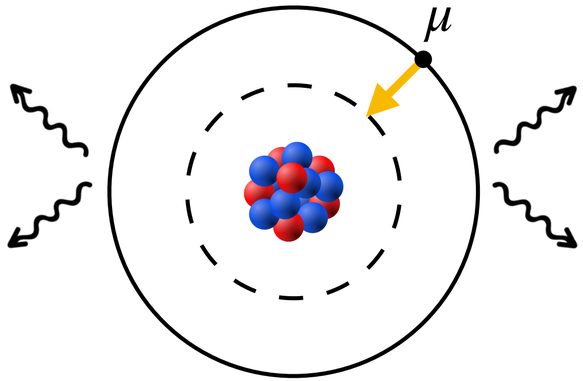
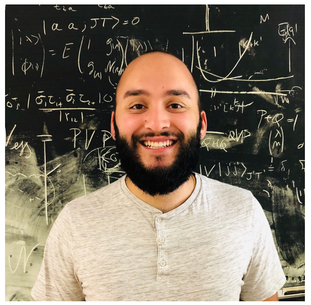
Extracting charge radii from precision spectroscopy

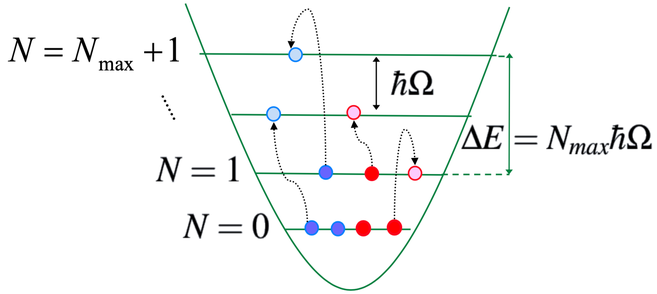
Ab initio calculations of polarizability of $^{6}\mathrm{Li}$ and $^{7}\mathrm{Li}$ atoms



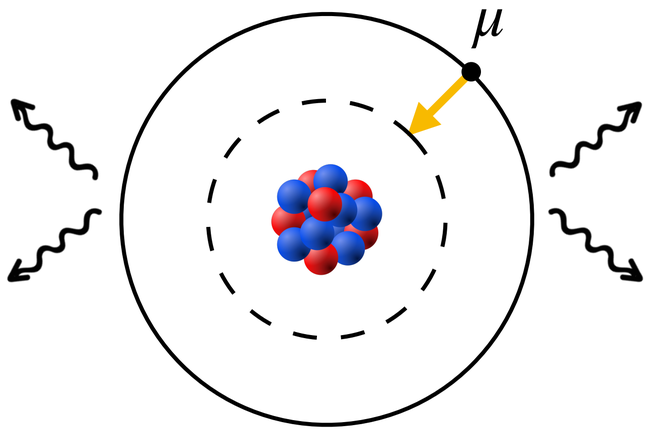


Michael Gennari

Petr Navratil



Muonic atoms as a precision probe



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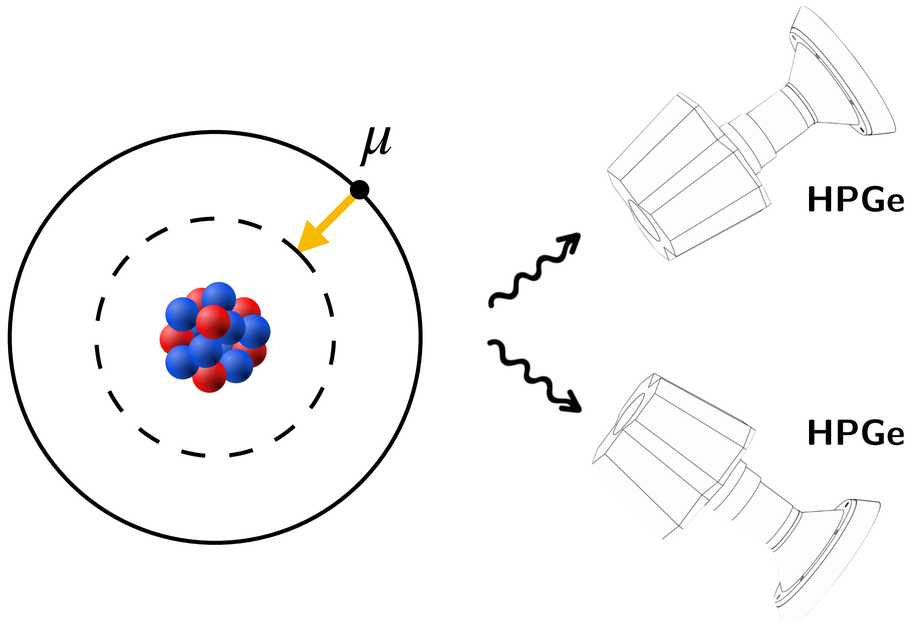

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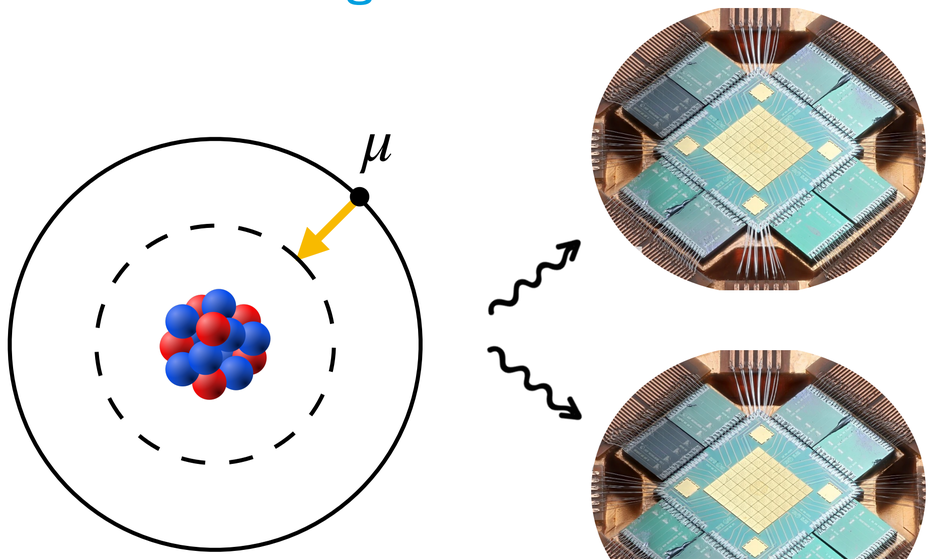

[Unger et al. J. Low Temp. Phys. (2024)]

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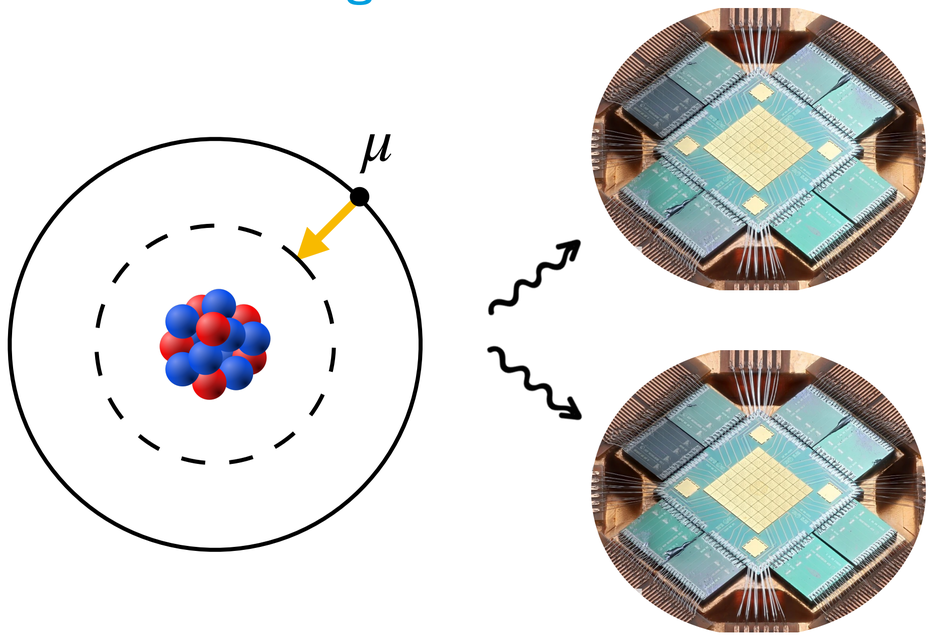

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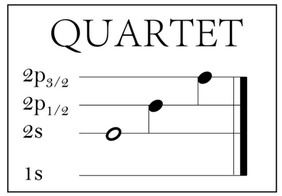

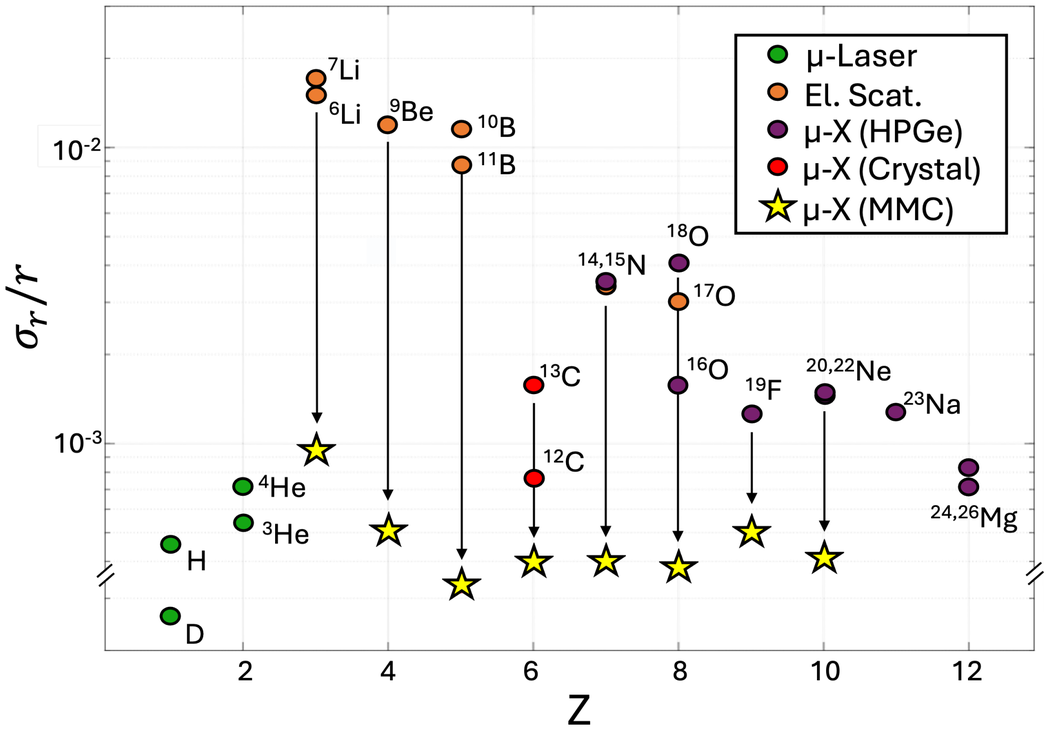

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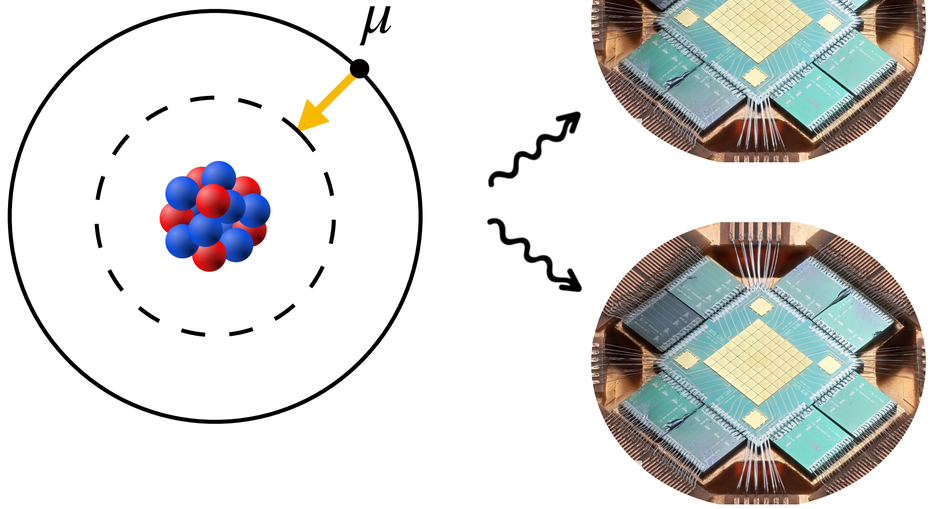

