Uncertainty quantification and inference with many-body perturbation theory

Isak Svensson

21 January 2026

Challenges in effective field theory descriptions of nuclei
Hirschegg, 18-24 January 2026

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Outline

1. UQ for many-body perturbation theory (MBPT)
2. Equation of state (EOS) inference from neutron-star data
3. Inference of 3N couplings from neutron-star data

1: Assessing truncation errors in many-body methods

arXiv > nucl-th > arXiv:2507.09079

Nuclear Theory

[Submitted on 11 Jul 2025]

A Bayesian approach for many-body uncertainties in nuclear structure: Many-body perturbation theory for finite nuclei

Isak Svensson, Alexander Tichai, Kai Hebeler, Achim Schwenk

A comprehensive assessment of theoretical uncertainties defines an important frontier in nuclear structure research. Ideally, theory predictions include uncertainty estimates that take into account truncation effects from both the interactions and the many-body expansion. While the uncertainties from the expansion of the interactions within effective field theories have been studied systematically using Bayesian methods, many-body truncations are usually addressed by expert assessment. In this work we use a Bayesian framework to study many-body uncertainties within many-body perturbation theory applied to finite nuclei. Our framework is applied to a broad range of nuclei across the nuclear chart calculated from two- and three-nucleon interactions based on chiral effective field theory. These developments represent a step towards a more complete and systematic quantification of uncertainties in \emph{ab initio} calculations of nuclei.

arXiv:2507.09079, accepted by Phys. Rev. C

All code and data: `https://github.com/svisak/manybody_uncertainties`

Motivation - errors in chiral effective field theory (χEFT)[1]

χEFT calculations of (e.g.) nuclei have several sources of uncertainty:

- Uncertainty in the determination of the LECs
- Truncation of the EFT expansion
- Limited model spaces
- Truncation of the many-body method

Goal:
Rigorous treatment of uncertainties arising from truncating the many-body perturbation theory (MBPT) expansion at finite order, replacing ad-hoc/expert assessments

Model is heavily inspired by the BUQEYE model for EFT truncation errors, in particular Wesolowski, IS *et al.*, Phys. Rev. C **104** (2021)

[1] Weinberg, van Kolck, Kaplan, Savage, Wise, Bernard, Epelbaum, Kaiser, Meißner, ...

Inference framework

Ground-state binding energy E in MBPT:

$$E = E_{\mathrm{HF}} + \mathrm{MBPT}(2) + \mathrm{MBPT}(3) + \dots$$

We assume that the ratios of contributions are (roughly) constant, i.e.:

$$\frac{\mathrm{MBPT}(2)}{E_{\mathrm{HF}}} \approx \frac{\mathrm{MBPT}(3)}{\mathrm{MBPT}(2)} \approx \frac{\mathrm{MBPT}(4)}{\mathrm{MBPT}(3)} \approx \dots \approx \text{constant}$$

Similar to EFT truncation errors

MBPT data and input nuclear interaction models

Inference data: binding energies[2] for 37 nuclei from ^{14}O to ^{208}Pb

Use three different χEFT interactions:

- 1.8/2.0 (EM) ("Magic")[3]
- ΔN^{2}LO$_{\mathrm{GO}}$[4]
- 1.8/2.0 (EM7.5)[2]

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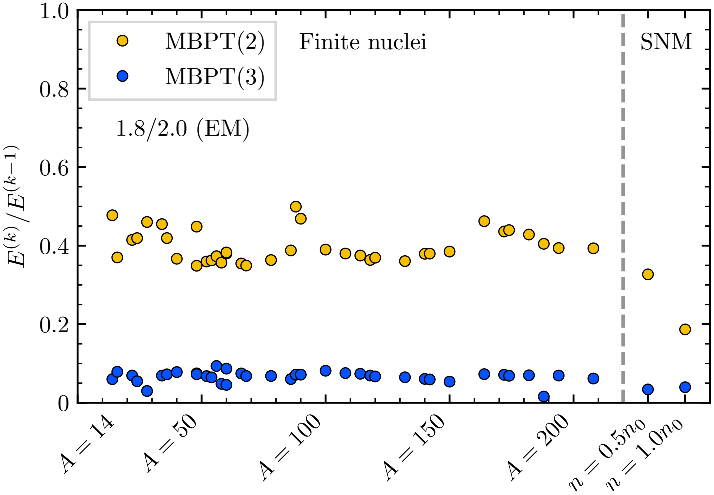
[2] P. Arthuis *et al.*, arXiv:2401.06675

