Multimodal superfluidity of neutron matter

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Outline

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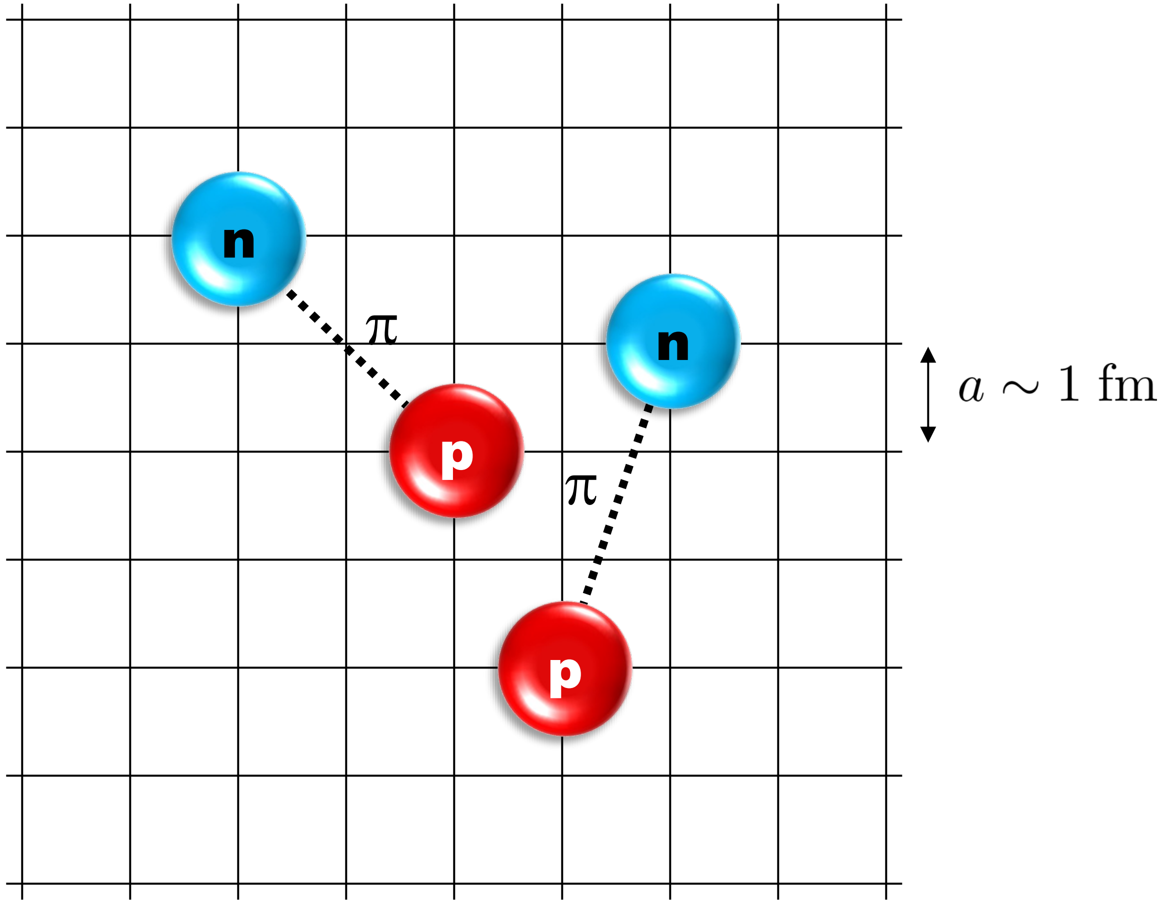
Lattice effective field theory

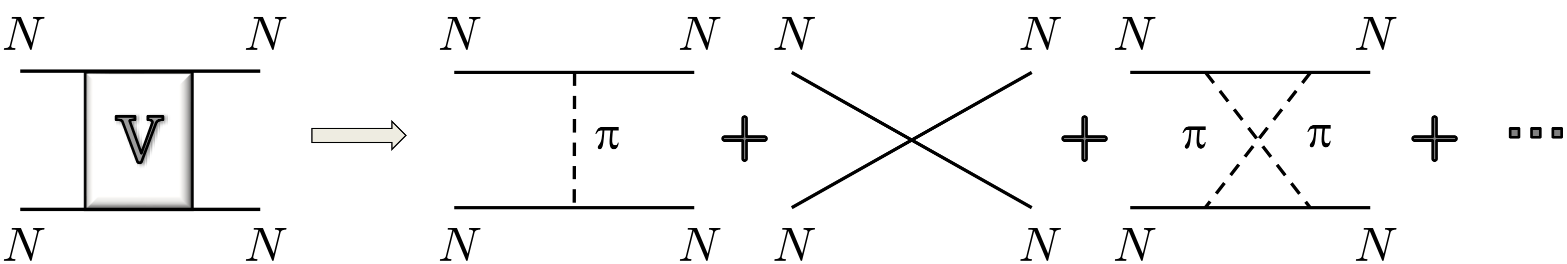


D.L, Annu. Rev. Nucl. Part. Sci. 75, 109 (2025)

Lähde, Meißner, Nuclear Lattice Effective Field Theory (2019), Springer

Chiral effective field theory

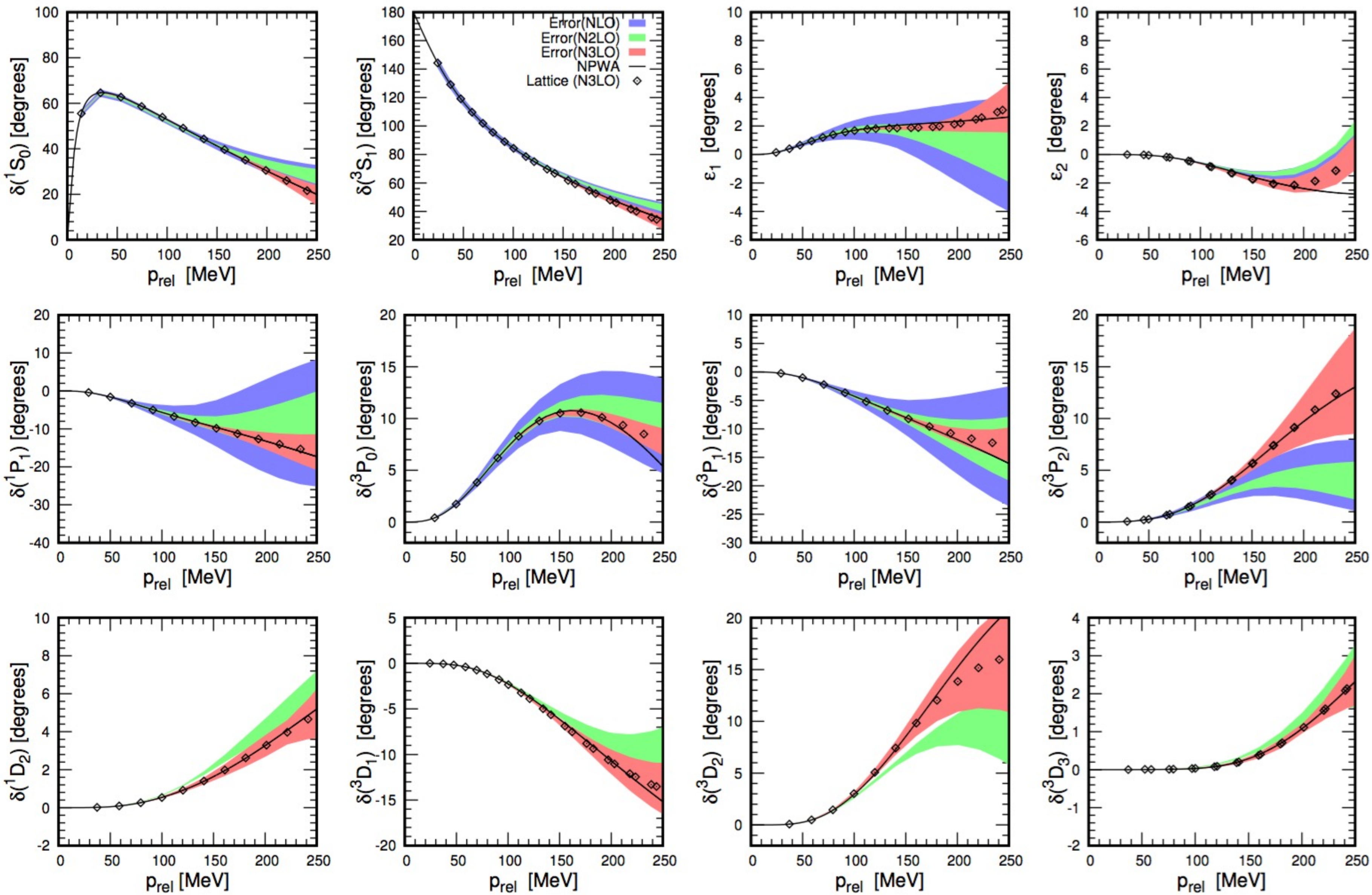
Construct the effective potential order by order



V^{OPEP} Contact interactions V^{TPEP}

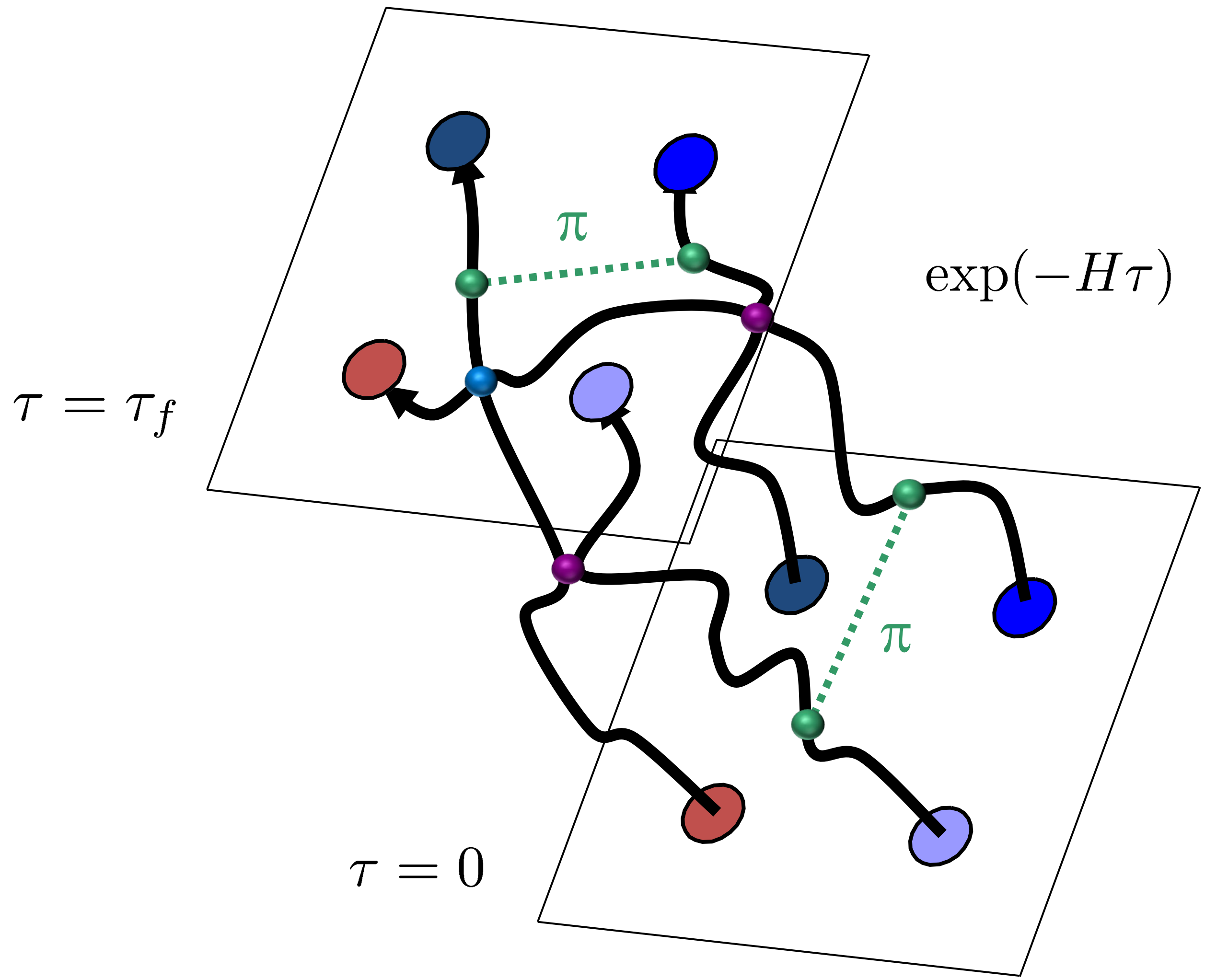
Leading order (LO) Next-to-leading order (NLO)