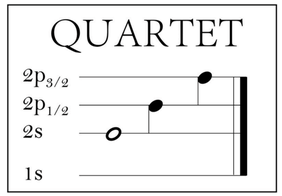

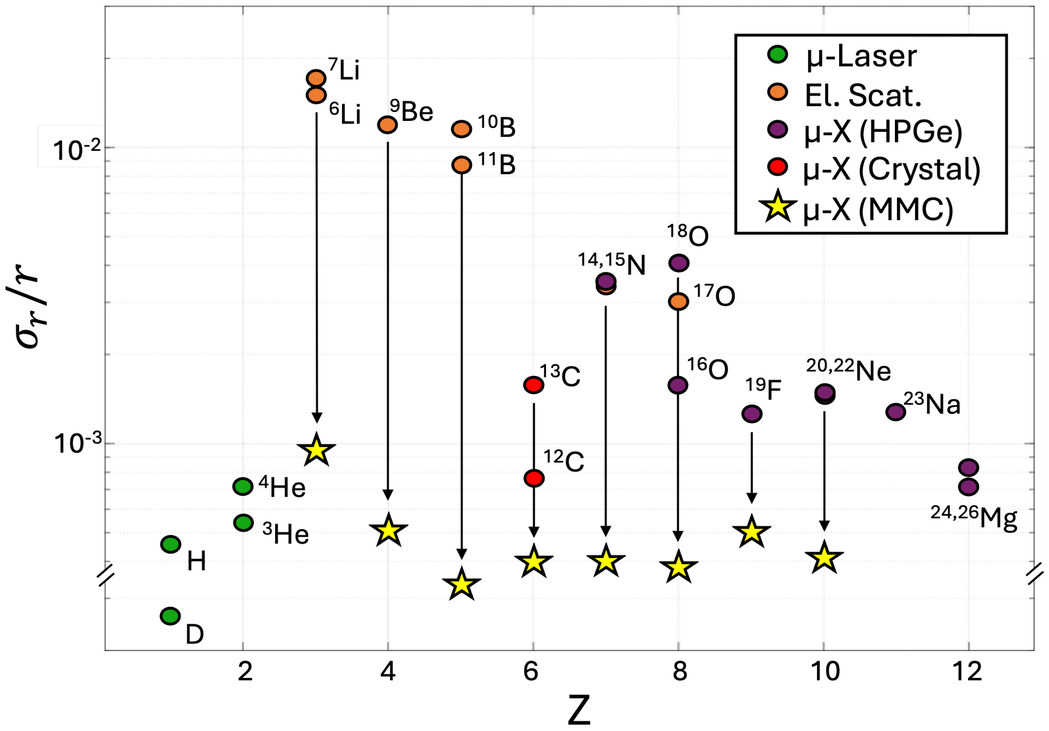

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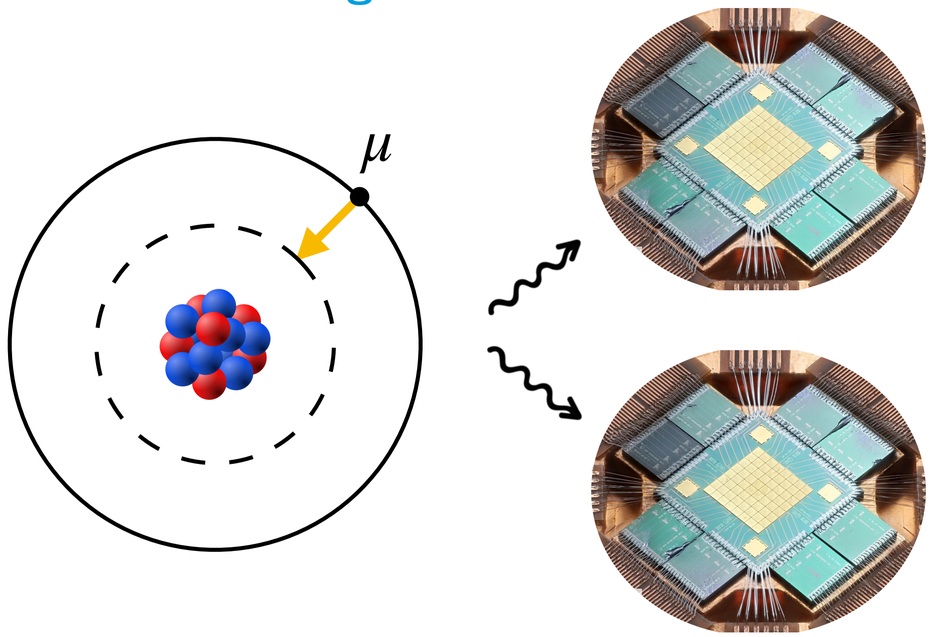

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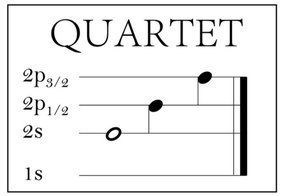

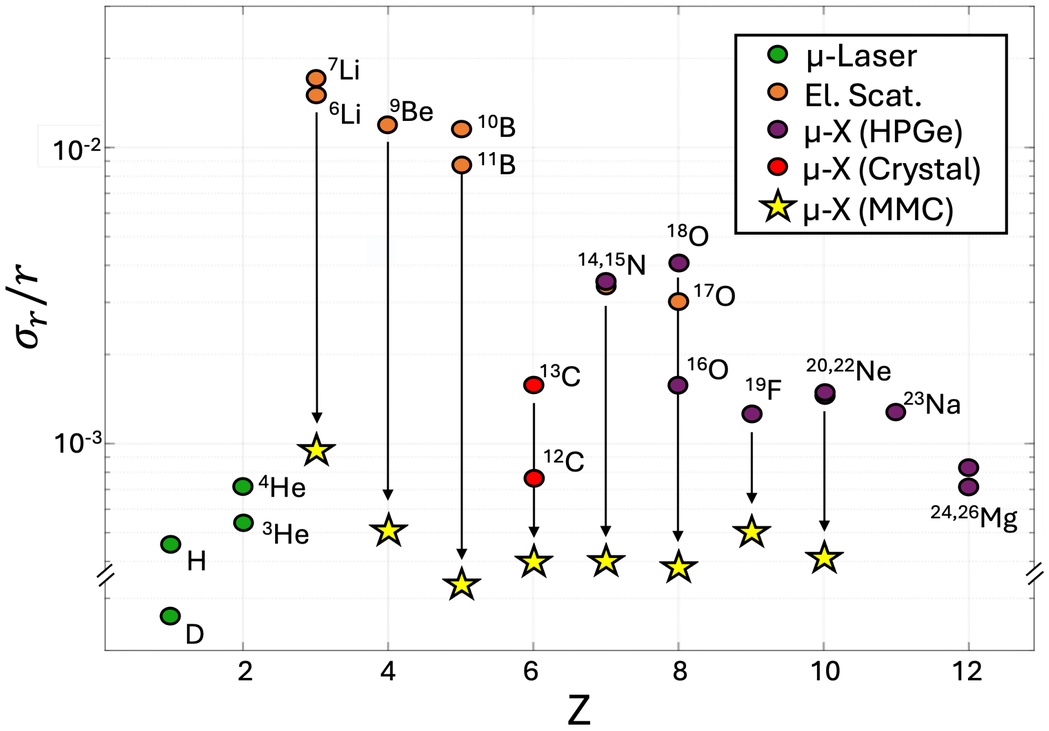

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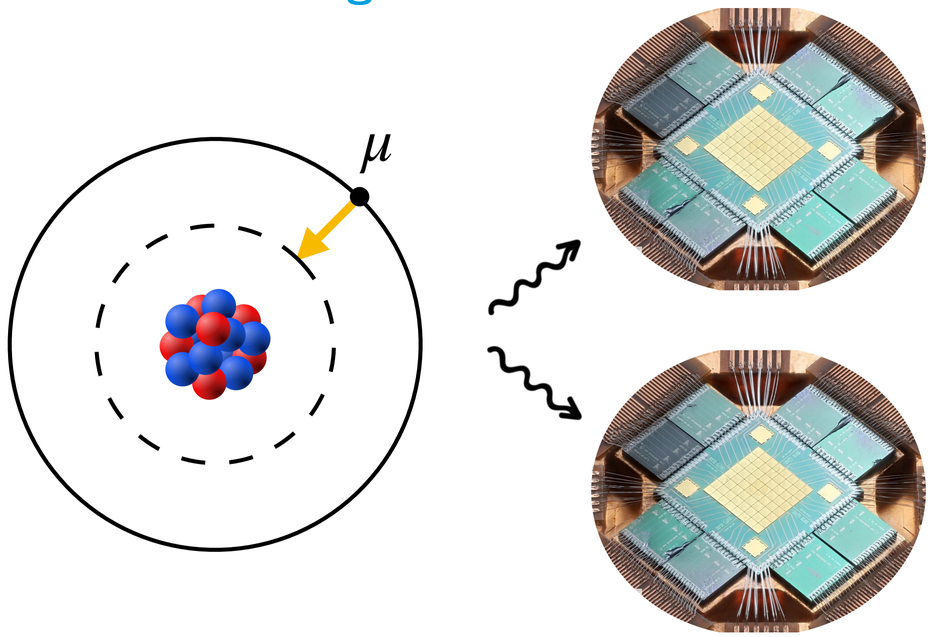

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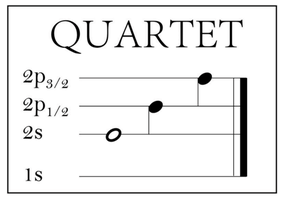

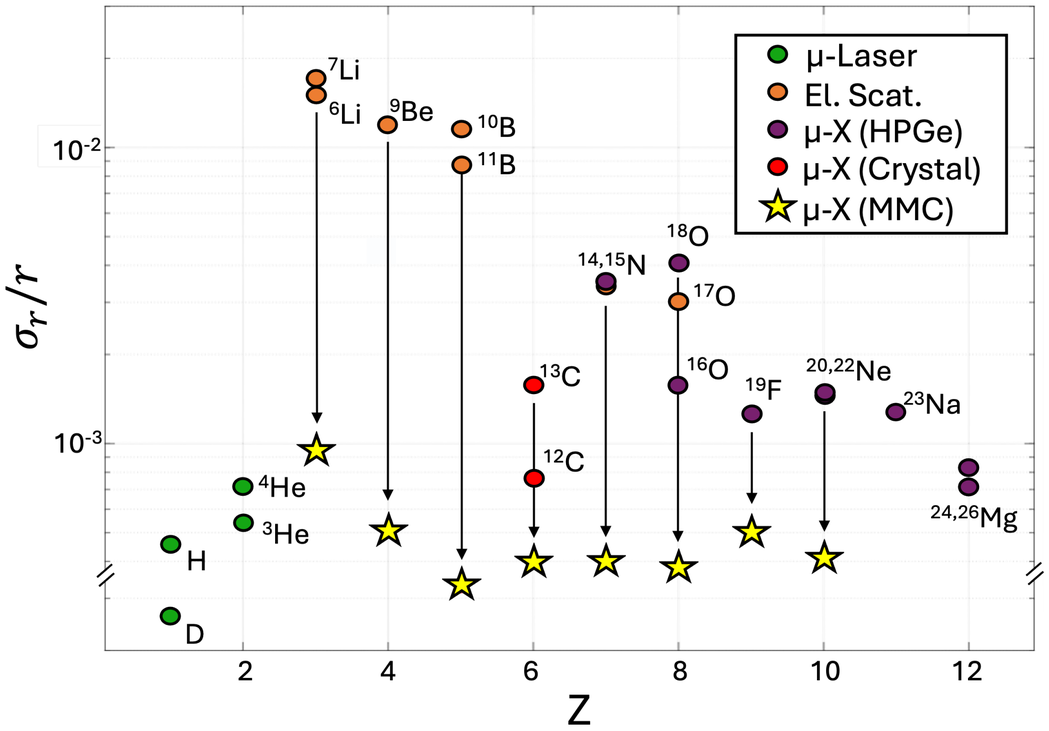

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Beyond Nuclear: δ_C estimation for $|V_{ud}|$ extraction

[V. Katyal et al. PRA (2025)] [C.-Y. Seng, M. Gorchtein, PLB (2023)]
[B. Ohayon, Atomic Data and Nuclear Data Tables (2025)]

The two-photon exchange nuclear correction

Radius extraction master formula

$$\delta_{\mathrm{E}} = \delta_{\mathrm{QED}} + \mathscr{C}\ r_c^2 + \delta_{\mathrm{NS}}$$

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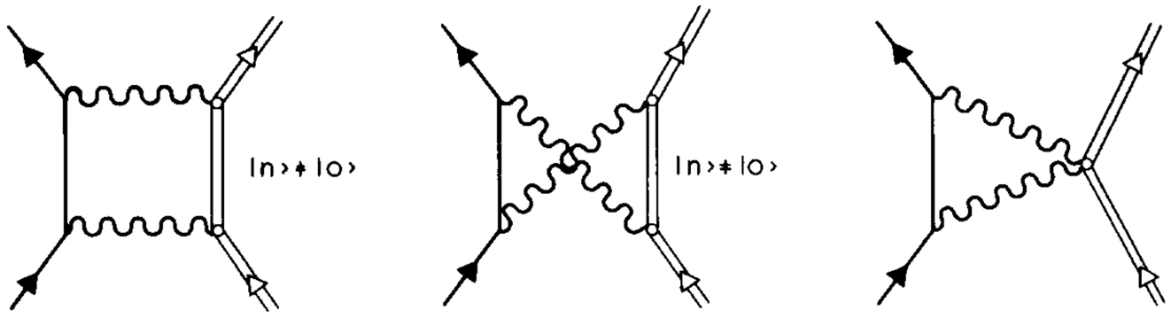
Two-photon polarizability contributions



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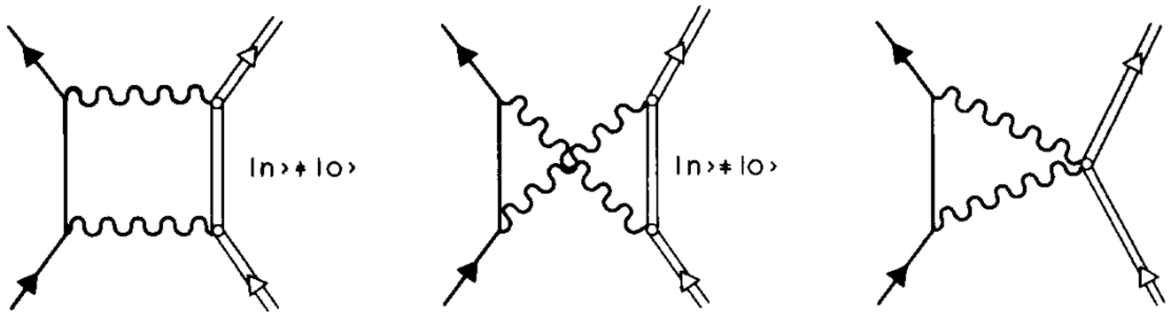

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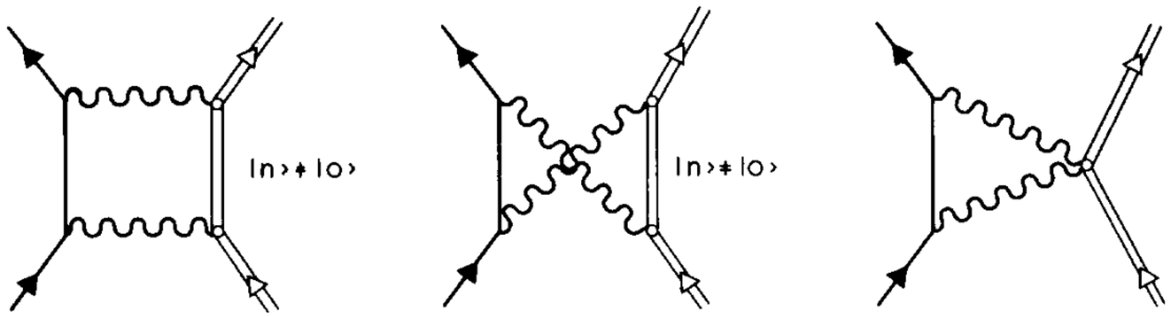