[3] K. Hebeler *et al.*, Phys. Rev. C **83** (2011)

[4] W. Jiang *et al.*, Phys. Rev. C **102** (2020)

MBPT ratios



Calculated [1.8/2.0 (EM)] ratios of MBPT corrections. Gives an idea of the convergence of the expansion as well as the correlation structure across nuclei. Symmetric nuclear matter (SNM) results are not used in the main inference.

Inference setup

We assume that the **MBPT expansion** can be expressed as

$$E = E_{\mathrm{ref}} \sum_{n=0}^{\infty} \gamma_n R^n$$

where $n = k - 1$ and k is the MBPT order (counting HF as $k = 1$)

Yields an **MBPT truncation error**: $\delta E = \mathcal{N}\left(0, E_{\mathrm{ref}}^2 \bar{\gamma}^2 \frac{R^{2(n+1)}}{1-R^2}\right)$

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(just like Q and $\bar{c}^2$ for EFT errors)

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(just like Q and $\bar{c}^2$ for EFT errors)

$\longrightarrow$ Bayesian posterior

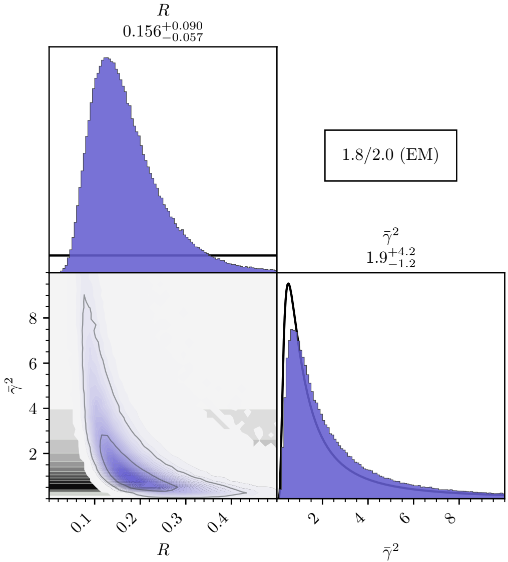
Posterior for the hyperparameters – 1.8/2.0 (EM)

We find the most likely value for $R \approx 0.15$ (with rather large uncertainty)

Most likely value for $\bar{\gamma}^2 \approx 1$

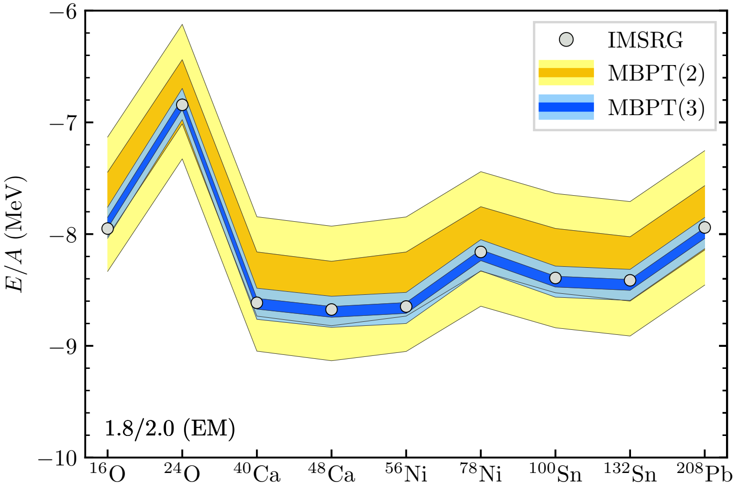
Naturally, R and $\bar{\gamma}^2$ are correlated (larger R can compensate for smaller $\bar{\gamma}^2$ etc.)

