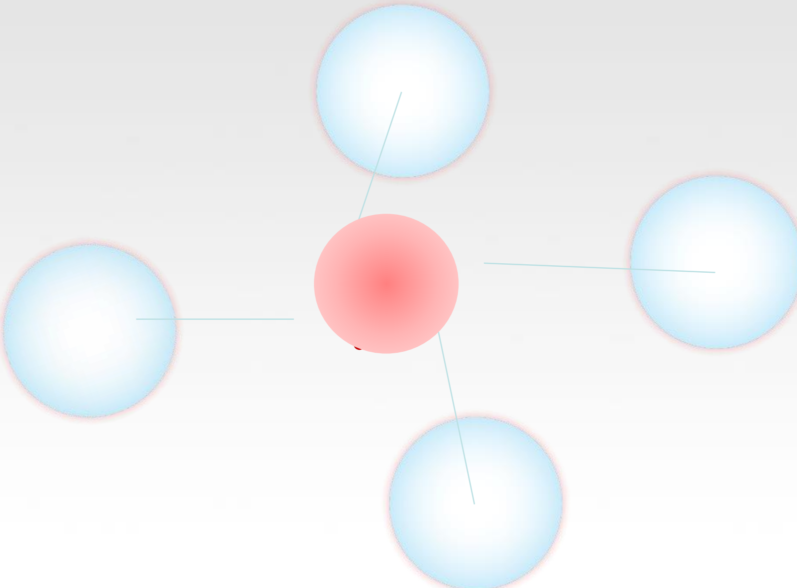
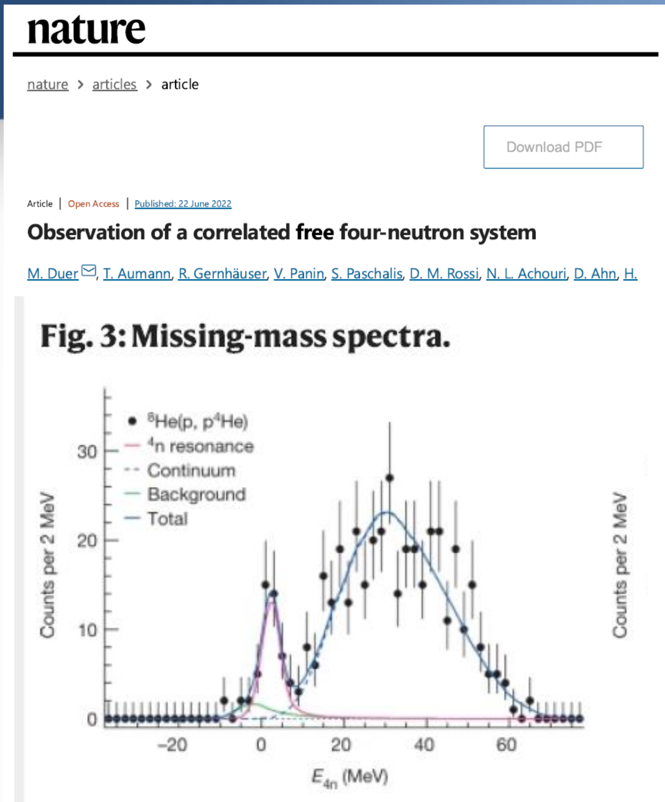


Description of 4n halo nuclei

and their decay

R. Lazauskas (IPHC)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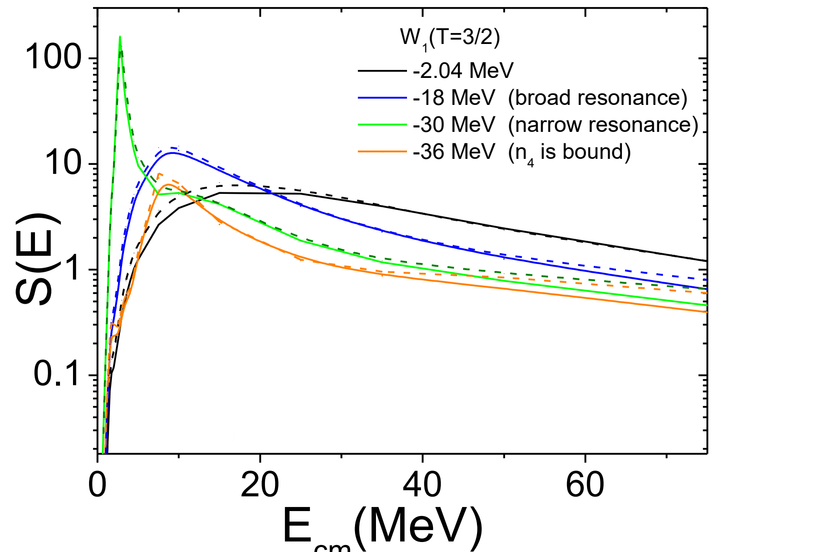
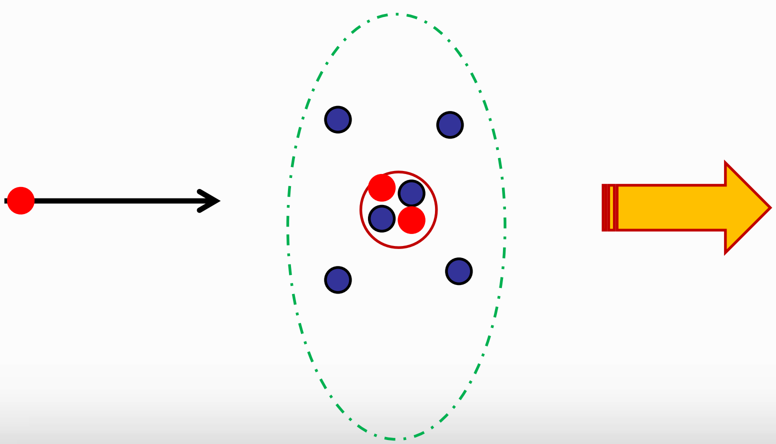
Observation of a correlated free four-neutron system

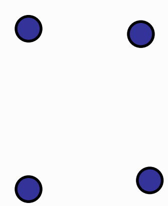
M. Duer, T. Aumann, R. Gernhäuser, V. Panin, S. Paschalis, D. M. Rossi, N. L. Achouri, D. Ahn, H.

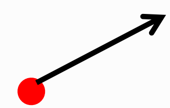
Fig. 3: Missing-mass spectra.



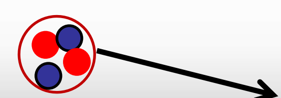
RL, J. Carbonell et al., Prog. Theor. Phys. 7, 073D03 (2017)



