Nucleon-nucleon correlation functions from different interactions in comparison

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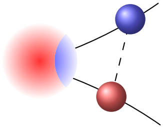

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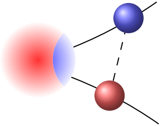
in collaboration with A. Kievsky

Correlation functions

- correlations between particle momenta can be measured in heavy-ion collisions (femtoscopy) Lisa, Pratt, Soltz, Wiedemann, ARNPS 55 (2005), Fabbietti, Mantovani Sarti, Vazquez Doce, ARNPS 71 (2021)
  - pratical definition of correlation: $C(\mathbf{k}_1, \mathbf{k}_2) = \frac{\mathcal{P}(\mathbf{k}_1, \mathbf{k}_2)}{\mathcal{P}(\mathbf{k}_1)\mathcal{P}(\mathbf{k}_2)}$ with probability distributions $\mathcal{P}$
  - have an imprint of creation process and of final-state interactions
    ➔ if the source is approximately known, we can use it to study the interaction



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  - have an imprint of creation process and of final-state interactions ➔ if the source is approximately known, we can use it to study the interaction
- theoretical counterpart: $C(\mathbf{k}_1, \mathbf{k}_2) = \frac{c_{NN}}{N} \sum_{m_1,m_2} \int d\mathbf{r}_1 \int d\mathbf{r}_2 S_1(\mathbf{r}_1) S_1(\mathbf{r}_2) |\Psi_{m_1,m_2}(\mathbf{k}_1\mathbf{r}_1, \mathbf{k}_2\mathbf{r}_2)|^2$ with single-particle source function S_1 and wave function Ψ
- integrating out the center-of-mass dependence yields the Koonin-Pratt formula: $C(\mathbf{k}) = \frac{c_{NN}}{N} \sum_{m_1,m_2} \int d\mathbf{r} S(\mathbf{r}) |\Psi_{m_1,m_2,\mathbf{k}}(\mathbf{r})|^2$
- averaging over the angle of $\boldsymbol{k}$ yields: $C(k) = \frac{c_{NN}}{4\pi N} \sum_{m_1,m_2} \int \mathrm{d}\Omega_{\boldsymbol{k}} \left\langle \Psi_{m_1,m_2,\boldsymbol{k}} \middle| S(\mathbf{r}) \middle| \Psi_{m_1,m_2,\boldsymbol{k}} \right\rangle$

Correlation functions II

- typically evaluated in partial-wave basis with truncations
  ➔ truncation parameters $j_{\mathrm{max,int}}$ and $j_{\mathrm{max,free}}$

$$C(k) = \frac{c_{NN}}{4\pi N} \sum_{0 \le j \le j_{\mathrm{max,int}}} \sum_{l,s,t,l'} w_j \int \mathrm{d}r\, r^2 S(r) \left| \Psi^{(l')}_{k;(l,s,)j,t,m_t}(r) \right|^2$$
$$+ \frac{c_{NN}}{4\pi N} \sum_{j_{\mathrm{max,int}} < j \le j_{\mathrm{max,free}}} \sum_{l,s,t} w_j \int \mathrm{d}r\, r^2 S(r) \left| \Psi^{(\mathrm{free})}_{k;(l,s,)j,t,m_t}(r) \right|^2$$

- source function is typically Gaussian: $S(r) = \left(4\pi\rho^2\right)^{-3/2} e^{-r^2/(4\rho^2)}$

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- source function is typically Gaussian: $S(r) = \left(4\pi\rho^2\right)^{-3/2} e^{-r^2/(4\rho^2)}$
- $\Psi^{(l')}_{k;(l,s,)j,t,m_t}(r)$ is obtained by solving the coupled-channel Schrödinger equation
  - nuclear tensor force couples spin-1 channels with $l = l_- = j-1$ and $l = l_+ = j+1$

$$\left(\partial_r^2 - \frac{\hat{l}(\hat{l}+1)}{r^2} + k^2\right) \boldsymbol{u}_{k;\alpha}(r) - 2\mu V_\alpha(r) \boldsymbol{u}_{k;\alpha}(r) = 0 \quad \text{with } \alpha = \{s, j, t, m_t\}$$

  - boundary condition: j_l or F_l as incoming wave in partial-wave l

$$\boldsymbol{u}^{(l)}_{k;\alpha}(r) = \begin{pmatrix} u^{(l)}_{k;l_-,\alpha}(r) \\ u^{(l)}_{k;l_+,\alpha}(r) \end{pmatrix} \to \begin{pmatrix} \delta_{l,l_-}\tilde{j}_l(kr) + T^{(l,l_-)}_{k;\alpha}\tilde{h}^+_{l_-}(kr) \\ \delta_{l,l_+}\tilde{j}_l(kr) + T^{(l,l_+)}_{k;\alpha}\tilde{h}^+_{l_+}(kr) \end{pmatrix}$$

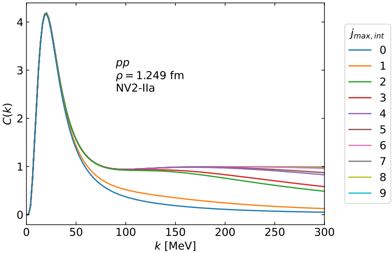
Nucleon-nucleon interactions

- Argonne V18 (AV18) Wirigna, Stoks, Schiavilla, PRC 51 (1995)
  - long-range part given by one-pion exchange
  - short-range part is phenomenological (terms: central, tensor, spin-orbit, quadratic spin-orbit, L^2)
  - fitted to phase shift data up to $T_{lab} = 350$ MeV
- Norfolk interactions Piarulli *et al.*, PRC 94 (2016)
  - following the chiral EFT approach
    - consistent with chiral symmetry
    - power counting determines which terms are present
  - N3LO
  - local formulation, Δ excitations included
  - versions differing in fit region
    - NV2-I: $T_{lab} \leq 125$ MeV
    - NV2-II: $T_{lab} \leq 200$ MeV
  - versions differing in regulation scale
    - a: $R_S = 0.8$ fm, $R_L = 1.2$ fm
    - b: $R_S = 0.7$ fm, $R_L = 1.0$ fm