Each new MBPT order contributes $\approx 15\%$ of the previous order



Posterior predictive distributions for nuclei

Uncertainty bands for selected nuclei, scaled by the mass number A. IMSRG(2) results for comparison as gray circles. Credibility intervals shown at the 68% and 90% levels.



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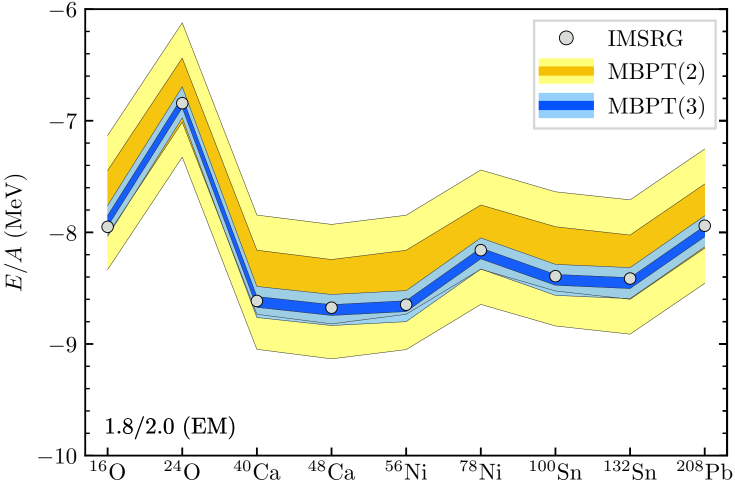
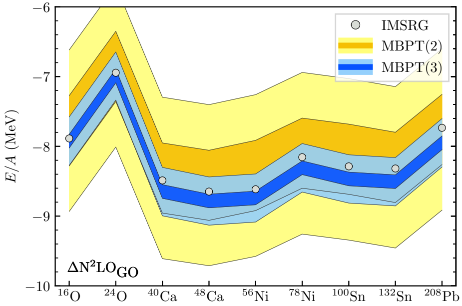
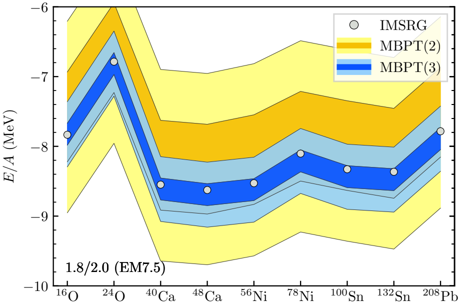


Interaction sensitivity – PPDs



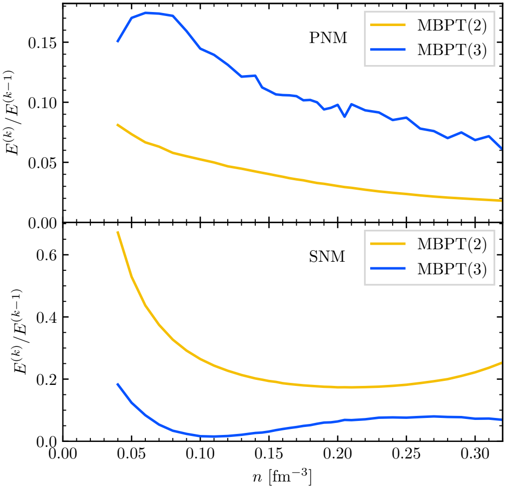


PPDs with ΔN^2LO_{GO} . . .

. . . and 1.8/2.0 (EM7.5)

These interactions are harder (converge slower), hence **larger uncertainties**

Future work: including nuclear matter



Data from F. Alp, Y. Dietz *et al.*, arXiv:2504.18259

For **symmetric nuclear matter** (bottom), ratios behave similarly to those of finite nuclei. We have tested the inference with two density points (0.5 and 1.0 n_0), with similar results to before.

