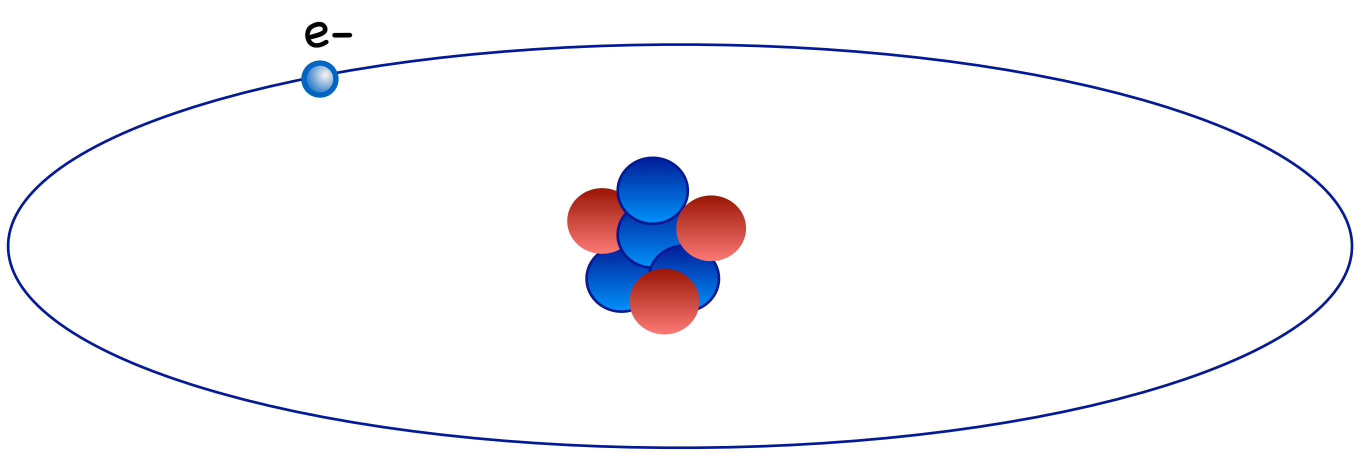
Nuclear electromagnetic structure in muonic atoms

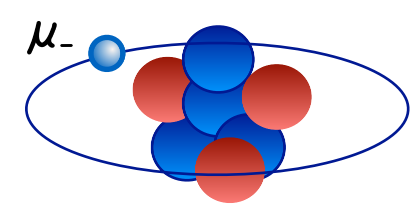
Sonia Bacca

Hydrogen-like muonic atoms

Ordinary atoms



Muonic atoms



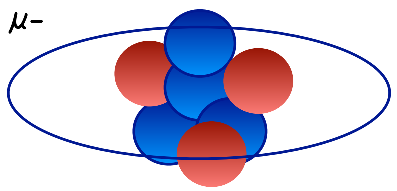
muon more sensitive to the nucleus

Can be used as a precision probe for the nucleus

- To extract charge radii from 2P-2S or 2P-1S data
- To extract Zemach radii from hyperfine splitting data

Extracting the charge radius

Strong experimental program at PSI to extract the nuclear charge radius from the Lamb shift measurement (2S-2P) in **muonic atoms**



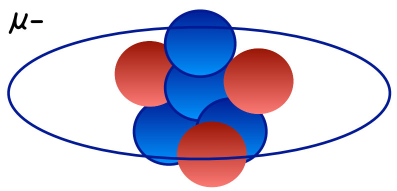
$$E_{2S-2P} = \delta_{\rm QED} + \frac{m_r^4 (Z\alpha)^4}{12} \langle r_c^2 \rangle + \delta_{\rm TPE}$$

what is measured

what you want to extract

Extracting the charge radius

Strong experimental program at PSI to extract the nuclear charge radius from the Lamb shift measurement (2S-2P) in **muonic atoms**



$$E_{2S-2P} = \delta_{\rm QED} + \frac{m_r^4 (Z\alpha)^4}{12} \langle r_c^2 \rangle + \delta_{\rm TPE}$$

well known | not well known

- μD $\longrightarrow$ results released in 2016
- μ^4He$^+$ $\longrightarrow$ results released in 2021
- μ^3He$^+$ $\longrightarrow$ results released in 2023

CREMA Collaboration

A matter of precision

The uncertainty of the extracted radius depends on the precision of the TPE

$$E_{2S-2P} = \delta_{\rm QED} + \mathcal{A}_{\rm OPE}\langle r_c^2\rangle + \delta_{\rm TPE}$$

Even though, roughly: 95% 4% 1%

The uncertainty on TPE exceeds the experimental precision, hence reducing uncertainties is important

Uncertainties comparison (as of 2013)

System	ΔE_{2S-2P}	$\Delta\delta_{TPE}$
μ^2H	0.003 meV	0.03 meV
μ^3He^+	0.08 meV	1 meV
μ^4He^+	0.06 meV	0.6 meV

A matter of precision

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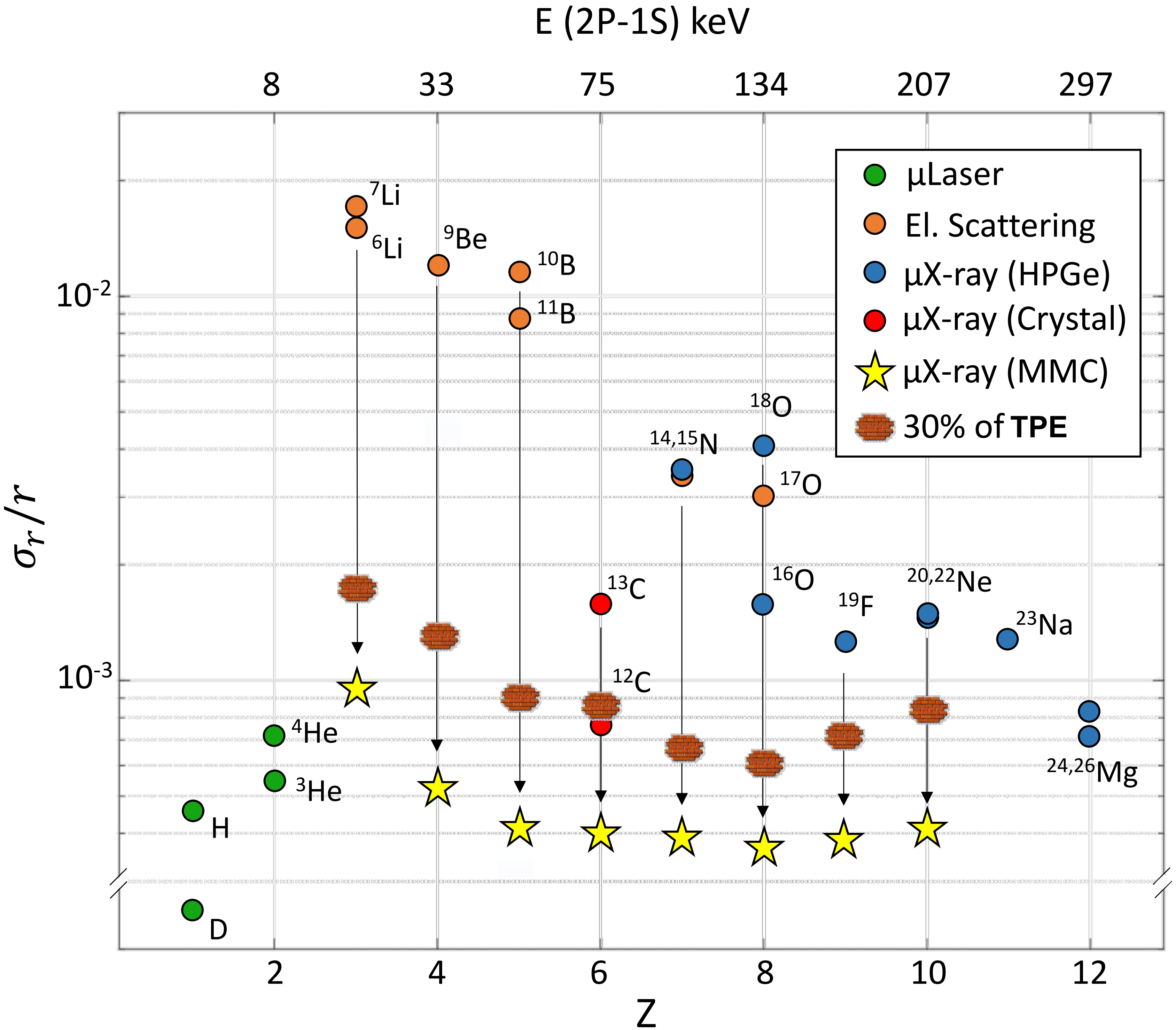
Uncertainties comparison (as of 2026)