$a = 1.315\,\mathrm{fm}$



Li, Elhatisari, Epelbaum, D.L., Lu, Meißner, PRC 98, 044002 (2018)

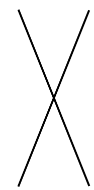
Euclidean time projection



Auxiliary field method

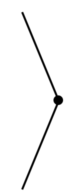
We can write exponentials of the interaction using a Gaussian integral identity

$$\exp\left[-\frac{C}{2}(N^\dagger N)^2\right]$$



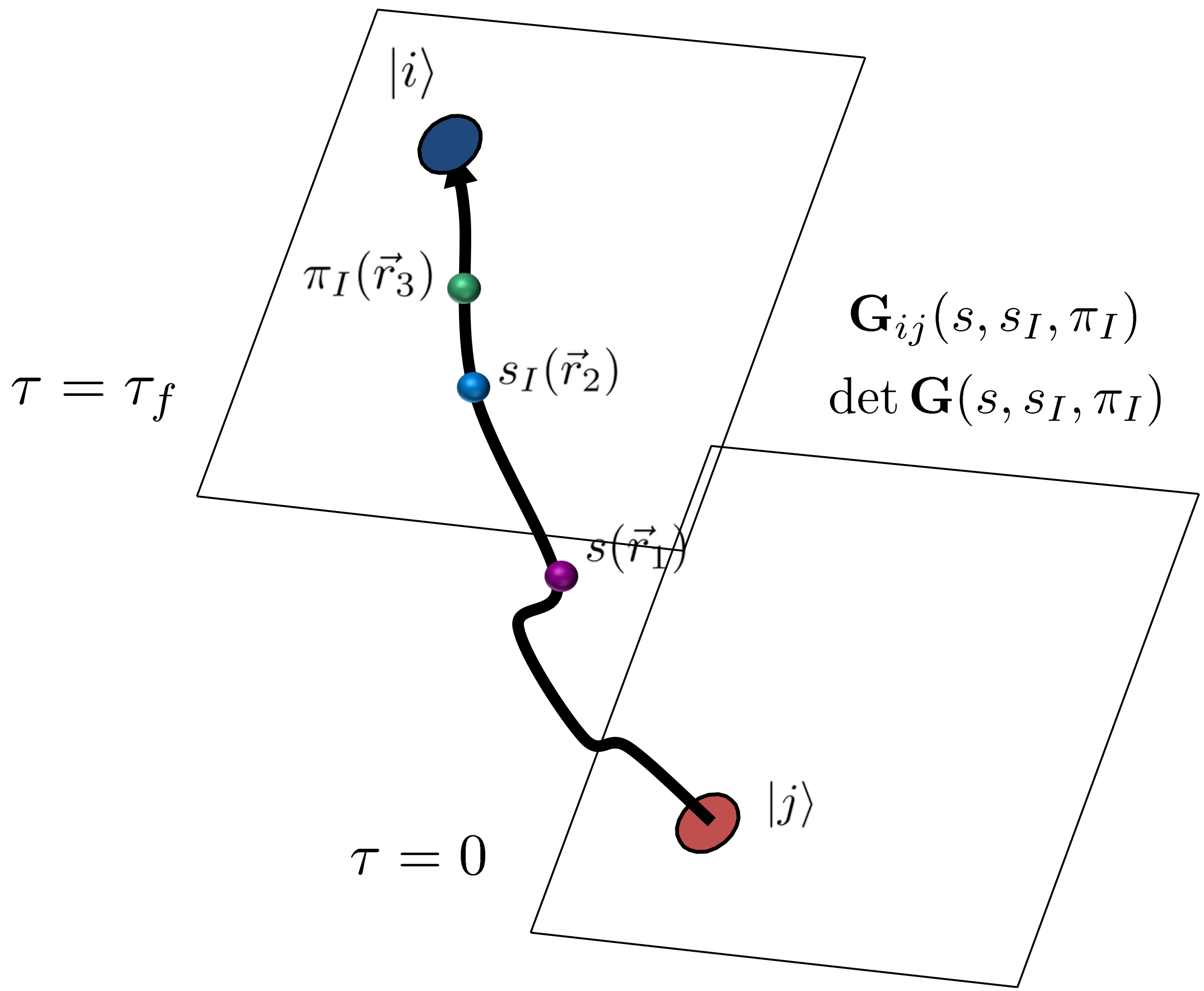
$(N^\dagger N)^2$

$$= \sqrt{\frac{1}{2\pi}} \int_{-\infty}^{\infty} ds \exp\left[-\frac{1}{2}s^2 + \sqrt{-C}\, s(N^\dagger N)\right]$$

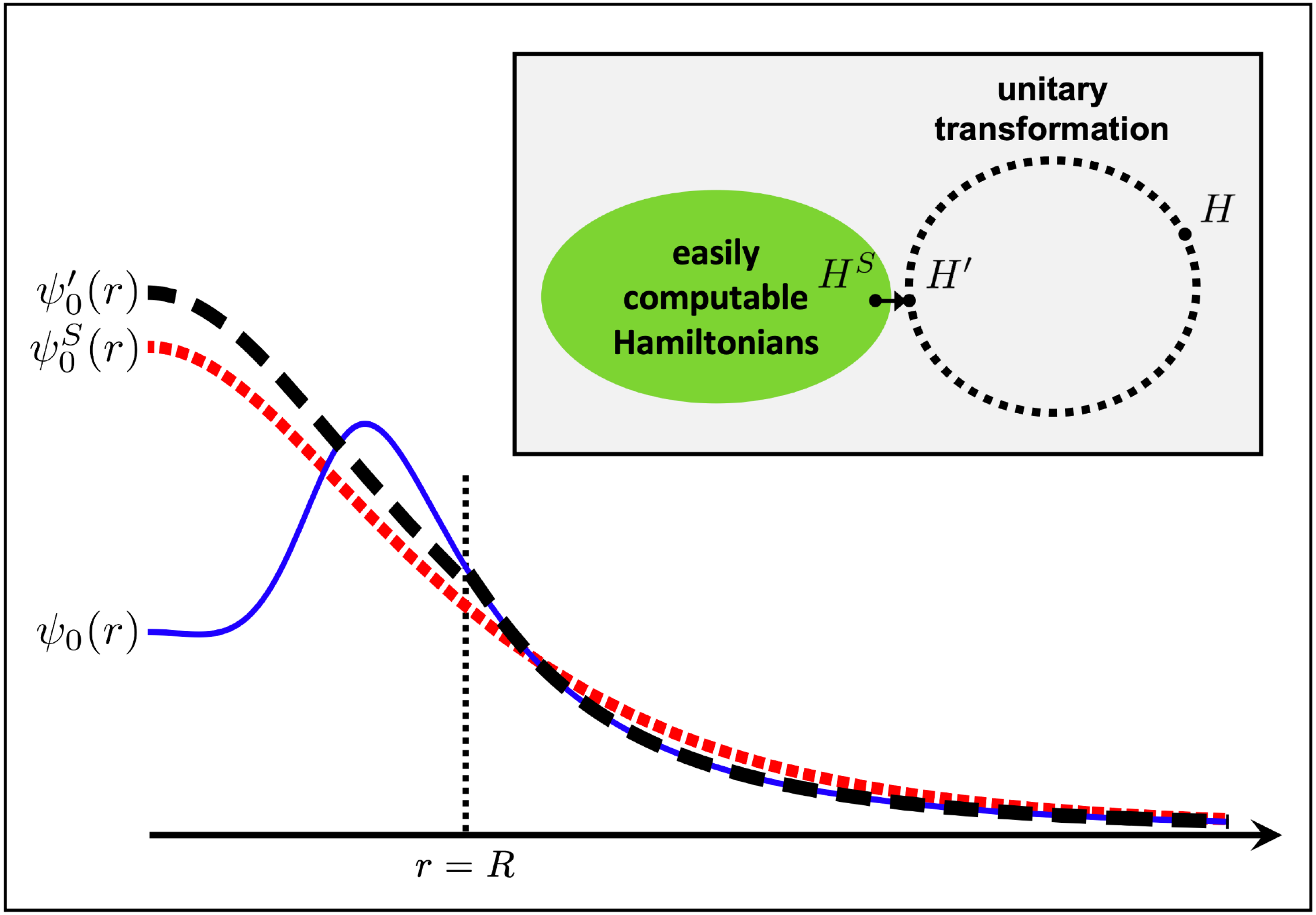


$sN^\dagger N$

We remove the interaction between nucleons and replace it with the interactions of each nucleon with a background field.

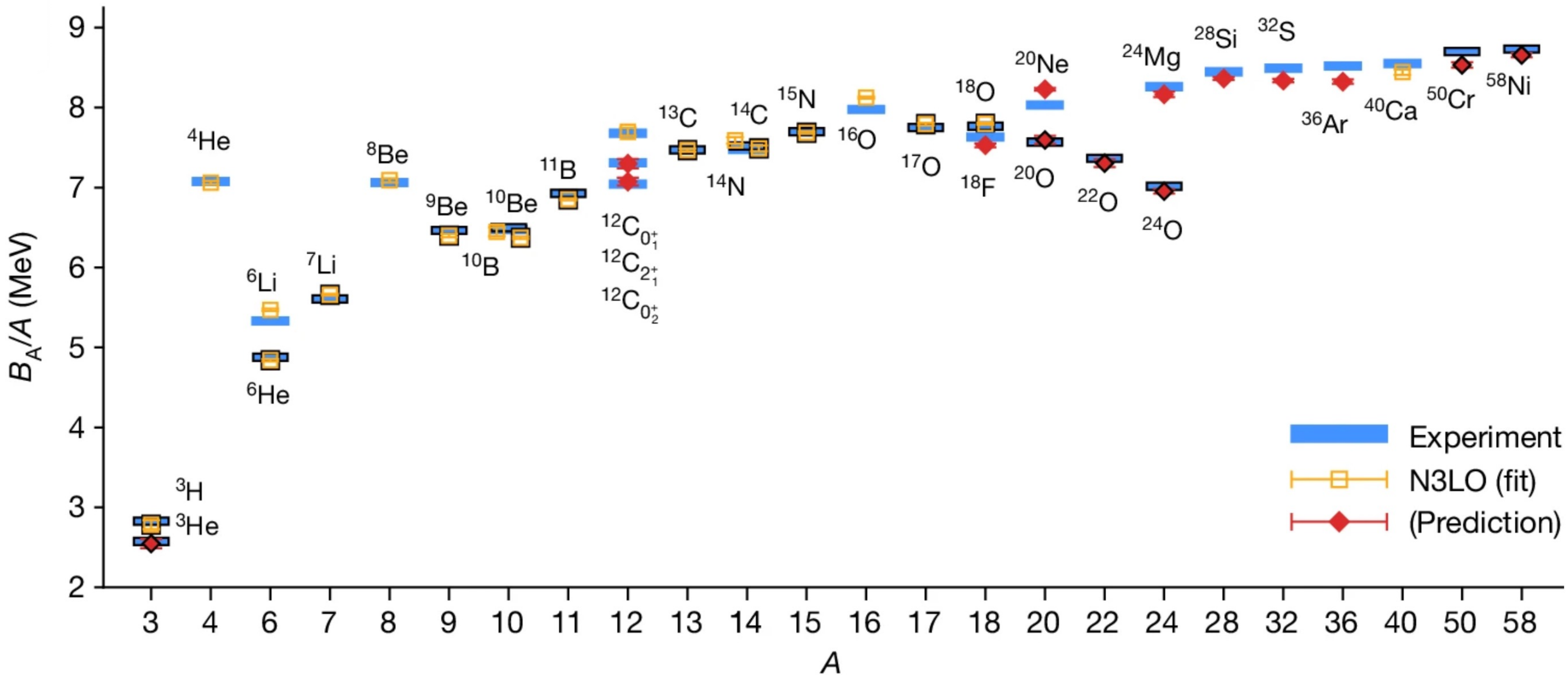

$|i\rangle$
$\pi_I(\vec{r}_3)$
$s_I(\vec{r}_2)$
$\tau = \tau_f$
$\mathbf{G}_{ij}(s, s_I, \pi_I)$
$\det \mathbf{G}(s, s_I, \pi_I)$
$s(\vec{r}_1)$
$|j\rangle$
$\tau = 0$