Pure neutron matter (top), on the other hand, behaves quite differently: the MBPT(3)/MBPT(2) ratio is **larger** than the MBPT(2)/E_{HF} ratio.

In the future, we would like to model this kind of data using **Gaussian processes**

2: Inferring the dense-matter EOS from multimessenger neutron-star observations

With Melissa Mendes, Hannah Göttling, Anna Hensel, Kai Hebeler, Achim Schwenk (in preparation)

See also Hannah's talk later today and Göttling *et al.*, arXiv:2512.19593

Constraints on the EOS across densities

Different theoretical and observational constraints in different density regimes

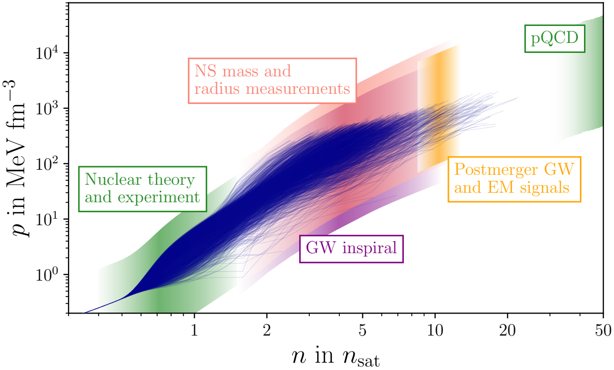


Figure from Koehn *et al.*, Phys. Rev. X **15** (2025)

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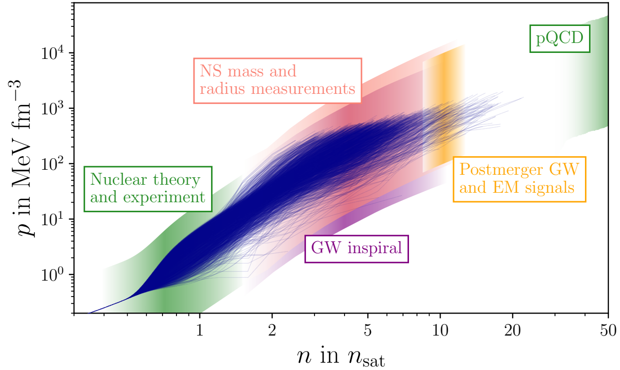

Figure from Koehn *et al.*, Phys. Rev. X **15** (2025)

Different theoretical and observational constraints in different density regimes

$\sim 0.5 - 1.5 n_{\mathrm{sat}}$: χEFT MBPT calculations from Keller *et al.*, Phys. Rev. Lett. **130** (2023) (see also Yannick's talk tomorrow)

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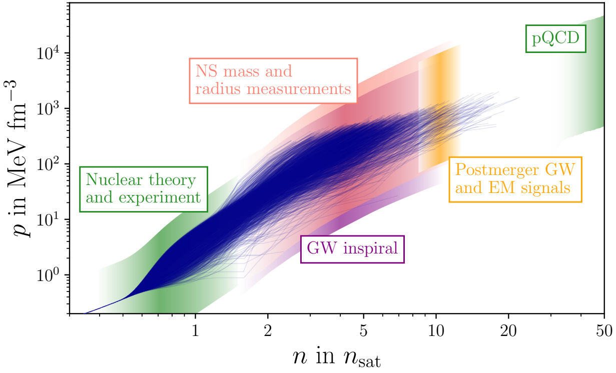

Figure from Koehn *et al.*, Phys. Rev. X **15** (2025)

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Two agnostic high-density extensions, "PP" and "CS"

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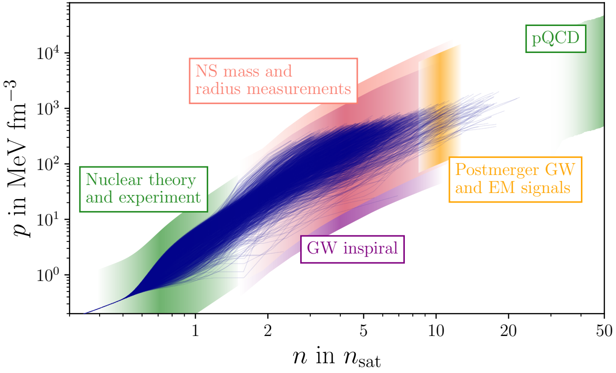