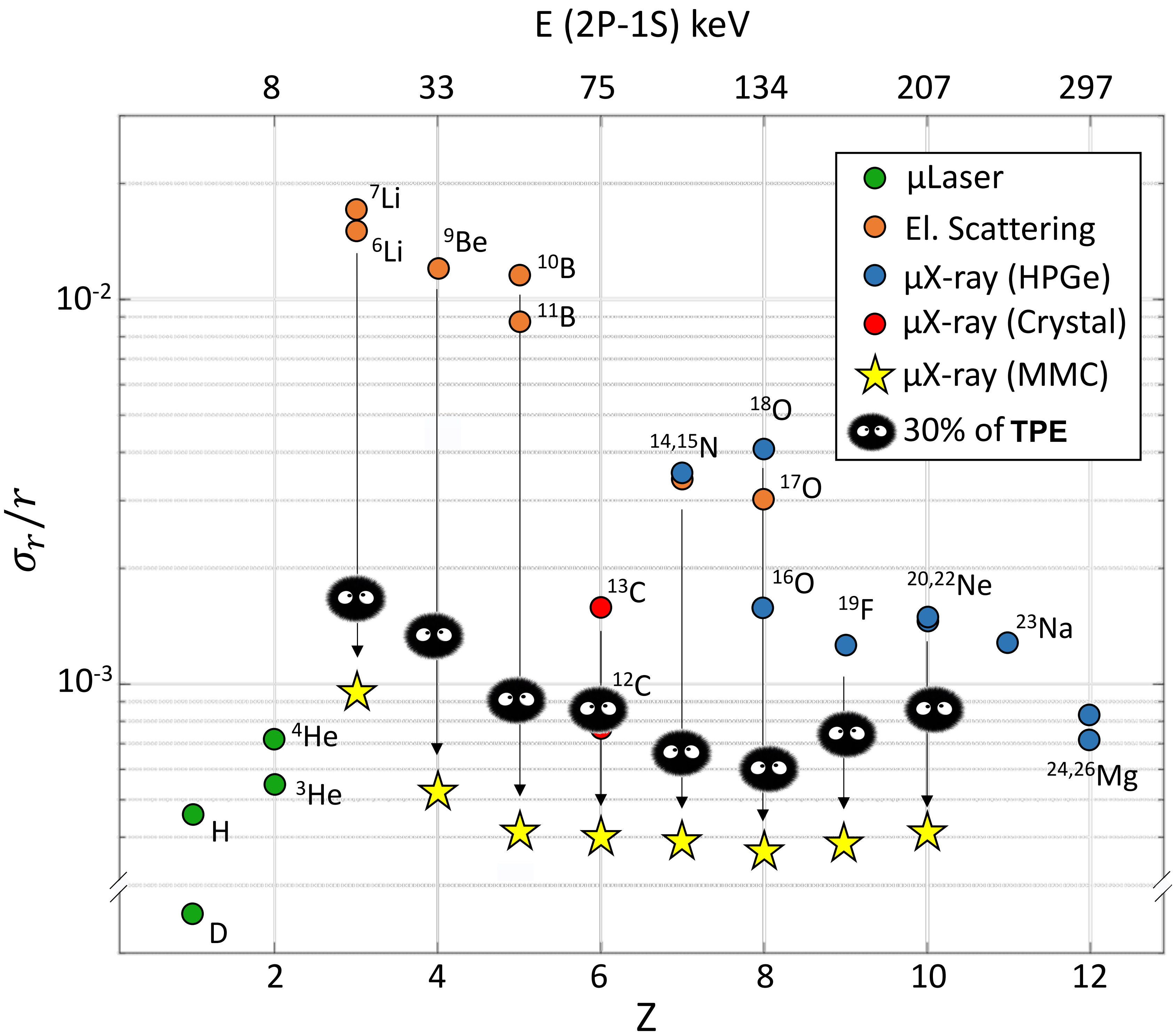
System	ΔE_{2S-2P}	$\Delta\delta_{TPE}$
μ^2H	0.003 meV	0.02 meV
μ^3He^+	0.08 meV	0.3 meV
μ^4He^+	0.06 meV	0.4 meV

Extracting the charge radius



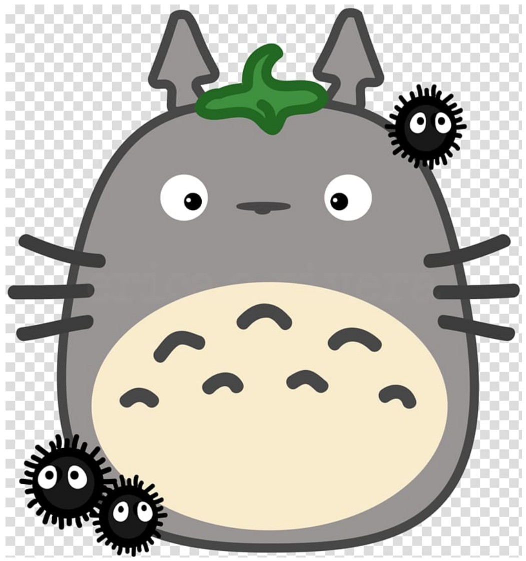
QUARTET Collaboration: curtesy of Ben Ohayon

Extracting the charge radius

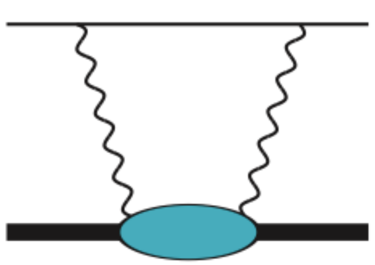


QUARTET Collaboration: curtesy of Ben Ohayon

Susuwatari/Totoro

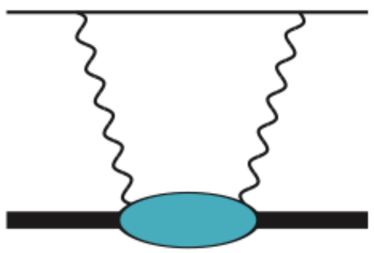


Two-photon exchange (TPE)



$$\delta_{\rm TPE} = \underbrace{\delta^A_{\rm Zem} + \delta^N_{\rm Zem}}_{\text{Elastic terms}} + \underbrace{\delta^A_{\rm pol} + \delta^N_{\rm pol}}_{\text{Inelastic terms}}$$

Two-photon exchange (TPE)



$$\delta_{\text{TPE}} = \delta^{A}_{\text{Zem}} + \delta^{N}_{\text{Zem}} + \delta^{A}_{\text{pol}} + \delta^{N}_{\text{pol}}$$