Composition of correlation functions

pp correlation function as an example for the composition of correlation functions

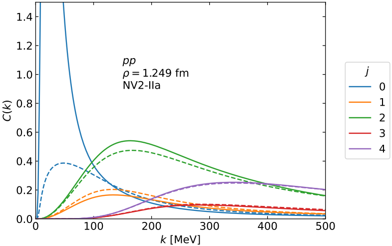
$$C(k) = \frac{c_{NN}}{4\pi N} \sum_{0\leq j\leq j_{max,int}} \sum_{l,s} \sum_{l'} \int \mathrm{d}r\, r^2 S(r) \left| \Psi^{(l')}_{k;(l,s),j,t}(r) \right|^2$$



- at low k dominated by lowest partial waves
- peak almost entirely given by $j = 0$ contribution (mostly s-wave)
- to get $C(k) = 1$ for $k \to \infty$: the larger the k, the more partial waves are necessary

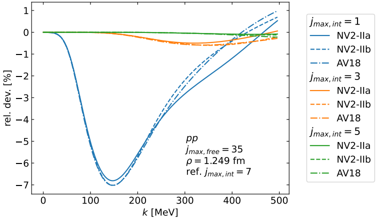
Composition of correlation functions
Difference between free and interacting contributions

- single components of the *pp* correlation function of specific *j*
  $C_j(k) = \frac{c_{NN}}{4\pi N} \sum_{l,s} \sum_{l'} \int \mathrm{d}r\, r^2 S(r) \left| \Psi^{(l')}_{k;(l,s),j,t}(r) \right|^2$
- free contribution (dashed line) vs. interacting contribution (solid line)



- the *s*-wave component displays largest difference
- the biggest deviation moves with higher *j* to higher *k*
  (interplay of centrifugal barrier (almost free at low *k*) and being almost free at high *k*)

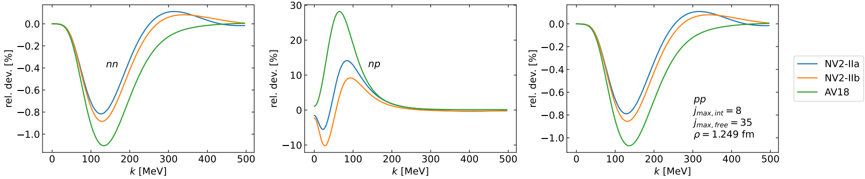
Convergence in the partial waves



- using $j_{max,int} = 5$ results in convergence better than 1 % for $k < 500$ MeV

Influence of the coupling between different channels

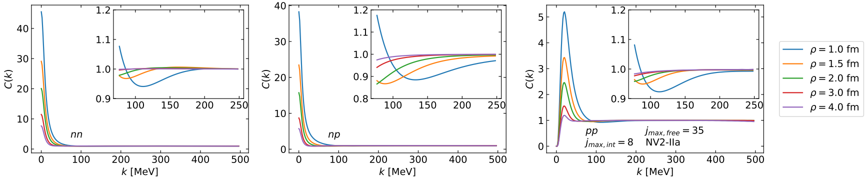
- effect of coupling often neglected ➔ quantification important
- quantify effect in terms of relative deviation $(C_{\text{cpld.}}(k)/C_{\text{uncpld.}}(k) - 1)$ in %



- effect of coupling due to the tensor force is much larger for *np* than for *nn* and *pp*
  - antisymmetrization condition: $(-1)^{l+s+t} = -1$
  - lowest coupled channels for *nn* and *pp*: $^3P_2$-$^3F_2$
  - lowest coupled channels for *np*: $^3S_1$-$^3D_1$

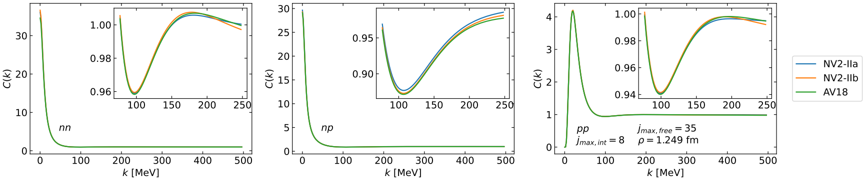
Correlation functions for different source radii ρ

- *pp* collisions lead to smaller source radii compared to, e.g., Pb-Pb collisions
  - *pp* @ $\sqrt{s} = 13$ TeV: $\rho = 1.249$ fm Acharya *et al.*, PLB 805 (2020)
  - Pb-Pb @ $\sqrt{s_{NN}} = 2.76$ TeV: $\rho \approx 4$ fm (depends also on selected m_T) Adam *et al.*, PRC 92 (2015)

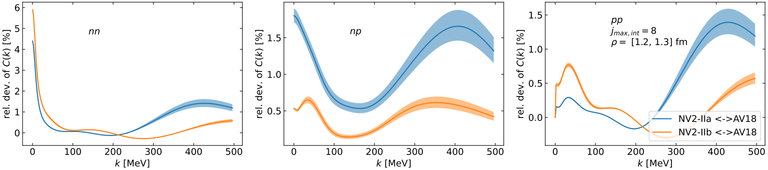


- smaller ρ lead to
  - less pronounced peak
  - smaller "dip" at intermediate k

Sensitivity to the *NN* interaction

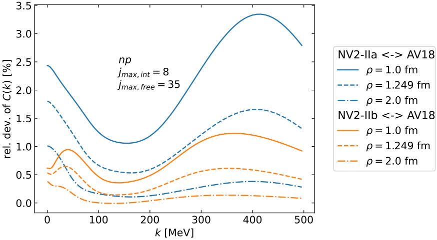


quantify effect in terms of relative deviation $(C_{\mathrm{NV2-IIx}}(k)/C_{\mathrm{AV18}}(k) - 1)$ in %