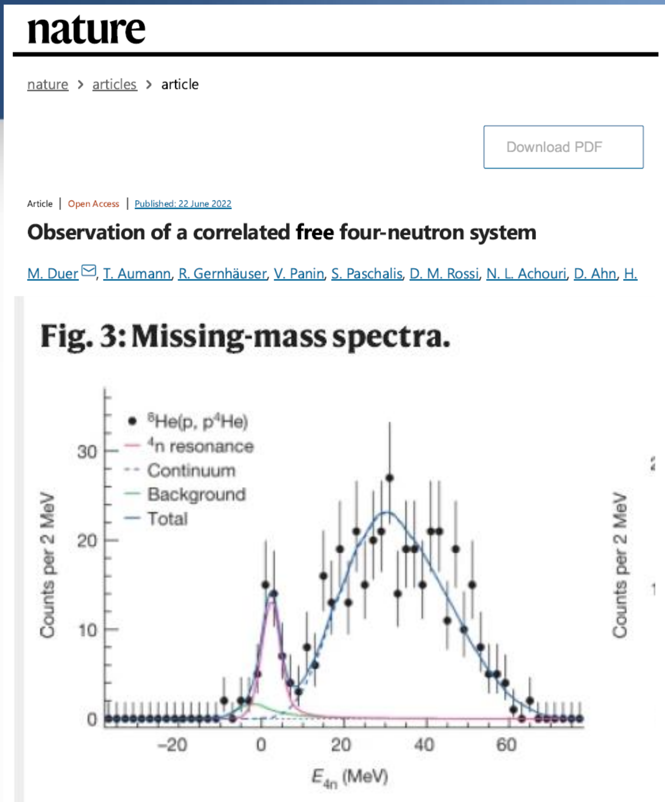




$$S(E) = \left|\langle \Psi_f^{4n}(E) | \Psi_i^{4n} \rangle\right|^2$$



Introduction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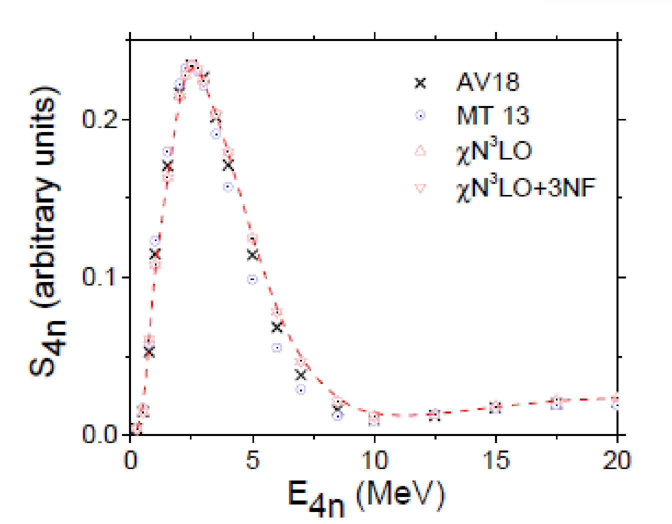
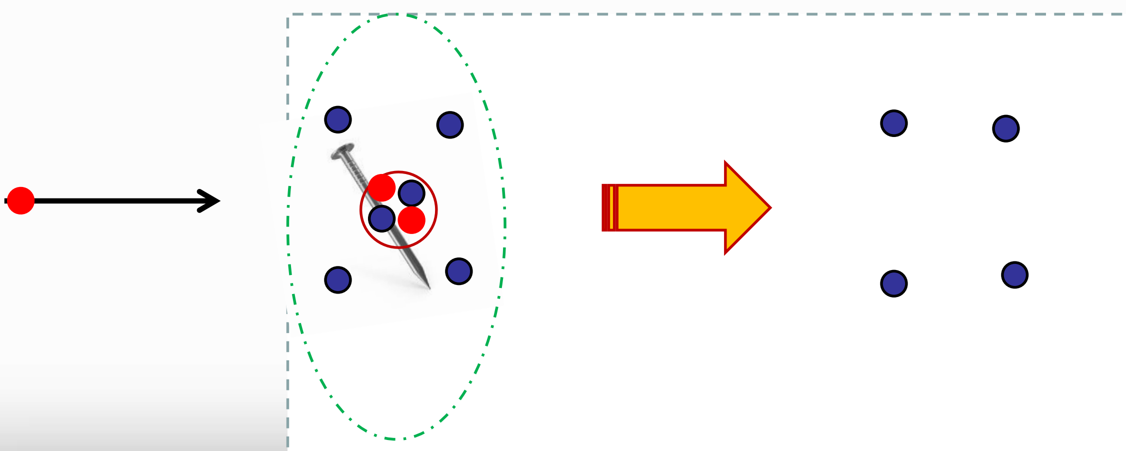
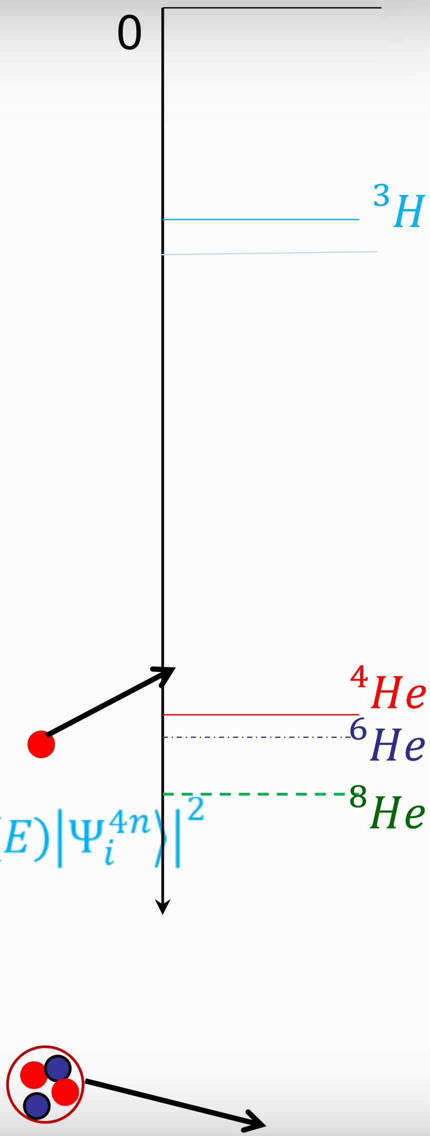
R.L et al., Phys.Rev.Lett. 130 (2023) 10







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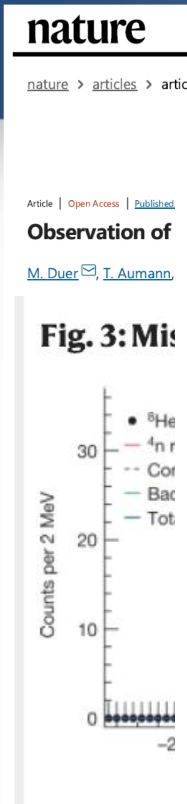
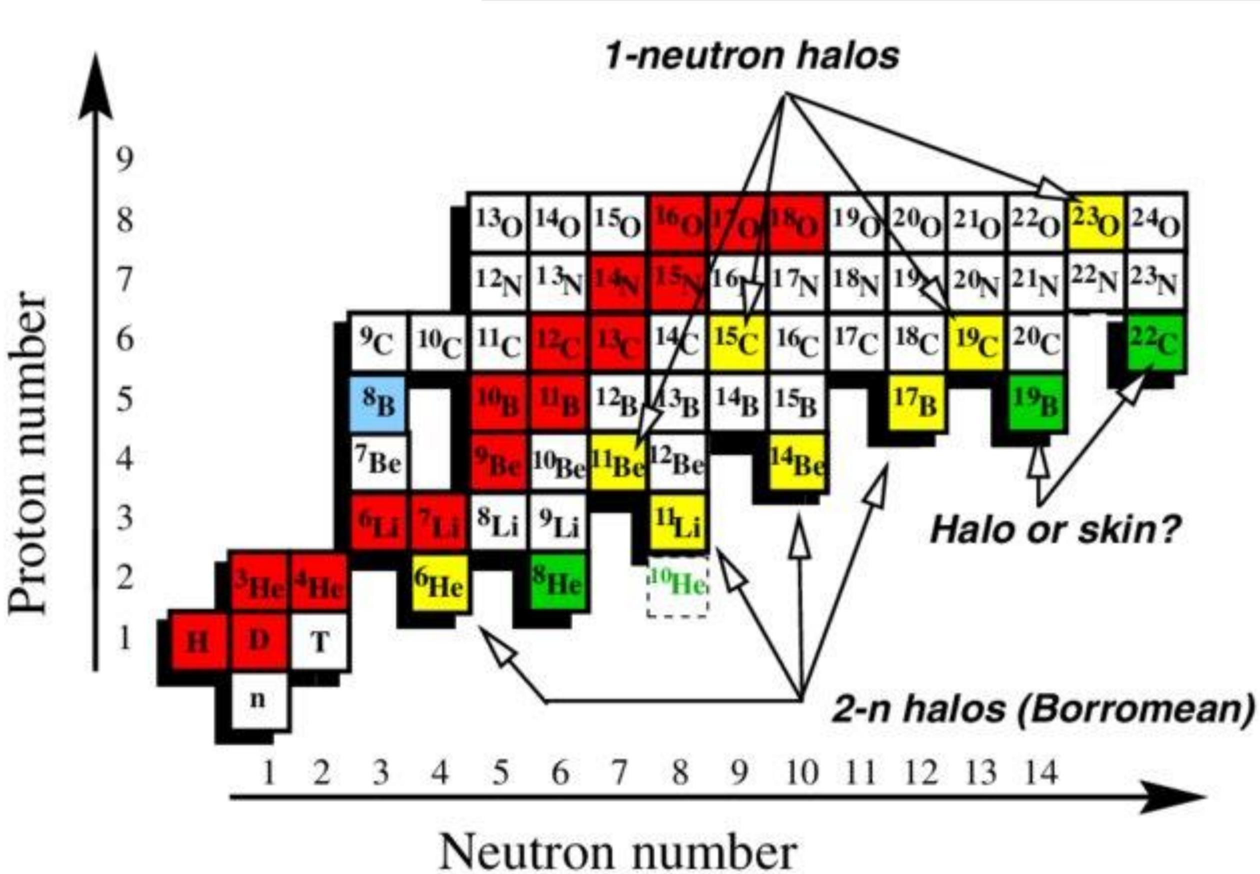
Introduction

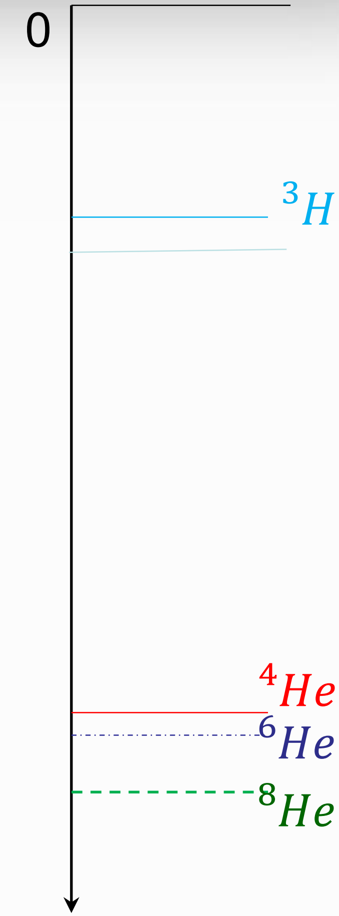


Ques

✓ If

✓ W

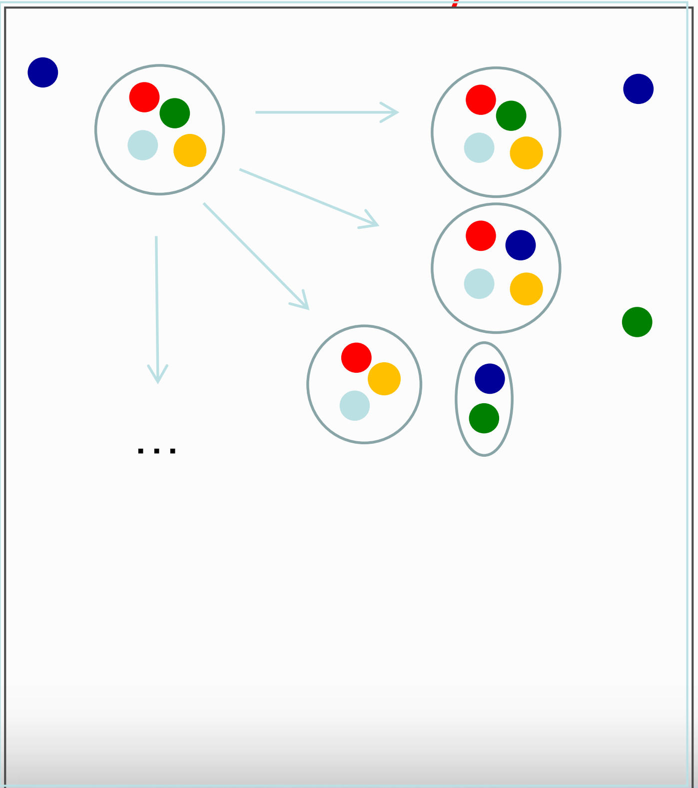




F.M. Marqués et al., Phys. Rev. C 65, 044006 (2002).

