Electromagnetic and exotic nuclear moments

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In collaboration with: B. C. Backes, J Bonnard, G. Danneaux, R.F. Garcia Ruiz, R.P. de Groote, R. Han, W. C. Haxton, W. Jiang, M. Kortelainen, A. Nagpal, A. Restrepo-Giraldo, P.L. Sassarini, A.E. Stuchbery, X. Sun, and H. Wibowo

Hirschegg 2026 - Challenges in effective field theory descriptions of nuclei, Hirschegg, Austria, 18-24 January 2026

Outline

1. Nuclear shapes and currents
2. Experimental data
3. Methodology
   a) Self-consistency
   b) Polarization
   c) Symmetry restoration
4. Ground states of near doubly magic nuclei
5. Ground and excited states of open-shell nuclei
6. Meson exchange currents for magnetic dipole moments
7. Electric dipole moments
8. Anapole moments
9. Conclusions

Nuclear shapes and currents

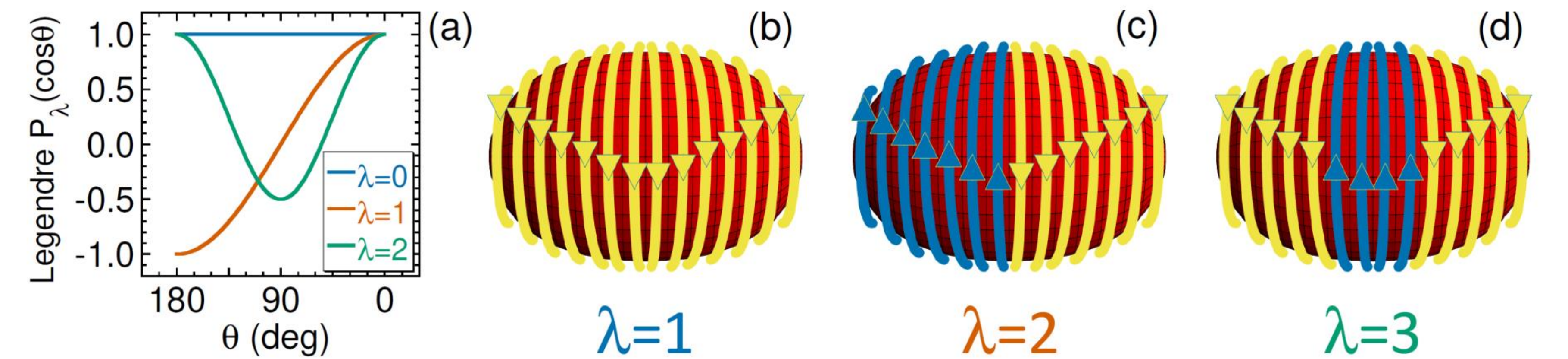


Figure 1: Legendre polynomials, $P_\lambda(\cos\theta)$ for $\lambda = 0, 1, 2$, (a), and the patterns of the current flows for the magnetic dipole (b), quadrupole (c), and octupole (d) moments of symmetry-broken aligned ground states of odd nuclei.

Nuclear shapes

$$R(\theta) = R_0 \sum_{\lambda=0,1,2,3\ldots}^{\lambda_{\max}} \sqrt{\tfrac{2\lambda+1}{4\pi}}\beta_\lambda\, P_\lambda(\cos\theta), \qquad \bar{\rho}(\boldsymbol{r}) = \begin{cases} \rho_0 & \text{for } r \leq R(\theta) \\ 0 & \text{for } r > R(\theta) \end{cases}$$

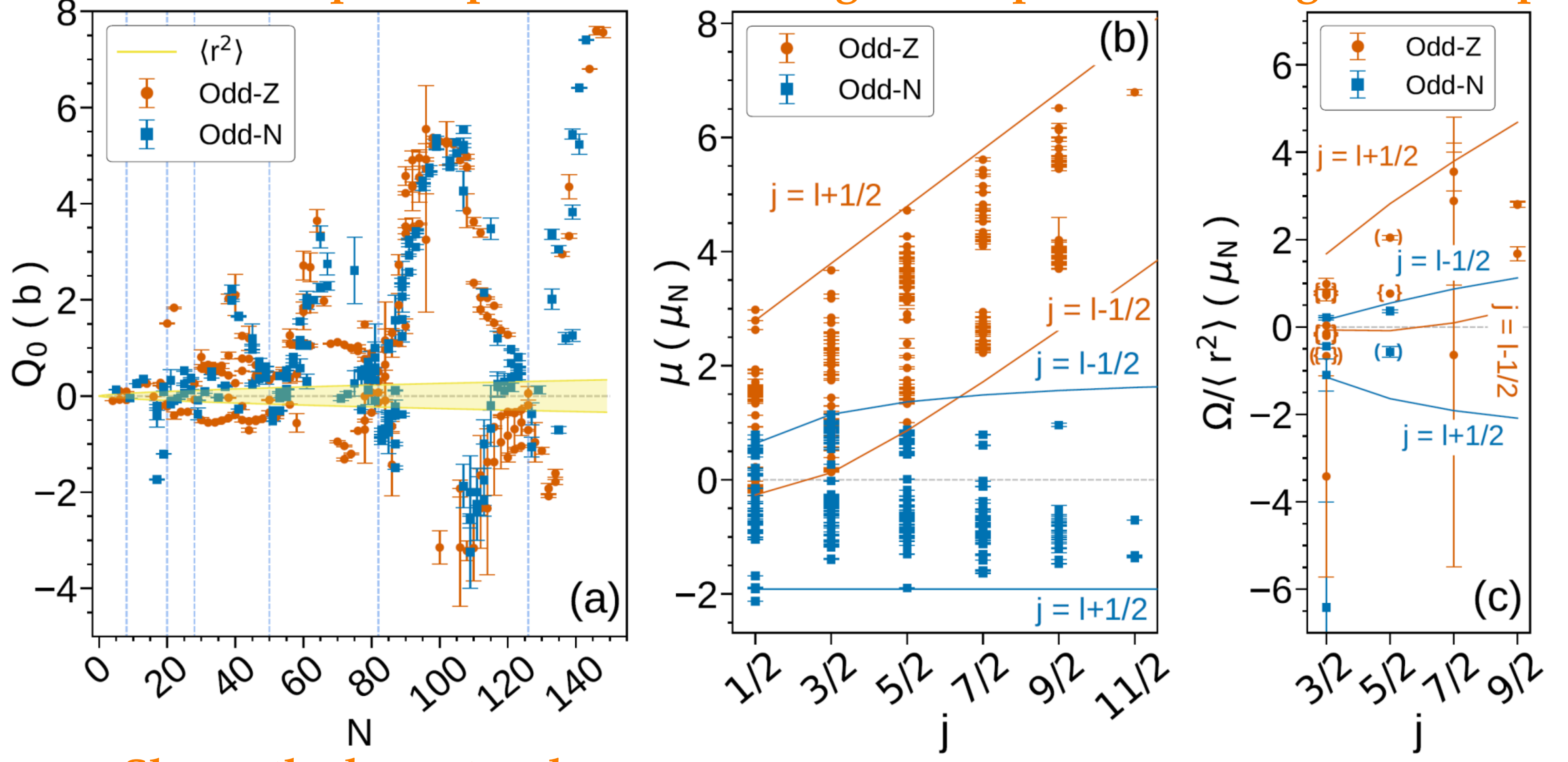
Nuclear currents

$$j_\phi(z,\eta) = \sum_{\lambda=1,2,3\ldots}^{\lambda_{\max}} j_\lambda(r)\, P_{\lambda-1}(\cos\theta), \quad \text{with } r = \sqrt{\eta^2 + z^2}, \quad \cos\theta = \frac{z}{r},$$

J. Dobaczewski *et al.*, arXiv:2511.04632

Experimental data

Electric quadrupole | Magnetic dipole | Magnetic octupole



J. Dobaczewski *et al.*, arXiv:2511.04632

Shows the lowest-order deformation:
$Q_0 > 0$ (prolate)
$Q_0 = 0$ (spherical)
$Q_0 < 0$ (oblate)

Shows the lowest-order orbital and spin current flow

Shows the higher-order orbital and spin current flow

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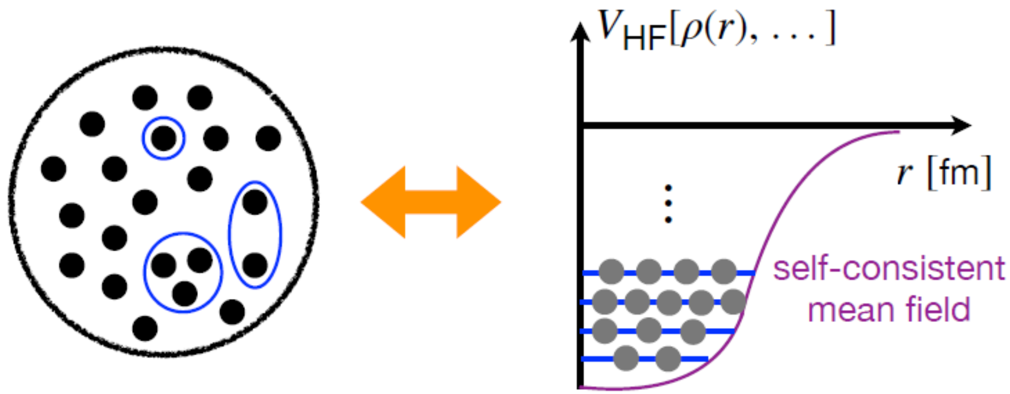
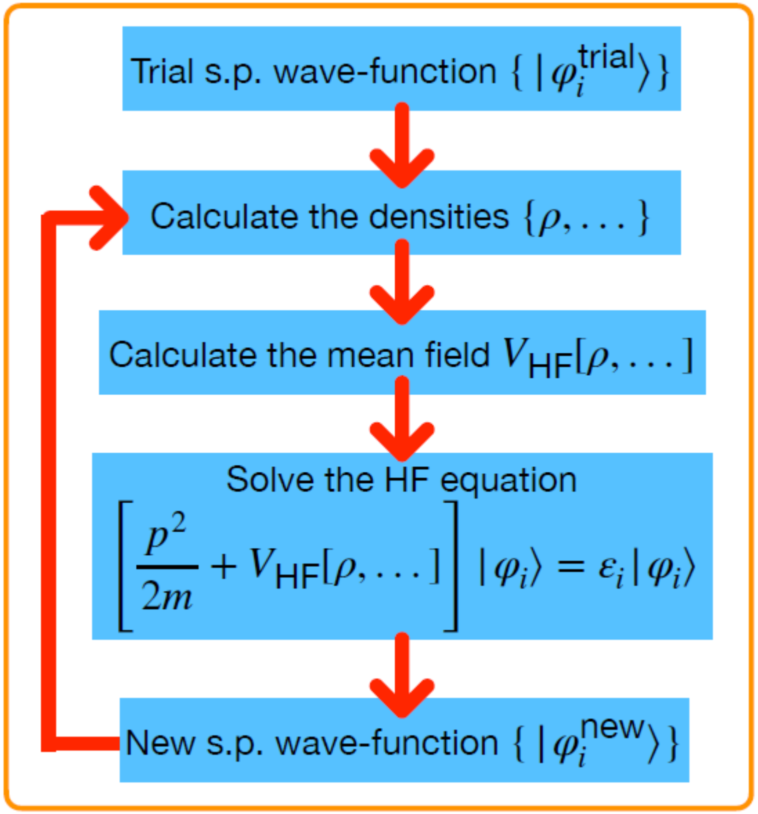
Nuclear density functional theory





Energy density functional

$$\mathscr{E}[\rho(\mathrm{r}), \mathrm{s}(\mathrm{r}), \tau(\mathrm{r}), \mathrm{T}(\mathrm{r}), \mathrm{j}(\mathrm{r}), \overleftrightarrow{J}(\mathrm{r})]$$

Coupling constants

$$T\text{-even}: \; C_t^{\rho}, C_t^{\Delta\rho}, C_t^{\tau}, C_t^{J}, C_t^{\nabla J}$$

$$T\text{-odd}: \; C_t^{s}, C_t^{\Delta s}, C_t^{T}, C_t^{j}, C_t^{\nabla j}$$

Parametrization: UNEDF1

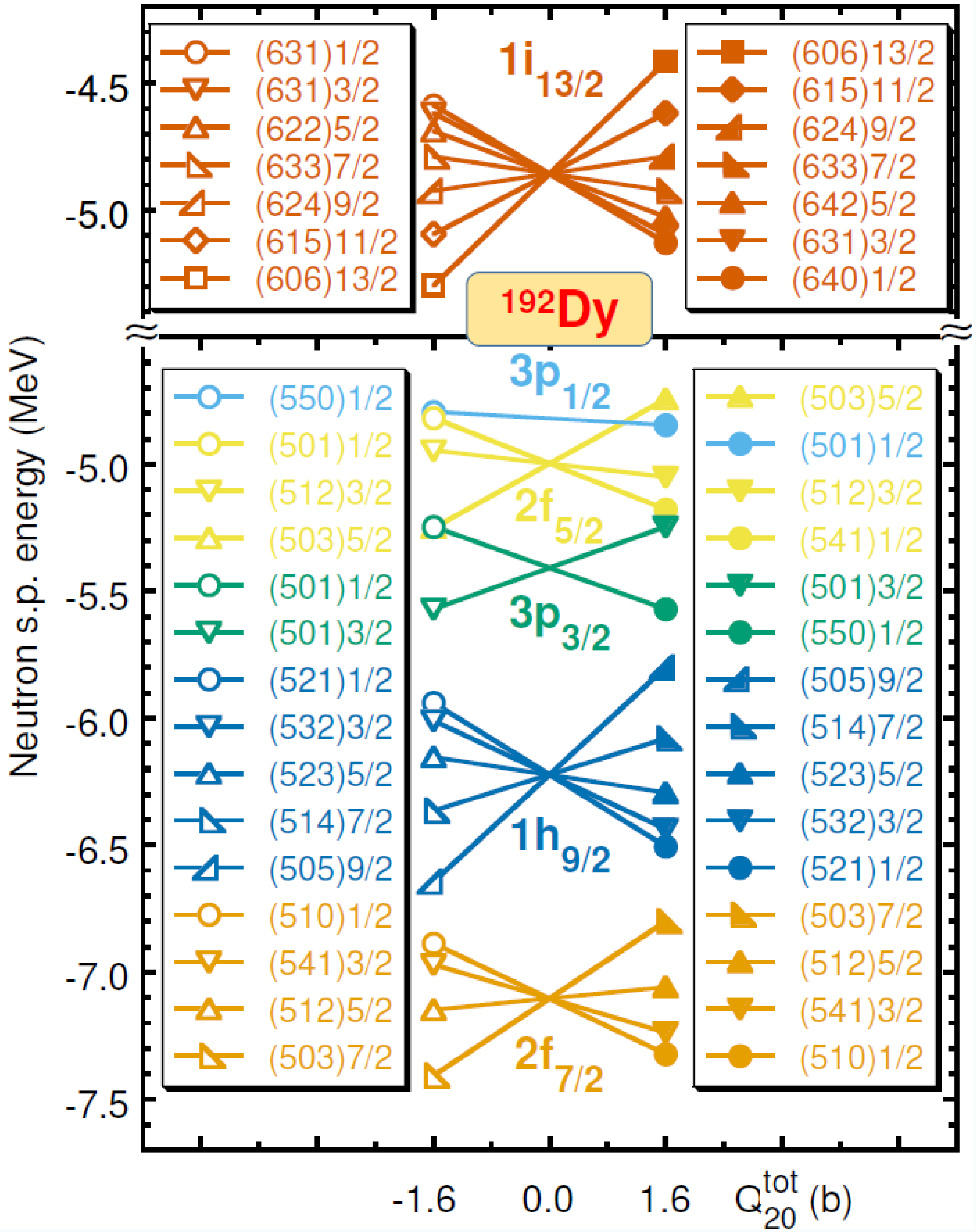
→ Hartree-Fock (HF) equation

M. Kortelainen et al., Phys. Rev. C **85**, 024304 (2012)

Picture courtesy of H. Wibowo

Self-consistent equations are solved iteratively, which includes the polarization effects summed up to all orders without recurring to the lowest order perturbative coupling.

How to calculate odd nuclei in nuclear DFT?



without pairing

A even, $p > A$, $h \leq A$

$$|\Psi\rangle^{\text{even}}_{\text{HF}} = a^+_A \dots a^+_2 a^+_1 |0\rangle$$

$$|\Psi\rangle^{\text{odd}}_{\text{HF}} = \begin{cases} a^+_p |\Psi\rangle^{\text{even}}_{\text{HF}} \\ a_h |\Psi\rangle^{\text{even}}_{\text{HF}} \end{cases}$$

with pairing

$$|\Psi\rangle^{\text{even}}_{\text{HFB}} = \prod_{\mu>0} \left(u_\mu + v_\mu a^+_{\bar{\mu}} a^+_\mu\right) |0\rangle$$

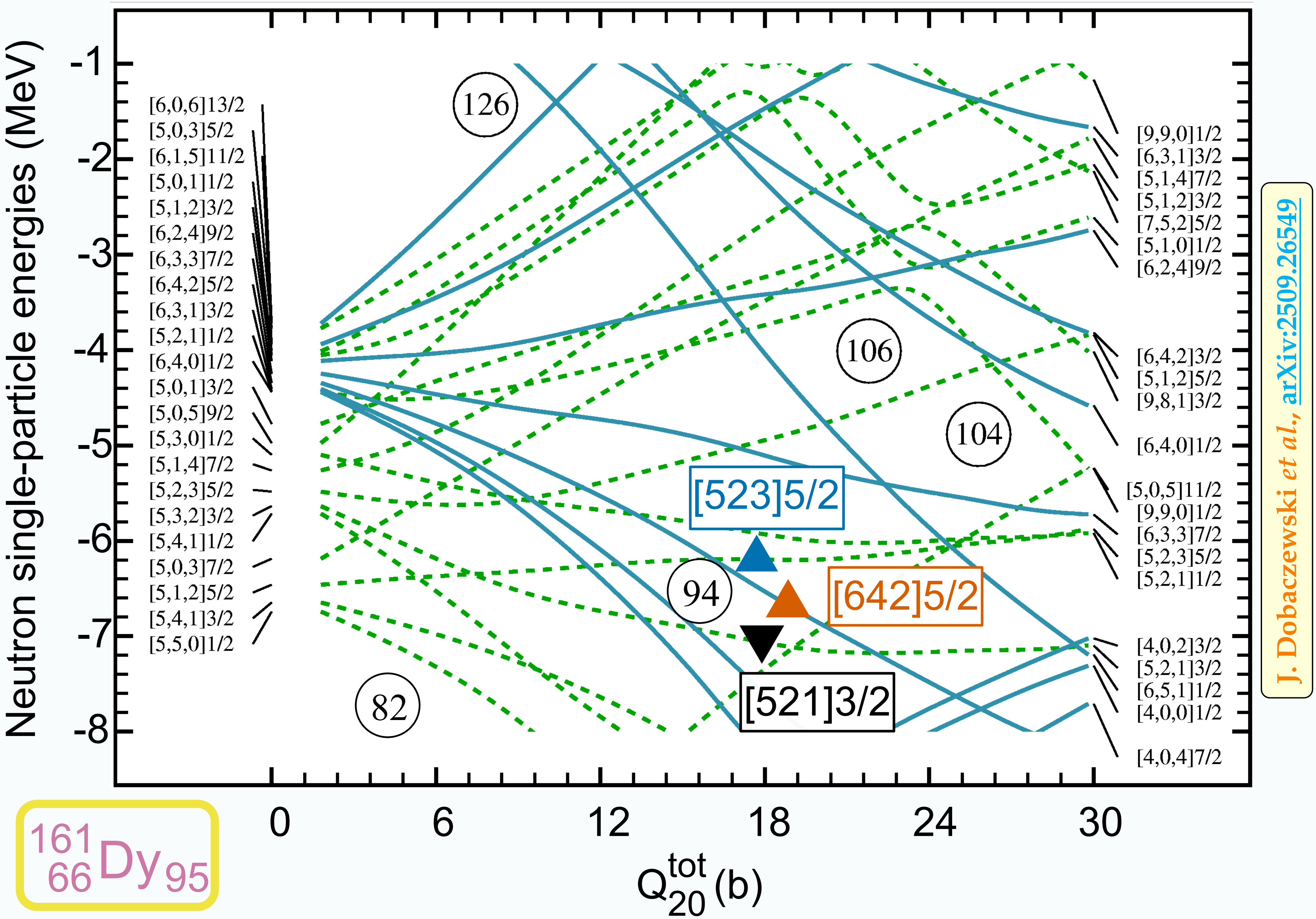
$$|\Psi\rangle^{\text{odd}}_{\text{HFB}} = \beta^+_\nu |\Psi\rangle^{\text{even}}_{\text{HFB}}$$

$$= a^+_\nu \prod_{\nu\neq\mu>0} \left(u_\mu + v_\mu a^+_{\bar{\mu}} a^+_\mu\right) |0\rangle$$

tagging quasiparticle states

$$\max_\mu \left\{ \langle \varphi_\nu | \phi^{\text{upper}}_\mu \rangle, \langle \varphi_\nu | \phi^{\text{lower}}_\mu \rangle \right\}$$

J. Dobaczewski *et al.*, arXiv:2509.26549



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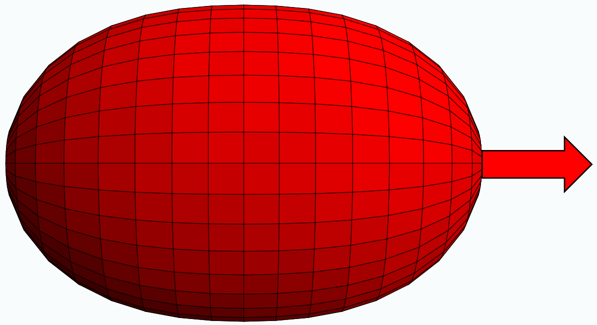
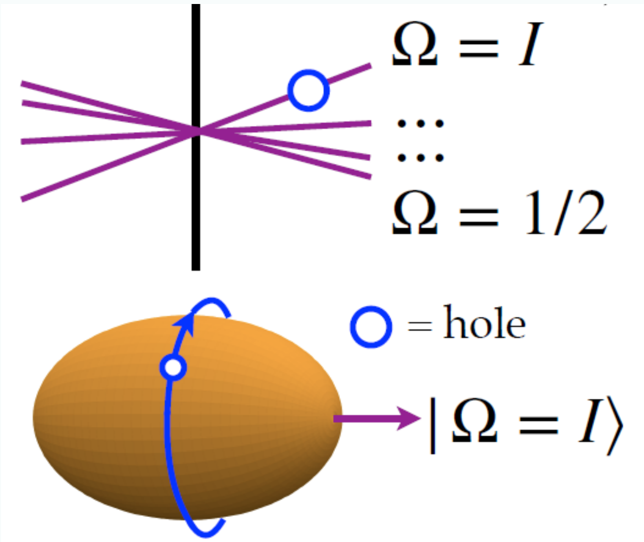
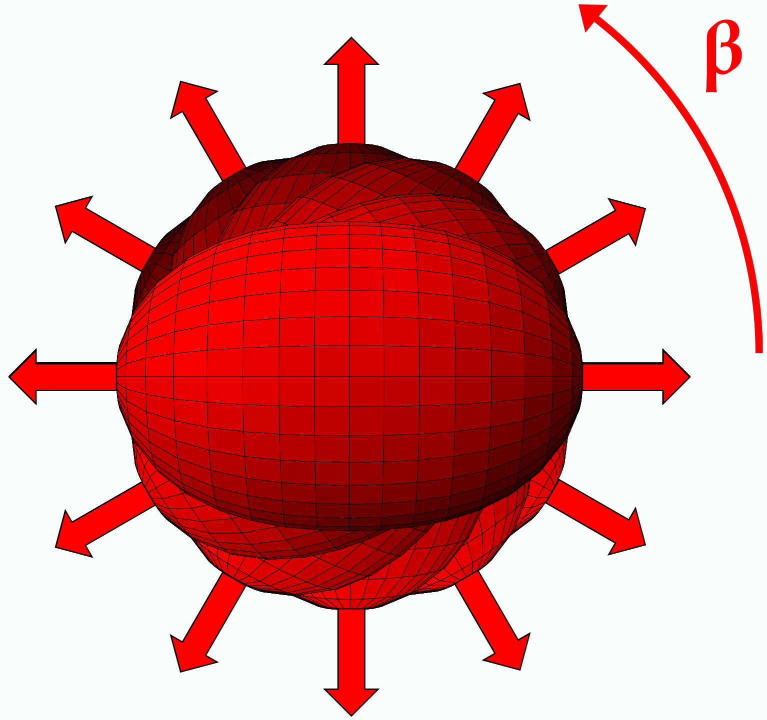
Time-odd spin alignment & symmetry restoration

"Intrinsic"
Symmetry broken







"Laboratory"
Symmetry restored

$$|IM\rangle = \mathcal{N}_I \int_{\beta=0}^{\pi} \mathrm{d}\beta\, d^I_{M\Omega}(\beta)|\Omega,\beta\rangle$$

J. A. Sheikh *et al.*, J. Phys. G48, 123001 (2021)

Spectroscopic moments are determined for symmetry-restored wave functions without using effective charges or effective g-factors and compared with experimental data.

