FACULTY OF PHYSICS
UNIVERSITY OF WARSAW

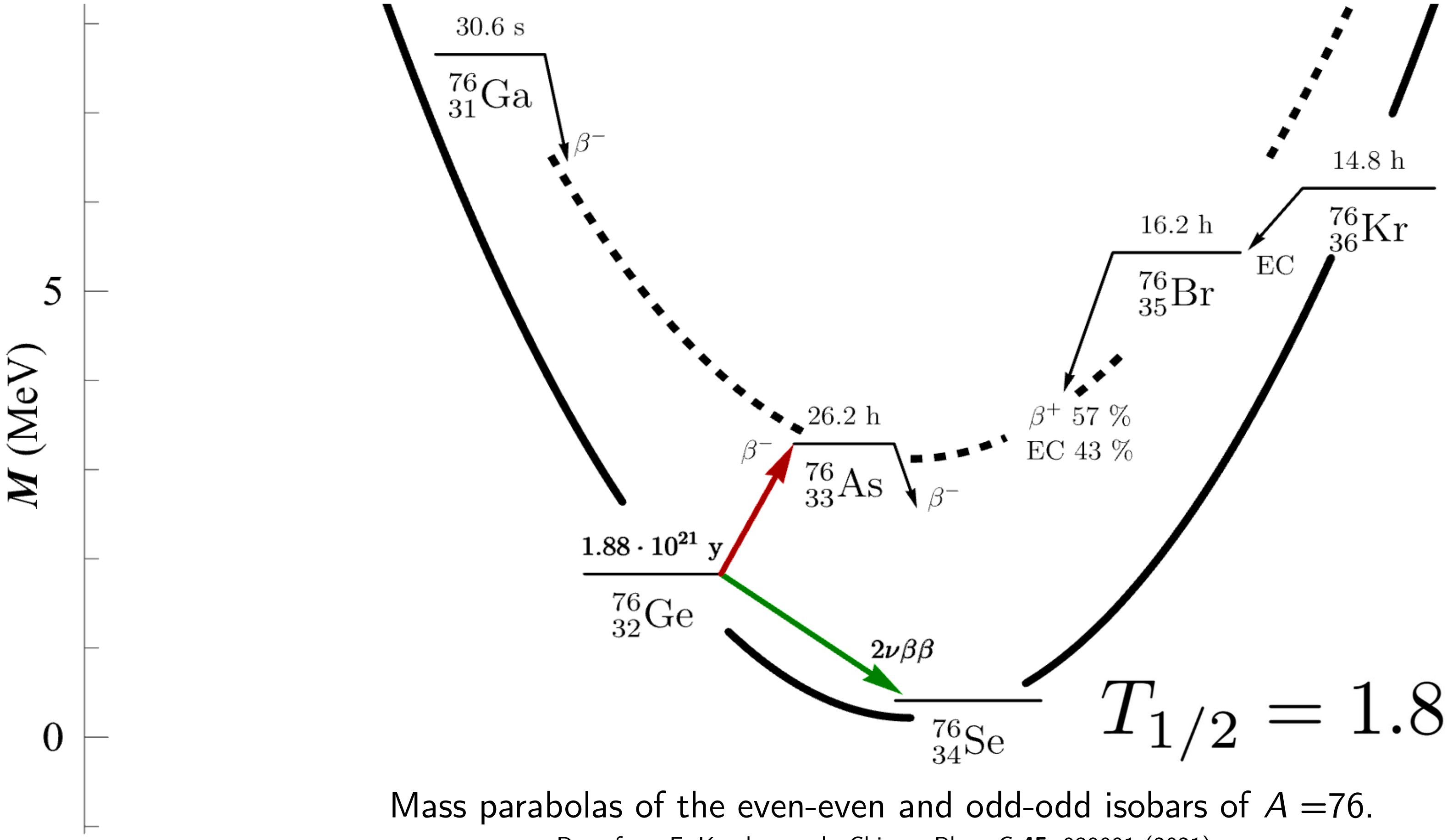
# Double beta decay within the nuclear density functional theory

J. Miśkiewicz[1], M. Konieczka[1], W. Satuła[1]

[1] *University of Warsaw, Faculty of Physics, Department of Complex Systems Modelling*

