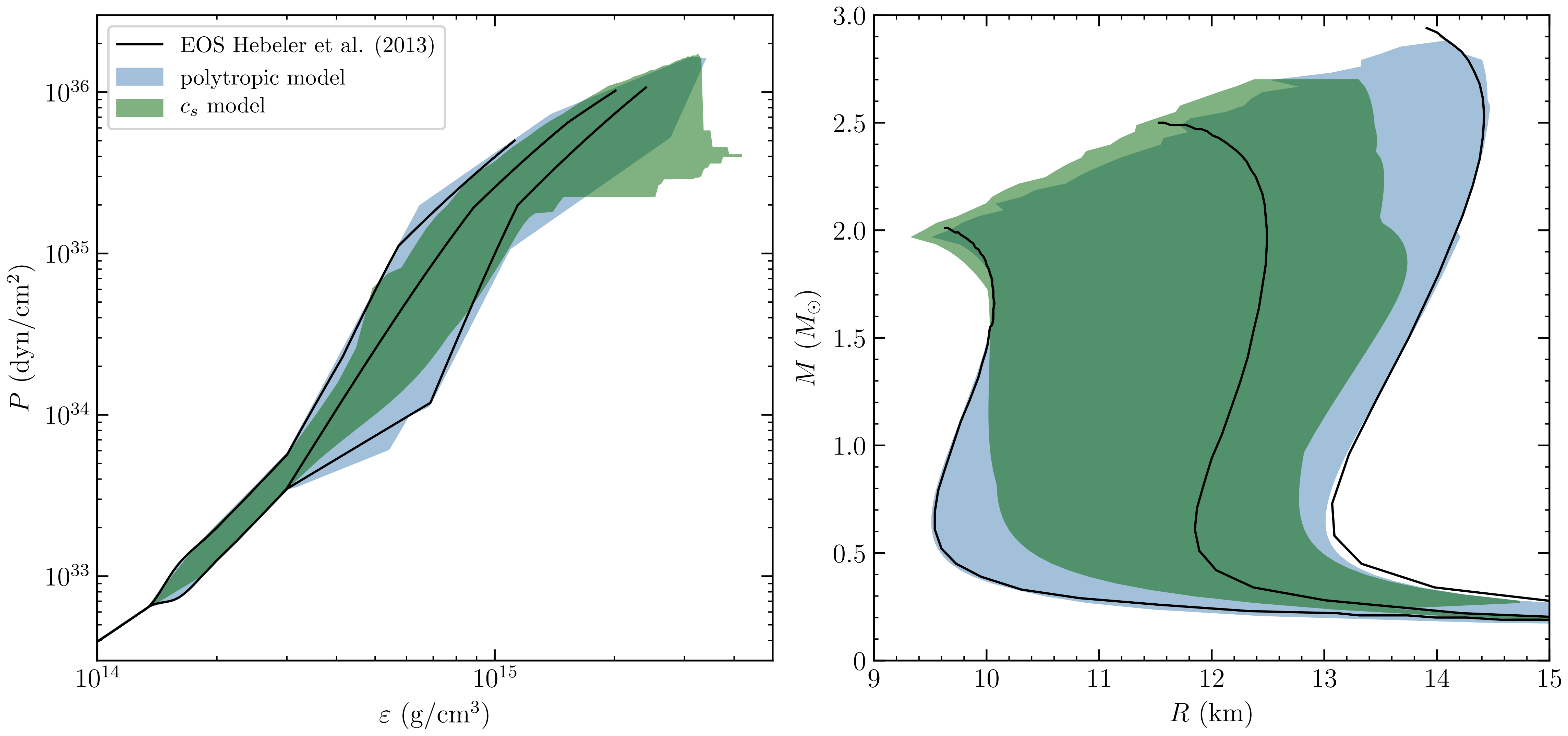
Nuclear matter calculations from chiral interactions

Yannick Dietz

Together with Faruk Alp, Kai Hebeler, Achim Schwenk

Nuclear Equation of State



Greif *et al.*, MNRAS (2019)

- Equation of state (EOS) relates pressure P and energy density ε (or here: $\frac{E}{A}$ with n)
- Understanding astrophysical phenomena requires knowledge about EOS
- This talk: calculating low-density EOS with many-body perturbation theory from chiral EFT interactions
- “low-density” for us: $0.3n_0 \leq n \leq 2n_0$

Low-density EOS from chiral EFT interactions

Drischler *et al.*, PRL (2019), Keller *et al*. PRC (2021), Keller *et al.*, PRL (2023)

- Use Chiral EFT as low-density effective theory of strong interaction
- Describe infinite, uniform, isotropic nuclear matter with grand-canonical potential $\Omega(T, \mu_n, \mu_p)$
- Perturbative expansion of Ω using MBPT
- Obtain free energy through Legendre transformation, pressure through derivative,…

Nuclear interaction
Chiral EFT: $H = (H_0 + U) + \lambda(V_{NN} + V_{3N} - U)$

↓

Grand-canonical potential
$\Omega(T, \mu) = -\frac{1}{\beta} \ln \mathrm{Tr}\left(e^{-\beta(H - \mu_n N_n - \mu_p N_p)}\right)$

↓

Approximation strategy
Many-body perturbation theory

↓

Gaussian process emulation

↓

Equation of state (EOS)
$F(x, T, n),\ P(x, T, n),\ \ldots$

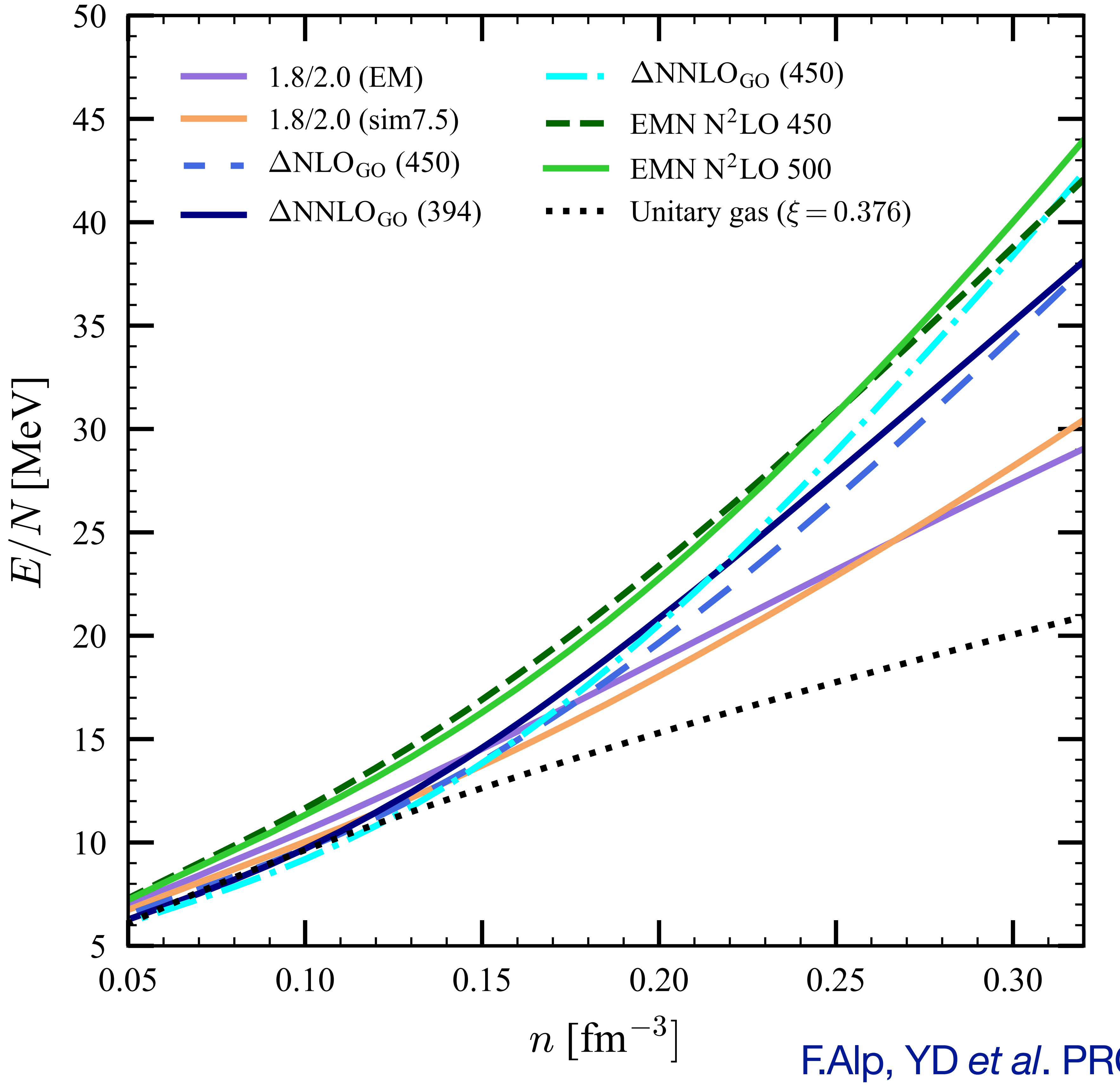
Low-density EOS from chiral EFT interactions

- Recently: Comparison between different families of chiral potentials and analysis of EOS F.Alp, YD *et al*. PRC (2025)
- Interactions: Δ_{GO}(450), -(394) family, EMN 500, EMN 450, 1.8/2.0 (EM), 1.8/2.0 (EM7.5) and 1.8/2.0 (sim7.5) Arthuis *et al*. (arXiv:2401.06675)
- Chiral order up to N3LO in NN and N2LO in 3N
- Restriction to pure neutron matter (PNM) and symmetric nuclear matter (SNM) at $T = 0$