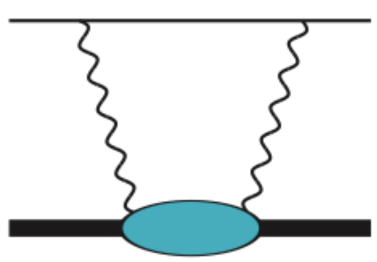
A: Few-nucleon terms

N: Single-nucleon terms

Inelastic terms (cfr. Pachucki)

Leading non-relativistic term

Take non-relativistic kinetic energy in muon propagator neglecting the Coulomb force in the intermediate state



$$P \simeq \frac{m_r^3 (Z\alpha)^5}{12} \sqrt{\frac{2m_r}{\omega}} \left[|\boldsymbol{R}-\boldsymbol{R}'|^2 - \frac{\sqrt{2m_r\omega}}{4} |\boldsymbol{R}-\boldsymbol{R}'|^3 + \frac{m_r\omega}{10} |\boldsymbol{R}-\boldsymbol{R}'|^4 \right]$$

η expansion $\quad \delta^{(0)} \quad \delta^{(1)} \quad \delta^{(2)}$

★ $|\boldsymbol{R}-\boldsymbol{R}'|$ "virtual" distance traveled by the proton between the two-photon exchange

★ Uncertainty principle $|\boldsymbol{R}-\boldsymbol{R}'| \sim \frac{1}{\sqrt{2m_N\omega}}$

★ $\eta = \sqrt{2m_r\omega}|\boldsymbol{R}-\boldsymbol{R}'| \sim \sqrt{\frac{m_r}{m_N}} \sim 1/3$

expand the muon matrix elements in powers of η up to the second order

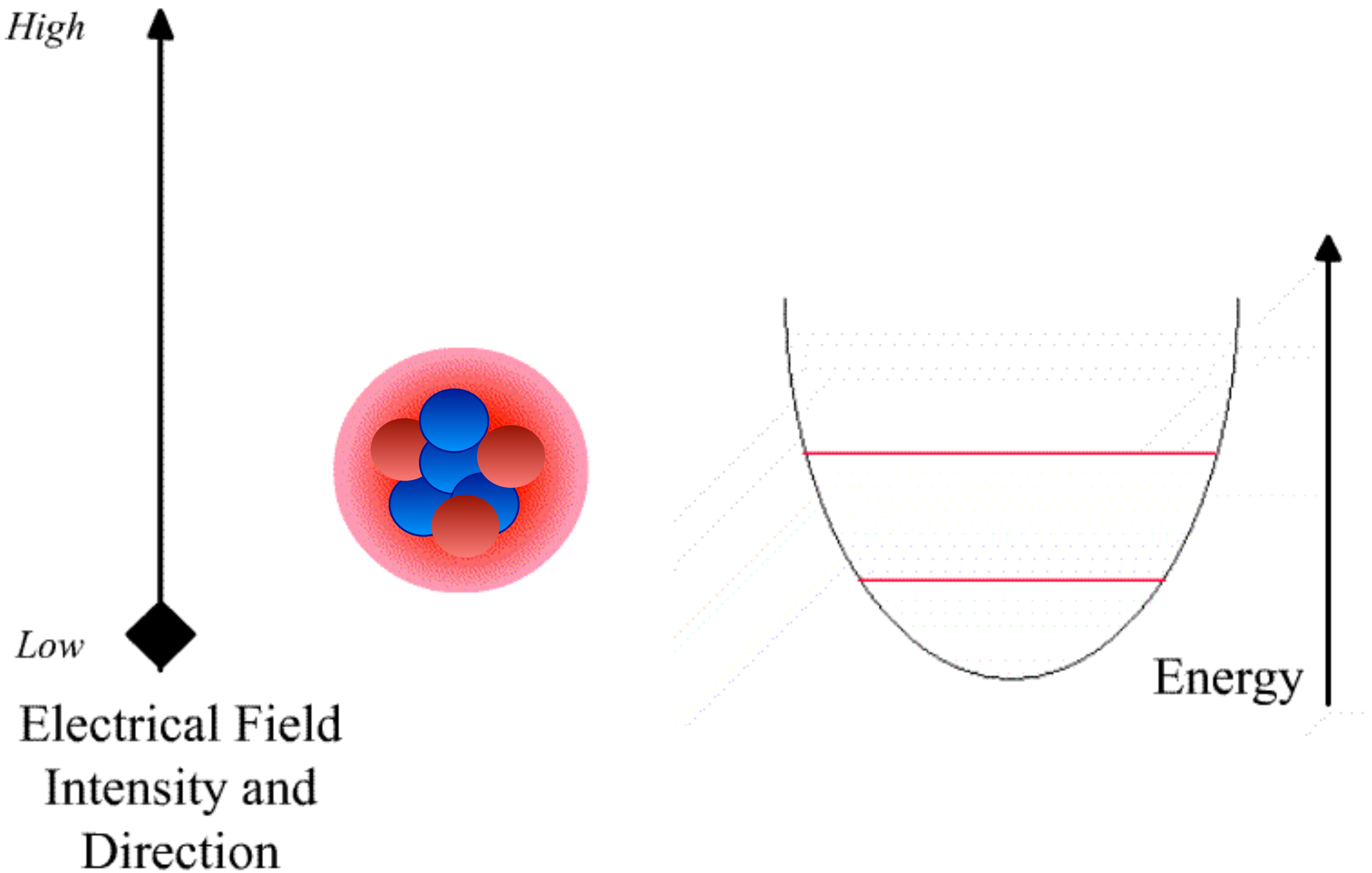
Example of muonic ^{4}He

Ji, Nevo Dinur, S.B. and Barnea, PRL **111**, 143402 (2013)

[meV]	AV18/UIX	χEFT*
$\delta^{(0)}$	-3.743	-3.981
$\delta^{(1)}$	0.741	0.809
$\delta^{(2)}$	0.077	0.101
δ_{NS}	0.517	0.530
δ_{pol}	-2.408	-2.542

Polarizability

$$\delta^A_{pol} = \delta^{(0)} + \delta^{(1)} + \delta^{(2)} + \delta_{NS} + \dots$$



Curtesy of V. Pascalutsa

What about uncertainties?

C.Ji et al., JPG: Part. Nucl. **45,** 093002 (2018)

Relative % error

	μ^2H			μ^3H			μ^3He$^+$			μ^4He$^+$		
	δ^A_{pol}	δ^A_{Zem}	δ^A_{TPE}	δ^A_{pol}	δ^A_{Zem}	δ^A_{TPE}	δ^A_{pol}	δ^A_{Zem}	δ^A_{TPE}	δ^A_{pol}	δ^A_{Zem}	δ^A_{TPE}
Numerical	0.0	0.0	0.0	0.1	0.0	0.1	0.4	0.1	0.1	0.4	0.3	0.4
Nuclear-model	0.3	0.5	0.4	1.3	2.4	1.7	0.7	1.8	1.5	3.9	4.6	4.4
ISB	0.2	0.2	0.2	0.7	0.2	0.5	1.8	0.2	0.5	2.2	0.5	0.5
Nucleon-size	0.3	0.8	0.0	0.6	0.9	0.2	1.2	1.3	0.9	2.7	2.0	1.2
Relativistic	0.0	—	0.0	0.1	—	0.1	0.4	—	0.1	0.1	—	0.0
Coulomb	0.4	—	0.3	0.5	—	0.3	3.0	—	0.9	0.4	—	0.1
η-expansion	0.4	—	0.3	1.3	—	0.9	1.1	—	0.3	0.8	—	0.2
Higher $Z\alpha$	0.7	—	0.5	0.7	—	0.5	1.5	—	0.4	1.5	—	0.4
Total	1.0	0.9	0.8	2.3	2.2	2.0	4.2	2.2	2.1	5.5	5.1	4.6