- sensitivity of up to 5.9 % for *nn*
- sensitivity of up to 1.8 % for *np*, 1.4 % for *pp*

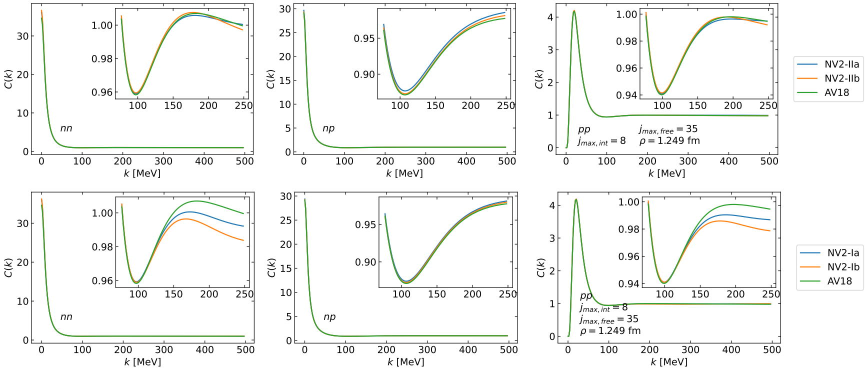
Sensitivity to the *NN* interaction
Source-radius-dependence



- the larger the source radius, the smaller the sensitivity

Sensitivity on the interaction
Connection to phase shifts

- compare NV2-I and NV2-II interactions
  - NV2-I: fitted phase shifts up to $T_{lab} = 125$ MeV $\leftrightarrow k \approx 242$ MeV
  - NV2-II: fitted phase shifts up to $T_{lab} = 200$ MeV $\leftrightarrow k \approx 307$ MeV



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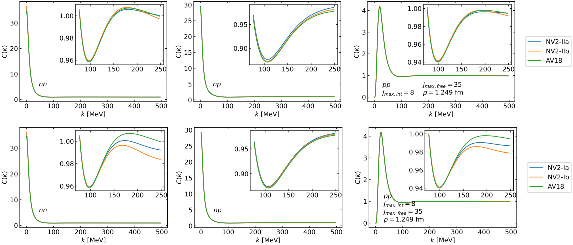

- in regions where phase shifts are reproduced better the correlation functions are more similar

➔ correlation functions and phase shifts seem to capture similar information
  - "unphysical" off-shell behavior might be also captured, but this illustrates the strong connection to the phase shifts
  - $C(k)$ might be especially useful in systems with scarce scattering data

Simplified approaches for calculating correlation functions
Gaussian representation

- one might want to infer effective-range-expansion parameters directly from $C(k)$
- simplified approaches are useful for this purpose
- one approach: Gaussian representation
  local Gaussian potential: $V(r) = V_G e^{-(r/r_G)^2}$ (plus Coulomb interaction)

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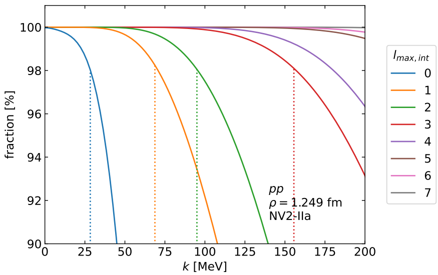

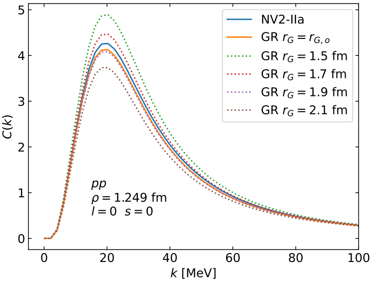

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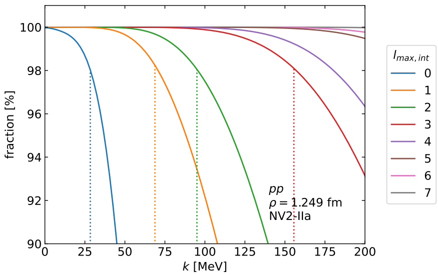

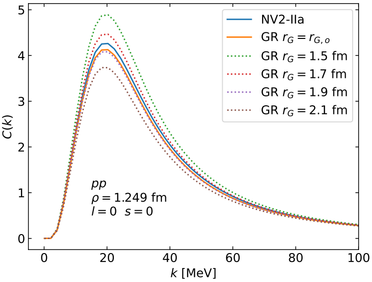

- Gaussian parameterization has an accuracy of about 3 % in the peak region

Simplified approaches for calculating correlation functions
Lednický-Lyuboshitz approach

idea

- use asymptotic wave function over complete range of r to calculate $C(k)$ Lednický, PPN 40 (2009)
- asymptotic wave function determined by scattering amplitude

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correlation function as sum of spin singlet (S) and spin triplet (T) contribution

$$C(k) = \frac{1}{4}\int \mathrm{d}\boldsymbol{r}\, S(\boldsymbol{r}) \left| \Psi^{(S)}_{-\boldsymbol{k}}(\boldsymbol{r}) + \Psi^{(S)}_{+\boldsymbol{k}}(\boldsymbol{r}) \right|^2 + \frac{3}{4}\int \mathrm{d}\boldsymbol{r}\, S(\boldsymbol{r}) \left| \Psi^{(T)}_{-\boldsymbol{k}}(\boldsymbol{r}) - \Psi^{(T)}_{+\boldsymbol{k}}(\boldsymbol{r}) \right|^2$$

$$\Psi^{(i)}_{-\boldsymbol{k}}(\boldsymbol{r}) = e^{\mathrm{i}\delta_c}\sqrt{A_c(\eta)}\left[e^{-\mathrm{i}\boldsymbol{kr}} F(-\mathrm{i}\eta, 1, \mathrm{i}\xi) + \delta_{i,S} f_{c,0}(k) \tilde{G}_0(kr, \eta)/r\right]$$

