combined fields (laser + plasma waves)
4) Electron dynamic in turbulent plasma waves
5) PIC simulations
6) Conclusions

Introduction

- **Short laser pulse:** $\tau \sim \lambda_{pe} / 2c$ **standard LWFA – laser wake field acceleration**



proposed by T. Tajima and J. M. Dawson (1979)

$$\lambda_{pe}(\mu\mathrm{m}) = 2\pi c/\omega_{pe} = 3.3\times10^{10}\,[n_e\,(\mathrm{cm}^{-3})]^{-1/2}$$

Fig. Plasma density perturbation**:**
adopted from E. Esarey, Rev. Mod. Phys. **81,** 1229 (2009).

- **Long laser pulses:** $c\tau > \lambda_{pe} \quad P > P_c, \quad P_c = 17(\omega_L/\omega_{pe})^2 GW$

SM (self-modulated) WFA

Maximum field :

$$E_{x,\max}(c\tau = \lambda_{pe}/2) \approx \frac{m_e c\omega_{pe}}{e}\frac{a_0^{\,2}/2}{\sqrt{1+a_0^{\,2}/2}} \approx \frac{m_e c\omega_{pe}}{\sqrt{2}e} a_0, \quad a_0 >> 1.$$

Maximum energy limited by dephasing : $W_{\max} \approx m_e c^2 \frac{n_{cr}}{n_e}\sqrt{1+a_0^2}.$

Self-modulation instability: N.E. Andreev et al, JETP Lett. **55** 571 (1992);
P. Mora, Phys. Fluids **4,** 1630 (1992); P. Sprangle and E. Esarey, 2241 (1992);
Raman Forward Scattering: Mori

Motivation

When conditions for LWFA and self-modulated LWFA are not optimal, high energy electron generation characterized by spectrum of thermal character (often two populations with two distinct temperatures) can be attributed to stochastic acceleration.

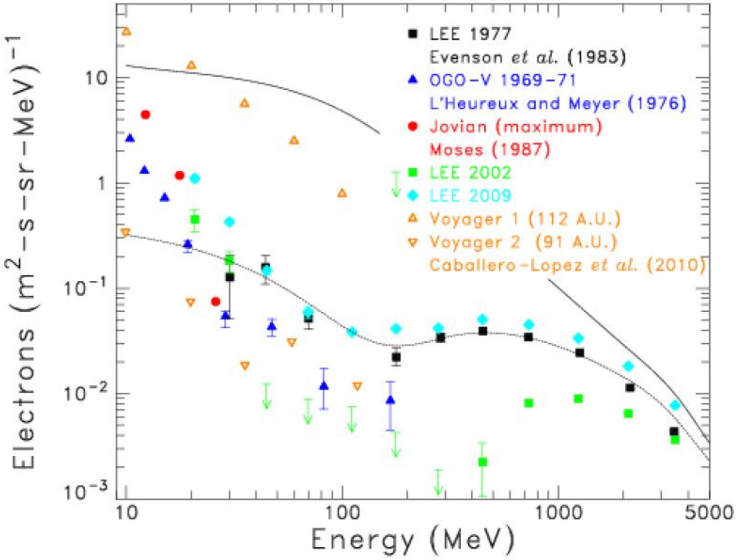
Possible applications:

1) ”table-top astrophysics”

2) radiation testing of spacecraft microelectronics.

Total charge can be higher than in the case of quasi-monoenergetic spectra of electrons!

Cosmic Ray Electron Spectrum in 2009



P. Evenson and J. Clem, Proceedings of the. 32nd International. Cosmic Ray Conf.

Stochastic electron heating

Laser pulse field + arbitrary additional field (E.M. fields, electrostatic field, coulomb, magnetic field)

Colliding laser pulses [Z.M. Sheng et al. PRE **69**, 016407 (2004)]

Incident and reflected light in preplasma [Y. Sentoku V.Yu. Buchenkov, Appl. Phys. B**74** 207 (2002)]

Incident and reflected light at sharp plasma-vacuum interface (vacuum heating) [V.S. Rastunkov and V.P. Krainov Laser Phys. **15** 262 (2005) **]**

Incident and SRS fields

Interaction of laser pulse with Coulomb field (e-i collisions in a strong e.m. field, interaction with nano/micro targets)

Electromagnetic field and quasi-static magnetic field

Laser pulse and plasma wave (wake field from a pulse front)

Lyapunov exponents: A.J. Lichtenberg, M.A. Lieberman Regular and Chaotic Dynamics, 2nd ed., Applied Mathematical Sciences, Vol. 38, New York

Stochastic electron acceleration with assistance of plasma waves

Test –particle model for Stochastic Acceleration in Combined Fields

$$\boxed{\frac{d}{dt}\left(\vec{p}-\frac{e\vec{A}}{c}\right)=-e\vec{E}-\frac{\vec{v}\times\vec{B}}{c}, \qquad \frac{d}{dt}\vec{r}=\frac{\vec{p}}{m_e\gamma}, \qquad \vec{E}=-\frac{\partial\Phi}{\partial\vec{r}}-\frac{1}{c}\frac{\partial\vec{A}}{\partial t}, \qquad \vec{B}=rot\ \vec{A},}$$

Laser pulse ***Scattered wave***

$$\frac{eA_y}{m_e c^2}=a_0(x/L,ct/L)\cos(\omega_L t-k_L x)+a_1(x/L,ct/L)\cos(\omega_{01}t-k_{01}x+\psi),$$

$$\mathrm{v}^0_{\mathrm{ph}}=\omega_L/k_L, \quad \mathrm{v}^1_{\mathrm{ph}}=\omega_{01}/k_{01},$$ → ***Plasma wave***

$$\Phi=\phi_0\cos(\omega_1 t-k_1 x+\varphi), \quad \omega_1/k_1\approx \mathrm{v}_{\mathrm{g}}<c,$$

$$t\longrightarrow t'+l \qquad \omega_L=\omega_S+\omega_l, \qquad k_L\to k_S\pm k_l\Rightarrow$$ ***RFS process***

$$\omega_L\approx\omega_S+\omega_{pe} \qquad k_l\approx\omega_{pe}/c$$

Plasma wave excitation through Raman Forward Scattering Instability!

Integrals of motion

If $\mathrm{v}_{\mathrm{ph}}^{1} = \mathrm{v}_{\mathrm{ph}}^{0} = \mathrm{v}_{\mathrm{g}} \quad \Rightarrow$ ***Equation of motion is integrable***

$$\boxed{J_0 = p_y - a_y, \quad J_1 = p_x - \frac{\gamma}{\mathrm{v}_{\mathrm{ph}}^{0}} + \phi, \quad J_2 = y - \int d\xi \frac{v_y}{\mathrm{v}_{\mathrm{ph}}^{0} - v_x}, \quad J_3 = \tau - \int d\xi \frac{\mathrm{v}_{\mathrm{ph}}^{0}}{\mathrm{v}_{\mathrm{ph}}^{0} - v_{\mathrm{x}}}, \quad x = \tau(\xi) - \xi,}$$

no plasma wave

$$\boxed{p_x = p_y^2 / 2, \quad p_y = a_y, \quad \gamma = p_x + 1}$$

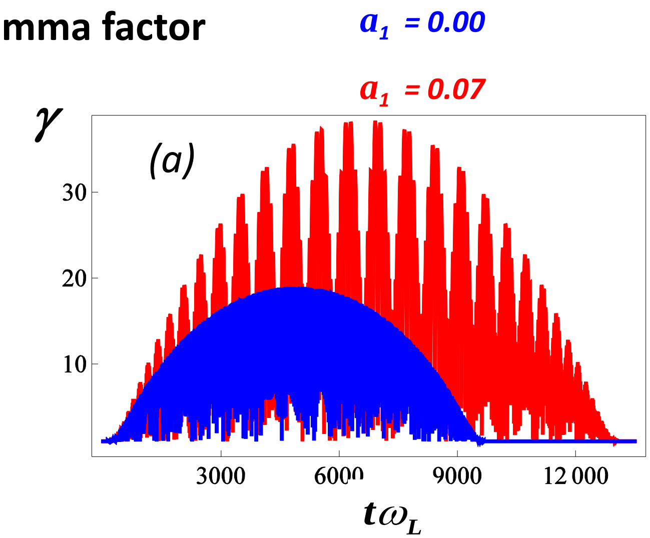
If system is not integrable then chaos, stochastic dynamics are possible for some value of plasma wave amplitude, as a result electrons can be strongly heated in stochastic manner!

$$\boxed{p_{x\ \mathrm{max}} \gg a_0^2 / 2, \quad \gamma_{\ \mathrm{max}} = a_0^2 / 2 + 1}$$

Numerical implementation of test-particle model: Boris scheme from MANDOR PIC 3D3V code

