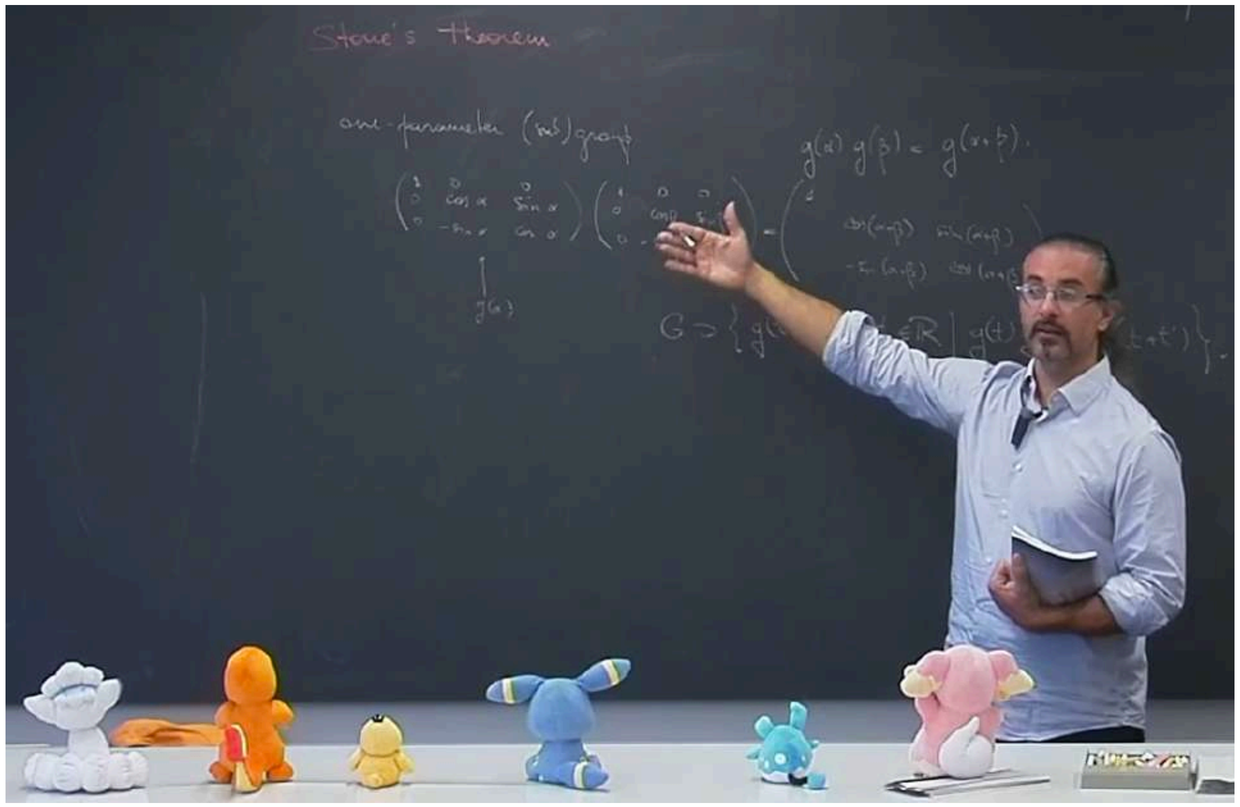
Multi-neutron systems and the large-charge expansion

Silas Beane

Hirschegg, January 2026

I will discuss work done in collaboration with

Domenico Orlando (INFN, Torino and Bern) and Susanne Reffert (Bern)



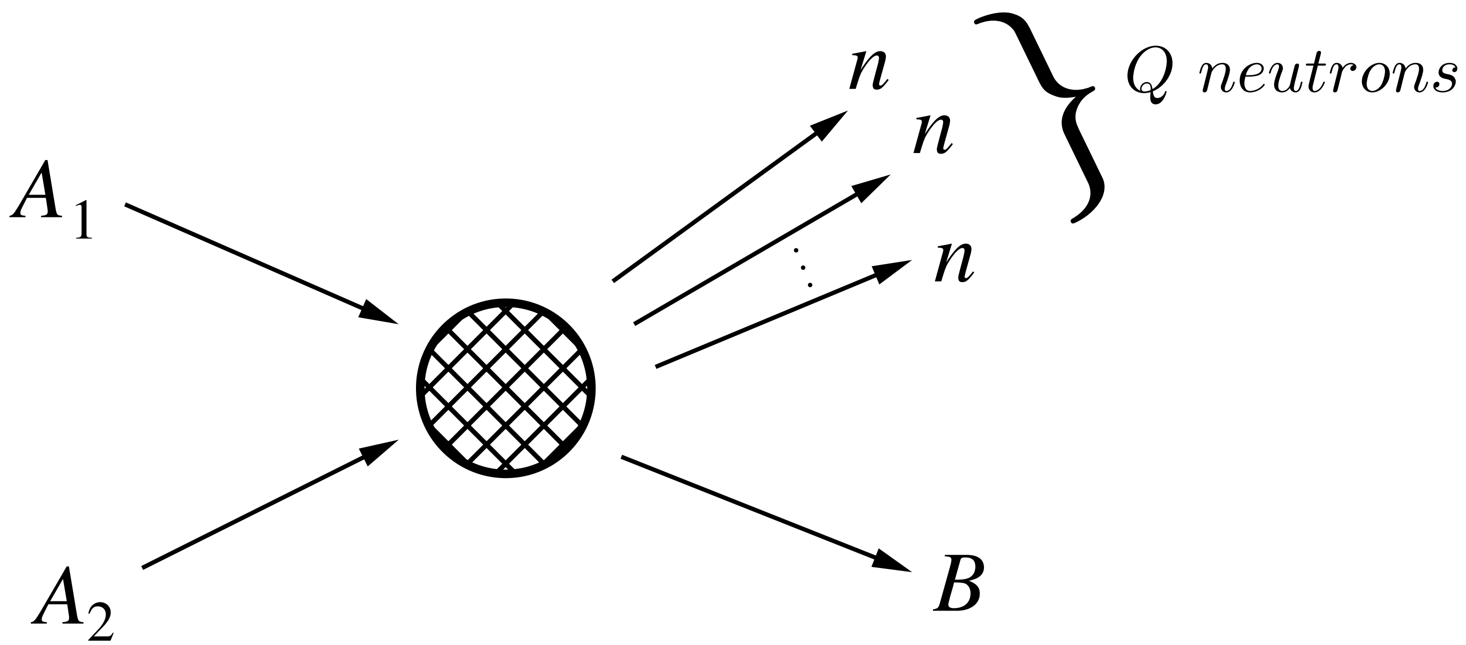
arXiv:2403.18898

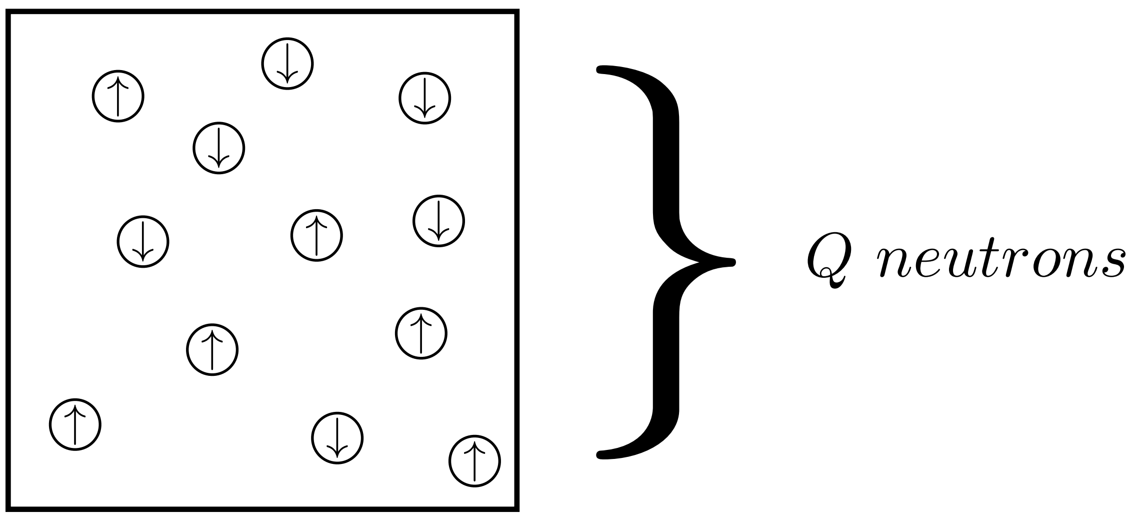
arXiv:2501.10505

arXiv:2309.15177

(Chowdhuri, Mishra,Son)

Multi-neutron systems:





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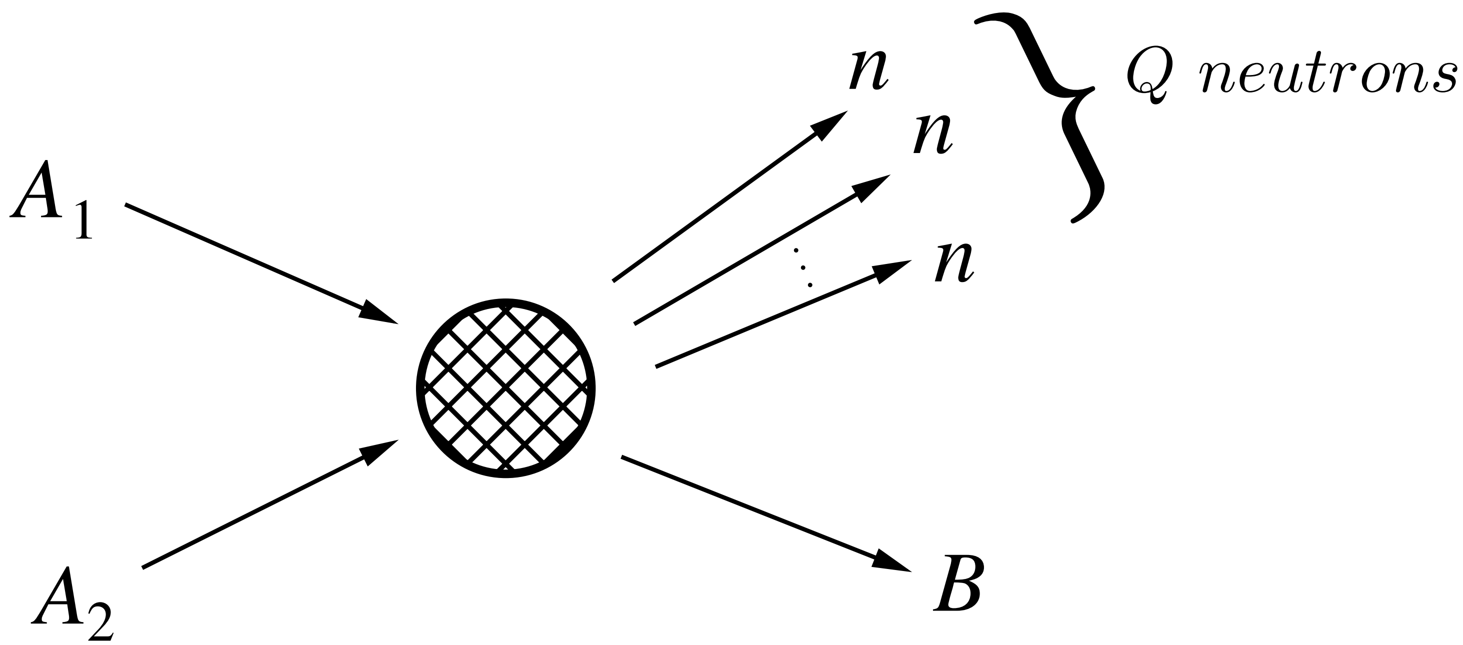
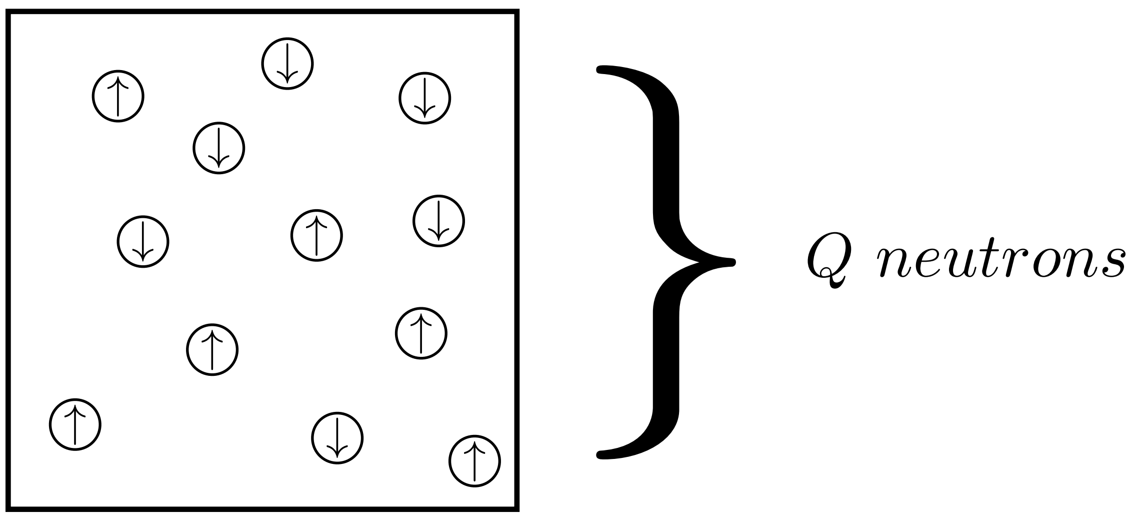


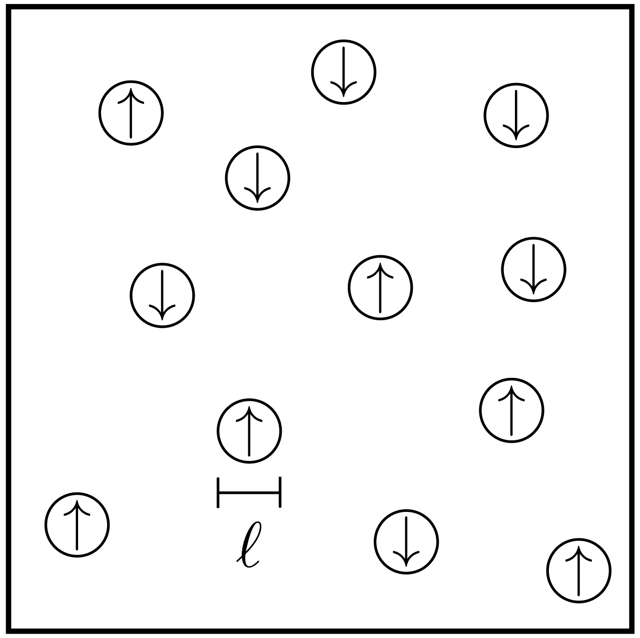


Will exploit two theoretical tools:

- Schrödinger symmetry and its breaking
- The large-charge expansion (superfluid EFT)

Overview: unitary Fermi gas and neutrons

Consider a gas of Q non-relativistic fermions of mass M with range of interaction ℓ



$$\rho\ell^3 \ll 1$$

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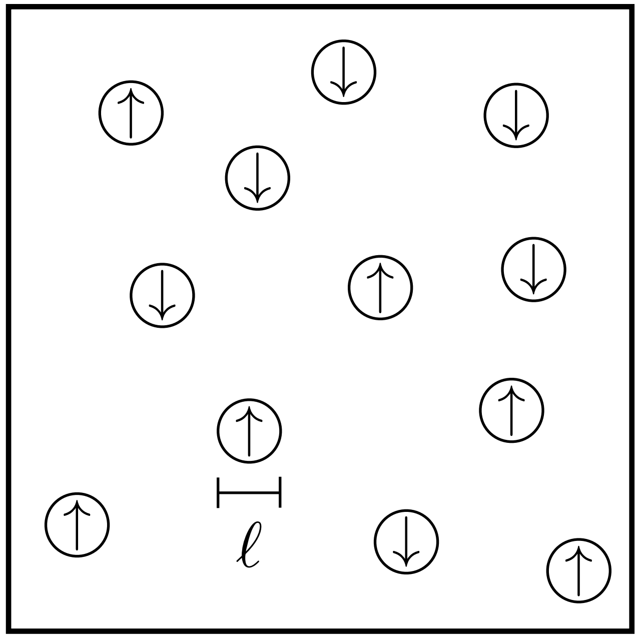

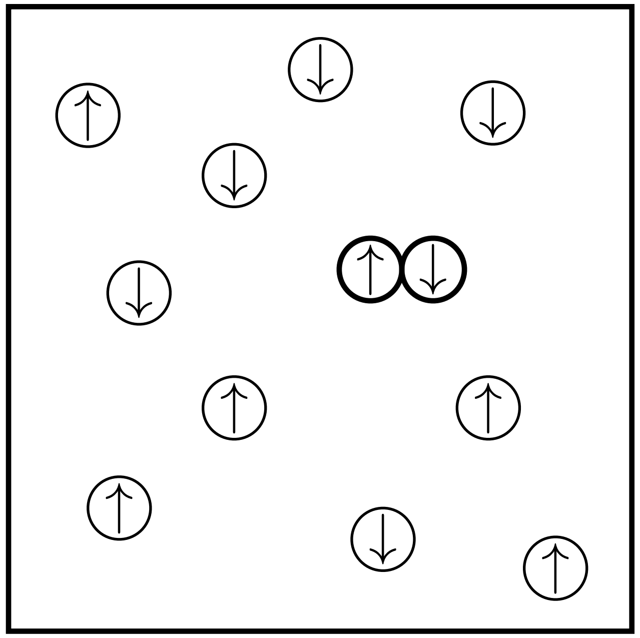

$$\rho \ell^3 \ll 1 \qquad\qquad S = e^{2i\delta(k)}$$

$$\boxed{k \cot \delta(k) = -\frac{1}{a} + \frac{1}{2} r k^2 + v_2 k^4 + v_3 k^6 + \ldots}$$

$$r \sim (v_2)^{1/3} \sim (v_3)^{1/5} \ldots \sim \ell$$

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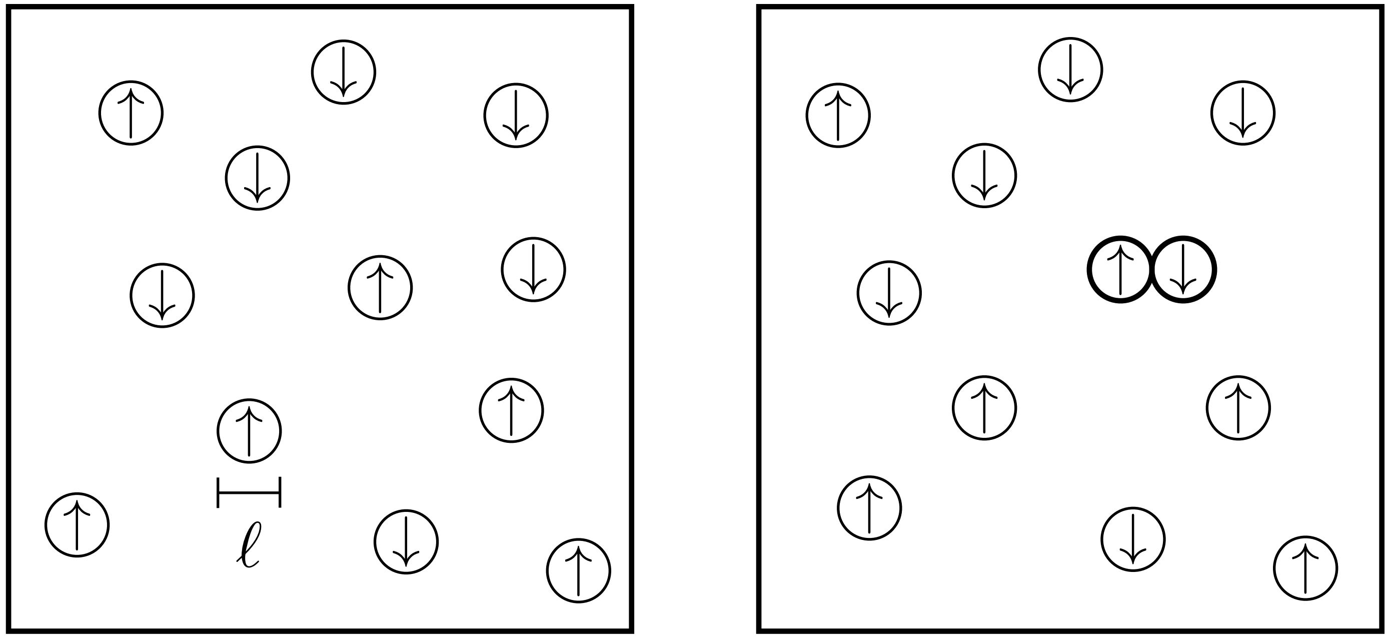


$\rho\ell^3 \ll 1$

$S \;=\; e^{2i\delta(k)}$

$$\boxed{k\cot\delta(k) \;=\; -\frac{1}{a} \;+\; \frac{1}{2}rk^2 \;+\; v_2k^4 \;+\; v_3k^6 \;+\; \ldots}$$