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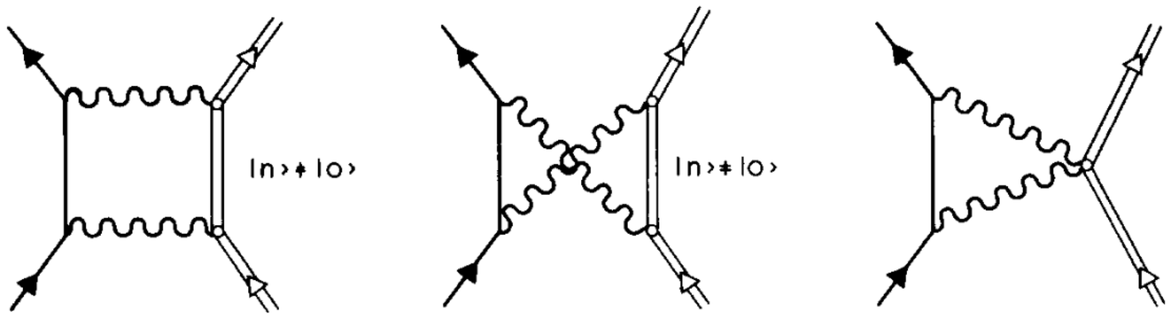

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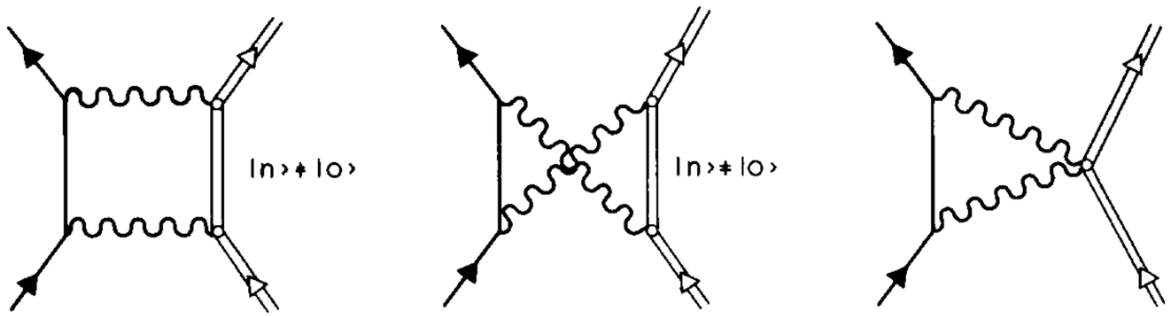

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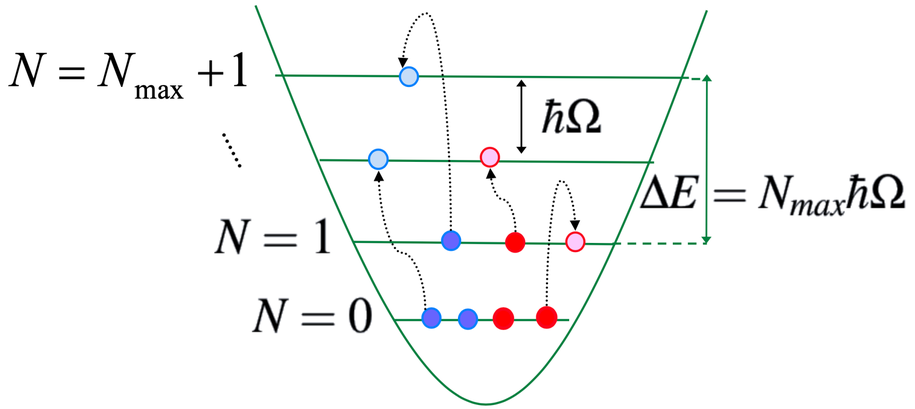
Ab initio nuclear corrections



Nuclear physics modelling

Model used for nuclear currents

- Electromagnetic current modelling
  - General one-body current for point-like particles
  - Nucleon charge form factor $F(q^2) = \left(1 + q^2/\Lambda^2\right)^{-2}$
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  - Harmonic oscillator Slater determinant
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  - N4LO-E7 and N3LO

  ➡ **Estimate interaction uncertainty**

- Model space
  - Harmonic oscillator Slater determinant
  - Vary many-body basis: $(\Omega, N_{\max})$

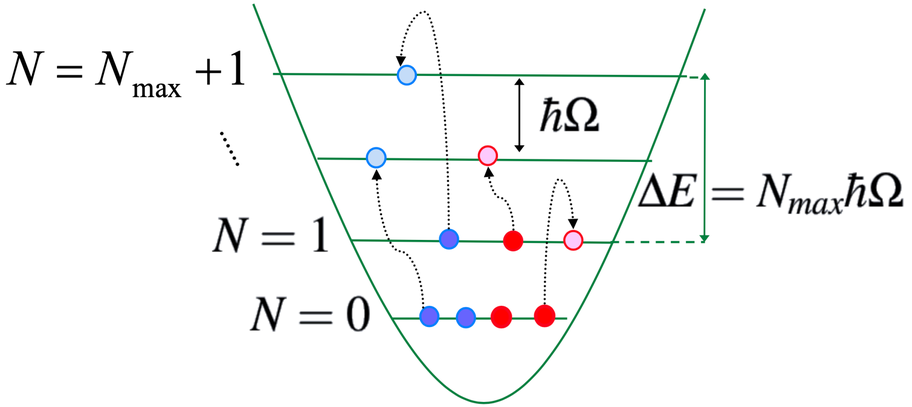
  ➡ **Estimate model space truncation uncertainty**

- Many-body approximation
  - No-Core Shell Model
  - More details in next section

  ➡ **Negligible many-body approximation uncertainty**

Ab initio No-Core Shell Model

Anti-symmetrized products of many-body HO states



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Lanczos tridiagonalization algorithm [Lanczos (1950)]

- Initialization: normalized pivot $|\phi_1\rangle$
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- Output:
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H in Lanczos basis

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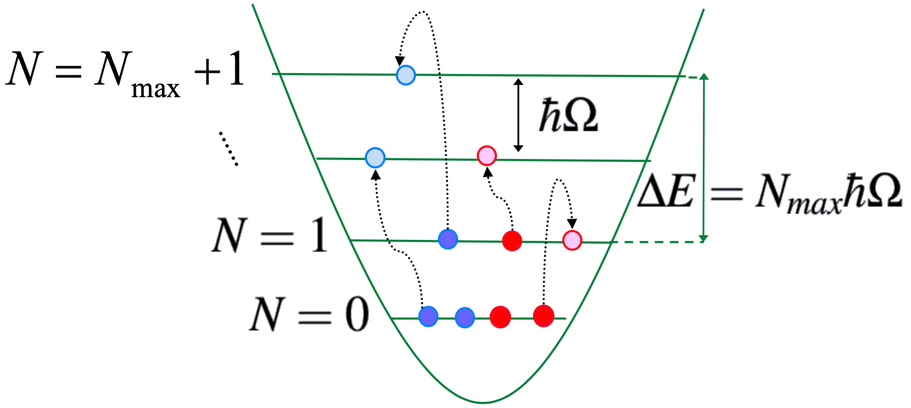

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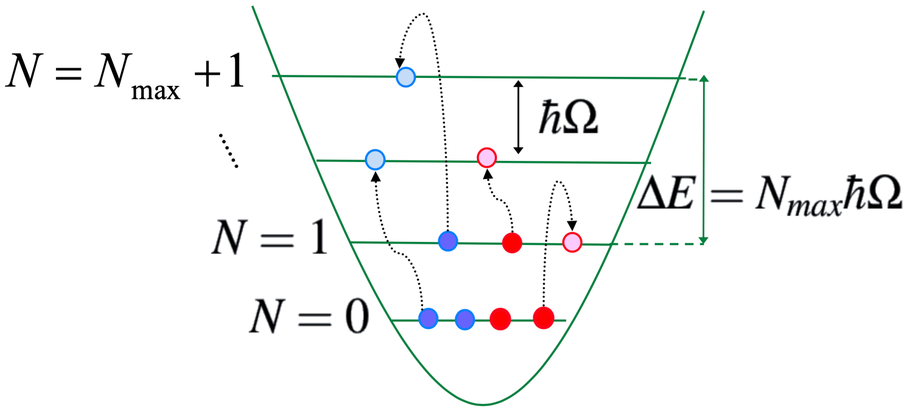

Application to nuclear structure

- Efficient calculation of low-lying spectra
  - Selection rules ⇒ **Fast matrix-vector multiplication**
  - In practice: $N_L \sim 100 - 200$ is sufficient
- Application to $^{6-7}$Li
  - $N_L = 200$ for $N_{\max} = 1$ to 9
  - Ground-state of $^{6-7}$Li $|\Psi\rangle$ ⇒ **Starting point for** δ^A_{pol}

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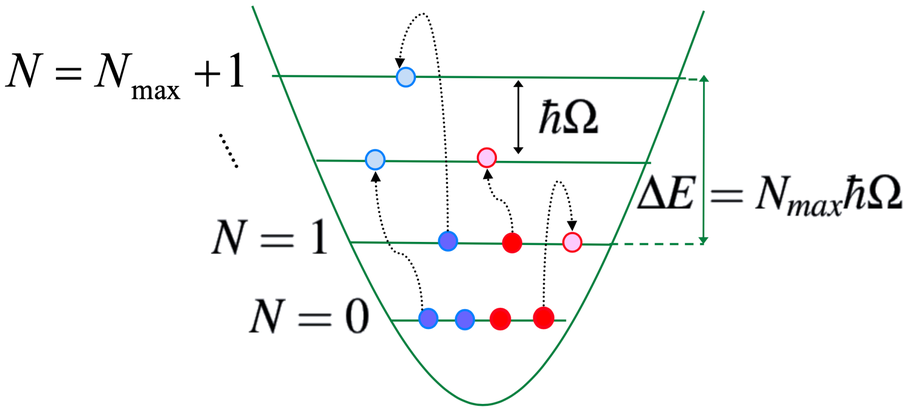

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Lanczos-strength algorithm for δ^A_{pol}

- Convergence problem
  - Often the **strength of** $S_O(\omega)$ **is fragmented**
  - And only low-lying states converged
- Lanczos strength algorithm
  - Pivot based on 1st Lanczos output: $|\phi'_1\rangle = O|\Psi\rangle$
  - Recovers **exactly** $\int d\omega\ \omega^n\ S_O(\omega)$ for $n \leq 2N_L - 1$

➡ **One additional NCSM run per operator**

Numerical results for $^{6-7}\mathrm{Li}$ isotopes

A first test case for N4LO-E7 and $N_{\max} = 7$

Numerical calculations

- $q_{\max} = 700$ MeV and $\Delta q = 10$ MeV
- 10 different operators for $J_{\max} = 3$

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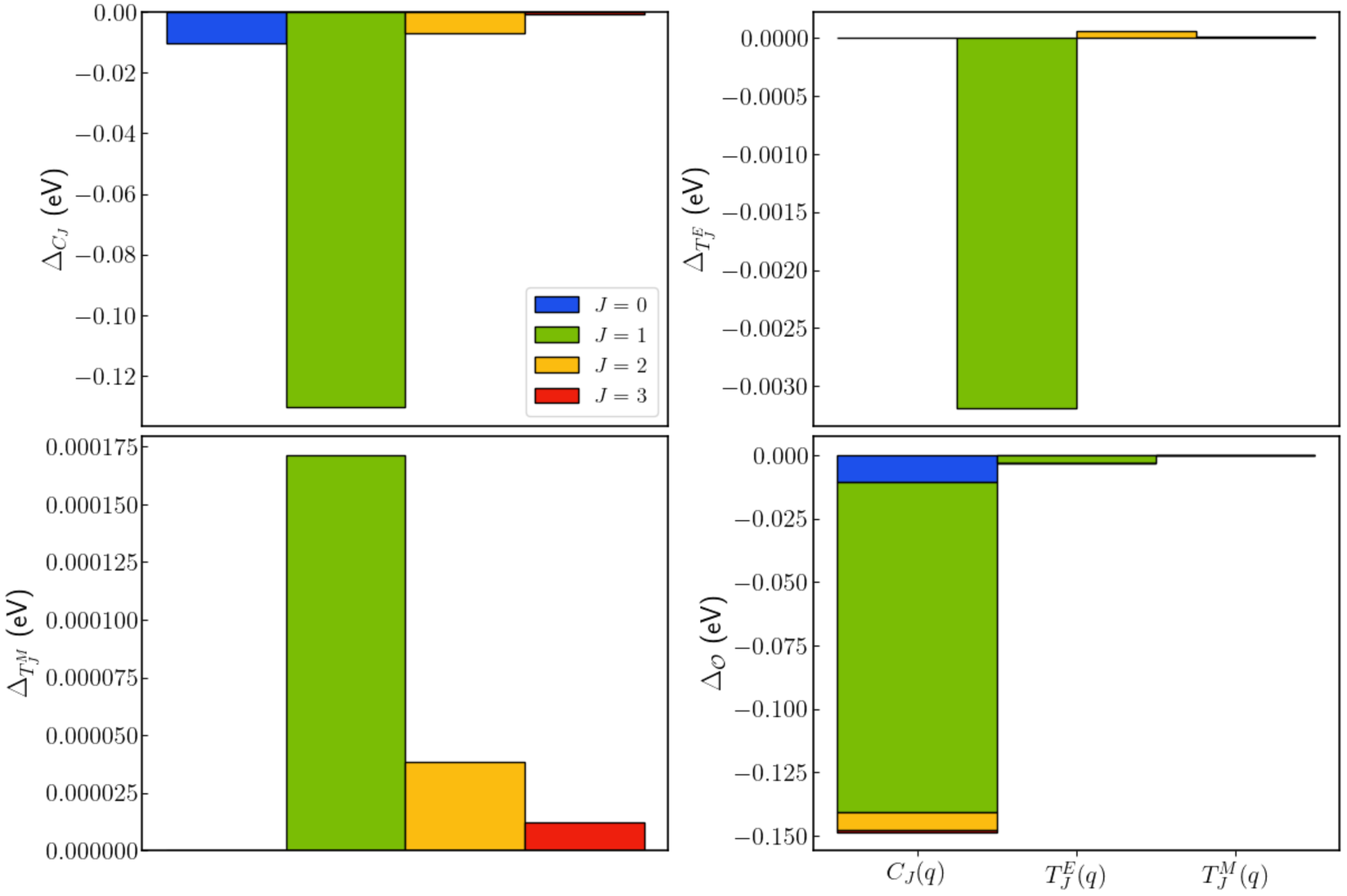