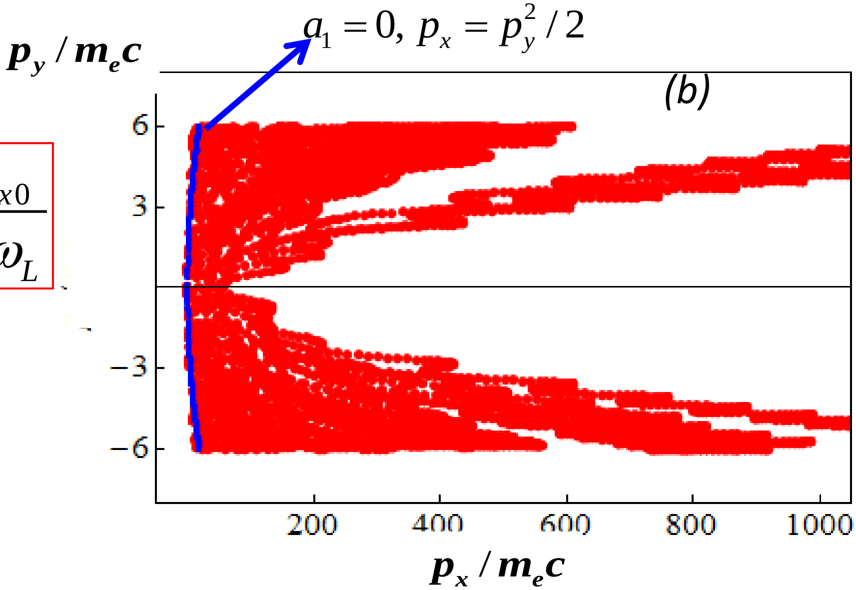
Test particle trajectories

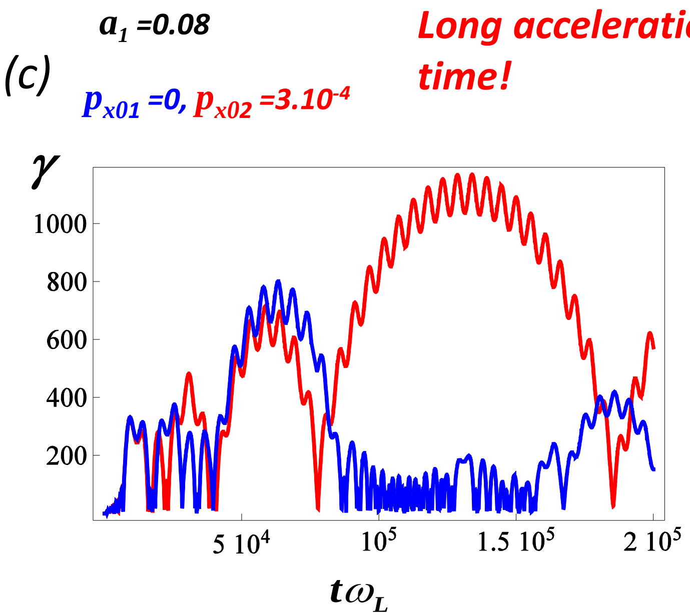
Regular trajectories, time evolution of electron gamma factor



$$a_1 = \frac{eE_{x0}}{m_e \omega_L}$$

Momentum space (p_x,p_y) for ten trajectories with various values (x_0,p_{x0})





Long acceleration time!

Fig. (a) -- Regular trajectories; Fig (b,c) demonstrate a high rate of separation of close trajectories;

Fig. (b) also demonstrates destruction of adiabatic invariants J_0,J_1. (a_1=0.08), and as a result considerable increase of maximum electron energy on its trajectory for (one can see at b,c).

I=5 •10^{19} W/cm², a_0=6, τ =700 fs, λ=1 μm,

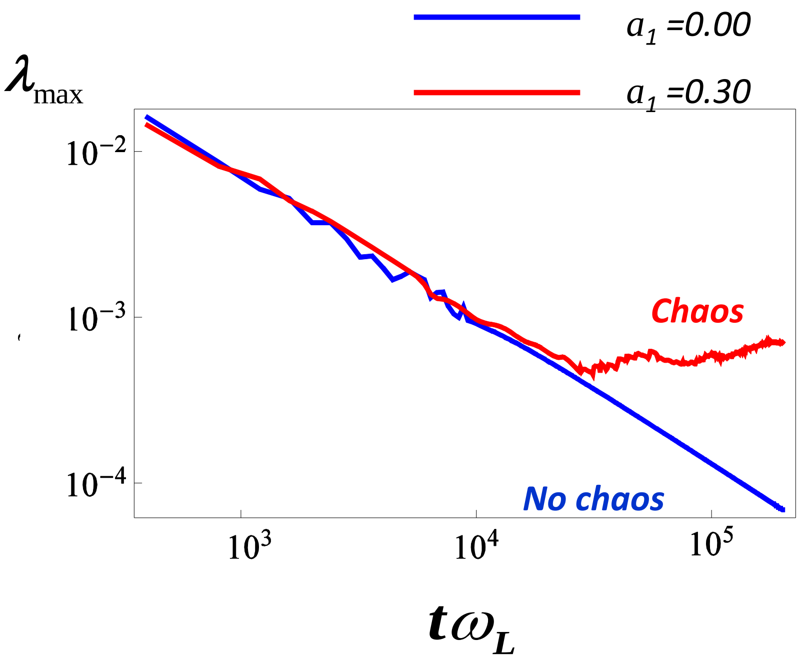
underdense plasma, n_e≈2 •$10^{-2}n_{cr}$,

Trajectory Stability Analysis

Lyapunov exponent:

$$\lambda_{\max} = \lim_{t\to\infty} \lim_{d(0)\to 0} \frac{1}{t} \ln \frac{d(\vec{x}_0,t)}{d(\vec{x}_0,0)}, \quad d \approx \exp(\lambda_{\max} t)$$

Criteria of stochastic motion: $\lambda_{\max} > 0$

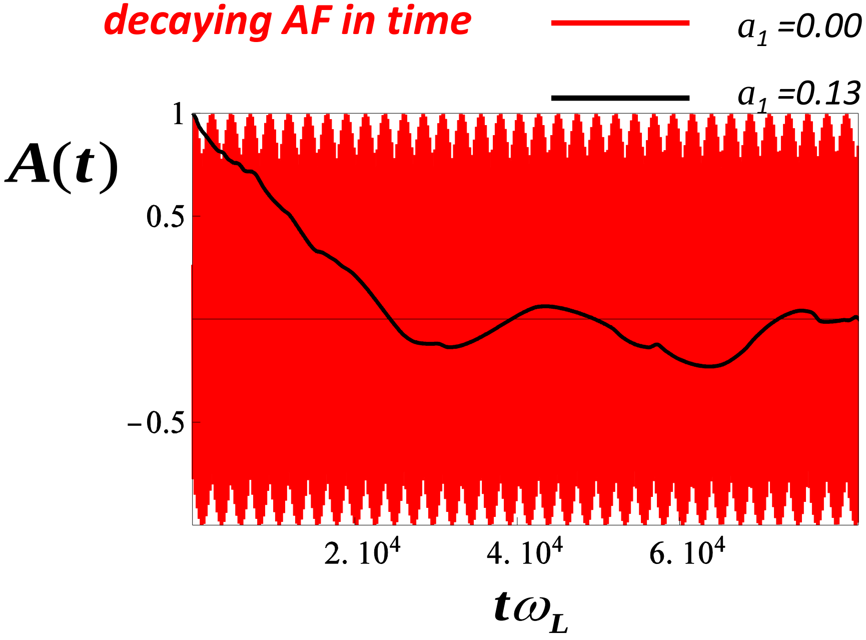


Maximum Lyapunov exponent vs. acceleration time

Autocorrelation Function (AF)

$$A(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_0^T p_y(t)\, p_y(t+\tau)\, dt$$

Onset of chaotic motion corresponds to decaying AF in time



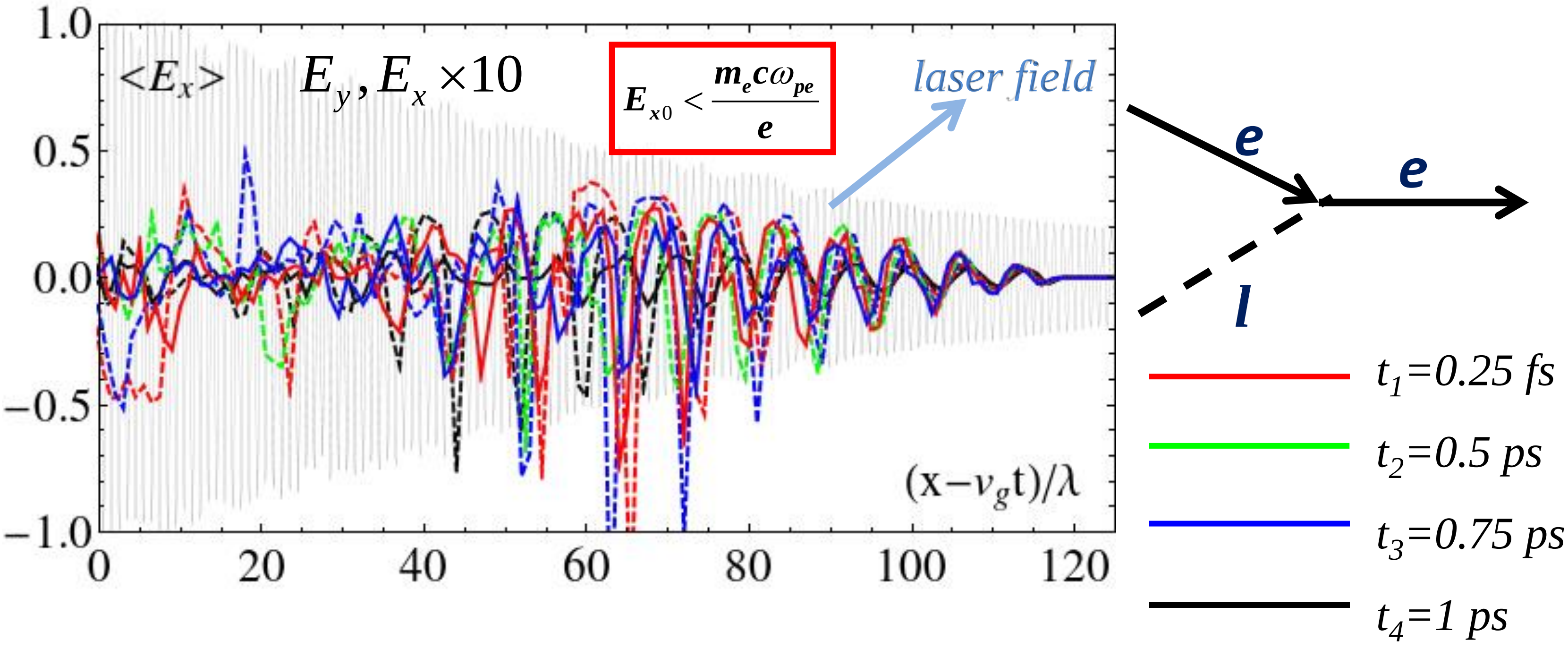
AF vs. acceleration time

***Parameters** : a_0=6, n_e≈0.04 n_{cr}, τ≈ 700 fs*

Unfortunatelly, there is a long acceleration time (10 ps)!

Turbulent plasma fields

Electric fields for moments of time 250,500,750,1000 fs, ..