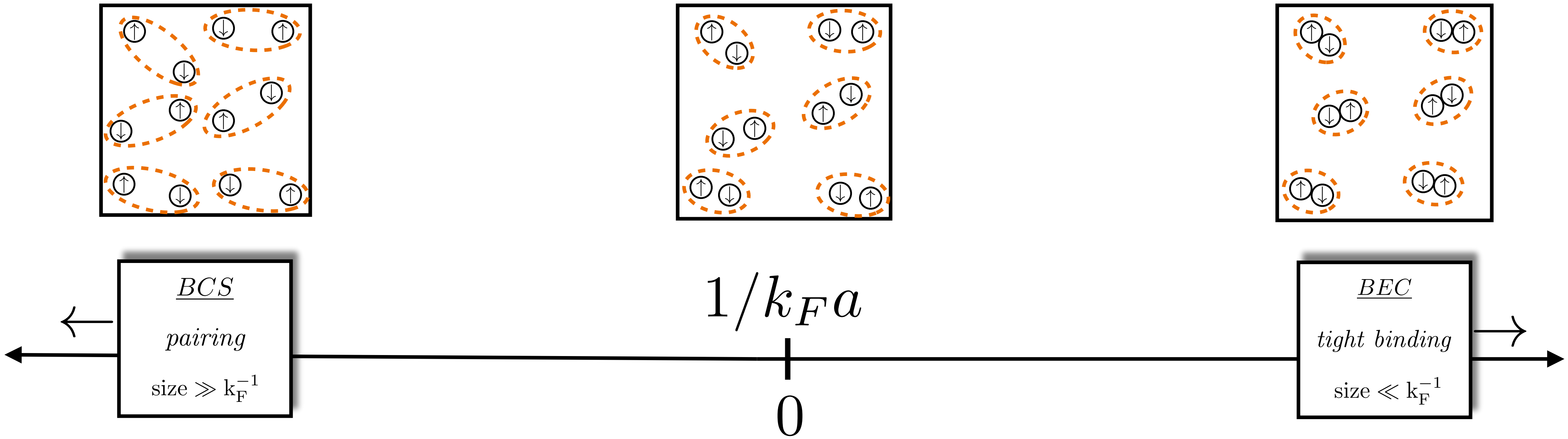
$r \sim (v_2)^{1/3} \sim (v_3)^{1/5} \ldots \sim \ell$

<u>Unitary Fermi gas</u>: $\ell \to 0 \;\;,\;\; a \to \infty$

Can exploit non-relativistic conformal symmetry (=Schrödinger symmetry)!

Q large builds a Fermi surface. If there is attraction, implies..

<u>Superfluidity/pairing</u>: spontaneous breaking of particle number



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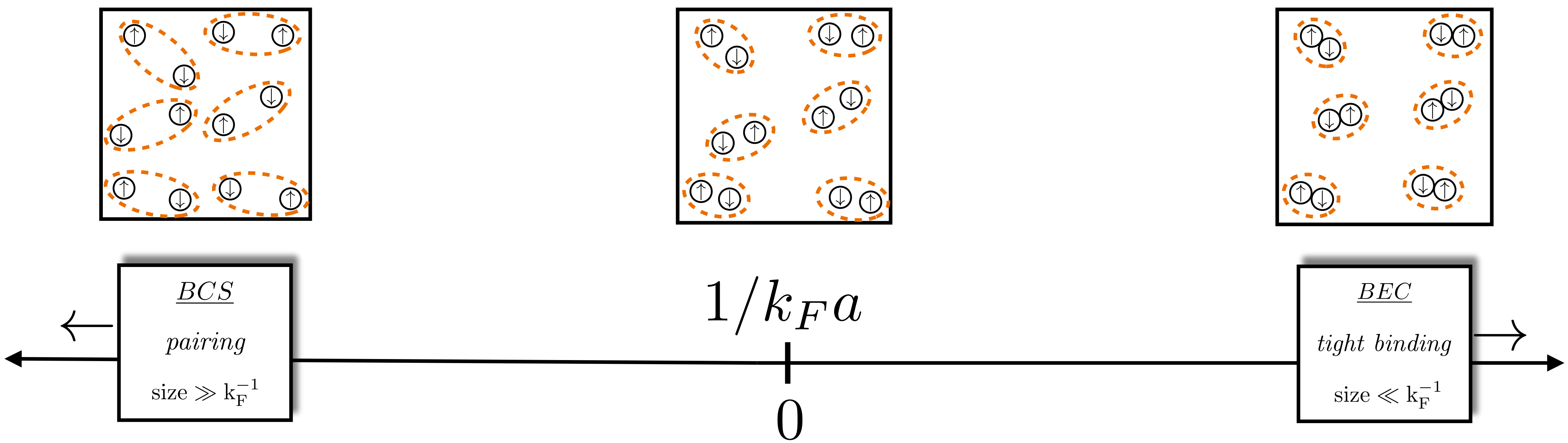

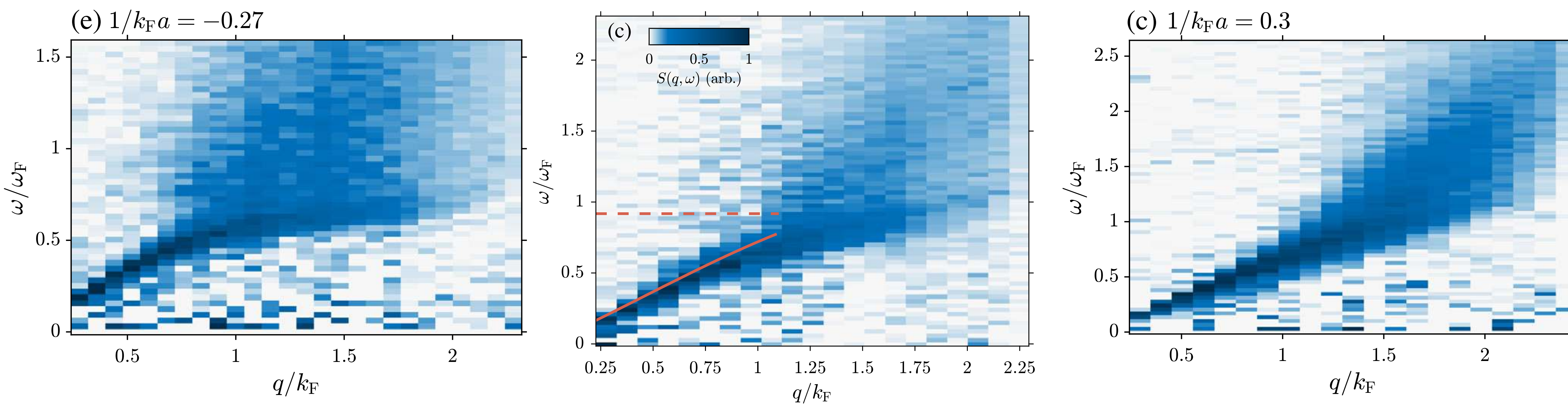

Bragg spectroscopy (Biss et al)

$$\boxed{\omega_q = c_s\, q\left(1 + \zeta \frac{q^2}{k_F^2} + \mathcal{O}(q^4 \log q)\right)}$$

$$\zeta^{\exp}_{a^{-1}=0} = -0.085(8)$$

"concave"

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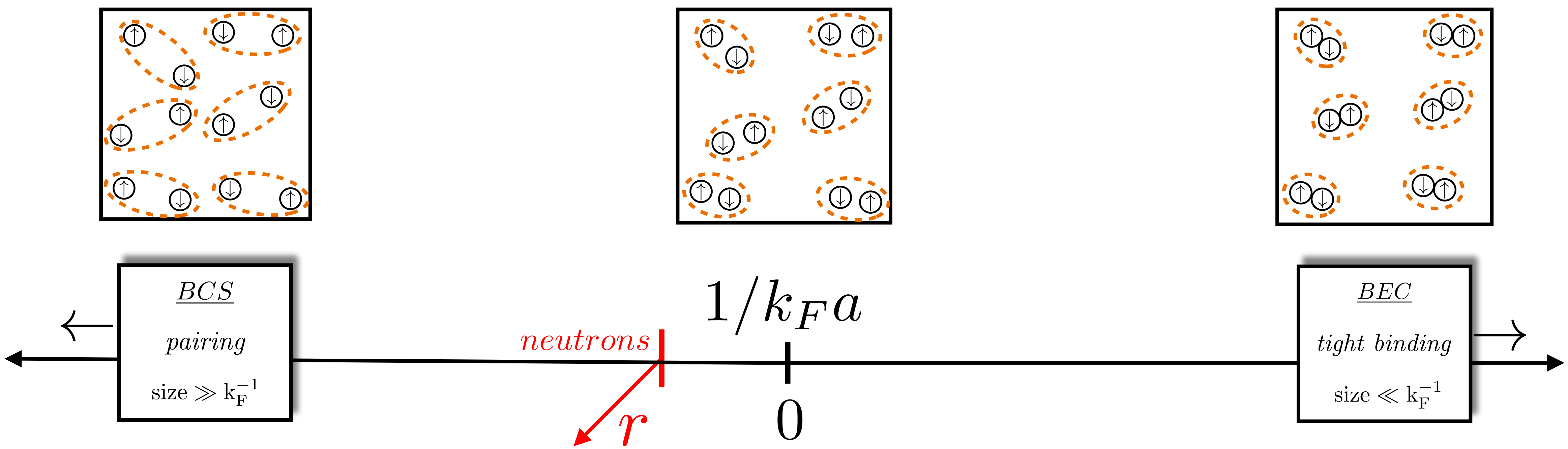