Figure from Koehn *et al.*, Phys. Rev. X **15** (2025)

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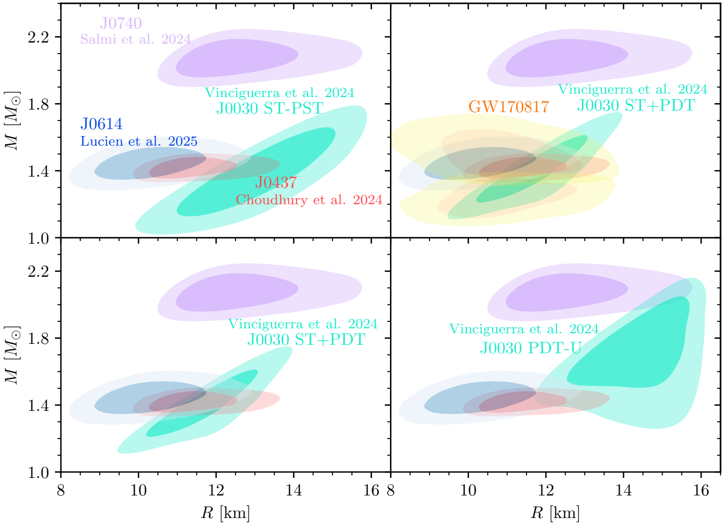
Two agnostic high-density extensions, "PP" and "CS"

Constraints from pQCD (Anna Hensel, Master thesis 2025)



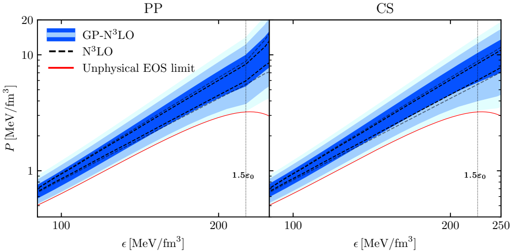
Neutron-star data from NICER[5], gravitational wave data[6]

[5] Riley *et al.*, ApJ Lett. **887** (2019), Salmi *et al.*, ApJ **941** (2022), Vinciguerra *et al.*, ApJ **961** (2024), Choudhury *et al.*, ApJ Lett. **971** (2024), Lucien Mauviard *et al.*, ApJ **995** (2025)

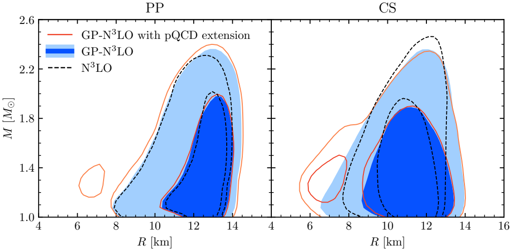
[6] Abbott *et al.*, PRL **119** (2017)

EOS priors in the pressure-density and mass-radius planes

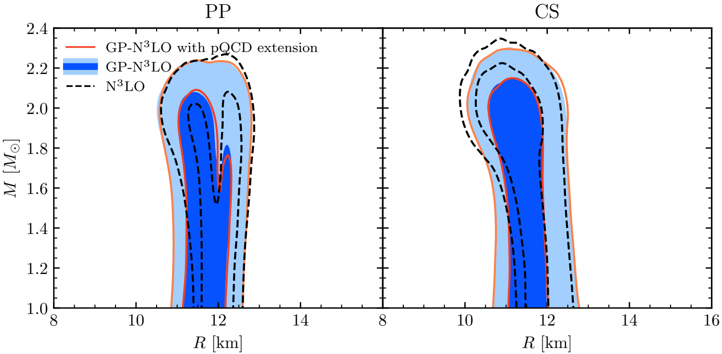
Top: Prior in pressure vs energy density with "old" uniform N^3LO band (black dashed) and new Gaussian process-based prior