*Challenges in effective field thery descriptions of nuclei, Hirschegg 2026*

# Double beta decay

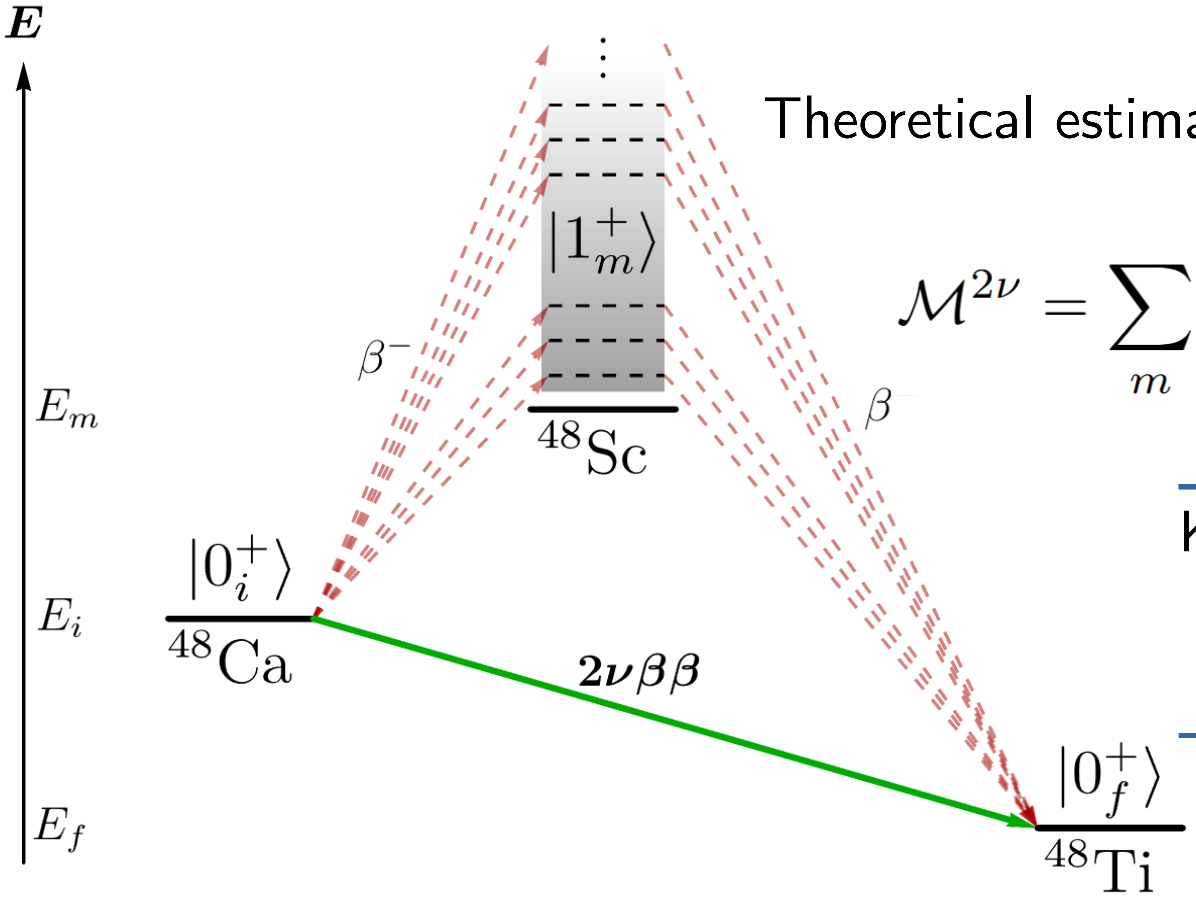



$$T_{1/2} = 1.88 \cdot 10^{21} \text{y}.$$

Mass parabolas of the even-even and odd-odd isobars of $A$ =76.

Data from F. Kondev et al., Chinese Phys. C **45**, 030001 (2021).

# Fermi golden rule of the 2nd order




Forbidden transitions contribute to the total decay rate.

Theoretical estimation via **virtual** transitions:

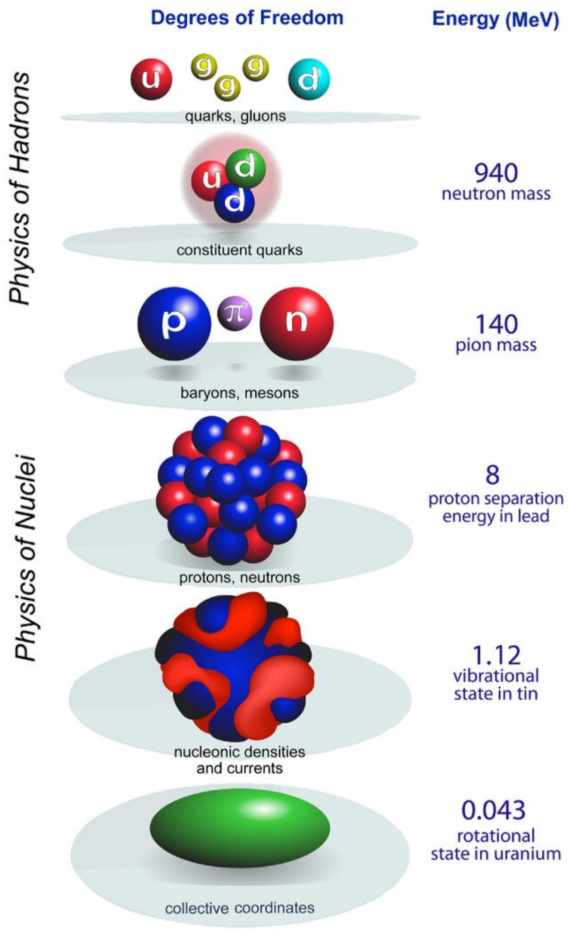
$$\mathcal{M}^{2\nu} = \sum_m \frac{\langle 0_f^+ | \hat{H}_\beta | \mathbf{1_m^+} \rangle \langle \mathbf{1_m^+} | \hat{H}_\beta | 0_i^+ \rangle}{\boldsymbol{\Delta E_m} + \frac{1}{2} Q_{\beta\beta} + \Delta M}$$

Key difficulty: constructing

$|0_i^+\rangle, |1_m^+\rangle, |0_f^+\rangle$

(strong interaction)

# Dealing with strong interaction




Hartree-Fock → Kohn-Sham framework

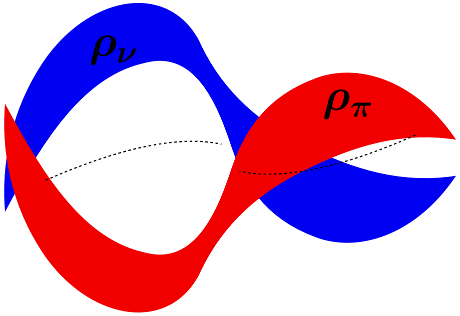
We are here

Hierarchy of the nuclear degrees of freedom.