Simulations witness that turbulent electric fields are generated during subpicosecond laser pulse plasma interaction. Such fields accelerate electrons in stochastic manner.

1D-2D simulations demonstrate rapid stochastic electron heating. A substantial fraction of the background plasma electrons can be accelerated through this process for reasonable period of time (1 ps)!

Diffusion model of stochastic particle acceleration

$$\frac{\partial f_e}{\partial t} + \vec{V}_e \frac{\partial f_e}{\partial \vec{R}_e} + \vec{F}_L \frac{\partial f_e}{\partial \vec{P}_e} = \frac{\partial}{\partial P_i}\left(D_{ij} \frac{\partial f_e}{\partial P_j} \right),$$

Fokker-Plank equation

$$D_{ij} = 8\pi^2 e^2 \int W(\vec{k}) \frac{k_i k_j}{k^2} \delta(\omega - \vec{k}\vec{V}_e) d^3 k$$

Diffusion coefficient

$$W(\vec{k}) = \frac{E_k^2}{8\pi}$$

1D limit of diffusion equation

$$\frac{\partial f_e}{\partial t} + (V_x - v_g c)\frac{\partial f_e}{\partial \zeta} = \frac{\partial}{\partial P_x}\left(D_{xx} \frac{\partial f_e}{\partial P_x} \right), \quad \zeta = X - v_g ct$$

Parameters of plasma wave spectrum were taken from results of PIC simulations !

Analytical estimations and numerical solution of diffusion equation for EDF demonstrate that this can explain a relatively short acceleration period which was detected in PIC simulations!

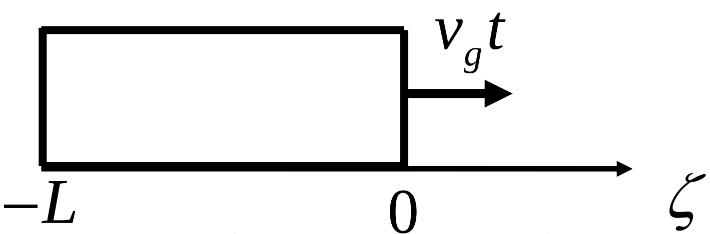
Diffusion model of electron stochastic heating

1D model

$$\cancel{\frac{\partial f_e}{\partial t}} + (V_x - v_g c)\frac{\partial f_e}{\partial \zeta} = \frac{\partial}{\partial P_x}\theta(-\zeta)D_{xx}\frac{\partial f_e}{\partial p}, \quad \zeta = X - v_g ct,$$

$$D_{xx} = \pi e^2 \int dk \cdot \delta\left(\omega - k \cdot v\right) E_k^2,$$

Quasi stationary solution



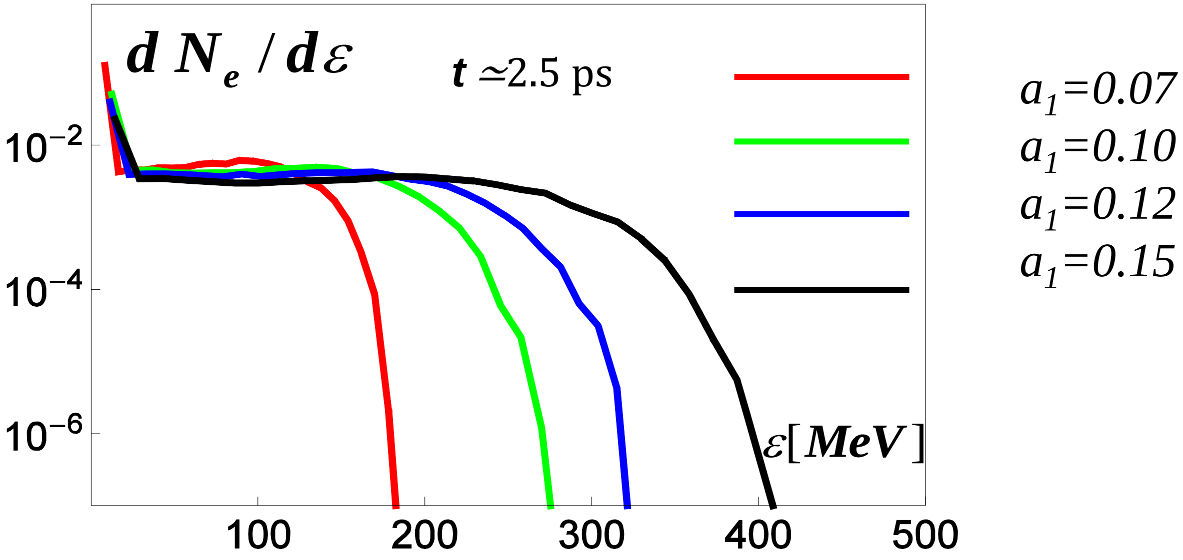
L – laser pulse width

$$\boxed{\begin{aligned} &V_x > cv_g, \quad D_{xx} = D_0 \approx const, \\ &D_0 = e^2 (E^2)_{k_e} / |V_x|, \quad k_e \approx k_{e0}. \end{aligned}}$$

$$v_g < V_x \approx c \quad \Rightarrow$$

$$\boxed{\begin{aligned} &f_e \propto \frac{n_{e0}}{\sqrt{\pi m_e T_e}} \exp\left(-\frac{P_x^2}{m_e T_e}\right), \quad T_e = \frac{4|\zeta| D_0}{m_e c(1 - v_g)}, \\ &t_{ac} \approx 1.3 ps, \quad T_{e\max} \approx 100 - 200\ MeV \end{aligned}}$$

Electron energy spectra (heating in stochastic plasma fields)



$$a_1 = \frac{eE_{x0}}{m_e \omega_L}$$

E_{av} = 200 MeV at t=0.9 ps

Simulation demonstrates rapid stochastic electron heating. A substantial fraction of the background plasma electrons can be accelerated through this process for reasonable period of time (1 ps)!

Test Particle Simulation

Stochastic plasma waves

$$E_x(t,x) = \sum_{j=-N}^{N} E_{0,j} \cos\left(\omega_{pe} t - k_{p,j} X + \varphi_{0,j}\right) \;,$$

$\varphi_{0,j}$ ***- stochastic (random) phases***

$$k_{p,j} = \{k_{pe} + j\,\delta k/N\},\; j = \{-N ..., 0, ... N\}$$

Model spectrum of plasma waves (from PIC data)

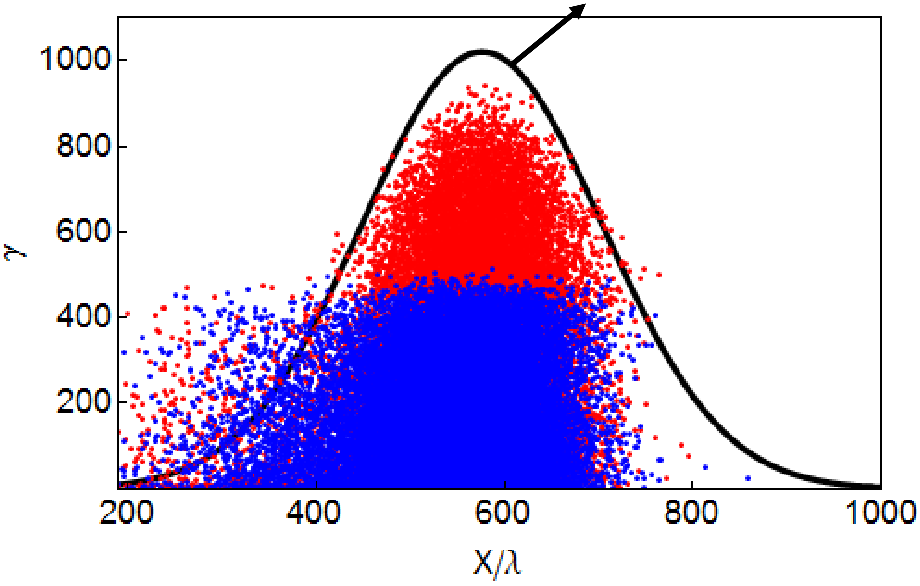
$$E_{0,j}^2 = \hat{I}(k_{p,j}) \,, \quad \hat{I}(k_{p,j}) = \frac{\hat{I}_0}{[1 + ((k_{p,j} - k_{pe})/\Delta k_e)^\alpha]} \,,$$

$$\alpha \approx 5/2,\; k_{pe} \approx 0.15 k_0,\; \delta k \approx k_{pe} \,,\; \Delta k_e \approx 0.02 k_0,\; N = 10 \,.$$

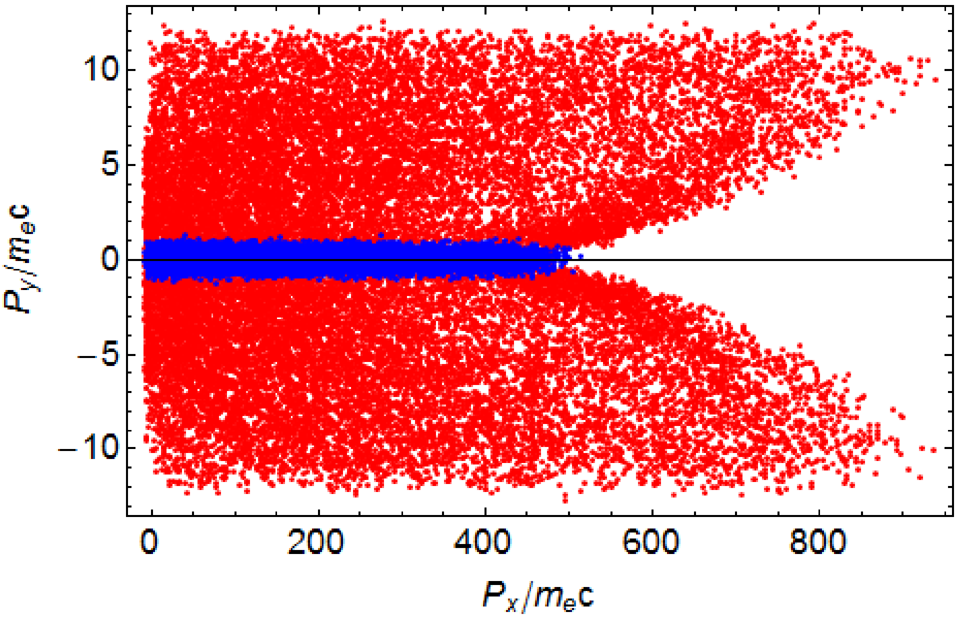
Electrons are initially at rest

$$f_{e0} = n_0 \delta(P_{x0}) \delta(P_{y0}) \delta(P_{z0}) \theta(|\Delta_x^2 - X_0^2|) \theta(|\Delta_y^2 - Y_0^2|) \delta(Z_0)$$