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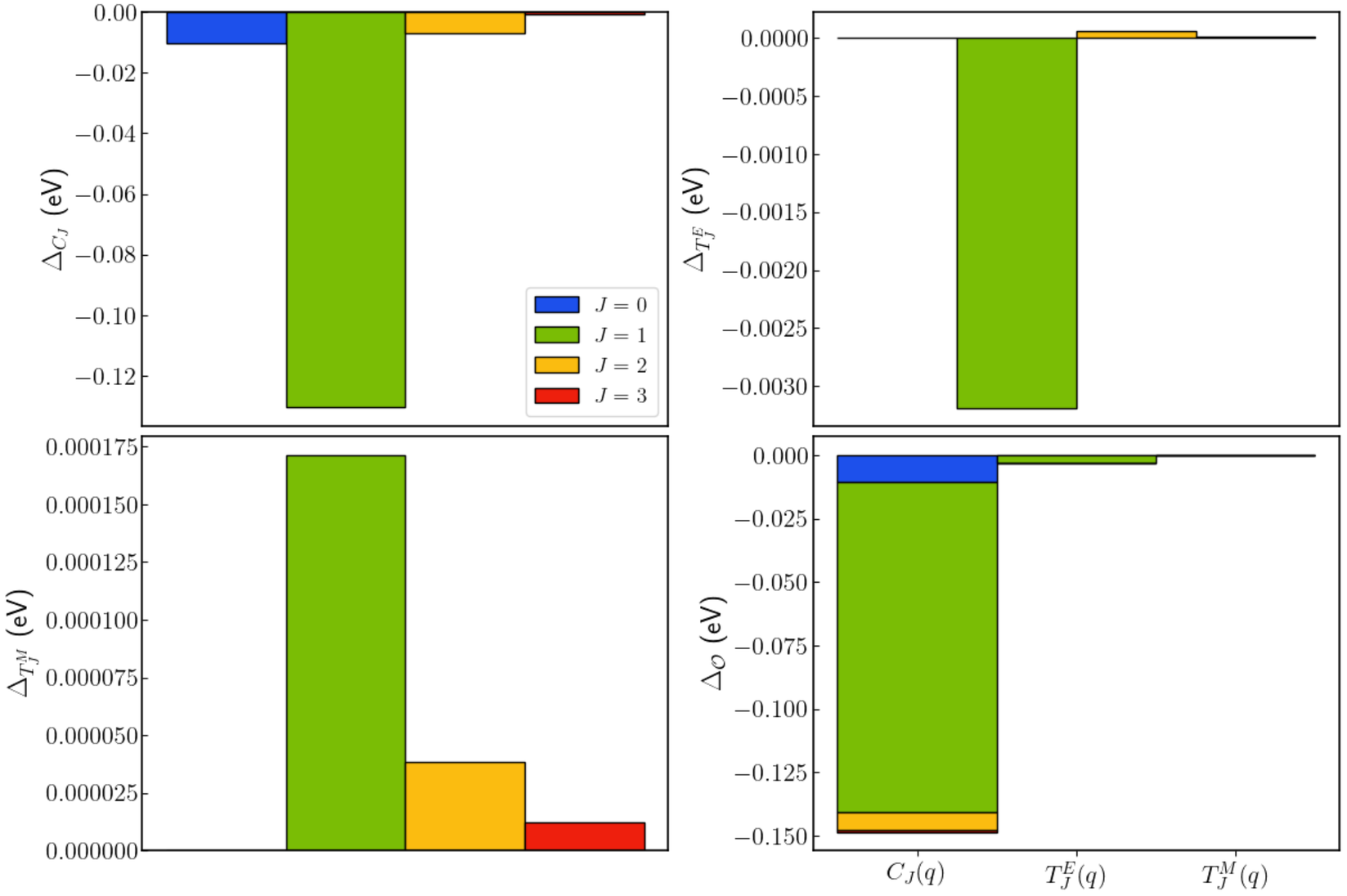

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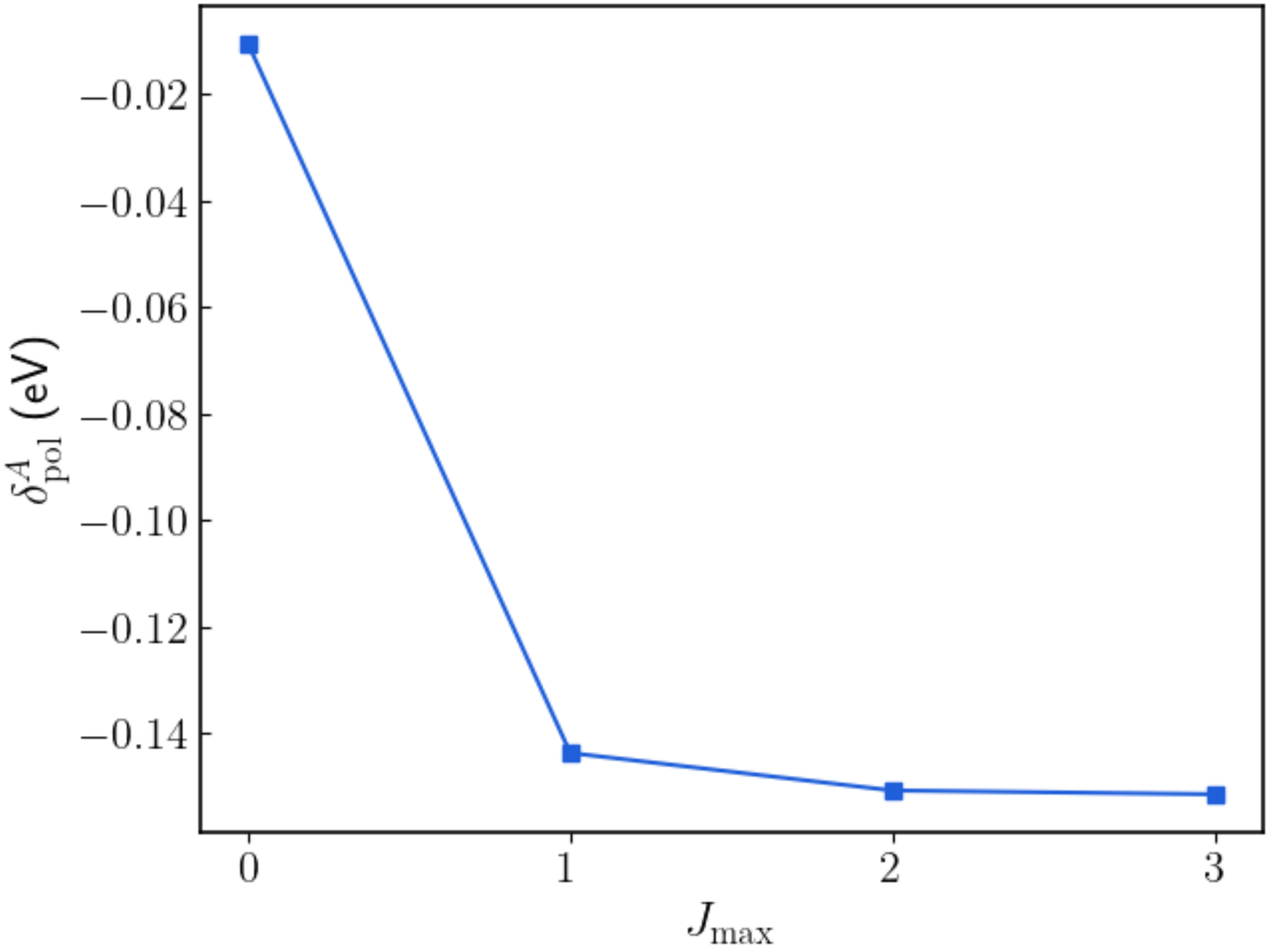
Observations

- Contribution repartitions
  - Well-known **dipole** dominance
  - **Charge** contributions are dominant
- Negligible contributions
  - TM is negligible for any J
  - TE is relevant only for $J = 1$

➡ **Only half the operators are relevant**

Checking convergence in $J_{\max}$



Results

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- All other cases are similar

➡ **Fast exponential convergence**

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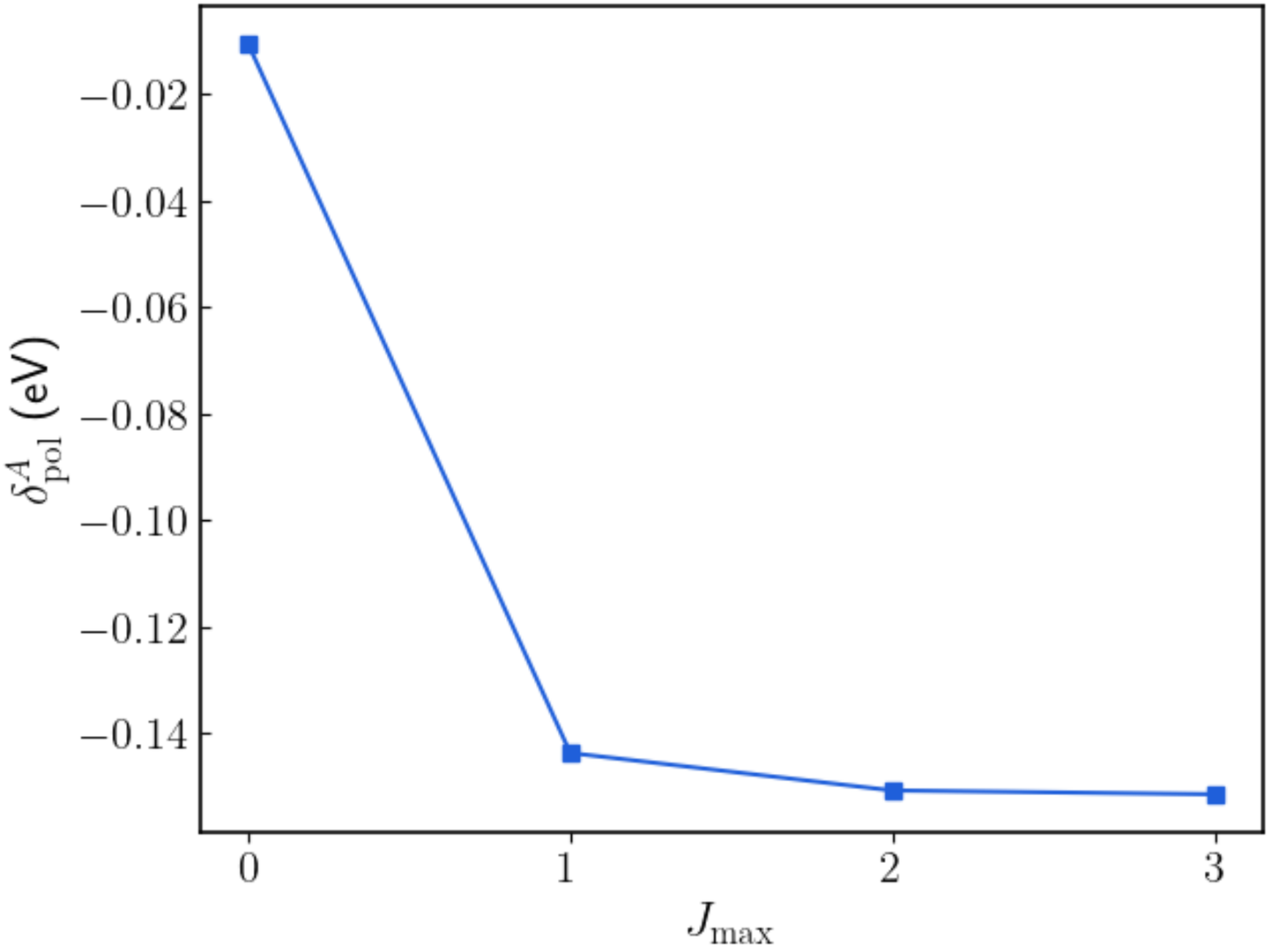