Wavefunction matching



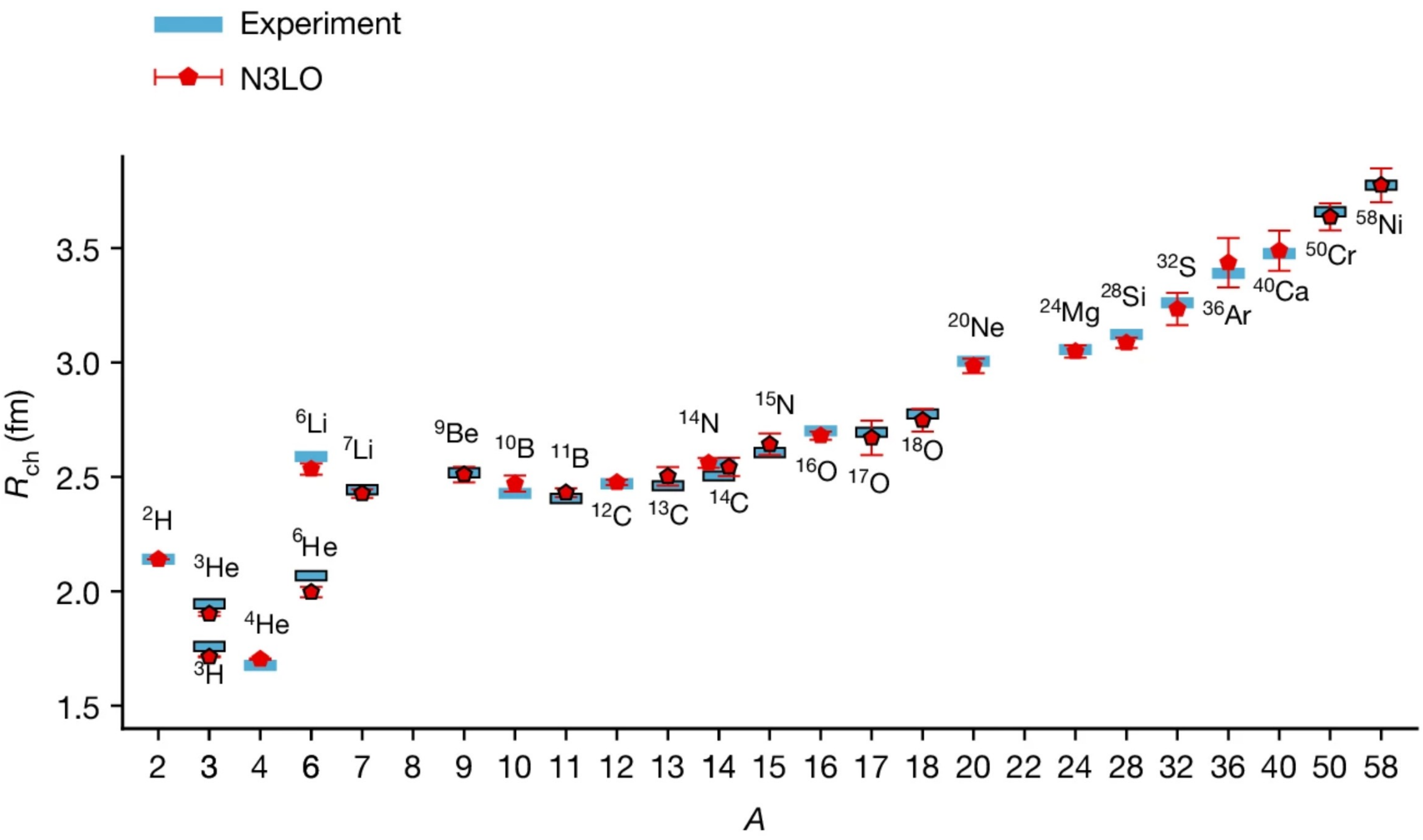
Elhatisari, Bovermann, Ma, Epelbaum, Frame, Hildenbrand, Krebs, Lähde, D.L., Li, Lu,
M. Kim, Y. Kim, Meißner, Rupak, Shen, Song, Stellin, Nature 630, 59 (2024)

Binding energies



Elhatisari, Bovermann, Ma, Epelbaum, Frame, Hildenbrand, Krebs, Lähde, D.L., Li, Lu, M. Kim, Y. Kim, Meißner, Rupak, Shen, Song, Stellin, Nature 630, 59 (2024)

Charge radii



Elhatisari, Bovermann, Ma, Epelbaum, Frame, Hildenbrand, Krebs, Lähde, D.L., Li, Lu,
M. Kim, Y. Kim, Meißner, Rupak, Shen, Song, Stellin, Nature 630, 59 (2024)

Neutron and nuclear matter

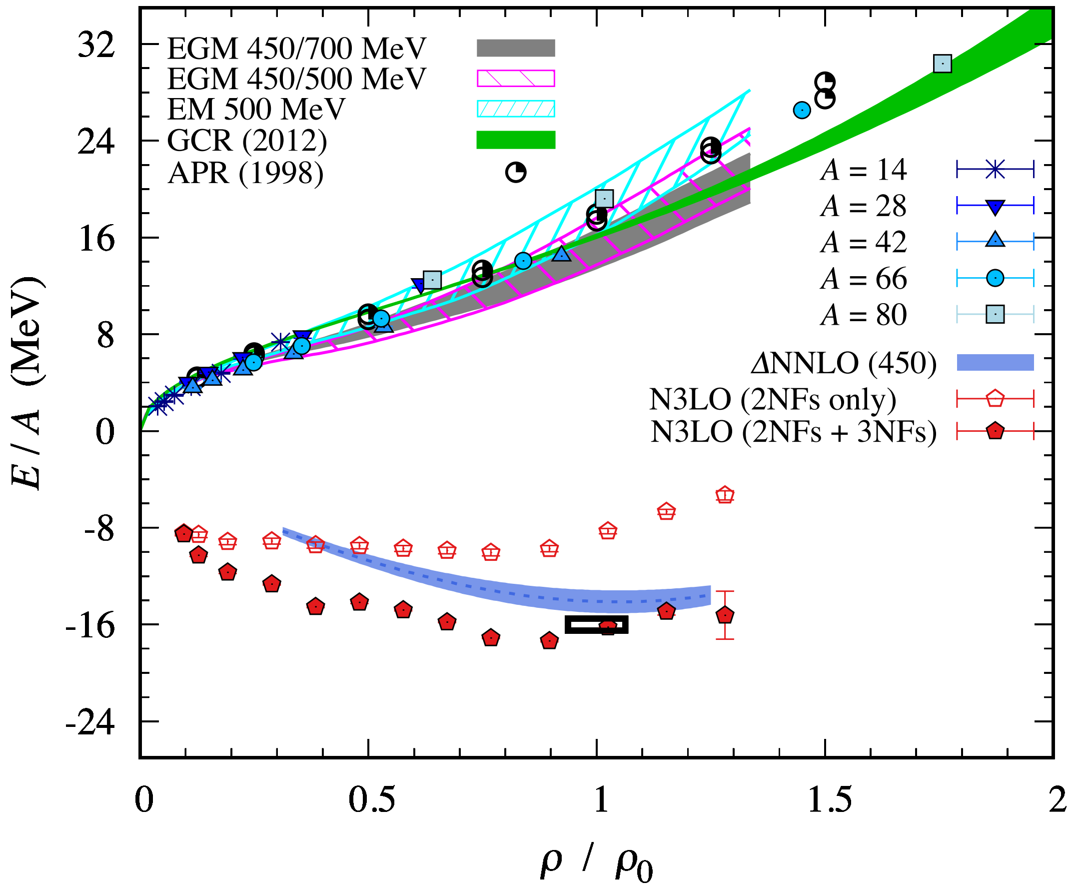


Figure adapted from Tews, Krüger, Hebeler, Schwenk, Phys. Rev. Lett. 110, 032504 (2013)

Elhatisari, Bovermann, Ma, Epelbaum, Frame, Hildenbrand, Krebs, Lähde, D.L., Li, Lu, M. Kim, Y. Kim, Meißner, Rupak, Shen, Song, Stellin, Nature 630, 59 (2024)

Superfluid condensation

Bosonic superfluidity

$$\langle \Psi_0 | a^\dagger(\mathbf{r}) a(\mathbf{0}) | \Psi_0 \rangle$$

Fermionic superfluidity (S-wave)

$$\langle \Psi_0 | a^\dagger_\downarrow(\mathbf{r}) a^\dagger_\uparrow(\mathbf{r}) a_\uparrow(\mathbf{0}) a_\downarrow(\mathbf{0}) | \Psi_0 \rangle$$

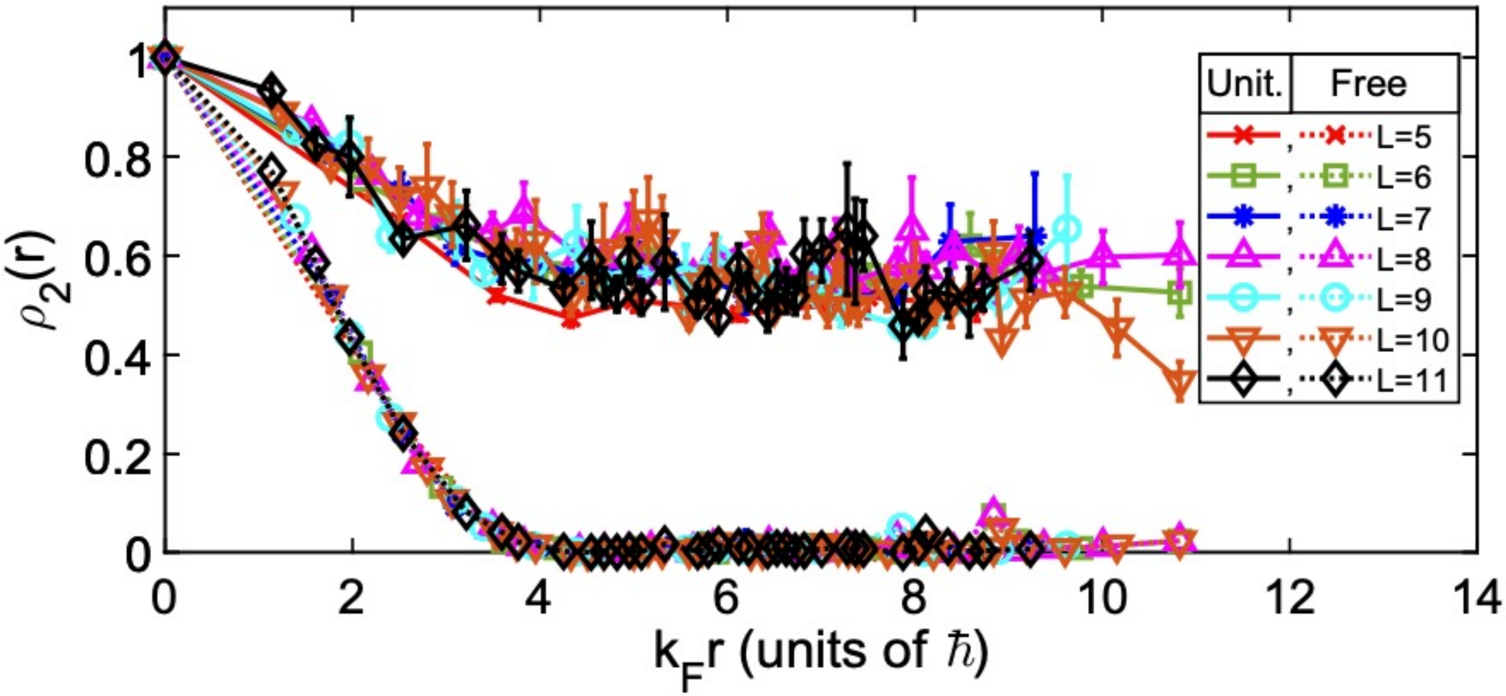
Fermionic superfluidity (P-wave)