Picture taken from E. Litvinova and C. Robin (INPC2016).

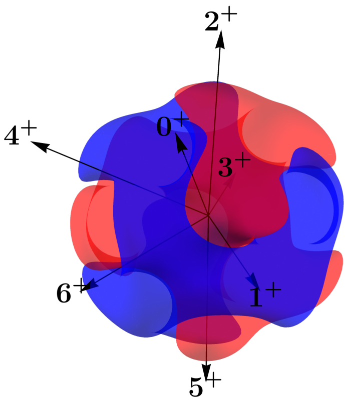
# DFT-NCCI in a nutshell

**Skyrme SV density functional**




$$\frac{\delta}{\delta\rho}$$

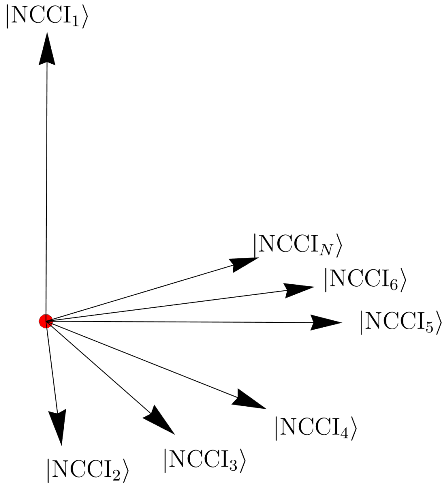
**Slater determinant (mean-field)**




**no good quantum numbers**
~~$\gamma$, $\beta$ **transitions**~~

$I^\pi$**-projection**
**conf. mixing**
**Hill-Wheeler**

**NCCI basis**



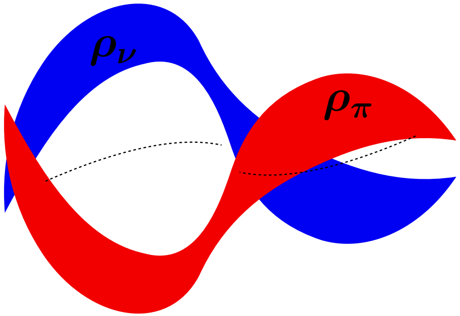

**good quantum numbers**
$\gamma$, $\beta$ **transitions**

# Skyrme SV functional

$$V_{\text{Skyrme}}(\mathbf{x_1},\mathbf{x_2}) = t_0(1+x_0\hat{P}_\sigma)\delta(\vec{r}_1-\vec{r}_2) + \frac{1}{2}t_1(1+x_1\hat{P}_\sigma)\left[\overleftarrow{k}^2\delta(\vec{r}_1-\vec{r}_2) + \delta(\vec{r}_1-\vec{r}_2)\overrightarrow{k}^2\right]$$
$$+ t_2(1+x_2\hat{P}_\sigma)\overleftarrow{k}\delta(\vec{r}_1-\vec{r}_2)\overrightarrow{k} + \frac{1}{6}t_3(1+x_3\hat{P}_\sigma)\rho^\alpha\left(\frac{\vec{r}_1+\vec{r}_2}{2}\right)\delta(\vec{r}_1-\vec{r}_2)$$
$$+ iW_0(\vec{\sigma}_1+\vec{\sigma}_2)(\overleftarrow{k}\times\delta(\vec{r}_1-\vec{r}_2)\overrightarrow{k}),$$

SV

$$\overrightarrow{k} = \frac{1}{2i}(\vec{\nabla}_1-\vec{\nabla}_2), \qquad \overleftarrow{k} = -\frac{1}{2i}(\vec{\nabla}_1-\vec{\nabla}_2)$$



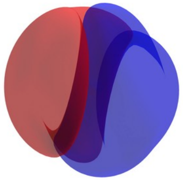

**The only density-independent parametrization → no singularities during *I*/*T*-projection**

# Modelling 2νββ of 48Ca

## 1) Mean – field configuration classes of each nucleus:

### 48Ca (mother)

1. Seniority-zero *nn*-pairs

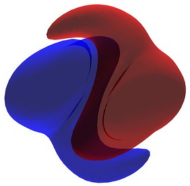


**24**

configurations

### 48Sc (virtual)

1. Seniority-two in $f_{7/2}$
2. Single *n*-excitations across $N$=28
3. Single *p*-excitations across $Z$=28

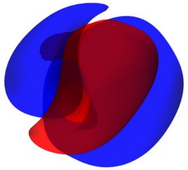


**54**

configurations

### 48Ti (daughter)

1. Seniority-zero *nn*- and *pp*-pairs in $f_{7/2}$
2. *np*-pairing in $f_{7/2}$
3. Seniority-zero *nn*-pairs across $N$=28 shell
4. Seniority-zero *pp*-pairs across $Z$=28 shell