Nuclear quadrupole & dipole moments

Spectroscopic electric quadrupole Q and magnetic dipole μ moments are :

$$Q = \sqrt{\frac{16\pi}{5}} \langle II \,|\, \hat{Q}_{20} \,|\, II \rangle \quad \text{and} \quad \mu = \sqrt{\frac{4\pi}{3}} \langle II \,|\, \hat{M}_{10} \,|\, II \rangle \,.$$

P. Ring and P. Schuck, *The Nuclear Many-Body Problem*

$$\hat{Q}_{20} = \sqrt{\frac{5}{16\pi}} e \sum_{i=1}^{A} \left(\frac{1}{2} - t_3^{(i)}\right) \left\{3z_i^2 - r_i^2\right\}; \quad \hat{M}_{10} = \sqrt{\frac{3}{4\pi}} \mu_N \sum_{i=1}^{A} \left\{ g_s^{(i)} s_{zi} + g_\ell^{(i)} \ell_{zi} \right\};$$

$$g_s^{(i)} = g_p(g_n) = 5.59(-3.83)$$
$$g_\ell^{(i)} = 1(0)$$

Intrinsic moments = moments of the symmetry-broken state
Spectroscopic moments = moments of the symmetry-restored state

Spectroscopic moments = moments measured experimentally

Open shell Gd-Os and near doubly magic nuclei

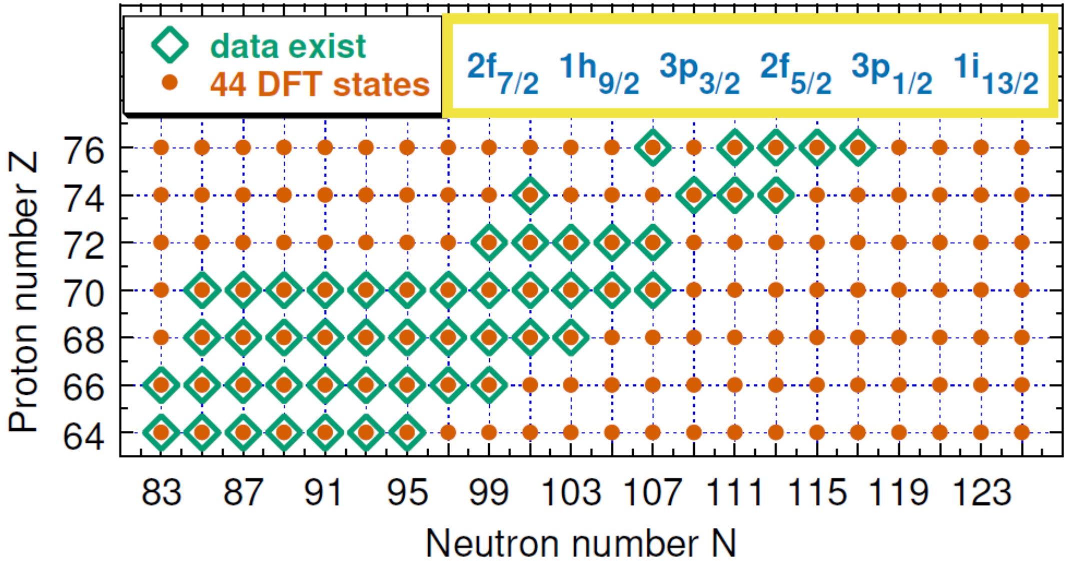


FIG. 1. Diagram illustrating the set of odd-N, even-Z open-shell elements and isotopes considered in this study for which (i) the calculations were performed (dots) and (ii) experimental data exist (diamonds).

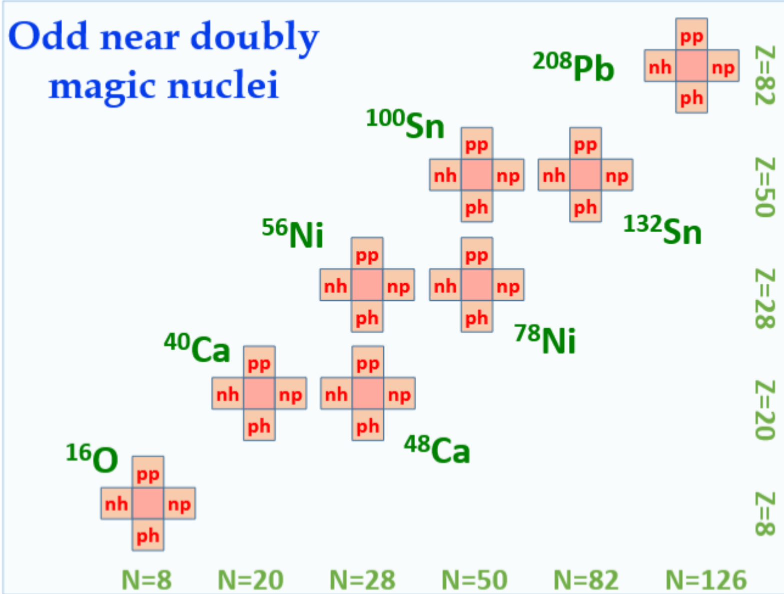


FIG. 1. Diagram illustrating the set of odd near doubly magic nuclei considered in this study. The symbols pp, ph, np, and nh represent the one-proton-particle, one-proton-hole, one-neutron-particle, and one-neutron-hole neighbors of the eight doubly magic nuclei.

Summary of results obtained near doubly magic nuclei

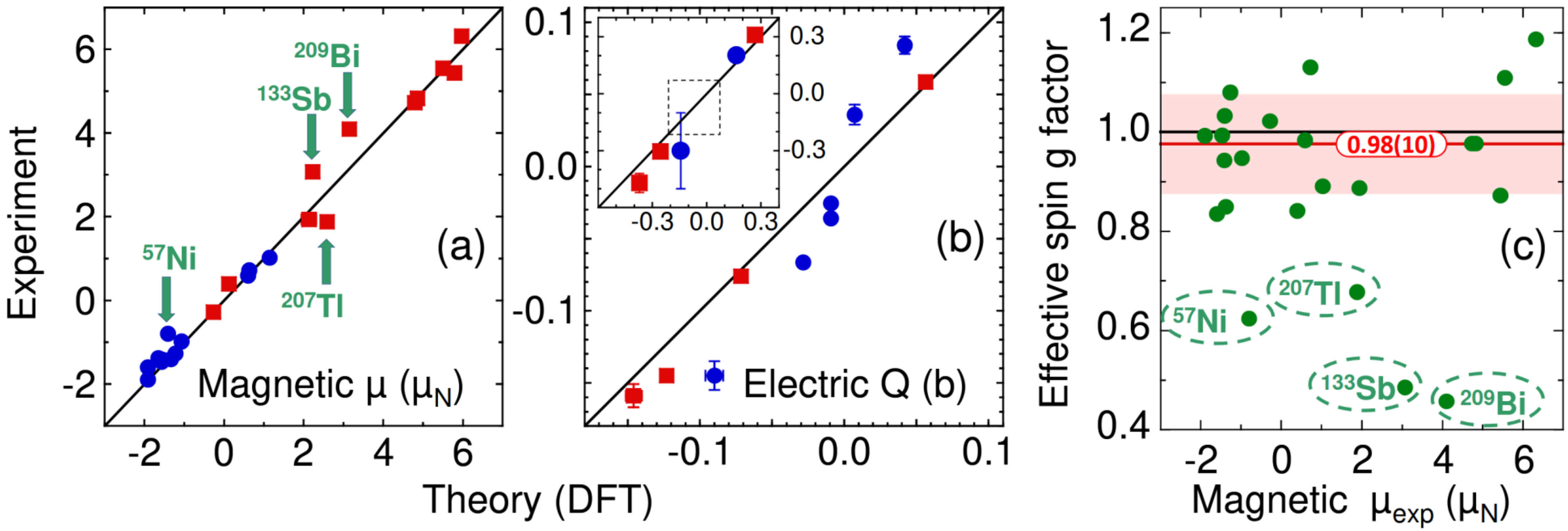


Figure 4: Calculated magnetic dipole moments μ (a) compared with 23 experimentally measured values (the arrows mark the outlier cases discussed in the text). Full circles (squares) show results obtained for odd-N (odd-Z) nuclei. Panel (b) shows the electric quadrupole moments Q compared with 15 experimentally measured values (the inset shows values that are outside the area of the main plot, as visualised by the dashed-line square drawn inside). Panel (c) shows the effective spin g factors described in the text, with ovals marking the outliers shown in panel (a).

P.L. Sassarini *et al.* J. Phys. G 49 (2022) 11LT01

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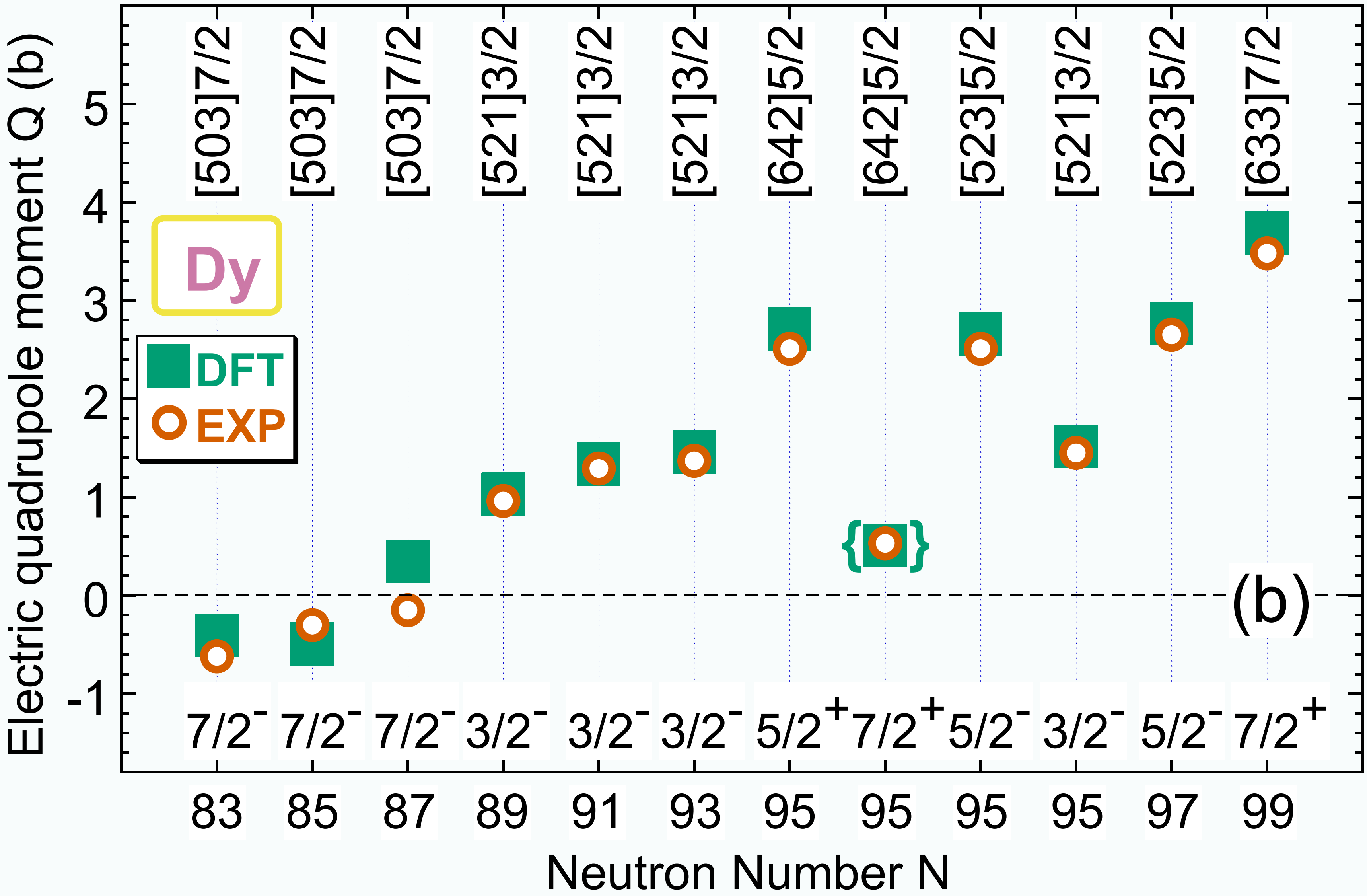
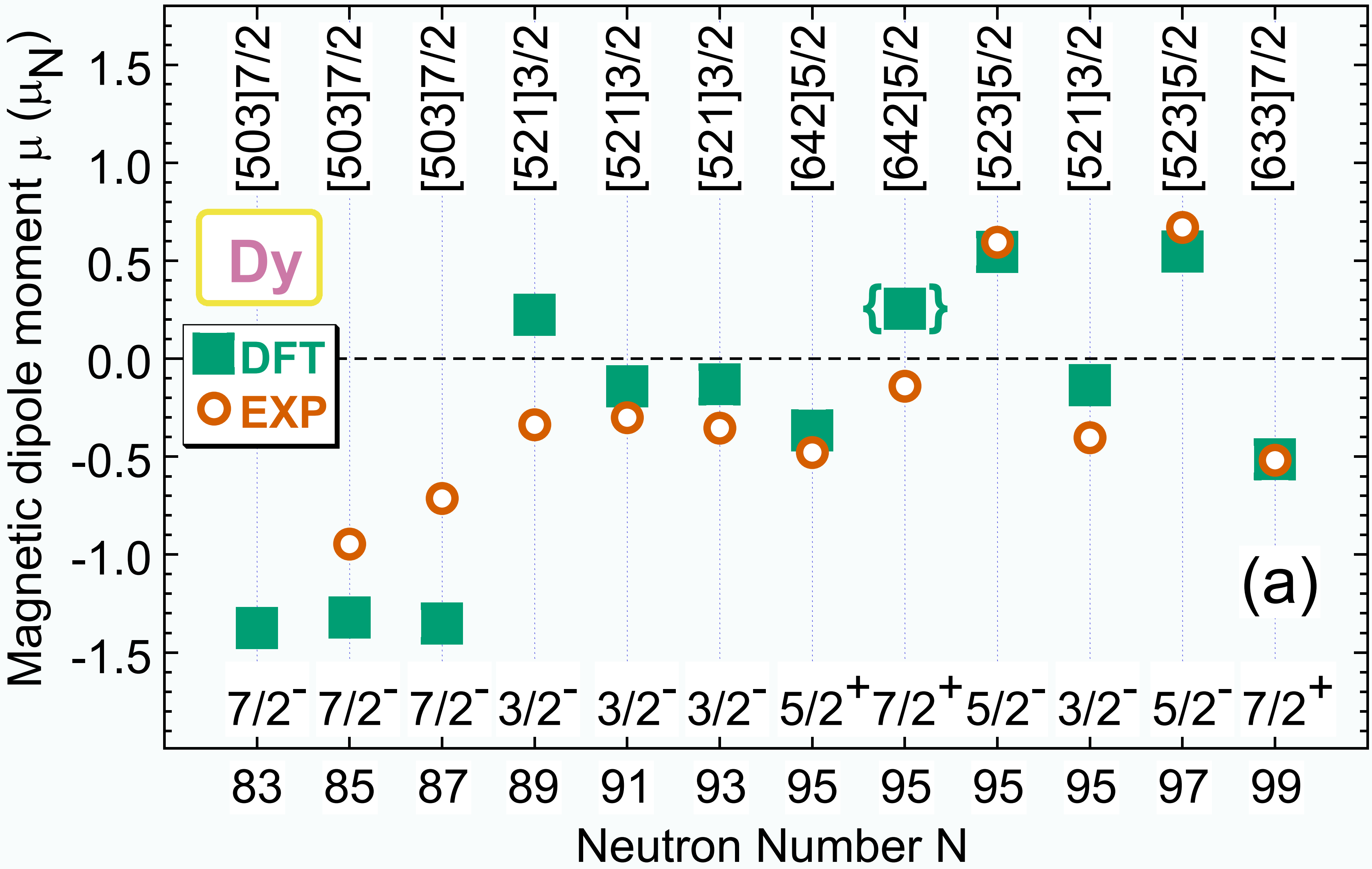
Dysprosium electric quadrupole moments vs. data



J. Dobaczewski *et al.*, arXiv:2509.26549

Dysprosium magnetic dipole moments vs. data



J. Dobaczewski *et al.*, arXiv:2509.26549

Summary of results obtained in Gd-Os nuclei

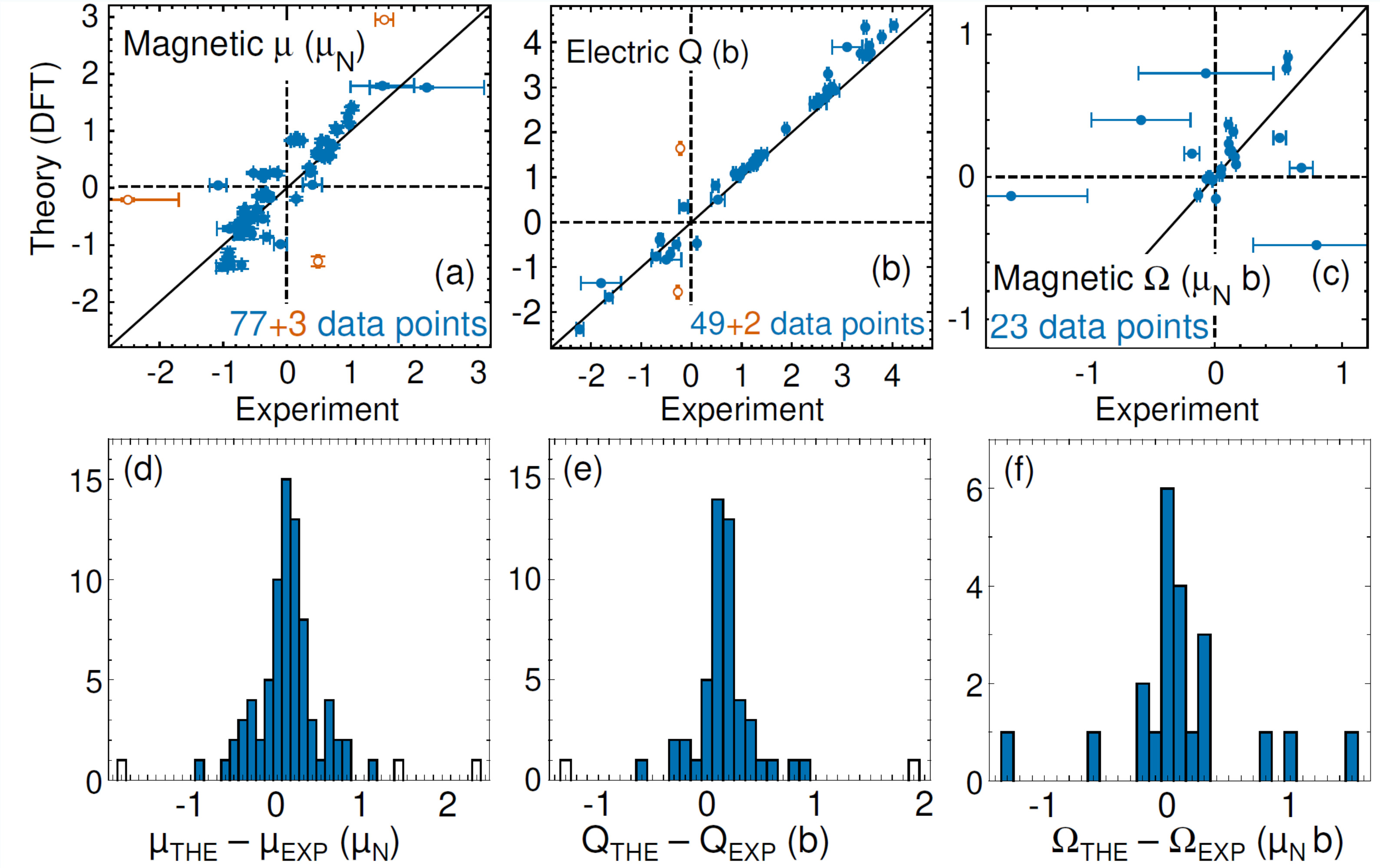
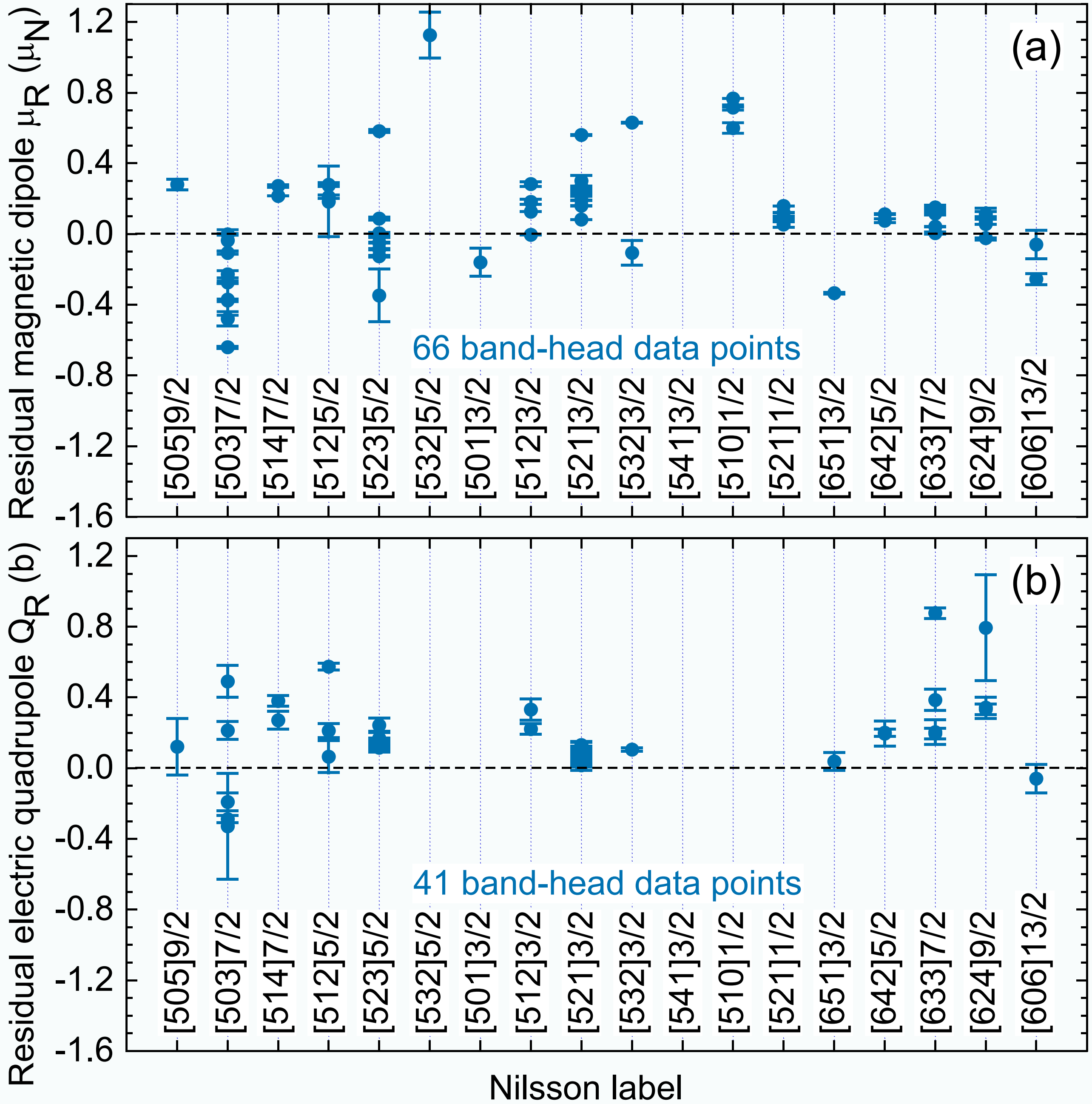