Non-relativistic Collisions

- In configuration space wave functions extend to infinity!
- Increasingly complex asymptotic behaviour for A>2 systems!!



How to take care of the boundary condition?

✓ Conceptual difficulties to uncouple different particle channel, to constrain assyptotes of the solutions in all directions and thus get unique (physical) solution to the Schrödinger eq.

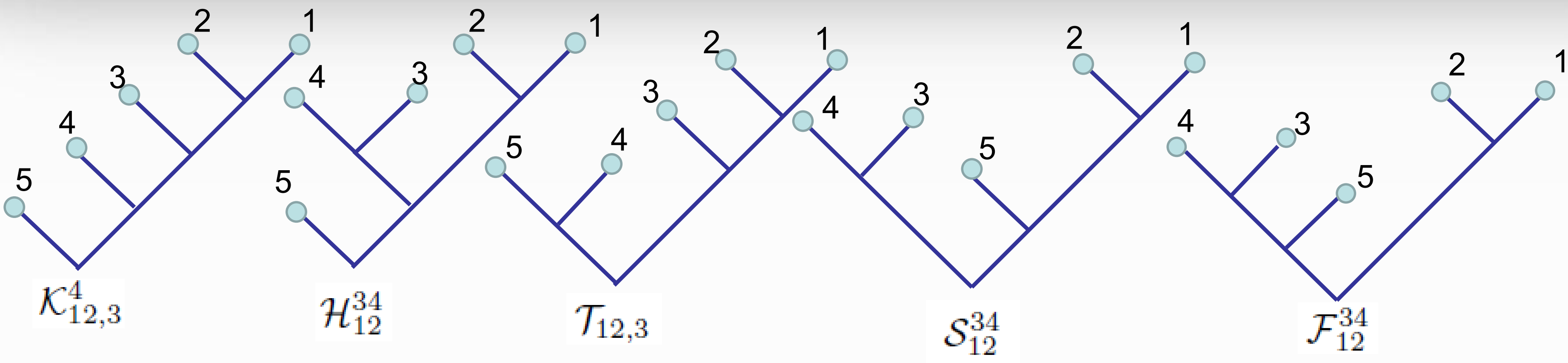
- It is ok, as long as there is single particle channel (elastic plus target/projectile excitations)
- Mathematically Ill-conditioned problem when several particle channels are open

✓ Faddeev-Yakubovsky equations efficiently separates assymptotes of the binary channels

L. D. Faddeev, Zh. Eksp. Teor. Fiz. **39**, 1459 (1960). [Sov. Phys. JETP **12**, 1014(1961)].

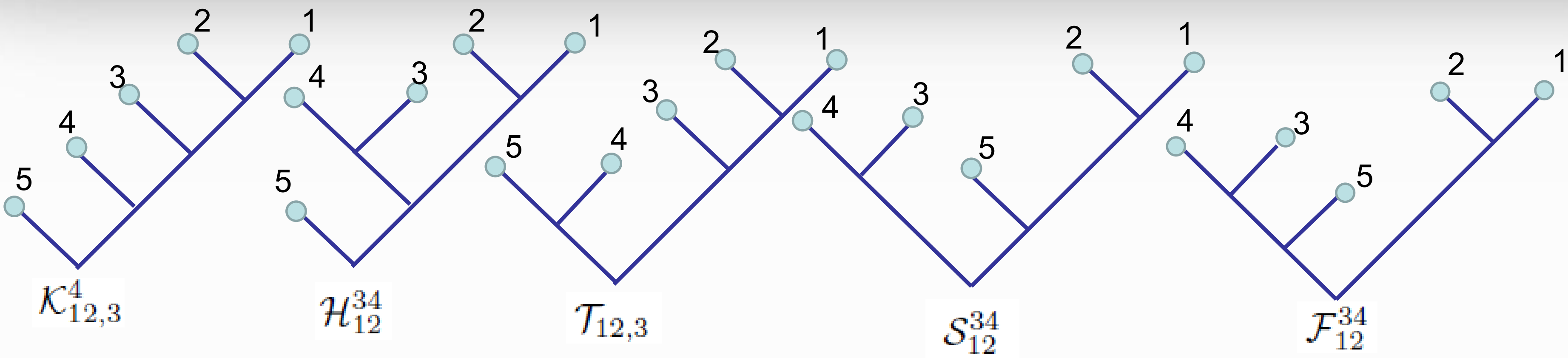
O. A. Yakubovsky, Sov. J. Nucl. Phys. **5**, 937 (1967).

Faddeev-Yakubovski eq. (5-body example)



$$
\begin{aligned}
(E - H_0 - V_{12})\,\mathcal{K}^4_{12,3} &= V_{12}\,(\mathcal{K}^4_{13,2} + \mathcal{K}^4_{23,1} + \mathcal{K}^5_{13,4} + \mathcal{K}^5_{23,4} + \mathcal{K}^2_{13,4} + \mathcal{K}^1_{23,4} \\
&\quad + \mathcal{T}_{13,4} + \mathcal{T}_{23,4} \\
&\quad + \mathcal{H}^{24}_{13} + \mathcal{H}^{14}_{23} + \mathcal{S}^{24}_{13} + \mathcal{S}^{14}_{23} + \mathcal{F}^{24}_{13} + \mathcal{F}^{14}_{23}) \\
(E - H_0 - V_{12})\,\mathcal{H}^{34}_{12} &= V_{12}\,(\mathcal{H}^{12}_{34} + \mathcal{K}^2_{34,1} + \mathcal{K}^1_{34,2} + \mathcal{K}^5_{34,1} + \mathcal{K}^5_{34,2} \\
&\quad + \mathcal{T}_{34,1} + \mathcal{T}_{34,2}) \\
(E - H_0 - V_{12})\,\mathcal{T}_{12,3} &= V_{12}\,(\mathcal{T}_{13,2} + \mathcal{T}_{23,1} \\
&\quad + \mathcal{H}^{45}_{13} + \mathcal{H}^{45}_{23} + \mathcal{S}^{45}_{13} + \mathcal{S}^{45}_{23} + \mathcal{F}^{45}_{13} + \mathcal{F}^{45}_{23}) \\
(E - H_0 - V_{12})\,\mathcal{S}^{34}_{12} &= V_{12}\,(\mathcal{F}^{12}_{34} + \mathcal{S}^{15}_{34} + \mathcal{S}^{25}_{34} \\
&\quad + \mathcal{F}^{15}_{34} + \mathcal{F}^{25}_{34} \\
&\quad + \mathcal{H}^{15}_{34} + \mathcal{H}^{25}_{34}) \\
(E - H_0 - V_{12})\,\mathcal{F}^{34}_{12} &= V_{12}\,(\mathcal{S}^{12}_{34} + \mathcal{K}^1_{34,5} + \mathcal{K}^2_{34,5} + \mathcal{T}_{34,5})
\end{aligned}
$$

Faddeev-Yakubovski eq. (5-body example)



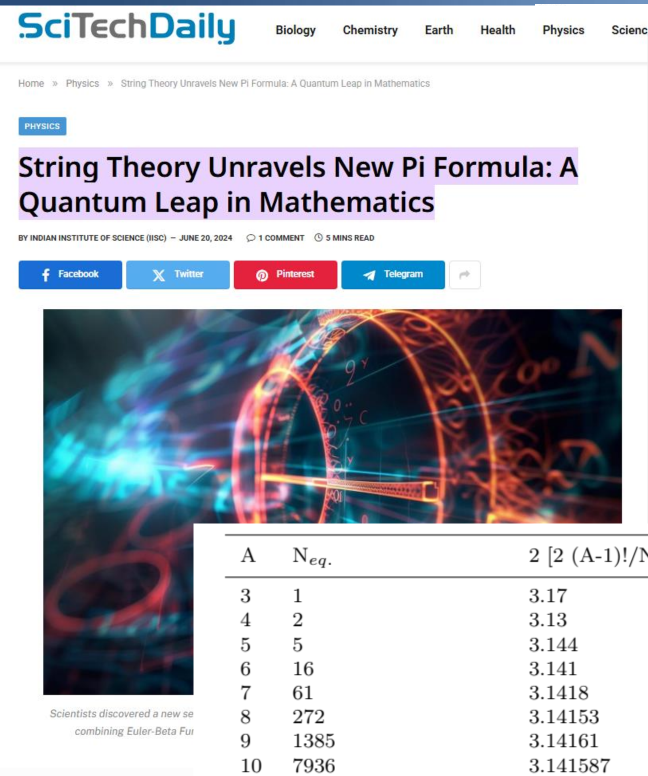
Merits:

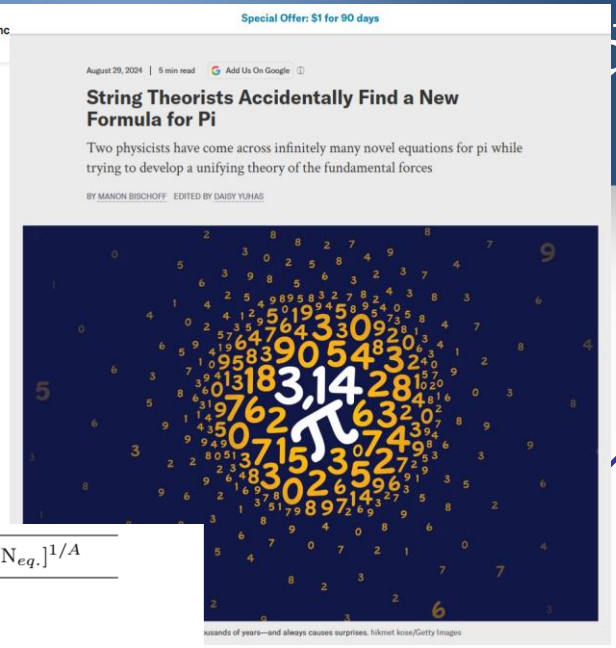
- ✓ Handling of symmetries
- ✓ Boundary conditions for binary channels
- ✓ Reduction to subsystems, teir full control
- ✓ Multipile ways to handle problem of resonant states (scattering, CS, ACCC..)