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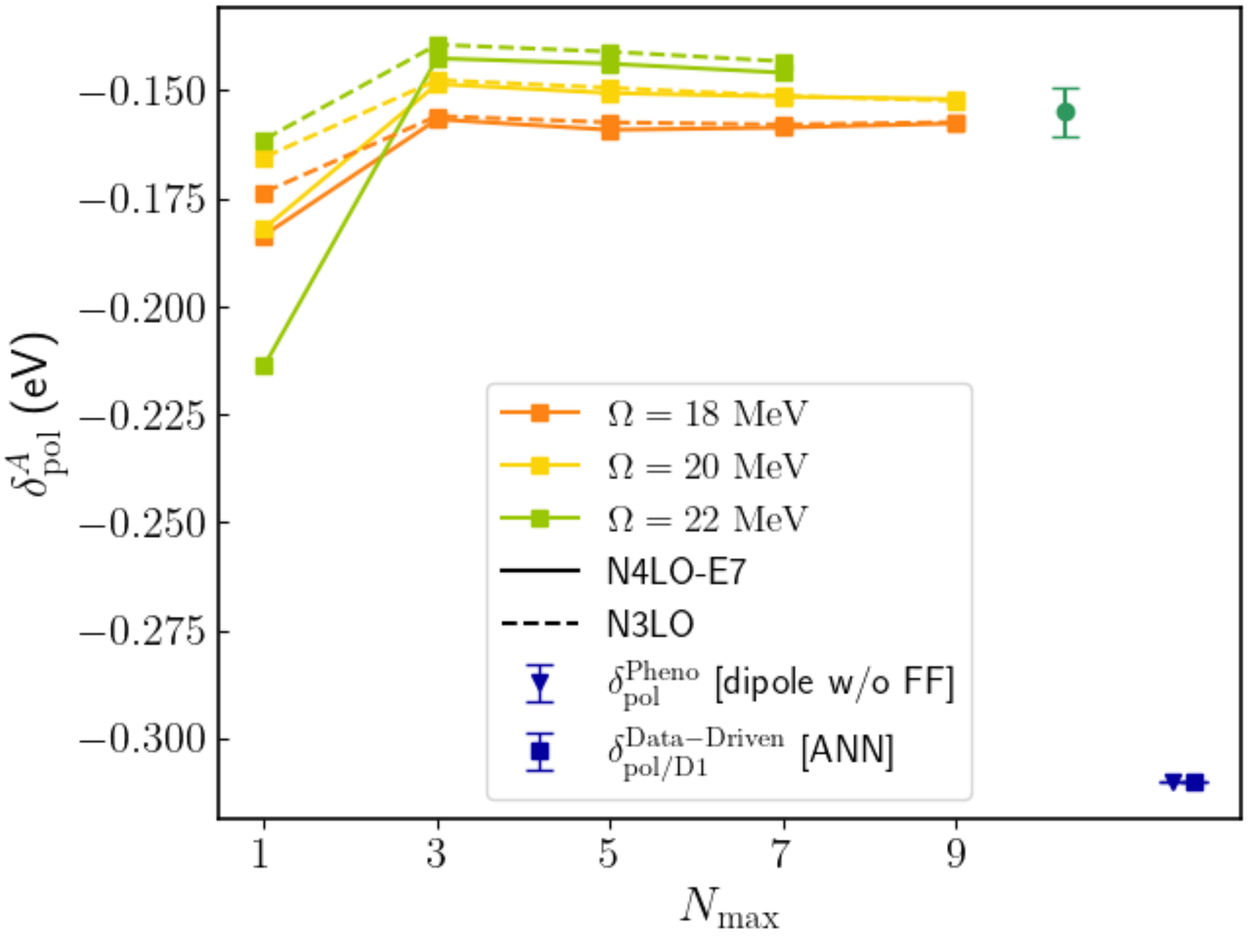
$$\epsilon_{J_{\max}} \lesssim 1 - 10 \text{ meV}$$

Multipole truncation ⇒ Negligible uncertainty

Dependence on $(\Omega, N_{\max})$ and the interaction



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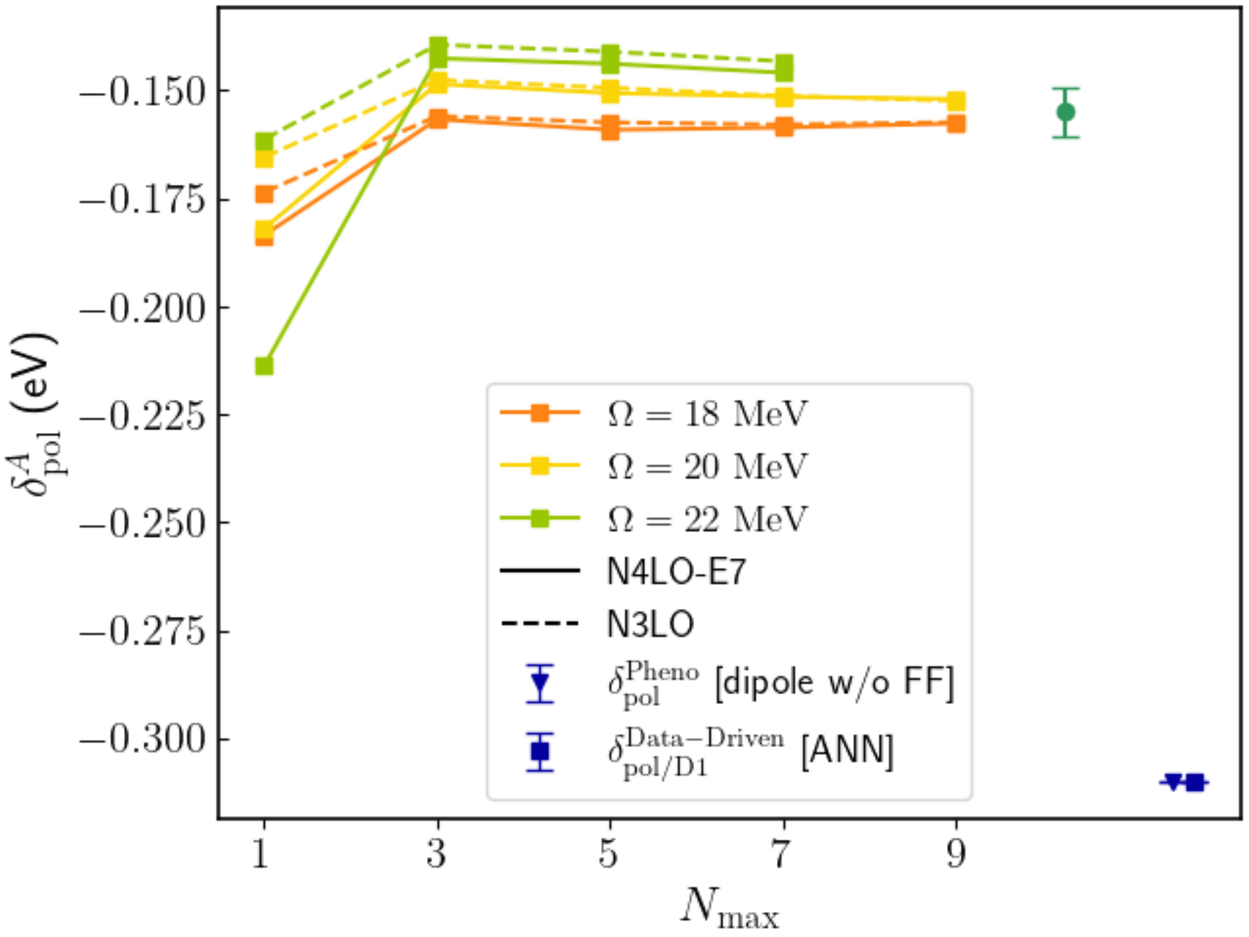

Preliminary numerical results

- Model-space dependence
  - Optimal frequency around 20 MeV
  - Run calculations for $\Omega = 18,\ 20,\ 22$ MeV
  - Truncations for $N_{\max} = 1 - 9$
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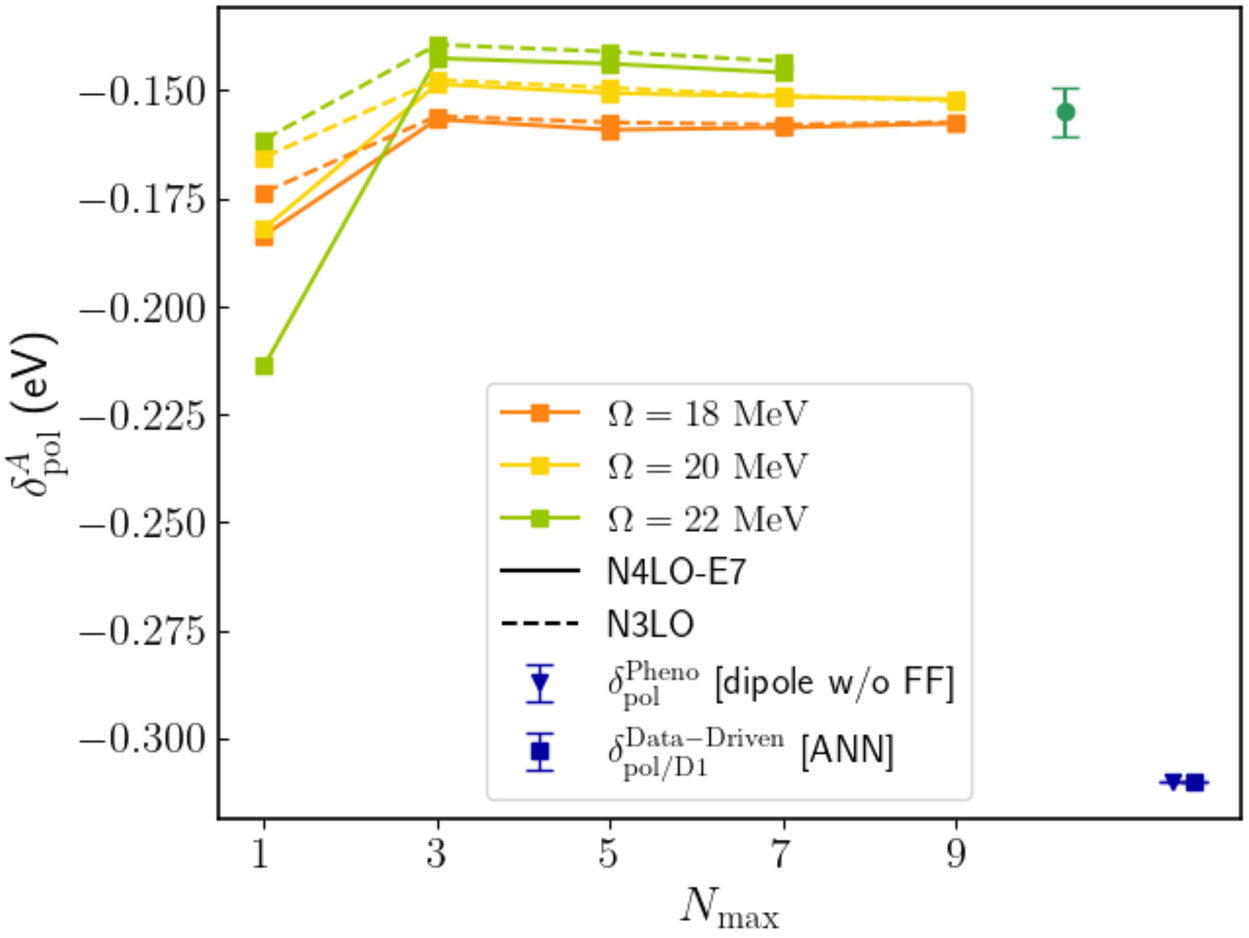

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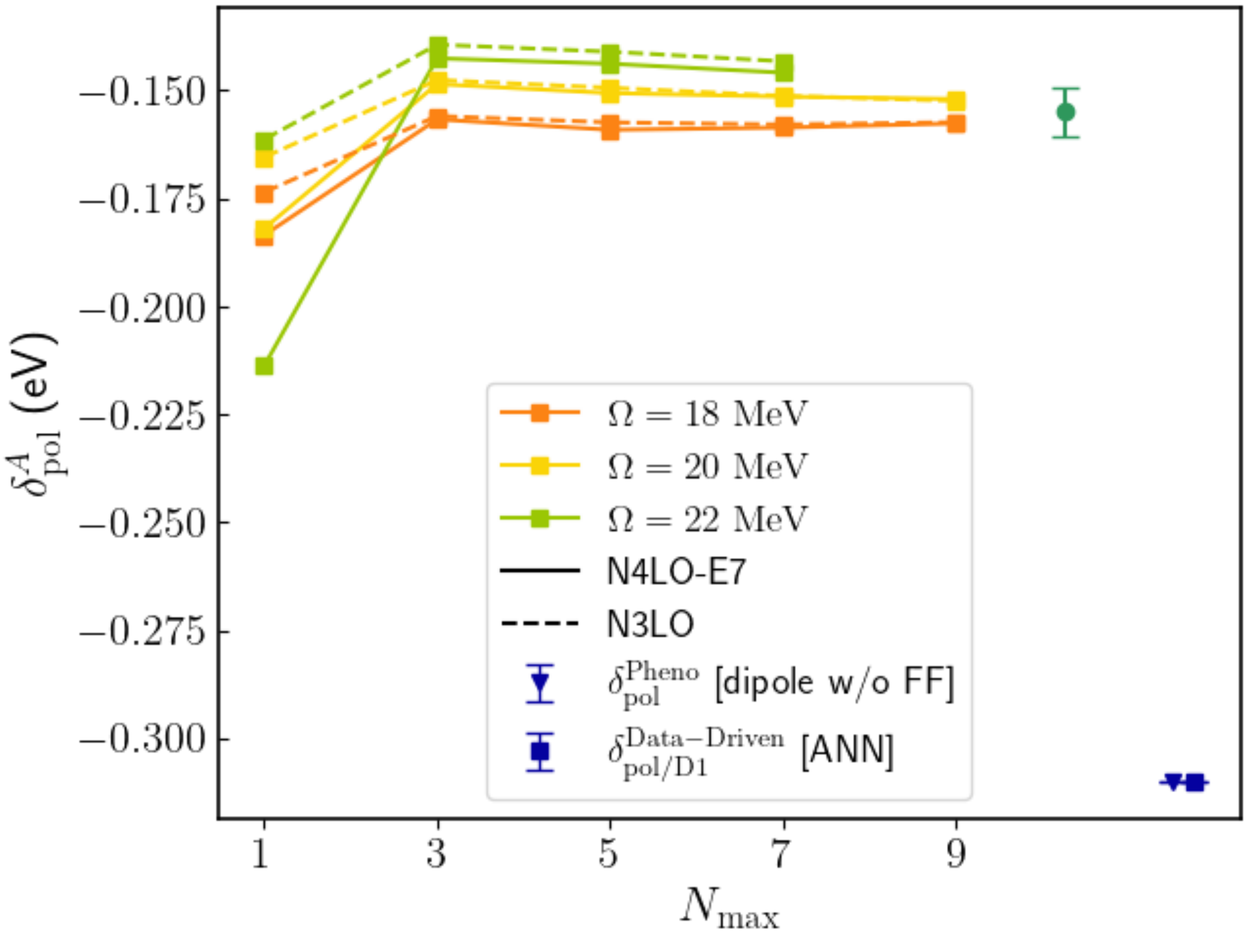