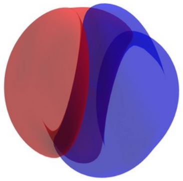
**49**

configurations

# Modelling 2νββ of 48Ca

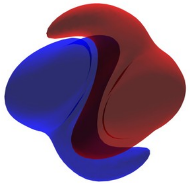
## 2) $I^\pi$ - projection:

**48Ca** $|\boldsymbol{\Psi}_i\rangle$



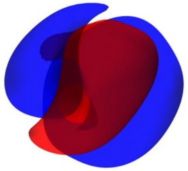
$\xrightarrow{\hat{P}^I_{MK}}$ $|\boldsymbol{\Psi}_{i1};0^+\rangle \;\vdots\; |\boldsymbol{\Psi}_{i\alpha};0^+\rangle \;\vdots$

**48Sc** $|\boldsymbol{\Psi}_m\rangle$



$\xrightarrow{\hat{P}^I_{MK}}$ $|\boldsymbol{\Psi}_{m1};1^+\rangle \;\vdots\; |\boldsymbol{\Psi}_{m\beta};1^+\rangle \;\vdots$

**48Ti** $|\boldsymbol{\Psi}_f\rangle$



$\xrightarrow{\hat{P}^I_{MK}}$ $|\boldsymbol{\Psi}_{f1};0^+\rangle \;\vdots\; |\boldsymbol{\Psi}_{f\gamma};0^+\rangle \;\vdots$

## 3) Partial Gamow-Teller elements:

$$\mathcal{M}_{fm} = \langle \boldsymbol{\Psi}_{m\alpha};0^+|\hat{H}_{\mathrm{GT}}|\boldsymbol{\Psi}_{m\beta};1^+\rangle$$

$$\mathcal{M}_{mi} = \langle \boldsymbol{\Psi}_{m\beta};1^+|\hat{H}_{\mathrm{GT}}|\boldsymbol{\Psi}_{i\alpha};0^+\rangle$$

# Watch out for orthogonality!

In the projection framework GT-matrix element is expressed via single-particle transition densities $\boldsymbol{\rho}$:

$$\langle \boldsymbol{\Psi_i}; I_i^\pi | \hat{O}_{\mathrm{GT}} | \boldsymbol{\Psi_j}; I_i^\pi \rangle \sim \int \langle \Psi_i | \hat{\tau}_{1\mu} \hat{\sigma}_{1\nu} | \tilde{\Psi}_j(\Omega) \rangle \cdot D^{I*}_{K\text{-num.}}(\Omega) d\Omega$$

Generalized Wick's Theorem

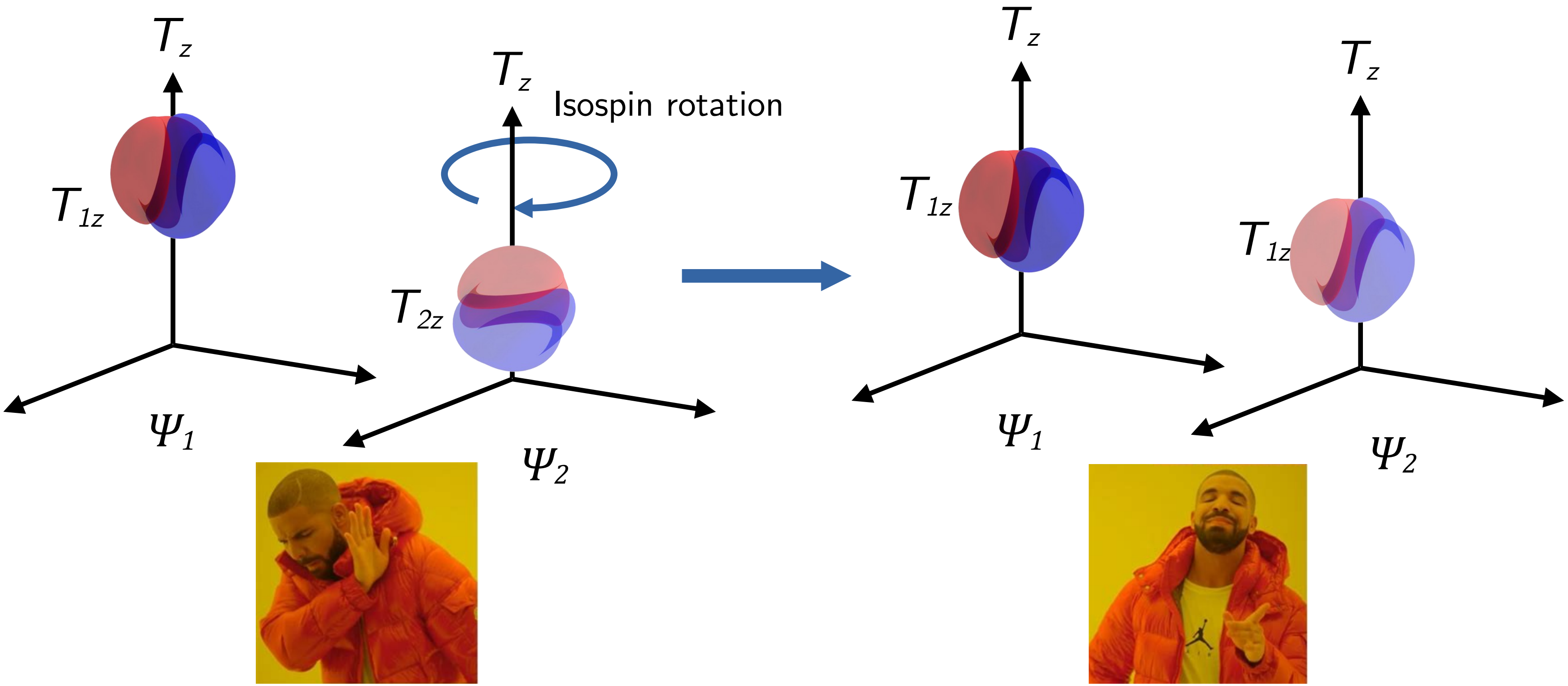
$$\sim \underline{\langle \Psi_i(T_z') | \tilde{\Psi}_j(T_z) \rangle} \sum_{aa';\sigma\sigma'} \langle a | \hat{\tau}_{1\mu} | a' \rangle \langle \sigma | \hat{\sigma}_{1\nu} | \sigma' \rangle \tilde{\boldsymbol{\rho}}(\vec{r}, a', \sigma'; \vec{r}, a, \sigma)$$