Price

- ✓ Overcomplexity with A!!

Problem	Number eq. (identical particles)	Number eq. (different particles)	First solution (Year)
A=2	1	1	
A=3	1	3	1973
A=4	2	18	1984
A=5	5	180	2015
A=6	16	2700	
A=N	$\mathrm{nint}(\frac{2(N-1)!}{(\pi/2)^N})$	$\frac{N!\,(N-1)!}{2^{N-1}}$	





$\mathcal{F}^{34}_{12}$

A	$N_{eq.}$	$2\,[2\,(A\text{-}1)!/N_{eq.}]^{1/A}$
3	1	3.17
4	2	3.13
5	5	3.144
6	16	3.141
7	61	3.1418
8	272	3.14153
9	1385	3.14161
10	7936	3.141587
11	50521	3.141594
12	353792	3.1415921
13	2702765	3.1415928
14	22368256	3.14159260
15	199360981	3.14159266
16	1903757312	3.141592649
17	19391512145	3.141592655
18	209865342976	3.1415926531
19	2404879675441	3.1415926537
20	29088885112832	3.14159265354
21	370371188237525	3.14159265360
22	4951498053124096	3.141592653585
23	69348874393137901	3.141592653591
24	1015423886506852352	3.1415926535893
25		3.1415926535899
26		3.14159265358974
27		3.14159265358980
28		3.141592653589788
29		3.141592653589794
30		3.1415926535897927
31		3.1415926535897933
32		3.14159265358979317
33		3.14159265358979324
34		3.141592653589793218

✓ Ha

✓ Bo hannels

✓ Re l control

✓ Mu n of

resona C..)

Price

✓ Ov

Problem	Number eq. (identical particles)	Number eq. (different particles)
A=2	1	1
A=3	1	3
A=4	2	18
A=5	5	180
A=6	16	2700
A=N	$\mathrm{nint}(\frac{2(N-1)!}{(\pi/2)^N})$	$\frac{N!\,(N-1)!}{2^{N-1}}$

Complicate structure of He isotopes

Many studies, mostly structure & using simplistic interactions:

M. V. Zhukov et al., Phys. Rev. C 50, R1(R) (1994)

Y. Kanada-En'yo, Phys. Rev. C 76, 044323 (2007)

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Y. Yamaguchi et al. Phys. Rev. C 108, L011304 (2023)

T. Myo et al. Phys. Rev. C 107, 034305 (2023); Phys. Rev. C 104, 044306 (2021)

but also ab-initio

S. Baroni et al., Phys. Rev. Lett. 110, 022505 (2013)

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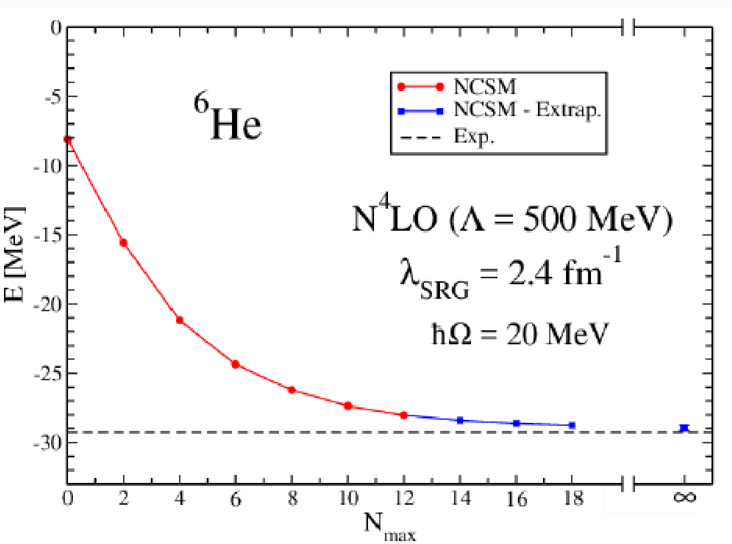

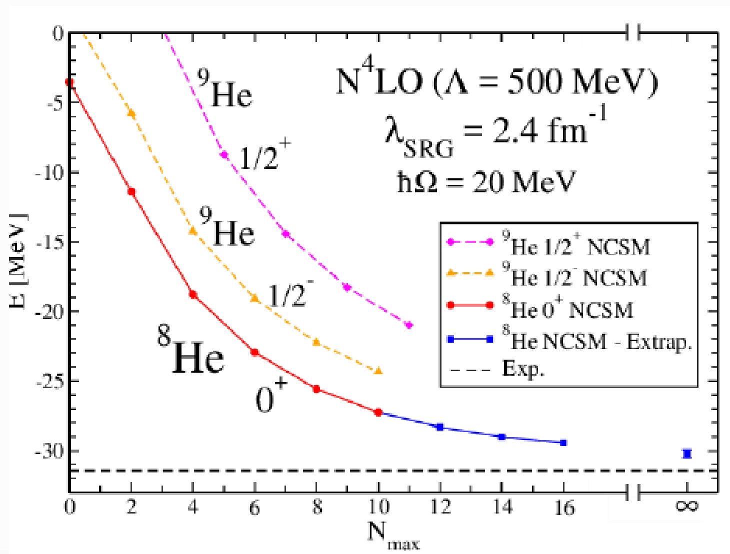

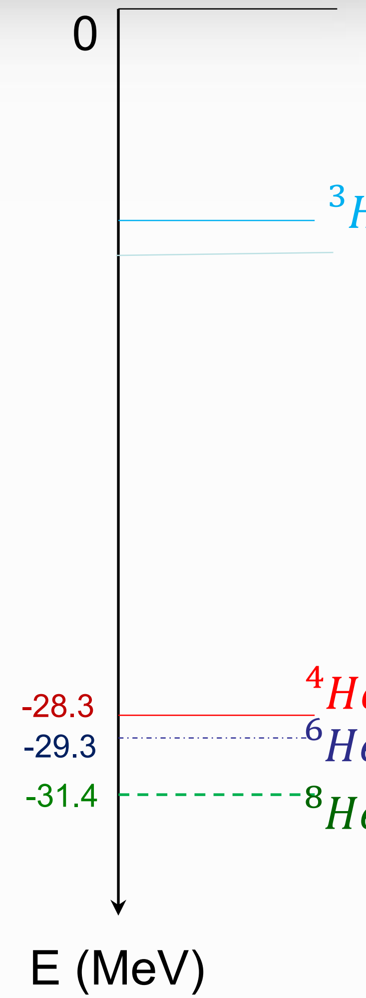

M. Vorrabi et al., Phys. Rev. C 97, 034314 (2018),
//indico.ectstar.eu/event/1/contributions/48/attachments/36/42/Vorabbi.pdf

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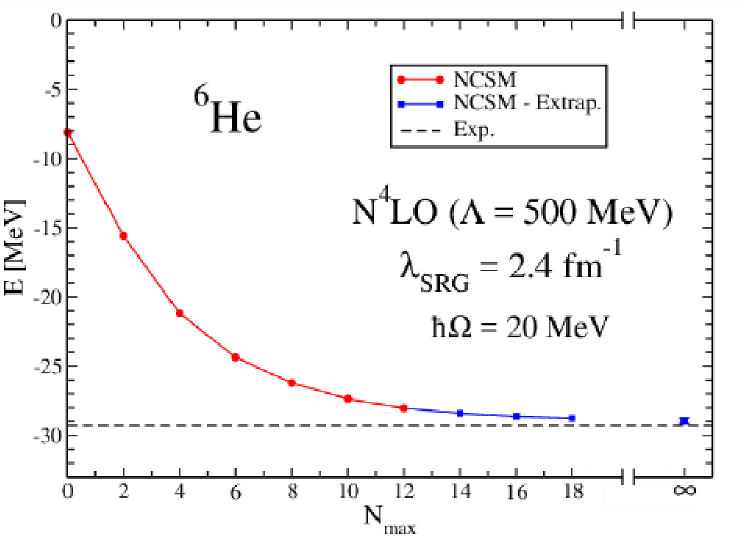
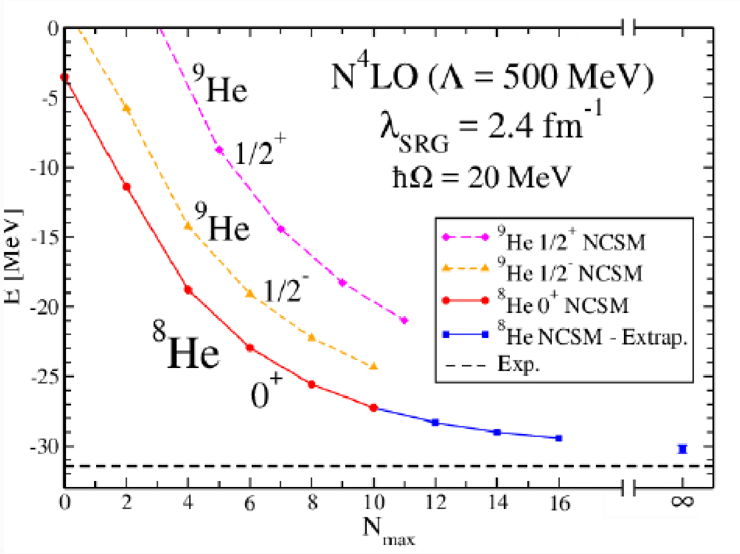
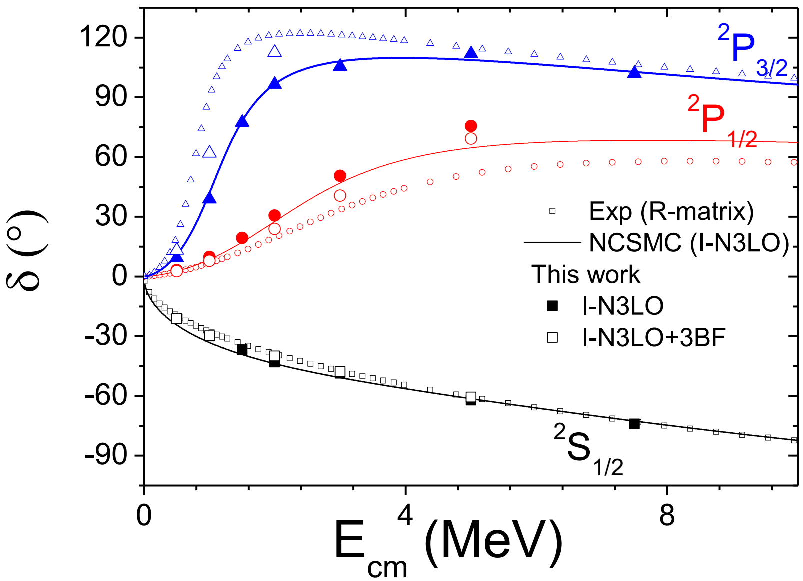