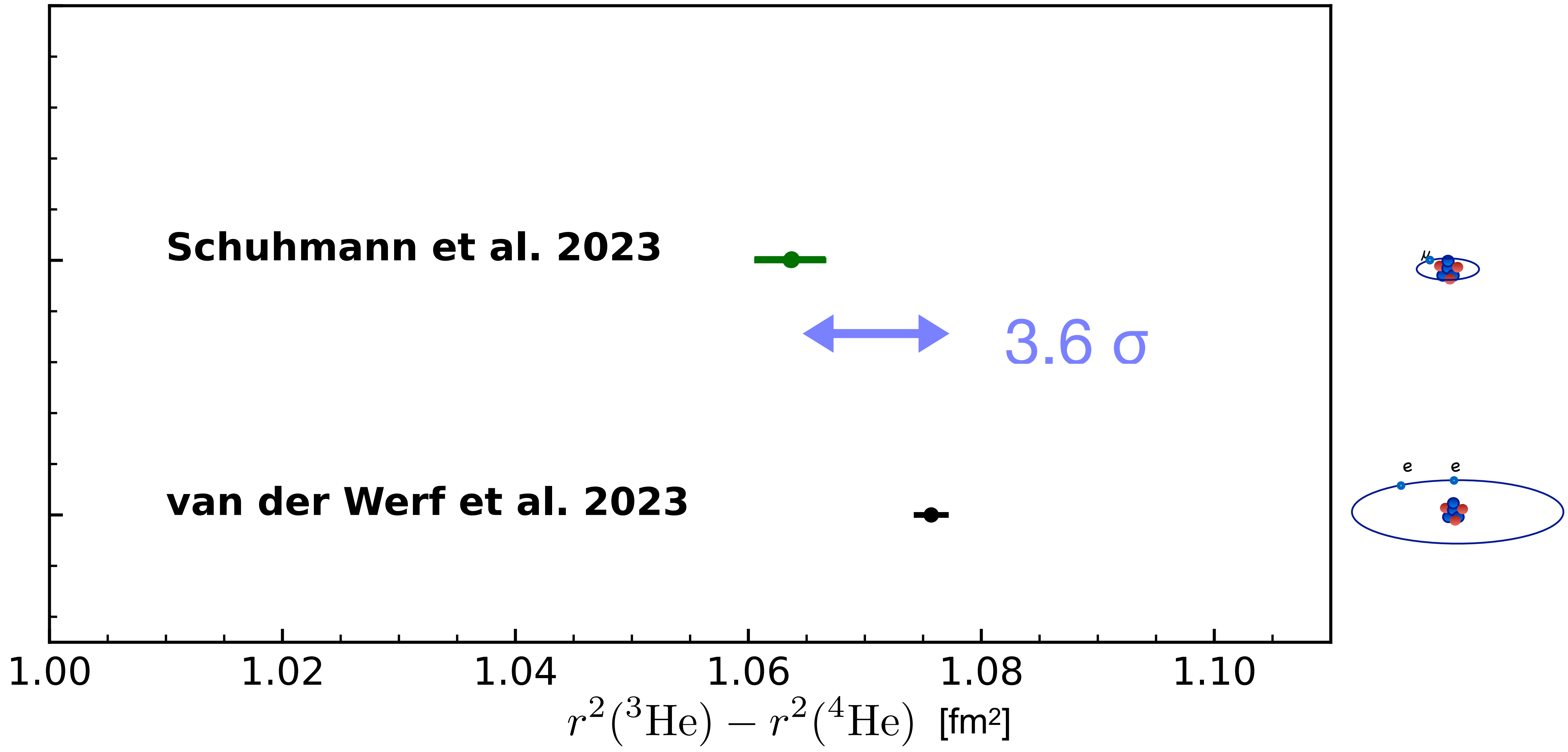
Nuclear model error is dominating for ^{3,4}He, but we had not performed yet an order-by-order analysis in chiral EFT

Recent progress

- Bayesian uncertainty quantification (published)
- Ideas for heavier systems (unpublished)

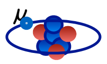
The Helium Isotope Shift puzzle

Status in 2023 using our input for TPE in muonic atoms

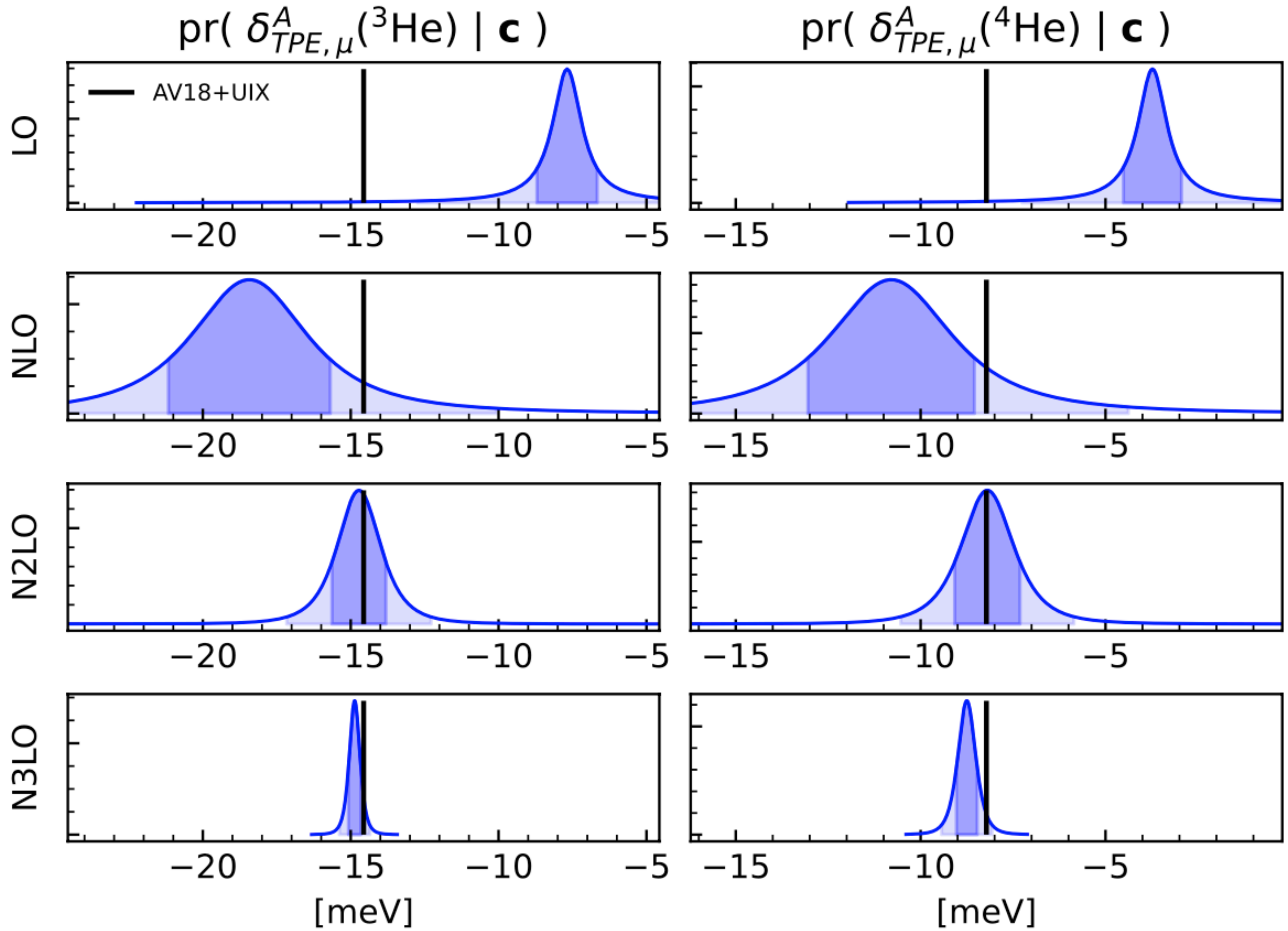


The Helium Isotope Shift puzzle

Simone Li Muli's PhD thesis (2023)