- confluent hypergeometric function F (stems from sum over F_l)
- $\tilde{G}_l = \sqrt{A_c(\eta)}\,(F_l + \mathrm{i}G_l)$, a combination of Coulomb wave functions

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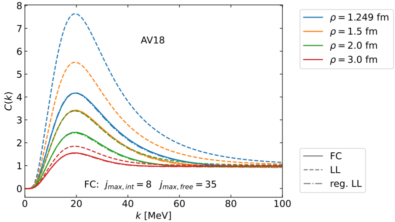
remarks

- takes all free waves into account
- extension beyond s-wave due to divergence structure of G_l problematic

Simplified approaches for calculating correlation functions
Lednický-Lyuboshitz approach II

comparison of *pp* correlation function based on AV18 and based on LL approach (using corresponding ERE parameters)



- for smaller source radii LL approach works not so well
  (about 40 % dev. in peak region for $\rho = 2$ fm, about 20 % dev. for $\rho = 3$ fm)
- similar findings were reported for *pd* system Rzesa, Stefaniak, Pratt, PRC 111 (2025)
- possible fix: use regularized G_l: $G_l \to G_l \left(1 - e^{-\gamma r}\right)$
  - works quite well for $\gamma = 1.0$ fm^{-1}
  - has scale dependence, Gaussian parameterization might be the better approach

Conclusion & Outlook

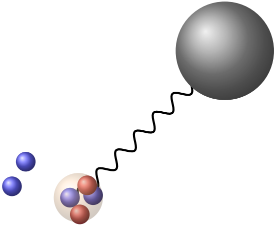
Conclusion

- detailed study of *NN* correlations based on different interactions (also chiral ints.)
- studied **convergence behavior** in detail ➔ important for **accurate results**
- correlation functions are **sensitive to nuclear interactions**, captures similar information as phase shift
  ➔ especially important in sectors with scarce scattering data
- benchmarked more approximate, effective approaches such as Lednický-Lyuboshitz approach or Gaussian representation

Outlook

- study hyperon-nucleon correlation function
- calculation of Λd correlation function using hyperspherical harmonics formalism ($\Lambda + p + n$)

Part II



*E*1 strength distributions
following Coulomb dissociation
&
finite-range interactions

in collaboration with H.-W. Hammer and D. R. Phillips

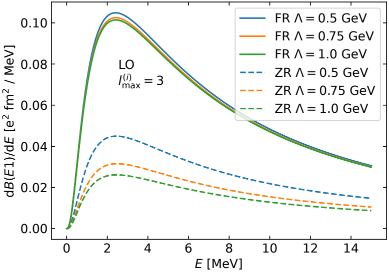
^{6}He in Halo EFT

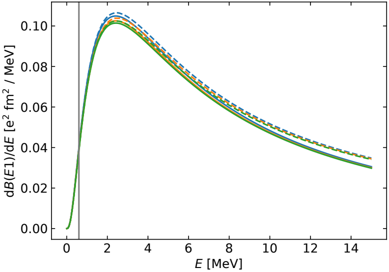
- $E1$ strength distribution $\mathrm{d}B(E1)/\mathrm{d}E$ is an interesting and experimentally well accessible observable
- $\mathrm{d}B(E1)/\mathrm{d}E$ of ^{6}He has also in theory been extensively investigated
  - often in cluster models , e.g., Cobis *et al.*, PRL 79 (1997), Danilin *et al.*, NPA 632 (1998), Forssén *et al.*, NPA 697 (2002), Grigorenko *et al.*, PRC 102 (2020)
  - recently also in cluster E(F)Ts Bertulani, PRC 108 (2023), Pinilla *et al.*, PRC 112 (2025)
- calculate $\mathrm{d}B(E1)/\mathrm{d}E$ of ^{6}He in Halo EFT
  - EFT for halo nuclei Hammer *et al.*, JPG 44 (2017)
  - α core and two neutrons as degrees of freedom
  - two-body interactions parameterized in terms of effective-range expansion (ERE) parameters
  - three-body interaction for renormalizing three-body system
  - systematic improvability and uncertainty estimates
- three-body dynamics are described in terms of Faddeev equations
  - three-body wave functions can afterwards be assembled from Faddeev amplitudes
- speciality: use finite-range interactions, wherever more than one ERE parameter needs to be fitted

  ➔ avoids difficulties with energy-dependent interactions

Leading-order results
(preliminary)

- compare zero-range (ZR) and finite-range (FR) results for different cutoffs Λ
  - $\Lambda_1 = 500$ MeV, $\Lambda_2 = 750$ MeV, $\Lambda_3 = 1000$ MeV
- left: $\mathrm{d}B(E1)/\mathrm{d}E$ itself; right: shape of $\mathrm{d}B(E1)/\mathrm{d}E$





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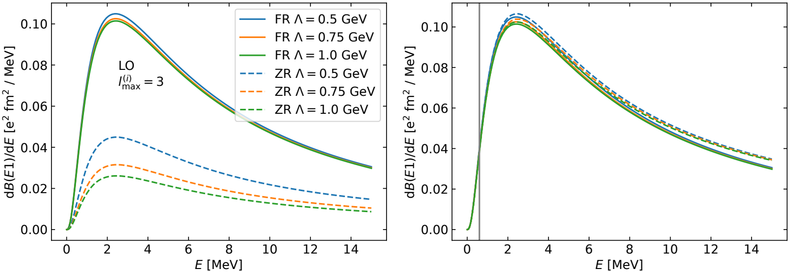