Low-density EOS from chiral EFT interactions

PNM Results

- Consistent results up to $n \approx 0.16 \mathrm{fm}^{-3}$, larger spread beyond
- SRG-evolved family leads to softer EOS than unevolved interactions
- Unevolved interactions follow similar trend



F.Alp, YD *et al*. PRC (2025)

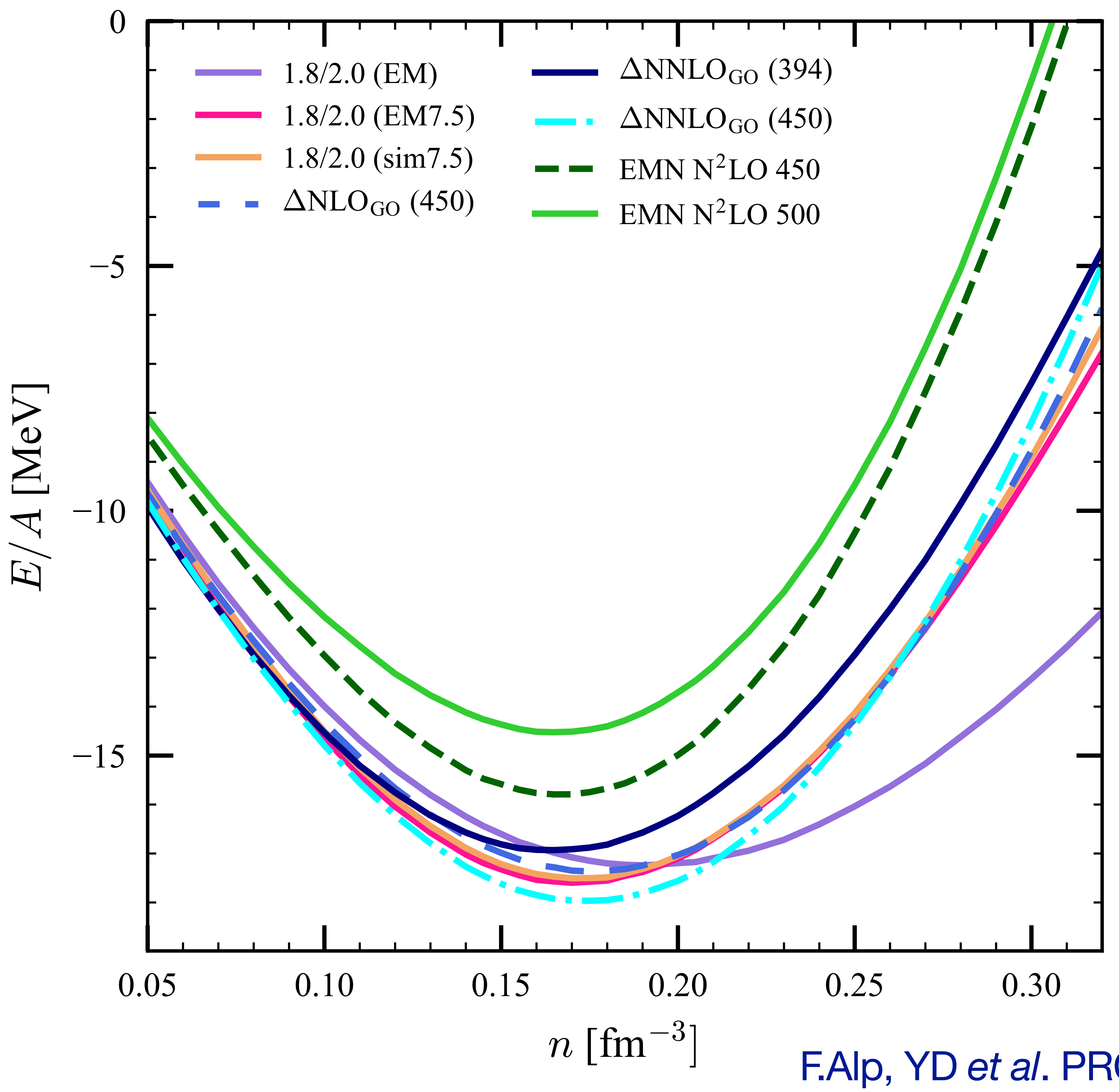
TECHNISCHE UNIVERSITÄT DARMSTADT

SFB 1245

Low-density EOS from chiral EFT interactions

SNM Results

- 1.8/2.0 (EM) interaction considerably more binding for high n
- N2LO EMN potentials saturate at higher E_{sat}
- Shift of LECs in 1.8/2.0 (EM7.5) induces stiffening



F.Alp, YD *et al*. PRC (2025)

TECHNISCHE UNIVERSITÄT DARMSTADT

SFB 1245

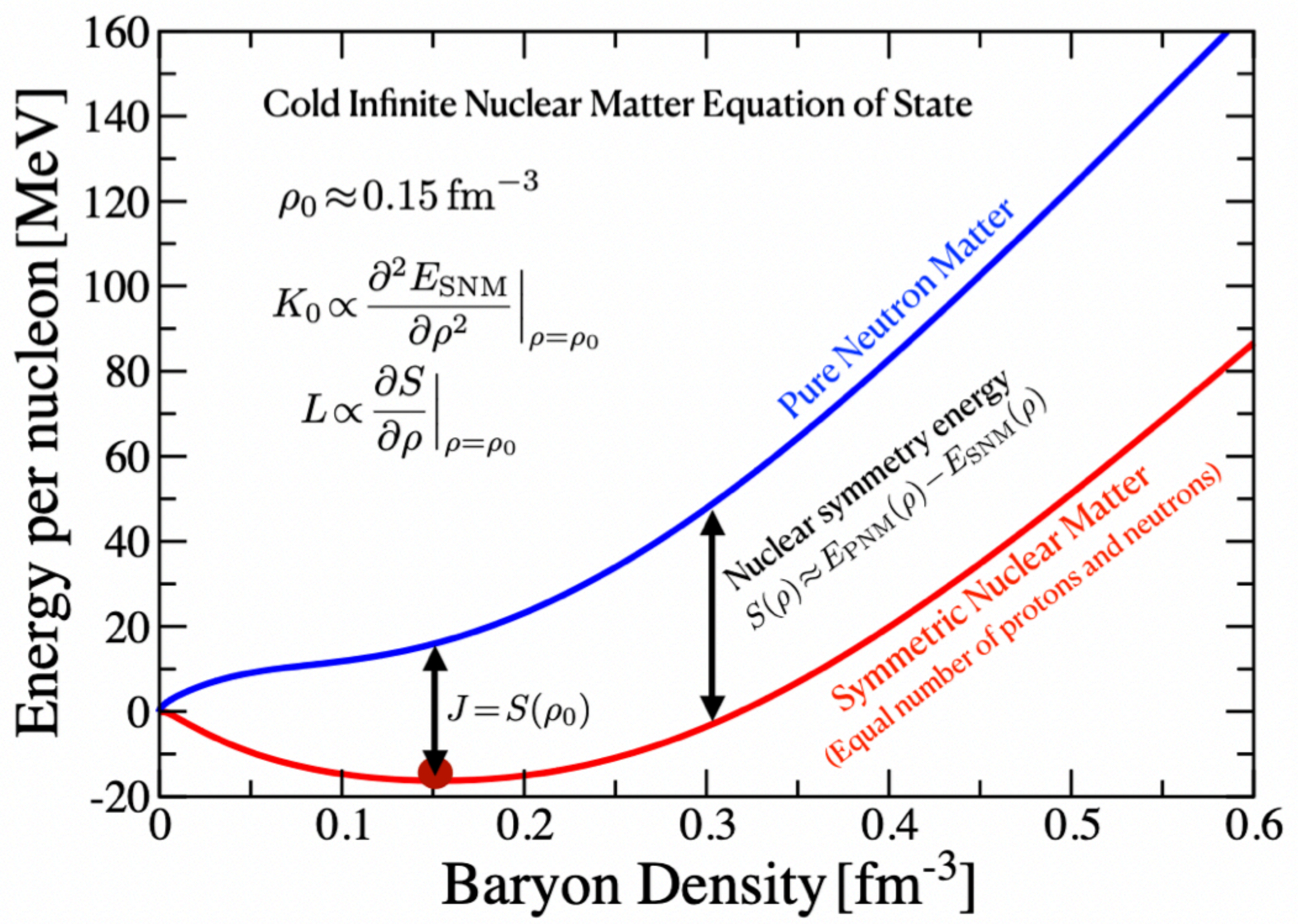
Quadratic expansion of EOS of asymmetric matter

- Capture essential EOS features in approx: $E(n,\alpha) = (\varepsilon_0 + \frac{1}{2}K\tilde{n}^2) + S(n)\alpha^2 + \mathcal{O}(\tilde{n}^3) + \mathcal{O}(\alpha^3)$

$$S(n) = (E_{\rm sym} + L\tilde{n} + \frac{1}{2}K_{\rm sym}\tilde{n}^2 + \mathcal{O}(\tilde{n}^3))$$

$$\tilde{n} = \frac{n - n_0}{3n_0}, \alpha = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

$$L = 3n_0 \frac{\partial S}{\partial n}\Big|_{n=0.16{\rm fm}^{-3}}, K = 9n_0^2 \frac{\partial^2 E(n,\alpha)}{\partial^2 n}\Big|_{\alpha=0, n=n_0}$$