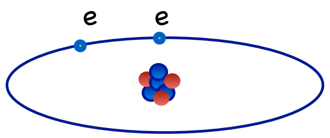
Nuclear structure corrections to muonic atoms 

Bayesian uncertainty quantification $\delta = \delta_{\mathrm{ref}} \sum_{n=0}^{\infty} c_n (Q/\Lambda)^n$



The Helium Isotope Shift puzzle

Nuclear structure corrections to ordinary atoms



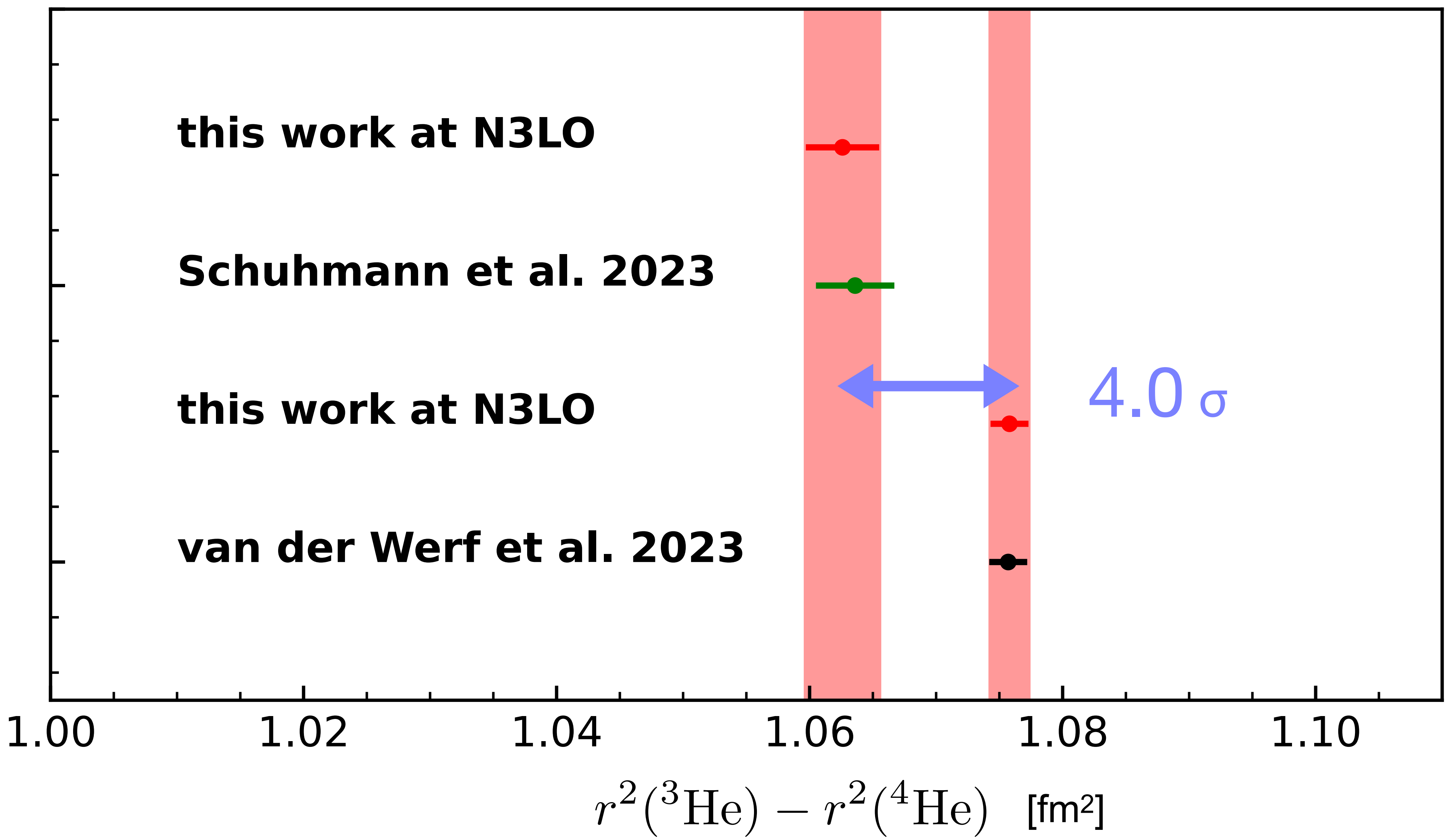
$$\delta^A_{\mathrm{TPE},\,e} = -\frac{2}{3} m (Z\alpha)^2 \phi^2_{nS}\ \tilde{\alpha}_{\mathrm{pol,e}}$$

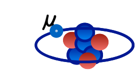
$$\tilde{\alpha}_{\mathrm{pol,e}} = \sum_{N\neq 0} |\langle N|\,\mathbf{D}\,|0\rangle|^2 \left[\frac{19}{6\omega_N} + \frac{5\ln(2\omega_N/m)}{\omega_N}\right]$$

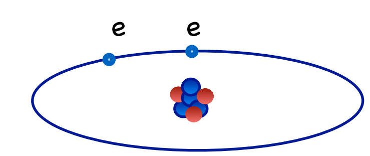
In meV	^{3}He	^{4}He
Our at N3LO	3.514(68)	1.909(96)
Pachucki, Moro (2007)	3.560(360)	2.070(200)

The Helium Isotope Shift puzzle

Li Muli, Richardson, SB, PRL **134** 032502 (2025)