➔ zero-range (ZR) approach has convergence issue in Λ, related to $V(E)$

➔ finite-range (FR) approach is convergent

➔ results for the shape agree

Going to NLO

- inclusion of the different NLO effects in the finite-range approach ($^2S_{1/2}$ nc int., r_0-term of $^1S_0$ nn int., (UT of $^2P_{3/2}$ nc int. in FR already LO))

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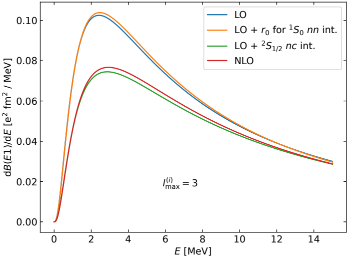

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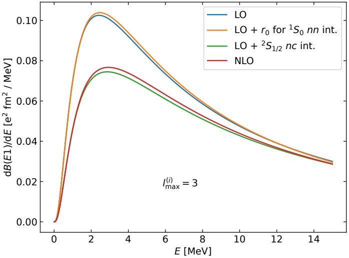

➔ NLO corrections have the expected size

➔ NLO correction from $^2S_{1/2}$ *nc* int. much stronger than from *nn* r_0 term

Final-state interactions and partial waves

- calc. FSI via Møller operator: e.g. *nn* FSI:
  ${}_c\langle p, q; \Omega_c | \, \Omega_{nn} = {}_c\langle p, q; \Omega_c | + {}_c\langle p, q; \Omega_c | \, t_{nn}(E_p) G_0(E_p)$
- also apply multiple FSIs by using products of Møller operators: e.g. $\Omega_{nn}\Omega_{nc;1,3/2}$ Göbel, Acharya, Hammer, Phillips, PRC 107 (2023)

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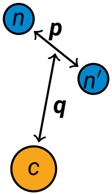
- *a priori* the matrix element of the t_i acting in the *jk* subsystem is known for i $(=\mathcal{S}_i)$ as spectator
  ${}_{i}\langle p,q;\Omega|t_i(E_3)|p',q';\Omega'\rangle_{i}$

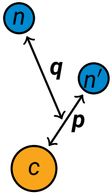
➔ recoupling between states of different spectators and different partial waves in some cases necessary

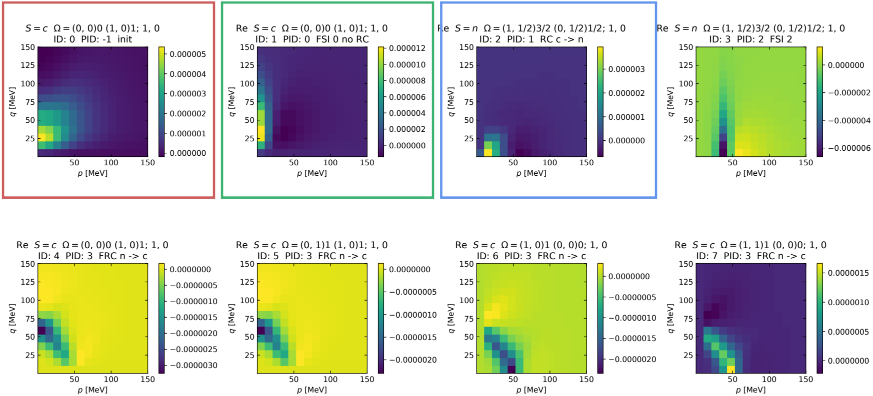
- strategy: make use of relation for
  ${}_{\mathcal{S},\mathcal{S}'}\mathcal{T}^{p,q|p',q'}_{\Omega,\Omega'} f(p',q') :=$
  $\int \mathrm{d}p'\, p'^2 \int \mathrm{d}q'\, q'^2\, {}_{\mathcal{S}}\langle p,q;\Omega|p',q';\Omega'\rangle_{\mathcal{S}'}\, f(p',q')$
  - simplify analytically to reduce number of numerical integrations (Wigner 3nj symbols etc.)

