Quantifying uncertainties for the nuclear equation of state in β-equilibrium

Hannah Göttling

in collaboration with Luis Hoff, Kai Hebeler and Achim Schwenk

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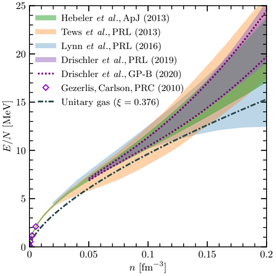
The nuclear equation of state (EOS)
From nucleons to neutron stars

- connect microscopic calculations of dense matter to astrophysical observations

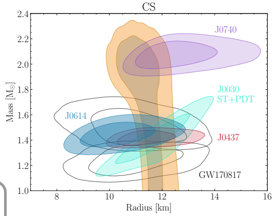


Huth et al., PRC (2021)

The equation of state (EOS) is ...

... given by pressure as a function of density

... key to neutron stars and mergers

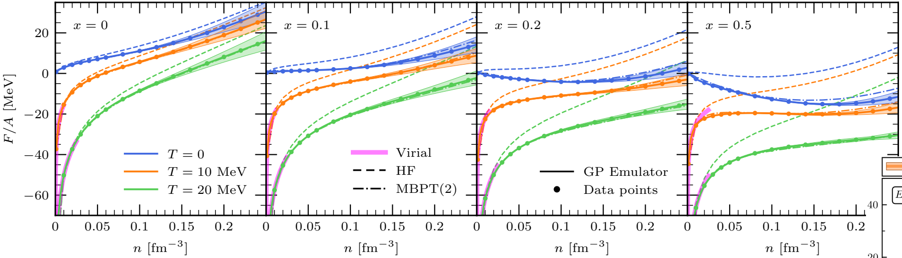


from I. Svensson, M. Mendes, N. Rutherford

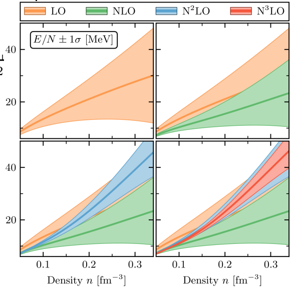
- **goal:** provide microscopic information with quantified uncertainties

EOS developments and uncertainties

- nucleonic matter results for different proton fractions (x) and temperatures (T)



Keller et al., PRL (2023)



- order-by-order corrections from effective field theory (EFT)
- quantify truncation uncertainties with Gaussian processes (GP)

Drischler et al., PRC (2020)

goal: expand GP uncertainties to arbitrary *x*

GP uncertainty quantification

following Melendez et al. PRC (2019)

- collection of random variables: $f(x) \sim GP[m(x), \kappa(x, x'; \theta)]$
- kernel: $\kappa(x, x'; \theta) = \bar{c}^2 \, exp\left[-\frac{1}{2}(x - x')^T L (x - x')\right]$

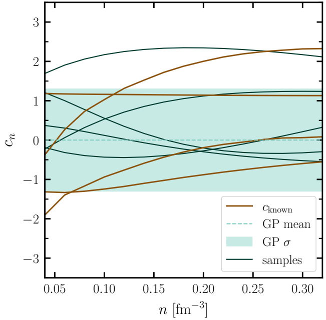
2D correlations

$$L = \begin{pmatrix} \ell_n^{-2} & 0 \\ 0 & \ell_x^{-2} \end{pmatrix}$$

- consider EFT expansion for observable

$$E(x) = E_{\text{ref}}(x)\left[\underbrace{\sum_{i=0}^{k} c_i(x) Q^i(x)}_{\text{known}} + \underbrace{\sum_{i=k+1}^{\infty} c_i(x) Q^i(x)}_{\Delta E(x)}\right]$$

$\rightarrow$ reference energy E_{ref} and expansion parameter $Q = \frac{k_F}{\Lambda_b}$



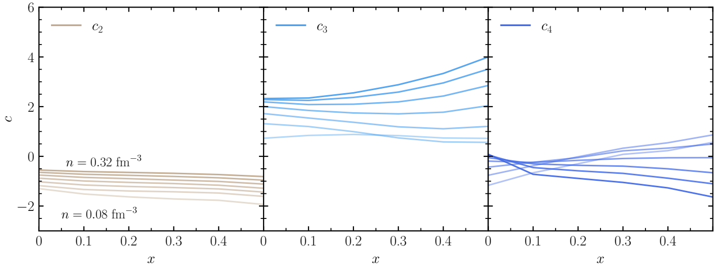
$\Rightarrow$ **hyperparameters $\bar{c}^2, \ell$ are determined from expansion coefficients**