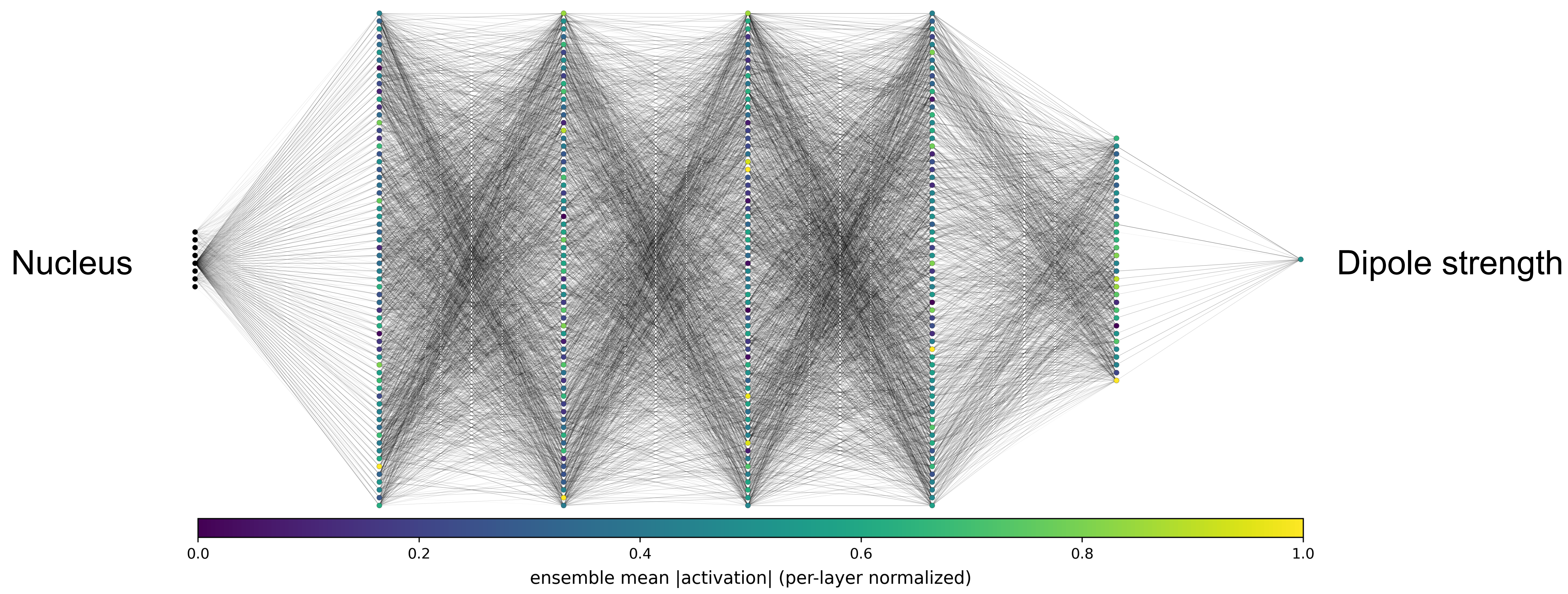


Problem was in QED (missing off-diagonal hyperfine interaction effects)
→ Qi et al, Phys. Rev. Research **7**, L022020 (2025)

How to address heavier muonic atoms?

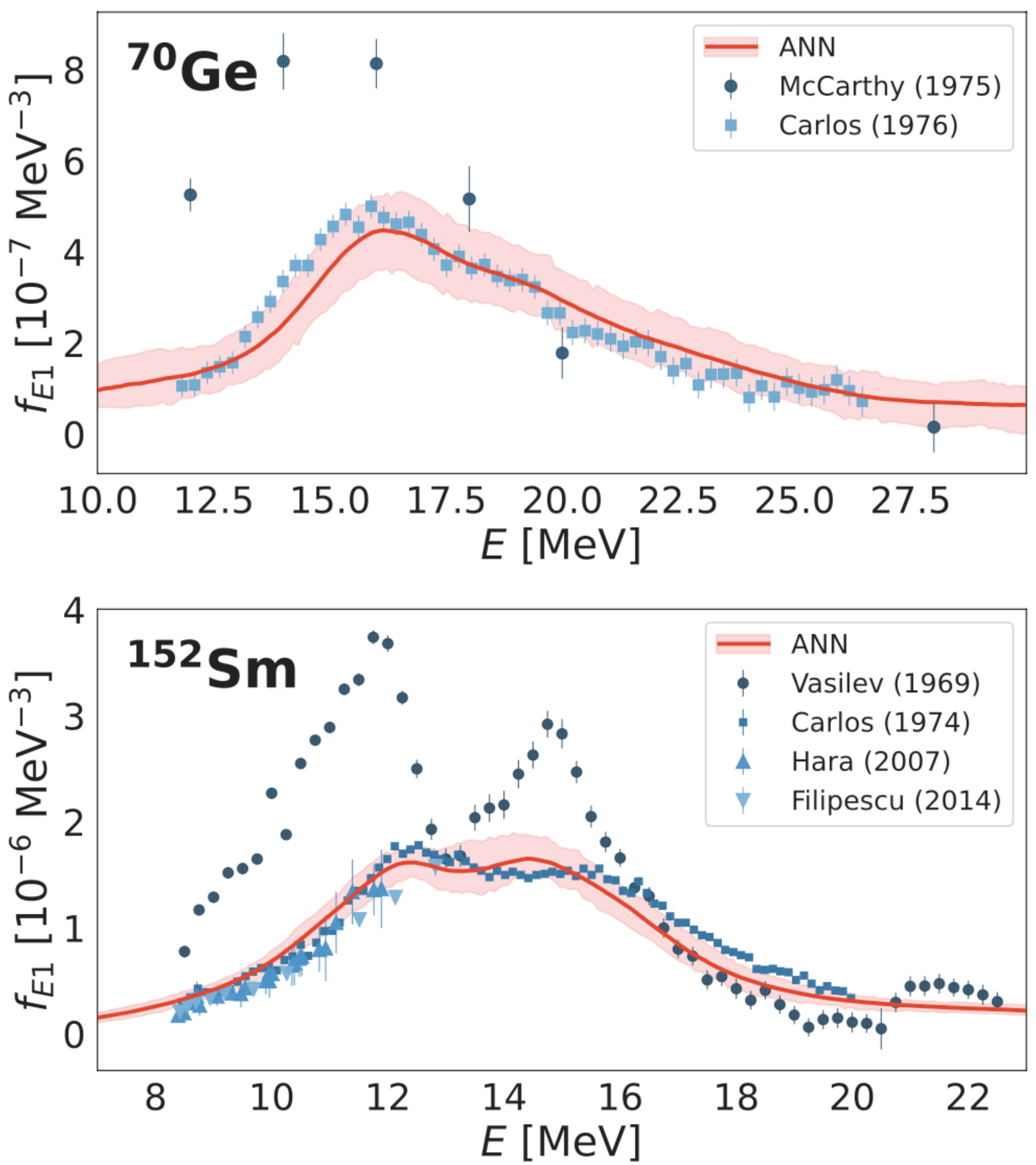
- Use phenomenological approaches
- Use other ab-initio methods (CC theory)

Phenomenological approach (ANN)



Phenomenological approach (ANN)

Jiang, Egert, SB, Bonaiti, von Neumann-Cosel, Phys.Rev.C 111 (2025) 5, L051308



Phenomenological approach (ANN)

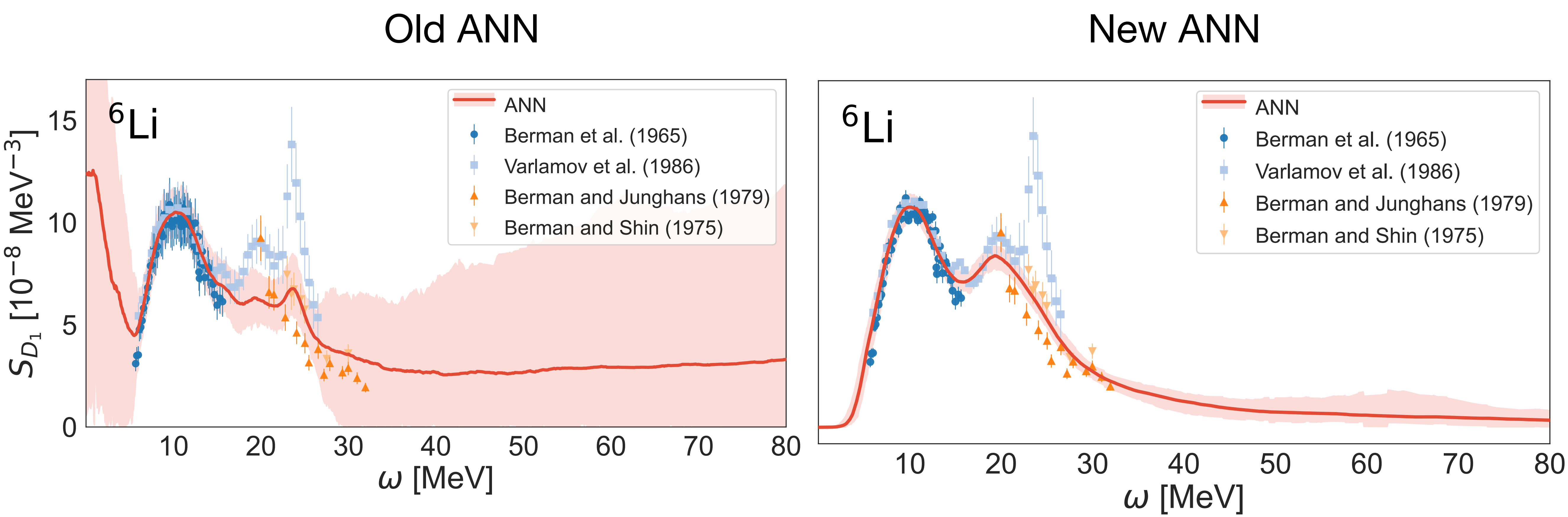
Optimize the ANN for light nuclei

Tim Egert’s master thesis

- Take data with error bars only for A<50
- Cross section set to zero before threshold
- Impose quasi-deuteron model on the tail