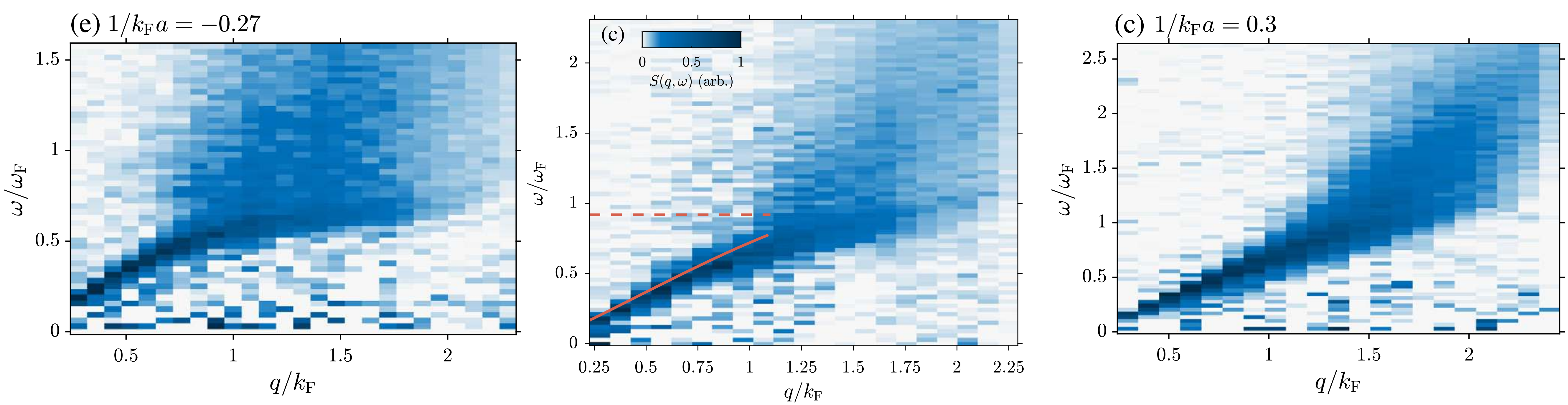

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$a \sim -24\,\mathrm{fm}$ 🙂

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$a \sim -24\,\mathrm{fm}$ 🙂 $\quad r \sim 3\ \mathrm{fm}$ 😭 $\quad v_2 \sim 0.4\,\mathrm{fm}^3$ 🙂 $\quad v_3 \sim 0.7\,\mathrm{fm}^5$ 🙂

Fundamental EFT defined

Introduce auxiliary field s. At unitarity CFT defined by:

$$\boxed{\mathcal{L}_{CFT} = \psi_\sigma^\dagger\left[i\partial_t + \frac{\overrightarrow{\nabla}^2}{2M}\right]\psi_\sigma + \frac{1}{C_\star}s^\dagger s + \psi_\downarrow^\dagger\psi_\uparrow^\dagger s + s^\dagger\psi_\uparrow\psi_\downarrow}$$

$\sigma = \uparrow, \downarrow$

$$C_0^{\mathrm{PDS}}(\nu) \;=\; \frac{4\pi}{M}\frac{1}{1/a \;-\; \nu} \;\underset{a\to\infty}{\longrightarrow}\; C_\star$$

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$$C_0^{\mathrm{PDS}}(\nu) = \frac{4\pi}{M} \frac{1}{1/a - \nu} \xrightarrow[a \to \infty]{} C_\star$$

Deformed CFT is defined by:

$$\boxed{\mathcal{L} = \mathcal{L}_{CFT} + \frac{M}{4\pi a} s^\dagger s - \frac{M^2 r}{16\pi} s^\dagger \left(i \overleftrightarrow{\partial_t} + \frac{\overrightarrow{\nabla}^2 + \overleftarrow{\nabla}^2}{4M} \right) s}$$

$[s] = [O_2] = 2$

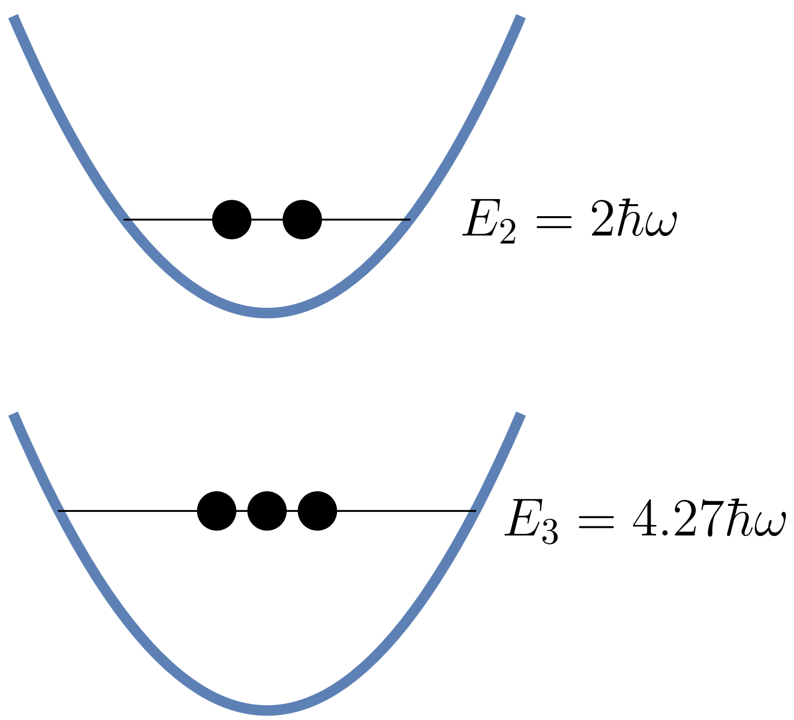
relevant deformation

irrelevant deformation

State-operator correspondence

(Nishida,Son)

The dimension of a primary operator = energy of state in harmonic potential



$\Longrightarrow$

$\Longrightarrow$

@unitarity	@free
$\Delta_2 = 2$	3
$\Delta_3 = 4.27$	$\frac{11}{2}$

Correspondence valid only in the symmetry limit!

Consider subsectors of fixed charge Q (canonical)

$$G(x_1, x_2, \ldots, x_N) \;=\; -i\langle 0|T\left(\mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\ldots\mathcal{O}_N(x_N)\right)|0\rangle$$

n-point functions constrained by Schrödinger symmetry

$$\boxed{G(x_1, x_2) = \delta_{Q_1,Q_2}\delta_{\Delta_1,\Delta_2}\,\theta(\tau_{12})\,\tau_{12}^{-\Delta_1}\,\exp\left(-\frac{Q_1 M \mathbf{x}_{12}^2}{2\tau_{12}}\right)\Psi_2 + \ldots}$$

$$\boxed{\begin{aligned} G(x_1, x_2, x_3) = \delta_{Q_1+Q_2,-Q_3}\,\theta(\tau_{13})\,\theta(\tau_{23})\,\tau_{13}^{-\Delta_{13,2}/2}\,\tau_{23}^{-\Delta_{23,1}/2}\,\tau_{12}^{-\Delta_{12,3}/2} & \\ \times \exp\left(-\frac{Q_1 M \mathbf{x}_{13}^2}{2\tau_{13}} - \frac{Q_2 M \mathbf{x}_{23}^2}{2\tau_{23}}\right)\Psi_3(v_{123}) + \ldots & \end{aligned}}$$

$$\tau_{ij} \equiv \tau_i - \tau_j,\; \mathbf{x}_{ij} \equiv \mathbf{x}_i - \mathbf{x}_j \qquad \Delta_{ij,k} \equiv \Delta_i - \Delta_j - \Delta_k \qquad v_{123} \equiv \frac{1}{2}\frac{\left(\mathbf{x}_{13}\tau_{23} - \mathbf{x}_{23}\tau_{13}\right)^2}{\tau_{12}\tau_{13}\tau_{23}}$$

Superfluid EFT and large charge expansion

(Son,Wingate)
(Greiter,Wilczek,Witten)

BCS instability + Fermi surface

$$\boxed{U(1) \to \emptyset \quad : \quad \langle \psi\psi \rangle = |\langle \psi\psi \rangle| \, e^{-2i\theta}}$$

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Galilean invariant building block:

$$X = D_t\theta - \frac{(\partial_i\theta)^2}{2M} \quad , \quad D_t\theta = \dot{\theta} - A_0 \quad , \quad A_0 = M\omega^2 r^2/2$$

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At unitarity have NR CFT:

$$\boxed{\mathcal{L}_{LO} = c_0\, M^{3/2} X^{5/2}} \qquad c_0 = \frac{2^{5/2}}{15\pi^2\xi^{3/2}}$$

Homogeneous ground state (grand canonical)

$$\boxed{EOM: \quad \partial_t \rho \; - \; \frac{1}{M}\partial_i \left(\partial_i \theta \rho\right) = 0}$$

$$\Rightarrow \quad \theta(x) \; = \; \mu \, t \; - \; \phi(x)$$

symmetry breaking scale

phonon

$$\rho \; = \; \frac{5}{2} c_0 \, M^{3/2} X^{3/2}$$

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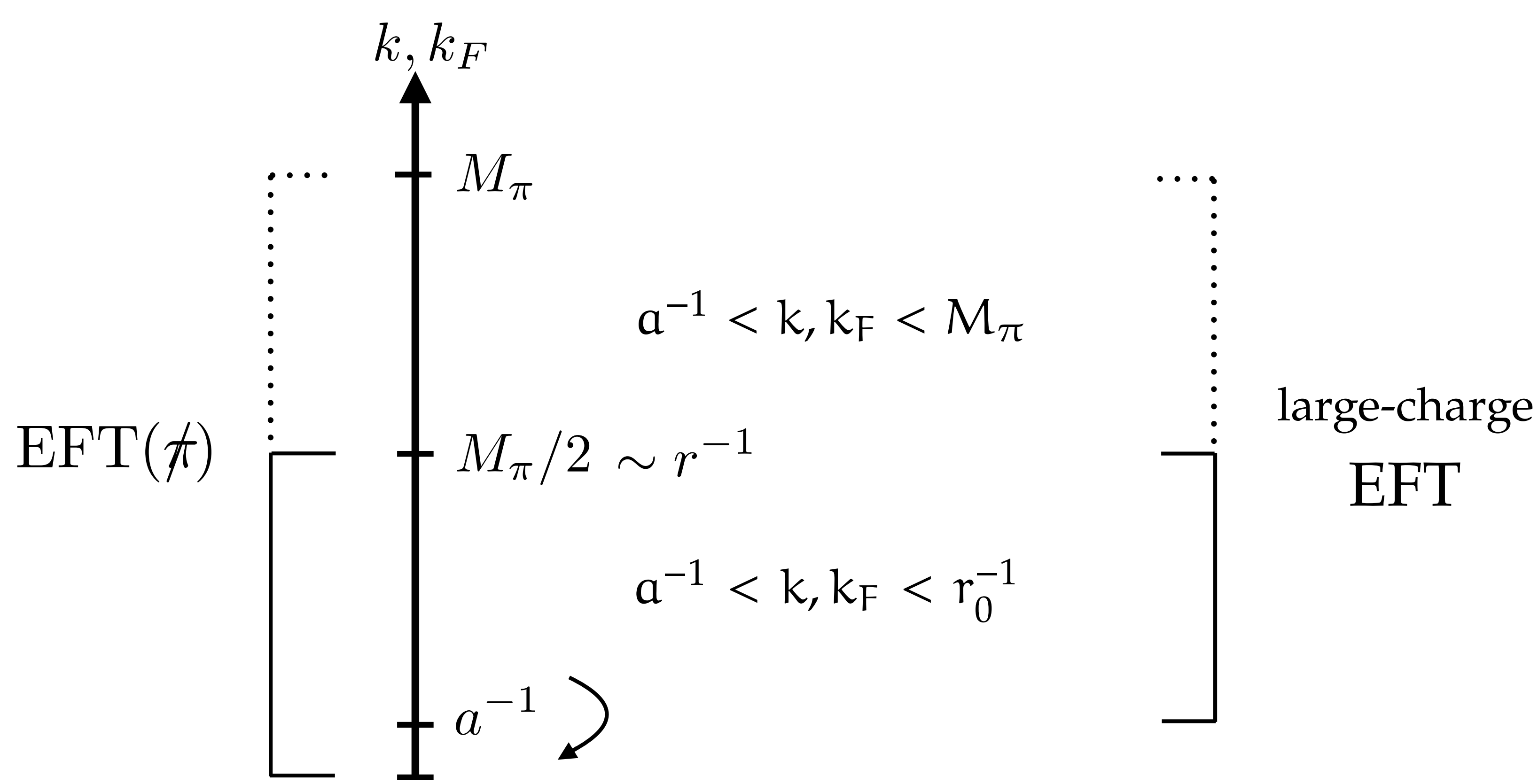
<u>Power counting</u>: derivative expansion <u>or</u> large μ