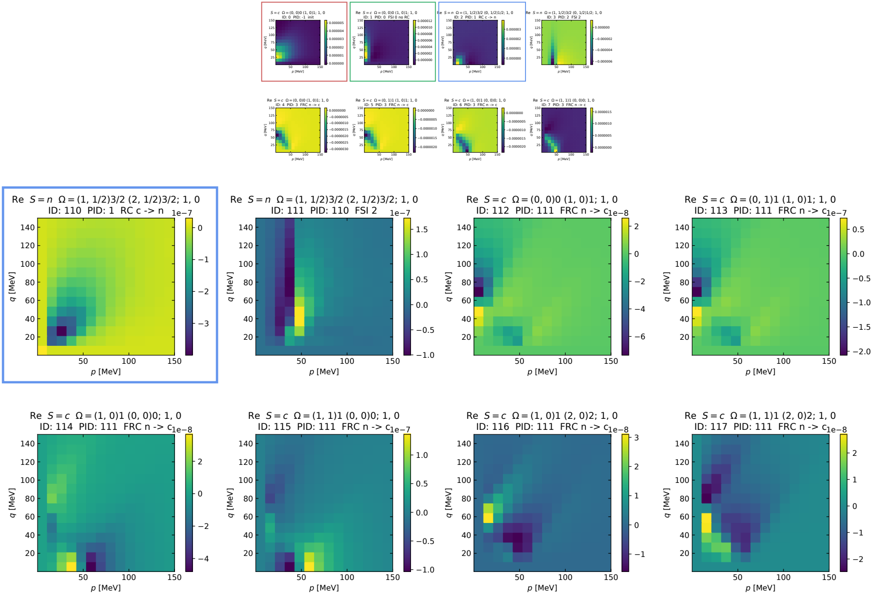
spectator: *c*



spectator: *n*

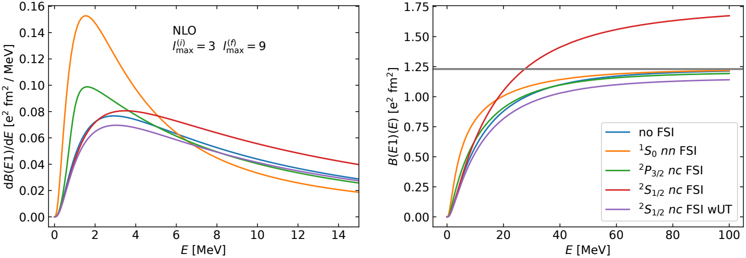



S = c Ω = (0, 0)0 (1, 0)1; 1, 0
ID: 0 PID: -1 init
Re S = c Ω = (0, 0)0 (1, 0)1; 1, 0
ID: 1 PID: 0 FSI 0 no RC
Re S = n Ω = (1, 1/2)3/2 (0, 1/2)1/2; 1, 0
ID: 2 PID: 1 RC c -> n
Re S = n Ω = (1, 1/2)3/2 (0, 1/2)1/2; 1, 0
ID: 3 PID: 2 FSI 2
Re S = c Ω = (0, 0)0 (1, 0)1; 1, 0
ID: 4 PID: 3 FRC n -> c
Re S = c Ω = (0, 1)1 (1, 0)1; 1, 0
ID: 5 PID: 3 FRC n -> c
Re S = c Ω = (1, 0)1 (0, 0)0; 1, 0
ID: 6 PID: 3 FRC n -> c
Re S = c Ω = (1, 1)1 (0, 0)0; 1, 0
ID: 7 PID: 3 FRC n -> c
q [MeV]
p [MeV]


Re S = n Ω = (1, 1/2)3/2 (2, 1/2)3/2; 1, 0
ID: 110 PID: 1 RC c -> n
Re S = n Ω = (1, 1/2)3/2 (2, 1/2)3/2; 1, 0
ID: 111 PID: 110 FSI 2
Re S = c Ω = (0, 0)0 (1, 0)1; 1, 0
ID: 112 PID: 111 FRC n -> c
Re S = c Ω = (0, 1)1 (1, 0)1; 1, 0
ID: 113 PID: 111 FRC n -> c
Re S = c Ω = (1, 0)1 (0, 0)0; 1, 0
ID: 114 PID: 111 FRC n -> c
Re S = c Ω = (1, 1)1 (0, 0)0; 1, 0
ID: 115 PID: 111 FRC n -> c
Re S = c Ω = (1, 0)1 (2, 0)2; 1, 0
ID: 116 PID: 111 FRC n -> c
Re S = c Ω = (1, 1)1 (2, 0)2; 1, 0
ID: 117 PID: 111 FRC n -> c
q [MeV]
p [MeV]

NLO results with FSIs I

- all FSIs (also $^2S_{1/2}$ *nc* FSI) in comparison on the basis of the NLO ground state
- overall $E1$ strength obtained from $\langle r_c^2 \rangle$ via sum rule

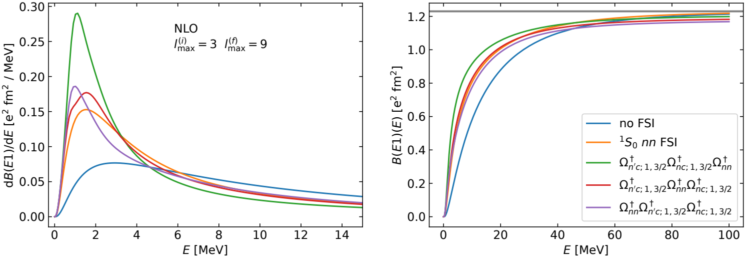


➔ sum rule fulfilled except for one FSI, which is missing unitarity term (UT) according to power counting

➔ *nn* FSI is more important than $^2P_{3/2}$ *nc* and $^2S_{1/2}$ *nc* FSIs

NLO results with FSIs II

- now also FSI based on products of three Møller operators (third order)



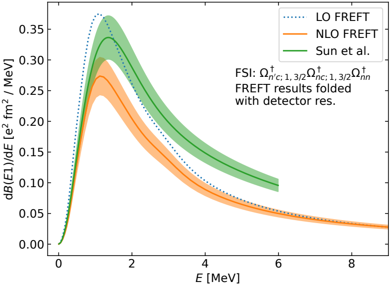
Göbel, Hammer, Phillips, in preparation

➔ effects of going to third order in FSI approximation quite strong

➔ strong variation between different orders of applying the single FSIs; in any case stronger than *nn* FSI only

NLO results in comparison with experimental data

- folded with detector resolution
- uncertainty estimated according to power counting
- in comparison with experimental data from Sun *et al.*, PLB 814 (2021)



Göbel, Hammer, Phillips, in preparation

→ acceptable agreement
→ including FSI up to third order important for agreement

Conclusion and outlook

Conclusion

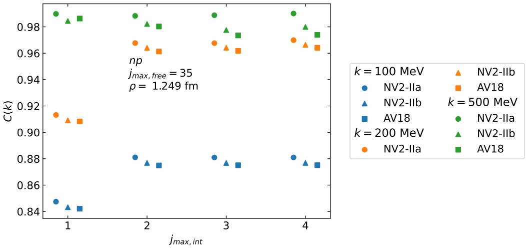
- finite-range interactions for *p*-wave *nc* interaction work well
  ➔ convergence of the *E*1 distribution in cutoff Λ
- NLO correction due to *s*-wave *nc* interaction important ➔ reduces *E*1 strength
- FSI effects beyond leading order in the Møller operator significant ➔ increases *E*1 strength
- acceptable agreement with experimental data