M. Vorrabi et al., Phys. Rev. C 97, 034314 (2018),
//indico.ectstar.eu/event/1/contributions/48/attachments/36/42/Vorabbi.pdf

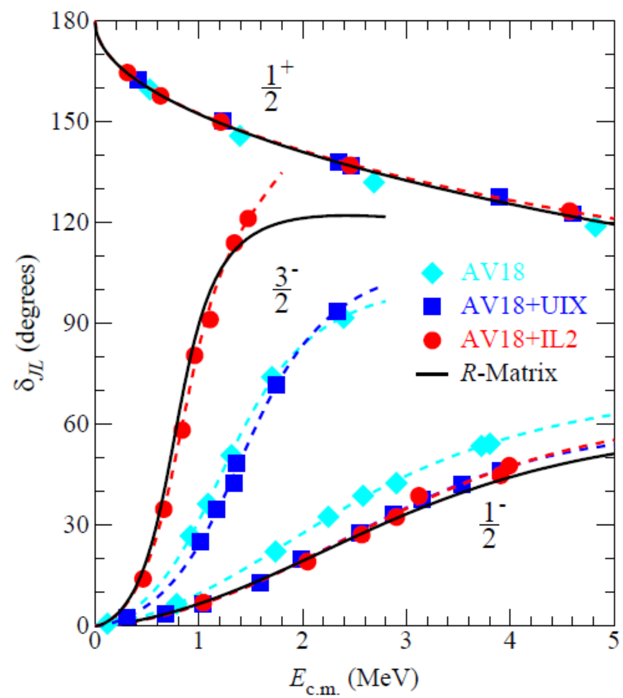
Complicate structure of He isotopes

^{4}He-n atypical case:

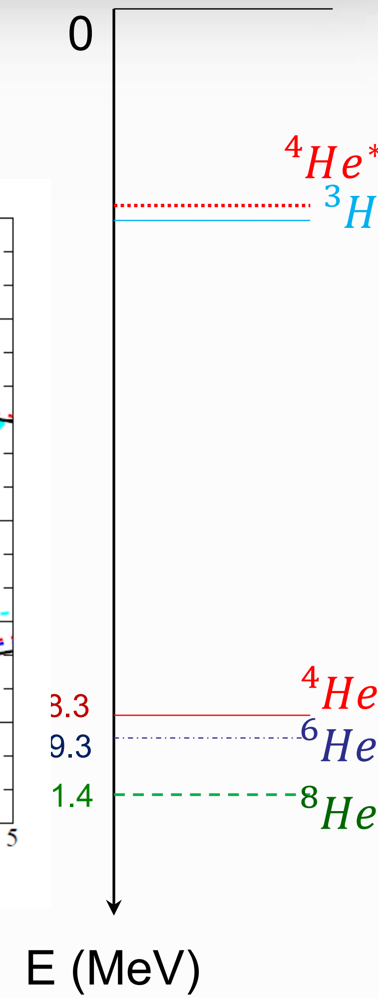


R. Lazauskas, Phys. Rev. C **79**, 054007 (2009).

P. Navratil et al., Physica Scripta **91** (2016) 053002



K.M. Nollett et. al., Phys.Rev.Lett.99:022502,2007

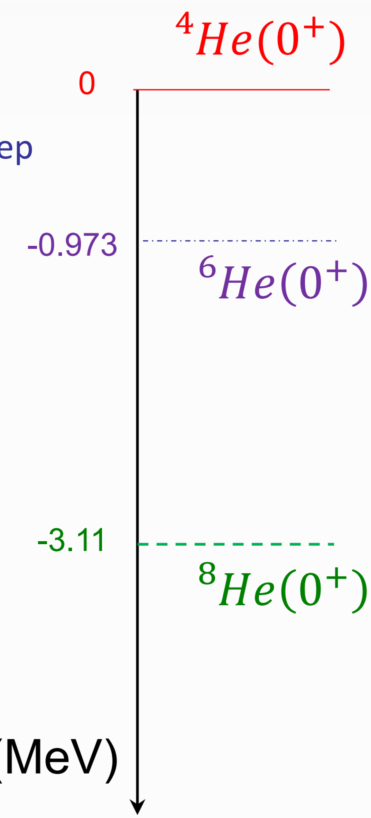


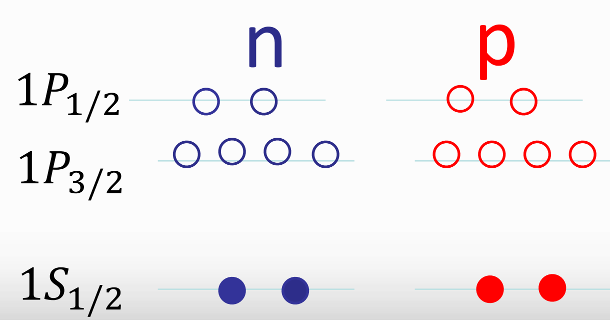
Struggles with a model

- There exist several 2-body models for α-n interaction
  - Major difference how to account for repulsion produced by Pauli principle, leading to repulsive S-wave
    - Local pot: J.M. Bang and C. Gignoux, Nucl. Phys. **A313**, 119 (1979) – produces a deep

$$V(r) = \frac{V_C}{\left[1+\exp\left(\frac{r-R_C}{a_C}\right)\right]} + \frac{(l.s)}{r}\frac{d}{dr}\frac{V_{SO}}{\left[1+\exp\left(\frac{r-R_{SO}}{a_{SO}}\right)\right]}$$: (6 parameters)

Spurious S-wave state: -9.84 MeV $\rightarrow +\infty$





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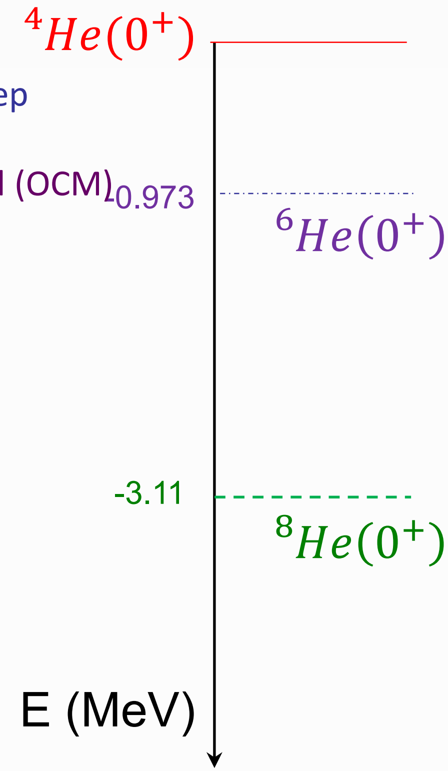
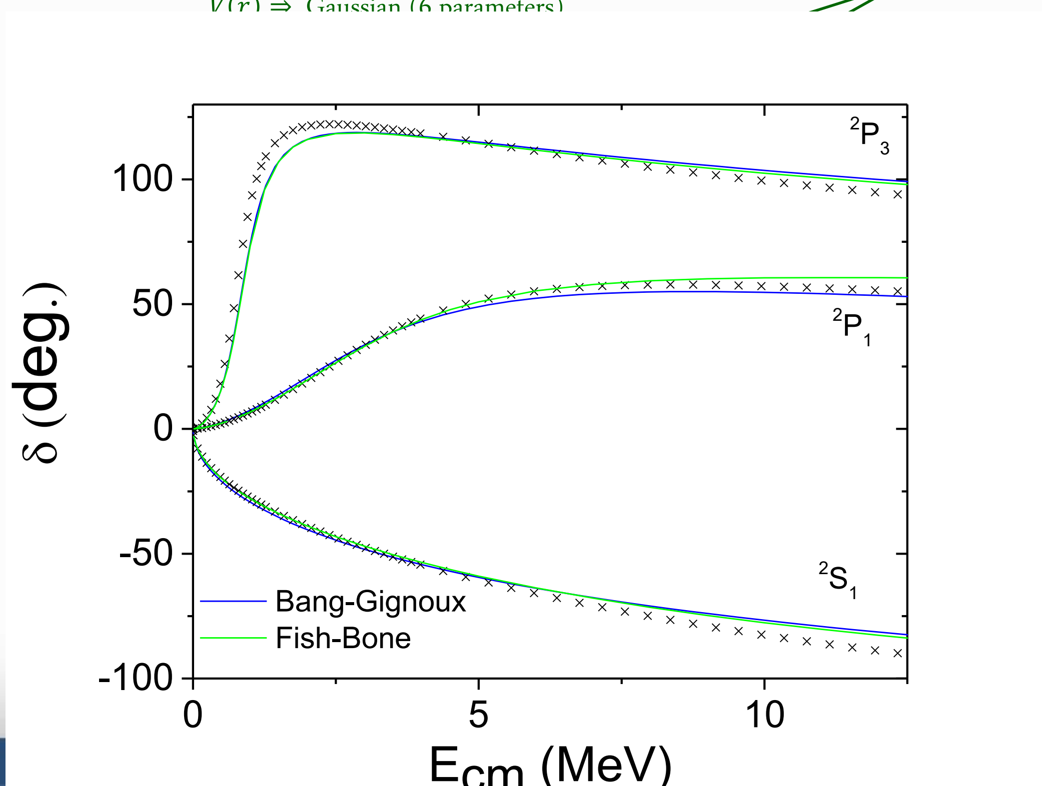
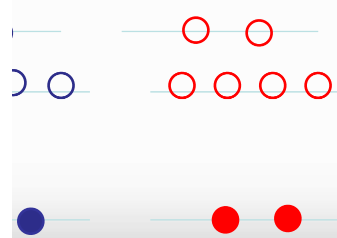
- KKNN: H. Kanada et al., **PTP 61,** 1327(1979); In S-wave a $^2S_1$ HO state is eliminated (OCM)
- Fish-bone: E. Smith, R. Woodhouse, and Z. Papp, Phys. Rev. C **86**, 067001(2012) ;