(Favrod,Orlando,Reffert)

(Kravec,Pal)

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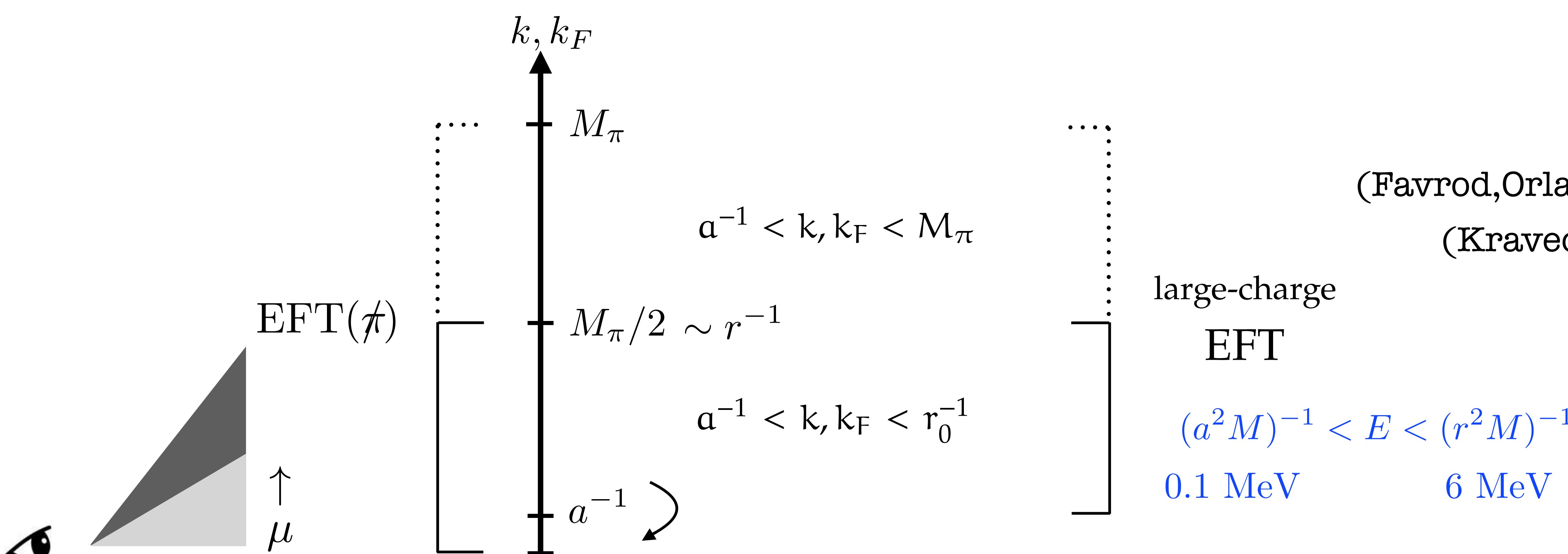
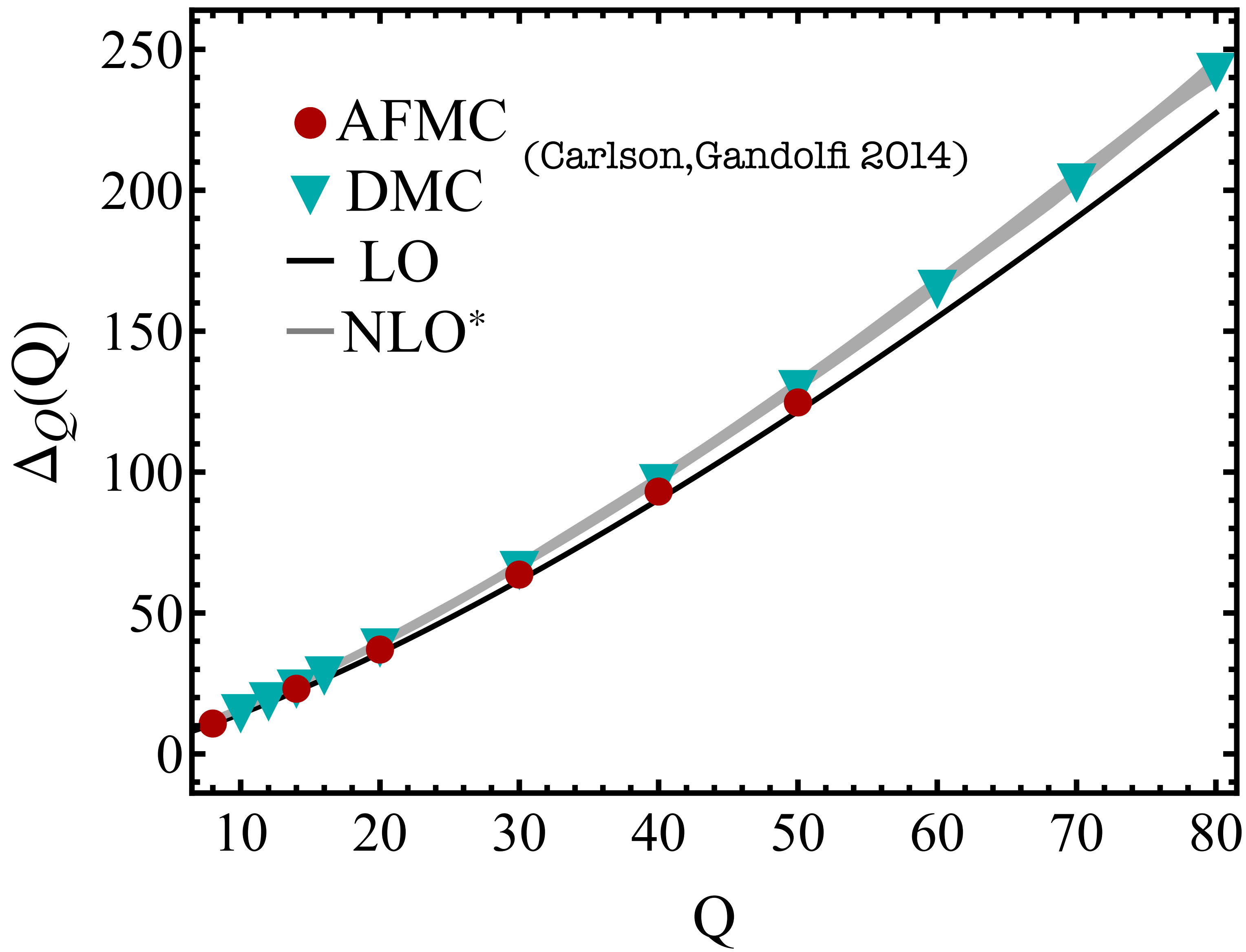


(Favrod,Orlando,Reffert)

(Kravec,Pal)

State-operator correspondence:
energy of Q fermions in a harmonic trap



$$\boxed{\Delta_Q(Q) = \frac{3^{4/3}}{4}\xi^{1/2}Q^{4/3} - 3^{2/3}\sqrt{2}\pi^2\xi\, c_{NLO}\, Q^{2/3} + O(Q^{5/9}) + \ldots + \frac{1}{3\sqrt{3}}\log Q}$$

(Son,Wingate) (Hellerman et al)

Not useful when including Schrödinger-symmetry breaking effects..

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But..exact large-charge solution can be found!

$$\boxed{\mathcal{O}_{\Delta,Q} = \mathcal{N}\, X^{\Delta/2} \exp\left(iQ\theta\right)}$$

normalization

primary operator

Now need Euclidean path integral formulation

$$\boxed{G_Q(x_1,x_2) \;=\; \int \mathcal{D}\theta\, \mathcal{O}_{\Delta,Q}(x_2)\mathcal{O}_{\Delta,-Q}(x_1)\, e^{-\int \mathrm{d}^4x \mathcal{L}_I} \;=\; \mathcal{N}^2 \int \mathcal{D}\theta\, e^{-\int \mathrm{d}^4x \mathcal{L}}}$$

$$\mathcal{L} = \mathcal{L}_I - \frac{\Delta}{2} \log X \left[\delta^4(x-x_2) + \delta^4(x-x_1)\right] - iQ\,\theta\left[\delta^4(x-x_2) - \delta^4(x-x_1)\right]$$

EOM: continuity equation with source

$$\partial_\tau \rho + \frac{1}{M}\partial_i \left(i\partial_i\theta\rho\right) = Q\left[\delta^4(x-x_2) - \delta^4(x-x_1)\right]$$

Saddle point instanton solution: master field

(Orlando,Reffert,SB)
(Son,Stephanov,Yee)

$$\theta_s(\tau,\mathbf{x}) = \frac{i}{2}\gamma\log\left(\frac{\tau_1-\tau}{\tau-\tau_2}\right) - \frac{i}{4}M\left[\frac{(\mathbf{x}-\mathbf{x}_2)^2}{(\tau-\tau_2)} - \frac{(\mathbf{x}-\mathbf{x}_1)^2}{(\tau_1-\tau)}\right]$$