Outlook

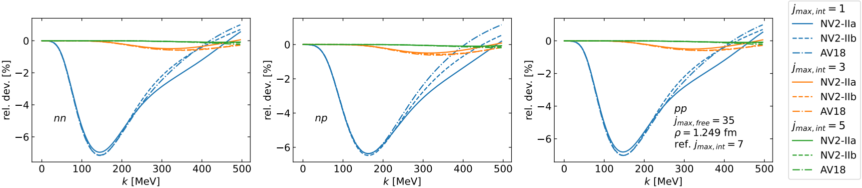
- full three-body calculation of FSI work in progress
- use these interactions also to calculate *nn* relative-energy distribution at NLO

Backup slides

Convergence in partial waves



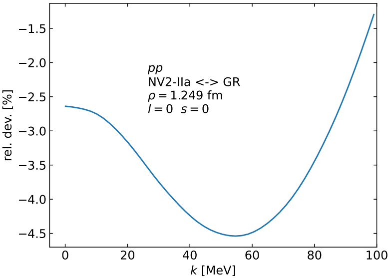
Convergence in partial waves II



- convergence behavior is similar for all three systems

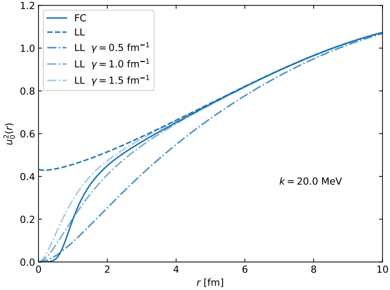
Gaussian representation

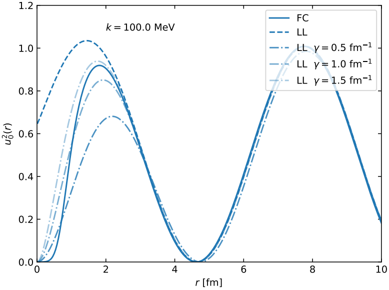
relative deviation between result from Gaussian representation and full calculation



Wave functions of regularized Lednický-Lyuboshitz approach

squares of radial wave functions from direct calculation, from LL approach, and from regularized LL approach in comparison





$E1$ strength distributions in halo EFT

$E1$ strength as an interesting observable

- parameterizes the Coulomb dissociation cross section: $\frac{\mathrm{d}\sigma}{\mathrm{d}E} \propto \frac{\mathrm{d}B(E1)}{\mathrm{d}E}$
- characteristic property of halo nuclei
- for $2n$ halos reltd. to a large core distance r_c

see, e.g., Forssén, Efros, Zhukov, NPA 697 (2002), Acharya, Phillips, EPJ Web Conf. 113 (2016), Hagen, (2014)
review of low-energy dipole response in Aumann, EPJA 55 (2019)

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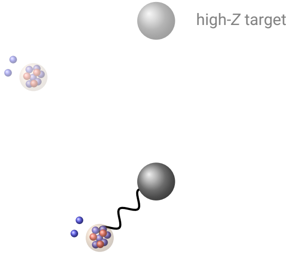
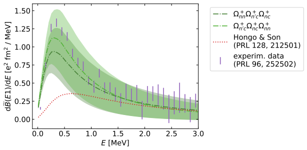


$E1$ strength of ^{11}Li in halo EFT

good agreement with experimental data from Nakamura et al., PRL 96 (2006) was found



Göbel, Acharya, Hammer, Phillips, PRC 107 (2023)

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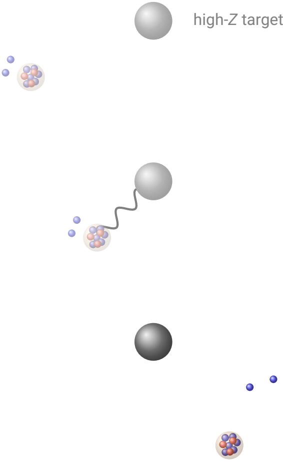

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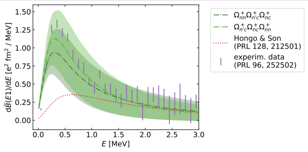

Göbel, Acharya, Hammer, Phillips, PRC 107 (2023)

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    solutions of H @ different E are in fact solutions of different H

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  ➔ corrections $\propto \partial_E V$ to expectation values and normalization are necessary
  - probability density of ^{6}He: $\forall\, p, q < M_{hi}$: corrections to the normalization are most important Göbel, Hammer, Ji, Phillips, FBS 60 (2019)

Finite-range EFT

- implication for $\frac{\mathrm{d}B(E1)}{\mathrm{d}E}$ in zero-range halo EFT
  - shape: no problem
  - absolute value: would require correction terms
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- finite-range interactions in use: Yamaguchi (YM) interactions Yamaguchi, PR 95 (1954)
  - work well in momentum-space Faddeev calculations
  - have already two parameters ➔ ideal for p-wave $n\alpha$ int.
  - extension of YM form factors ➔ more parameters ➔ reproducability of more ERE terms
- Yamaguchi interaction is a rank-one separable interaction:
  $\langle p, l | V_{\bar{l}} | p', l' \rangle = \delta_{l,l'} \delta_{l,\bar{l}} g_l(p) \lambda_l g_l(p')$
  with YM form factors $g_l(p) := p^l \frac{\beta_l^4}{\left(p^2+\beta_l^2\right)^2}$