$$\langle \Psi_0 | a^\dagger_\uparrow(\mathbf{r}) a^\dagger_\uparrow(\mathbf{r} + \Delta\mathbf{r}) a_\uparrow(\Delta\mathbf{r}) a_\uparrow(\mathbf{0}) | \Psi_0 \rangle$$

We can also perform calculations in momentum space. But we need to compute cumulants to obtain irreducible contributions only.

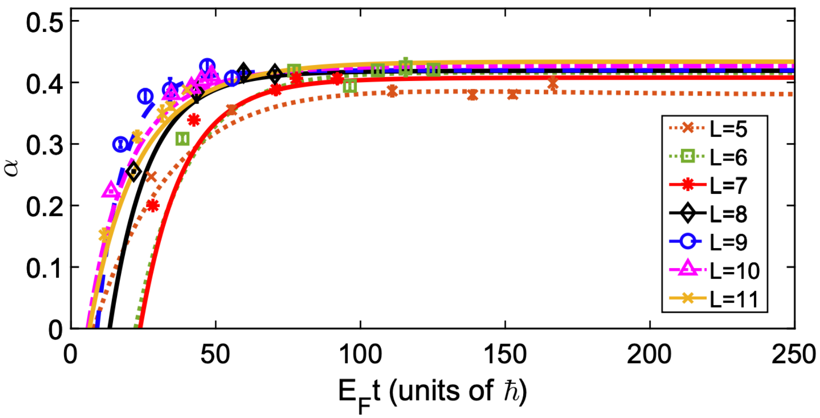
Yang, RMP **34**, 694 (1962)

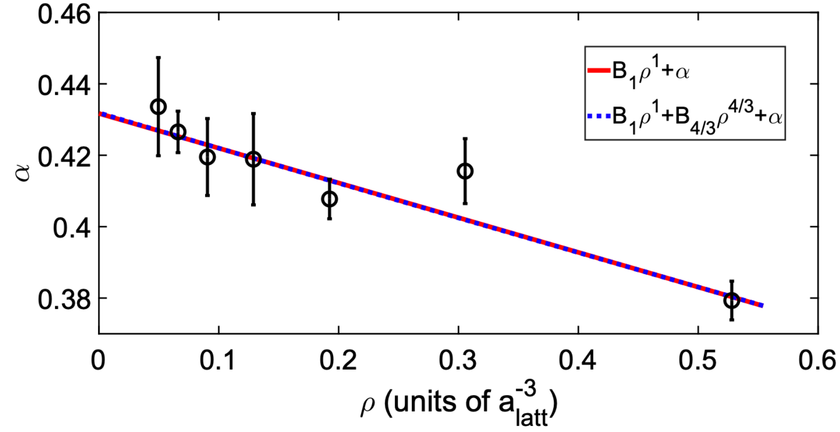
Unitary limit



He, Li, Lu, D.L., Phys. Rev. A 101, 063615 (2020)

Unitary limit





condensate fraction = 0.43(2)

He, Li, Lu, D.L., Phys. Rev. A 101, 063615 (2020)

^{6}Li experiments: 0.46(7) [1, 2] and 0.47(7) [3]

[1] Zwierlein, Stan, Schunck, Raupach, Kerman, Ketterle, PRL 92, 120403 (2004).
[2] Zwierlein, Schunck, Stan, Raupach, Ketterle, PRL 94, 180401 (2005).
[3] Kwon, Pace, Panza, Inguscio, Zwerger, Zaccanti, Scazza, Roati, Science 369, 84 (2020).

Can a many-body system have S-wave and P-wave superfluid condensation at the same time?

Attractive extended Hubbard models

We consider attractive extended Hubbard models for two-component fermions in 1, 2, 3 dimensions

$$H = H_{\text{free}} + \tfrac{1}{2}C_2 \textstyle\sum_{\mathbf{n}} \tilde{\rho}(\mathbf{n})^2$$

$$\tilde{\rho}(\mathbf{n}) = \textstyle\sum_{j=\uparrow,\downarrow} \tilde{a}_j^\dagger(\mathbf{n})\tilde{a}_j(\mathbf{n}) + s_L \sum_{|\mathbf{n}-\mathbf{n}'|=1} \sum_{j=\uparrow,\downarrow} \tilde{a}_j^\dagger(\mathbf{n}')\tilde{a}_j(\mathbf{n}')$$

$$\tilde{a}_j(\mathbf{n}) = a_j(\mathbf{n}) + s_{NL} \textstyle\sum_{|\mathbf{n}-\mathbf{n}'|=1} \; a_j(\mathbf{n}')$$

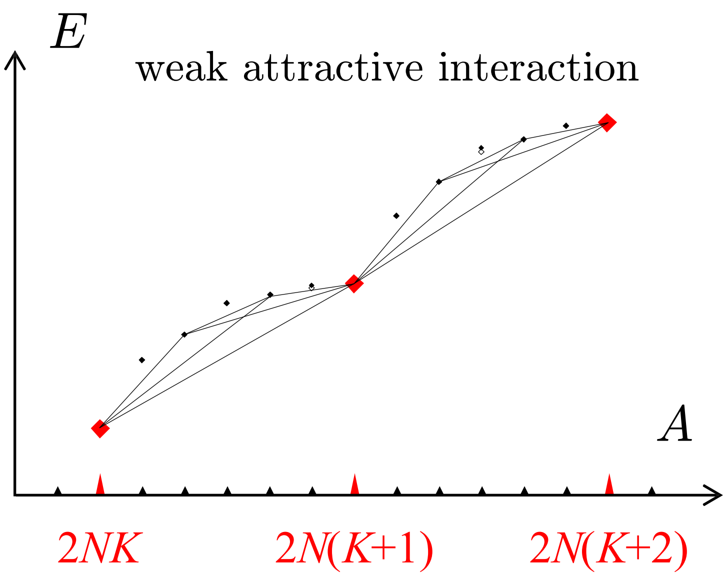
This interaction produces both S-wave and P-wave attraction. The symmetry group is

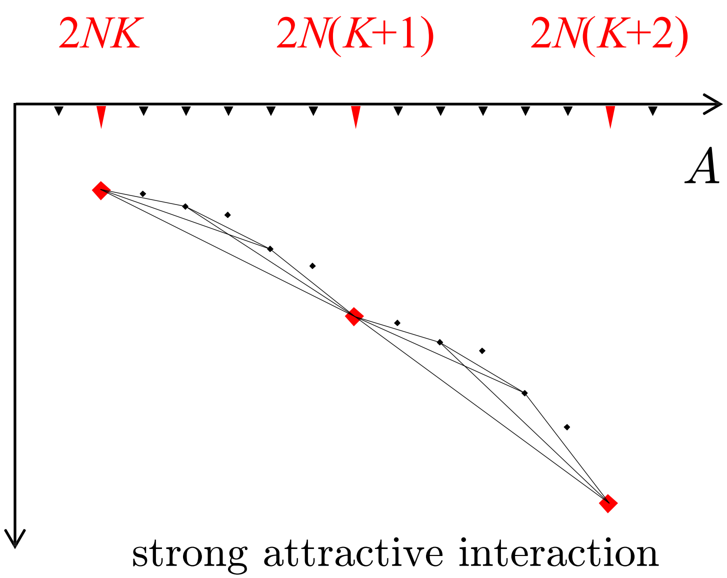
$$G = U(1) \times SU(2)_{\text{spin}} \times SO(3)_{\text{orb}}$$

Spectral convexity theorem

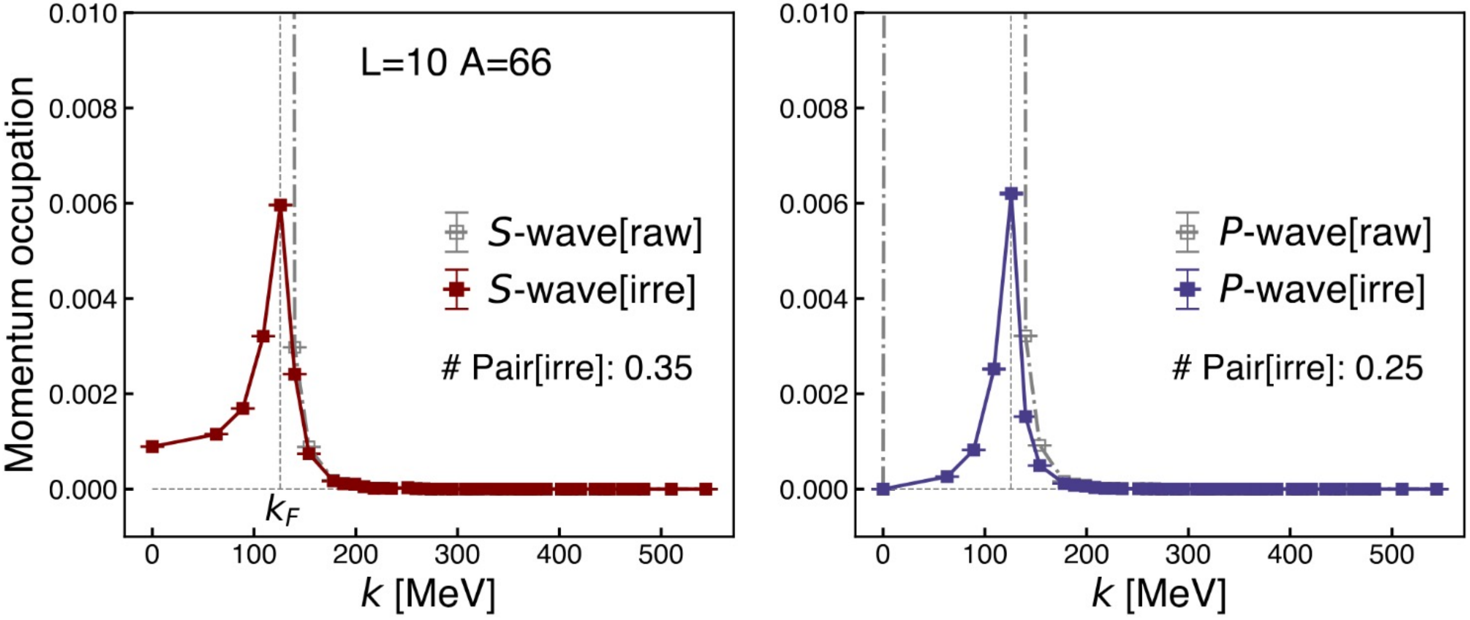
Consider any fermionic system with an SU($2N$) symmetry that can be simulated with no sign problem using one auxiliary field. It must obey the SU($2N$) convexity bounds illustrated below, and the SU($2N$) symmetry is not spontaneously broken.