EOM: continuity equation with source

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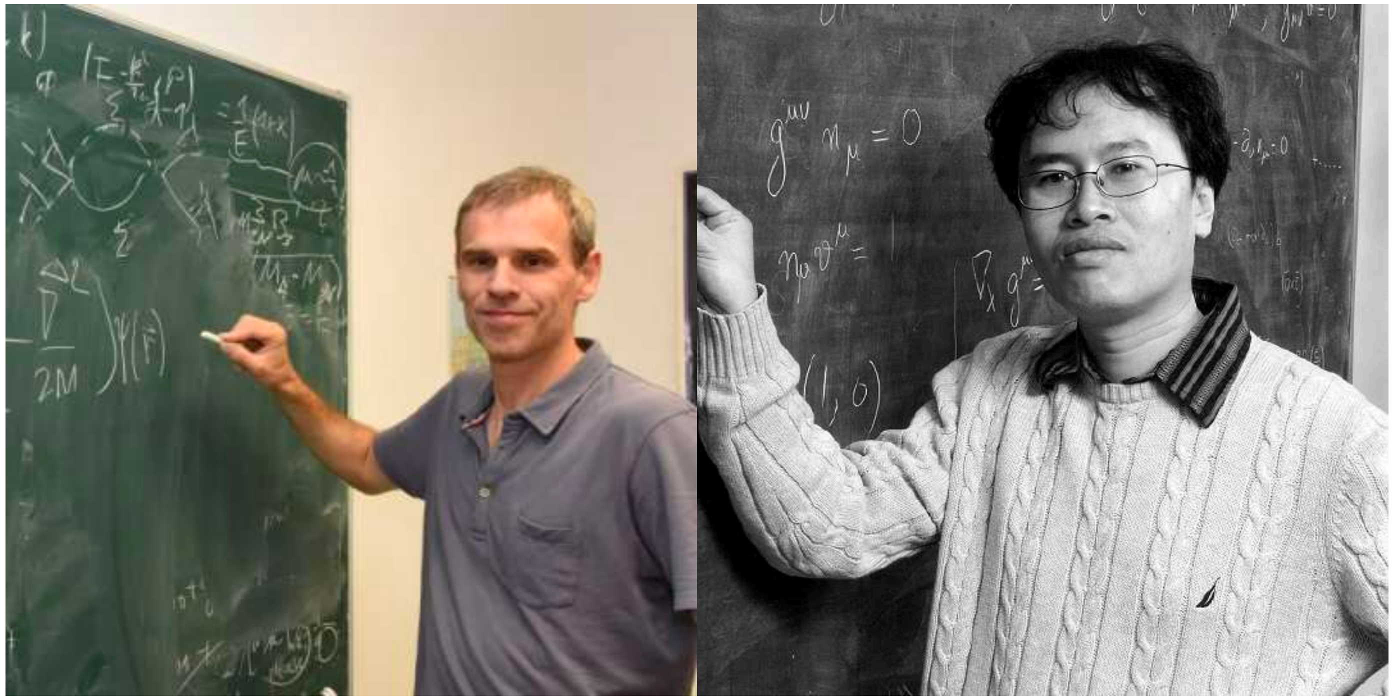
Emergent time-dependent harmonic trap

$$X = \bar{\mu}(\tau) - \frac{1}{2}M\bar{\omega}(\tau)^2 r^2 \qquad = 0 \quad @ \quad r = \bar{R}(\tau)$$

$$\bar{\mu}(\tau) = \frac{1}{2}\gamma\frac{(\tau_1-\tau_2)}{(\tau-\tau_2)(\tau_1-\tau)} \qquad \bar{\omega}(\tau) = \frac{1}{2}\frac{(\tau_1-\tau_2)}{(\tau-\tau_2)(\tau_1-\tau)}$$

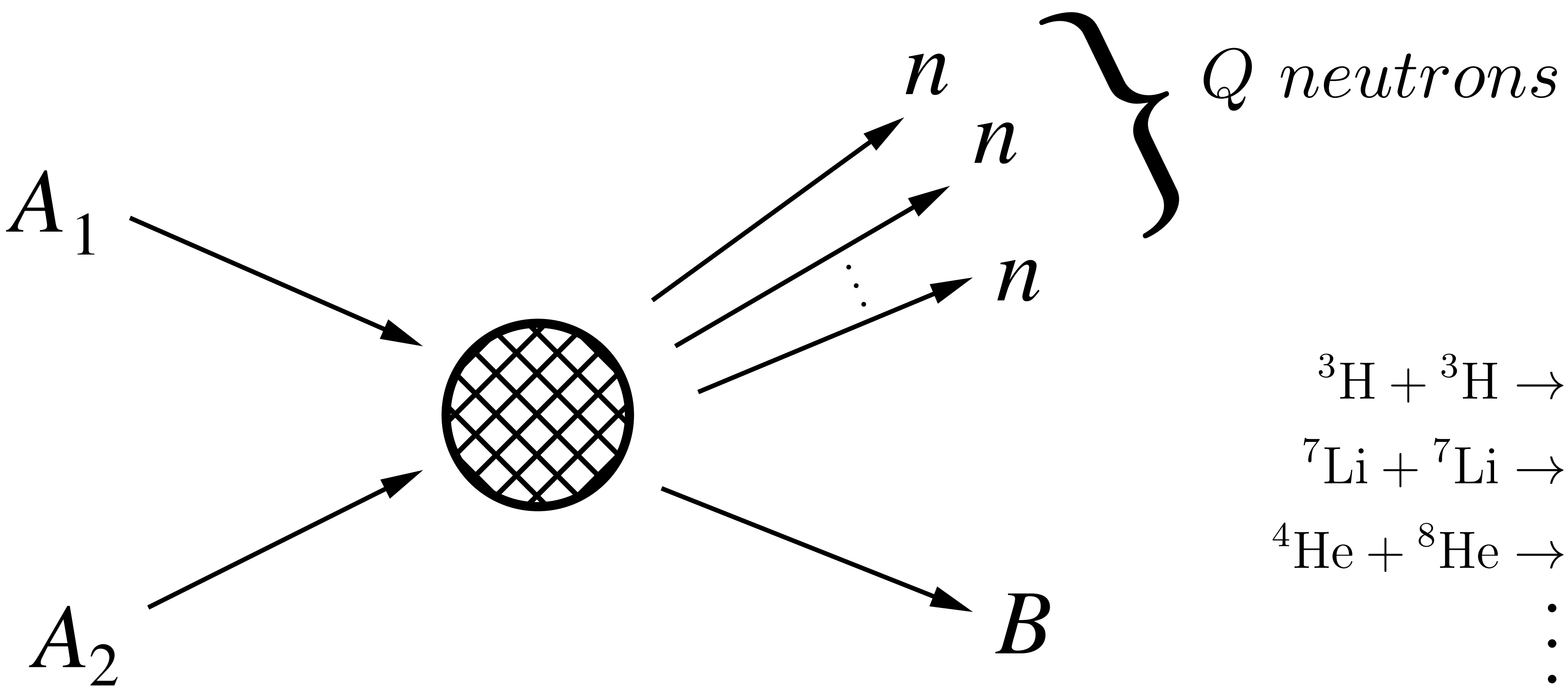
Recovers state-operator correspondence

Unnuclear physics



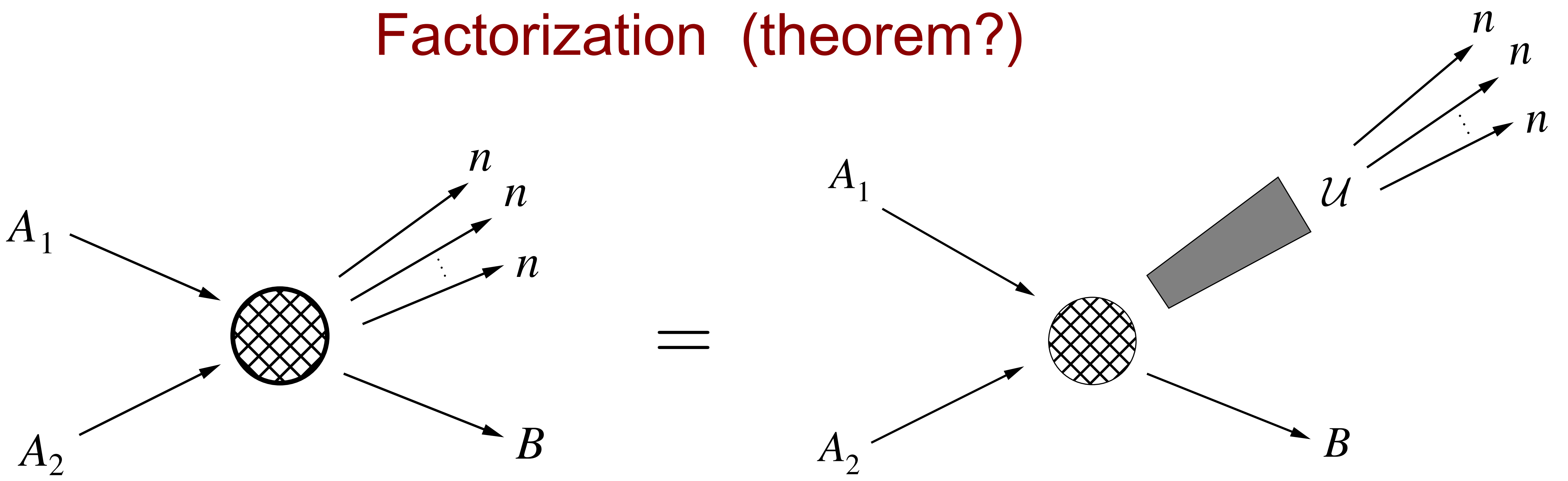
(Hammer,Son)

(Chowdhuri, Mishra,Son)



$$^{3}\mathrm{H} + {}^{3}\mathrm{H} \to {}^{4}\mathrm{He} + 2n$$
$$^{7}\mathrm{Li} + {}^{7}\mathrm{Li} \to {}^{11}\mathrm{C} + 3n$$
$$^{4}\mathrm{He} + {}^{8}\mathrm{He} \to {}^{8}\mathrm{Be} + 4n$$
$$\vdots$$

Factorization (theorem?)



$$P(A_1 + A_2 \to B + Qn) = P(A_1 + A_2 \to B + \mathcal{U})P(\mathcal{U} \to Qn)$$