Phenomenological approach (ANN)

Optimize the ANN for light nuclei



Phenomenological approach (ANN)

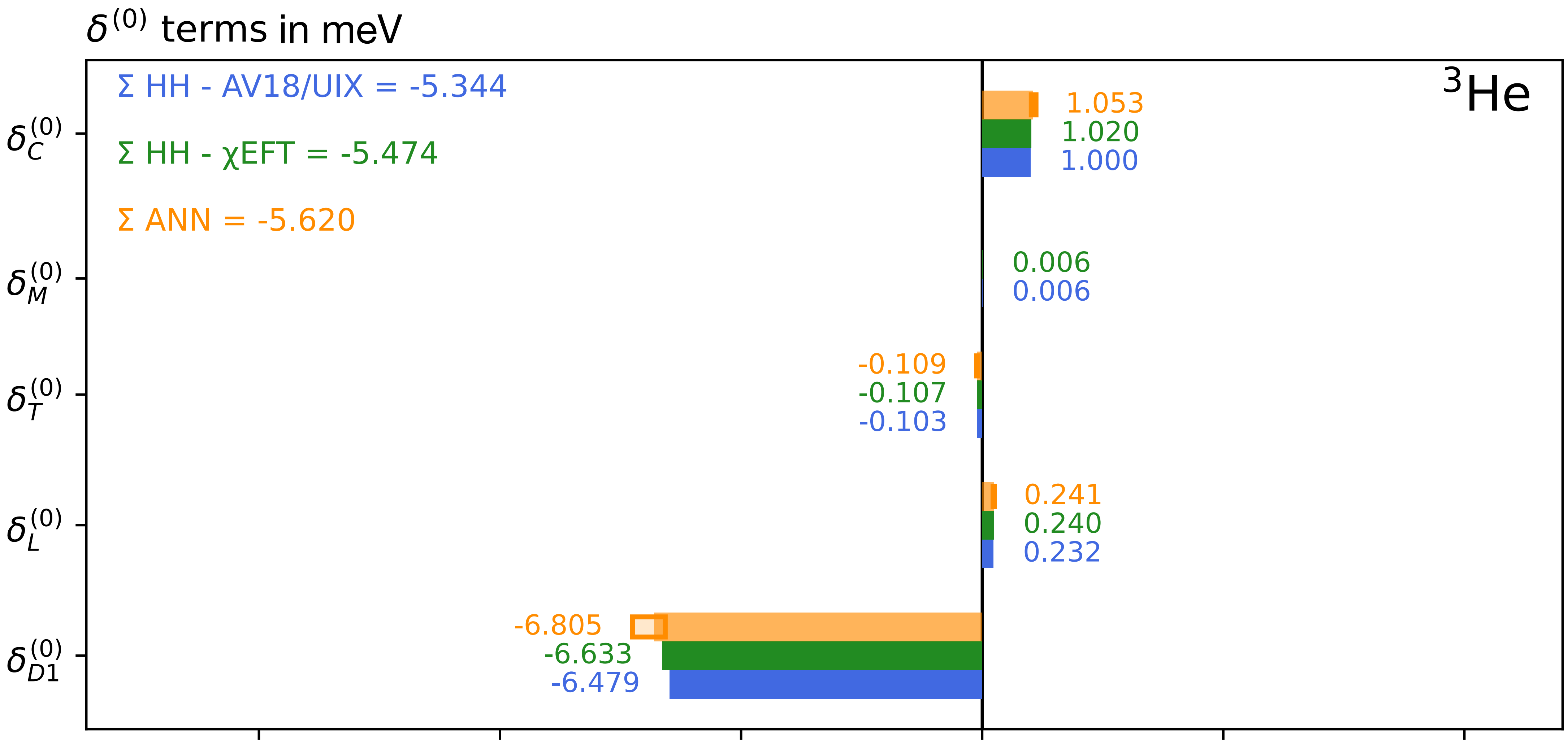
Application to muonic atoms

$$\underbrace{\delta^{(0)} = \delta^{(0)}_{D1} + \delta^{(0)}_{C} + \delta^{(0)}_{L} + \delta^{(0)}_{T}}_{} + \delta^{(0)}_{M}$$

Depend on electric dipole response function

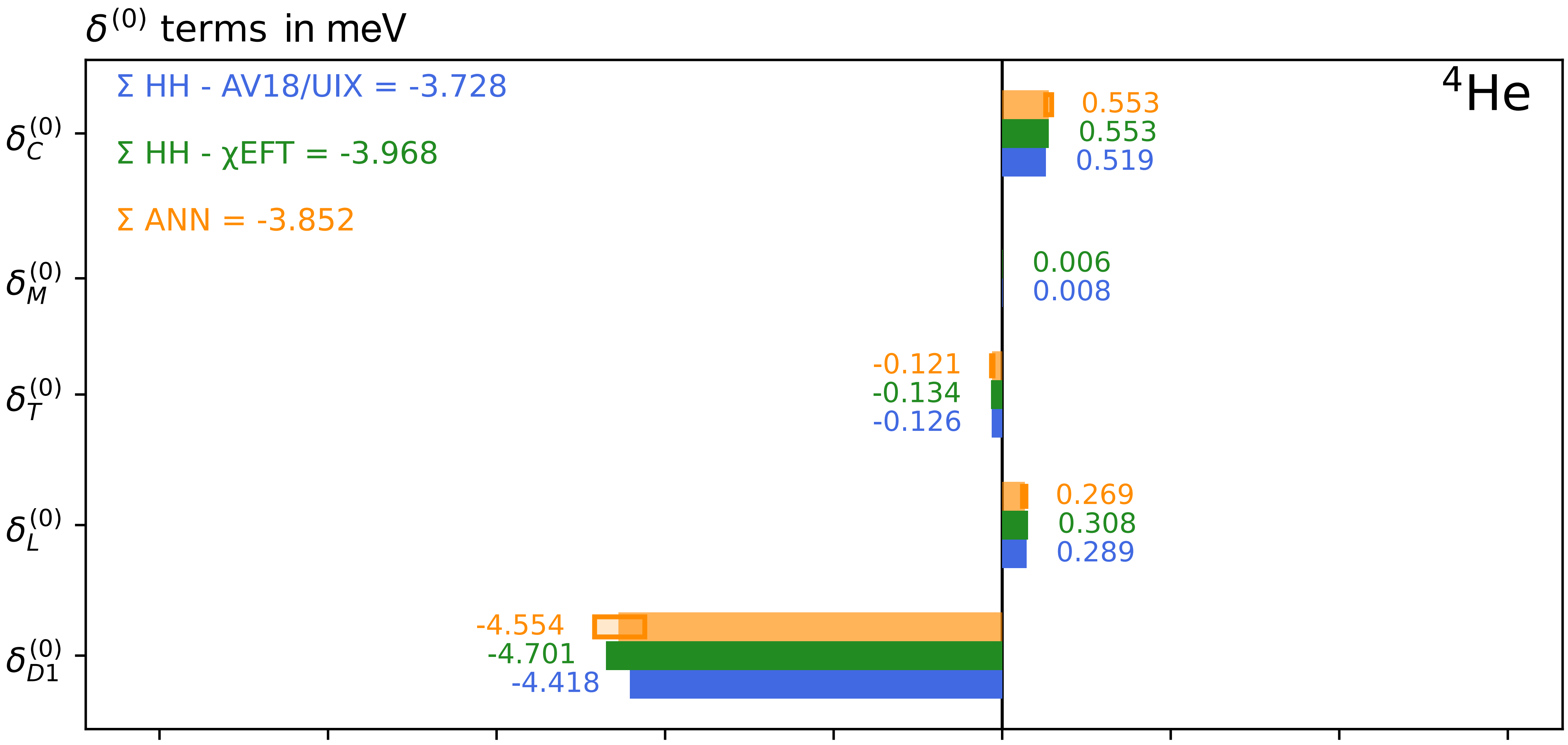
Phenomenological approach (ANN)