Figure 6: Summary comparison of the experimental and theoretical DFT magnetic dipole moments μ, (a) and (d), electric quadrupole moments Q, (b) and (e), in odd-N, even-Z isotopes between gadolinium and osmium (panels adapted from Reference (67). Panels (c) and (f) show the analogous comparison of the magnetic octupole moments Ω across the mass chart. Open symbols and bars denote the outliers, see text.

J. Dobaczewski *et al.*, arXiv:2511.04632

Residuals in Gd-Os nuclei



J. Dobaczewski *et al.*, arXiv:2509.26549

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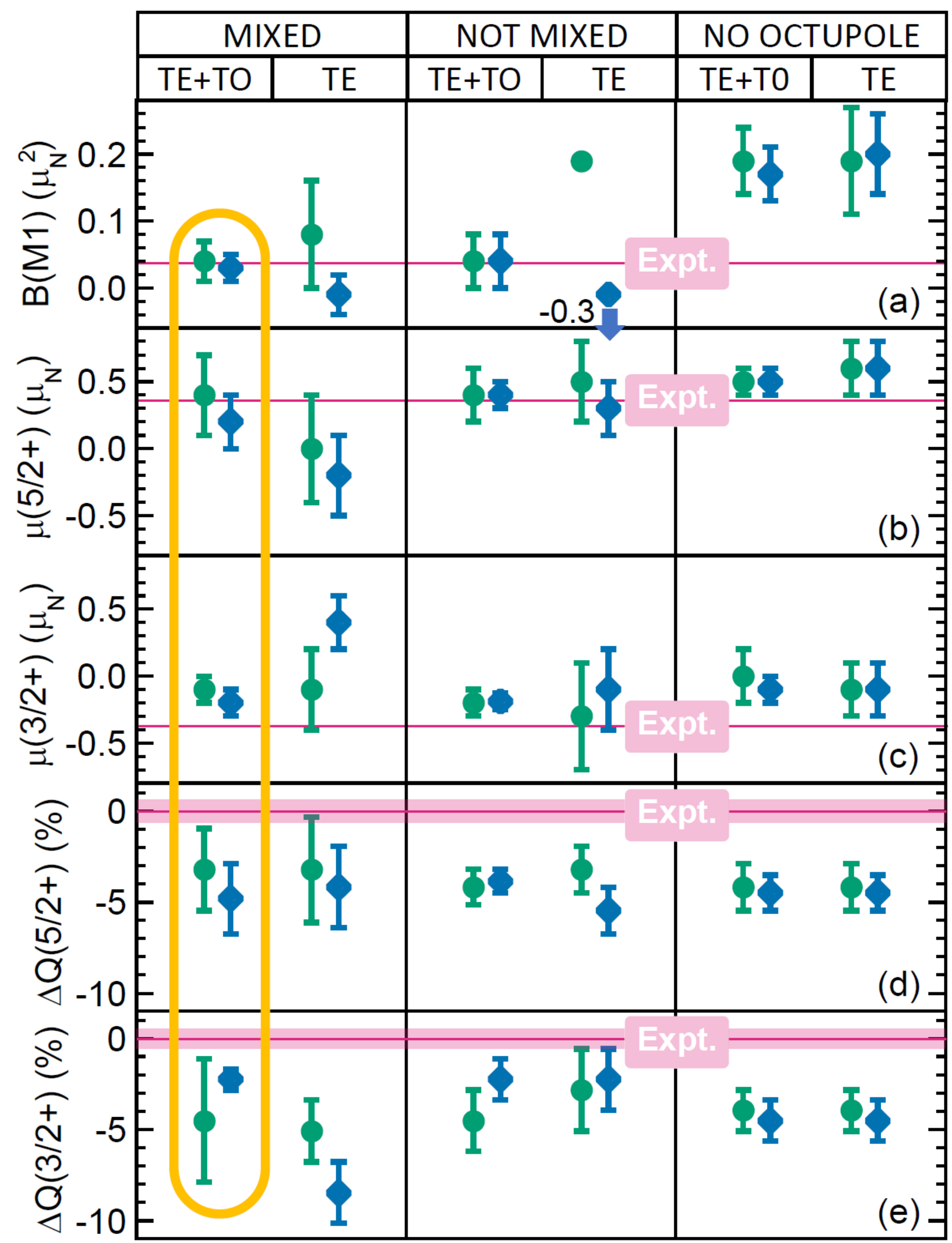
Radiative decay and electromagnetic moments in ^{229}Th

We determined the magnetic dipole transition strength B(M1:3/2+→5/2+) between the isomeric and ground states of ^{229}Th and discussed the effects of parity breaking, configuration mixing, and time-odd core polarization. Without parameter adjustment, the obtained results favorably compare with the experimental data but also indicate the need to systematically adjust the octupole degrees of freedom in future functional parametrizations.



A. Restrepo-Giraldo *et al.*, to be published

Meson exchange currents in nuclear DFT

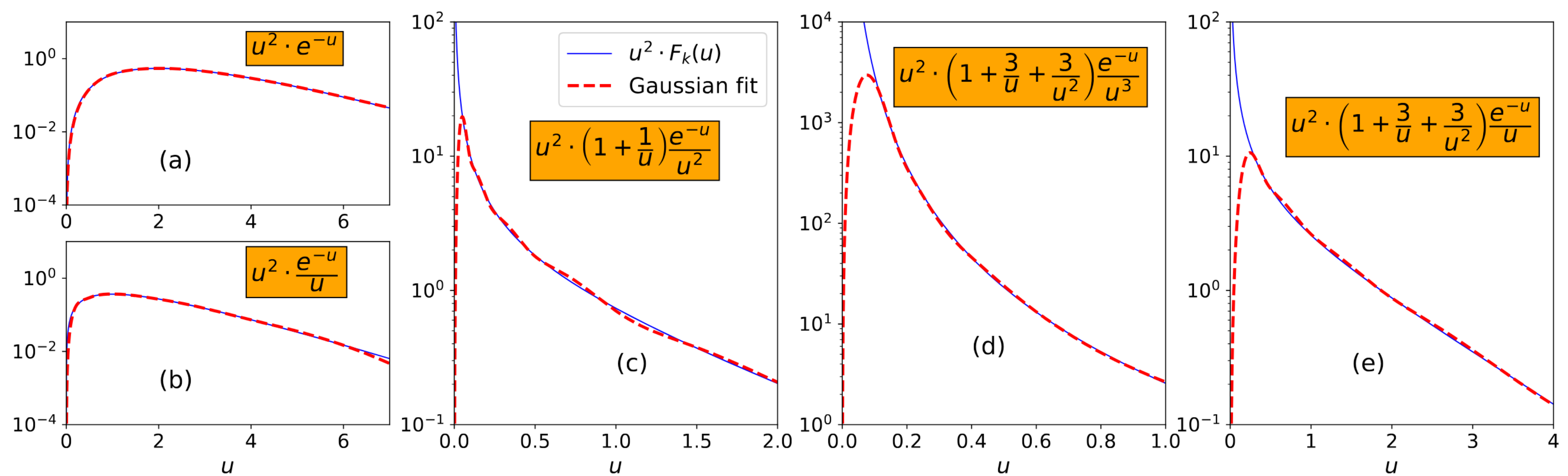
The two-body operator (40) has two terms, called the intrinsic term,

$$\hat{\boldsymbol{\mu}}_{2b}^{\text{int}}(\boldsymbol{r}_1,\boldsymbol{r}_2) = -\frac{g_A^2 m_\pi}{32\pi F_\pi^2}(\hat{\boldsymbol{\tau}}_1 \times \hat{\boldsymbol{\tau}}_2)_z \left\{ \left(1+\frac{1}{u}\right) [(\hat{\boldsymbol{\sigma}}_1 \times \hat{\boldsymbol{\sigma}}_2)\cdot\hat{\boldsymbol{r}}]\,\hat{\boldsymbol{r}} - (\hat{\boldsymbol{\sigma}}_1 \times \hat{\boldsymbol{\sigma}}_2) \right\} e^{-u},$$

and the Sachs term,

$$\hat{\boldsymbol{\mu}}_{2b}^{\text{Sachs}}(\boldsymbol{r}_1,\boldsymbol{r}_2) = -\frac{m_\pi^3 g_A^2}{96\pi F_\pi^2}(\hat{\boldsymbol{\tau}}_1 \times \hat{\boldsymbol{\tau}}_2)_z (\boldsymbol{R}\times\boldsymbol{r}) \left[\hat{S}_{12}\left(1+\frac{3}{u}+\frac{3}{u^2}\right) + \hat{\boldsymbol{\sigma}}_1\cdot\hat{\boldsymbol{\sigma}}_2 \right] \frac{e^{-u}}{u}.$$

R. Seutin *et al.* Phys. Rev. C 108 (2023) 054005; T. Miyagi *et al.* Phys. Rev. Lett. 132 (2024) 232503



H. Wibowo *et al.*, to be published

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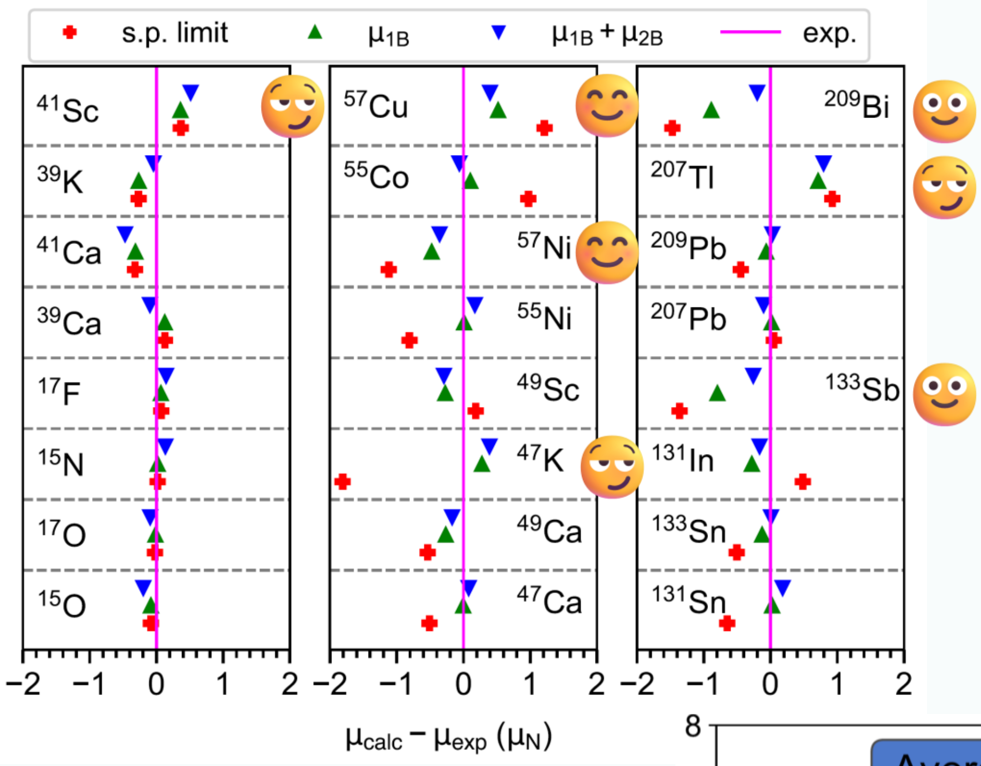
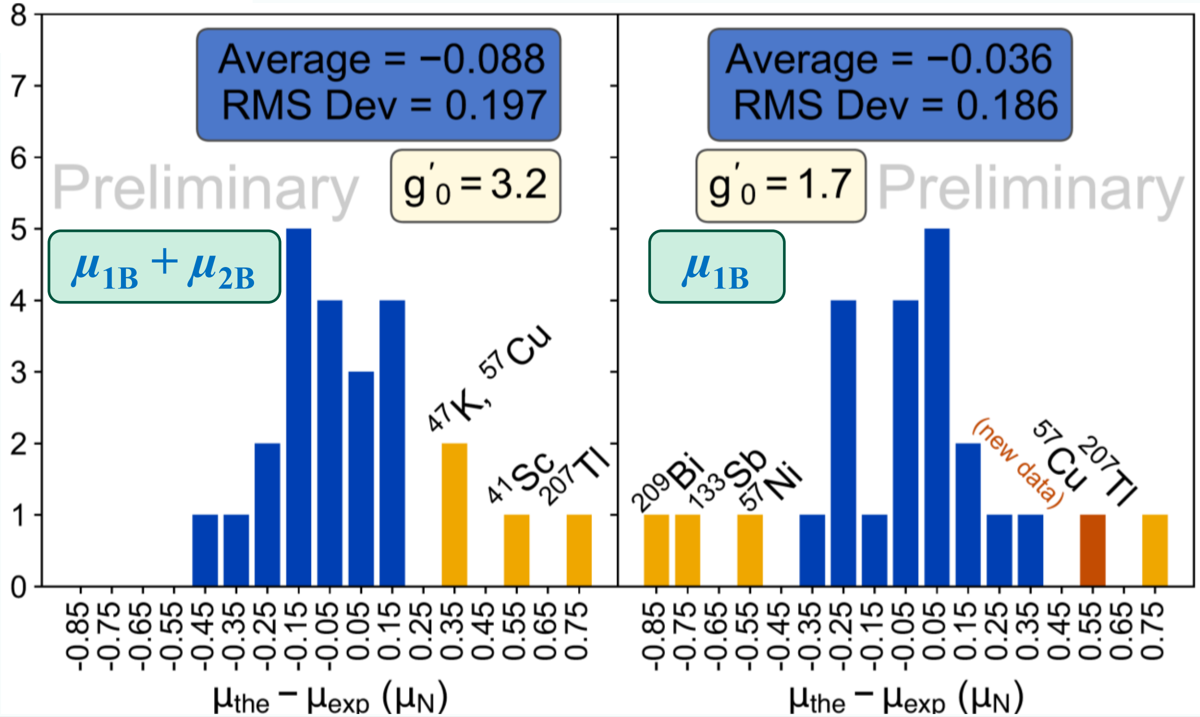
Meson exchange currents in nuclear DFT





H. Wibowo *et al.*, to be published

Electric dipole moments in nuclear DFT

The nuclear Schiff operator is defined as follows,

$$\hat{S}_z = \frac{e}{10}\sum_p \left(r_p^2 - \frac{5}{3}\langle r^2\rangle_{\mathbf{ch}} \right) z_p \qquad 1.$$

The laboratory Schiff moment, $S_z^{\mathbf{lab}}$, is determined using second-order perturbation theory,

$$S_z^{\mathbf{lab}} \approx \sum_{i\neq 0} \frac{\langle \Psi_0|\hat{S}_z|\Psi_i\rangle\langle\Psi_i|\hat{V}_{\mathbf{PT}}|\Psi_0\rangle}{E_0 - E_i} + \mathbf{c.c.}, \qquad 2.$$

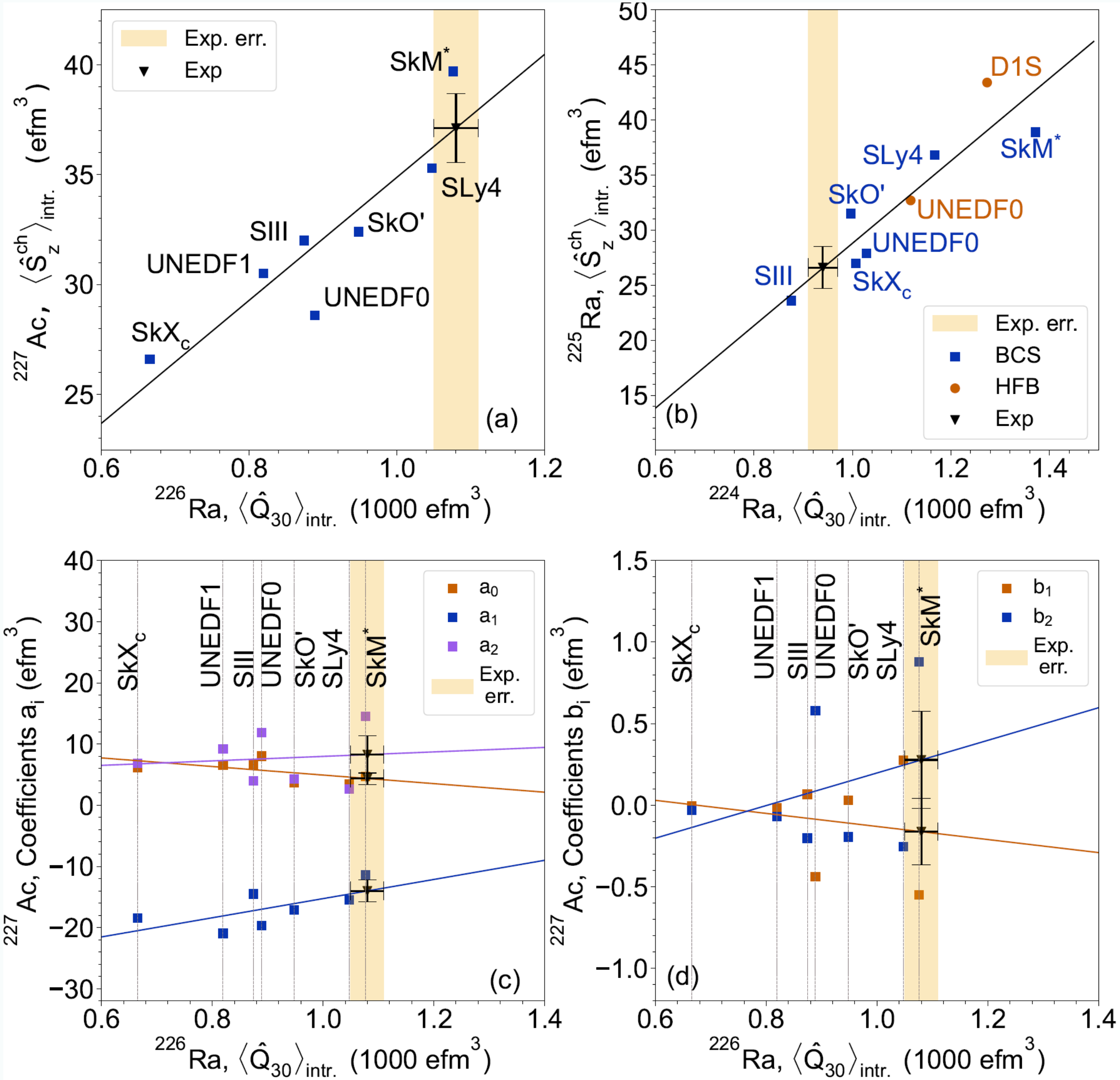
where $|\Psi_0\rangle$ is the ground state and the sum is over excited states. The P,T-violating NN interaction, $\hat{V}_{\mathbf{PT}}$ reads as follows,

$$\begin{aligned}\hat{V}_{\mathbf{PT}}(\boldsymbol{r}_1 - \boldsymbol{r}_2) = -\frac{g m_\pi^2}{8\pi m_N} \Big\{ (\hat{\boldsymbol{\sigma}}_1 - \hat{\boldsymbol{\sigma}}_2)\cdot(\boldsymbol{r}_1 - \boldsymbol{r}_2)\Big[\bar{g}_0 \hat{\vec{\tau}}_1 \cdot \hat{\vec{\tau}}_2 - \frac{\bar{g}_1}{2}(\hat{\tau}_{1z} + \hat{\tau}_{2z}) + \\ + \bar{g}_2(3\hat{\tau}_{1z}\hat{\tau}_{2z} - \hat{\vec{\tau}}_1\cdot\hat{\vec{\tau}}_2)\Big] - \frac{\bar{g}_1}{2}(\hat{\boldsymbol{\sigma}}_1 + \hat{\boldsymbol{\sigma}}_2)\cdot(\boldsymbol{r}_1 - \boldsymbol{r}_2)(\hat{\tau}_{1z} - \hat{\tau}_{2z})\Big\} \\ \times \frac{e^{-m_\pi r}}{m_\pi r^2}\left[1 + \frac{1}{m_\pi r}\right] + \frac{1}{2m_N^3}\left[\bar{c}_1 + \bar{c}_2 \hat{\vec{\tau}}_1\cdot\hat{\vec{\tau}}_2\right](\hat{\boldsymbol{\sigma}}_1 - \hat{\boldsymbol{\sigma}}_2)\cdot\boldsymbol{\nabla}\delta^3(\boldsymbol{r}_1 - \boldsymbol{r}_2). \qquad 3.\end{aligned}$$