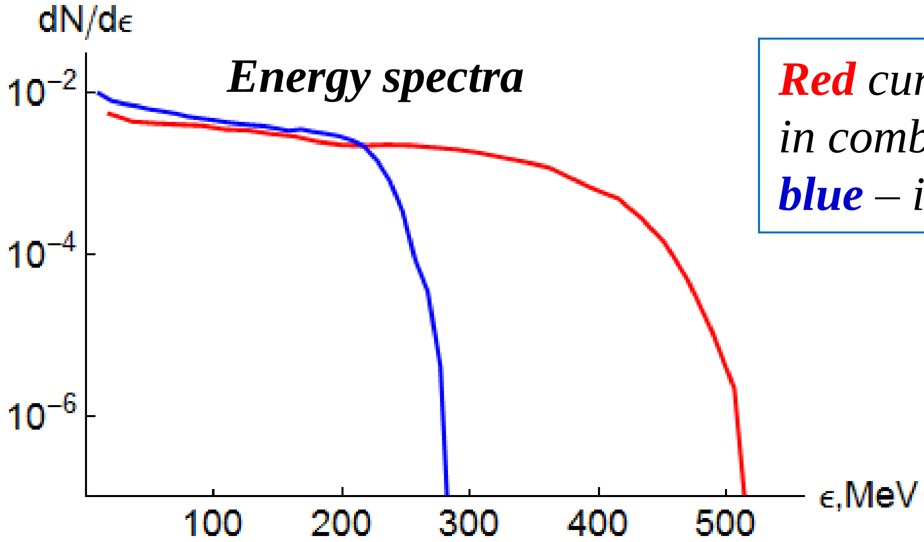
Test Particle Simulation(2)



Envelope of laser pulse



Phase plane



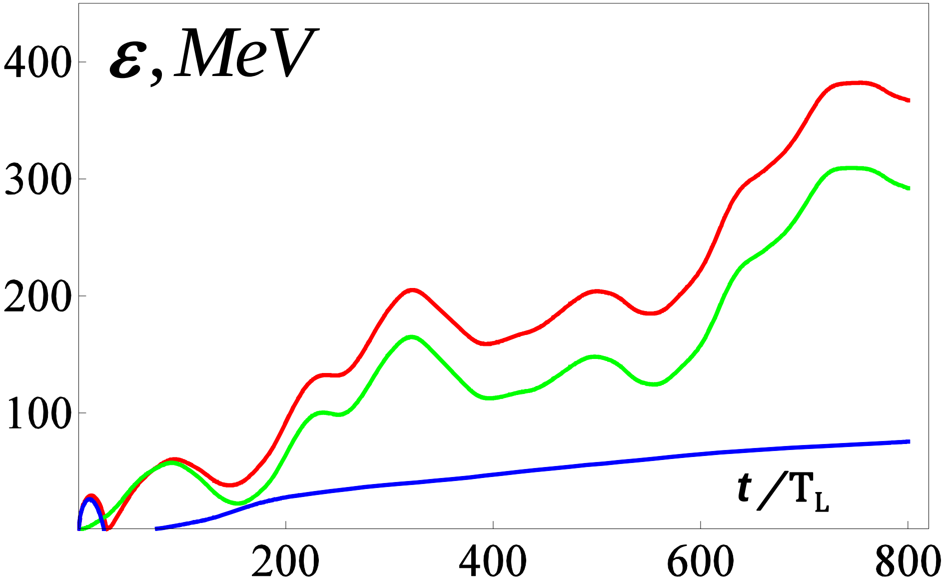
Energy spectra

Red *curve and points are correspond to dynamics in combined fields (laser pulse+plasma waves),* ***blue*** *– in plasma waves only.*

Average electron energy
150 MeV and 100 MeV

Stochastic acceleration in combined fields

Energy of accelerating test electron vs. time



$$m_e c^2 \gamma(t) = -e \int_0^t d\tau (E_{\|} V_{\|} + E_{\perp} V_{\perp})$$

🟥 $E_{\|} \neq 0, \quad E_{\|} \neq 0,$

🟩 $E_{\|} \neq 0, \quad E_{\|} = 0,$

🟦 $E_{\|} = 0, \quad E_{\|} \neq 0,$

Most energetic electrons gain energy in the longitudinal plasma fields rather than they are accelerated directly by laser pulse!

PIC simulations

3D3V fully relativistic PIC code "Mandor"

D.V. Romanov, V.Yu. Bychenkov, W. Rozmus, et al. PRL 93 215004 (2004).
http://mandor.ilc.edu.ru/mandor3

Simulation parameters:

Size of simulation box : (X,Y) :1000-1500 μm x 100 μm,

Size of cell - 0.1 μm,

Number of macroparticles of each per cell – 1÷ 4,

time step is 0.2 of Kurant's number, periodic in y and absorbing in x.

Parameters of laser and plasma:

Linearly polarized laser pulse interacts with underdense plasma

I=5 •10^{19} W/cm^2, a_0=6, t =700 fs, n_e=0.02-0.1 n_{cr}

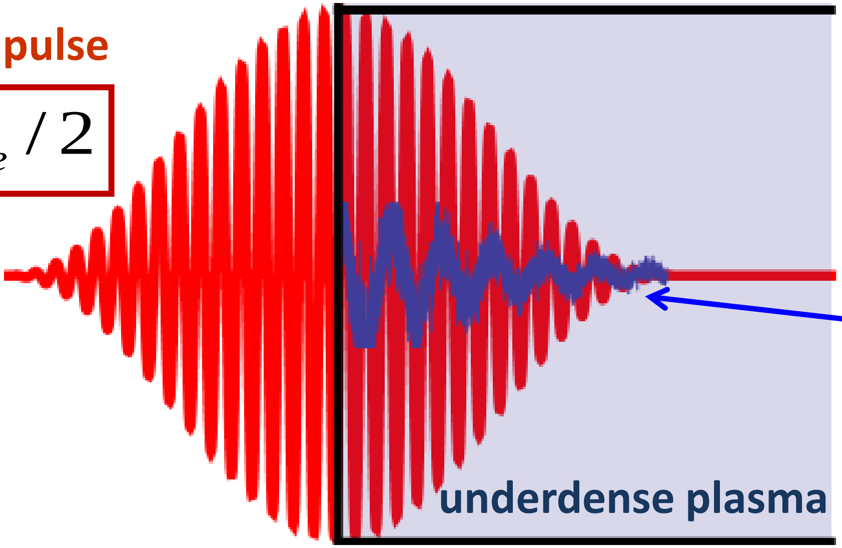
L = 350 ÷ 1000 μm, L - plasma length

Plasma : hydrogen and electrons, ions are fully mobile

Stochastic electron heating in self modulation regime (SM LWFA)

long laser pulse

$$\boxed{c\tau >> \lambda_{pe} / 2}$$



$$t \rightarrow l + t'$$

Plasma waves excitation through <u>Forward Raman Scattering</u>

$$\boxed{k_e \approx \omega_{pe} / c}$$

Laser Plasma Parameters:

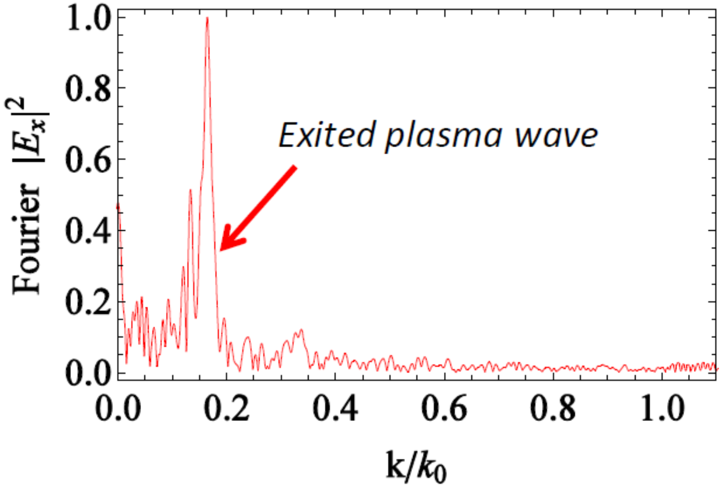
I=5 •10^{19} W/cm^2, a_0=6, τ=700 fs

n_e≈2 •$10^2 n_{cr}$,

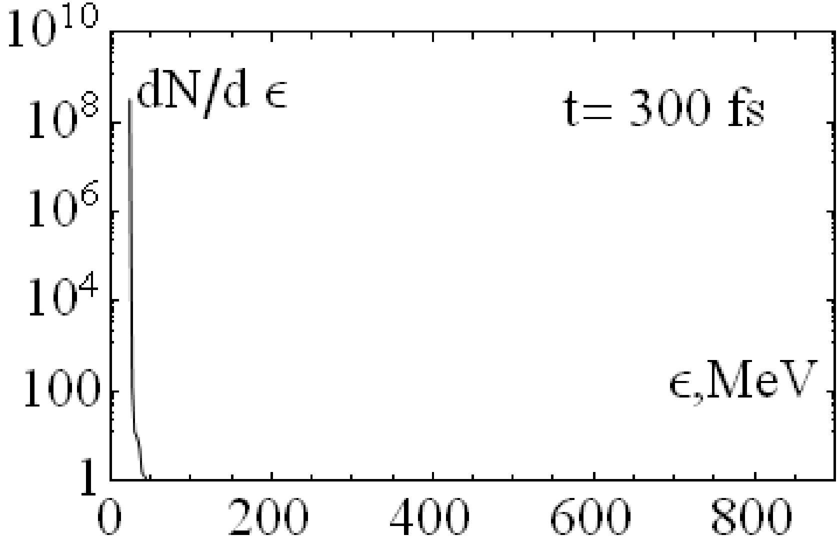
2D PIC simulation:

Spectra of plasma waves

Longitudinal component



Spectrum of electrons in 1D PIC



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long laser pulse

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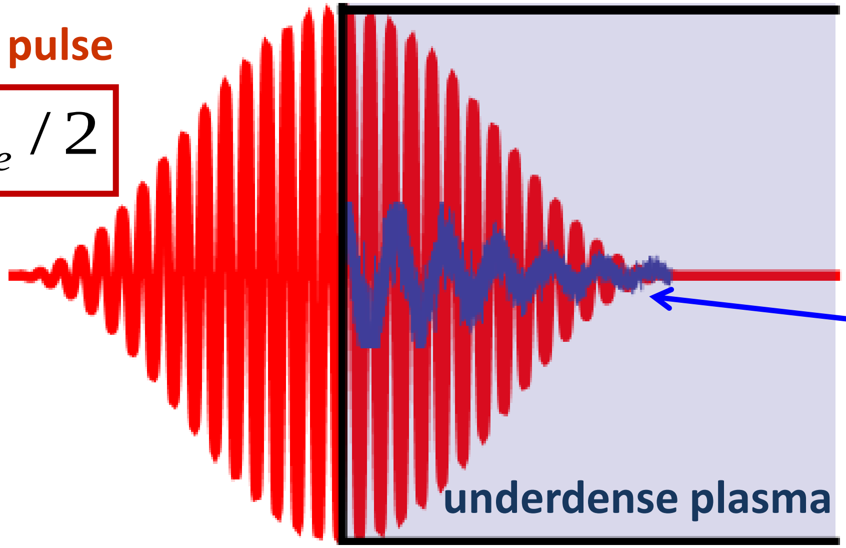

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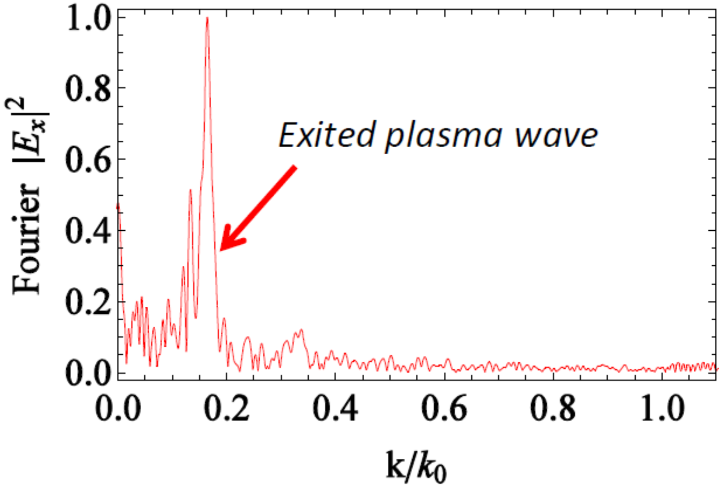

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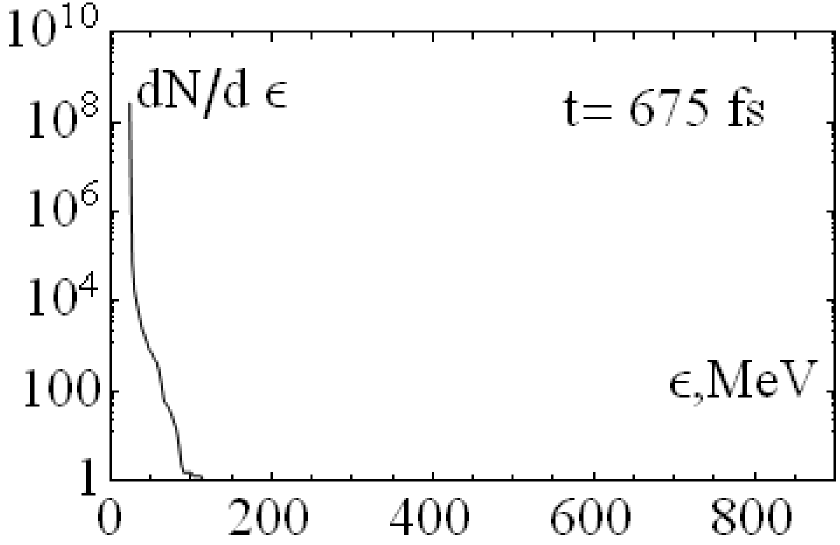

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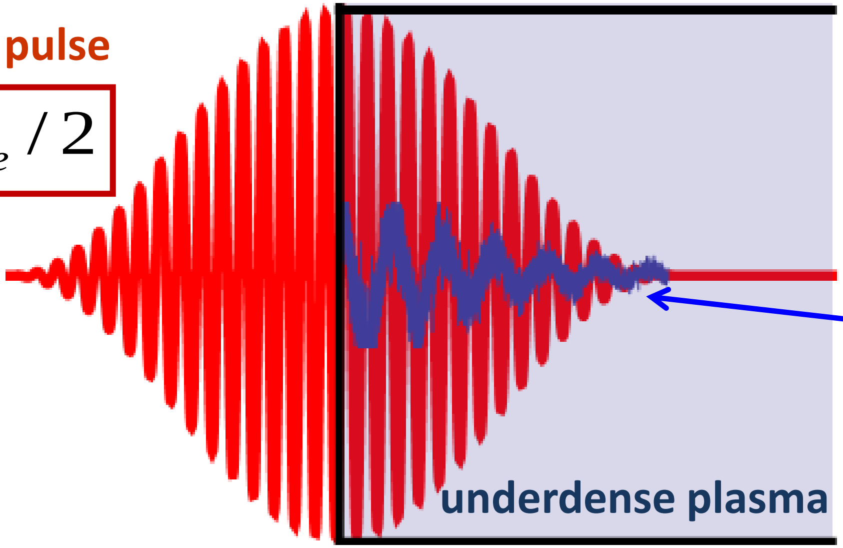

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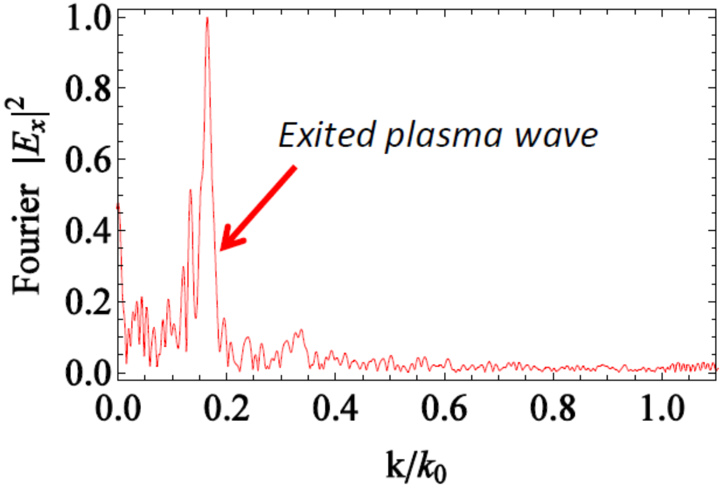
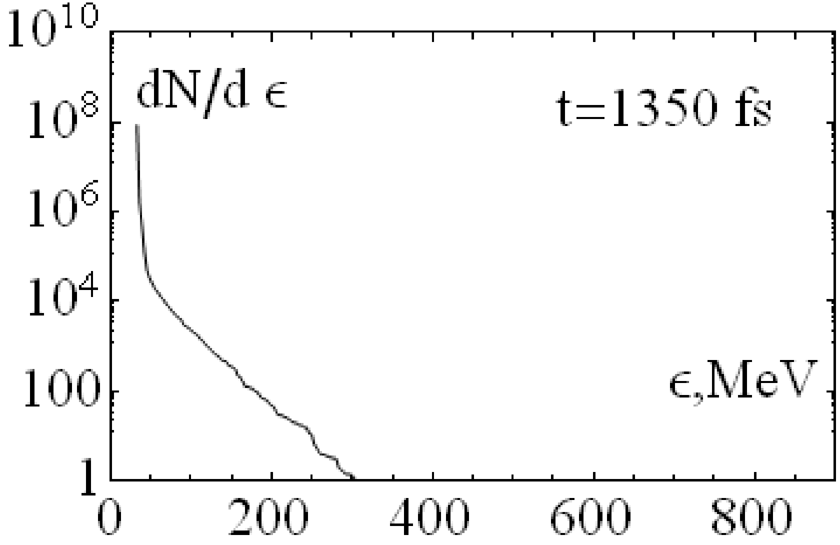