**orthogonal!**

But we've got a workaround for that ...

# Watch out for orthogonality!

Workaround: rotate in isospace, so both $\Psi_1$ and $\Psi_2$ have the same $T_z$:



# Modelling 2νββ of 48Ca

## 4) Configuration mixing of the projected states:

**48Sc:** $|\mathbf{\Psi}_{m1};1^+\rangle$ $\vdots$ $|\mathbf{\Psi}_{m\beta};1^+\rangle$ $\vdots$


$|\mathrm{NCCI}_{m1};1^+\rangle$
$|\mathrm{NCCI}_{m2};1^+\rangle$
$\vdots$
$|\mathrm{NCCI}_{m\beta};1^+\rangle$
$\vdots$

*+ same for* ***48Ca*** *and* ***48Ti***

## 5) Calculation of the cumulative 2νββ matrix element :

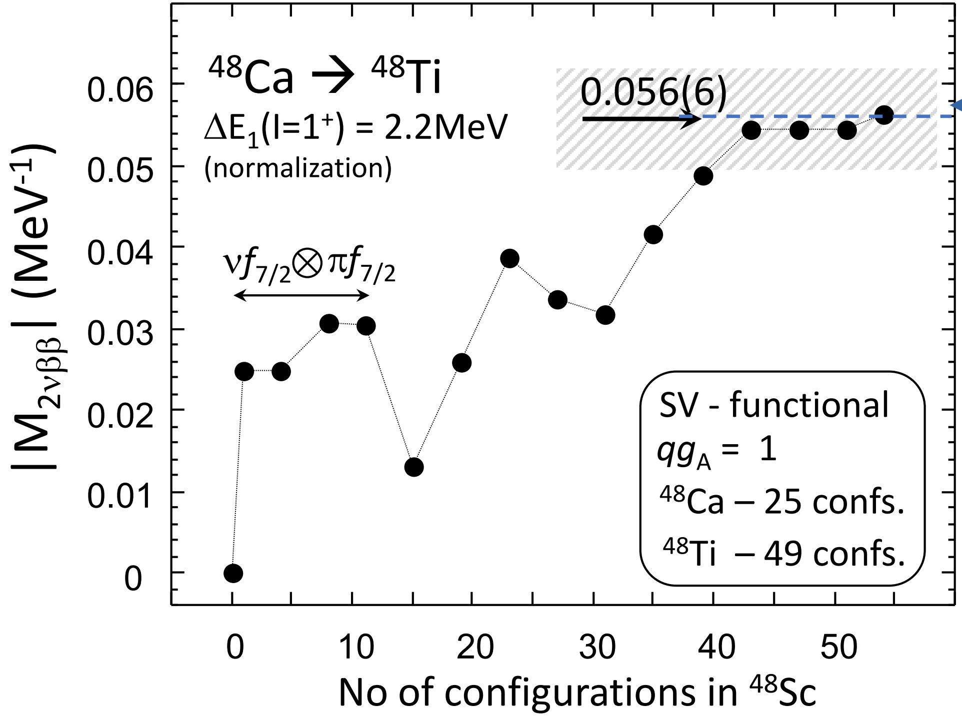
$$\mathcal{M}_{2\nu\beta\beta}=\sum_m \frac{\langle \mathrm{NCCI}_f|\hat{H}_{\mathrm{GT}}|\mathrm{NCCI}_m\rangle\langle \mathrm{NCCI}_m|\hat{H}_{\mathrm{GT}}|\mathrm{NCCI}_i\rangle}{\underbrace{\Delta E_j+\frac{1}{2}Q_{\beta\beta}+\Delta M}_{\text{spectrum shifted to experiment}}}$$

*The whole procedure has been performed* ***numerically*** *within DFT-based* ***HFODD*** *code (open access).*

# NME(48Ca → 48Ti) : virtual dependence

**Resulting cumulative $|\mathcal{M}_{2\nu\beta\beta}|$:**

$\Delta E_1(I^\pi=1^+)=2.2$MeV (normalization)




saturation at **0.056** (MeV$^{-1}$)