$V(r) \Rightarrow$ Gaussian (6 parameters)







Struggles with a model

- There exist several 2-body models for α-n interaction
  - Major difference how to account for repulsion produced by Pauli principle, leading to repulsive S-wave

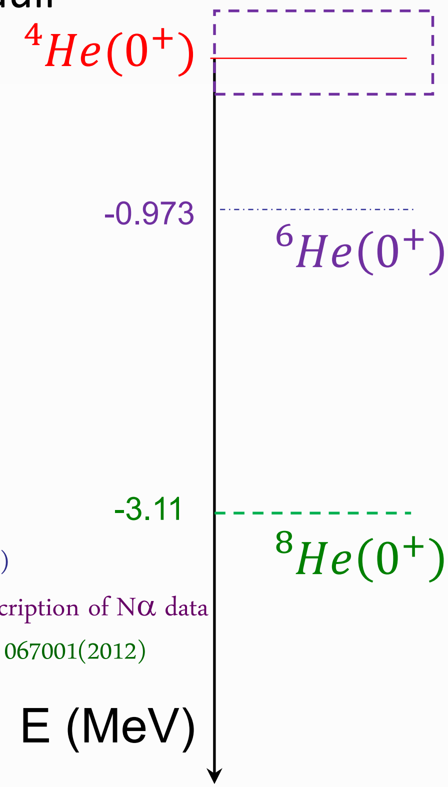
	Model	E(6He)-E(4He),MeV
Bang/Gignoux	Repulsive S wave	Not bound
	Excluding S wave	-0.199
KKNN	OCM	-0.755*
Papp	Fish-bone	-0.171
Exp.		-0.973

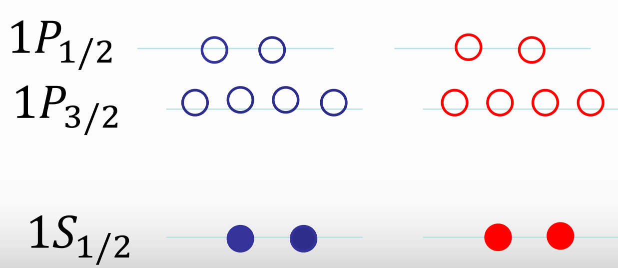
Av18 for nn interaction

J.M. Bang and C. Gignoux, Nucl. Phys. **A313**, 119 (1979)

H. Kanada et al., **PTP 61,** 1327(1979); *less accurate description of Nα data

E. Smith, R. Woodhouse, and Z. Papp, Phys. Rev. C **86**, 067001(2012)





Struggles with a model

- There exist several 2-body models for α-n interaction
  - Major difference how to account for repulsion produced by Pauli principle, leading to repulsive S-wave
- Conventional way to improve description of He-isotopes introduce (α-n-n) 3-body force, but…

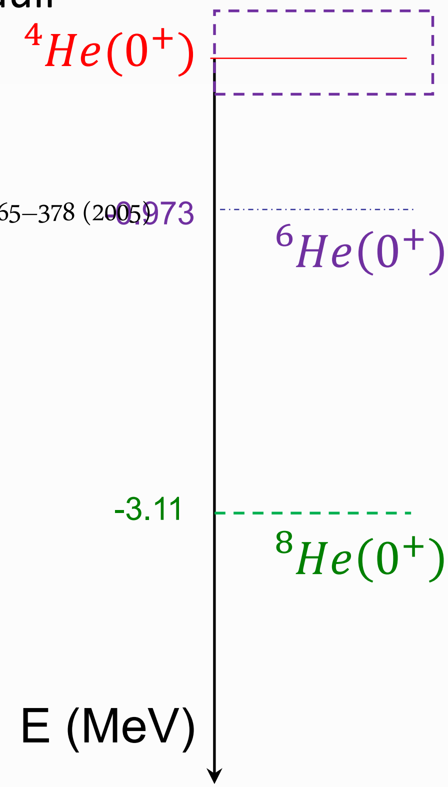
E. Garrido, D.V. Fedorov , and A.S. Jensen, Eur. Phys. J. A 25, 365–378 (2005)

M.D. Higgins, C. H. Greene, Phys. Rev. C **111**, 014001 (2025)

RESONANCES AND COLLISIONAL PROPERTIES OF …

TABLE III. Parameters for the spin-dependent three-body interaction between the ^{4}He $-nn$ triad. Note that $r_{\alpha_m^{(S)}} = 1/\sqrt{\alpha_m^S}$ and $r_{\beta_m^{(S)}} = 1/\sqrt{\beta_m^S}$.

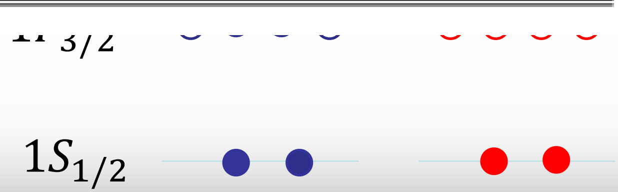
Singlet interaction			
m	$V_m^{(0)}$ [MeV]	$r_{\alpha_m^{(0)}}$ [fm]	$r_{\beta_m^{(0)}}$ [fm]
$m = 1$	-20.8792	2.47917	7.66667
$m = 2$	12.0000	4.50000	7.00000
Triplet interaction			
m	$V_m^{(1)}$ [MeV]	$r_{\alpha_m^{(1)}}$ [fm]	$r_{\beta_m^{(1)}}$ [fm]
$m = 1$	-5.08000	2.50000	8.00000



Properties of $^6He(0^+)$ state

Model	B; MeV	$\sqrt{< r_n^2 >}$	$\sqrt{< r_p^2 >}$
KKNN	0.755	2.73	1.89
Exp.	0.973	2.9(1)*	1.88

S.C. Pieper et al., Phys. Rev. C **64**, 014001(2001)



Struggles with a model

There exist several 2-body models for α-n interaction

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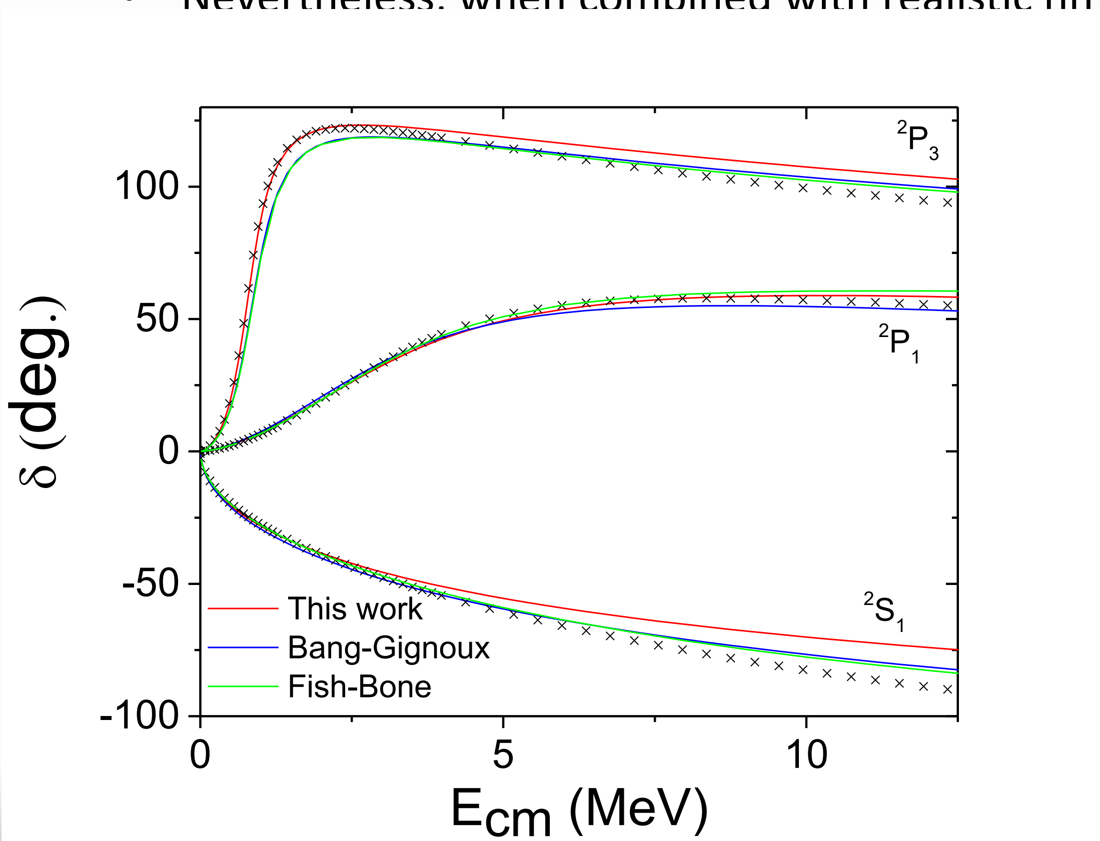
- Nevertheless, when combined with realistic nn interactions,