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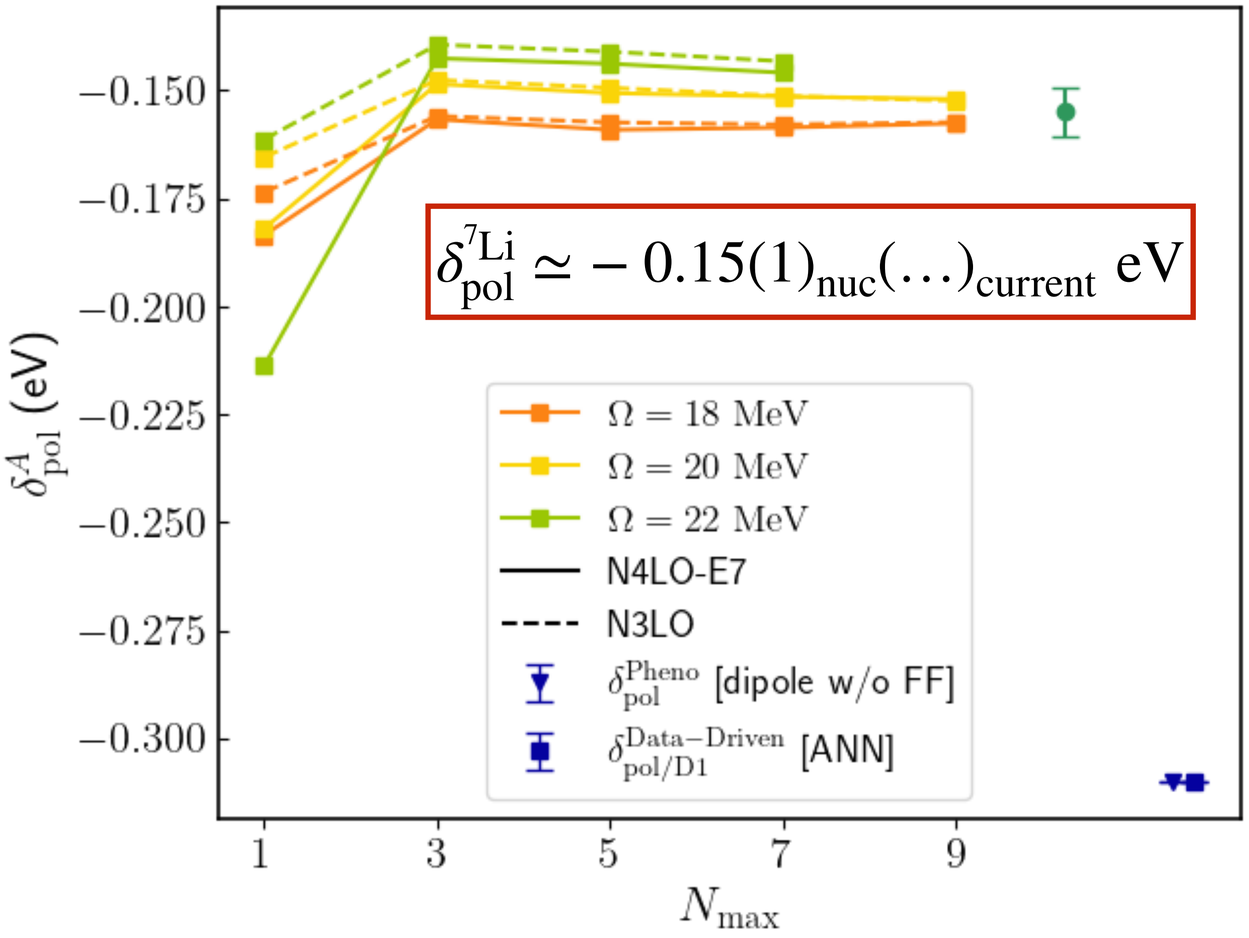

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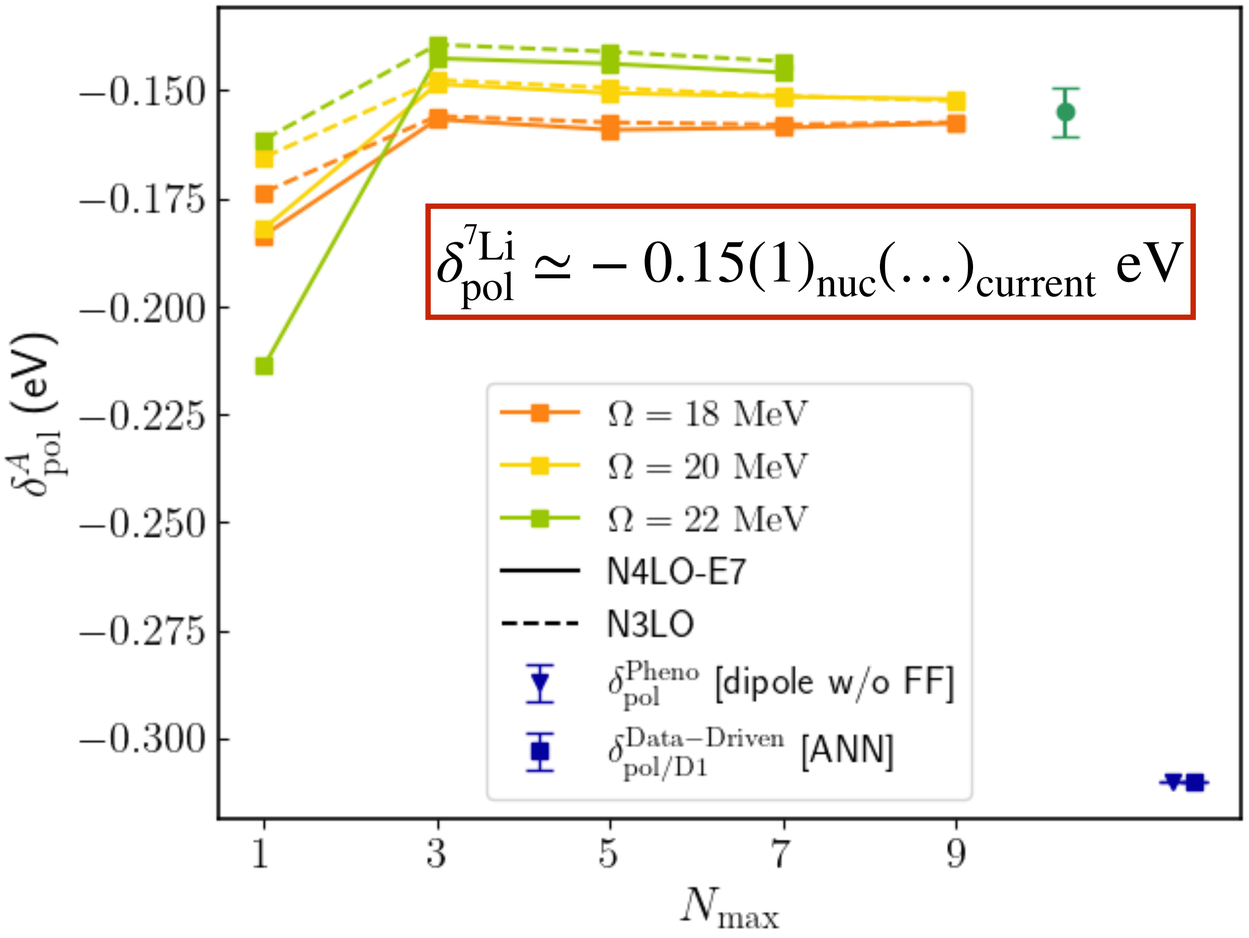

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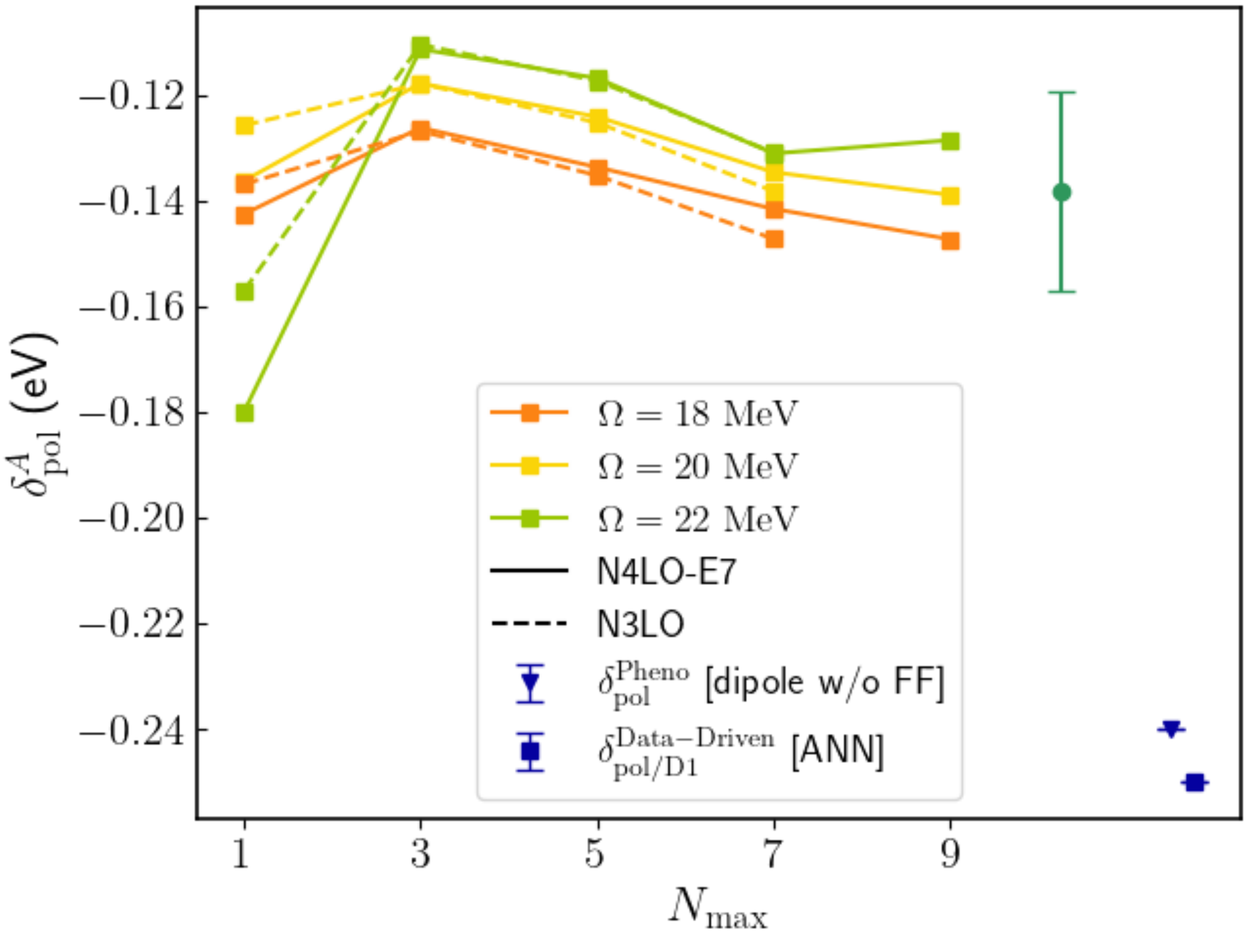
0.05 eV precision seems reachable for nuclear structure corrections!

Model-space dependence for $^{6}\mathrm{Li}$



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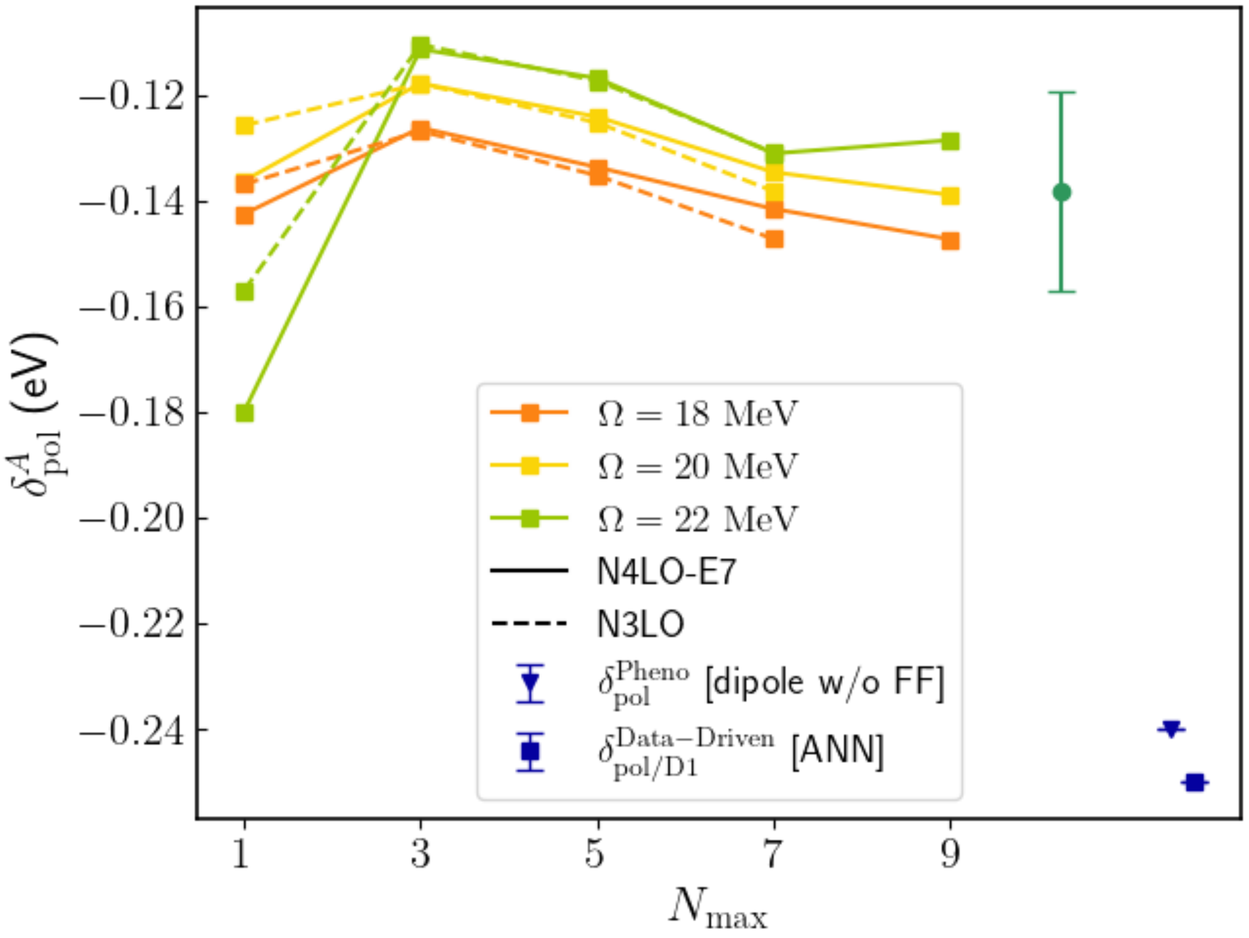