The linearity of $\hat{V}_{\mathbf{PT}}$ allows us to express the laboratory Schiff moment, $S_z^{\mathbf{lab}}$, as a linear combination of the unknown coupling constants with the coefficients calculated in DFT, that is,

$$S_z^{\mathbf{lab}} = a_0 g\bar{g}_0 + a_1 g\bar{g}_1 + a_2 g\bar{g}_2 + b_1\bar{c}_1 + b_2\bar{c}_2. \qquad 4.$$

Electric dipole moments in nuclear DFT



J. Dobaczewski *et al.*, Phys. Rev. Lett. 121 (2018) 232501

M. Athanasakis-Kaklamanakis *et al.*, Nature 648 (2025) 562

J. Dobaczewski *et al.*, arXiv:2511.04632

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Anapole moments in nuclear DFT

The nuclear anapole operator is the lowest-order parity non-conservation (PNC), time-reversal-odd multipole operator defined by Haxton, Liu, and Ramsey-Musolf as follows,

$$\hat{a}_\eta = -\frac{M^2}{9}\int \mathbf{d}^3\boldsymbol{r}\, r^2 \left[\hat{j}_\eta(\boldsymbol{r}) + \sqrt{2\pi}\left[Y_2(\theta,\phi)\otimes\hat{\boldsymbol{j}}(\boldsymbol{r})\right]_{1\eta}\right], \qquad 5.$$

Analogous to the laboratory Schiff moment, S_z^{lab}, the laboratory anapole moment is

$$a_z^{\mathrm{lab}} \approx \sum_{i\neq 0}\frac{\langle\Psi_0|\hat{a}_z|\Psi_i\rangle\langle\Psi_i|\hat{V}_{\mathbf{PNC}}|\Psi_0\rangle}{E_0 - E_i} + \mathbf{c.c.}, \qquad 6.$$

where $\hat{V}_{\mathbf{PNC}}$ is used in the form of the Desplanques Donoghue, and Holstein (DDH) potential, and has the form as follows,

$$\begin{aligned}
\hat{V}^{\mathbf{DDH}}(\boldsymbol{r}) =\ & \tfrac{i}{2\sqrt{2}}h_\pi^1 g_{\pi NN}\left(\hat{\boldsymbol{\tau}}_1\times\hat{\boldsymbol{\tau}}_2\right)_z\left(\hat{\boldsymbol{\sigma}}_1+\hat{\boldsymbol{\sigma}}_2\right)\cdot\tfrac{1}{2M}\left[\hat{\boldsymbol{p}}_1-\hat{\boldsymbol{p}}_2,\omega_\pi(r)\right] \\
& - g_\rho\left(h_\rho^0\hat{\boldsymbol{\tau}}_1\cdot\hat{\boldsymbol{\tau}}_2 + \tfrac{1}{2}h_\rho^1(\hat{\boldsymbol{\tau}}_1+\hat{\boldsymbol{\tau}}_2)_z + \tfrac{1}{2\sqrt{6}}h_\rho^2(3\hat{\tau}_{1z}\hat{\tau}_{2z}-\hat{\boldsymbol{\tau}}_1\cdot\hat{\boldsymbol{\tau}}_2)\right) \\
& \quad\times\tfrac{1}{2M}\left((\hat{\boldsymbol{\sigma}}_1-\hat{\boldsymbol{\sigma}}_2)\cdot\left\{\hat{\boldsymbol{p}}_1-\hat{\boldsymbol{p}}_2,\omega_\rho(r)\right\} + i(1+\chi_V)(\hat{\boldsymbol{\sigma}}_1\times\hat{\boldsymbol{\sigma}}_2)\cdot\left[\hat{\boldsymbol{p}}_1-\hat{\boldsymbol{p}}_2,\omega_\rho(r)\right]\right) \\
& - g_\omega\left(h_\omega^0 + \tfrac{1}{2}h_\omega^1\left(\hat{\boldsymbol{\tau}}_1+\hat{\boldsymbol{\tau}}_2\right)_z\right) \\
& \quad\times\tfrac{1}{2M}\left((\hat{\boldsymbol{\sigma}}_1-\hat{\boldsymbol{\sigma}}_2)\cdot\left\{\hat{\boldsymbol{p}}_1-\hat{\boldsymbol{p}}_2,\omega_\omega(r)\right\} + i(1+\chi_S)(\hat{\boldsymbol{\sigma}}_1\times\hat{\boldsymbol{\sigma}}_2)\cdot\left[\hat{\boldsymbol{p}}_1-\hat{\boldsymbol{p}}_2,\omega_\omega(r)\right]\right) \\
& + \tfrac{1}{2}\left(\hat{\boldsymbol{\tau}}_1-\hat{\boldsymbol{\tau}}_2\right)_z\left(\hat{\boldsymbol{\sigma}}_1+\hat{\boldsymbol{\sigma}}_2\right)\cdot\tfrac{1}{2M}\left(g_\rho h_\rho^1\left\{\hat{\boldsymbol{p}}_1-\hat{\boldsymbol{p}}_2,\omega_\rho(r)\right\} - g_\omega h_\omega^1\left\{\hat{\boldsymbol{p}}_1-\hat{\boldsymbol{p}}_2,\omega_\omega(r)\right\}\right) \\
& - \tfrac{i}{2}g_\rho h_\rho^{1'}\left(\hat{\boldsymbol{\tau}}_1\times\hat{\boldsymbol{\tau}}_2\right)_z\left(\hat{\boldsymbol{\sigma}}_1+\hat{\boldsymbol{\sigma}}_2\right)\cdot\tfrac{1}{2M}\left[\hat{\boldsymbol{p}}_1-\hat{\boldsymbol{p}}_2,\omega_\rho(r)\right].
\end{aligned} \qquad 7.$$

W.C. Haxton *et al.*, Phys. Rev. C 65 (2002) 045502

B. Desplanques *et al.*, Ann. Phys. 124 (1980) 449

J. Dobaczewski *et al.*, arXiv:2511.04632

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Conclusions

1. For the first time, in nuclear theory, we can systematically calculate spectroscopic electromagnetic moments in odd open-shell nuclei with arbitrary particle numbers and (axial) deformations.
2. Large nuclear-DFT single-particle phase space (well beyond the valence space) allows for using the bare effective charges and g-factors. (No adjustable "effective" values are needed.)
3. The calculated magnetic dipole moments μ and electric quadrupole moments Q reproduce the known experimental data in heavy open-shell nuclei.
4. It is essential to take into account simultaneously:
   a) Self-consistency
   b) Polarization
   c) Symmetry restoration
5. The effects of the extended T-odd sector, triaxiality, octupolarity, two-body currents, K-mixing, and configuration interaction (…) remain to be studied.

Thank you

Recent publications:

1. **Nuclear DFT analysis of electromagnetic moments in odd near doubly magic nuclei:**
   P.L. Sassarini, J. Dobaczewski, J. Bonnard, R.F. Garcia Ruiz
   J. Phys. G 49 (2022) 11LT01
2. **Nuclear DFT electromagnetic moments in heavy deformed open-shell odd nuclei:**
   J. Bonnard, J. Dobaczewski, G. Danneaux, M. Kortelainen
   Phys. Lett. B843 (2023) 138014
3. **Electromagnetic moments in the Sn-Gd region determined within the nuclear DFT:**
   H. Wibowo, B.C. Backes, J. Dobaczewski, R.P. de Groote, A. Nagpal, A. Sánchez-Fernández, X. Sun, J. L. Wood
   J. Phys. G 52 (2025) 065104
4. **Electromagnetic moments of ground and excited states calculated in heavy odd-N openshell nuclei:**
   J. Dobaczewski, A.E. Stuchbery, G. Danneaux, A. Nagpal, P.L. Sassarini, H. Wibowo
   arXiv:2509.26549, submitted to Physical Review C
5. **Electromagnetic and exotic moments in nuclear DFT:**
   J. Dobaczewski, B.C. Backes R.P. de Groote, A. Restrepo-Giraldo, X. Sun and H. Wibowo
   arXiv:2511.04632, submitted to Annual Review of Nuclear and Particle Science
6. **Nuclear DFT meson-exchange contributions to magnetic dipole moments of atomic nuclei:**
   H. Wibowo, R. Han, B.C. Backes, G. Danneaux, J. Dobaczewski, W.C. Haxton, W. Jiang, and M. Kortelainen,
   to be published
7. **Radiative decay and electromagnetic moments in ^{229}Th determined within nuclear DFT:**
   A. Restrepo-Giraldo, J. Dobaczewski, J. Bonnard, and X. Sun,
   to be published

Basic definitions

The electric and magnetic moments are defined as

$$Q_{\lambda\mu} = \langle\Psi|\hat{Q}_{\lambda\mu}|\Psi\rangle = \int q_{\lambda\mu}(\vec{r})\, d^3\vec{r},$$

$$M_{\lambda\mu} = \langle\Psi|\hat{M}_{\lambda\mu}|\Psi\rangle = \int m_{\lambda\mu}(\vec{r})\, d^3\vec{r},$$

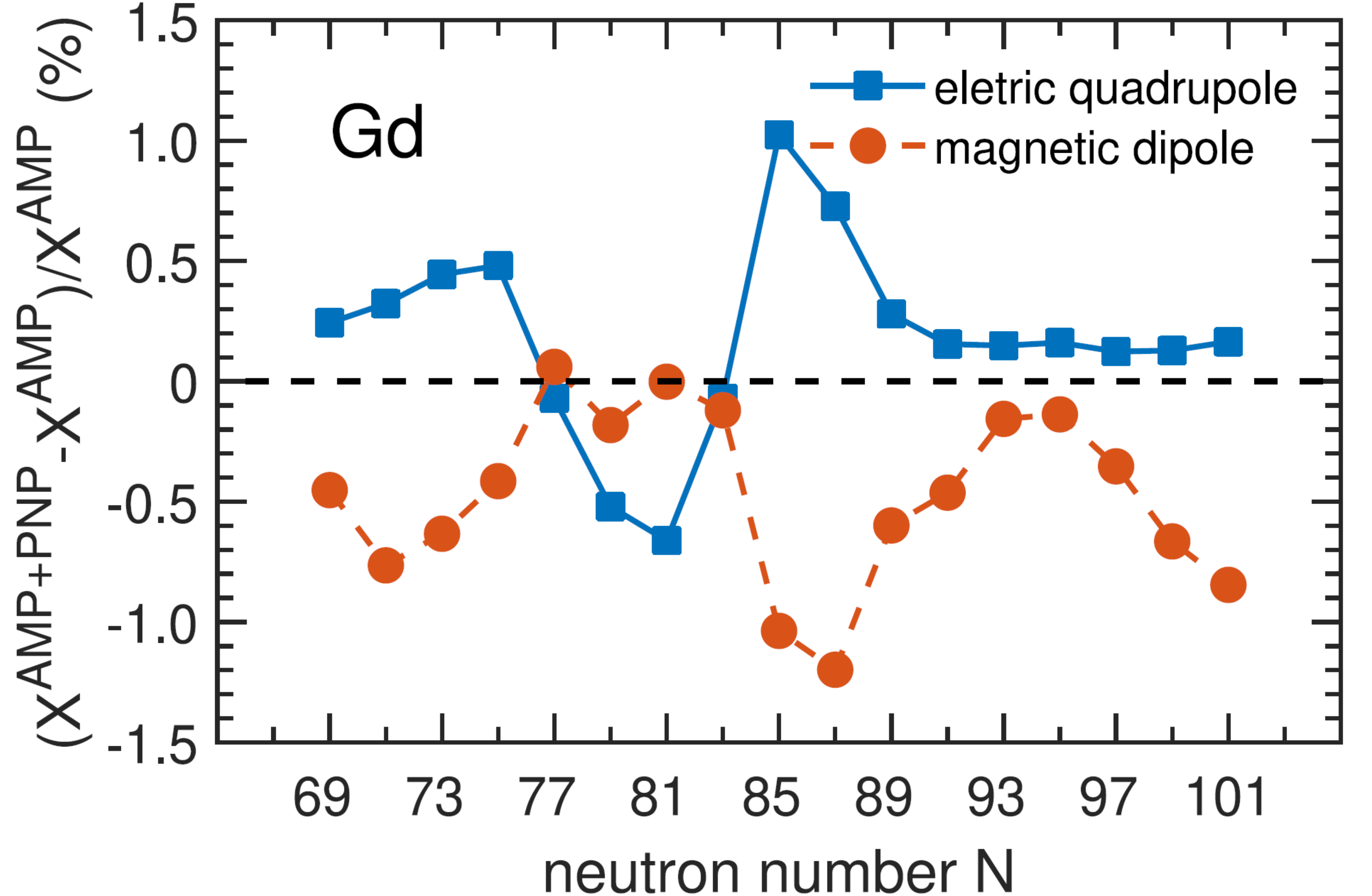
where $|\Psi\rangle$ is a many-body state, and $q_{\lambda\mu}(\vec{r})$ and $m_{\lambda\mu}(\vec{r})$ are the corresponding electric and magnetic-moment densities:

$$q_{\lambda\mu}(\vec{r}) = e\rho(\vec{r})Q_{\lambda\mu}(\vec{r}),$$

$$m_{\lambda\mu}(\vec{r}) = \mu_N\left[g_s\vec{s}(\vec{r}) + \tfrac{2}{\lambda+1}g_l\left(\vec{r}\times\vec{j}(\vec{r})\right)\right]\cdot\vec{\nabla}Q_{\lambda\mu}(\vec{r}),$$

and e, g_s, and g_l are the elementary charge, and the spin and orbital gyromagnetic factors, respectively. The multipole functions (solid harmonics) have the standard form: $Q_{\lambda\mu}(\vec{r}) = r^\lambda Y_{\lambda\mu}(\theta,\phi)$.

Particle-number symmetry restoration (PNP) is not worth the 100 × increase in CPU time



H. Wibowo *et al.*, J. Phys. G 52 (2025) 065104

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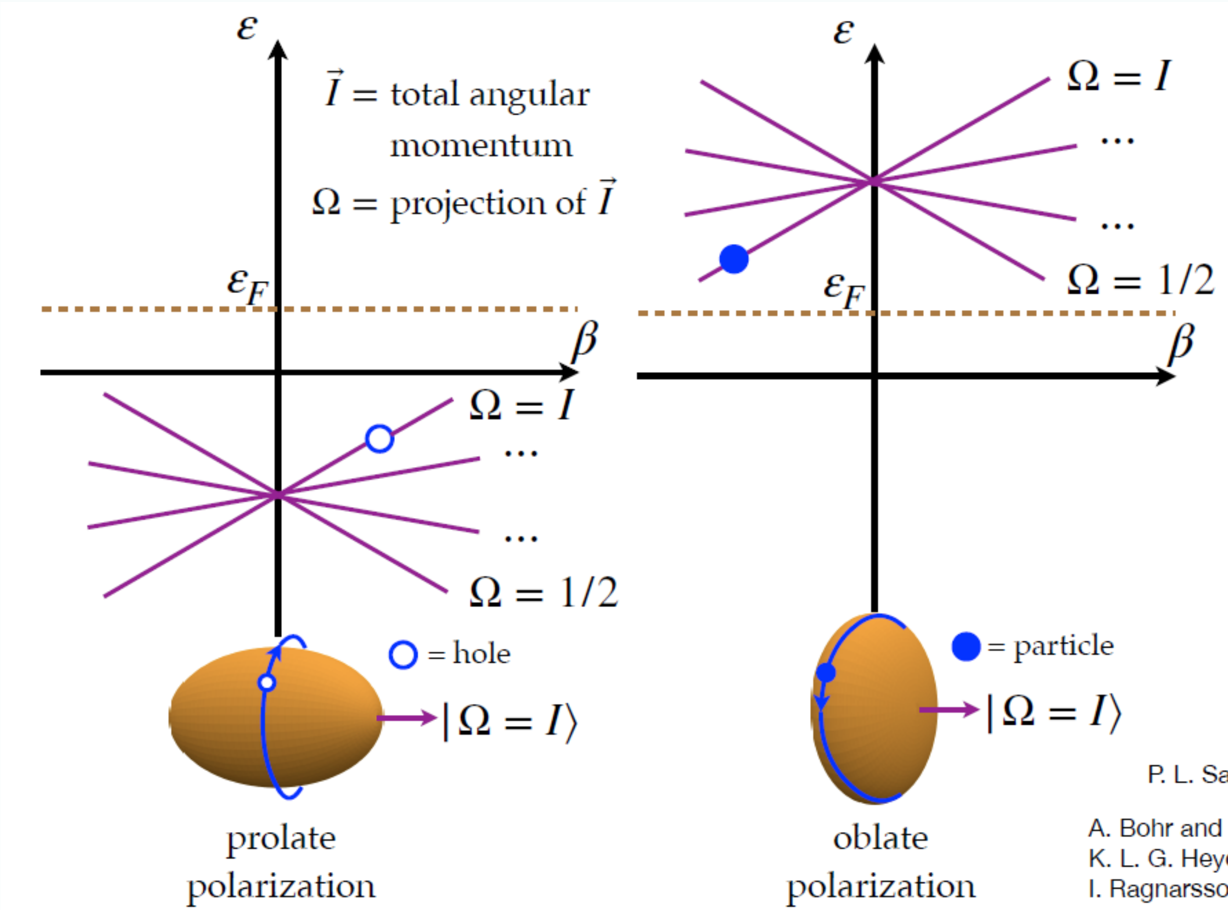
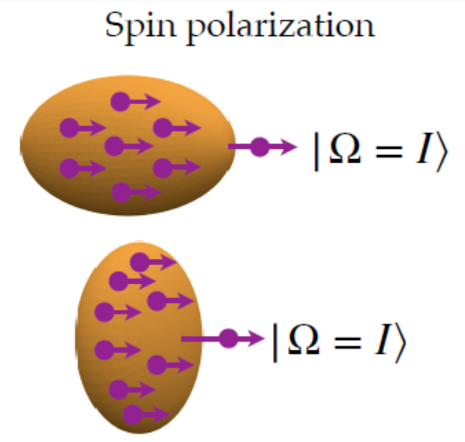
Shape and spin polarization





Landau parameter g_0' ($g_0' = 1.7$)

$$g_0' = N_0 \left(2C_1^s + 2C_1^T (3\pi^2\rho_0/2)^{2/3}\right)$$

$$\frac{1}{N_0} \approx 150 \frac{m}{m^*}\ \mathrm{MeV}\cdot\mathrm{fm}^3$$

P. L. Sassarini et al., J. Phys. G: Nucl. Part. Phys. **49**, 11LT01 (2022)

A. Bohr and B. R. Mottelson, *Nuclear Structure* Vol. 1
K. L. G. Heyde, *The Nuclear Shell Model*
I. Ragnarsson and S. G. Nilsson, *Shapes and Shells in Nuclear Structure*

Picture courtesy of H. Wibowo

In nuclear-DFT, we align the total angular momenta of odd nuclei along the intrinsic axial-symmetry axis with broken spherical and time-reversal symmetries. We fully account for the self-consistent charge, spin, and current polarizations, in particular through the inclusion of the crucial time-odd mean-field components of the functional.