D.L., Phys. Rev. Lett. 98 (2007) 182501

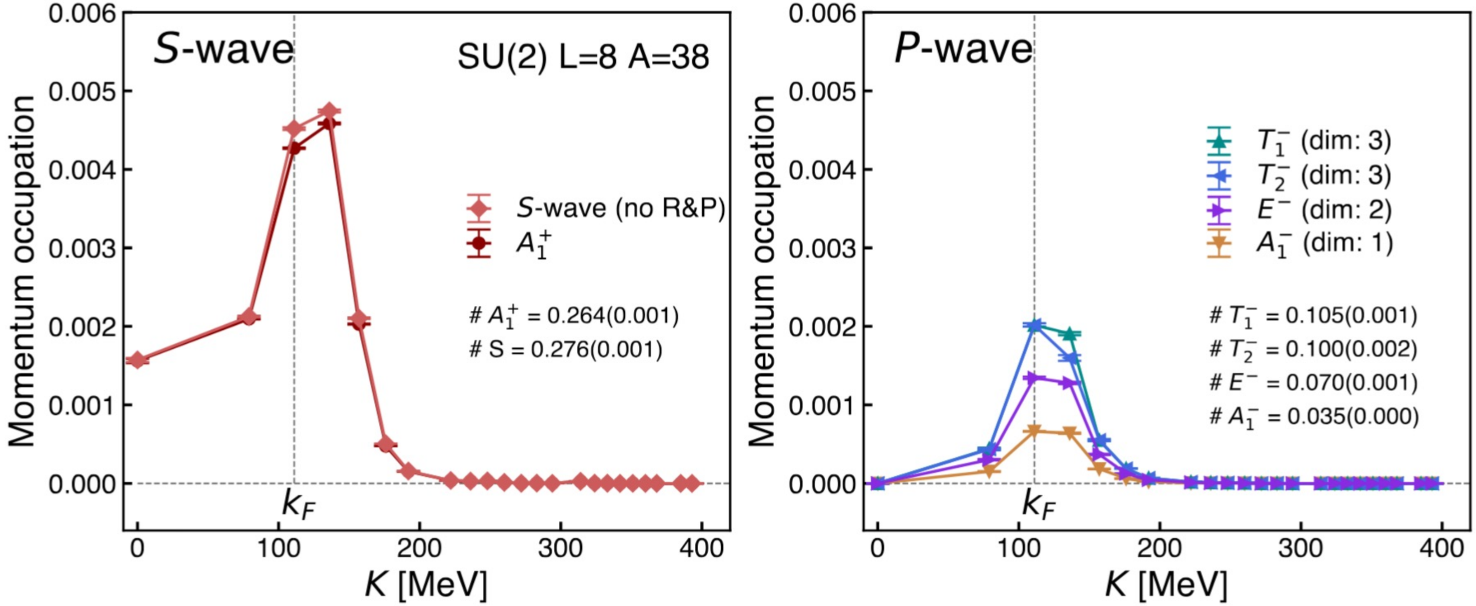


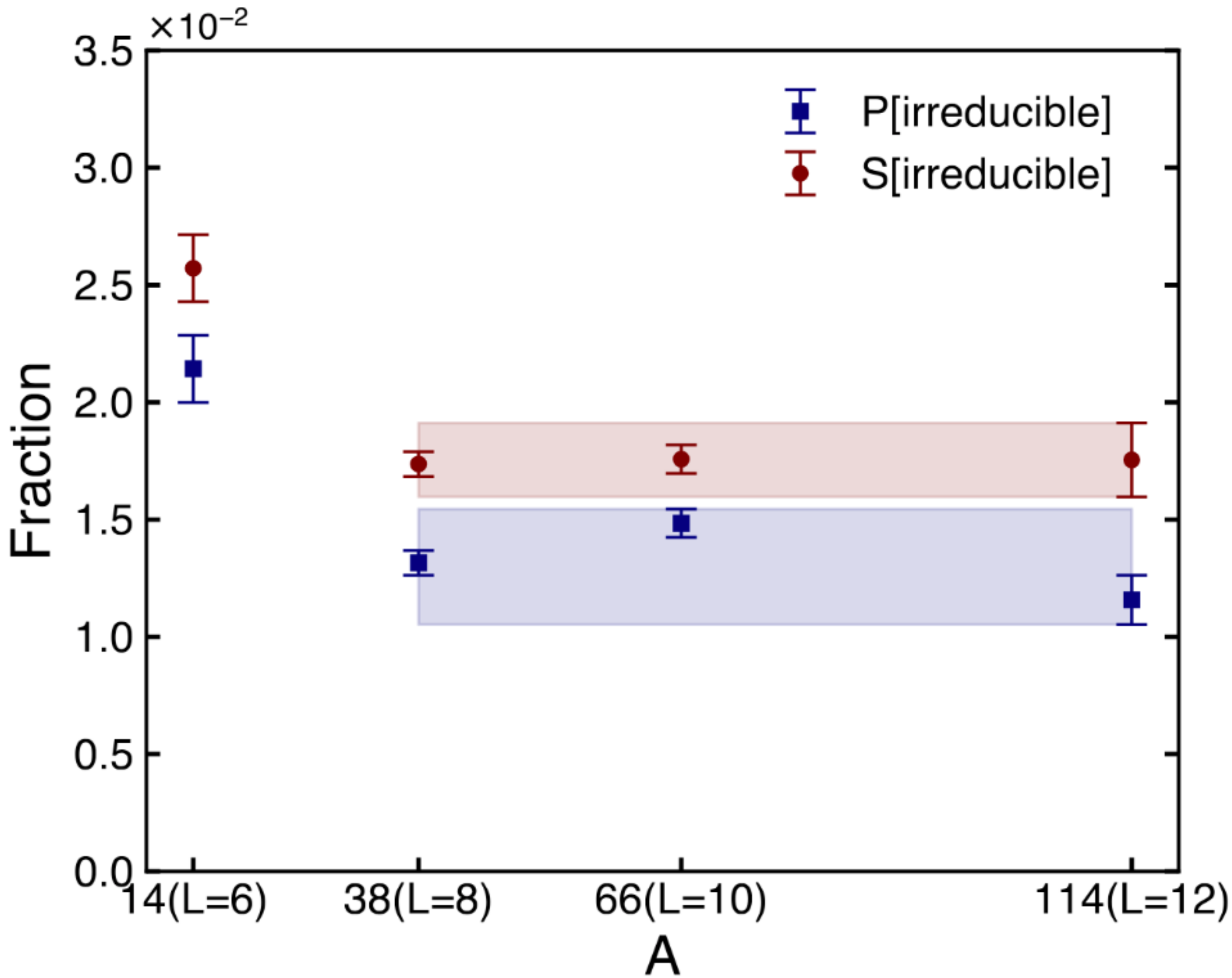


Attractive extended Hubbard model



Ma, Palkanoglou, Carlson, Gandolfi, Gezerlis, Given, D.L., Schmidt, Reddy, Yu, in progress

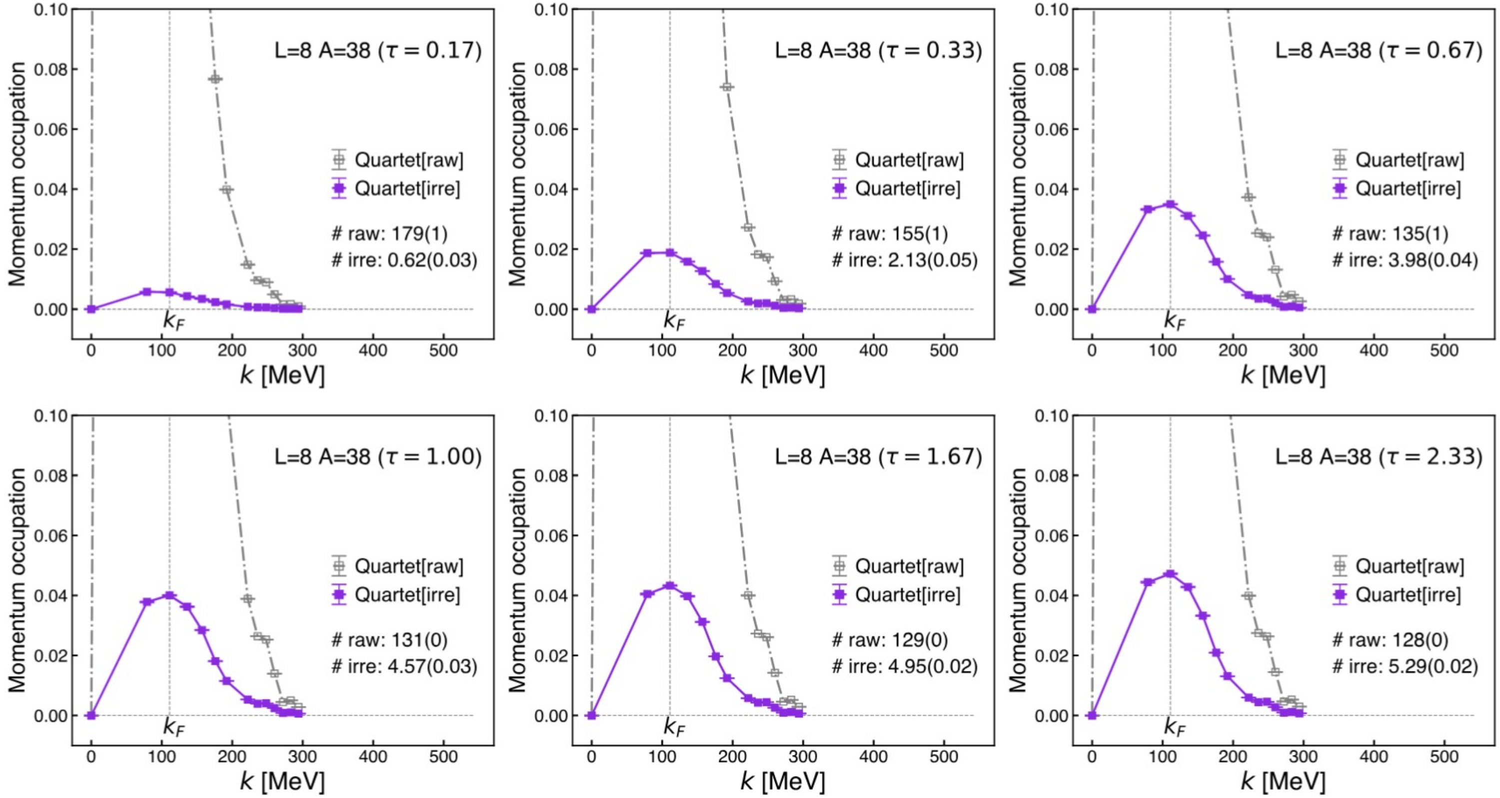

S-wave
SU(2) L=8 A=38
S-wave (no R&P)
A_1^+
A_1^+ = 0.264(0.001)
S = 0.276(0.001)
Momentum occupation
k_F
K [MeV]
P-wave
T_1^- (dim: 3)
T_2^- (dim: 3)
E^- (dim: 2)
A_1^- (dim: 1)
T_1^- = 0.105(0.001)
T_2^- = 0.100(0.002)
E^- = 0.070(0.001)
A_1^- = 0.035(0.000)
Momentum occupation
k_F
K [MeV]