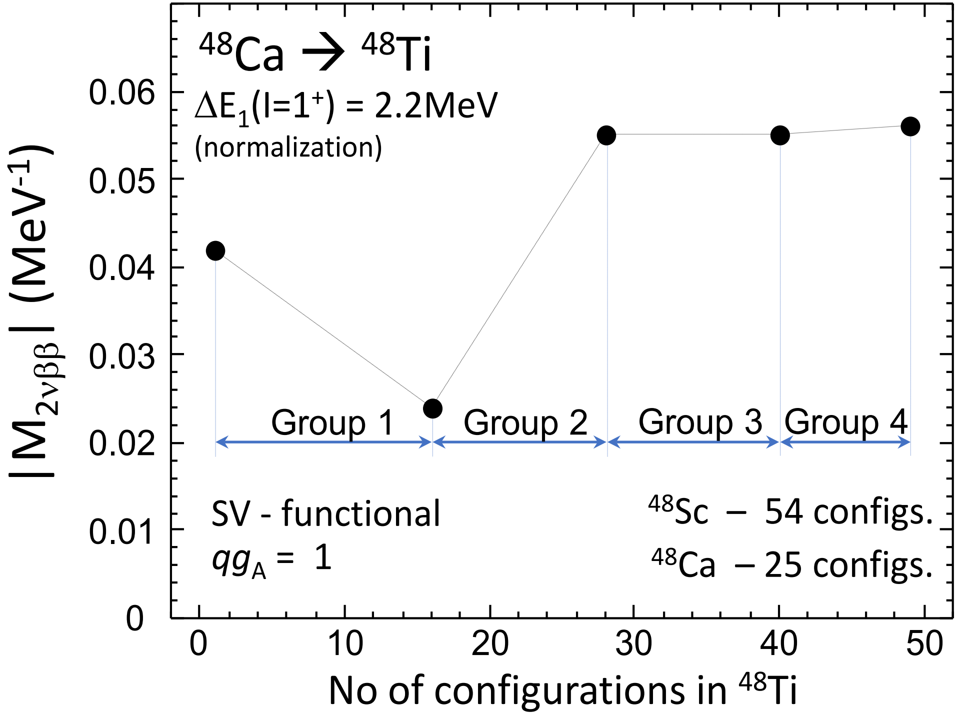
**0.029** (dimless)

$q \simeq 0.73$ – assumed from other DFT-NCCI works in the $A\simeq40$ region

# NME(48Ca → 48Ti) : daughter dependence

**Resulting cumulative** $|\mathcal{M}_{2\nu\beta\beta}|$**:**

$\Delta E_1(I^\pi{=}1^+){=}2.2$MeV (normalization)




Mixing of **crumble** GTs:

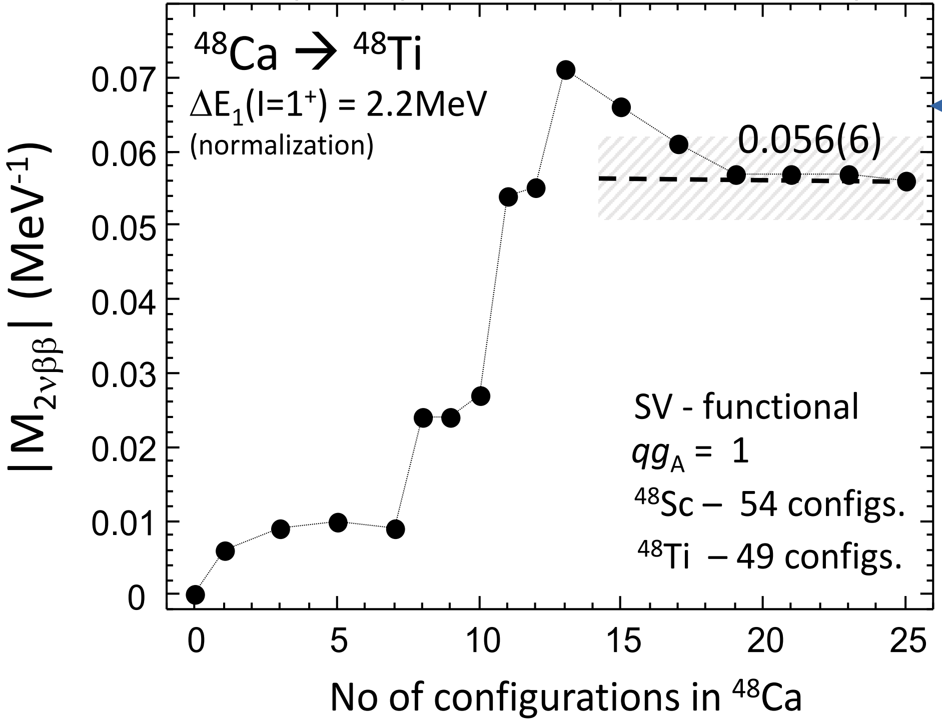
**88%** of the

total matrix element!