Credit: J. Piekarewicz

- Obtain K by quadratic fit to minimum,
  L by derivative at $n = 0.16{\rm fm}^{-3}$

TECHNISCHE UNIVERSITÄT DARMSTADT

SFB 1245

Quadratic expansion of EOS of asymmetric matter

In this work:

- Slope parameter: $46.7\mathrm{MeV} \leq L \leq 67.3\mathrm{MeV}$

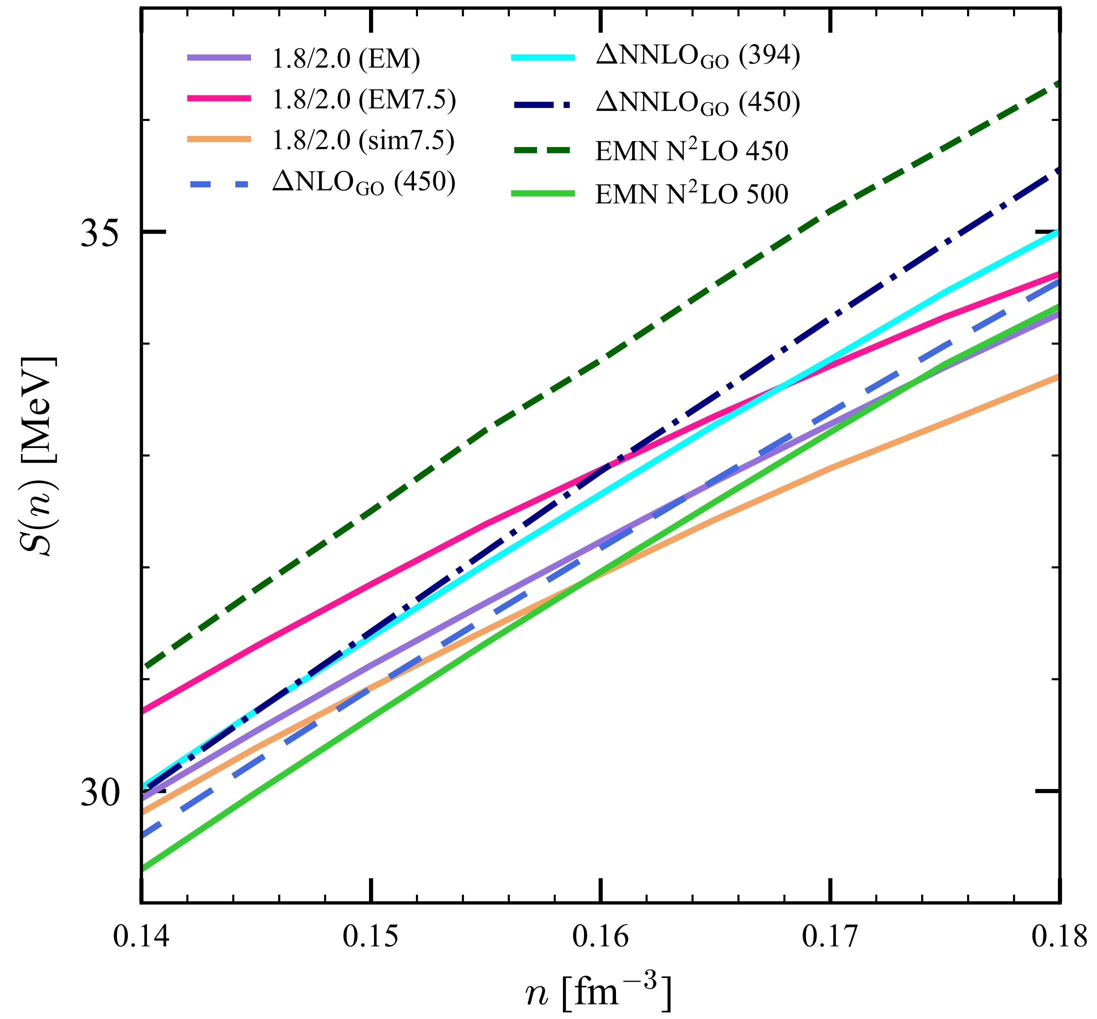
  $40\mathrm{MeV} \leq L \leq 65\mathrm{MeV}$*

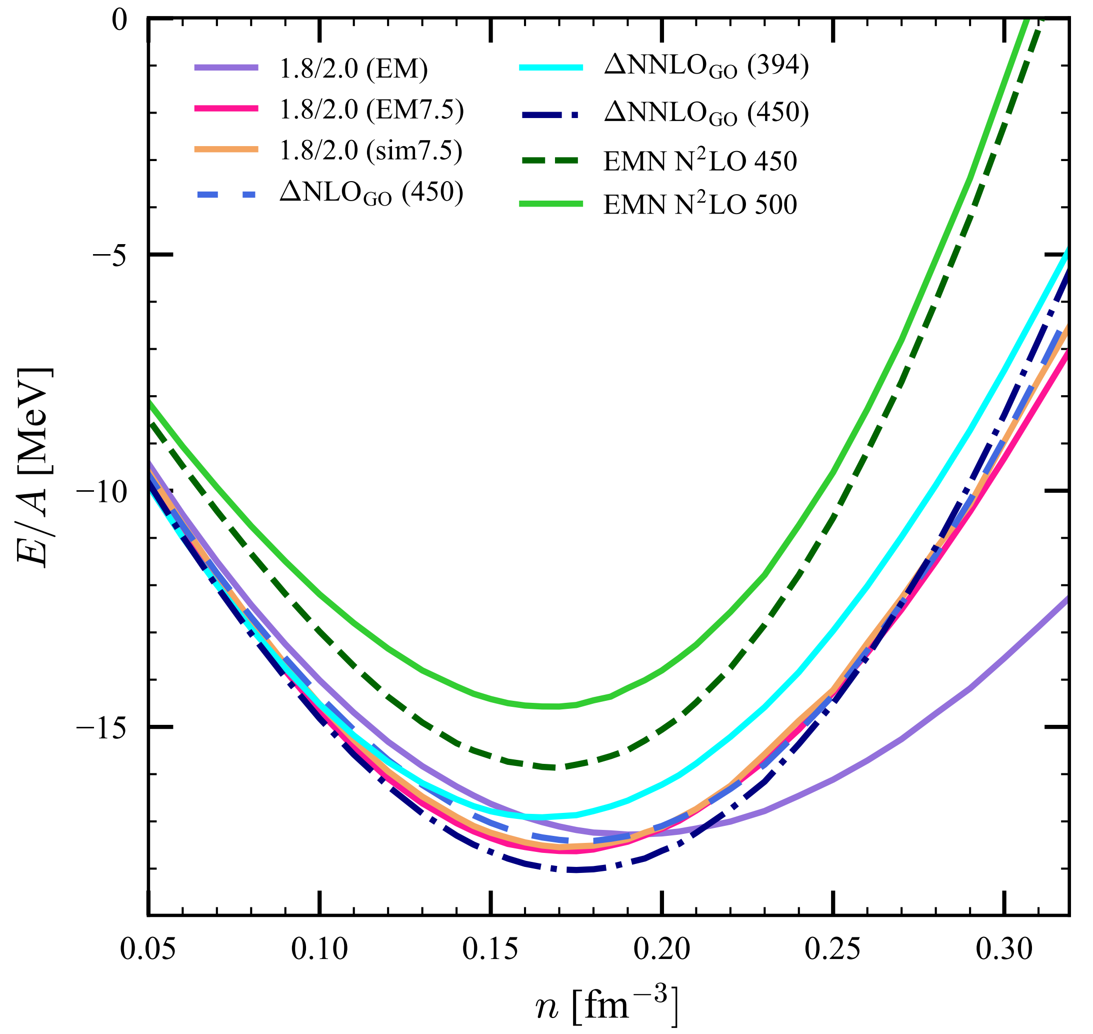
- Incompressibility: $251\mathrm{MeV} \leq K \leq 352\mathrm{MeV}$

  $K \simeq 230 \pm 20\mathrm{MeV}$*

- $\mathrm{NNLO}\Delta_{\mathrm{GO}}(450)$ yields highest L
  1.8/2.0 (sim7.5) yields smallest L