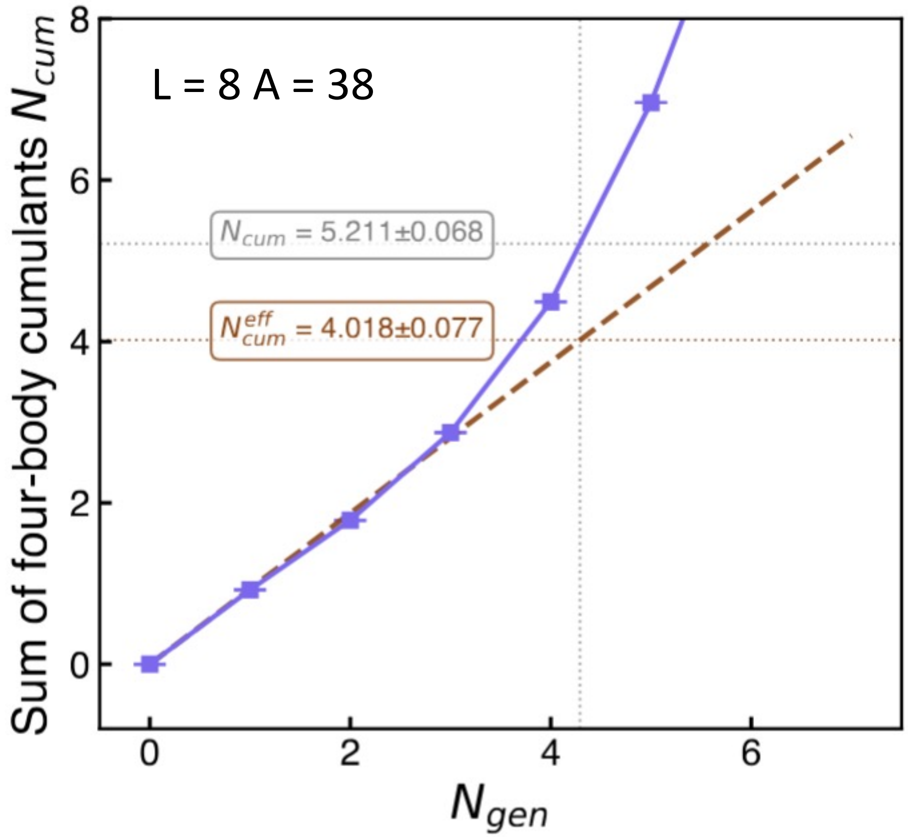

×10⁻²
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Fraction
P[irreducible]
S[irreducible]
14(L=6)
38(L=8)
66(L=10)
114(L=12)
A

How does the SU(2) spin symmetry remain unbroken if there is a P-wave condensate?

Why are the S-wave and P-wave condensates so small?

Attractive extended Hubbard model





L = 8 A = 38
N_{cum} = 5.211±0.068
N_{cum}^{eff} = 4.018±0.077
Sum of four-body cumulants N_{cum}
N_{gen}
0
2
4
6
8

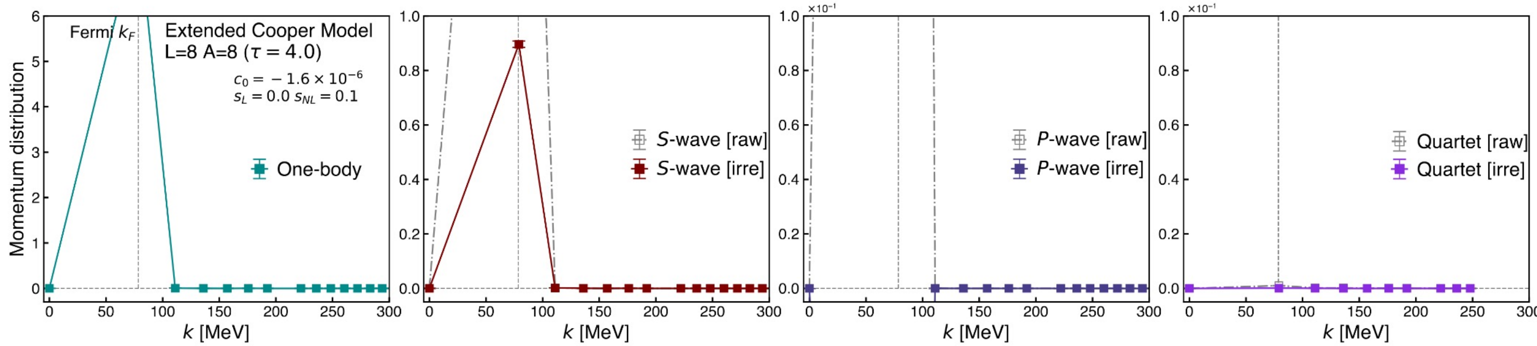
Table S4. Thermodynamic limit check with $c_0 = -1.6 \times 10^{-6}$ MeV^{-2}, $s_L = 0.5$ and $s_{NL} = 0.1$.

L	A	density (fm^{-3})	S-wave pairs	P-wave pairs	Quartets	S-wave/A	P-wave/A	Quartets/A
6	14	0.00844	0.180 (0.010)	0.150 (0.010)	0.011 (0.001)	0.0129	0.0107	0.0008
8	38	0.00966	0.330 (0.010)	0.250 (0.010)	4.018 (0.077)	0.0087	0.0066	0.1057
10	66	0.00859	0.580 (0.020)	0.490 (0.020)	7.729 (0.324)	0.0088	0.0074	0.1171
12	114	0.00859	1.000 (0.090)	0.660 (0.060)	-	0.0088	0.0058	-

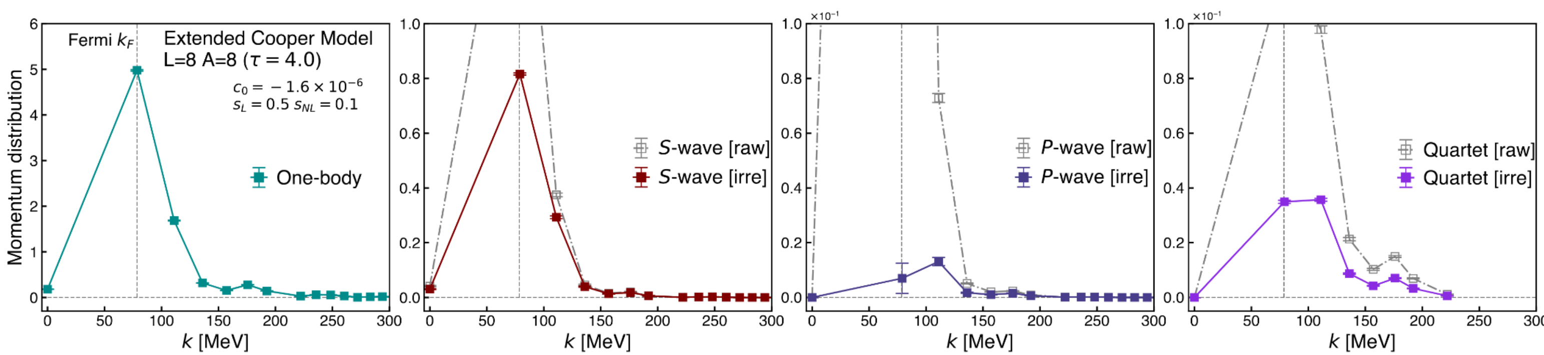
S-wave pairs	1.8(2)%
P-wave pairs	1.3(2)%
quartets	43(6)%