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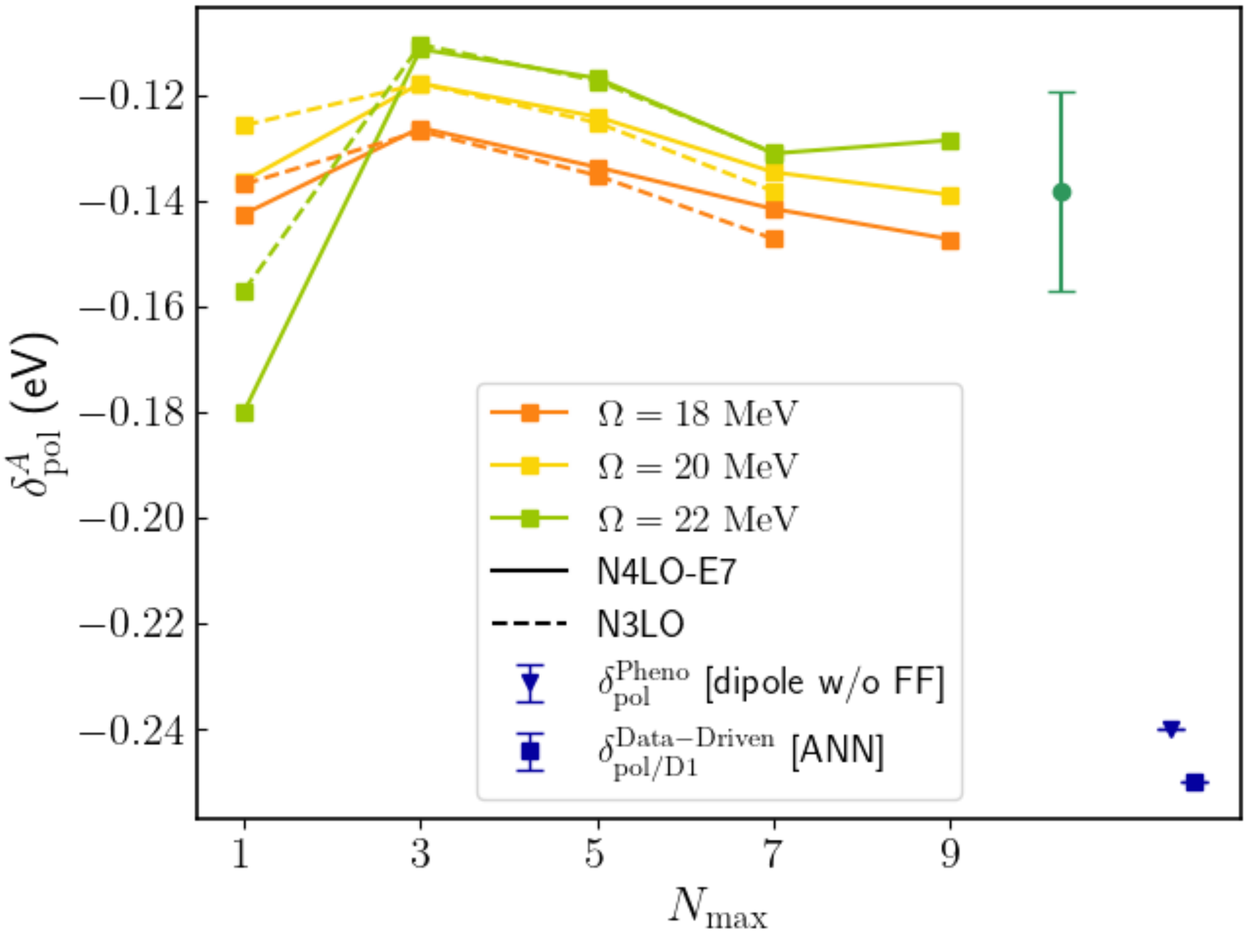

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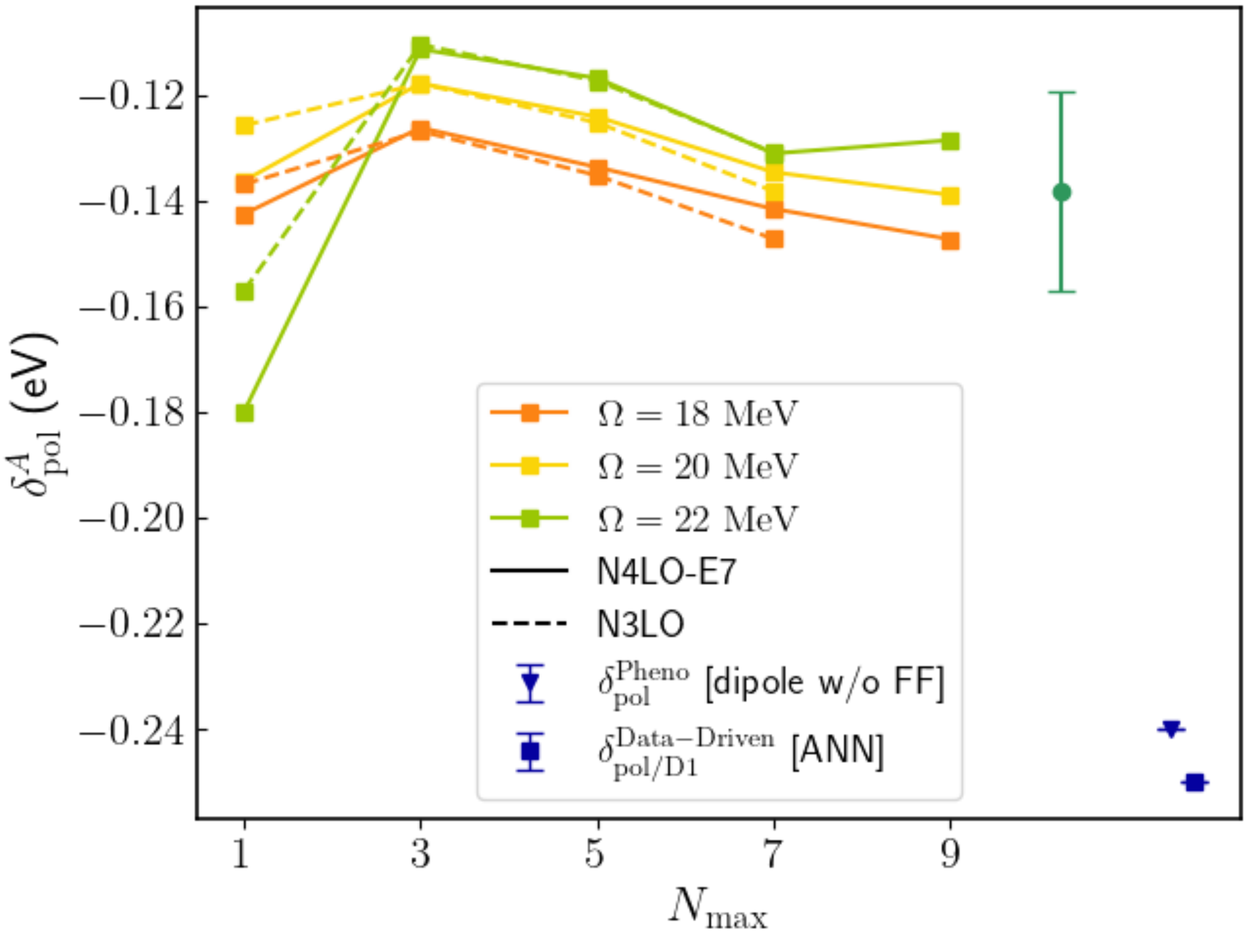

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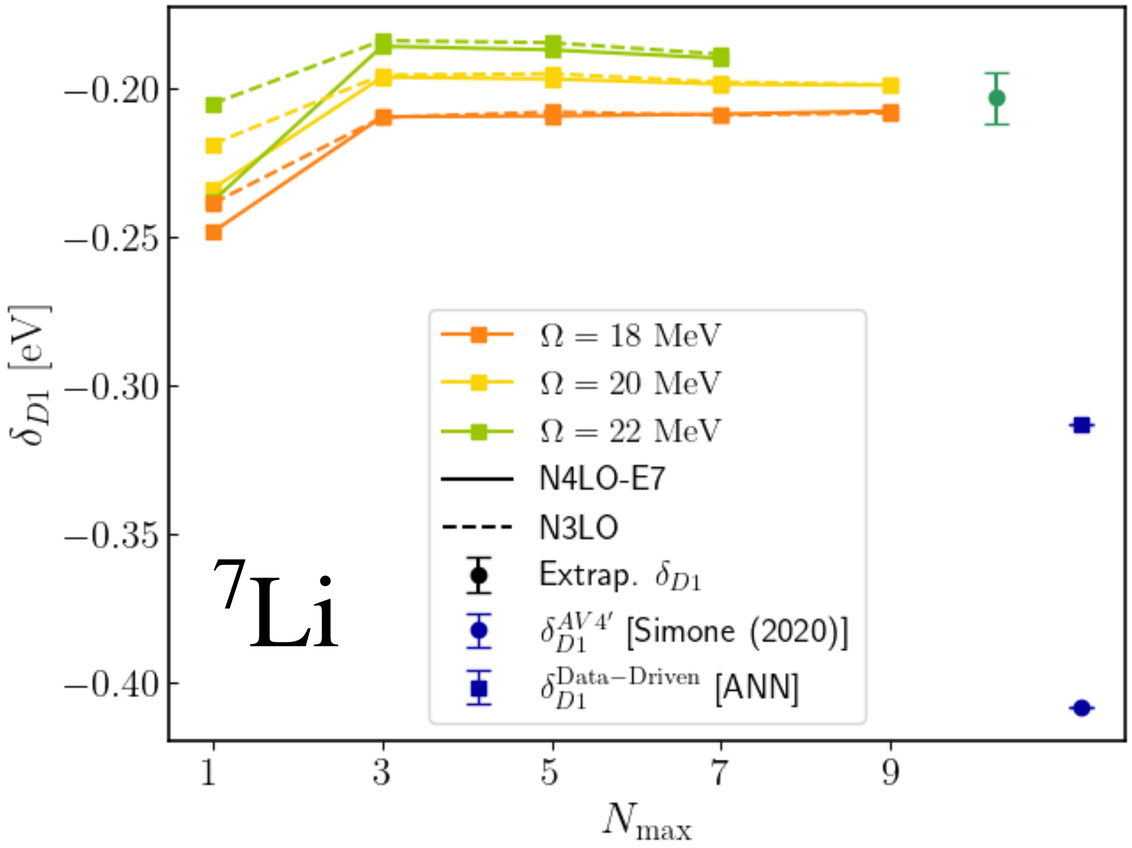
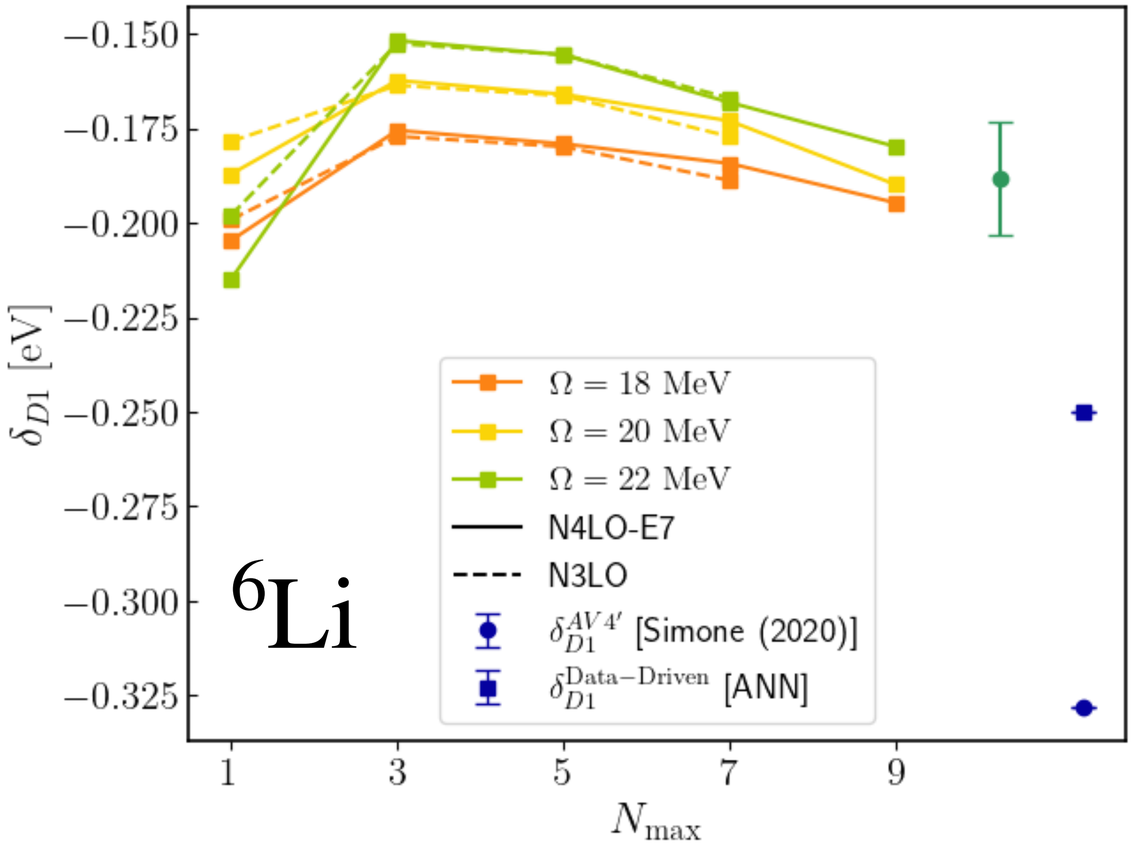
On-going results to be finalized : $\delta^{^6{\rm Li}}_{\rm pol} \sim -0.14\ (\ldots)_{\rm nuc}(\ldots)_{\rm current}$ eV