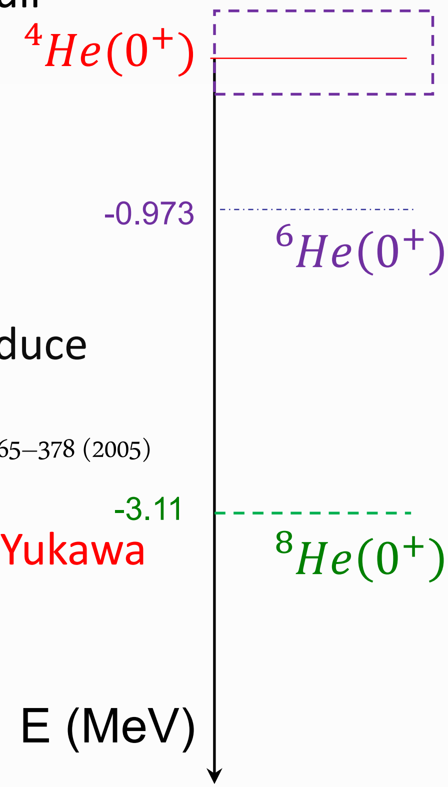
isotopes introduce
orce, but…

ısen, Eur. Phys. J. A 25, 365–378 (2005)

ı. C **111**, 014001 (2025)

states, simple Yukawa





Struggles with a model

How to resolve underbinding problem in α-n-n models?

- Ease projection of the deep (α-n) spurious states

  In OCM deep states are projected to E(α-n)=$+\infty$

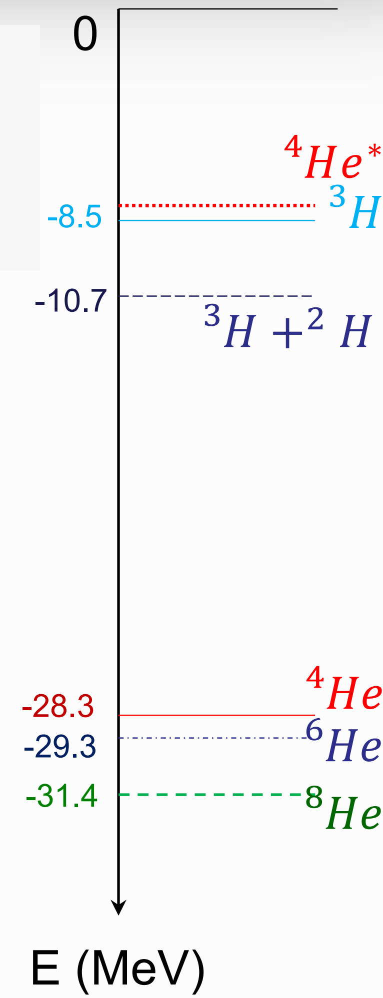
OCM

vs

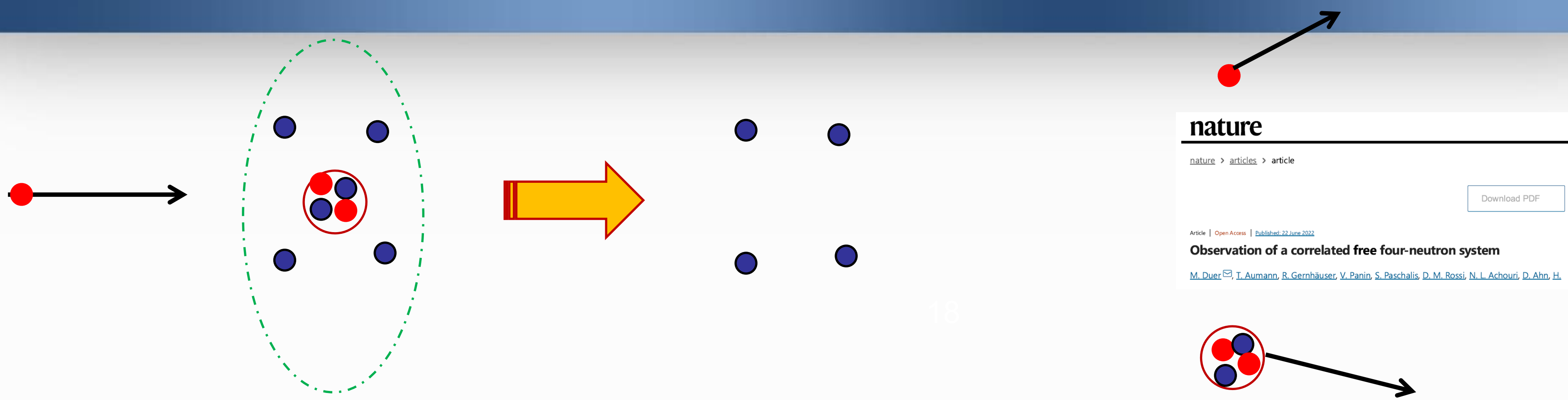
Energies relative to $^6He\ (0^+)$ threshold!

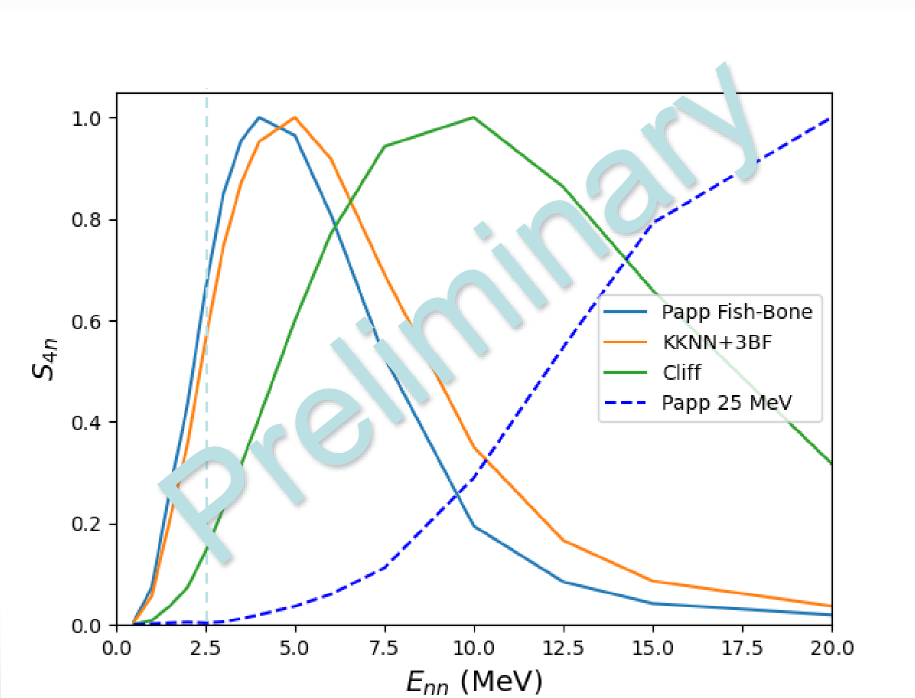
Model	$\sqrt{< r_n^2 >}; fm$ $^6He(0^+)$	$E(^6He(2^+))$ MeV	E($E(^7He(3/2^-))$ MeV	$E(^8He(0^+))$ MeV
KKNN	2.71	0.91-0.09i	0.315(5)-0.035(1)i	-2.98
Papp	2.88	0.82-0.10i	0.35(1)-0.041(1)i	-2.96
Cliff	2.18	0.99-0.07(1)i	0.512-0.088i	-2.5
Exp.	2.9(1)*	0.824-0.06(1)i	0.445-0.075(10)i	-3.11

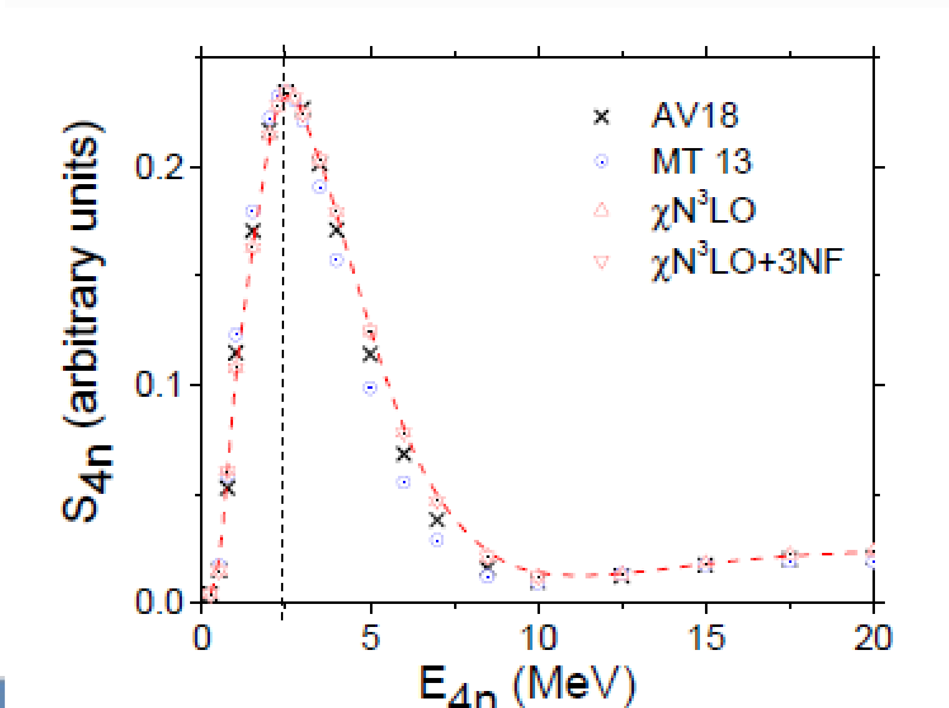
1 parameter is adjusted to reproduce $^6He(0^+)$ binding energy