Self-consistent Cooper model

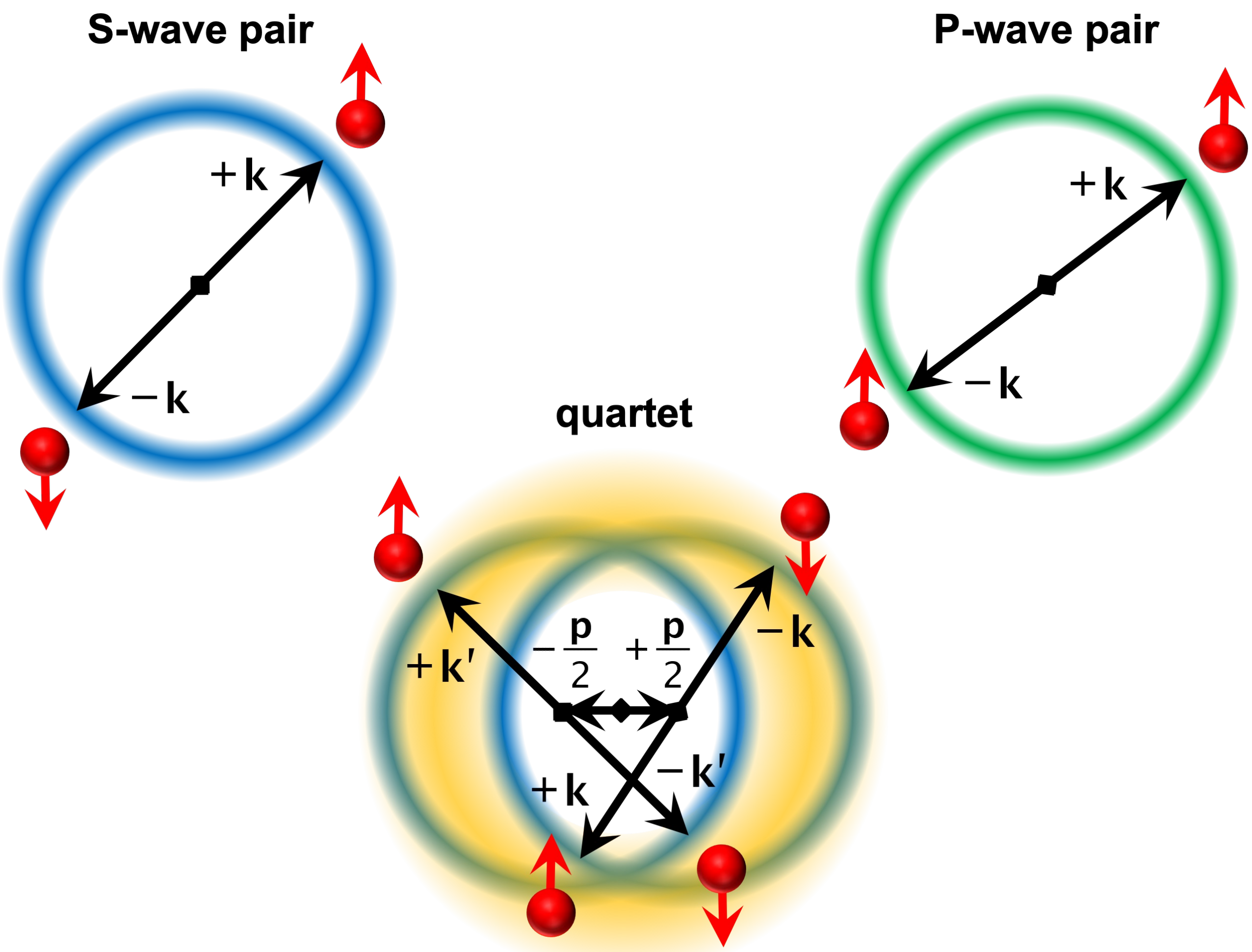
S-wave interactions only



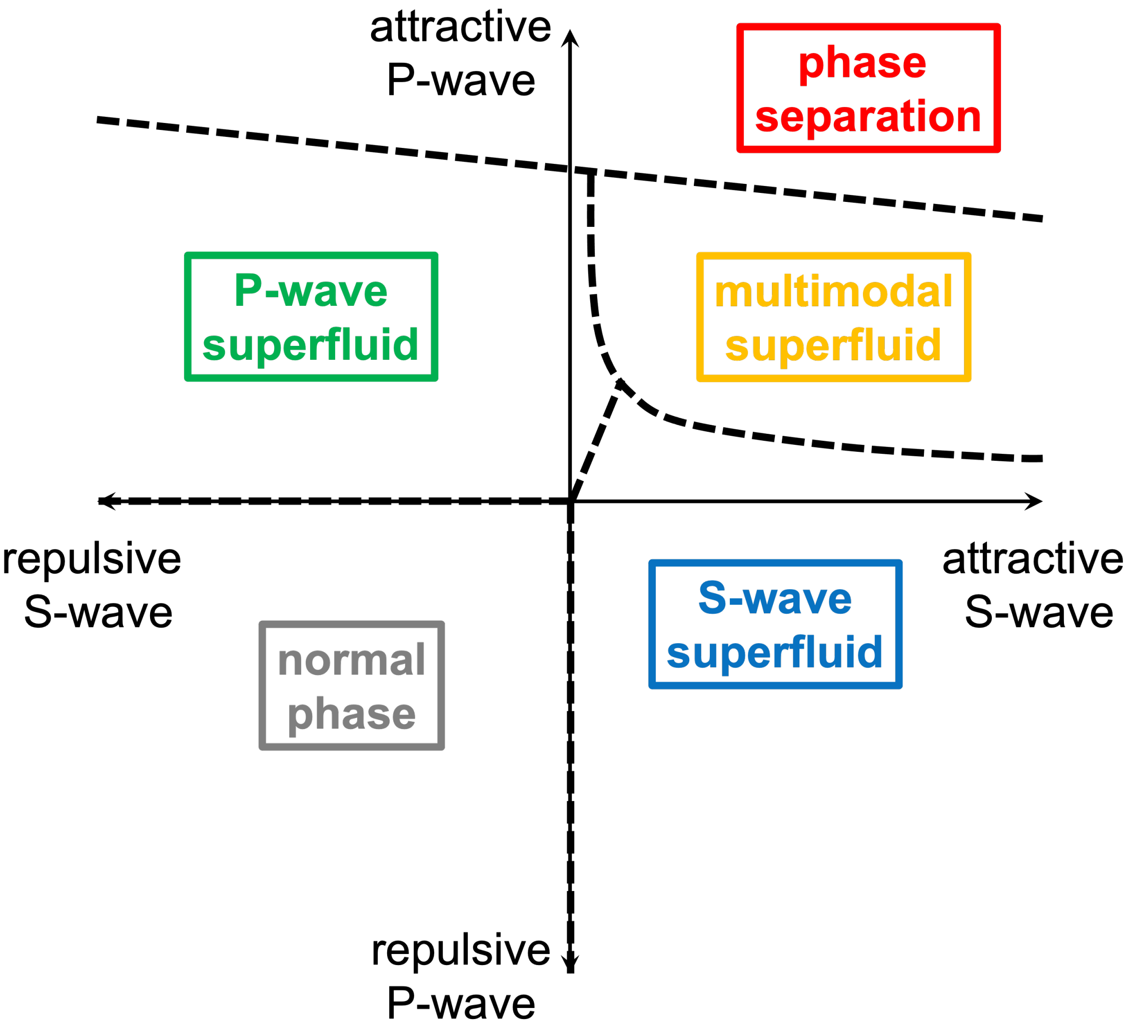
S-wave and P-wave interactions



Multimodal superfluidity

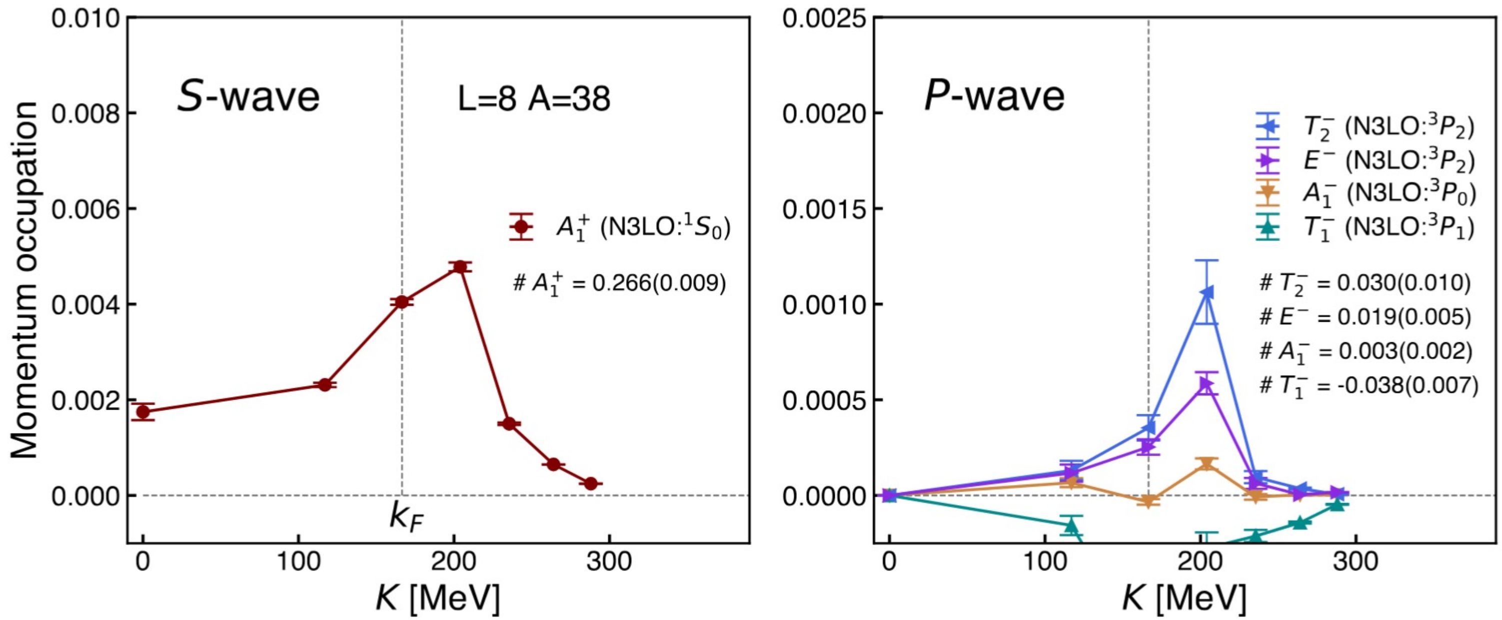


Quantum phase diagram



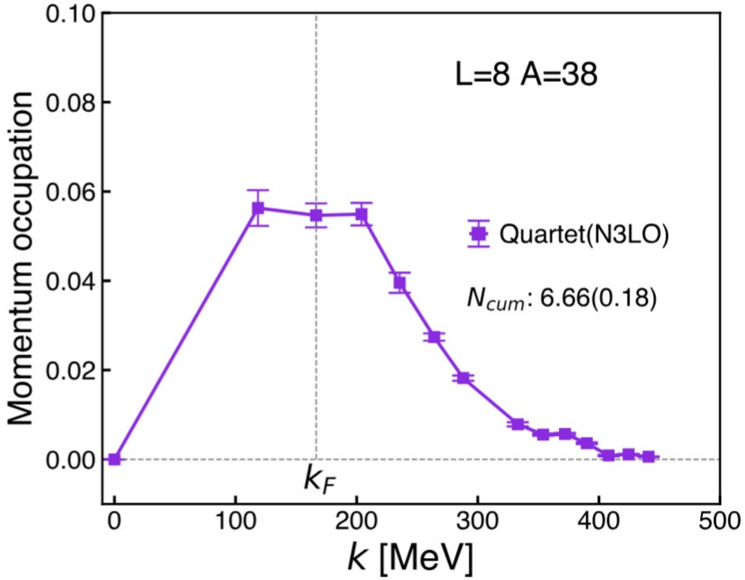
Neutron matter with N3LO interactions

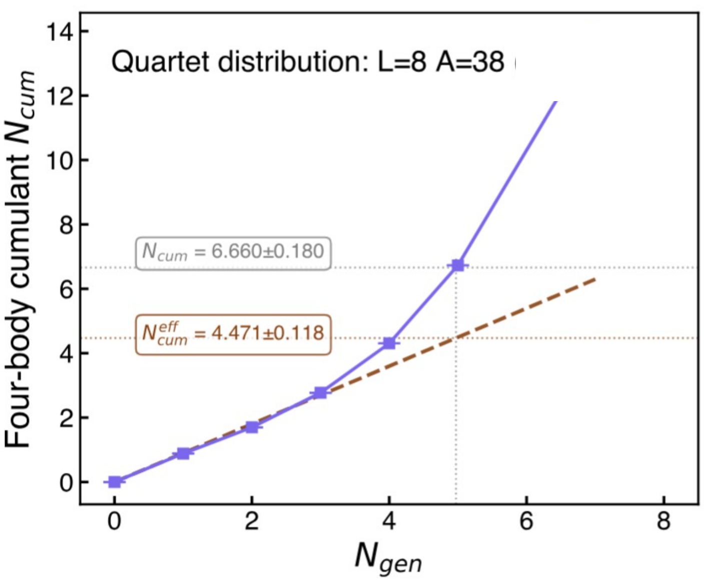
$$k_F = 167 \text{ MeV}$$



Neutron matter with N3LO interactions

$$k_F = 167 \text{ MeV}$$

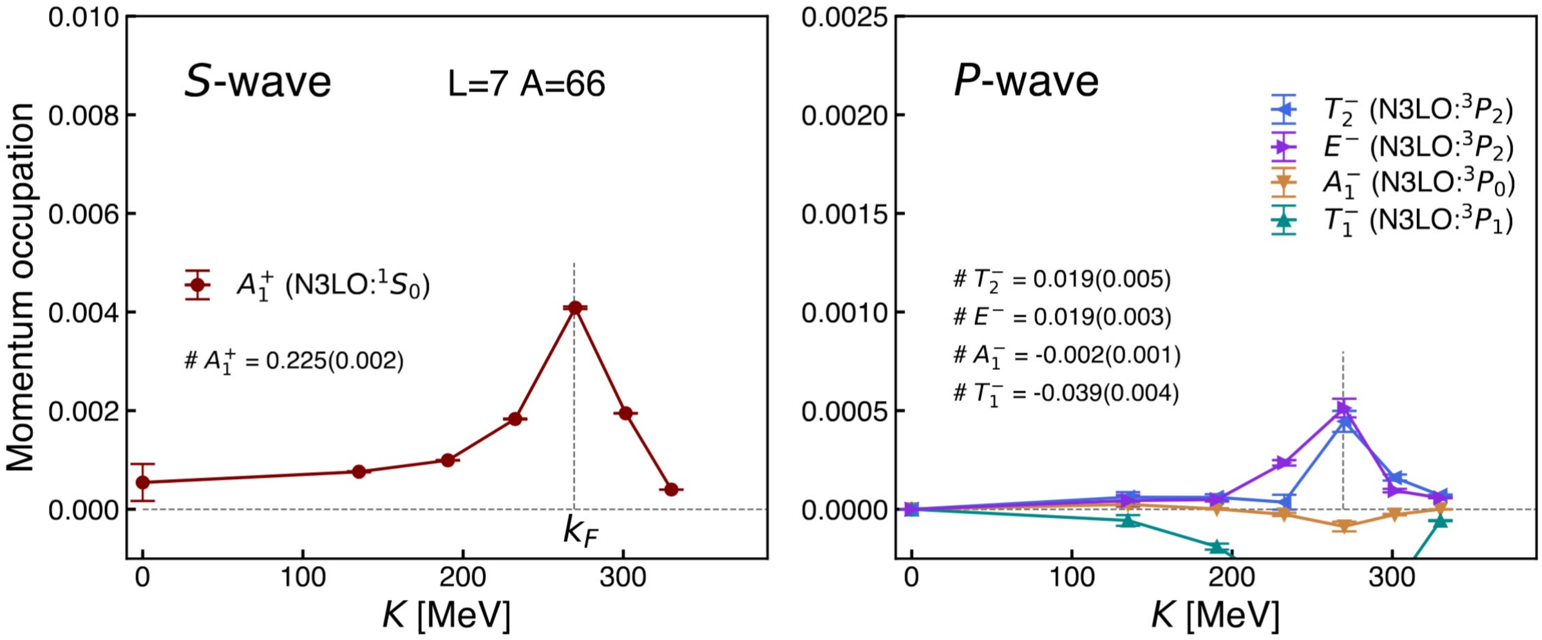




S-wave pairs	1.4(1)%
P-wave pairs	0.3(1)%
quartets	47(1)%

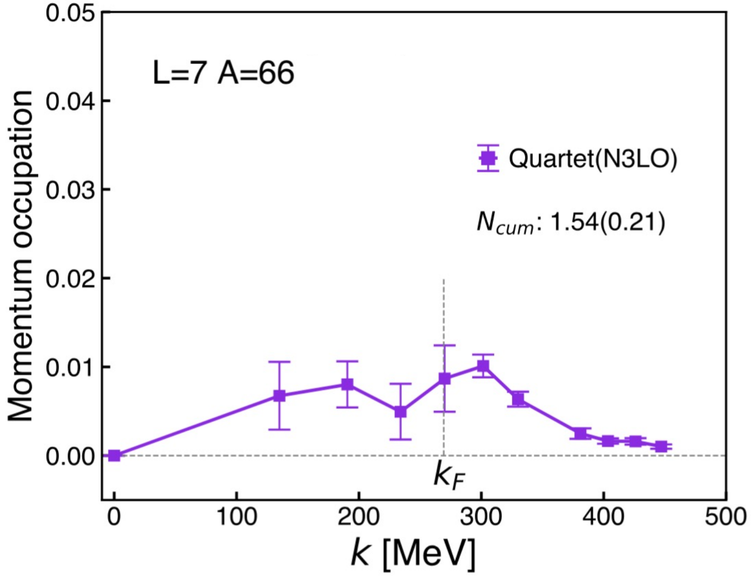
Neutron matter with N3LO interactions

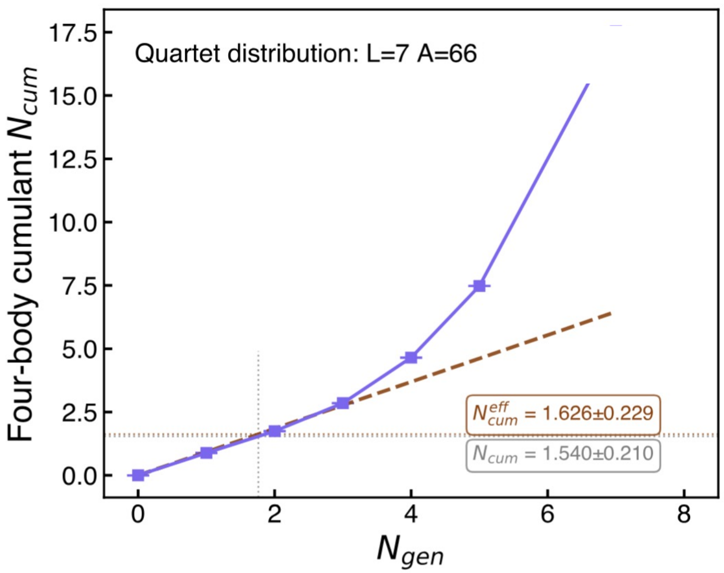
$$k_F = 269 \text{ MeV}$$



Neutron matter with N3LO interactions

$$k_F = 269 \text{ MeV}$$





S-wave pairs	0.68(1)%
P-wave pairs	0.12(2)%
quartets	10(1)%

Effective action

We can write the interactions using auxiliary fields for the S-wave and P-wave pairing channels:

$$\phi,\, A_{\mu,j} \qquad \begin{array}{r} \mu = 1,2,3 \text{ (orbital)} \\ j = 1,2,3 \text{ (intrinsic spin)} \end{array}$$

We integrate out the fermions and consider the terms in the low-energy effective action.

When the P-wave interaction is strong enough to bind two S-wave pairs, then we have asymptotic states composed of quartets. Our low-energy effective action must include an interpolating field for the quartet degrees of freedom.

$$Q_s \equiv \phi^2$$

$$D_1 \equiv \mathrm{Tr}(AA^T) = \sum_{\mu,j} A_{\mu j} A_{\mu j},$$

$$D_2 \equiv \mathrm{Tr}(A^\dagger A) = \sum_{\mu,j} A^*_{\mu j} A_{\mu j},$$

$$T^{(3)}_{jk} \equiv \sum_{\mu} A^*_{\mu j} A_{\mu k} - \frac{1}{3}\delta_{jk} D_2, \quad \text{(Spin Density Tensor)}$$

$$T^{(4)}_{\mu\nu} \equiv \sum_{j} A_{\mu j} A_{\nu j} - \frac{1}{3}\delta_{\mu\nu} D_1, \quad \text{(Orbital Pair Tensor)}$$

$$T^{(5)}_{jk} \equiv \sum_{\mu} A_{\mu j} A_{\mu k} - \frac{1}{3}\delta_{jk} D_1. \quad \text{(Spin Pair Tensor)}$$

$$\begin{aligned}
V_{\text{eff}} = {} & \mu_s |\phi|^2 + \frac{g_H}{2}|\phi|^4 + \alpha_s |Q_s|^2 + r_p D_2 + \sum_{i=1}^{2} \beta_i |D_i|^2 \\
& + \beta_3 \,\mathrm{Tr}[(T^{(3)})^2] + \beta_4 \,\mathrm{Tr}[(T^{(4)})^\dagger T^{(4)}] + \beta_5 \,\mathrm{Tr}[(T^{(5)})^\dagger T^{(5)}] \\
& - \frac{\eta_s}{2}\left[(\phi^\dagger)^2 Q_s + \text{h.c.}\right] + u_{sp}|\phi|^2 D_2 \\
& - \frac{\lambda_{sp}}{2}\left[Q_s^\dagger D_1 + \text{h.c.}\right] - \frac{\eta_{sp}}{2}\left[(\phi^\dagger)^2 D_1 + \text{h.c.}\right].
\end{aligned}$$

The S-wave pair field gets a nonzero expectation value.

$$\langle \phi \rangle = v = |v| e^{i\theta}$$

The couplings in the effective action produce a nonzero expectation value for the quartet field and double P-wave pair composite operator

$$\Sigma_s \equiv \langle Q_s \rangle = \left(\frac{\beta_1 \eta_s + \frac{1}{2} \lambda_{sp} \eta_{sp}}{2(\alpha_s \beta_1 - \frac{1}{4} \lambda_{sp}^2)} \right) v^2$$

$$\Sigma_1 \equiv \langle D_1 \rangle = \left(\frac{\alpha_s \eta_{sp} + \frac{1}{2} \lambda_{sp} \eta_s}{2(\alpha_s \beta_1 - \frac{1}{4} \lambda_{sp}^2)} \right) v^2$$