Benchmarking with pheno estimations





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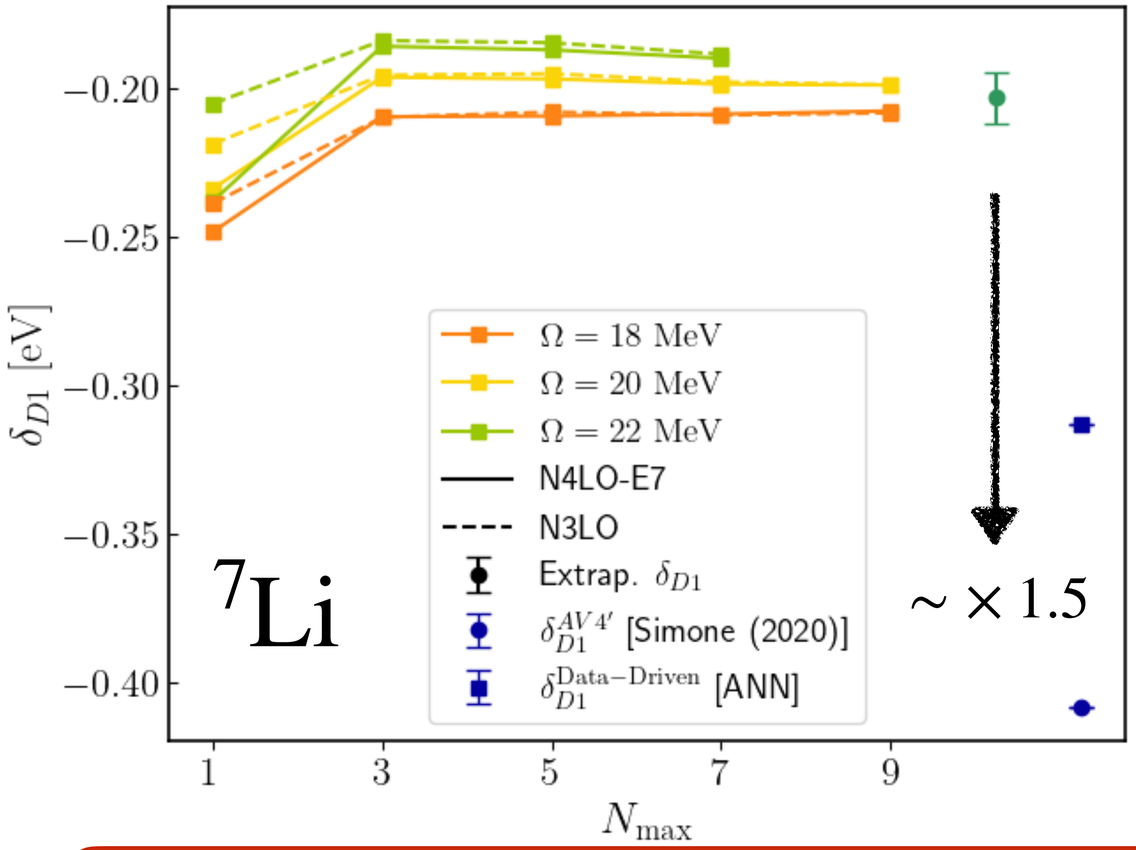

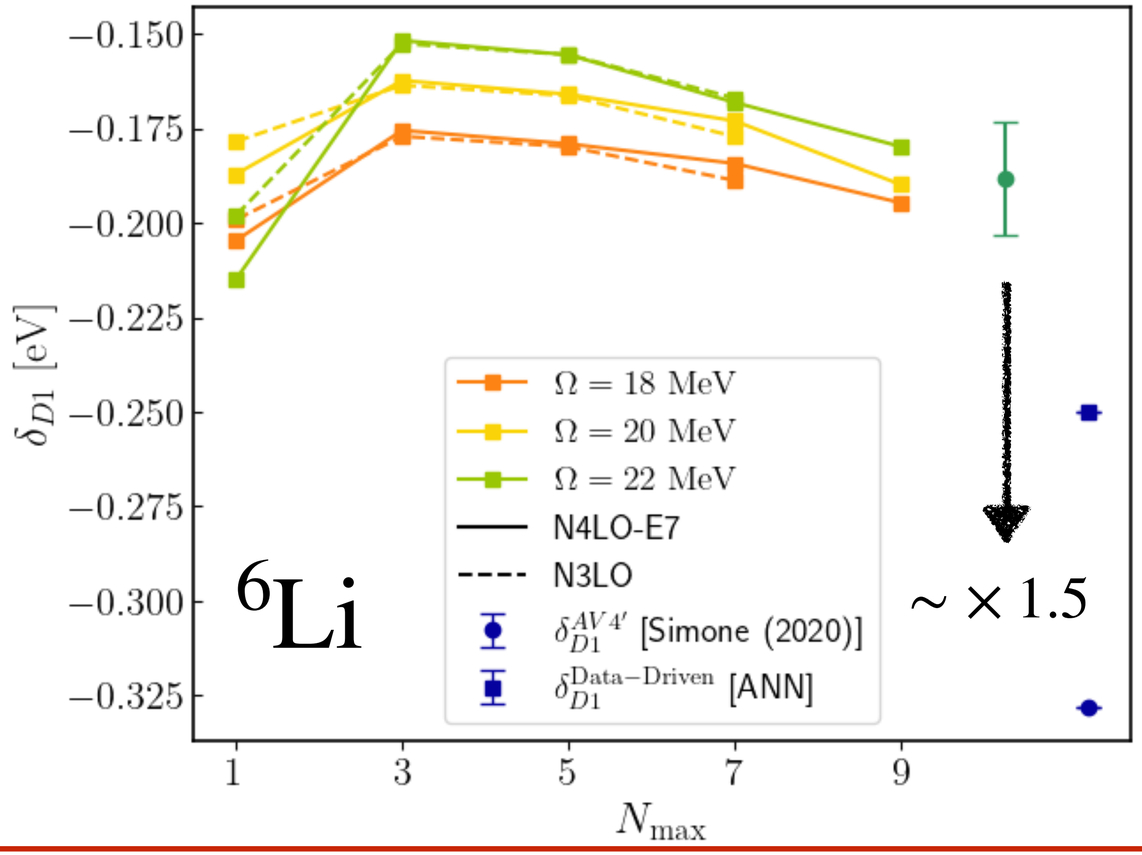

On-going investigation of discrepancy: *might* come from non-trivial (M_i, M) dependence

$$S_X(\omega, q) \equiv \sum_{J\geq 0} \sum_{N\neq 0} |\langle N | O_{X,J}(q) | \Psi \rangle|^2 \delta(E_N - E_0 - \omega)$$

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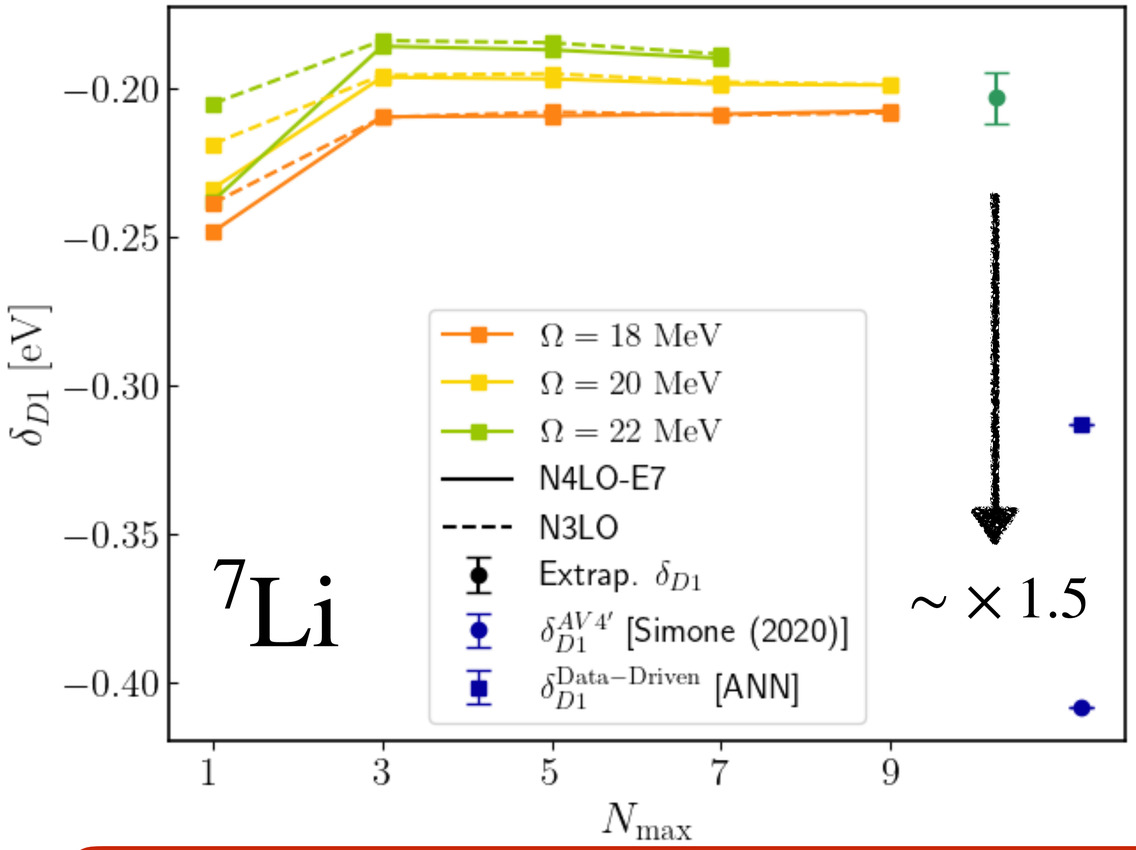

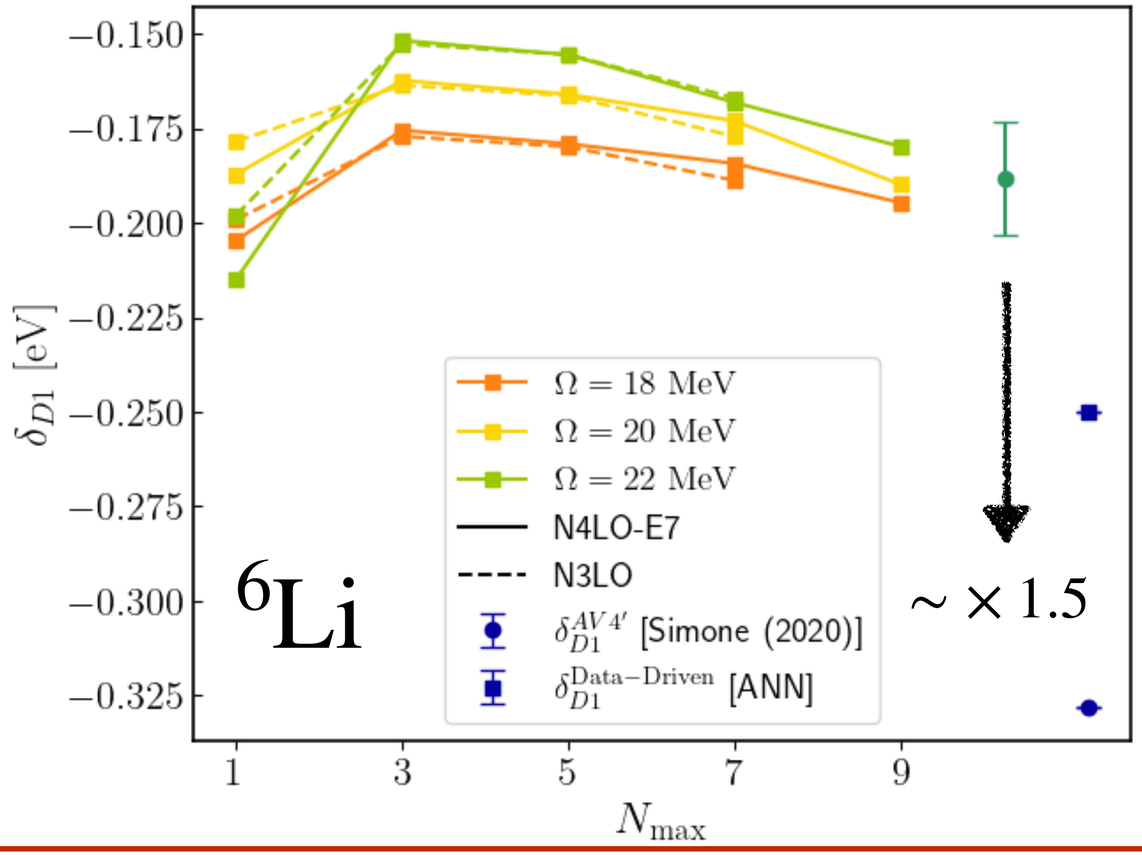

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- Muonic atoms: a precision probe for nuclear physics
  - Radii extraction: reference point + isotope-shift
  - Precise reference point: muonic atoms
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  - Complete (M_i, M) dependence analysis
  - Add current uncertainty quantification
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  - Atomic physics: three-photon exchange
  - Hadronic physics: more realistic model

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- Promising on going results for ^{6-7}Li:
  - Weak dependence between χEFT interactions
  - ➡ **NCSM**: **seems possible to reach $\epsilon_{\text{th}} \sim 0.05$ eV**

Outlook

- Completing on-going ab initio calculation
  - Complete (M_i, M) dependence analysis
  - Add current uncertainty quantification
  - Extension to ^{9}Be, $^{10-11}\text{B}$ ⇒ **new isotopic chains**
- Future modelling improvements
  - Nuclear physics: **higher-order currents**
  - Atomic physics: three-photon exchange
  - Hadronic physics: more realistic model
- Towards better controlling theoretical uncertainty
  - Shifting from pheno towards EFT approach
  - EFT based on **potential-NRQED** for $Z > 1$

[Peset et al., EPJA (2015)]

THANK YOU!

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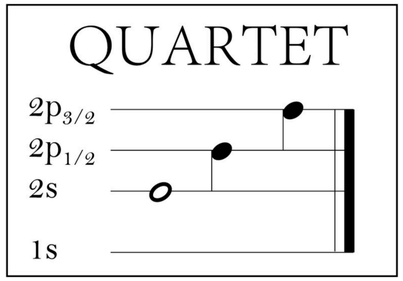

QUARTET Collaboration

JG|U

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