- N2LO EMN 450 yields highest K
  1.8/2.0 (EM) yields smallest K





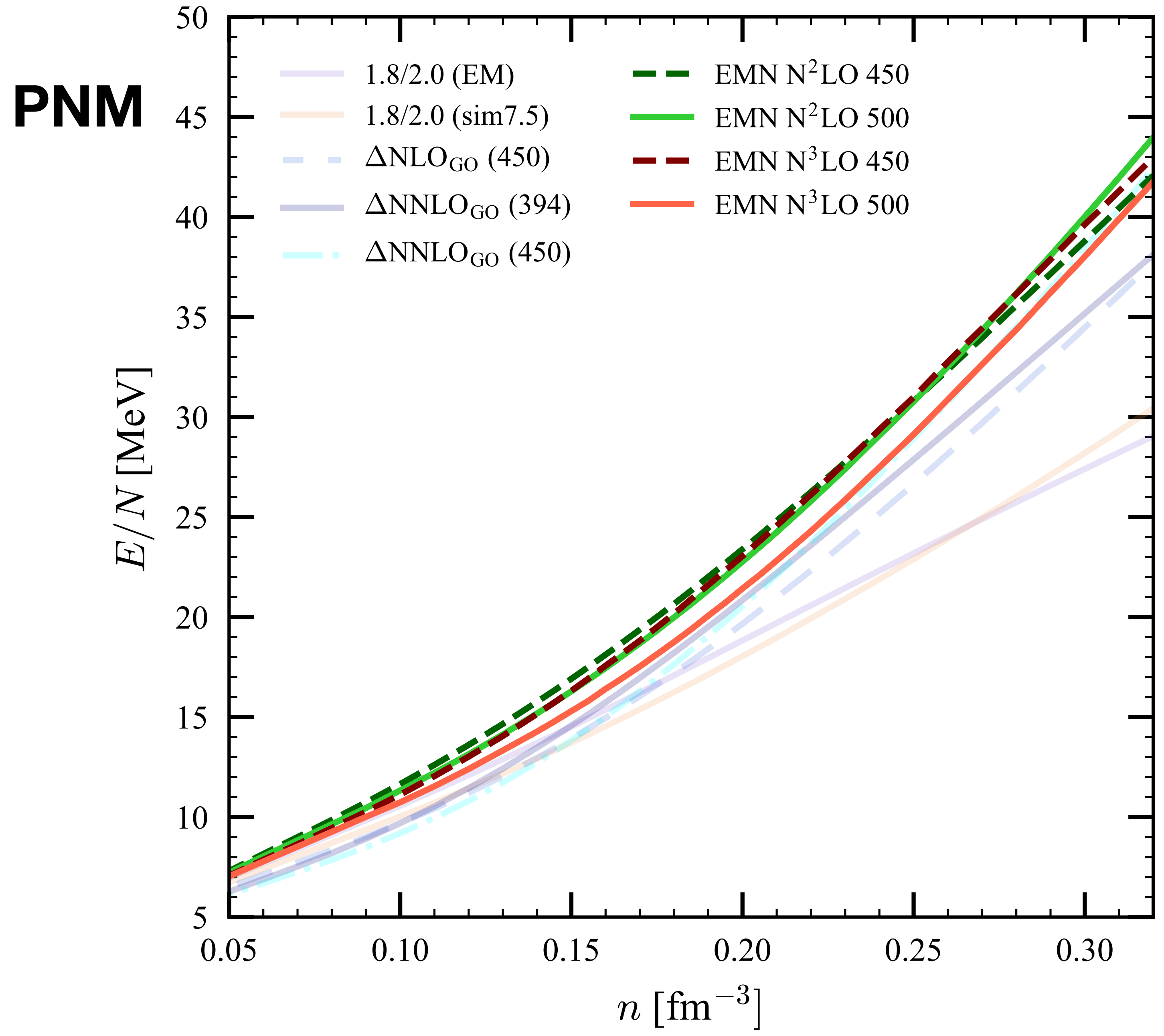
* Lattimer, arXiv:2308.08001

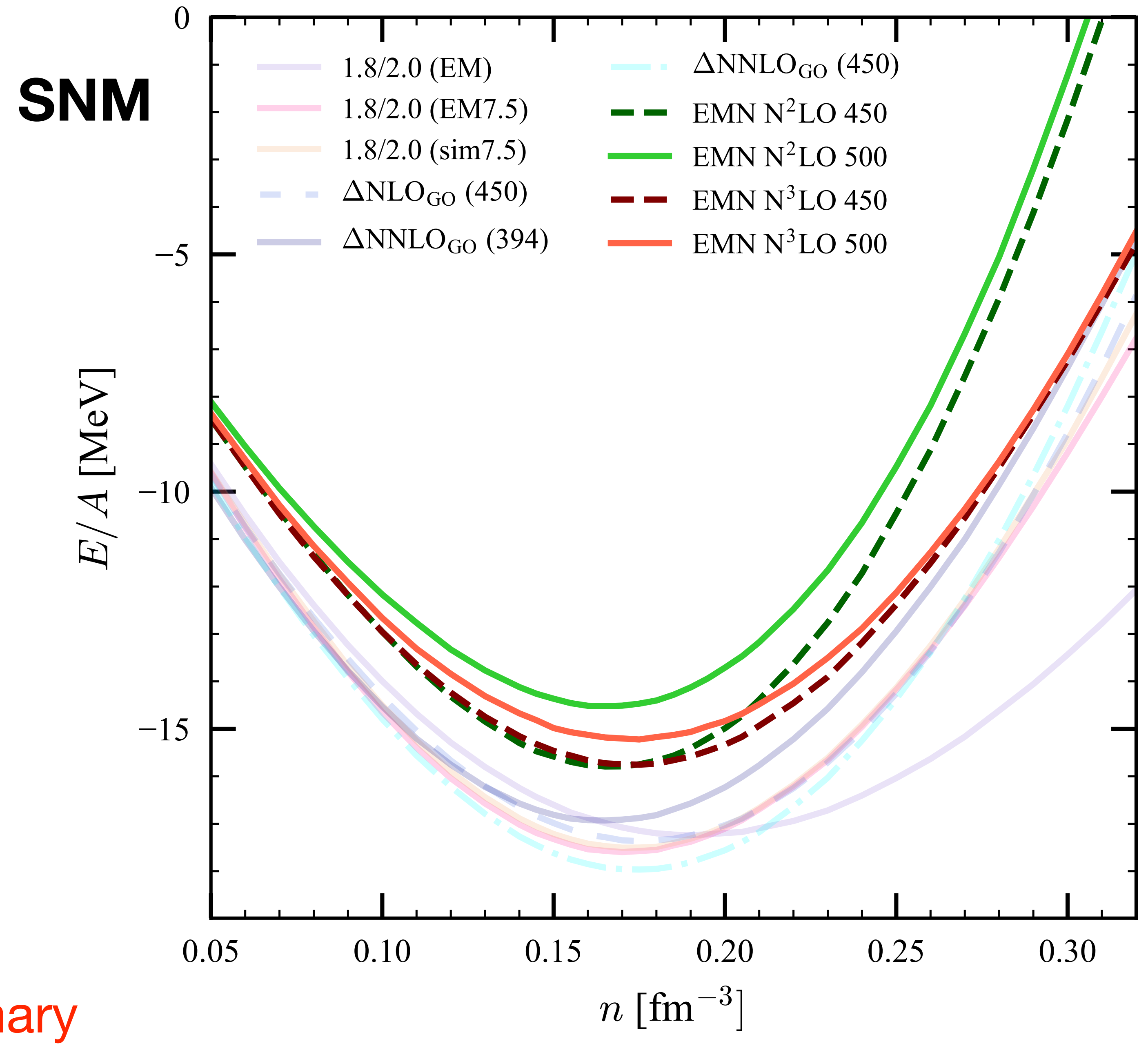
TECHNISCHE UNIVERSITÄT DARMSTADT

SFB 1245

Low-density EOS from N3LO chiral interactions

- Recently incorporated N3LO 3N topologies into framework
- Small extra contribution in PNM, bigger impact in SNM at high densities

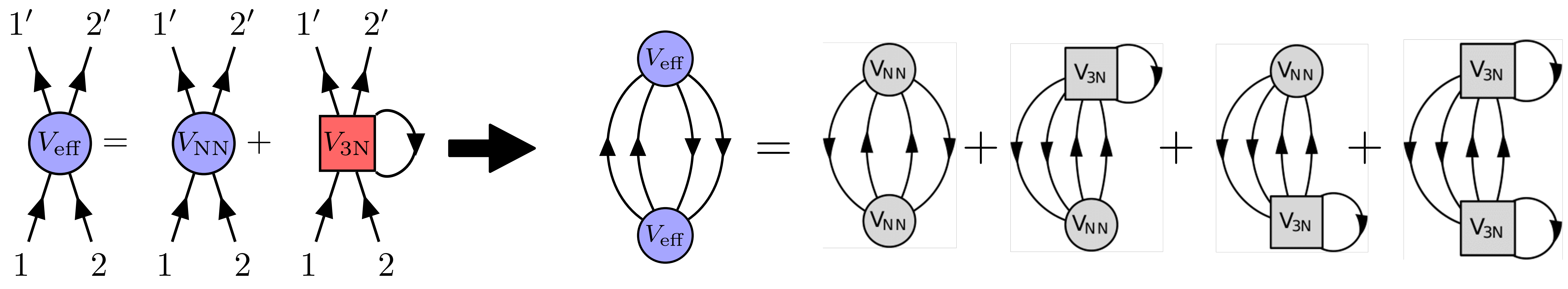




preliminary

Future developments

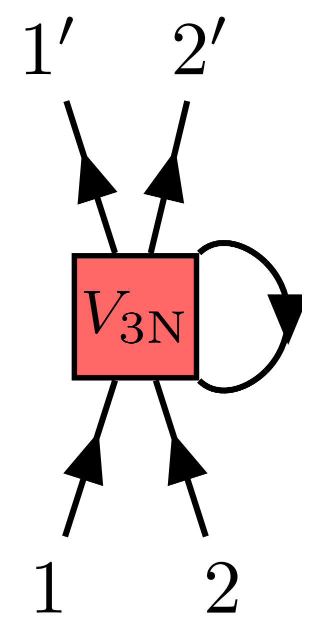
- Problem: Number of diagrams rises very fast with MBPT order → costly!
- Many diagrams involve self-closing loops
- Construct effective two-body vertex corresponding to NO2B approx.:

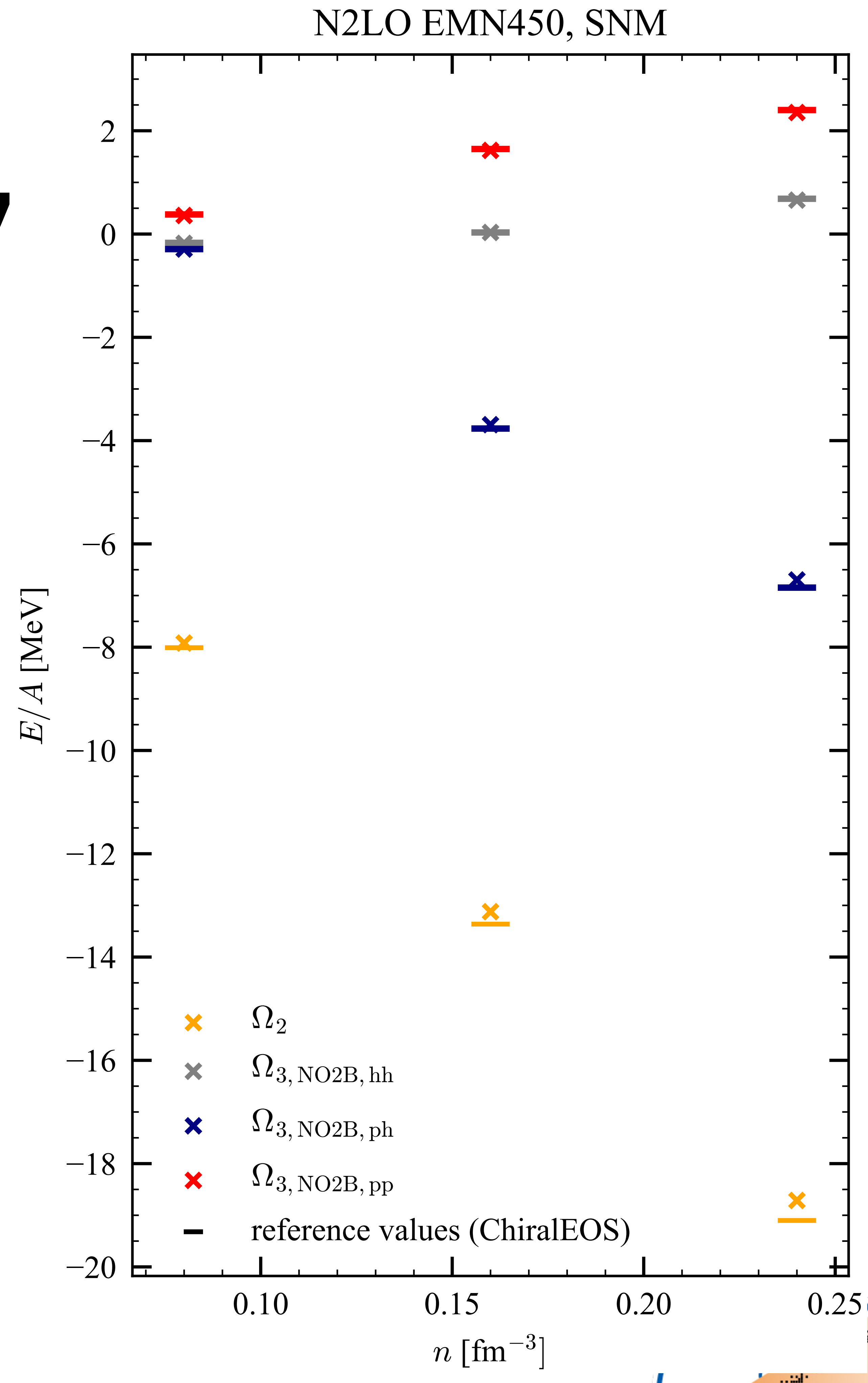


- Benefit: Reduce number of diagrams at high MBPT orders

Effective 2B-vertex: Benchmarks

- Precalculate eff. 2B-vertex on grid in $\mathbf{P}_{\mathrm{com}}, \mathbf{p}_{\mathrm{rel}}, \mathbf{p}'_{\mathrm{rel}}$ and interpolate when evaluating diagrams





- Eff. 2B-vertex within ~2% of full results in MBPT(2)/(3)
- Further interpolation strategies of vertex to be explored
- Future: MBPT(4) diagrams with eff. 2B vertex

Summary

- Use MBPT framework to compute nuclear matter properties
- Calculated PNM and SNM for different chiral interactions
- Extracted nuclear matter parameters from resulting EOSs
- Recently incorporated N3LO 3N force in matter
- Outlook: Calculation of effective 2B-vertex and MBPT(4) implementations

Thank you for your attention!

Backup slides

MBPT framework

Drischler *et al*., PRL (2019), Keller *et al*. PRC (2021), Keller *et al*., PRL (2023)

- Many-body method using expansion of grand-canonical potential

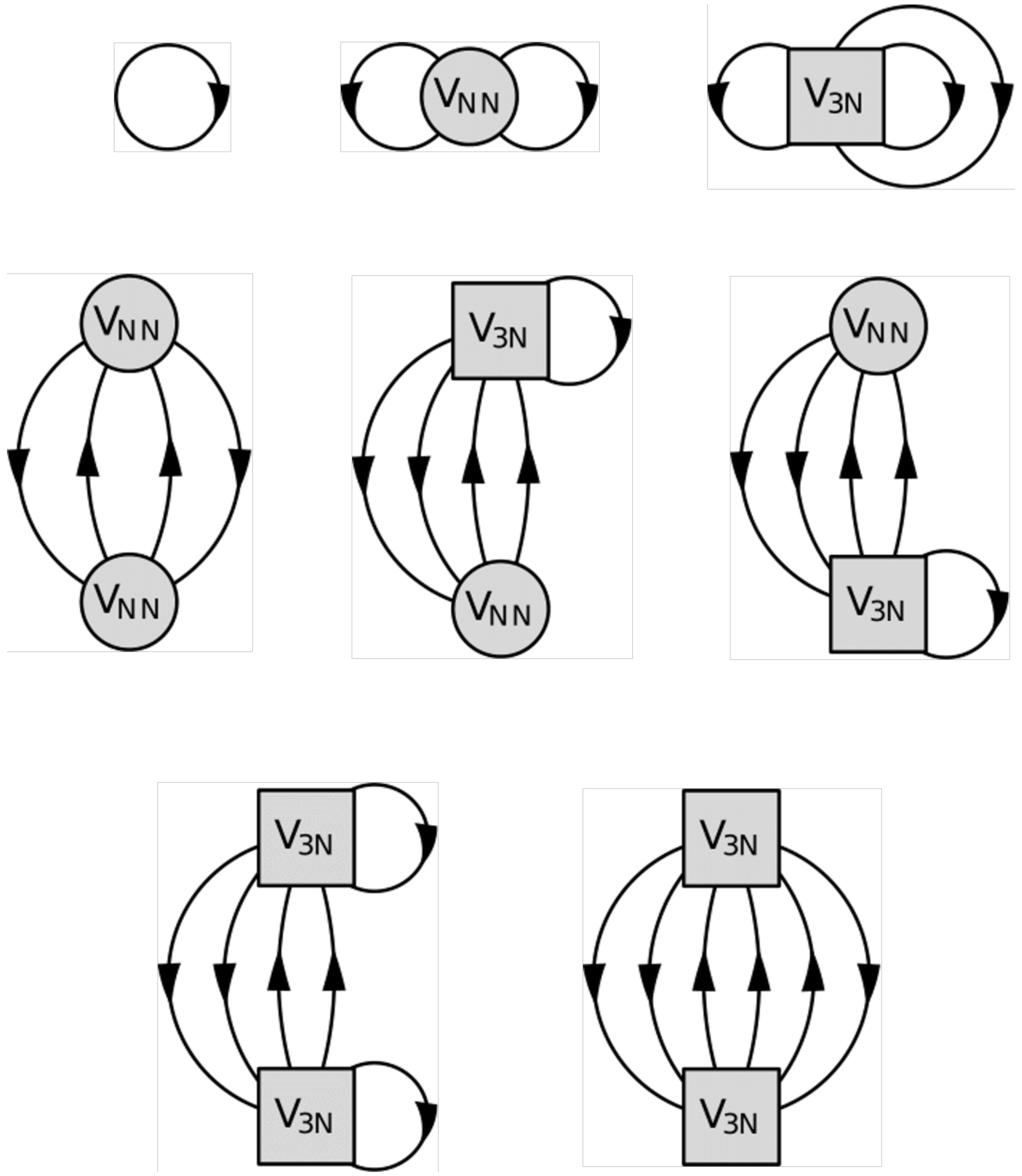
$$\Omega(T,\mu_n,\mu_p) = -\frac{1}{\beta}\log Z(T,\mu_n,\mu_p)$$

$$Z = \mathrm{Tr}\,\exp(-\beta[H-\mu_n N_n - \mu_p N_p])$$

- Power-series expansion

$$\Omega(T,\mu_n,\mu_p) = \sum_{l=0}^{\infty} \lambda^l \Omega_l$$

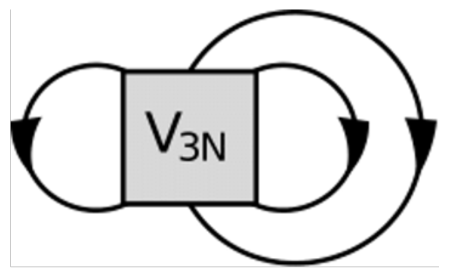
- Calculations conducted up to MBPT(3)