Bottom: Priors in mass-radius plane



Posteriors with all available data



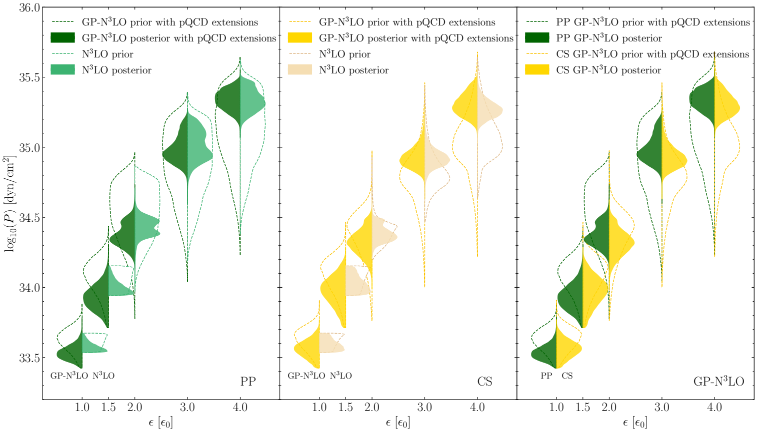
GP-based prior allow for a **wider range of radii at low masses**

pQCD constraint has **almost no effect**

Bimodal-like structure at $R = 12$ km (data effect)

Posteriors with all available data

GP vs no GP (PP) GP vs no GP (CS) PP vs CS

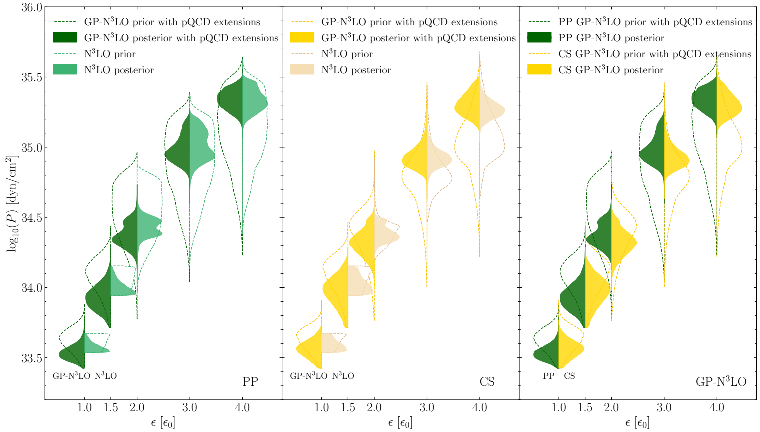


GP-based prior significantly more allowing at lower densities

High-density extension has only **minor impact**

Posteriors with all available data

GP vs no GP (PP) GP vs no GP (CS) PP vs CS



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High-density extension has only **minor impact**

3: Inferring three-nucleon couplings from multi-messenger neutron-star data

nature communications

Article https://doi.org/10.1038/s41467-025-64756-6

Inferring three-nucleon couplings from multi-messenger neutron-star observations

Received: 28 May 2025

Accepted: 24 September 2025

Published online: 06 November 2025

Rahul Somasundaram[1,2,9], Isak Svensson[3,4,5,9], Soumi De[2], Andrew E. Deneris[6], Yannick Dietz[3,4], Philippe Landry[7,8], Achim Schwenk[3,4,5] & Ingo Tews[2]

Based on Nature Commun. **16** (2025) 1, 9819

Code and data: `https://github.com/svisak/multimessenger_3N_constraints`

Inferring three-nucleon couplings from neutron-star (NS) data

χEFT: Low-momentum-scale (Q) expansion

New orders introduce **unknown low-energy constants** (LECs) that need to be fit to data:

- Two-nucleon (2N) LECs fit to 2N scattering data
- 3N LECs fit to properties of light nuclei
- Pion-nucleon (πN) LECs fit to πN scattering data
- This work: fit πN LECs (governing 3N forces) to multimessenger NS data

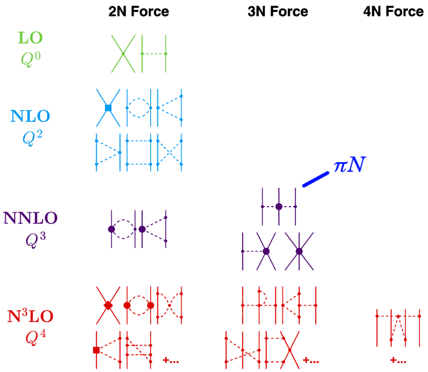


Figure adapted from Entem *et al.*, Phys. Rev. C **96**.2 (2017).

Bayesian inference

χEFT calculations of neutron matter depend on πN LECs c_1, c_3

$\longrightarrow$ In principle, neutron-star observables can constrain c_1, c_3

From LECs to NS observables:

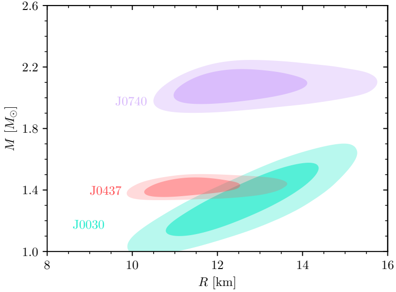
1. Input LECs into χEFT
2. Compute neutron-matter EOS using many-body perturbation theory
3. Solve TOV and quadrupolar tidal perturbation equations
4. Output: neutron-star masses, tidal deformabilities

With Bayesian inference, we can go from NS data to LECs

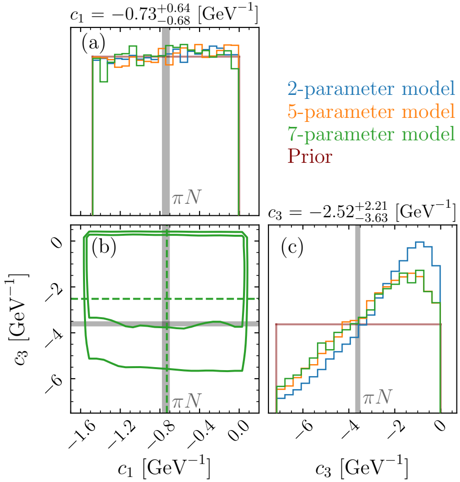
Problem: Huge computational cost
Solution: Emulators (PMMs, neural networks)

LEC posteriors using currently available data

We use NICER mass-radius data and GW170817 gravitational wave event



No constraints on c_1, but
clear preference for less negative c_3
(less repulsive $3N$ force)

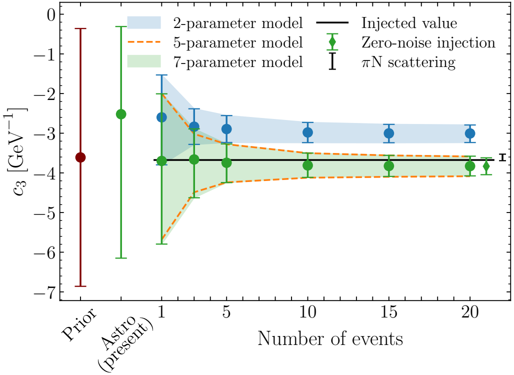


(πN result from Hoferichter *et al.*, Phys. Rept. **625** (2016))

Can we improve this with more&better GW data?

Next, use **simulated** next-generation GW data from Einstein Telescope[7] and Cosmic Explorer[8], ~ 1 year of observation

- Select 20 highest-SNR events, perform Bayesian inference
- c_3 converges quickly with number of observed events
- Final constraints almost comparable with πN scattering constraints



Uncertainties given as 90% credibility intervals.