Schmidt limits

The magnetic operator $\bar{\mu}$ is a one-body operator and the magnetic dipole moment μ is the expectation value of $\bar{\mu}_z$. The M1 operator acting on a composed state $|\mathrm{Im}\rangle$ can then be written as the sum of single particle M1 operators $\bar{\mu}_z(j)$ acting each on an individual valence nucleon with total momentum j:

$\mu = g_L\,\mathbf{L} + g_s\,\mathbf{s}$

$$\mu(I) \equiv \left\langle I(j_1, j_2, \ldots, j_n), m = I \left| \sum_{i=1}^{n} \bar{\mu}_z(i) \right| I(j_1, j_2, \ldots, j_n), m = I \right\rangle \tag{2.1}$$

The single particle magnetic moment $\mu(j)$ for a valence nucleon around a doubly magic core is uniquely defined by the quantum numbers l and j of the occupied single particle orbit [22]:

$$\text{for an odd proton:} \begin{cases} \mu = j - \frac{1}{2} + \mu_{\mathrm{p}} & \text{for } j = l + \frac{1}{2} \\ \mu = \dfrac{j}{j+1}\left(j + \dfrac{3}{2} - \mu_{\mathrm{p}}\right) & \text{for } j = l - \frac{1}{2} \end{cases} \tag{2.2}$$

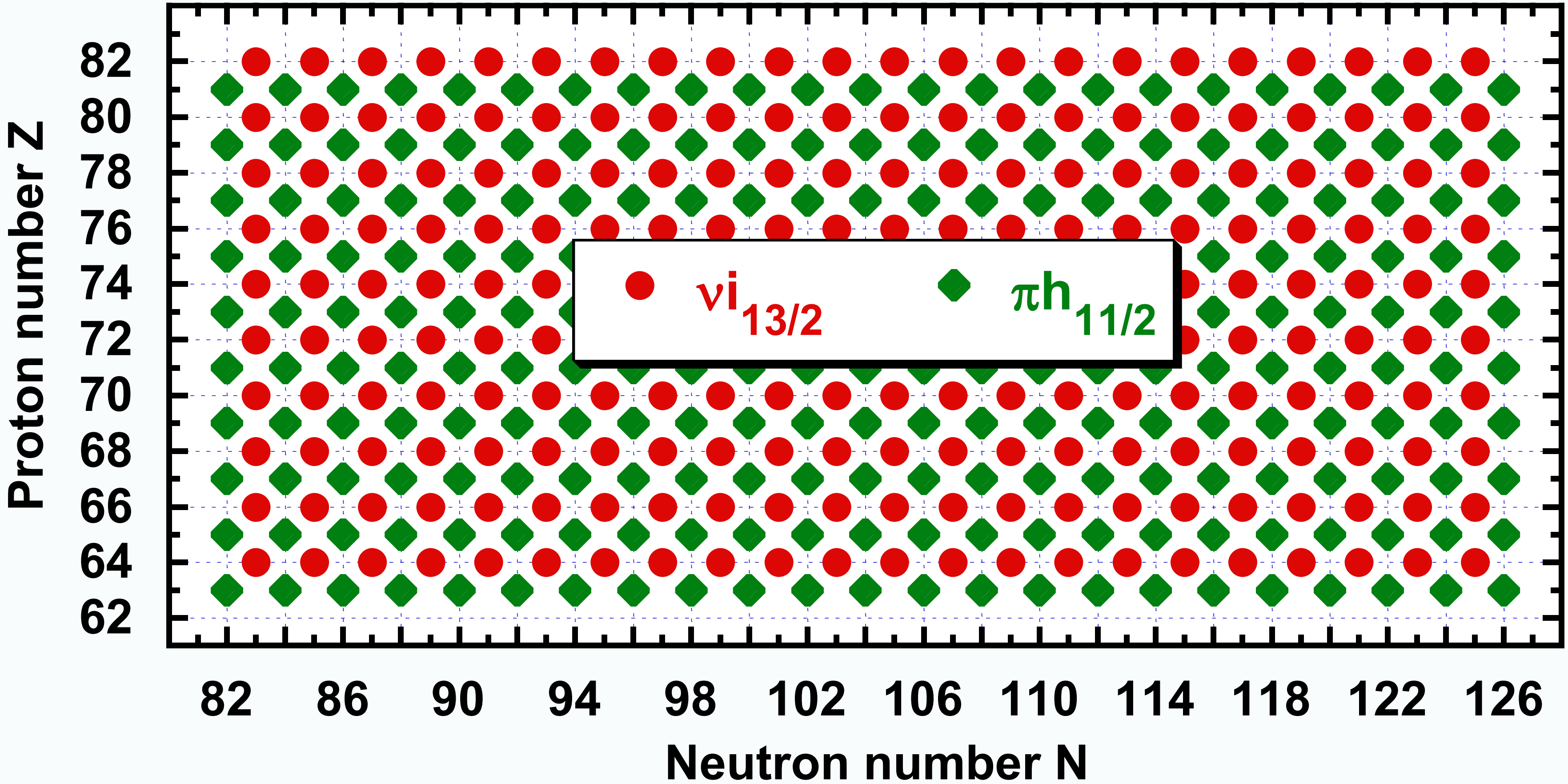
$$\text{for an odd neutron:} \begin{cases} \mu = \mu_{\mathrm{n}} & \text{for } j = l + \frac{1}{2} \\ \mu = -\dfrac{j}{j+1}\mu_{\mathrm{n}} & \text{for } j = l - \frac{1}{2} \end{cases} \tag{2.3}$$

Schmidt limits

These single particle moments calculated using the free proton and free neutron moments ($\mu_{\mathrm{p}} = +2.793$, $\mu_{\mathrm{n}} = -1.913$) are called the Schmidt moments. In a nucleus, the magnetic

Gerda Neyens, Rep. Prog. Phys. 66 (2003) 633–689

The first systematic nuclear-DFT analysis of the electromagnetic moments in heavy deformed open-shell odd nuclei



Blocked quasiparticles were tagged by the neutron $i_{13/2}$ (Ω=+13/2) or proton $h_{11/2}$ (Ω=+11/2) single-particle orbitals

J. Bonnard *et al.*, Phys. Lett. B 843 (2023) 138014

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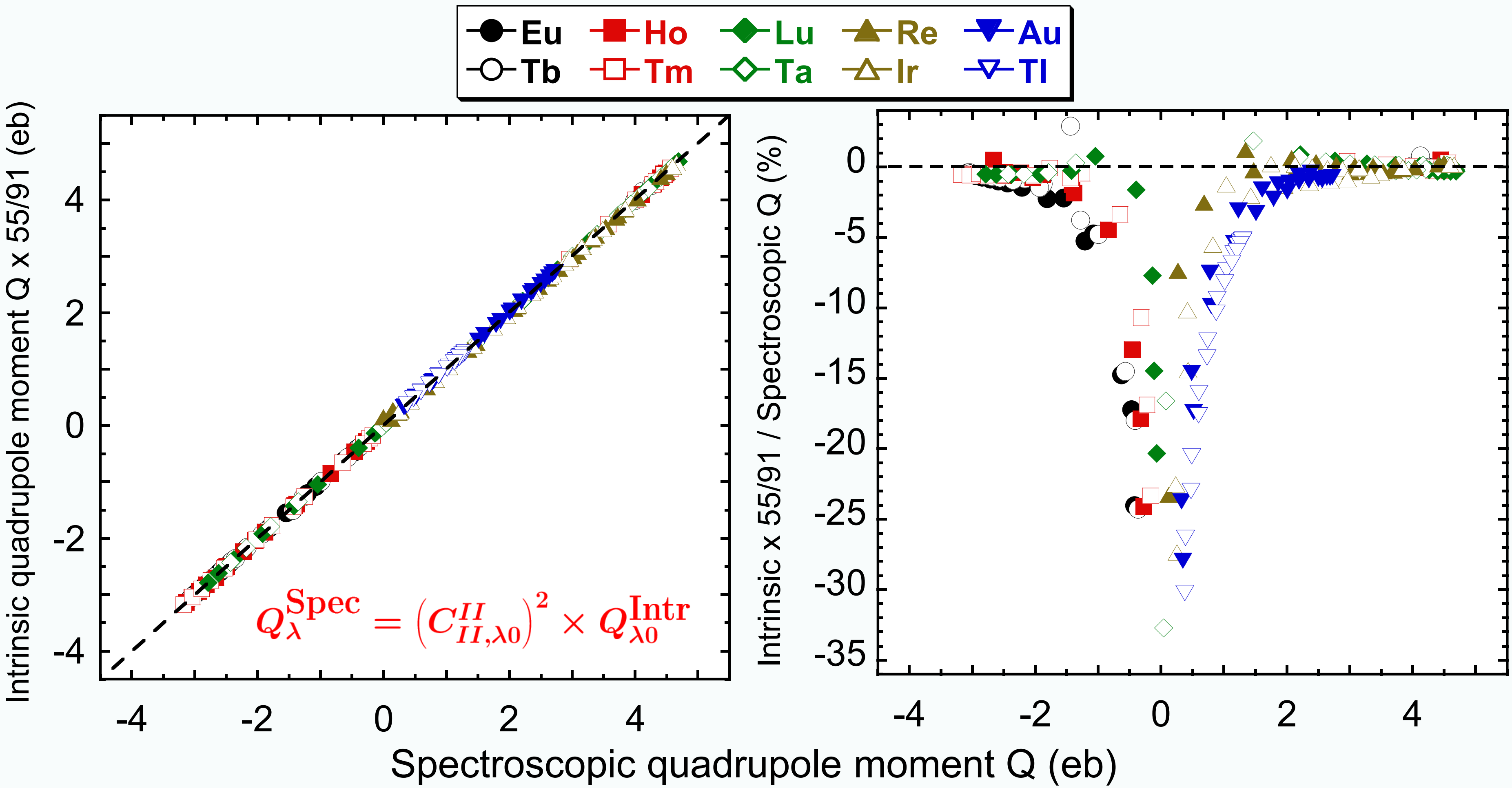
Heavy deformed π11/2⁻ odd-Z nuclei



J. Bonnard *et al.*, Phys. Lett. B 843 (2023) 138014

Conclusion:

Spectroscopic electric quadrupole moments can be inferred from the intrinsic ones at ~5% precision only at |Q|>1b)

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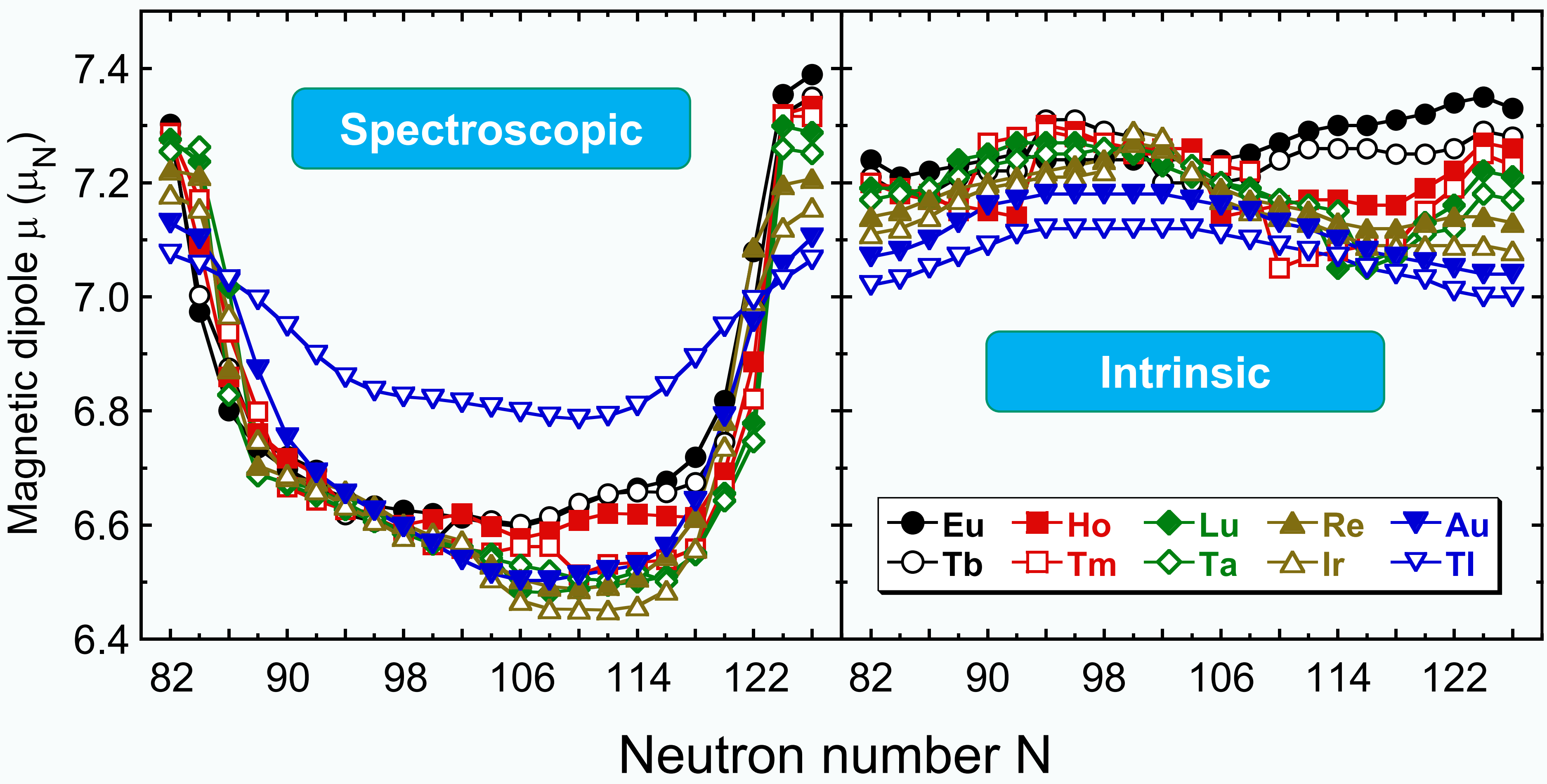
Heavy deformed π11/2⁻ odd-Z nuclei



J. Bonnard *et al.*, Phys. Lett. B 843 (2023) 138014

Conclusion:

Spectroscopic magnetic dipole moments cannot be inferred from the intrinsic ones

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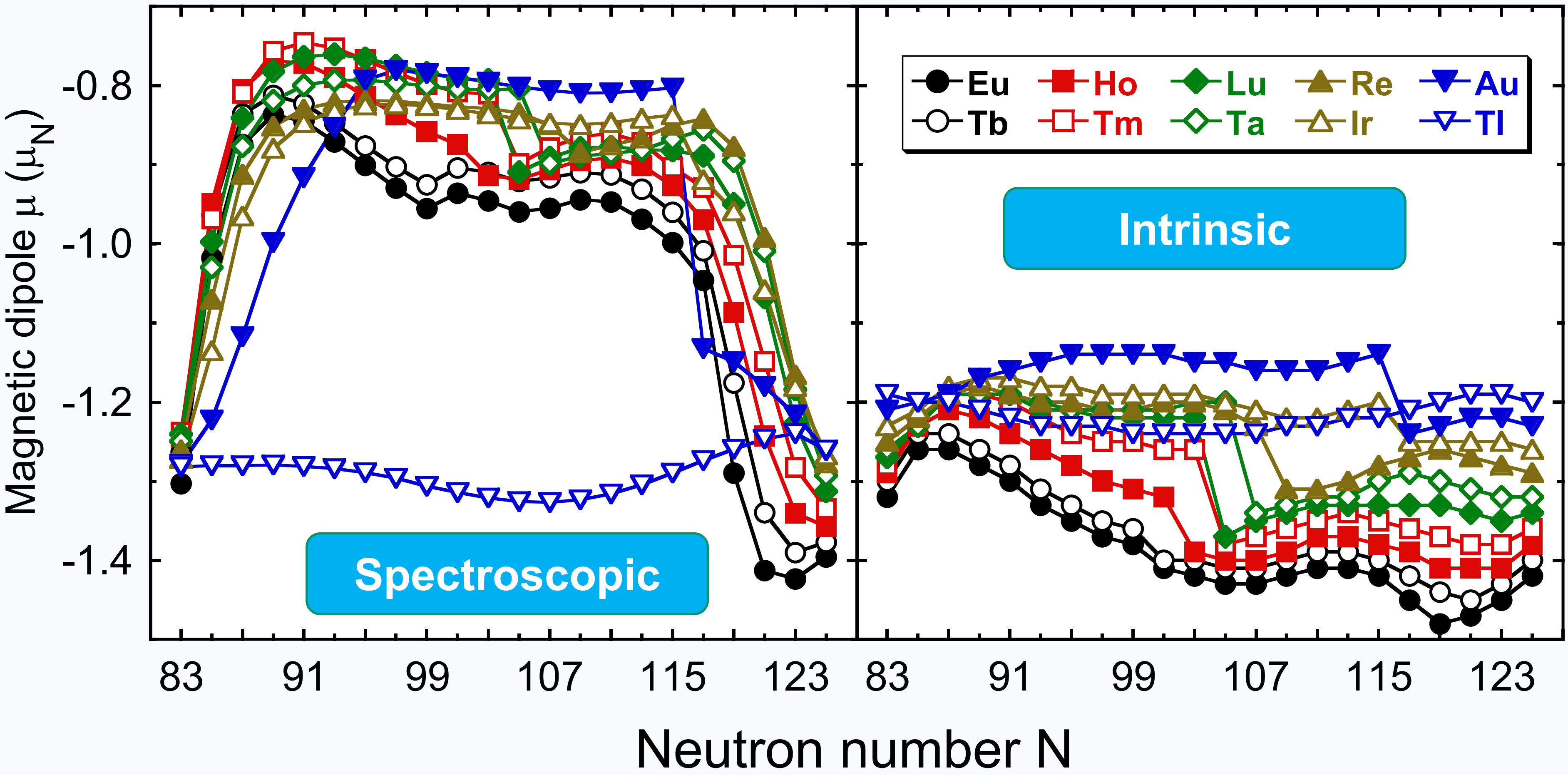
Heavy deformed ν13/2^{+} odd-N nuclei

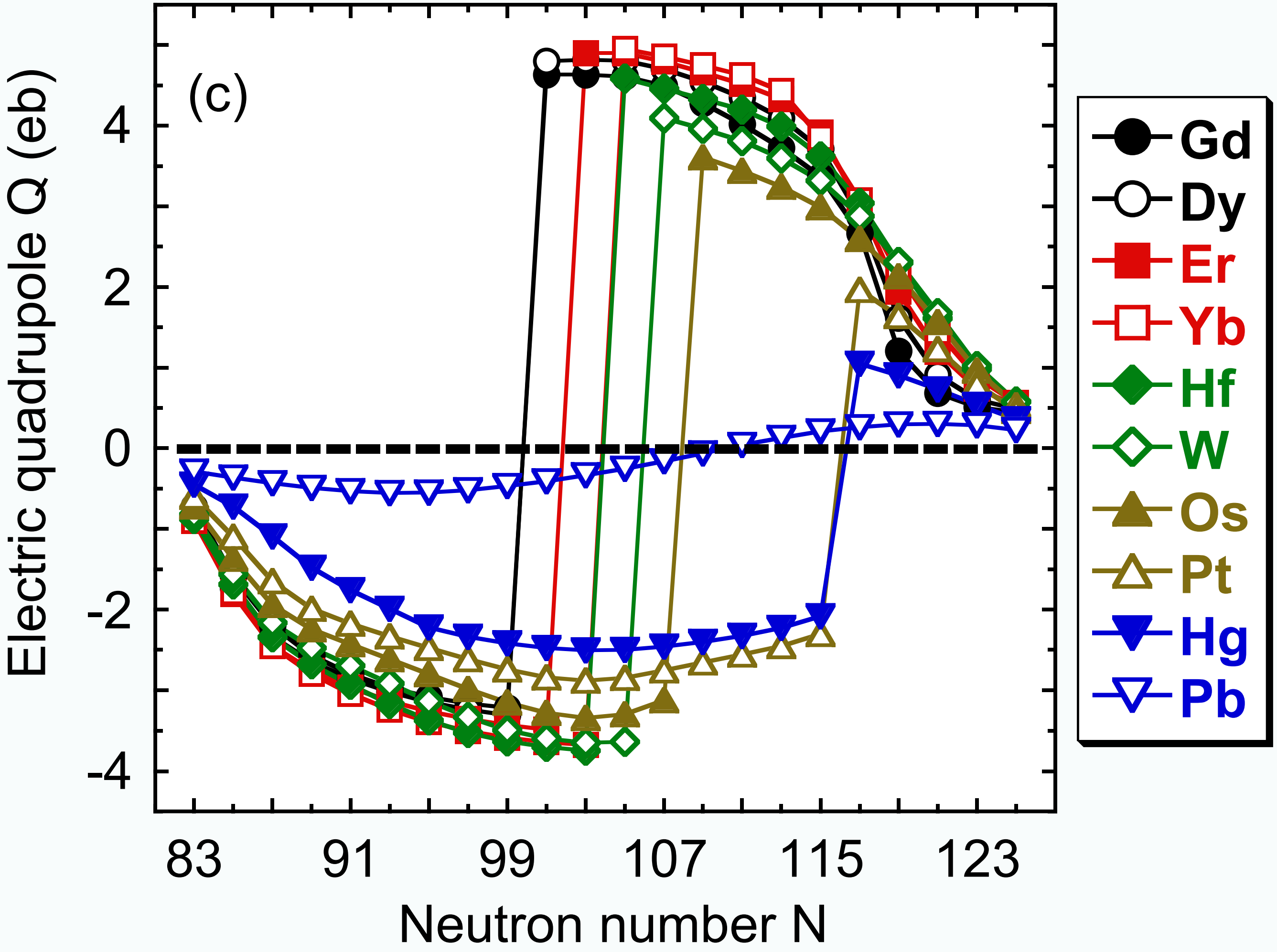


J. Bonnard *et al.*, Phys. Lett. B 843 (2023) 138014

Conclusion:

Spectroscopic magnetic dipole moments cannot be inferred from the intrinsic ones

Heavy deformed ν13/2^{+} odd-N nuclei

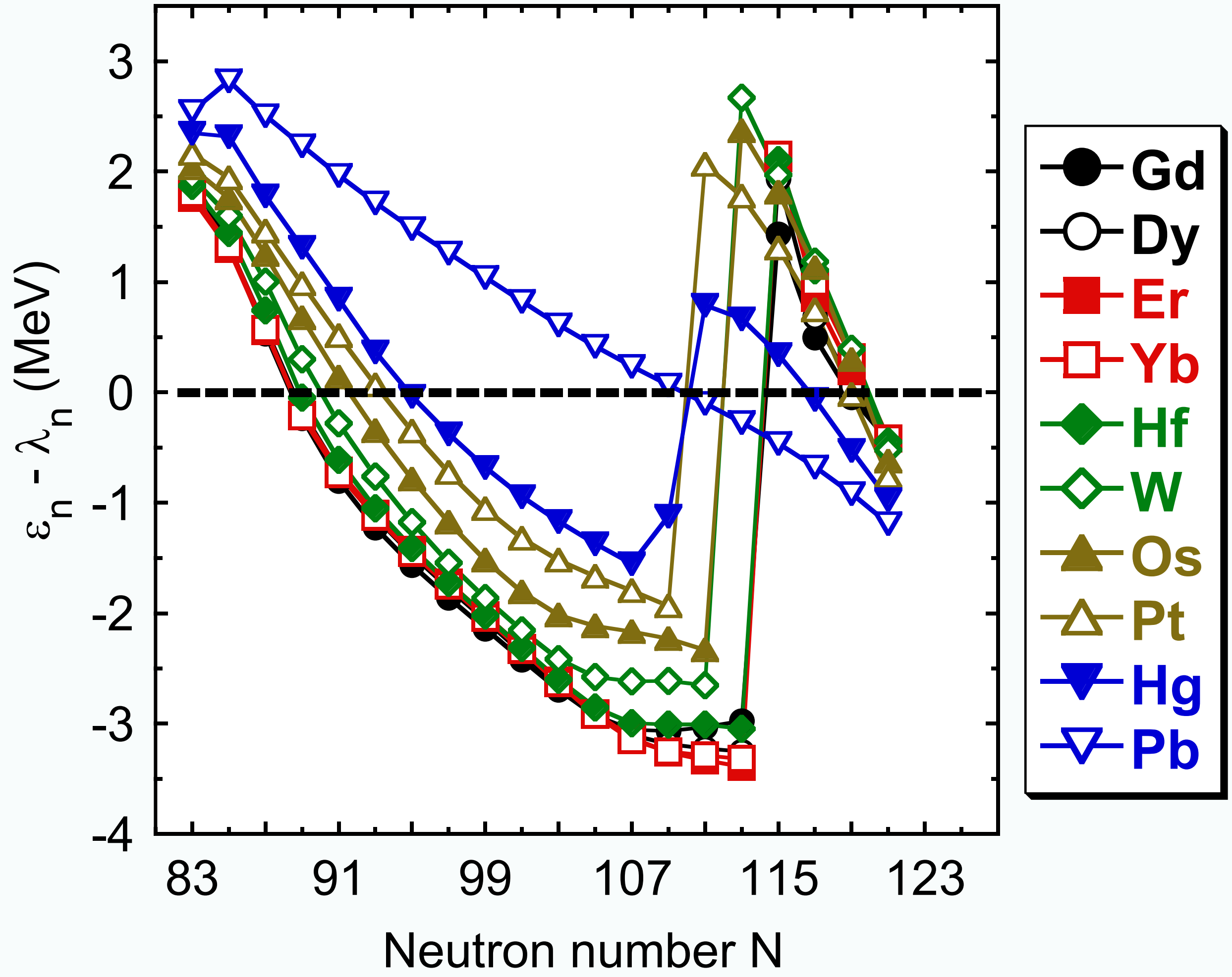


J. Bonnard *et al.*, Phys. Lett. B 843 (2023) 138014

Conclusion:

Rules of oblate and prolate polarizations do extend from the magicity towards the open shell systems.