Hebeler, Schwenk, PRC (2010)

Calculation of MBPT diagrams

- Calculation of MBPT diagrams:



$$\hat{=}\frac{1}{6}\mathrm{Tr}_{\sigma_1,\tau_1}\mathrm{Tr}_{\sigma_2,\tau_2}\mathrm{Tr}_{\sigma_3,\tau_3}\int\frac{d\mathbf{k_1}}{(2\pi)^3}\frac{d\mathbf{k_2}}{(2\pi)^3}\frac{d\mathbf{k_3}}{(2\pi)^3}n_{\mathbf{k_1}}n_{\mathbf{k_2}}n_{\mathbf{k_3}}f_R^2(p,q)\langle 123|V_{3\mathrm{N}}|123\rangle$$

- Number and dimensionality of higher-order diagrams rises very fast
  $\rightarrow$ Monte-Carlo integration and importance sampling

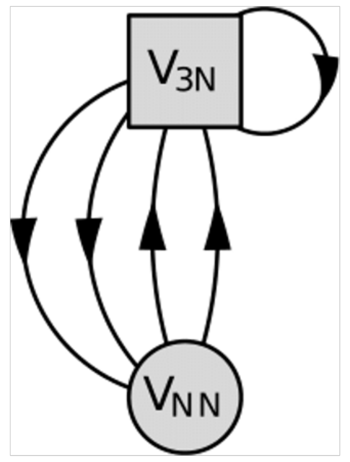
- $\langle 1'2'3'|V_{3\mathrm{N}}|123\rangle$ given in single-particle basis $|123\rangle = |k_1k_2k_3,\sigma_1\tau_1\sigma_2\tau_2\sigma_3\tau_3\rangle$

  e.g. $V_D = -\dfrac{g_A}{8f_\pi^2}\dfrac{c_D}{f_\pi^2\Lambda_\chi}\displaystyle\sum_{i\neq j\neq k}\dfrac{(\sigma_j\cdot\mathbf{q}_j)}{(q_j^2+m_\pi^2)}(\tau_i\cdot\tau_j)(\sigma_i\cdot\mathbf{q}_j)$

- Calculation of one grid point in (n, x, T) ~ 30min

Calculation of MBPT diagrams

- From 2nd order: need to evaluate self-energy of particles



$$\propto \Pi_i \left[\mathrm{Tr}_{\sigma_i,\tau_i} \int \frac{d\mathbf{k_i}}{(2\pi)^3} \right] f_R f_R' \langle 12|V_{\mathrm{NN}}|34\rangle\langle 345|V_{\mathrm{3N}}|125\rangle \frac{n_{\mathbf{k_1}} n_{\mathbf{k_2}}(1-n_{\mathbf{k_3}})(1-n_{\mathbf{k_4}}) - (1-n_{\mathbf{k_1}})(1-n_{\mathbf{k_2}})n_{\mathbf{k_3}} n_{\mathbf{k_4}}}{\epsilon_1+\epsilon_2-\epsilon_3-\epsilon_4} n_{\mathbf{k}}$$

- Individual diagrams solved to $5\mathrm{keV}$ accuracy

- Matter/ Hamiltonian needs to be perturbative for approach to work

TECHNISCHE UNIVERSITÄT DARMSTADT

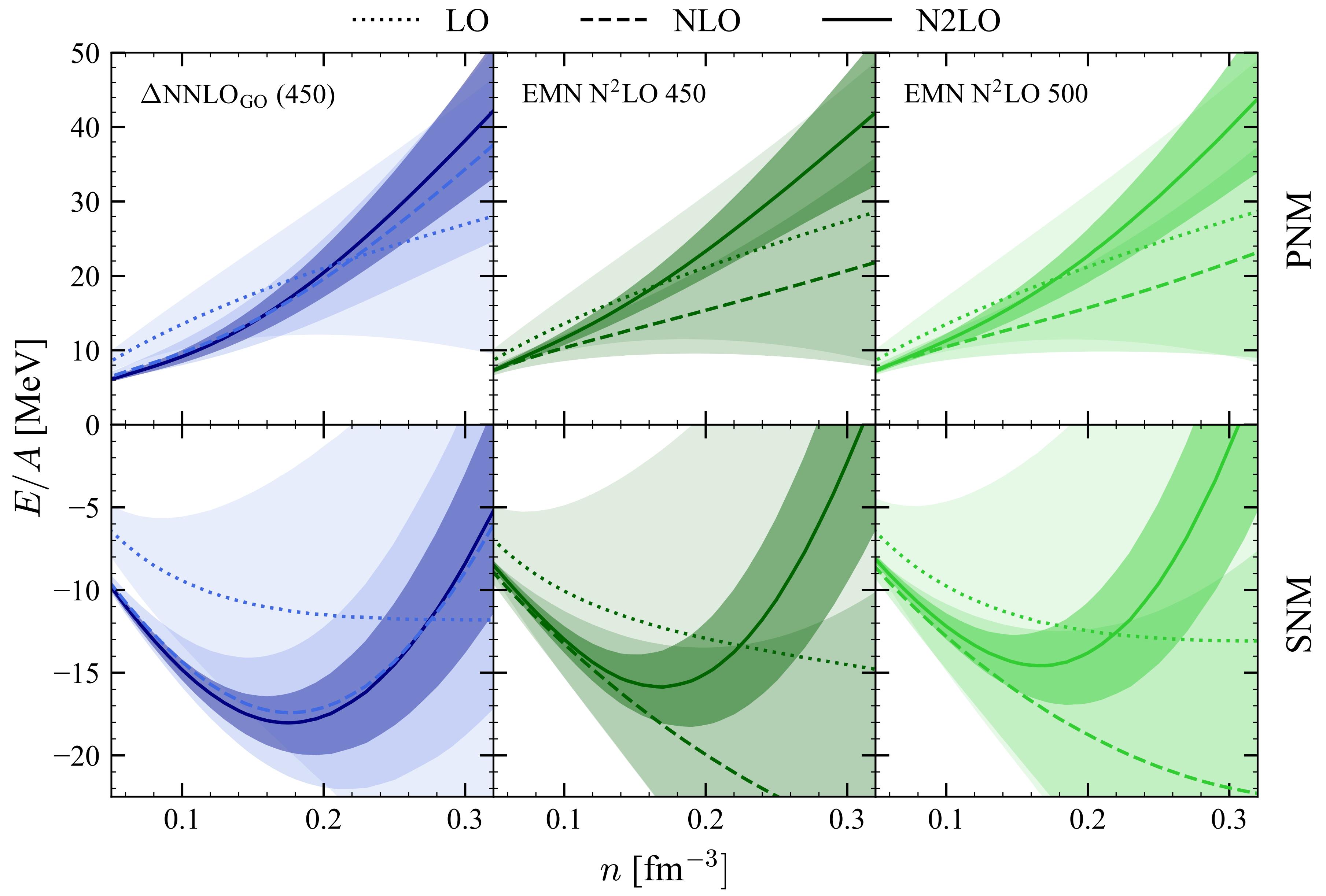
SFB 1245

Statistical uncertainties for chiral truncation

- Development of Gaussian Process framework to compute statistical uncertainties of EFT truncation

Work by Hannah Göttling

- Applicable to EMN 450, EMN 500 and $\Delta_{\mathrm{GO}}(450)$
- EMN uncertainties comparable to $\Delta_{\mathrm{GO}}(450)$ uncertainties in both PNM, SNM
- Yet, convergence behaviour of $\Delta_{\mathrm{GO}}(450)$ uncertainties improved compared to EMNs



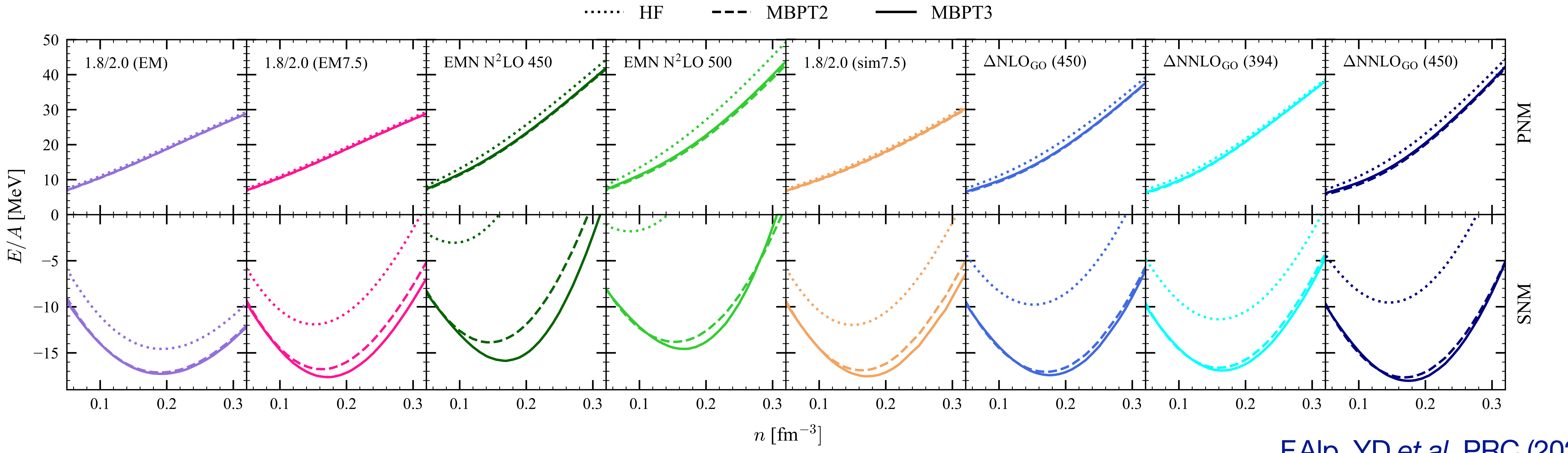
Many-body uncertainties

- Limited thorough tools to assess many-body uncertainties at the time
  → estimate by MBPT(3) contribution