Obtaining the $E1$ distribution of ^{6}He

- **approach:** ^{6}He in halo EFT
  1. calculate wave functions $\Psi_{c;\Omega}(p,q)$ (for different partial waves Ω)
  2. evaluate the $E1$ operator
  3. take final-state interactions into account
  4. obtain $E1$ strength distribution

$$\frac{\mathrm{d}B(E1)}{\mathrm{d}E} = \frac{1}{2J_i+1} \sum_{M_i,\mu} \int \mathrm{d}\tau_f \, \left| \langle f | \mathcal{M}(E1,\mu) | i; J_i, M_i \rangle \right|^2 \delta(E - E_f)$$

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  - $\not\pi$EFT
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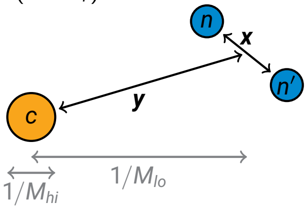
  - results are expanded in k/M_{hi}
    $\rightarrow$ systematic improvement possible



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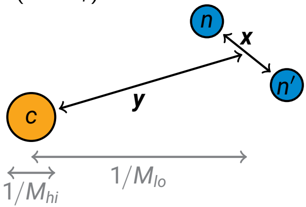
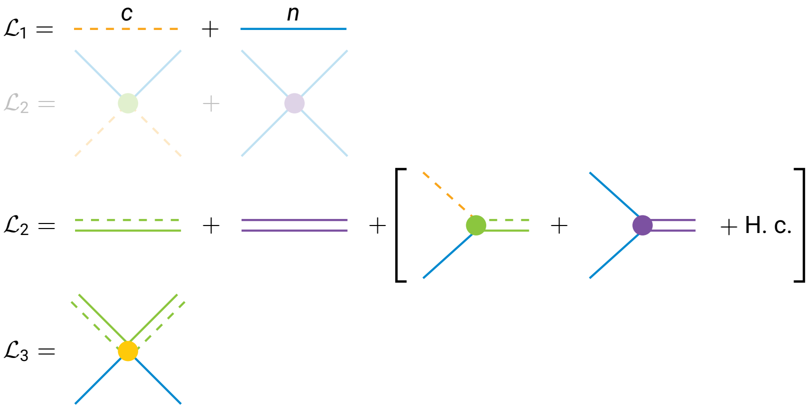


- **properties of ^{6}He**
  - Borromean $2n$ halo
  - separation of scales: $S_{2n} = 0.975\,\mathrm{MeV} \; < \; E_\alpha^* \approx 20\,\mathrm{MeV}$
  - quantum numbers: $J^\pi = 0^+$ (^{4}He: $J^\pi = 0^+$)
  - leading-order (LO) halo EFT interaction channels:
    nn: $^1S_0$; nc: $^2P_{3/2}$

halo EFT for ^{6}He formulated in Ji, Elster, Phillips, PRC 90 (2014)
review of halo EFT in Hammer, Ji, Phillips, JPG 44 (2017)

Leading-order Lagrangian



Leading-order Faddeev equations

use EFT in dimer formalism

1. step: obtain dressed dimer propagators

& renormalize using **input values**

a_1, r_1

a_0

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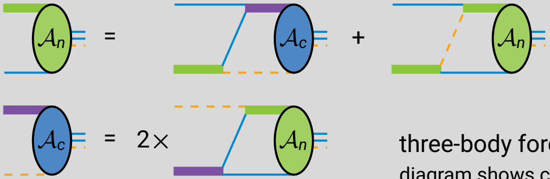
2. step: set up equations for Faddeev transition amplitudes



three-body force required for renormalization

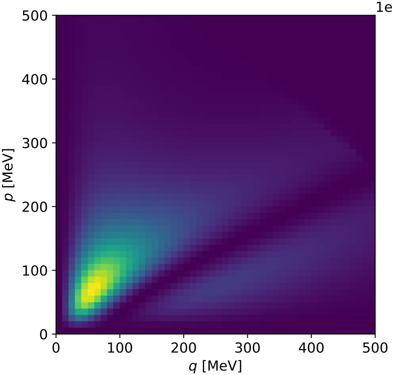
diagram shows case of vanishing three-body force

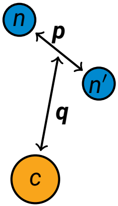
Results for the wave function

calculated ground-state wave functions and probability densities in halo EFT

Göbel, Hammer, Ji, Phillips, FBS 60 (2019)

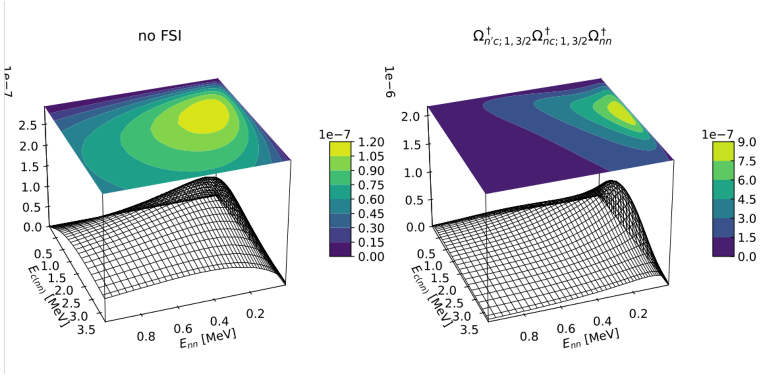
$\Psi_c^2(p,q)p^2q^2$





2D E1 distributions

definition: $\frac{\mathrm{d}B(E1)}{\mathrm{d}E_1 E_2} = \frac{1}{2J_i+1} \sum_{M_i,\mu} \int \mathrm{d}\tau_f \, | \langle f | \mathcal{M}(E1,\mu) | i; J_i, M_i \rangle |^2 \, \delta\left(E_1 - E_1^{(f)}\right) \delta\left(E_2 - E_2^{(f)}\right)$



➔ peak position fits expectation based on a_{nn}