Heavy deformed $\nu 13/2^+$ odd-N nuclei

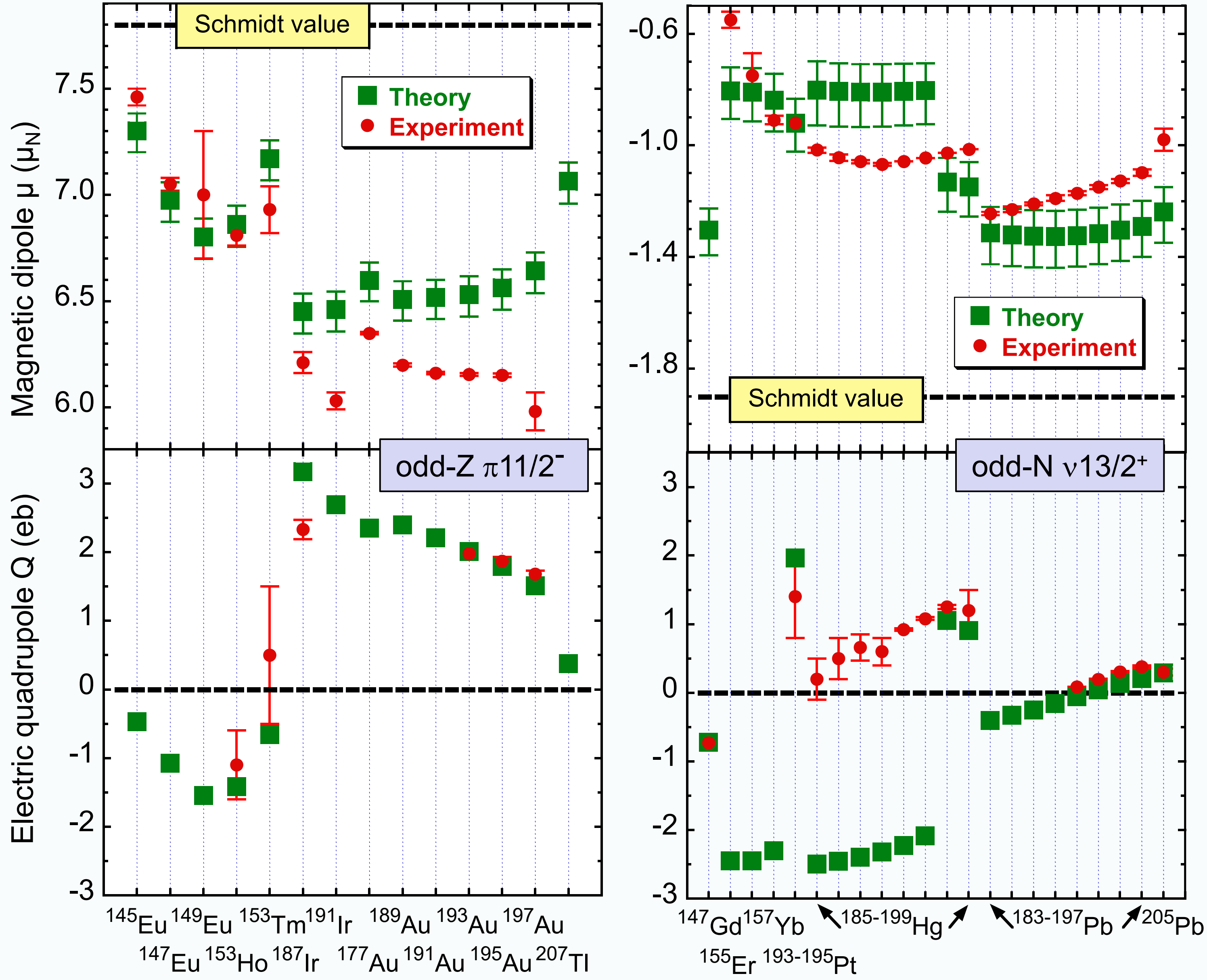


J. Bonnard *et al.*, Phys. Lett. B 843 (2023) 138014

Conclusion:

Rules of particle and hole polarizations do not extend from the magicity towards the open shell systems.

Spectroscopic moments: theory vs. experiment



J. Bonnard *et al.*, Phys. Lett. B 843 (2023) 138014

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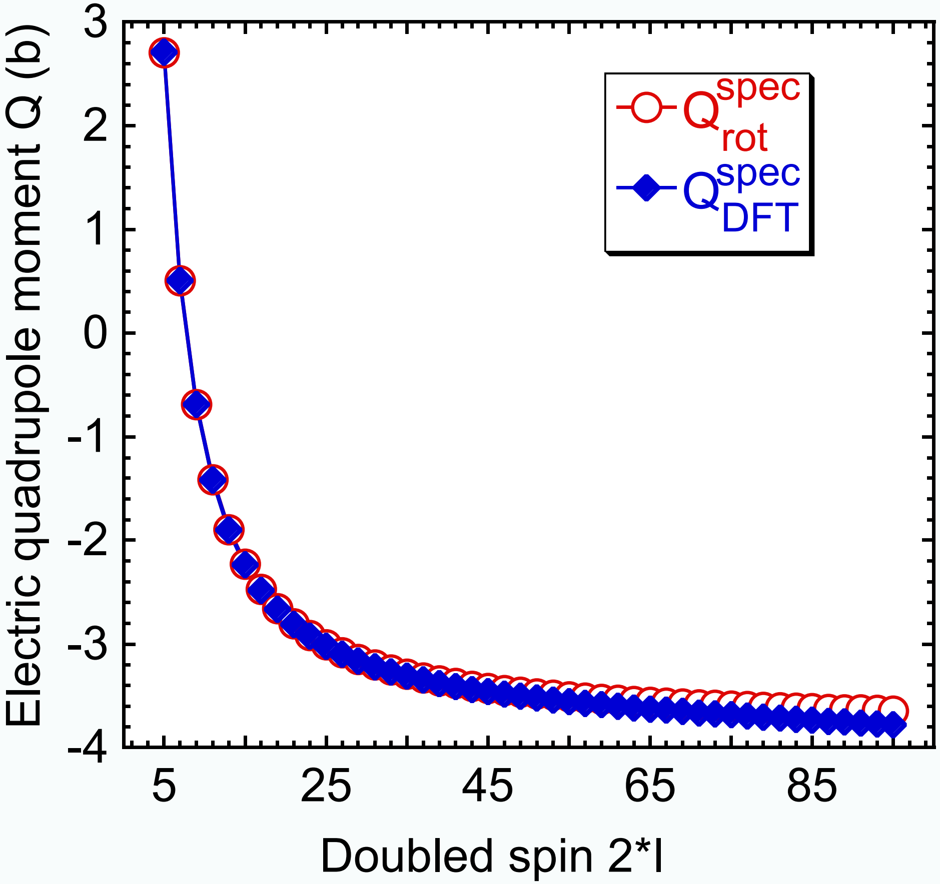
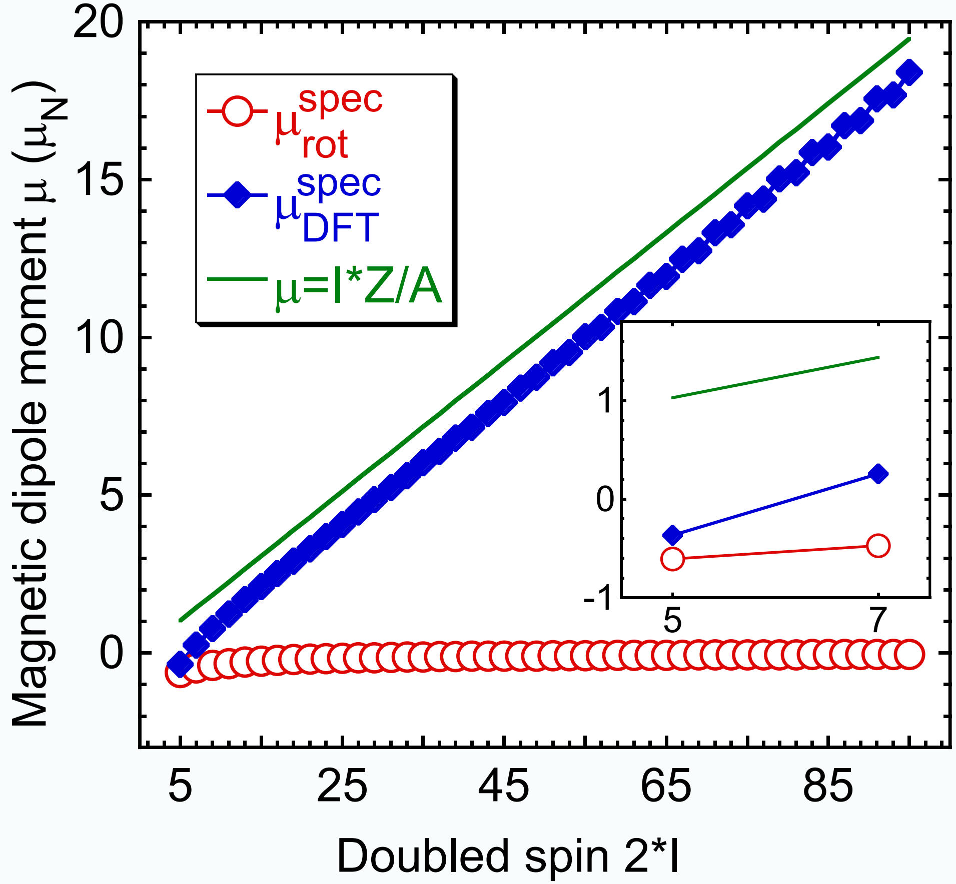
Electromagnetic moments – the rigid-rotor approximation

^{161}Dy 5/2$^+$ UNEDF1, g'_0=1.7

$$Q^{spec}_{rot} = Q^{intr}_{20} \times C^{II}_{II,20} \times C^{IK}_{IK,20} \qquad \mu^{spec}_{rot} = \mu^{intr}_{z} \times C^{II}_{II,10} \times C^{IK}_{IK,10}$$





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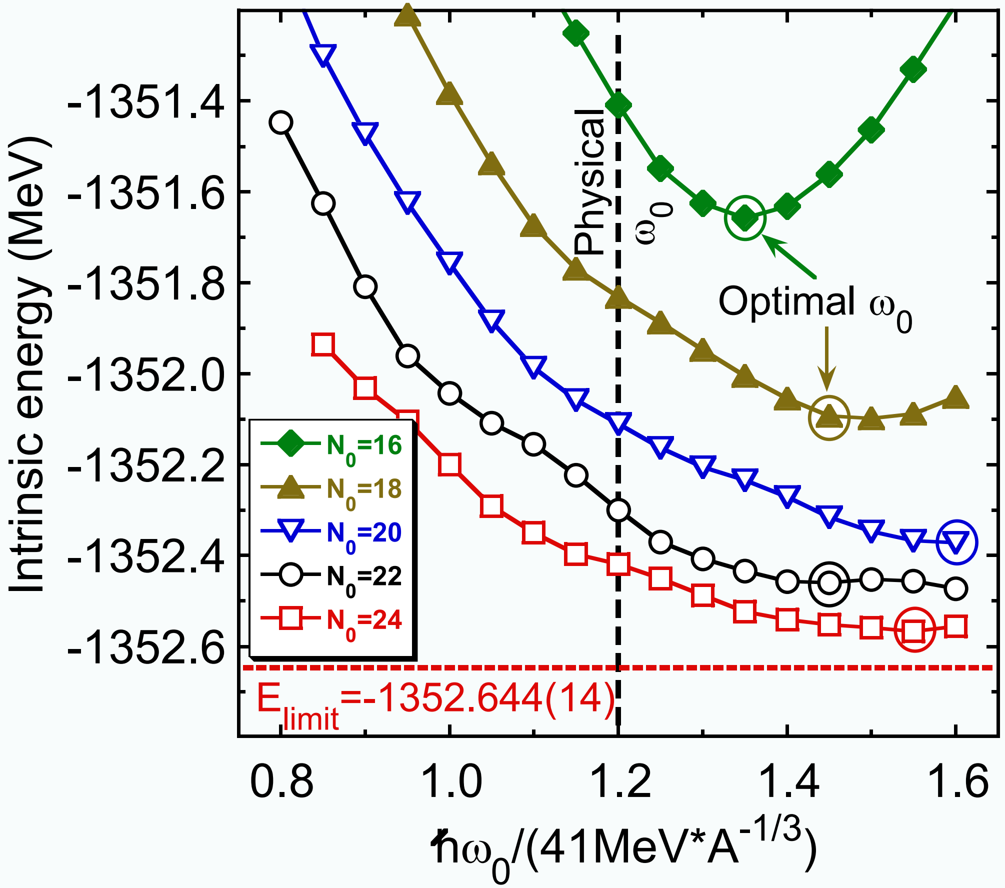
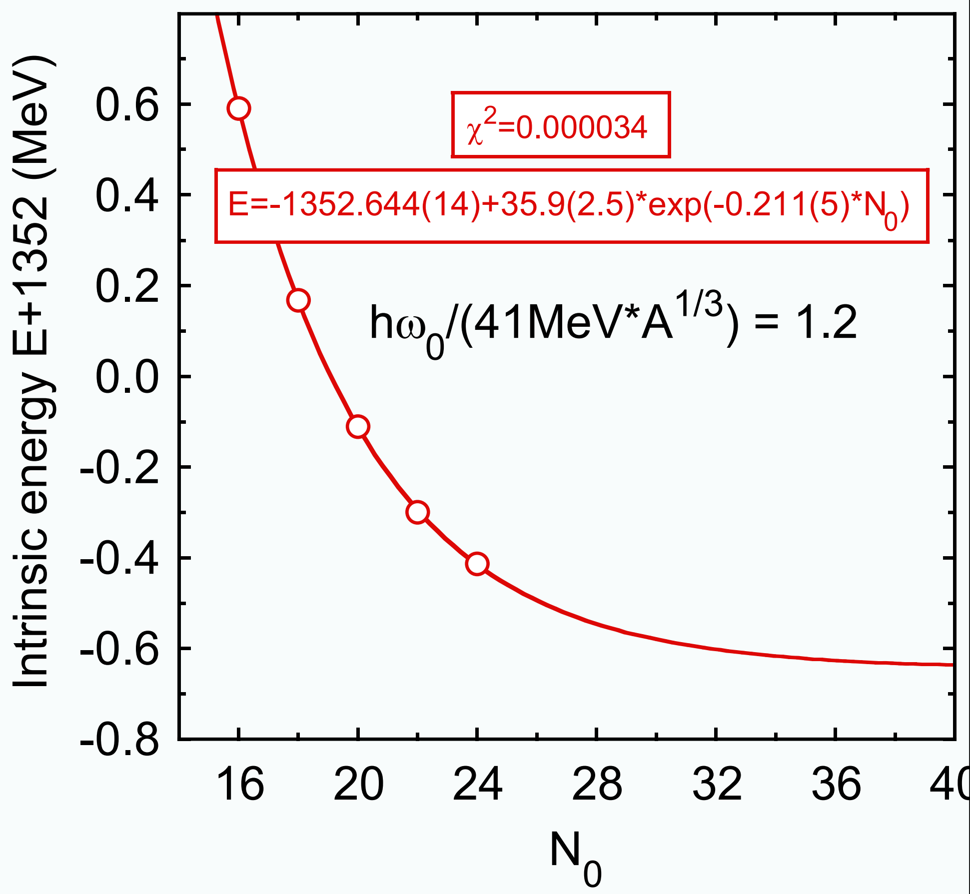
Convergence of the total HFB intrinsic energy