# NME(48Ca → 48Ti) : mother dependence

**Resulting cumulative $|\mathcal{M}_{2\nu\beta\beta}|$:**

$\Delta E_1(I^\pi{=}1^+){=}2.2$MeV (normalization)




saturation at **0.056** (MeV$^{-1}$)

**0.029** (dimless)

**Very surprising result in a spherical nucleus!**

**PRC 112, 055502 (Nov 2025)**

# Comparison with other models

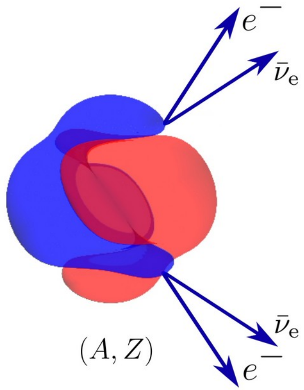
| Reference | Method | $\lvert\mathcal{M}^{2\nu}\rvert$ (MeV$^{-1}$) |
|---|---|---|
| A. S. Barabash (2020) | Experiment | $0.068 \pm 0.006$ |
| Y. Iwata et al. (2015) | Shell model | 0.0539 |
| F. Šimkovic et al. (2018) | ChER | 0.0832 |
| J. Kostensalo and J. Suhonen (2019) | Shell model | 0.100 |
| S. Novario et al. (2021) | Coupled cluster | 0.082 |
| J. Terasaki and Y. Iwata (2021) | Shell model | 0.0515 |
| | QRPA | 0.0745 |
| P. Veselý et al. (2024) | STDA | 0.1668 |
| **J. Miśkiewicz, M. Konieczka and W. Satuła (2025)** | **DFT-NCCI** | **0.056** |

Comparison of DFT-NCCI matrix element magnitude with other nuclear models. Values for other models converted from dimensionless units to [MeV$^{-1}$] assuming $qg_A$=1.

# Summary: Status and future plans

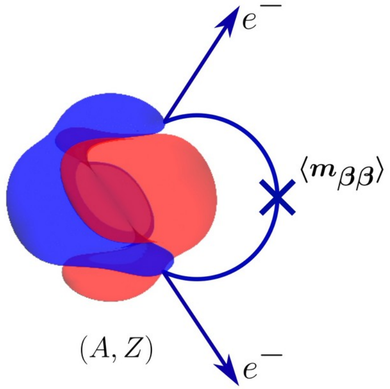
**2νββ**
(~ completed)

- **48Ca** (spherical)
- **76Ge** (deformed)
- 136Xe (heavy)

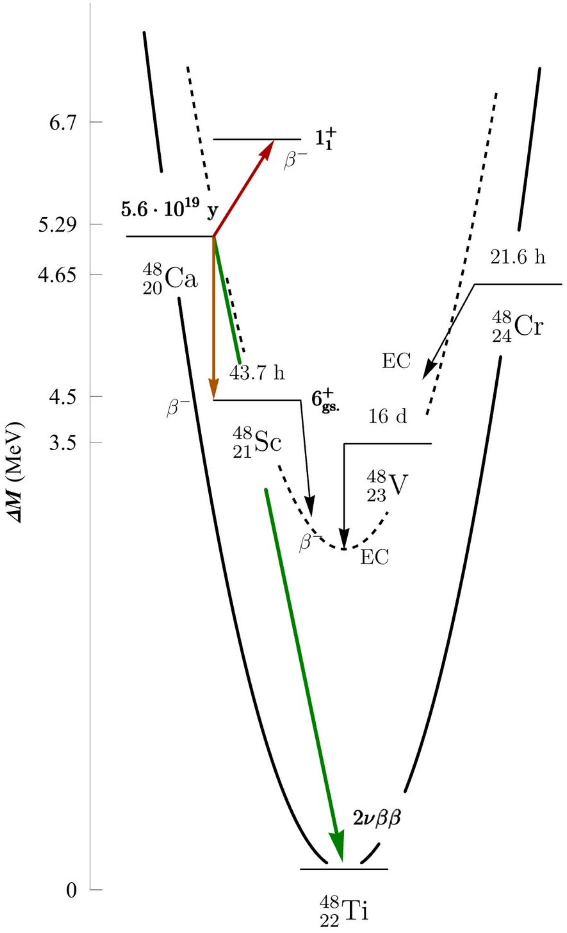



**0νββ**

- Extension of HFODD code: **forbidden transitions** (GWT)
- NMEs for 48Ca and 76Ge (closure approximation)
- Half-life estimation with modern constraints on $\langle m_{\beta\beta} \rangle$