Calculation of 4n energy distribution



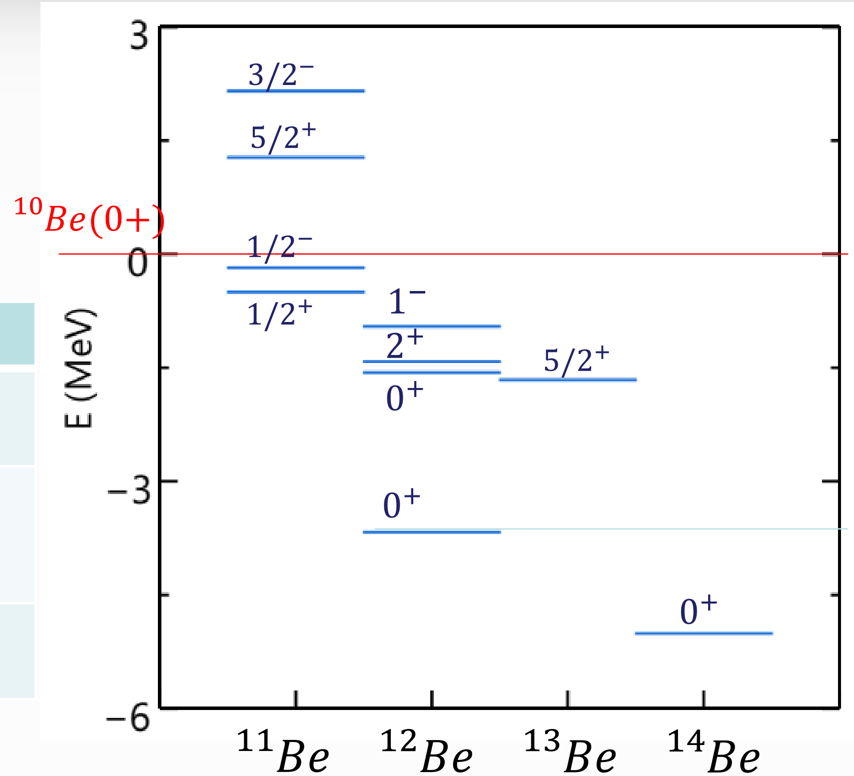




$^{11\text{-}14}$Be case

Models

✓ ^{10}Be-n interaction is a single Gaussian potential with OCM, parameters adjusted to reproduce ^{11}Be states

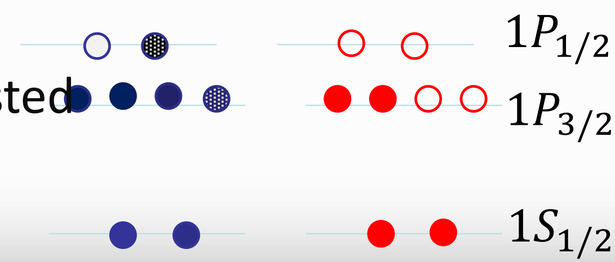


$$\widehat{V_\alpha}(r,r') = -\mathrm{V_0}\exp(-\lambda r^2)\delta(r,r') + \infty|\psi_{1s}^{\mathrm{OH}}(\mathrm{r}')><\psi_{1s}^{\mathrm{OH}}(r)|$$

Model	$\mathrm{V_0}$ (MeV)	λ (fm^{-2})	
A	99.0067	0.3547	$1/2^-$ state reproduced $3/2^-$ reproduced by shifting
B	59.5114	0.13	$1/2^-$, $3/2^-$ states reproduced by shifting a deep state
Capel pot*	*	*	$1/2^+$ deep state removed 3/2- deep state shifted

*Wood-Saxon potential from P.Capel et al., Phys. Rev C 70, 064605 (2004)

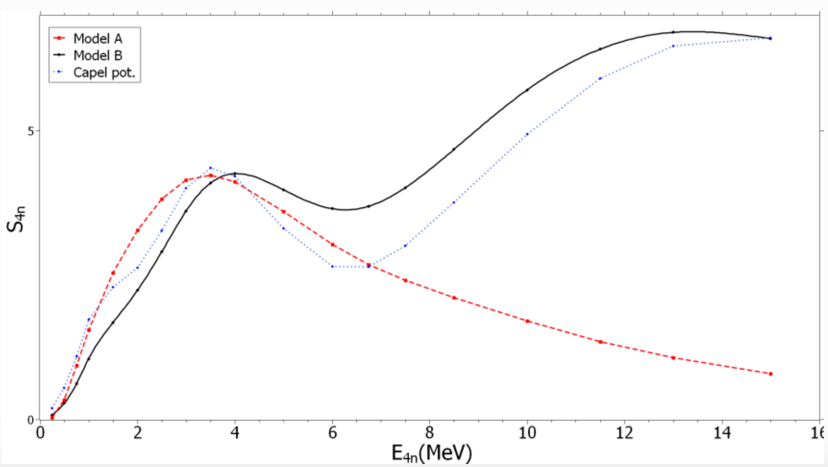
✓ ^{10}Be-n-n Hyperradial interaction adjusted to reproduce ^{12}Be states

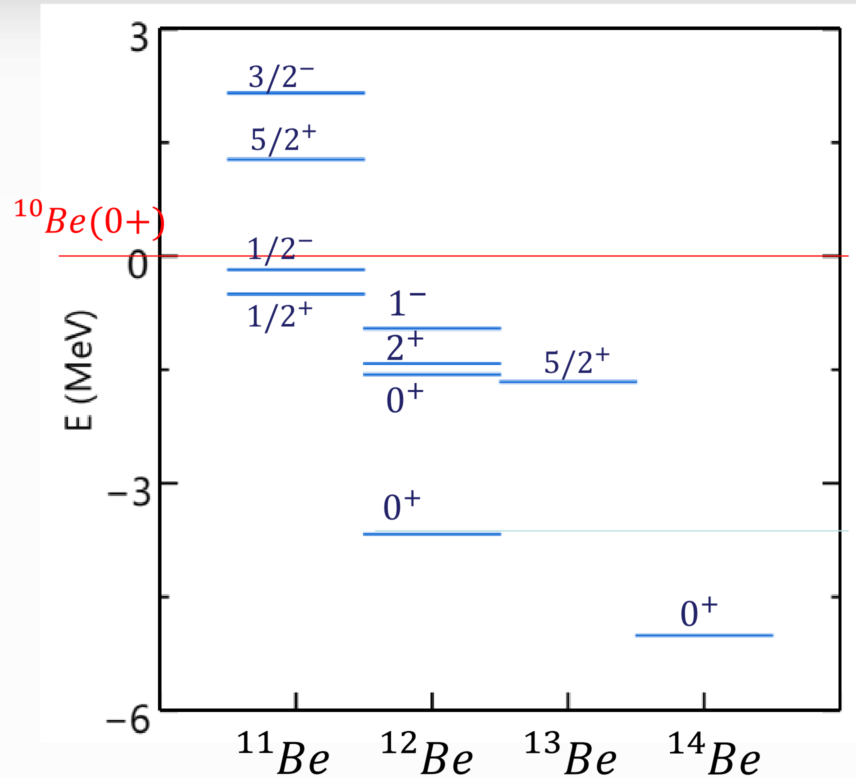


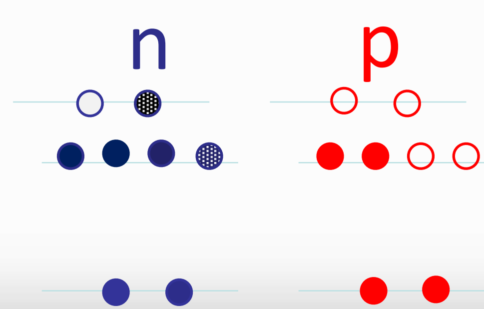
$^{11\text{-}14}$Be case

Model	$E(^{13}Be(5/2^{+}))$ MeV	$E(^{14}Be(0^{+}))$ MeV
A (this work)		-5.05
B (this work)	-1.84(1)-0.14(1)i	-4.84
Capel et al.	-3.92	-4.68
Exp.	-1.61-0.2(1)i	-5.01

*Wood-Saxon potential from P.Capel et al., Phys. Rev C 70, 064605 (2004)







Conclusions

- Neutron-halo states are ideal testground for cluster methods and phenomena related with Pauli exclusion principle
- Very striking dynamical phenomena was observed by Duer et al., where sharp low-energy peak appears naturally without any underlying ^4n resonant state
- The key in explaining this phenomena lies in understanding structure of ^8He nucleus

- I am trying to construct **A+4n** model able to sucessfuly describe 4n halos and their eventual decay emitting 4n
- It appears that if initial state contains weakly bound and peripheral 4n halo – presence of nn corelations leads to sharp low energy peak structures in total 4n energy distribution, without presence of underlying ^4n resonant state.