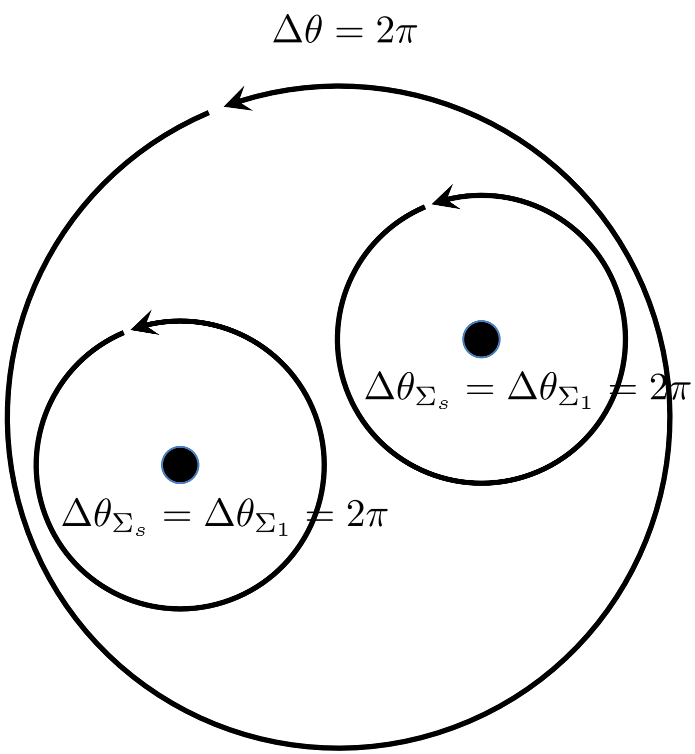
But the P-wave pair operator by itself has zero expectation value and the SU(2) spin symmetry and rotational symmetry are unbroken

$$\langle A_{\mu,j} \rangle = 0$$

We note that condensate phases are locked together:

$$\theta_{\Sigma_s} = \theta_{\Sigma_1} = 2\theta$$

We therefore have a rich phenomenology of vortices and half vortices with many metastable configurations that could appear in the crusts of neutron stars.



The phase locking of the S-wave pair condensate, P-wave double pair condensate, and quartet condensate results in a greater total superfluid stiffness. Because the coupled condensates must wind together, more energy is required to impose a macroscopic phase twist, which corresponds directly to an increase in the superfluid density.

The conversion between quartets and S-wave pairs and P-wave pairs introduces low energy excitations of the condensate known as Leggett modes, preventing the drastic reduction in heat capacity typical of a single-channel S-wave superfluid as it cools.

There appear to be many interesting consequences of multimodal superfluidity for neutron star crust cooling and glitch dynamics.

Experimental evidence

S-wave pair binding in MeV

Nucleus	Orbitals	$S_n(N_0+1)$	$S_n(N_0+3)$	$S_{2n}(\text{pair})$	$2\Delta_S^{(3)}$	$2\Delta_S^{(4)}$
^{18}O	$(1d_{5/2})^2$	4.143	3.957	12.188	+3.90	+4.09
^{42}Ca	$(1f_{7/2})^2$	8.363	7.933	19.814	+3.09	+3.52
^{92}Zr	$(2d_{5/2})^2$	7.194	6.733	15.829	+1.44	+1.90
^{134}Sn	$(2f_{7/2})^2$	2.402	2.050	5.990	+1.19	+1.54

Experimental evidence

P-wave pair binding in MeV

Nucleus	Orbitals	$S_n(N_0+1)$	S_{2n}(g.s.)	$E_x(1^+)$	$2\Delta_P^{(3)}$
^{18}O	$1d_{5/2}1d_{3/2}$	4.143	12.188	8.817	+0.17

Experimental evidence

Quartet binding in MeV

Set	Binding Energies	Method	$4\Delta_Q$
Helium $\{4,6,8\}$	$B(^4\text{He}) = 28.296$ $B(^6\text{He}) = 29.271$ $B(^8\text{He}) = 31.396$	three-point	$+1.16$
Oxygen $\{16\ldots22\}$	$B(^{16}\text{O}) = 127.619$ $B(^{18}\text{O}) = 139.807$ $B(^{20}\text{O}) = 151.372$ $B(^{22}\text{O}) = 162.027$	four-point	$+0.14$
Calcium $\{40\ldots46\}$	$B(^{40}\text{Ca}) = 342.052$ $B(^{42}\text{Ca}) = 361.895$ $B(^{44}\text{Ca}) = 381.029$ $B(^{46}\text{Ca}) = 398.568$	four-point (subset 1)	$+0.44$
Calcium $\{42\ldots48\}$	$B(^{48}\text{Ca}) = 415.990$	four-point (subset 2)	$+0.74$
Calcium $\{40\ldots48\}$		five-point	$+0.60$

Summary and outlook

Using nuclear lattice effective field theory, we presented evidence for a new phase of matter called multimodal superfluidity exhibits simultaneous S-wave, P-wave, and quartet condensation. We showed that this phase emerges in neutron matter with realistic interactions and is a general feature of two-component Fermi systems with attractive S-wave and P-wave interactions. We discussed experimental evidence for multimodal superfluidity in nuclei and its implications for neutron star crust cooling and glitch dynamics.