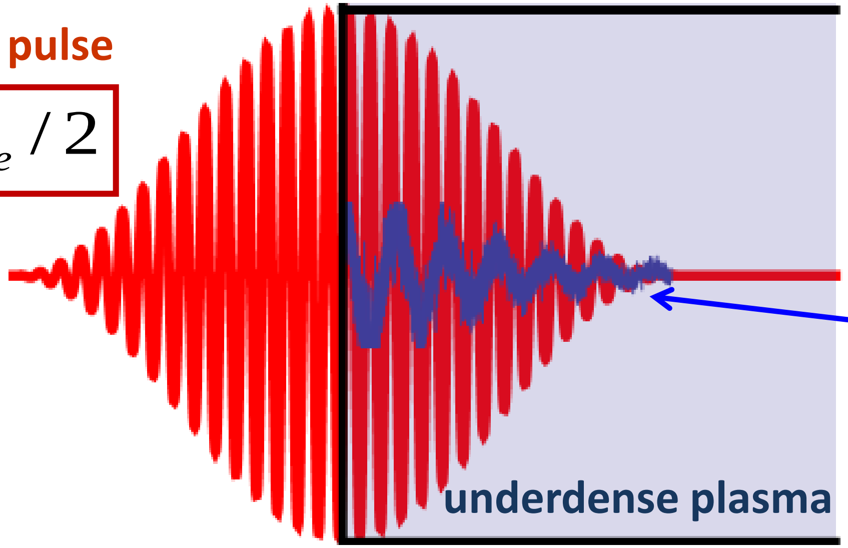
Spectrum of electrons in 1D PIC



Stochastic electron heating in self modulation regime (SM LWFA)

long laser pulse

$$c\tau >> \lambda_{pe} / 2$$



$$t \to l + t'$$

Plasma waves excitation through Forward Raman Scattering

$$k_{e0} \approx \omega_{pe} / c$$

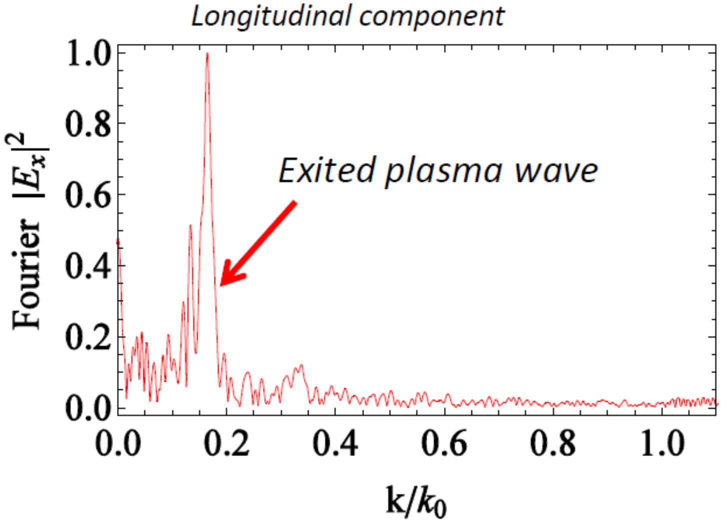
Laser Plasma Parameters:

$I = 5 \cdot 10^{19}$ W/cm^2, $a_0 = 6$, $\tau = 700$ fs

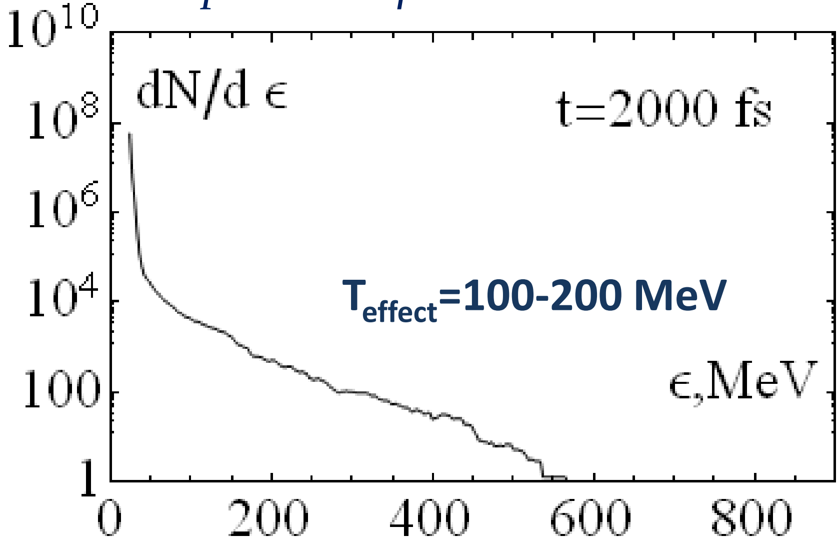
$n_e \approx 2 \cdot 10^2 n_{cr}$,

2D PIC simulation:

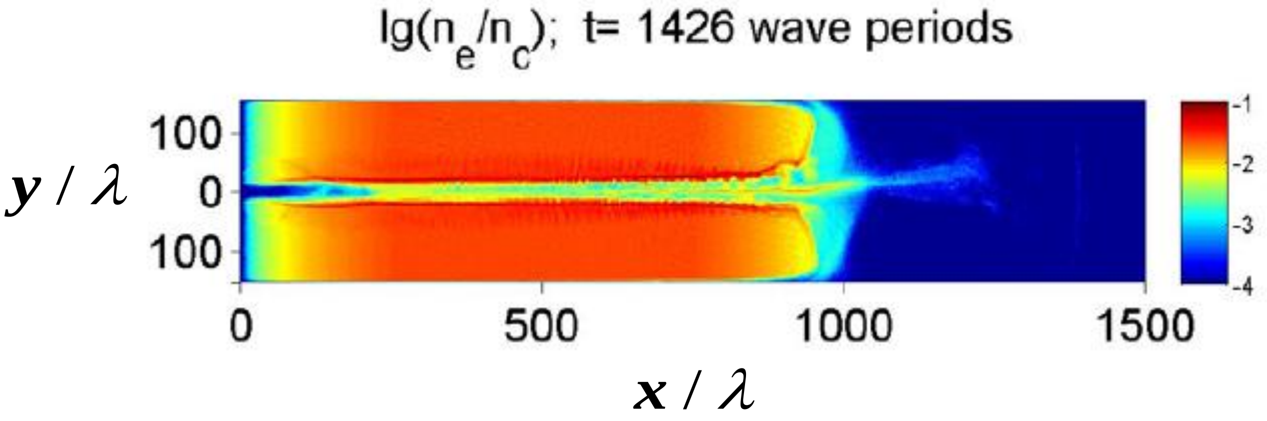
Spectra of plasma waves



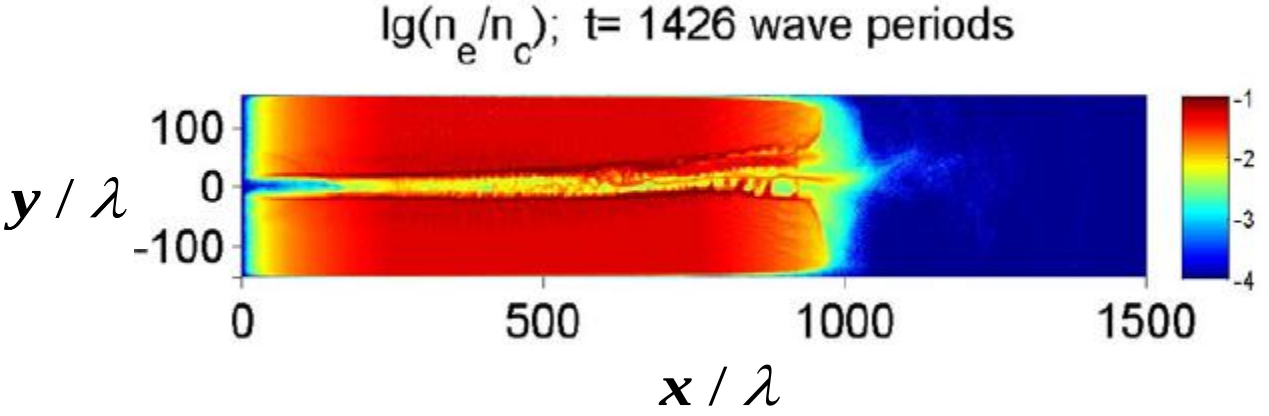
Spectrum of electrons in 1D PIC



Electron density 2D



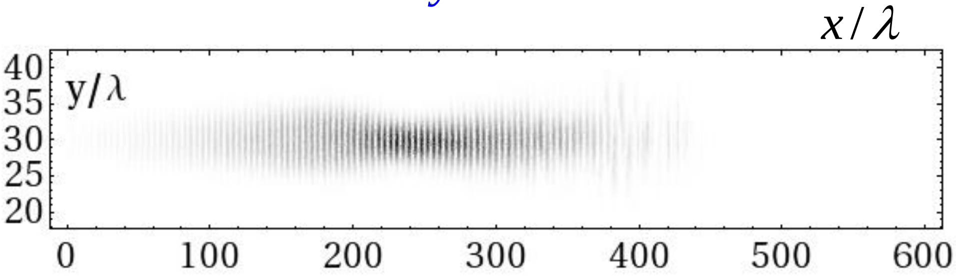
$n_e/n_c = 0.025$



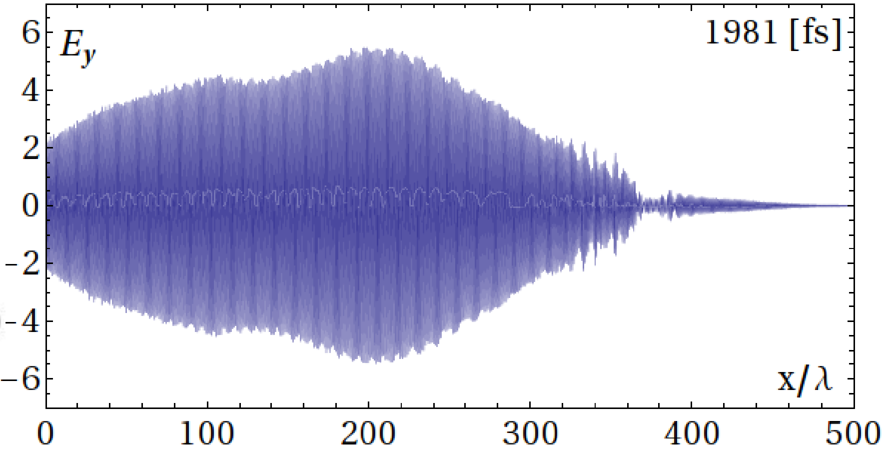
$n_e/n_c = 0.05$

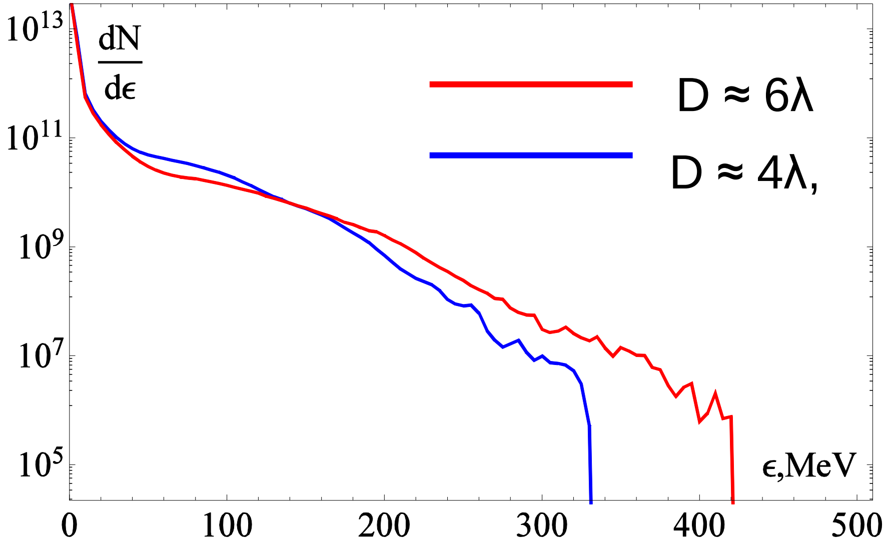
Electric Fields in 2D3V simulation

Intensity distribution



Transverse electric field





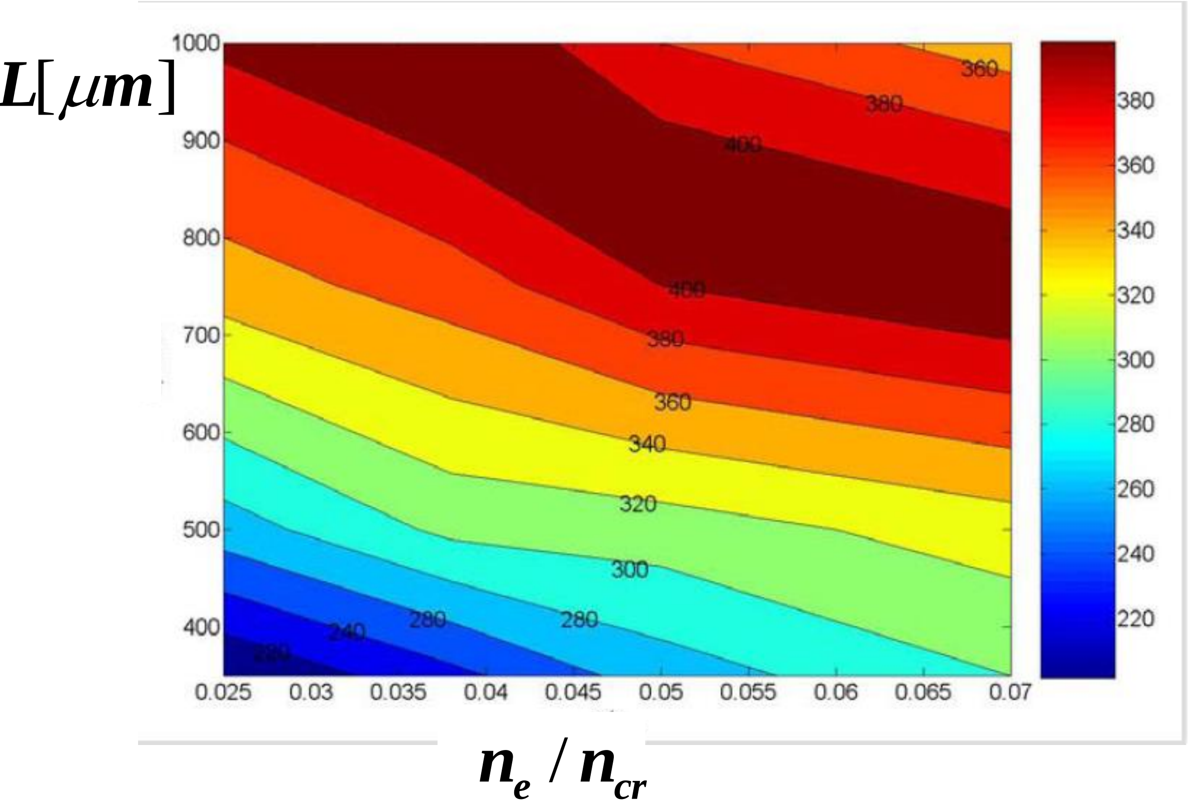
I = 5• 10^{19} W/cm^2, (a_0=6), D ≈ 6λ,

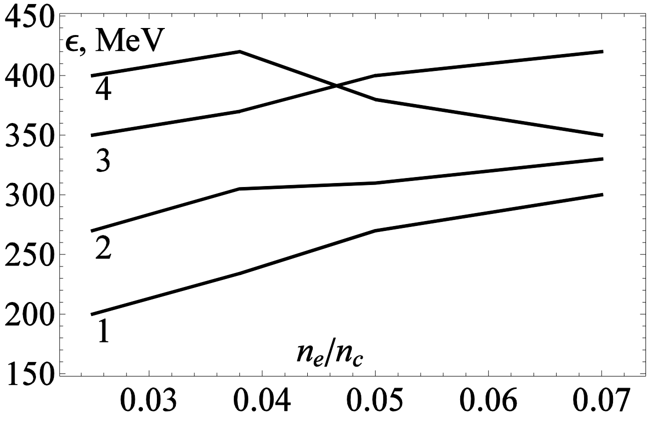
L = 800μm, n_e=5•10^{-2} n_c

One can see the formation of high –energy tail in the energy spectrum!

2D3V Mandor simulations - electrons

Density plot of maximum electron energy vs. plasma length and background plasma density





L/λ=350(1), 500(2), 750(3), 1000(4

Energy 20 J, duration 0.7 ps

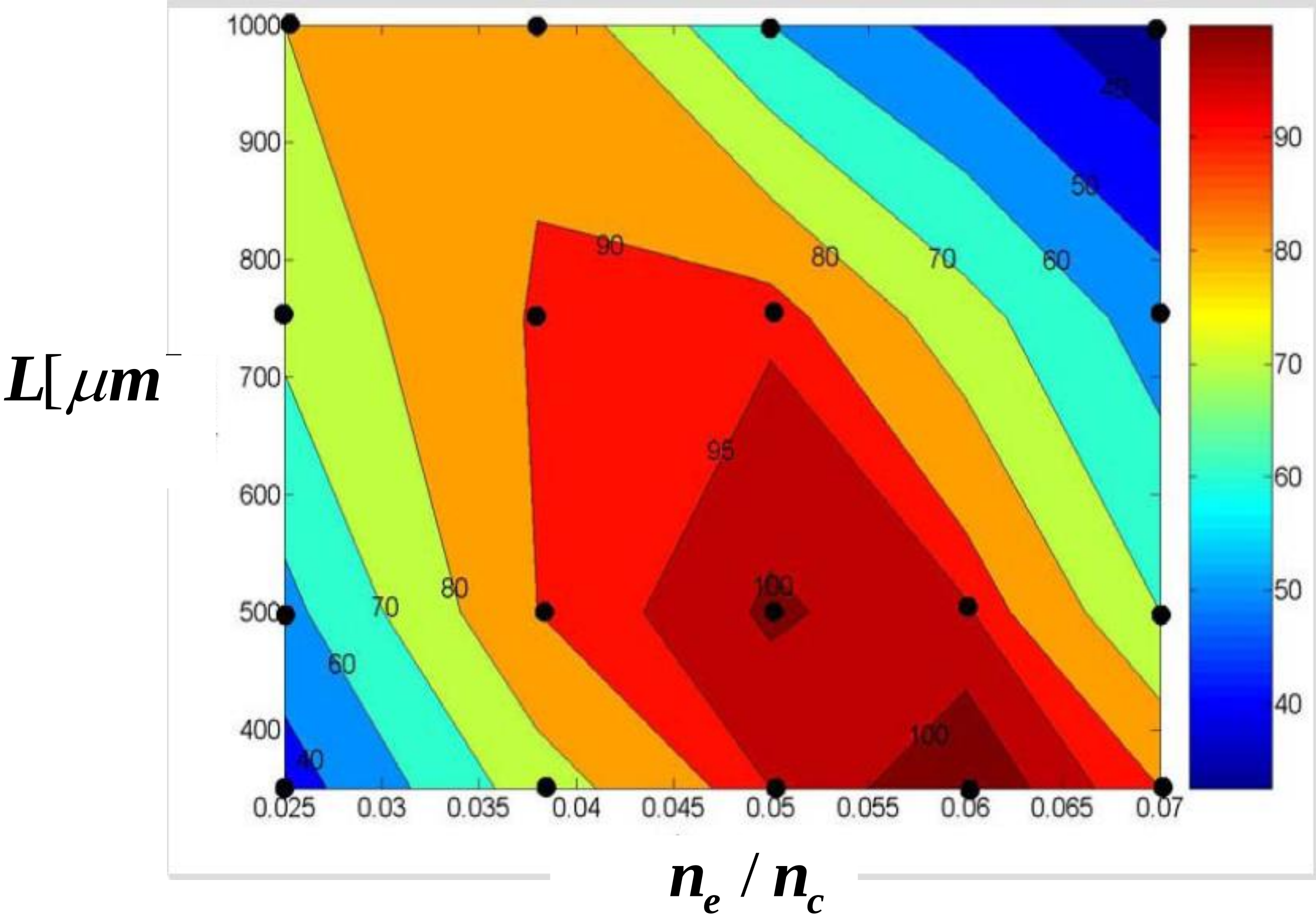
I = 5• 10^{19} W/cm^2, D ≈ 6λ, λ =1 μm.

Max energy is 400 MeV!

Maximum energy is given for a moment of time when laser pulse leaves right boundary of the simulation box

2D3V Mandor simulations - protons

Density plot of maximum proton energy vs. plasma length and plasma density



Max energy is 100 MeV!

Maximum energy is given for a moment of time when laser pulse passed right boundary of the simulation box

Conclusions

✓Stochastic acceleration in combined fields (laser wave, scattered wave and excited plasma wave).