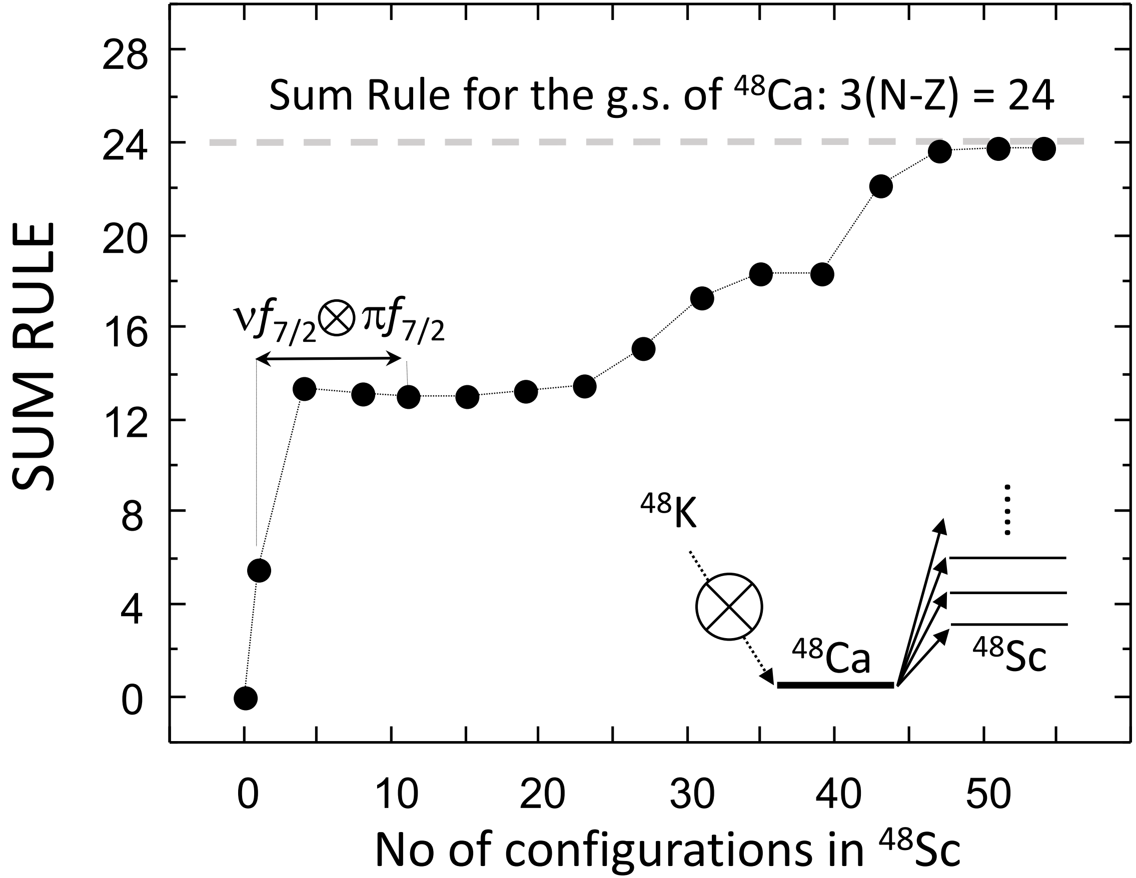


# 48Ca is unique among the 2νββ nuclei

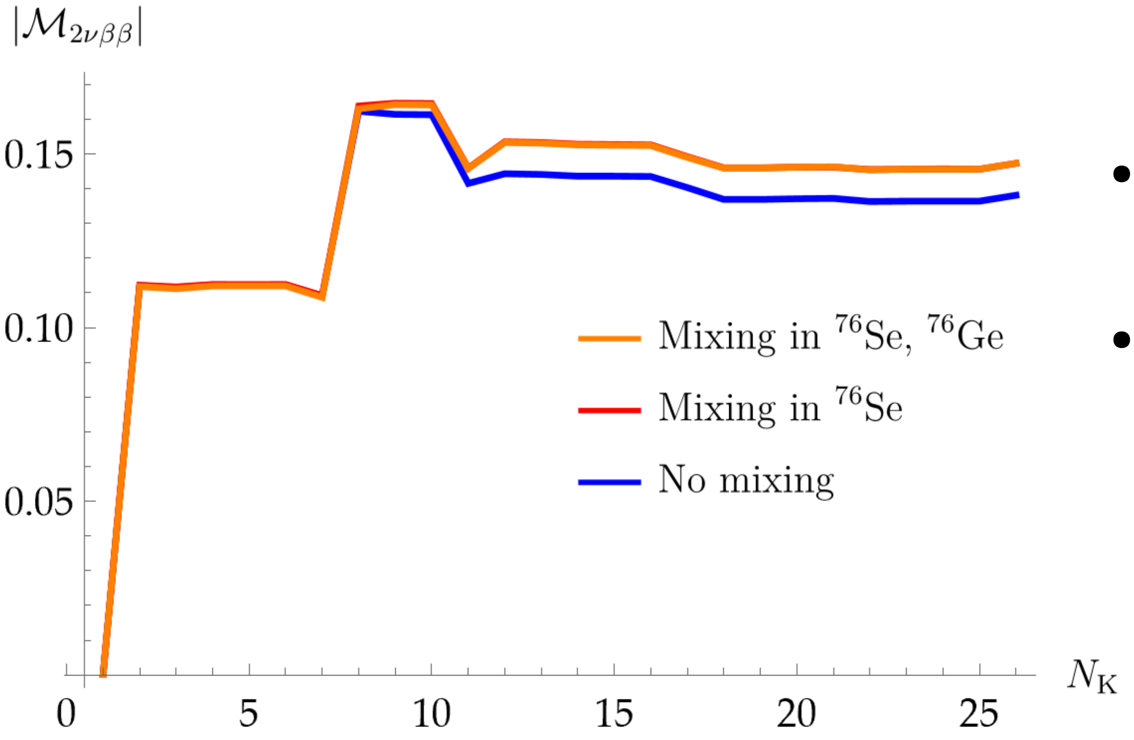



- Doubly magical
- The only spherical nucleus in the ***2νββ*** set
- Single ***β*** to $4^+$, $5^+$, $6^+$ possible (orange), but suppressed by ***2νββ*** (green)

# 48Ca Ikeda sum rule



# Preliminary estimation for 76Ge




Summing contributions from 76As configurations up to saturation. $\Delta E_1(I^\pi = 1^+) = 100$keV (normalization).

- High triaxiality (γ~27-30°) in each nucleus
- Single-state dominance scenario