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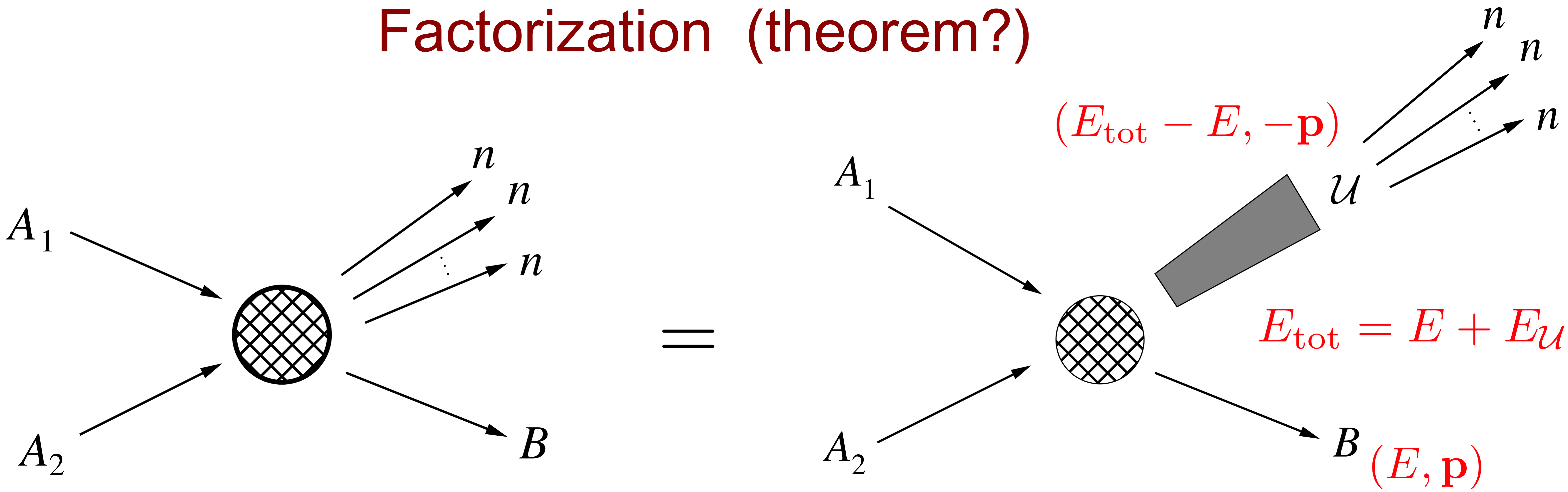
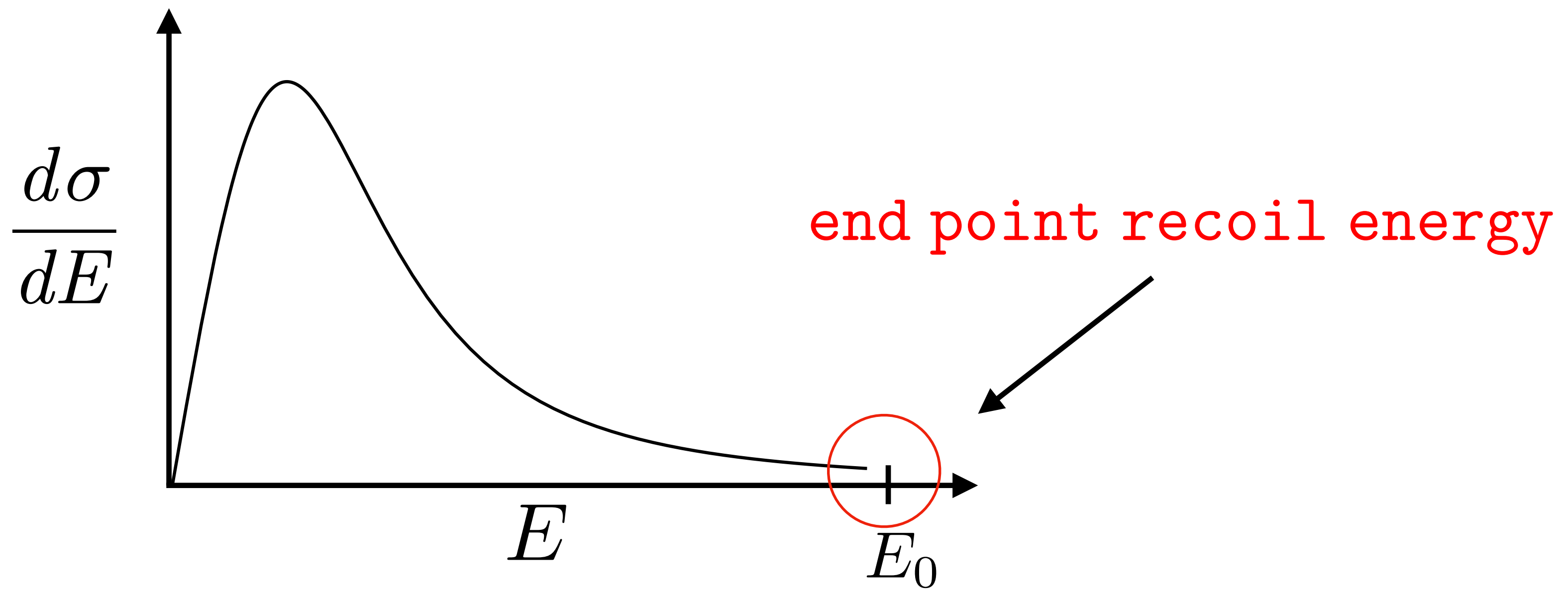


$$P(A_1 + A_2 \to B + Qn) = P(A_1 + A_2 \to B + \mathcal{U})P(\mathcal{U} \to Qn)$$

$$\boxed{\frac{d\sigma}{d\Omega} \sim |\mathcal{M}|^2 \sqrt{E}\, \mathrm{Im} G_{\mathcal{U}}(E_{\rm tot} - E, \mathbf{p})}$$

$$G_{\mathcal{U}}(t, \mathbf{x}) = -i\langle T\mathcal{U}(t, \mathbf{x})\mathcal{U}^\dagger(0, \mathbf{0})\rangle = C\, \frac{\theta(t)}{(it)^{\Delta_Q}} \exp\left(\frac{iM_{\mathcal{U}} x^2}{2t}\right)$$

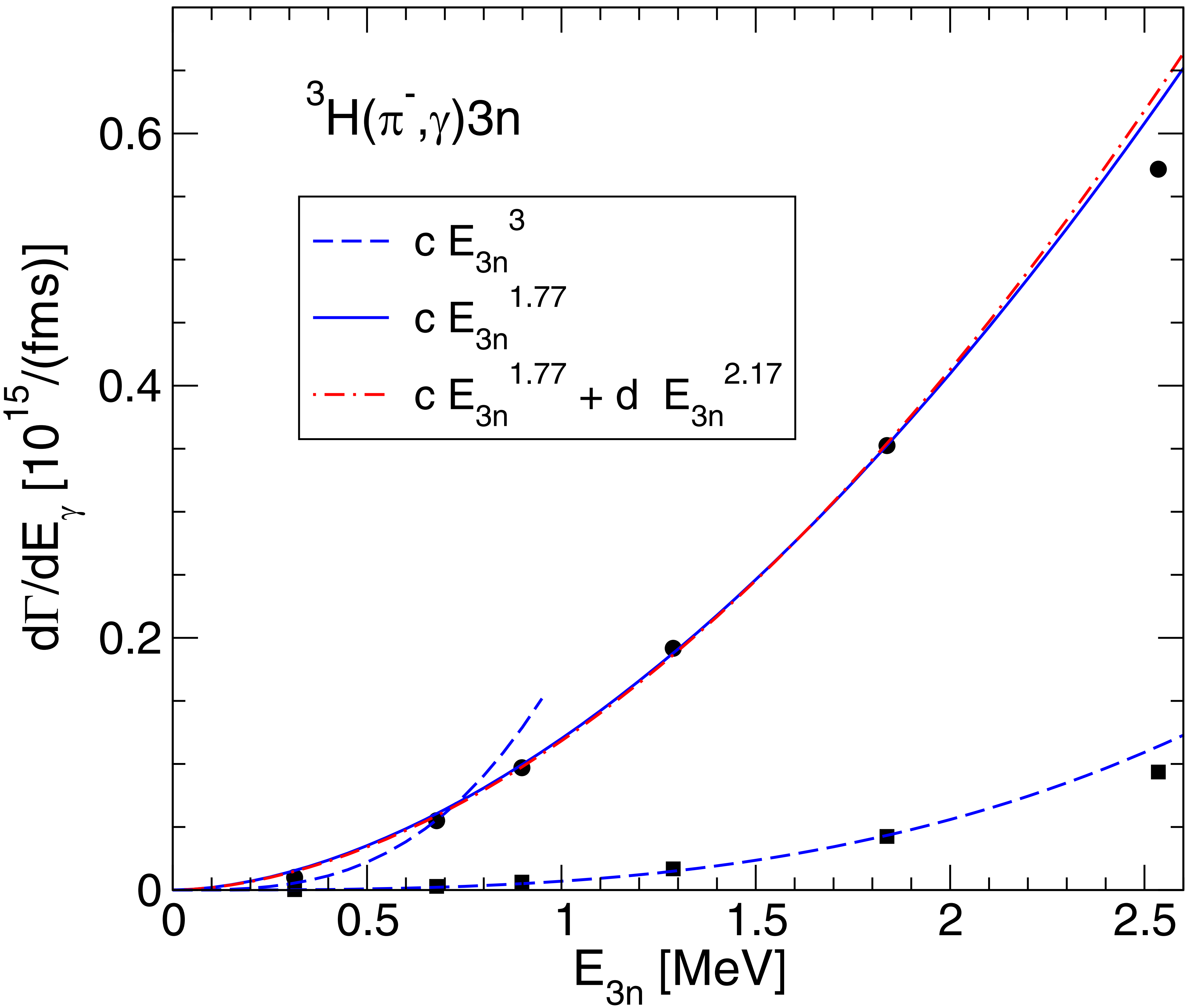


$$\boxed{\frac{d\sigma}{d\Omega} \sim (E_0 - E)^{\Delta_Q - 5/2}}$$

	Δ_Q	$\Delta_Q - \frac{5}{2}$	
$^3\mathrm{H} + {}^3\mathrm{H} \to {}^4\mathrm{He} + 2n$	2	-0.5	
$\pi^- + {}^3\mathrm{H} \to \gamma + 3n$	4.27	1.77	✓
$^4\mathrm{He} + {}^8\mathrm{He} \to {}^8\mathrm{Be} + 4n$	5.0	2.5	

$$\pi^- + {}^3\mathrm{H} \to \gamma + 3n$$

(Golak et al)



(Hammer,Son)

Deformation via conformal perturbation theory

(Chowdhuri, Mishra,Son)

$$\mathcal{L} = \mathcal{L}_{CFT} + \frac{M}{4\pi a} O_2^\dagger O_2 - \frac{M^2 r}{8\pi} O_2^\dagger \left(i\overleftrightarrow{\partial_t} + \frac{\overrightarrow{\nabla}^2 + \overleftarrow{\nabla}^2}{4M} \right) O_2$$

relevant deformation

irrelevant deformation

$$\mathrm{Im}\, G_Q(E, \mathbf{0}) \sim E^{\Delta_Q - 5/2} \left[1 + \frac{C_{rel}}{a\sqrt{ME}} + C_{irrel}\, r\sqrt{ME} \right]$$