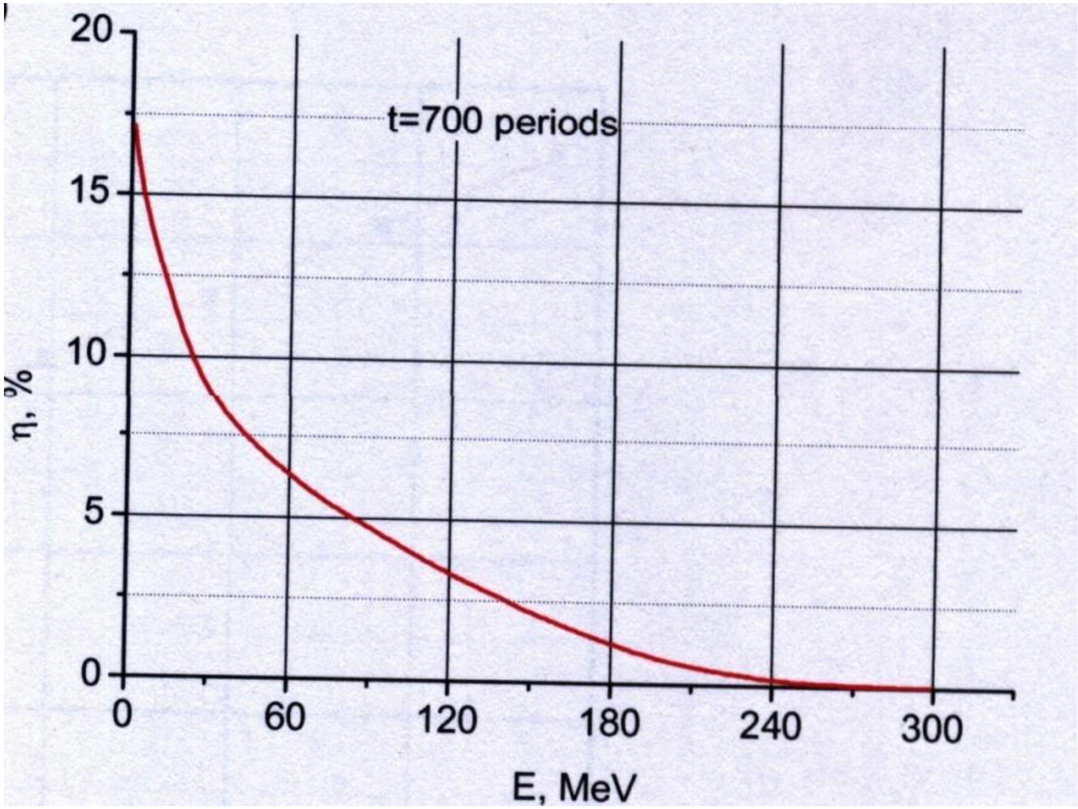
✓Revealed electron acceleration mechanism is stochastic heating in turbulent plasma waves!

✓Results of 1D and 2D PIC simulations are in qualitative agreement with developed theoretical model.

✓Effect of stochastic acceleration can be used for generation of fast electron and proton bunches from gas jet targets.

Conversion to high energy electrons

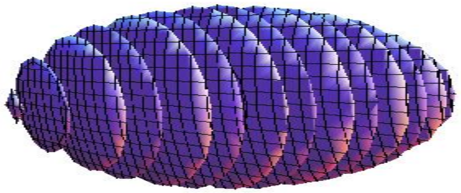
Laser-to-electron energy-conversion efficiency

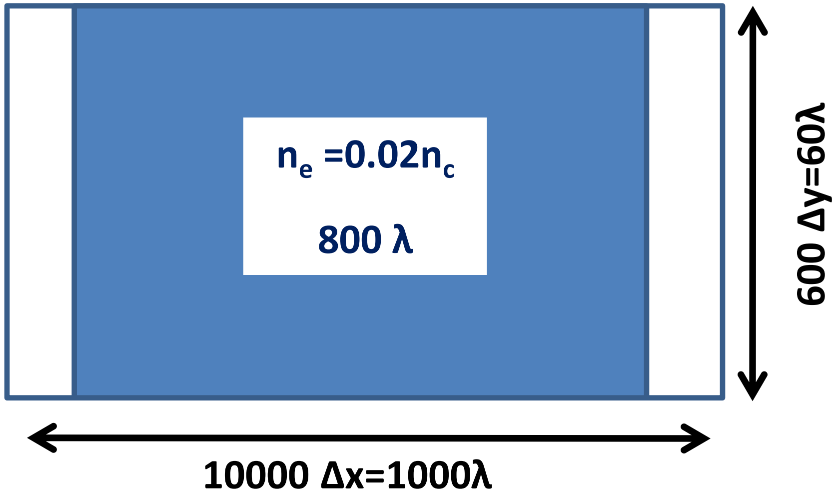


Simulations demonstrate high efficiency of laser energy transformation into fast electrons. The number of electrons with energy > 60 MeV is $5 \cdot 10^{10}$ (Q = 8 nC).

Thus, electrons can be accelerated to high energies, carrying a significant fraction of input laser energy (6% of laser energy converted to electrons with E > 60 MeV)!

2D Simulations of Electron Acceleration



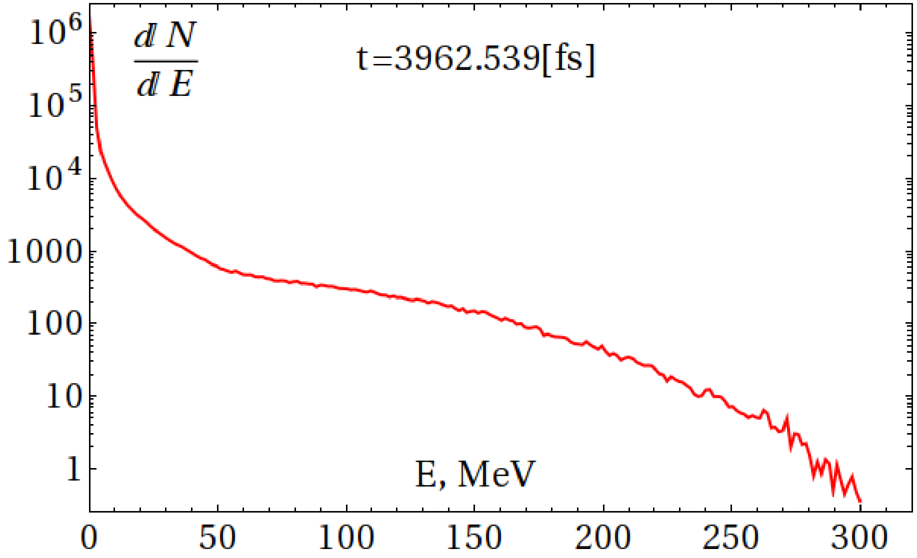


2 particles/cell.
Boundary conditions: periodic in y and absorbing in x.
Time step = 0.3 Courant number

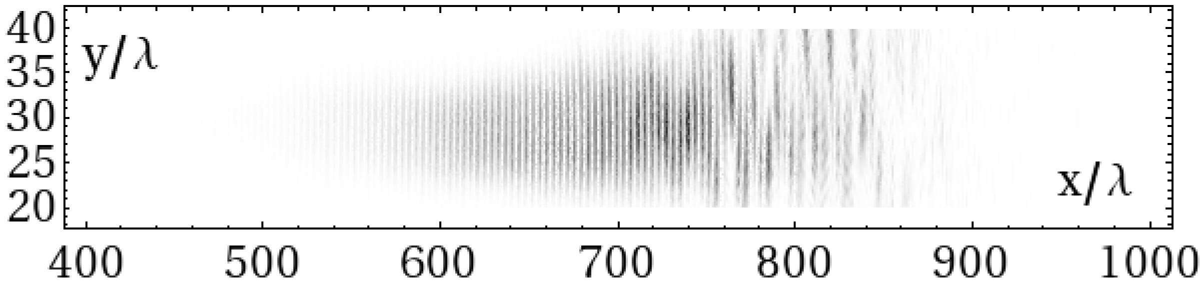
Target – plasma with immovable ions

Gaussian laser profile with 700 fs FWHM and $a_0 = 6$.
Gaussian transverse profile with 4 λ FWHM.
Laser pulse has been focused at the front plasma boundary.

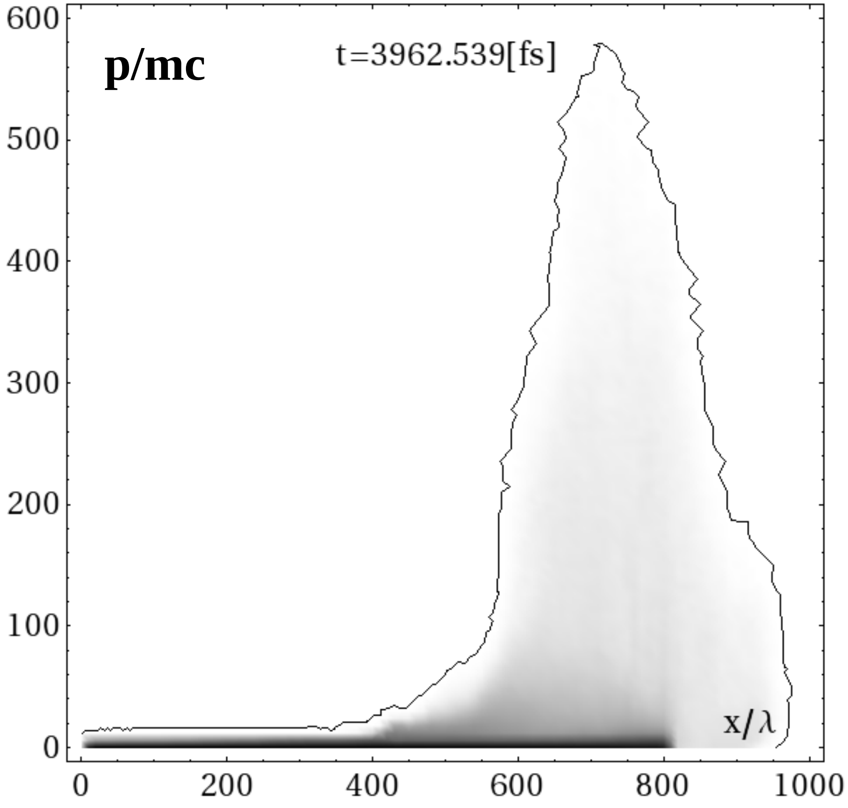
Electron spectrum et the end of simulation



Electron acceleration from 2D simulation



Laser pulse intensity



Electron phase space