Asymmetric matter

application of GP 2D to finite proton-fraction results - together with Luis Hoff

- consider asymmetry ($\alpha = 1 - 2x$) in reference scale

$$E_{\mathrm{ref}}^{\mathrm{BQ+3B}} = \underbrace{16\ \mathrm{MeV}\left(\frac{n}{n_0}\right)^{2/3}}_{\text{Drischler et al., PRC (2020)}} \underbrace{+\,4\ \mathrm{MeV}\left(\frac{n}{n_0}\right)^{6/3}(1-\alpha^2)}_{\text{3N informed}}$$



comparison

GP-1D:
individual $\bar{c}^2$

GP-2D:
one $\bar{c}^2$

choices

- stationary kernel
- inverse χ^2 prior for $\bar{c}^2$
- representative grid

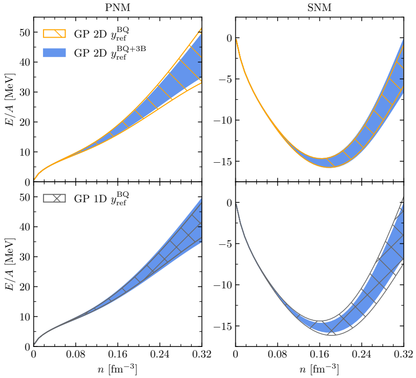
Asymmetric matter
comparison to GP 1D

GP-1D: different $\bar{c}^2$

GP-ND: one $\bar{c}^2$

- pure neutron matter (PNM) bands are increased
- symmetric nuclear matter (SNM) bands are decreased

$\rightarrow$ *x*-dependent reference scale counteracts this effect



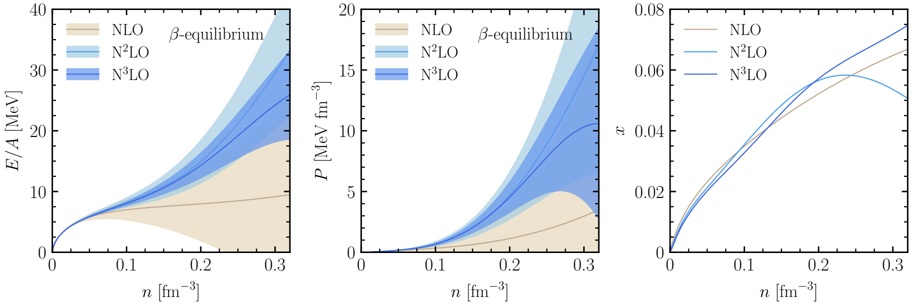
Neutron star matter

GP derivatives and uncertainty propagation

- constructed GP-2D kernel
  - $\rightarrow$ access uncertainties for arbitrary x
  - $\rightarrow$ access thermodynamic quantities $(P, \mu, ...)$

β-equilibrium:

- $\mu_n = \mu_p + \mu_e, \qquad \mu_{\nu,\bar{\nu}} = 0$
- $\mu_n - \mu_p \sim \partial_x E/A$



$\rightarrow$ **we provide the EOS of nuclear matter in β-equilibrium with quantified chiral uncertainties**

EOS application

Neutron star crust - with Luis Hoff

- extending the EOS distribution to a lower density regime

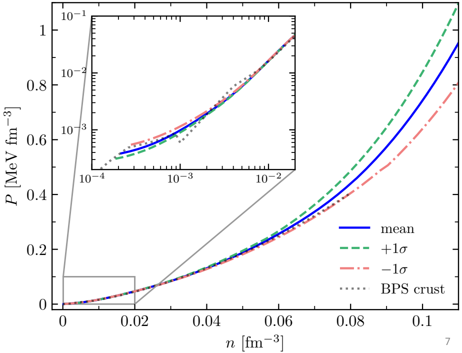
challenge:

- not only homogeneous matter anymore
- combining matter results with further conditions

method:

- providing P and μ by GP-2D
- taking the limits of the 68% bands
- create continuous low density extension

- phase coexistence allowing proton drip in all scenarios

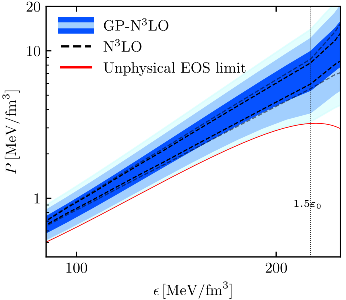


EOS application

Observation inference - together with Melissa Mendes and Isak Svensson

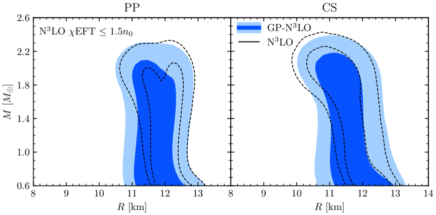
- integration of the EOS bands in an inference framework to compare with observations

EOS model: combined chiral EFT constraints with crust and high density extensions (PP, CS)
observations: pulsar mass measurements, NICER, and LIGO/Virgo



- create EOS samples from EFT band
- ensure the pressure to never decrease

→ posterior samples for the EOS in the mass-radius plane



Summary and outlook

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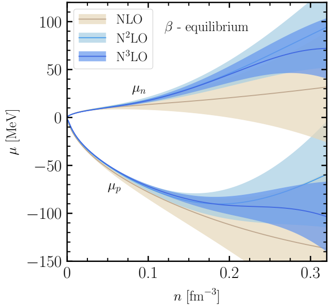
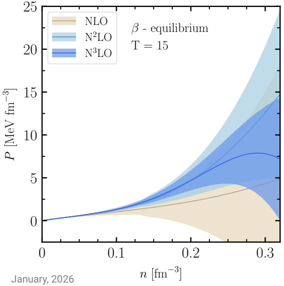
- extension of the GP uncertainty quantification to finite *x*
- accessing β-equilibrium and propagating uncertainties to other thermodynamic quantities





- application to crust extension and incorporation in astrophysical inference framework
- extend the GP to finite temperature results
- consider further sources of uncertainties