See Isak Svensson's talk

- SRG-evolved and deltafull potentials show better MBPT convergence
- MBPT(3) contribution dominated by chiral uncertainty (where applicable)

See Hannah Göttling's talk



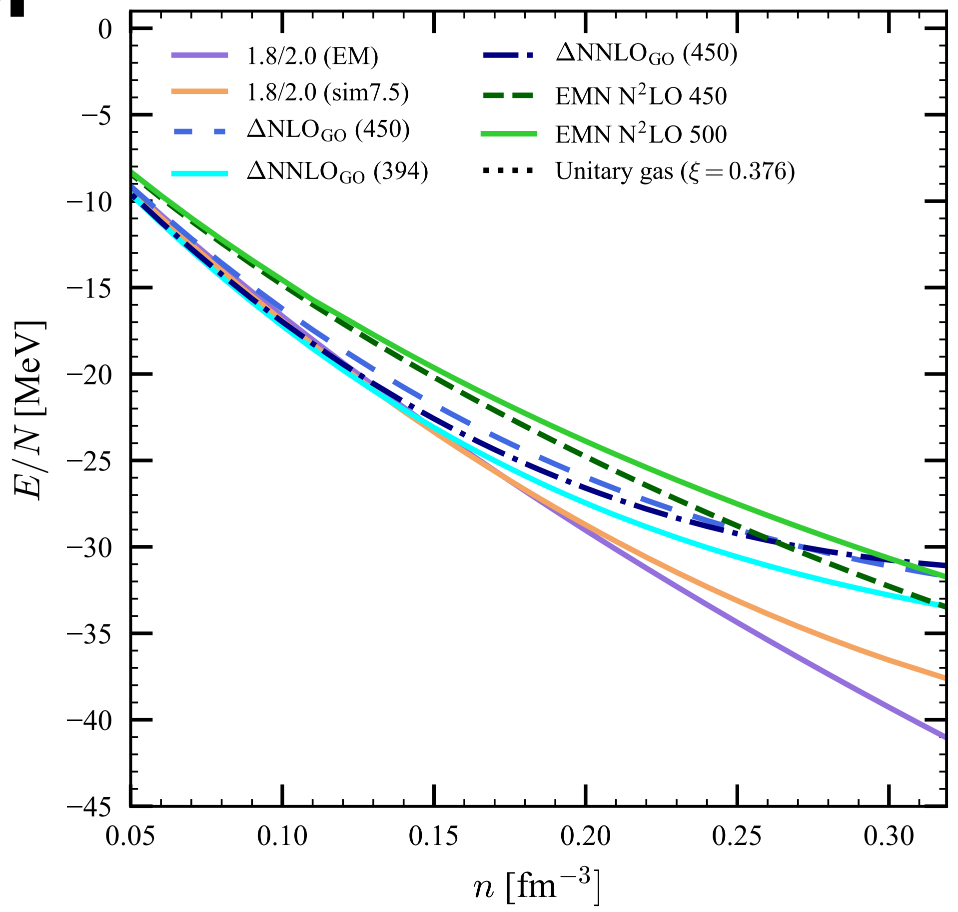
F.Alp, YD *et al*. PRC (2025)

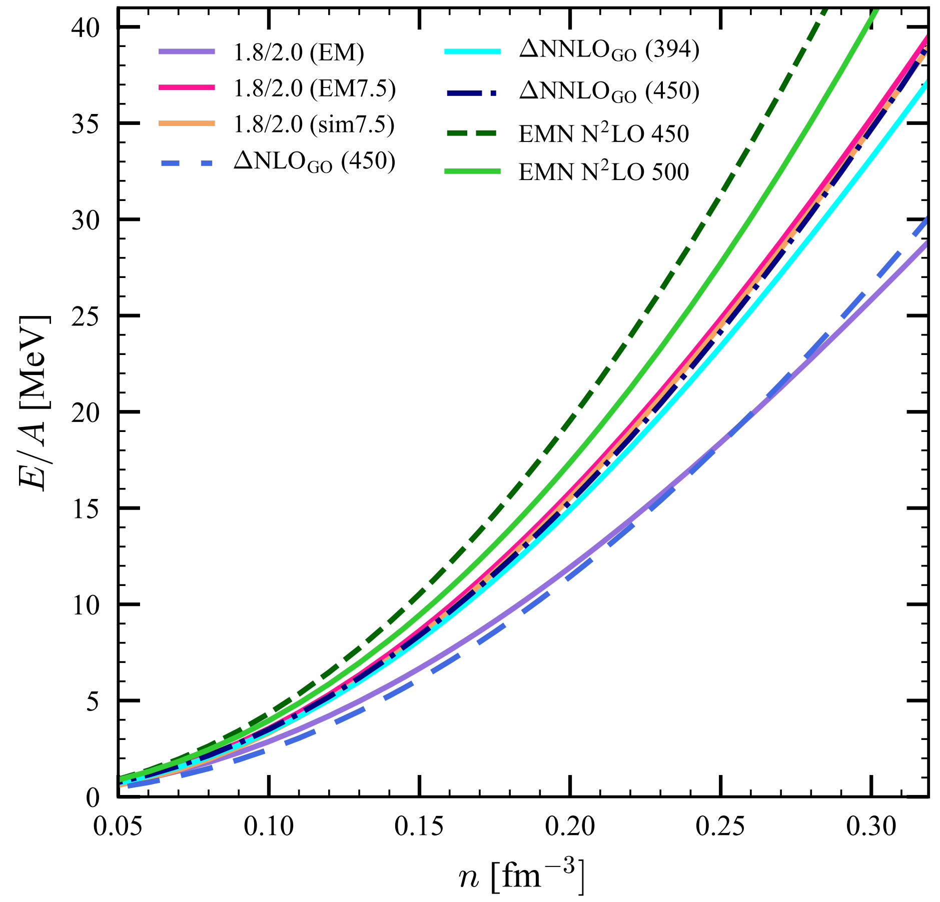
Low-density EOS from chiral EFT interactions