[7] Punturo *et al.*, Class. Quant. Grav. **27** (2010)

[8] Reitze *et al.*, Bull. Am. Astron. Soc. **51**, (2019)

Summary and outlook

- Bayesian error model for many-body perturbation theory works quite well.
  Outlook: nuclear matter, open-shell nuclei
- GP-based χEFT prior for EOS inference improves statistical rigor.
  Outlook: account for MBPT uncertainty?
- Inference of 3N couplings from NS data possible, new complementary constraint on c_3.
  Outlook: infer χEFT breakdown scale?

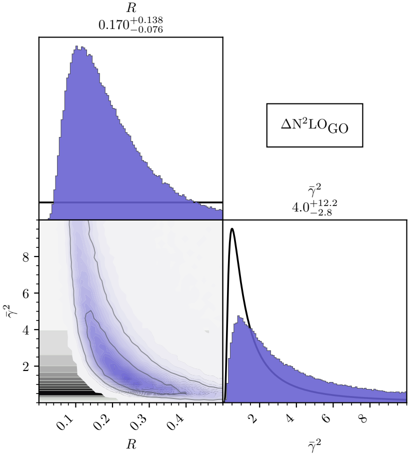
Collaborators:

Soumi De, Andrew E. Deneris, Yannick Dietz, Hannah Göttling, Kai Hebeler, Anna Hensel, Philippe Landry, Melissa Mendes, Achim Schwenk, Rahul Somasundaram, Ingo Tews, Alex Tichai

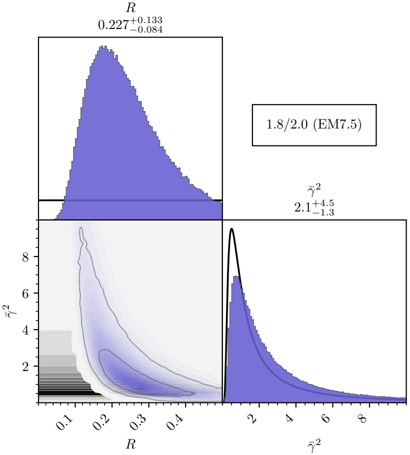
Thanks to Dick Furnstahl, Zhen Li, Pierre Arthuis, Matthias Heinz, Takayuki Miyagi, and Faruk Alp for discussions and input

Extra slides

Interaction sensitivity – hyperparameter posteriors



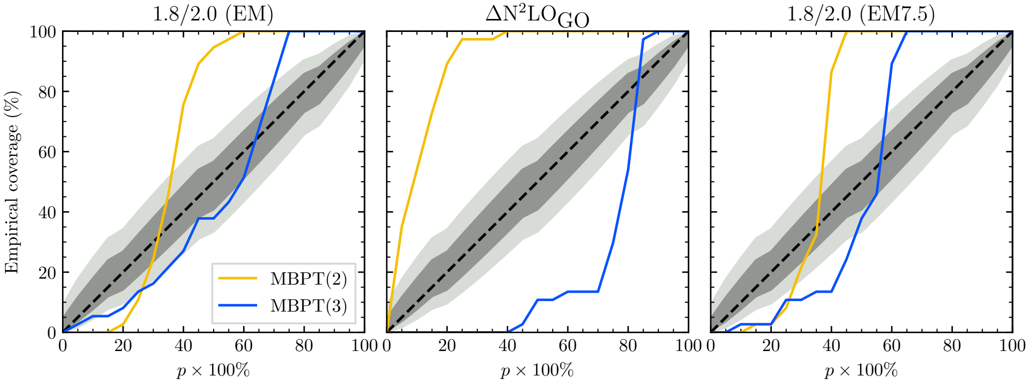
ΔN^2LO_{GO}



1.8/2.0 (EM7.5)

Note that R is larger than for 1.8/2.0 (EM) – **slower convergence**

Empirical coverage of MBPT bands with respect to IMSRG data



Empirical coverages. Measures the **observed** (y-axis) vs **expected** (x-axis) coverage of the credibility intervals. Ideal result is a diagonal line. Gray areas are confidence intervals that measure whether the observed result is compatible with the ideal.