Example: Q=3

$$\langle O_3(x) O_3^\dagger(0) \rangle = \frac{1}{Z} \int \mathscr{D}\psi\, O_3(x) O_3^\dagger(0) e^{iS_{\mathrm{CFT}} + \frac{i}{a}\int_y O_2^\dagger(y) O_2(y)}$$

$$= \langle O_3(x) O_3^\dagger(0) \rangle_0 + \frac{1}{a} \int dy\, \langle O_3(x) O_2^\dagger(y) O_2(y) O_3^\dagger(0) \rangle_0$$

$|1\rangle\langle 1|$ $|3\rangle\langle 3|$

$$\Longrightarrow$$

$$\Delta_3 = 4.27$$

$$C_{rel} = 2.64$$

$$C_{irrel} = 0 \;\; (!?)$$

For larger Q, perhaps it is more efficient to use the large-charge EFT. Naively, in conformal perturbation theory

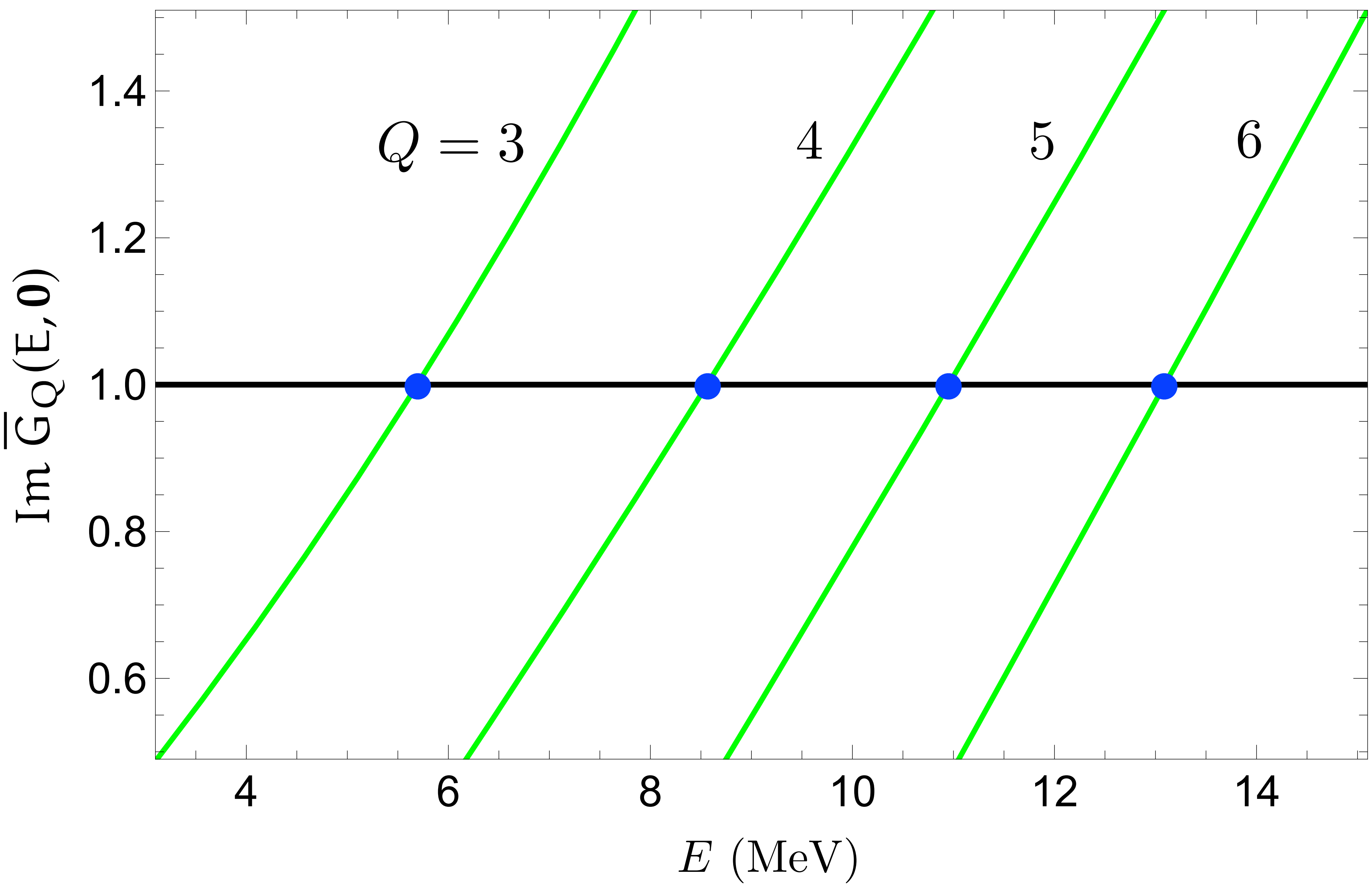
$$\boxed{G(x_1, x_2) \;=\; G_{CFT}(x_1, x_2) e^{-S_{\text{SB}}[\theta_S]}}$$

But there are boundary terms.. Need:

1. the LO action evaluated on the LO solution
2. the LO action evaluated on the NLO solution
3. the NLO action evaluated on the LO solution

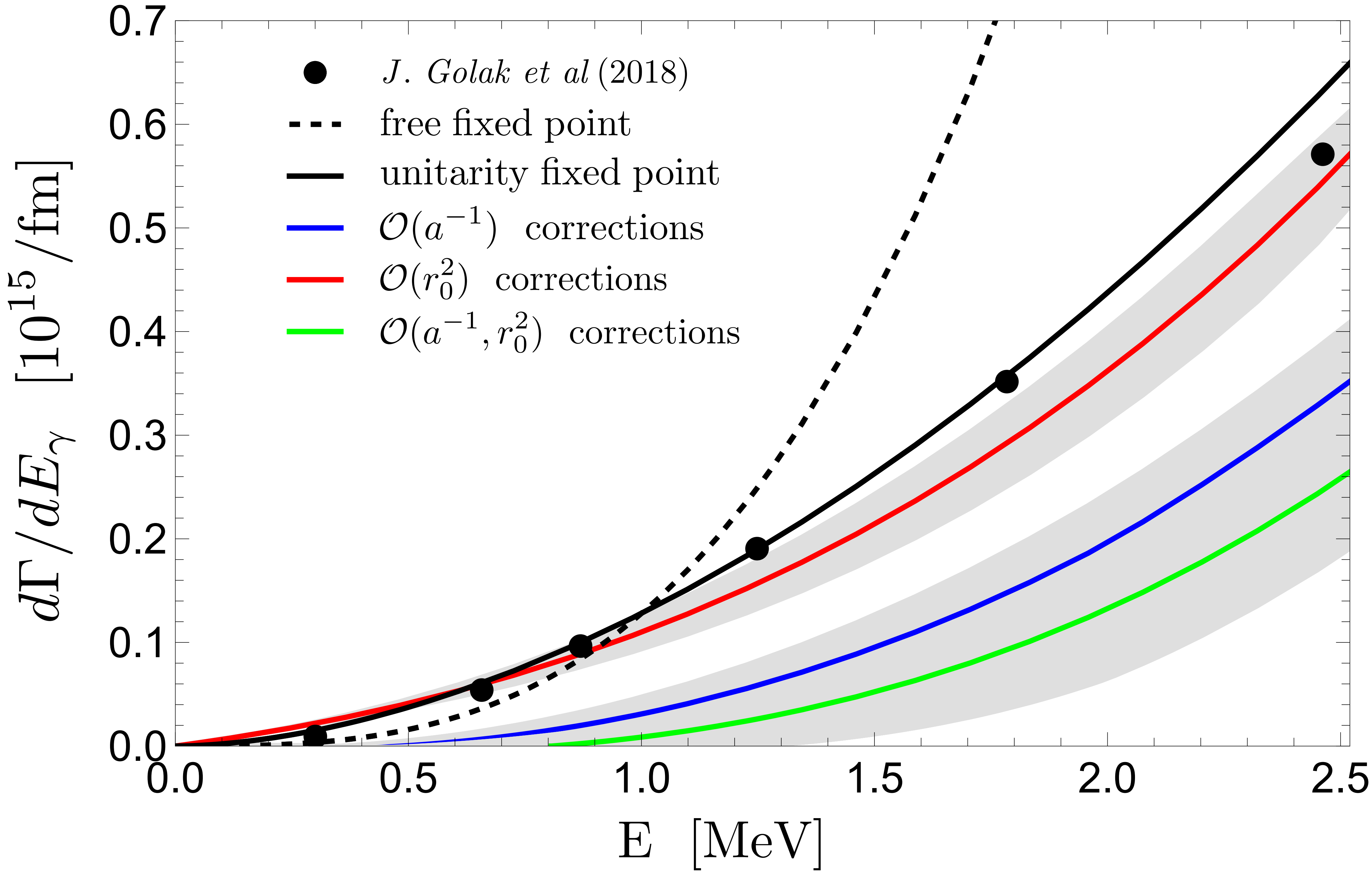
$$\mathrm{Im}\,\overline{G}_Q(E,\mathbf{0}) \equiv \mathrm{Im}\,G_Q(E,\mathbf{0})/\mathrm{Im}\,G_Q^{\mathrm{CFT}}(E,\mathbf{0}) = \left[1 + \mathcal{C}_Q\left(a\sqrt{ME}\right)^{-1} + \mathcal{C}_Q'' r_0^2 ME\right]$$

Neutron matter

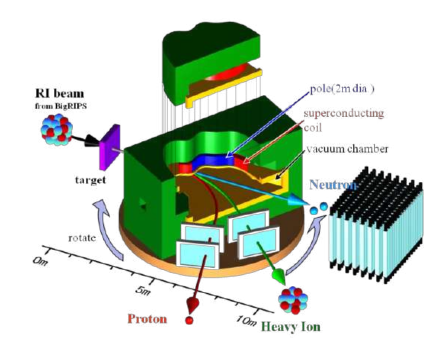


Need fine energy resolution

$\pi^- + {}^3\mathrm{H} \to \gamma + nnn$



Scattering length corrections not in perturbative regime:
need data at higher energies!



SAMURAI spectrometer

Summary

- The large-charge EFT provides a systematic (far infrared) description of many fermions near unitarity. Applicability to neutron matter is complicated by large effective range effects in the neutron-neutron interaction.

- The state-operator correspondence is a powerful tool for computing the dimensions of operators in an interacting NR CFT. However, including symmetry breaking is non-trivial.

- Large-charge master field is known; useful for computing Schrödinger-symmetry breaking corrections to correlation functions without detailed knowledge of few-body wavefunctions.

- Leading non-vanishing effects from relevant and irrelevant deformations of the non-relativistic CFT are under control in nuclear physics only in narrow energy windows.

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