^{167}Ho 11/2$^-$, UNEDF1, g'_0=1.7

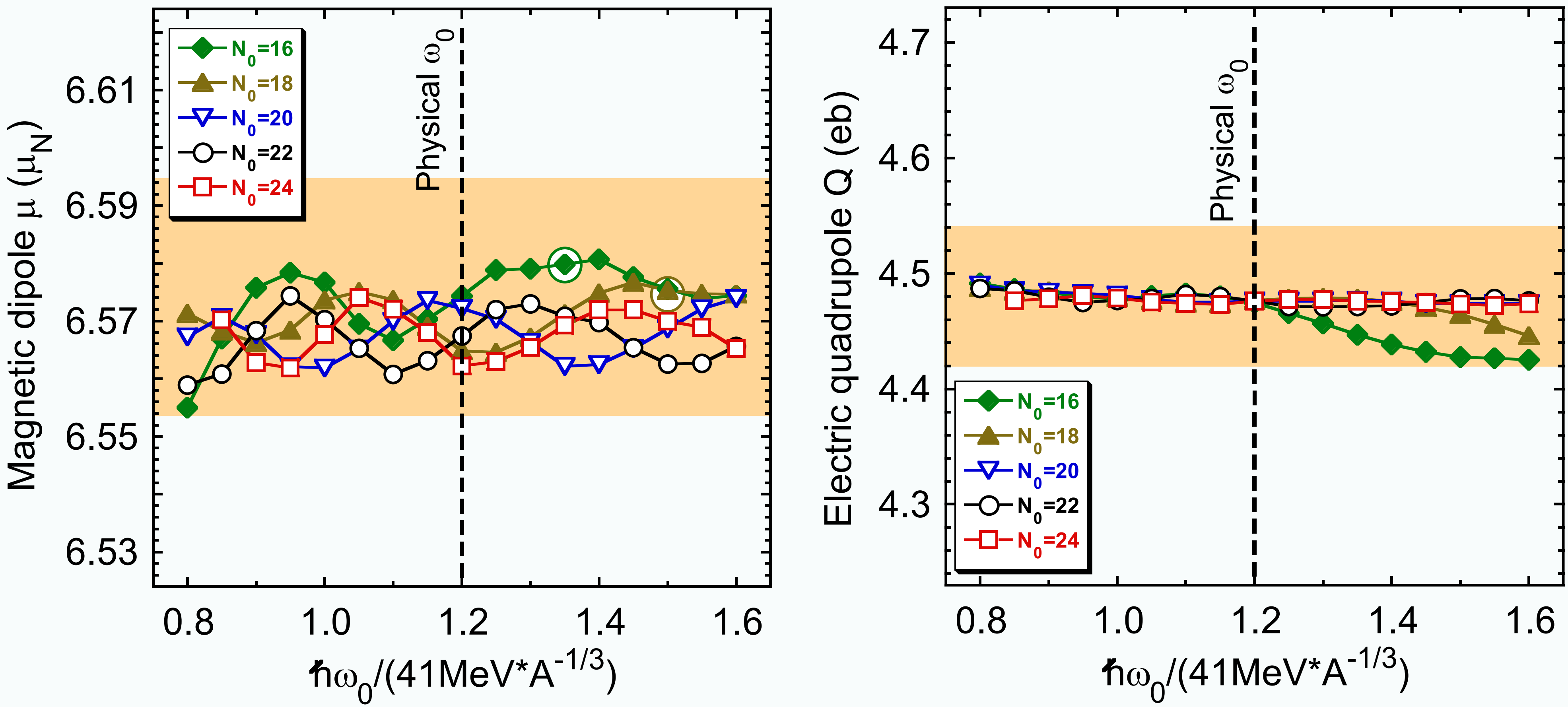




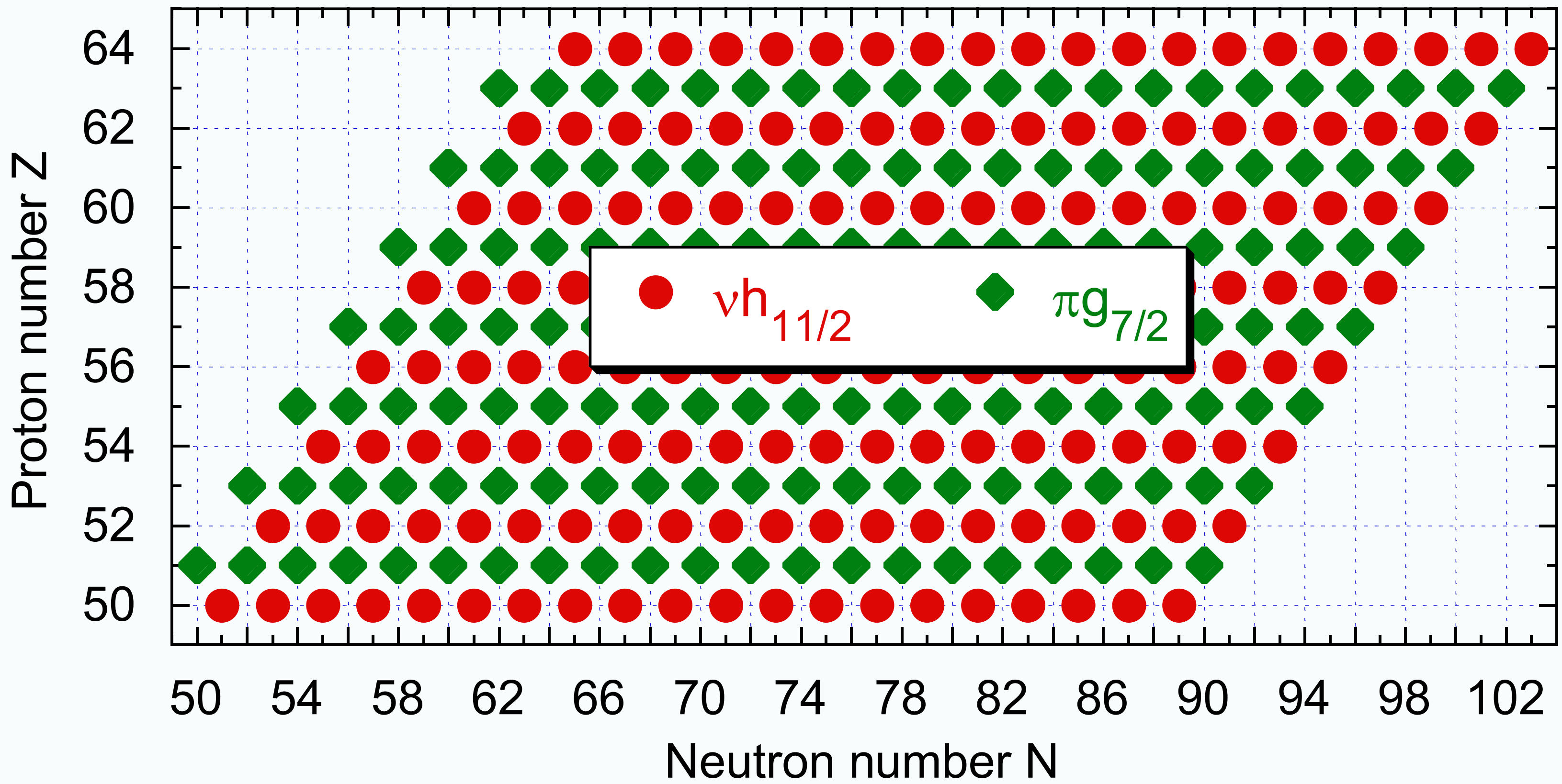
E_{exp}=-1357.77346

Convergence of the spectroscopic moments

^{167}Ho 11/2$^-$, UNEDF1, g'_0=1.7

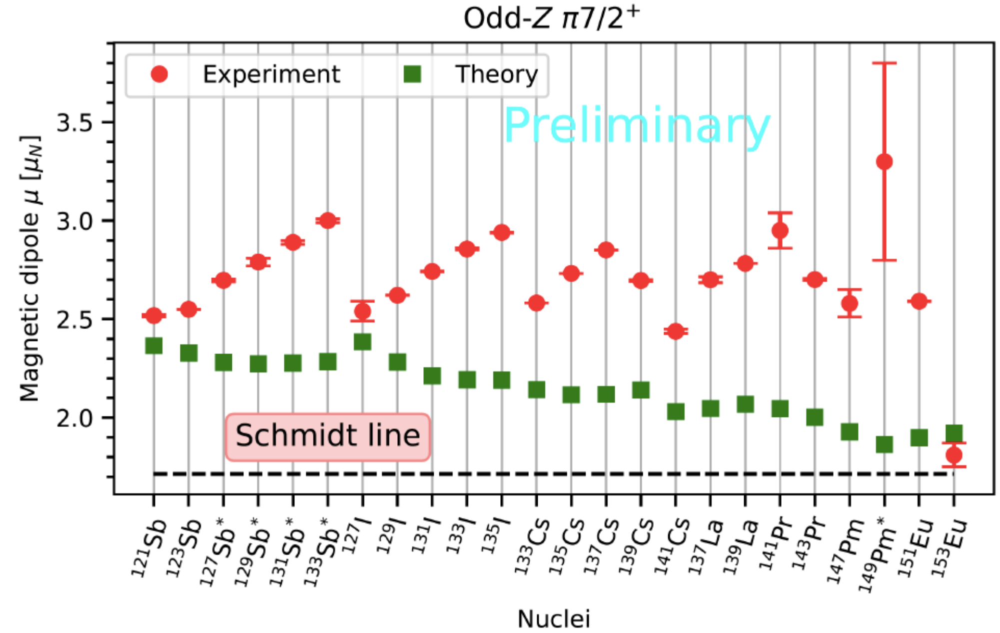


Nuclear-DFT analysis of electromagnetic moments between the Sn and Gd isotopes



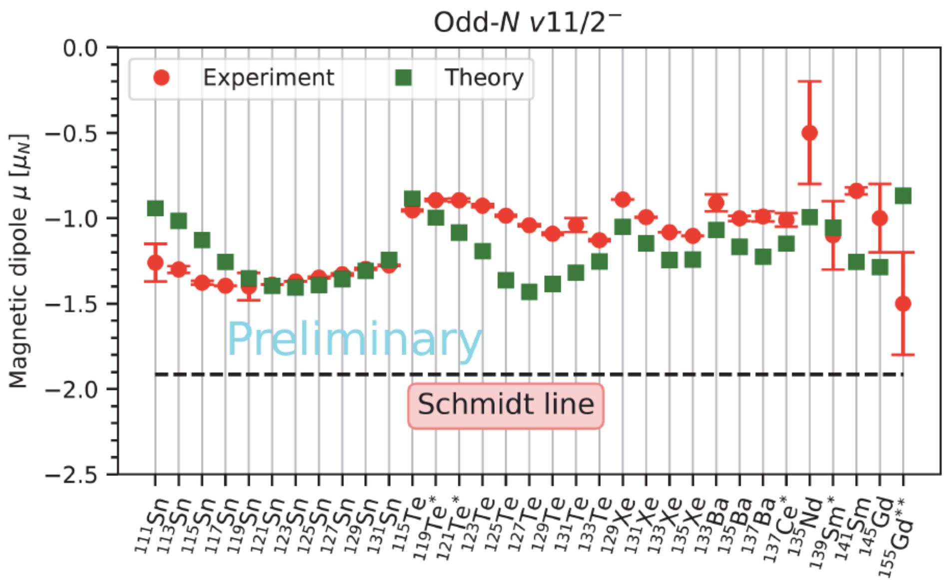
H. Wibowo *et al.*, to be published

Magnetic dipole moments: theory vs. experiment



N. J. Stone, *Table of nuclear magnetic dipole and electric quadrupole moments* (2014), INDC, report INDC(NDS)-0658

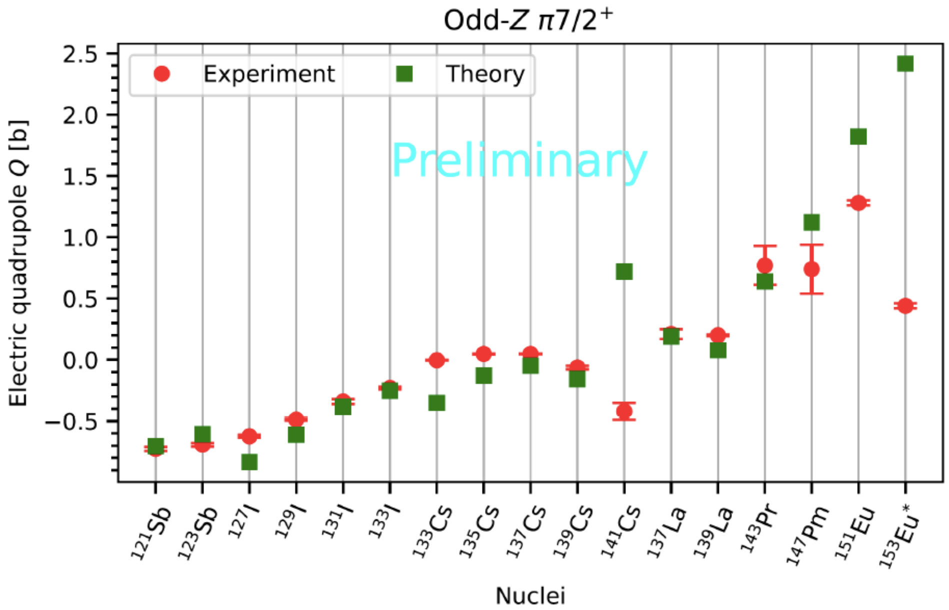
Schmidt lines represent the value of magnetic dipole moment of an odd-mass nucleus which is completely determined by the ℓ and j values of the unpaired nucleon (single-particle model).



Picture: courtesy H. Wibowo

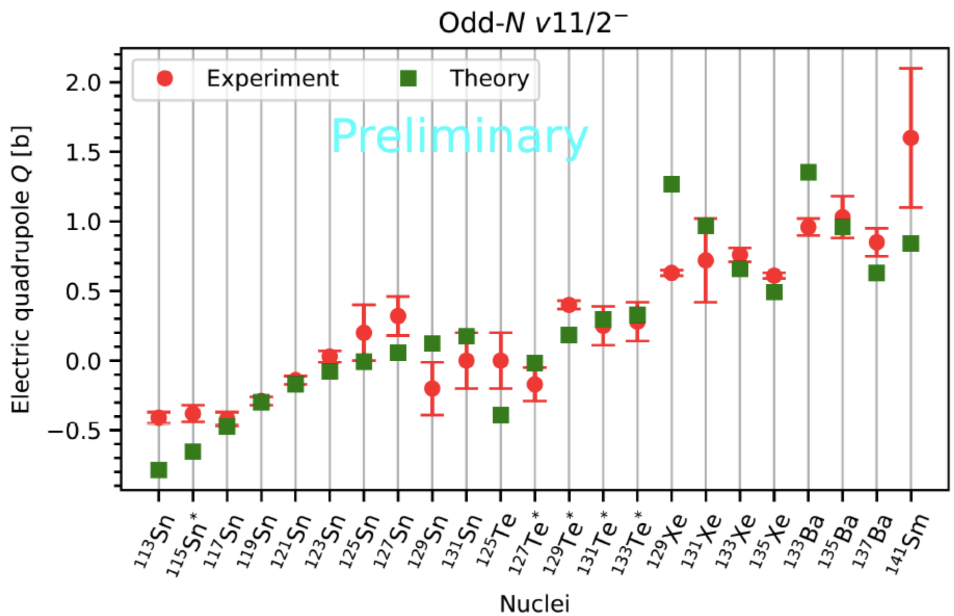
H. Wibowo *et al.*, to be published

Quadrupole moments: theory vs. experiment



N. J. Stone, *Table of nuclear magnetic dipole and electric quadrupole moments* (2014), INDC, report INDC(NDS)-0658

N. J. Stone, *Table of nuclear electric quadrupole moments, ADNDT 111-112*, 1 (2016)



Picture: courtesy H. Wibowo

H. Wibowo *et al.*, to be published

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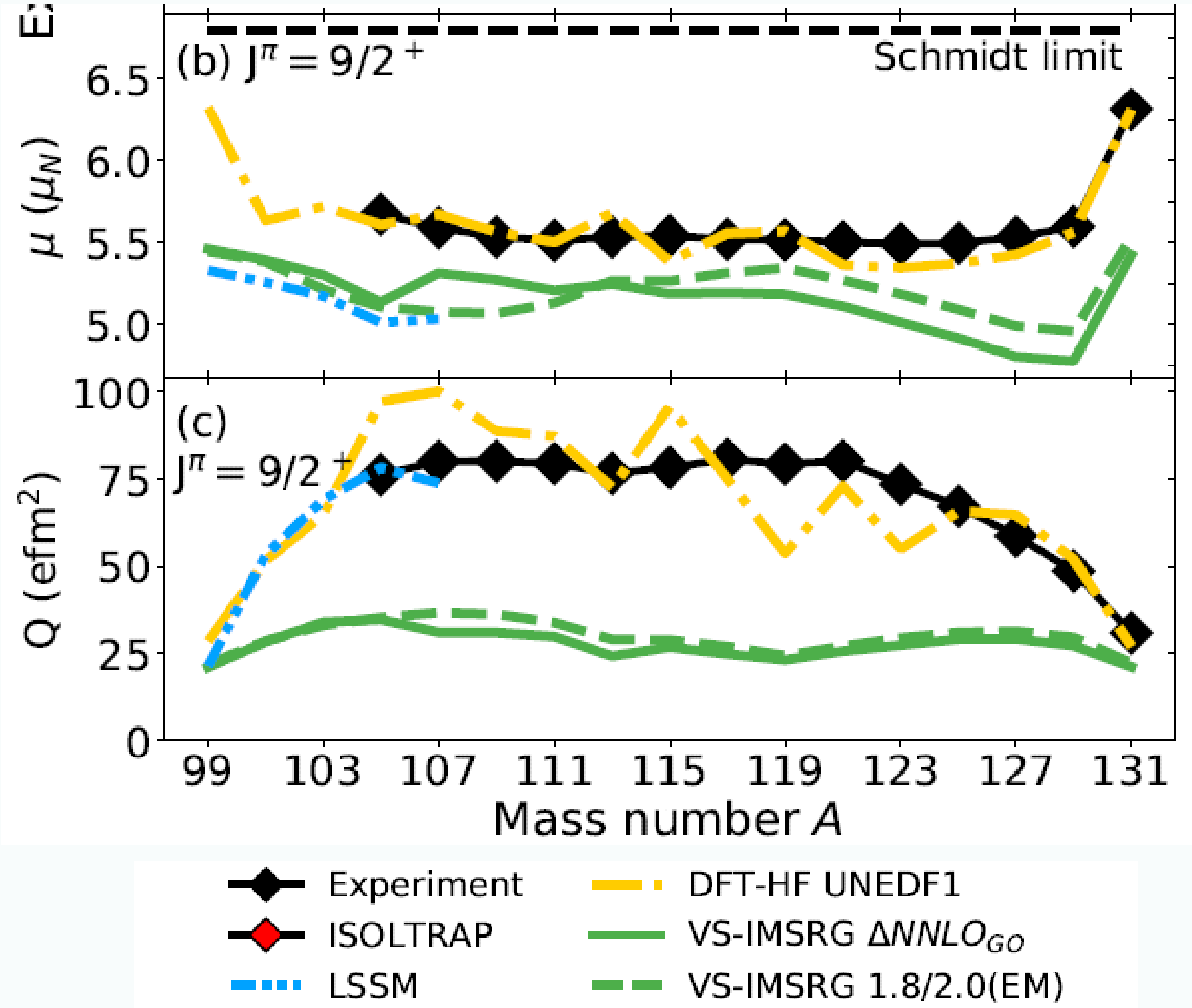
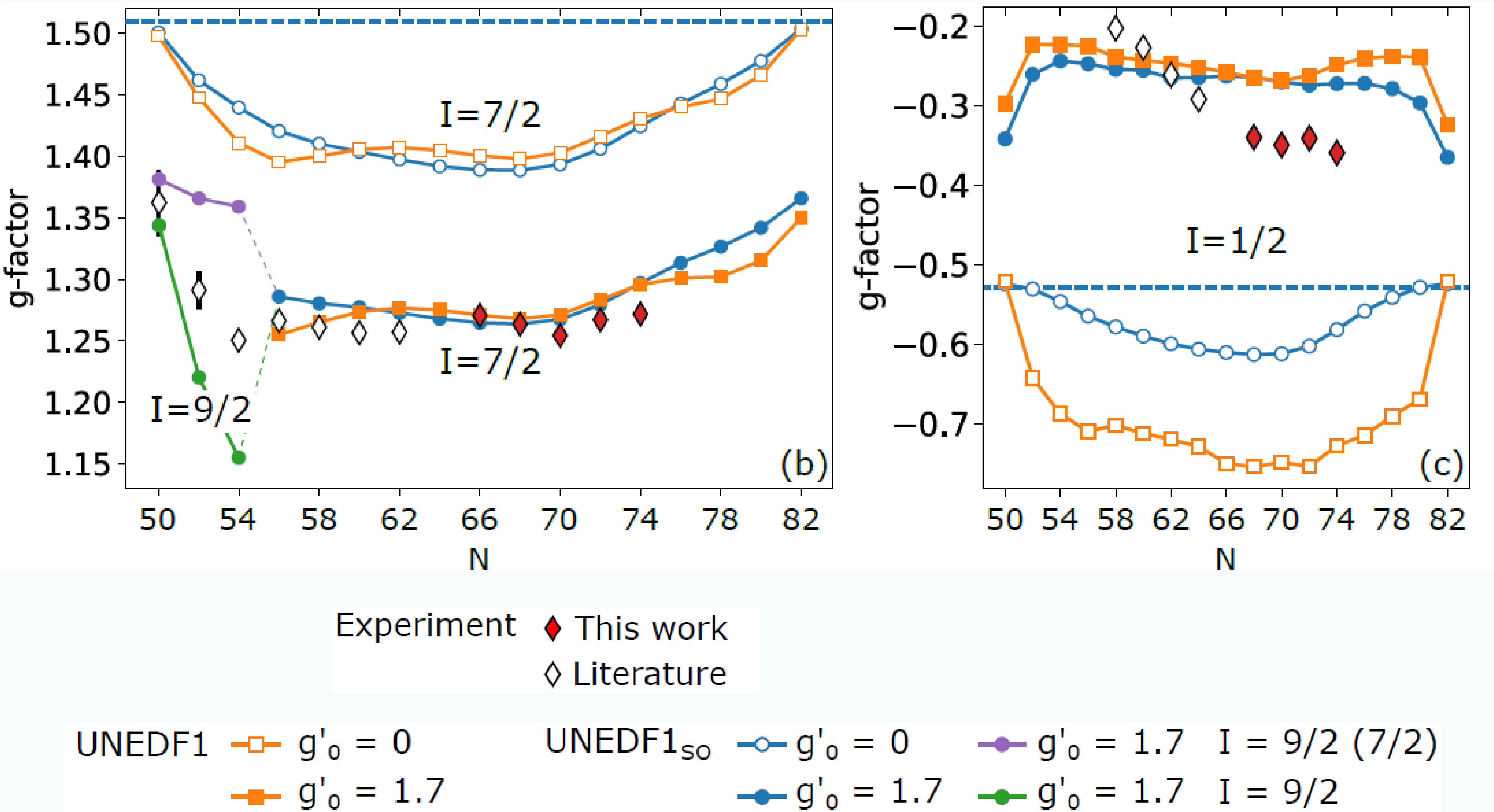
Moments of the 9/2 states in In



A.R. Vernon *et al.*, Nature 607, 260 (2022)

L. Nies *et al.*, Phys. Rev. Lett. 131, 022502 (2023)

Moments of the 1/2, 7/2 & 9/2 states in Ag



R. P. de Groote *et al.*, submitted to Phys. Lett. B

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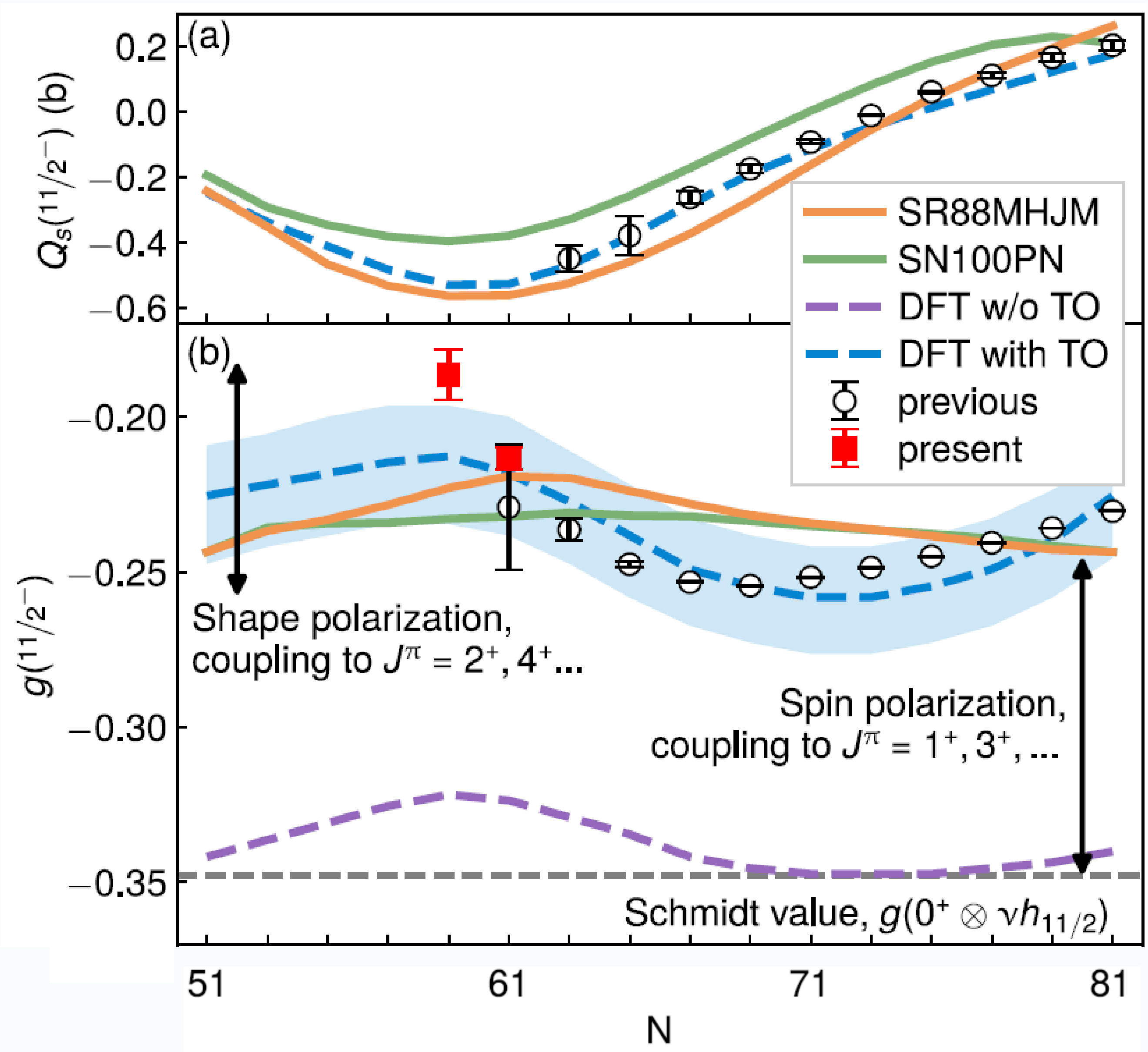
Moments of the $\nu h_{11/2}$ isomers in Sn