HF NN and HF 3N results

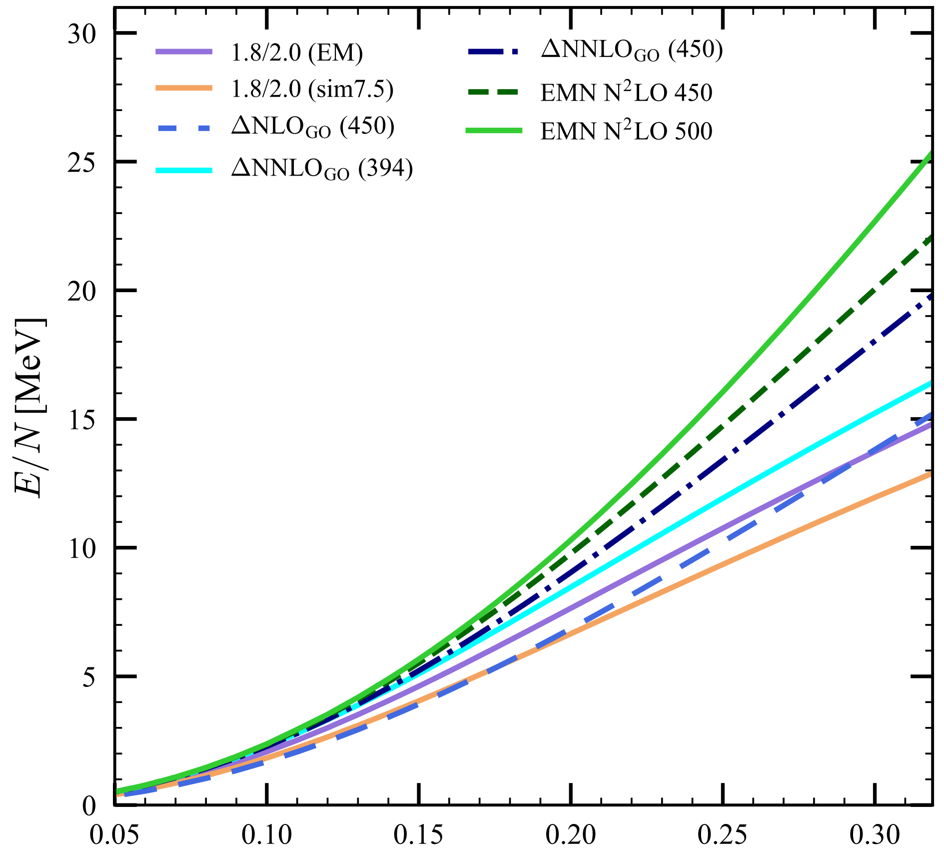
PNM **SNM**

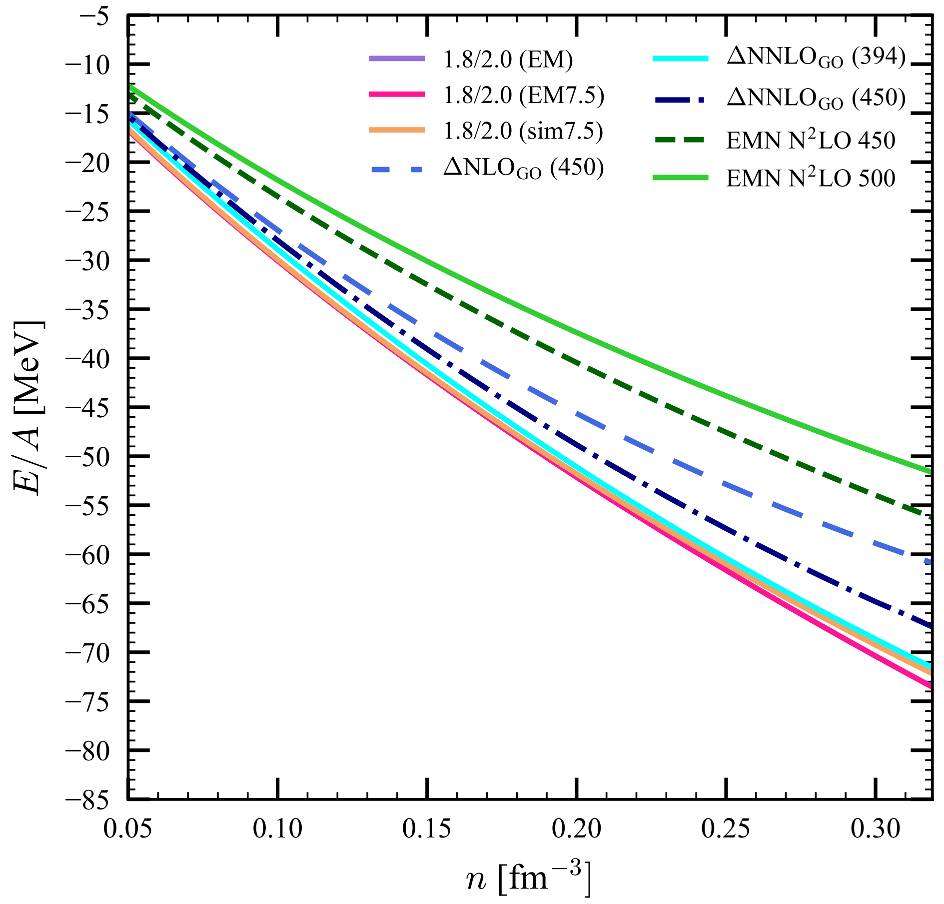
NN





3N

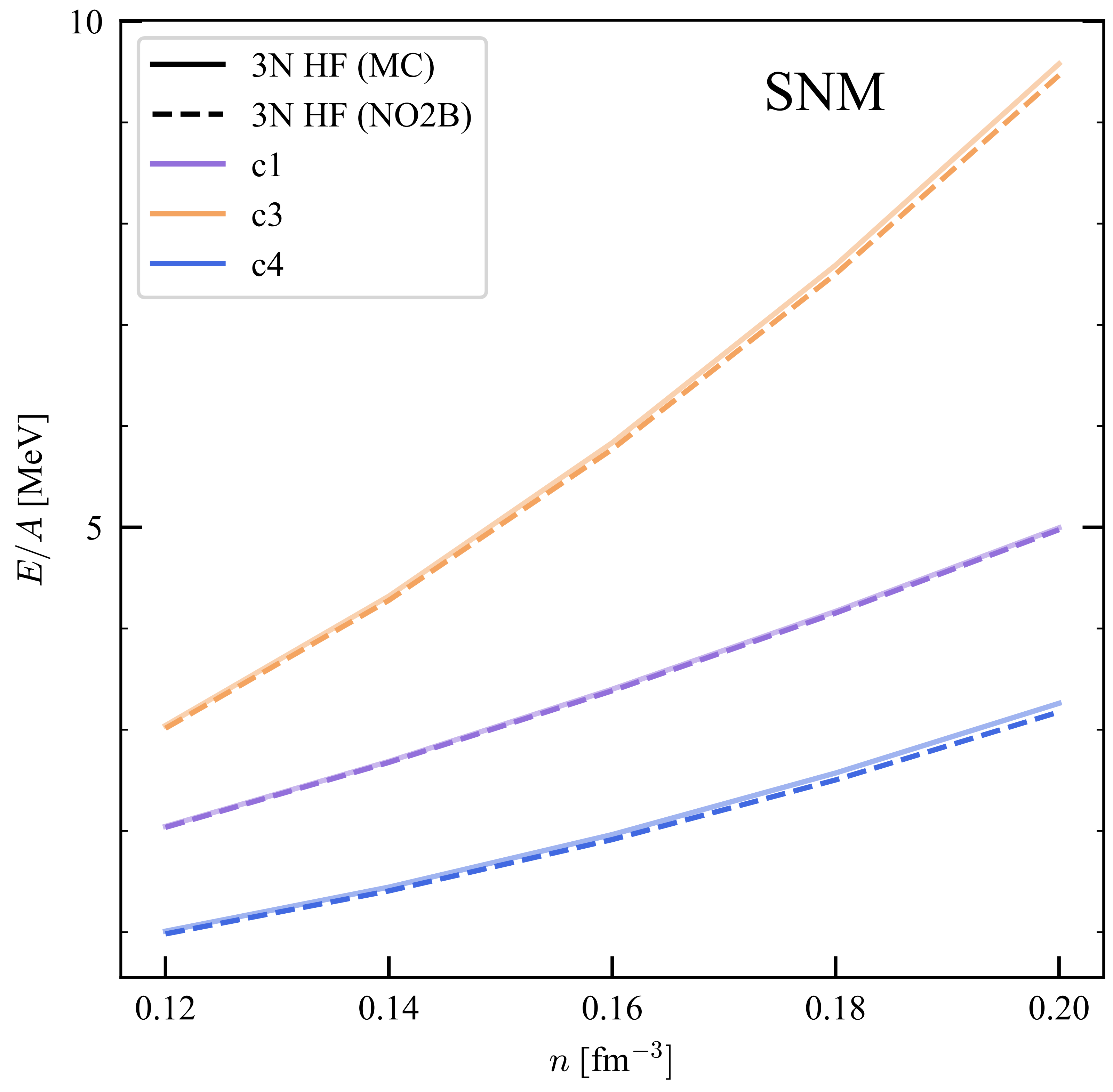




Effective 2B vertex: Benchmarks

- Conducted successful benchmarks at HF level for all N2LO topologies:

$$E_{3\mathrm{N,HF}} = \frac{1}{6}\mathrm{Tr}_{\sigma_1,\tau_1}\mathrm{Tr}_{\sigma_2,\tau_2}\int \frac{d\mathbf{k}_1}{(2\pi)^3}\frac{d\mathbf{k}_2}{(2\pi)^3} n_{\mathbf{k}_1} n_{\mathbf{k}_2} \overline{V}_{3\mathrm{N}}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_1,\mathbf{k}_2)$$



Open points:

- Implementation and benchmarking of N3LO topologies

ChiralEOS

- Code with which we calculate $\frac{E}{A}$:

Density range (fm^{-3})

Make sure to pick right interaction parameters

```
chiraleos -T 0 -n 0.04:0.13:0.01 -x 0:0.5:0.5 -O 3 -f parameter_files/EMN.csv -l 15 -o NLOEMN450_eos_low_J12.h5 -R -A
chiraleos -T 0 -n 0.14:0.21:0.005 -x 0:0.5:0.5 -O 3 -f parameter_files/EMN.csv -l 15 -o NLOEMN450_eos_sat_J12.h5 -R -A
chiraleos -T 0 -n 0.22:0.32:0.01 -x 0:0.5:0.5 -O 3 -f parameter_files/EMN.csv -l 15 -o NLOEMN450_eos_high_J12.h5 -R -A
```

Temperature (MeV)

MBPT order

Output file

Proton fraction

TECHNISCHE UNIVERSITÄT DARMSTADT

SFB 1245

ChiralEOS

- Output: energy values for different diagrams
- Higher orders give smaller contributions and (usually) take longer

```
------------------------------------------------------------------
Summary
------------------------------------------------------------------
x = 0.5
n = 0.16 fm^(-3)
T = 0 MeV

omega1_NN                     43.848610          8.62s
omega1_3N                     -15.219605         6.89s
omega2_NN_NN                  -2.306598          1.51m
omega2_3N_NN                  0.214546           2.38m
omega2_3N_3N                  -0.450606          60.00s
omega2_3N_res                 -0.096396          3.28m
omega3_pp                     -0.054936          2.95m
omega3_ph                     0.224539           1.96m
omega3_hh                     -0.130430          2.85m
omega3_pp_3N_NN_NN            0.125019           29.41s
omega3_ph_3N_NN_NN            -0.120508          41.48s
omega3_hh_3N_NN_NN            0.064537           19.18s
omega3_pp_3N_3N_NN            0.016576           40.19s
omega3_ph_3N_3N_NN            -0.281691          1.33m
omega3_hh_3N_3N_NN            -0.016115          47.31s
omega3_pp_3N_3N_3N            0.014052           1.40m
omega3_ph_3N_3N_3N            0.046065           1.36m
omega3_hh_3N_3N_3N            0.008648           1.41m
all diagrams                  25.885708

total free energy             -16.877018


Done with 66.6667%
Took 26.23m
```

Evaluation of full EOS ~ 10-16h

Yannick Dietz | Institut für Kernphysik, TU Darmstadt