g = μ/I



T.J. Gray *et al.*, Phys. Lett. B 847 (2023) 138268

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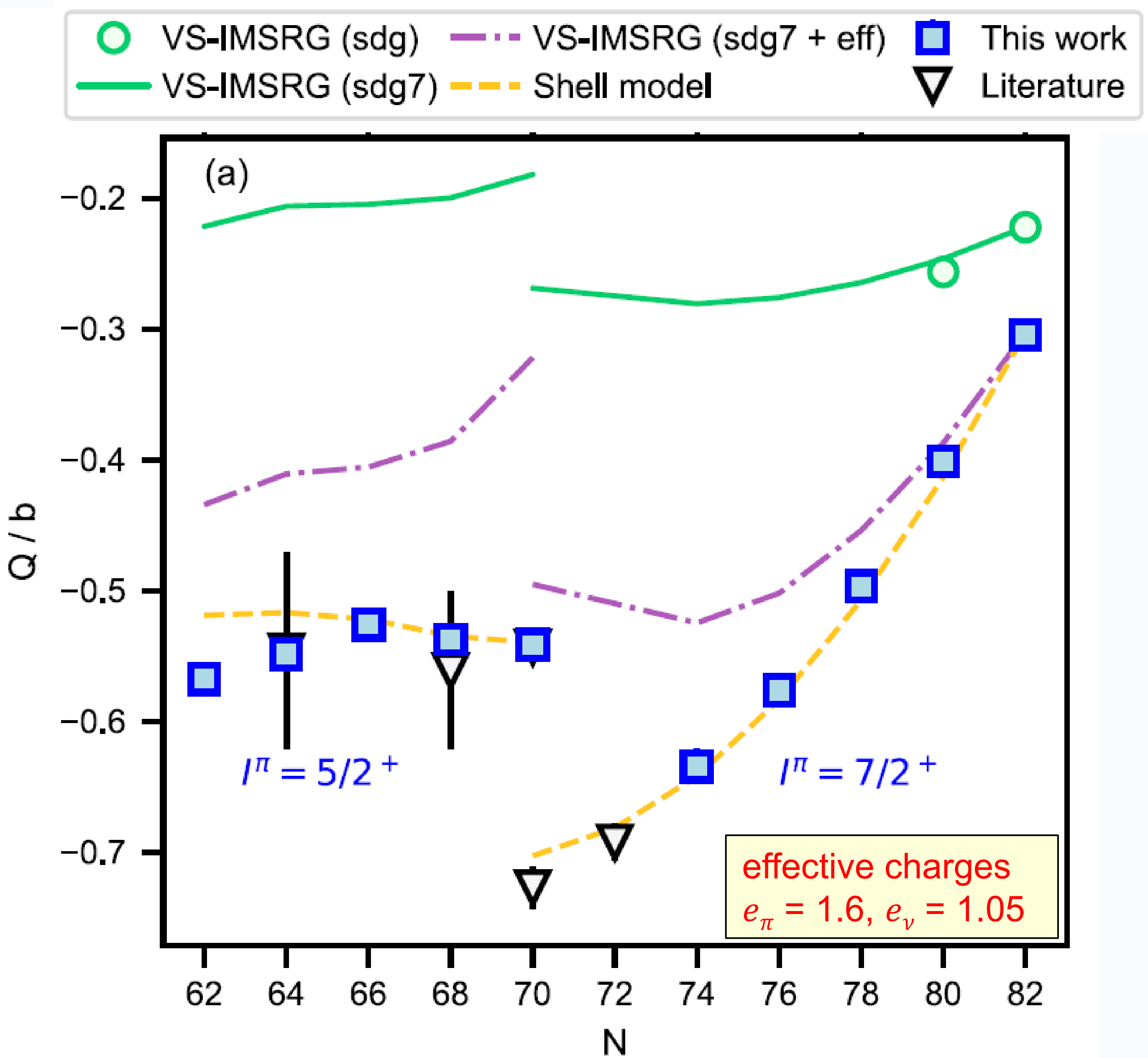
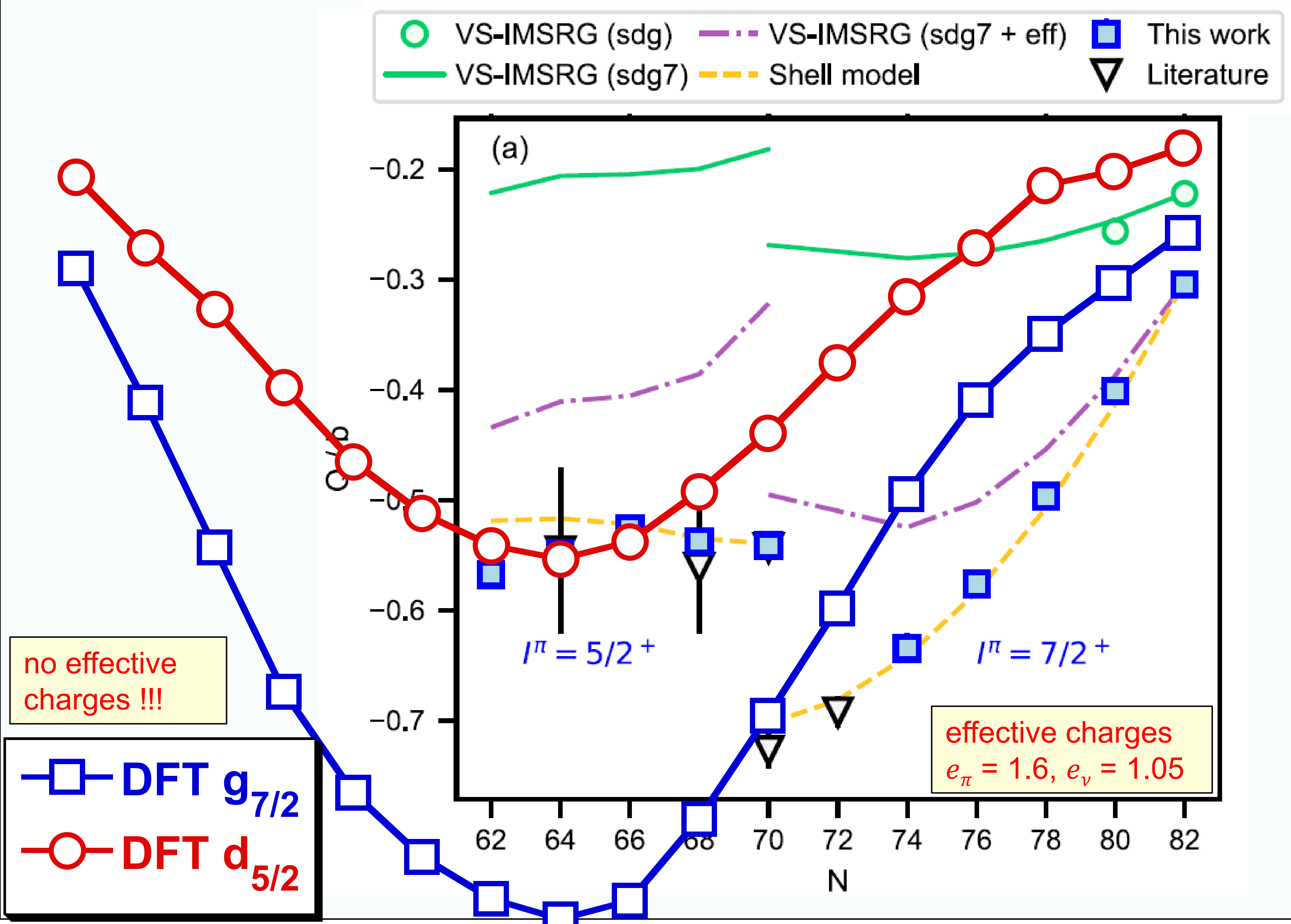
Quadrupole moments in Sb



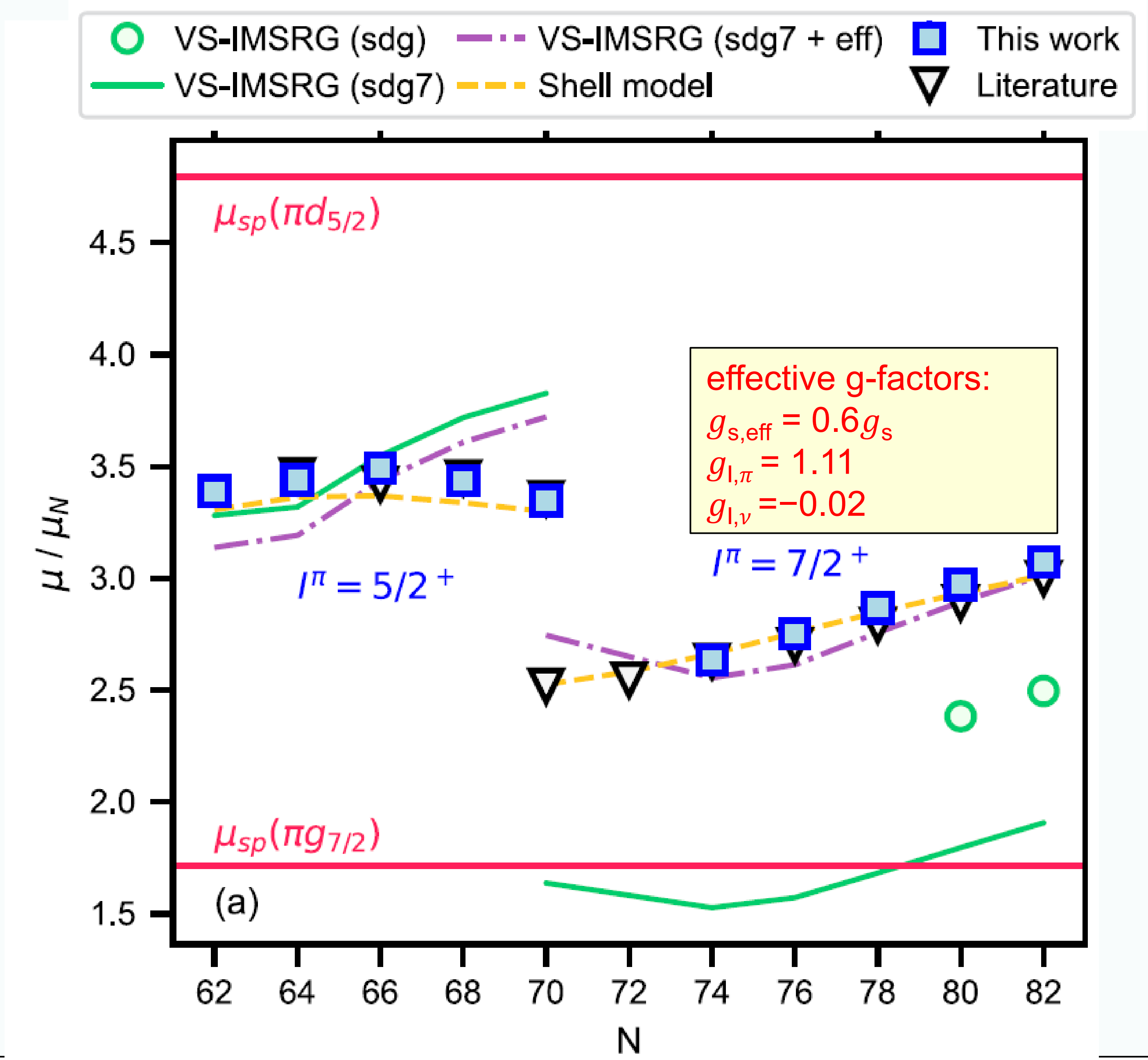
S. Lechner *et al.*, Phys. Lett. B 847 (2023) 138278

Quadrupole moments in Sb



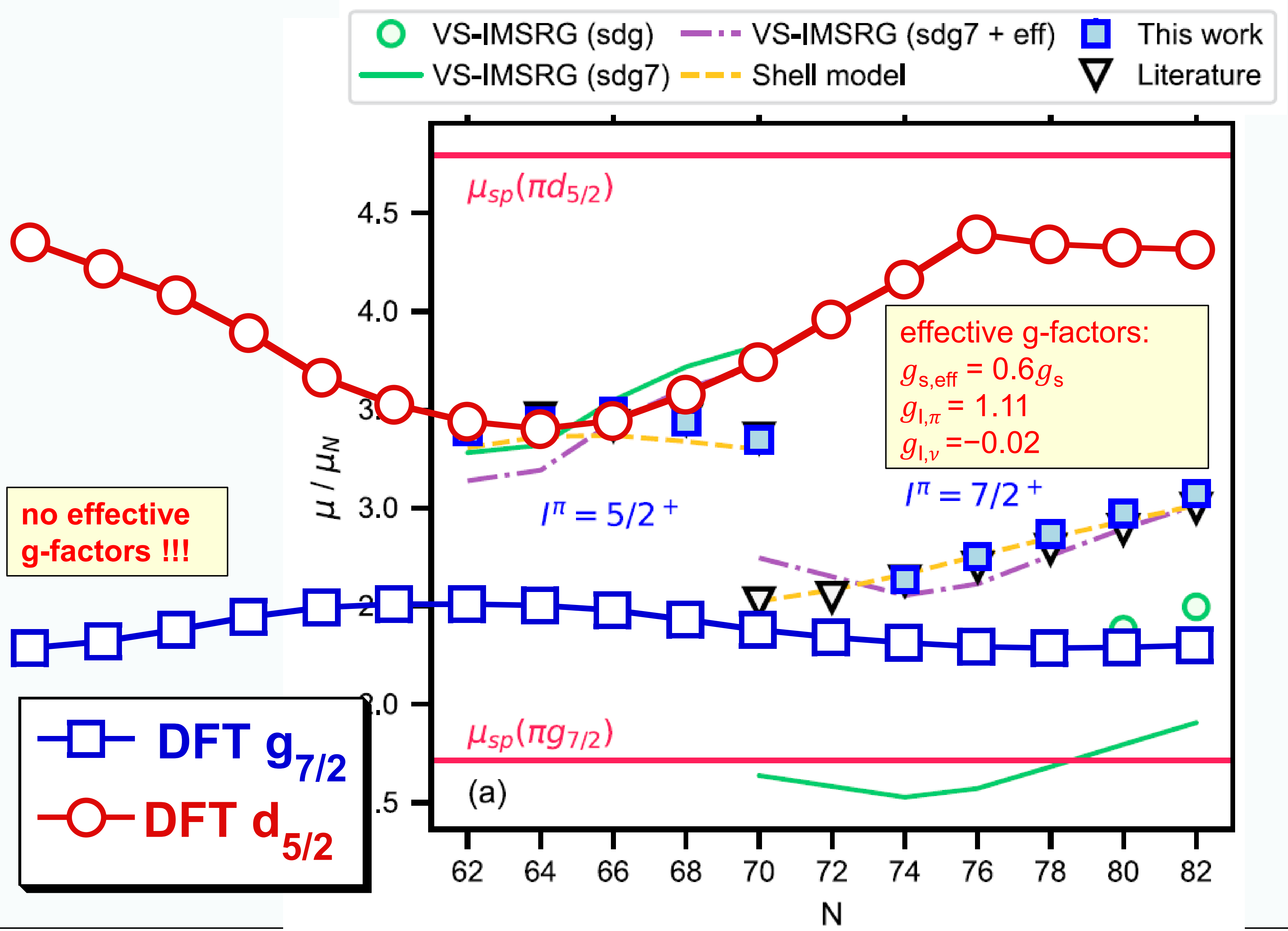
S. Lechner *et al.*, Phys. Lett. B 847 (2023) 138278

Magnetic dipole moments in Sb



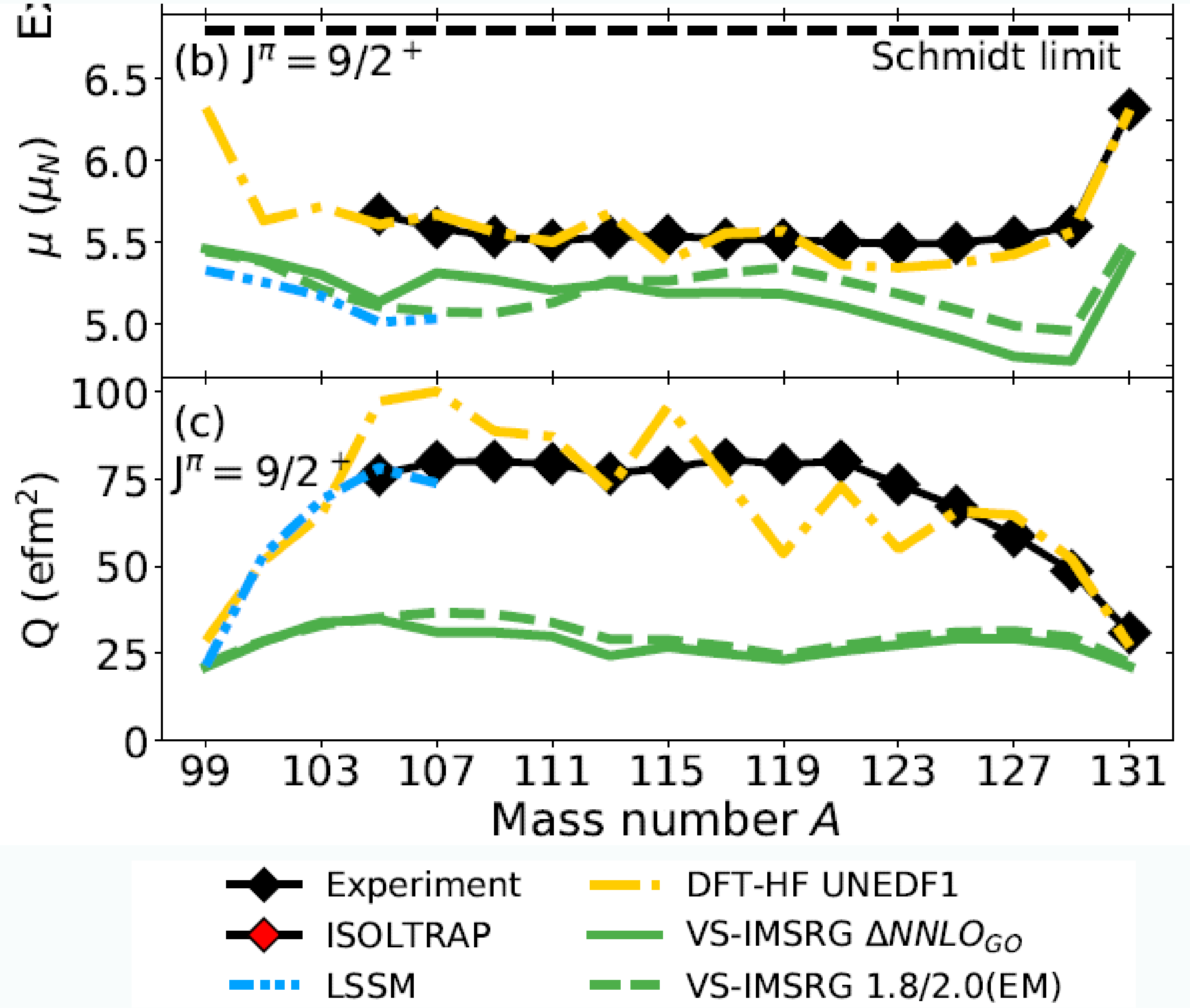
S. Lechner *et al.*, Phys. Lett. B 847 (2023) 138278

Magnetic dipole moments in Sb



S. Lechner *et al.*, Phys. Lett. B 847 (2023) 138278

Moments of the 9/2 states in In



A.R. Vernon *et al.*, Nature 607, 260 (2022)

L. Nies *et al.*, Phys. Rev. Lett. 131, 022502 (2023)

Two-body-current corrections to magnetic moments

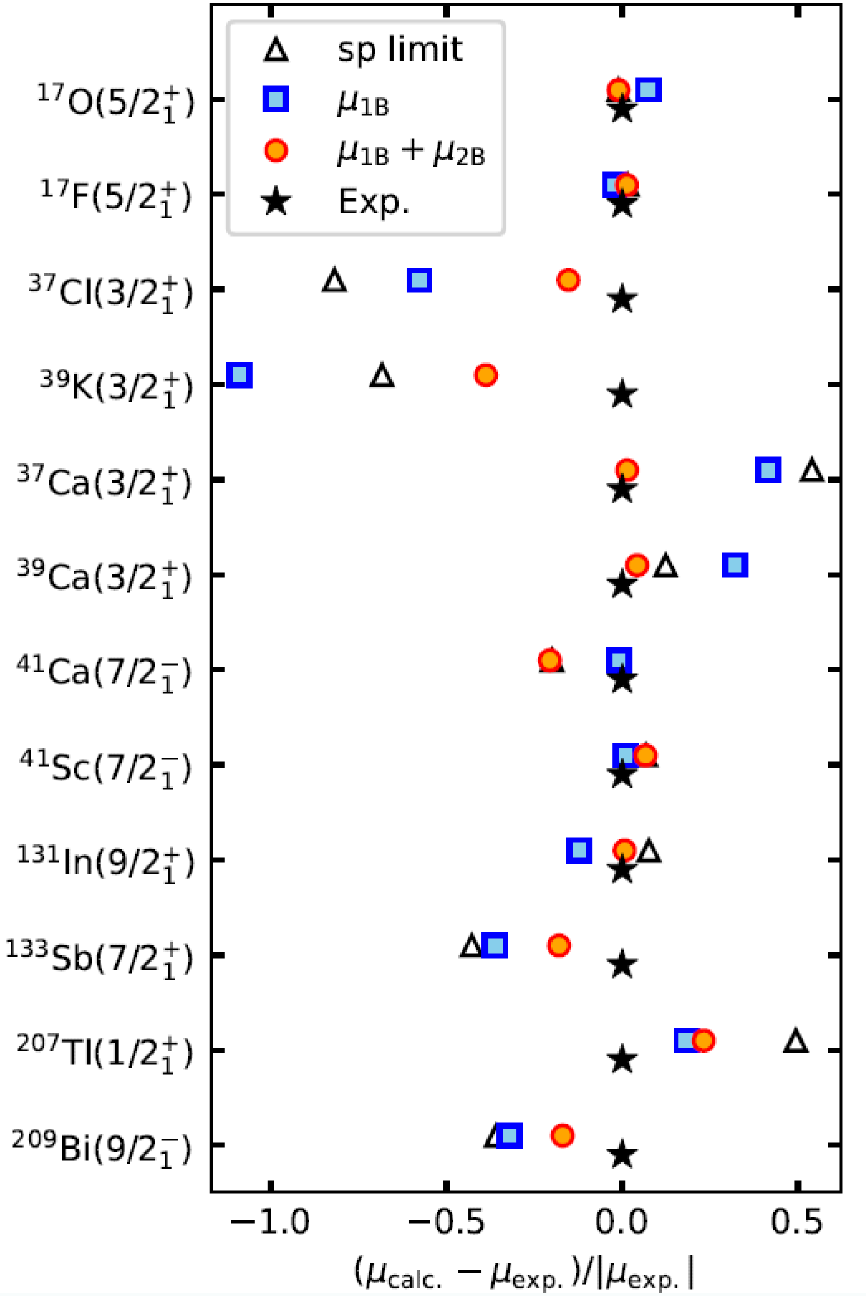


FIG. 1. Magnetic dipole moments of near doubly magic nuclei from $A = 17 - 209$ computed with the VS-IMSRG(2) relative to the experimental values. Results are shown at the one-body level, μ_{1B} (blue squares), and including 2BC, $\mu_{1B}+\mu_{2B}$ (red circles) based on the 1.8/2.0 (EM) NN+3N interactions. The experimental dipole moments (stars) are taken from Ref. [21, 35]. In addition, we show the simple single-particle (sp) limit (without many-body correlations and without 2BC).

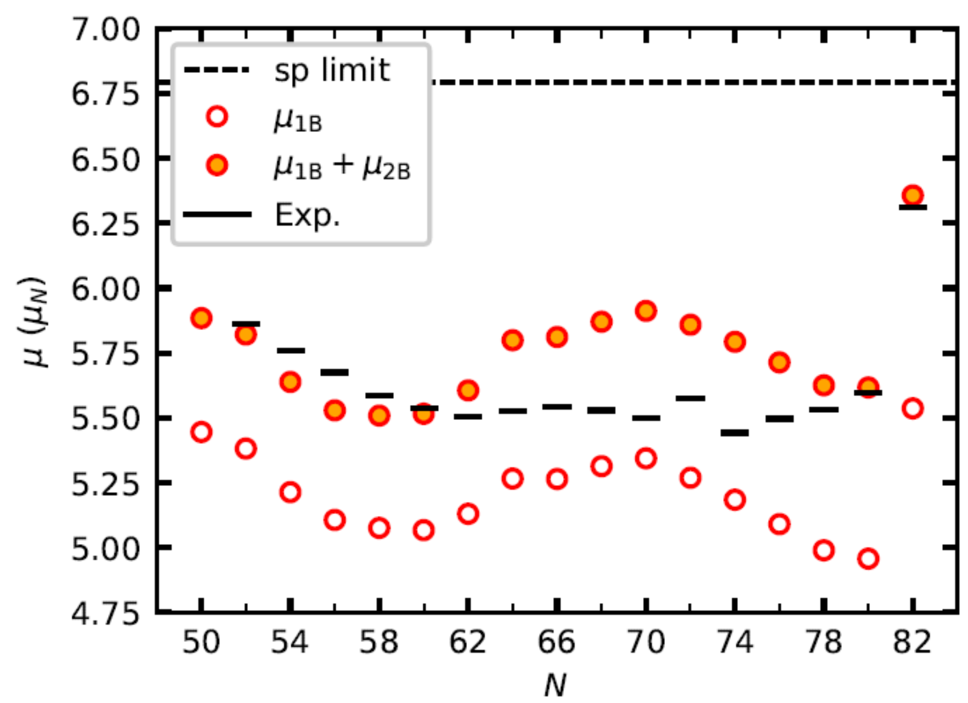


FIG. 4. Magnetic dipole moments of the $9/2^+$ ground state for the odd-mass indium isotopes isotopes computed with the VS-IMSRG(2) including 2BC, in comparison to experiment [21, 52].

T. Miyagi *et al.*, arXiv:2311.14383