Application to muonic atoms



Phenomenological approach (ANN)

Application to muonic atoms



Phenomenological approach (ANN)

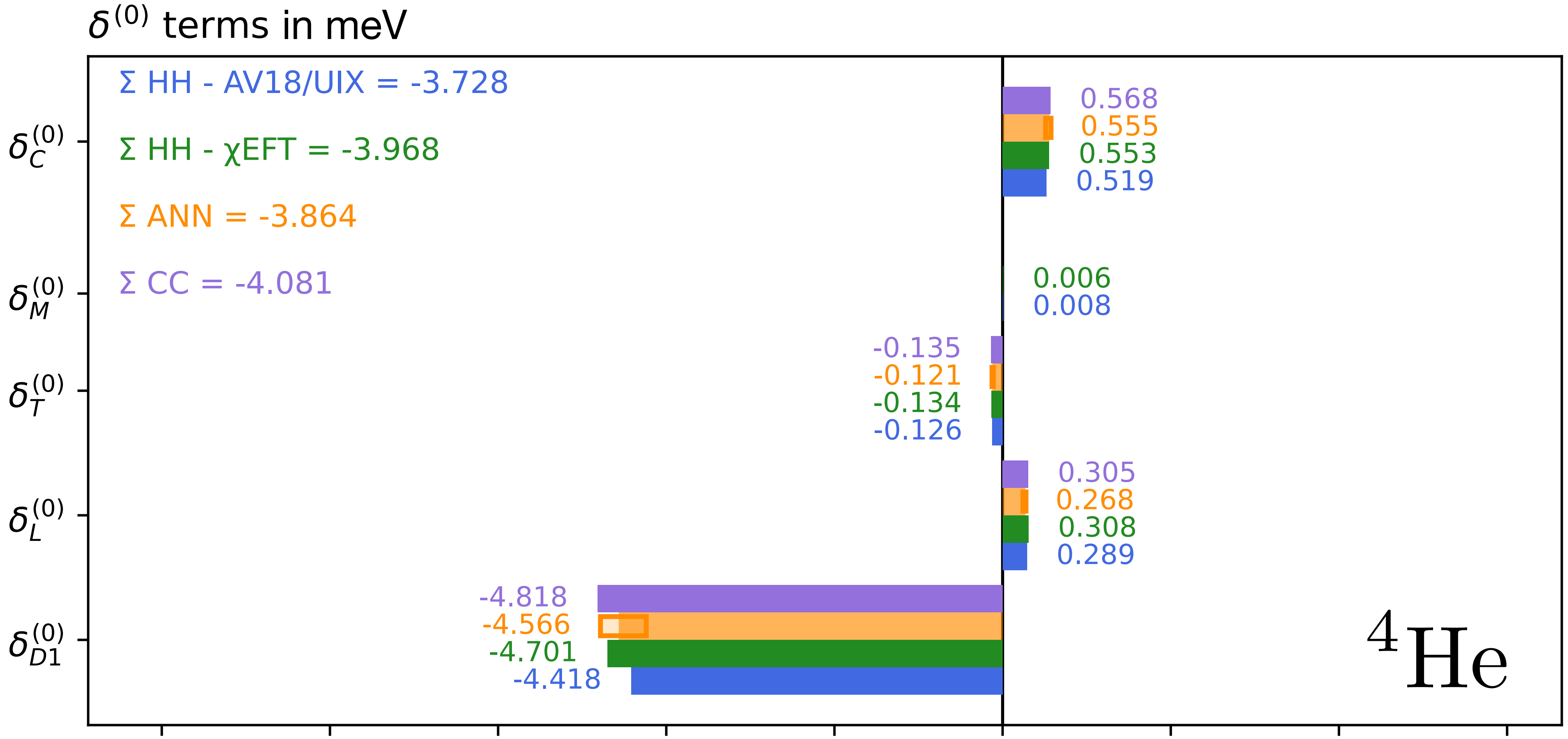
in meV

El.	$\delta^{(0)}_{\mathrm{D1}}$	$\delta^{(0)}_{\mathrm{C}}$	$\delta^{(0)}_{\mathrm{L}}$	$\delta^{(0)}_{\mathrm{T}}$
^{2}**H**	$-1.62^{+0.16}_{-0.20}$	$0.21^{+0.03}_{-0.03}$	$0.03^{+0.00}_{-0.01}$	$-0.01^{+0.00}_{-0.00}$
^{3}**H**	$-0.83^{+0.09}_{-0.06}$	$0.08^{+0.01}_{-0.00}$	$0.03^{+0.01}_{-0.01}$	$-0.01^{+0.00}_{-0.00}$
^{3}**He**	$-6.80^{+0.23}_{-0.45}$	$1.05^{+0.06}_{-0.04}$	$0.24^{+0.01}_{-0.02}$	$-0.11^{+0.01}_{-0.00}$
^{4}**He**	$-4.57^{+0.33}_{-0.22}$	$0.55^{+0.02}_{-0.04}$	$0.27^{+0.01}_{-0.03}$	$-0.12^{+0.01}_{-0.01}$
^{6}**Li**	$-30.67^{+1.30}_{-2.12}$	$6.29^{+0.33}_{-0.32}$	$1.03^{+0.06}_{-0.14}$	$-0.47^{+0.06}_{-0.03}$
^{7}**Li**	$-40.56^{+1.22}_{-2.16}$	$7.33^{+0.23}_{-0.35}$	$1.83^{+0.08}_{-0.09}$	$-0.83^{+0.04}_{-0.03}$
^{9}**Be**	$-146.93^{+11.71}_{-14.69}$	$36.52^{+2.55}_{-2.47}$	$4.97^{+1.13}_{-0.93}$	$-2.24^{+0.40}_{-0.51}$
^{10}**B**	$-170.15^{+24.15}_{-15.43}$	$41.74^{+2.85}_{-5.02}$	$7.36^{+1.35}_{-1.49}$	$-3.35^{+0.65}_{-0.61}$
^{12}**C**	$-496.21^{+48.45}_{-60.71}$	$126.10^{+14.59}_{-10.45}$	$24.20^{+3.82}_{-4.47}$	$-11.06^{+2.31}_{-1.40}$
^{13}**C**	$-608.41^{+48.16}_{-28.49}$	$166.27^{+5.92}_{-11.05}$	$24.45^{+2.36}_{-2.98}$	$-11.20^{+1.32}_{-0.96}$
^{14}**N**	$-1108.67^{+78.25}_{-73.39}$	$314.62^{+9.56}_{-29.57}$	$50.19^{+5.09}_{-6.26}$	$-23.05^{+2.84}_{-2.26}$
^{15}**N**	$-1194.61^{+98.42}_{-43.72}$	$343.20^{+7.33}_{-30.14}$	$53.21^{+3.57}_{-5.45}$	$-24.38^{+2.55}_{-1.43}$
^{16}**O**	$-1694.08^{+117.27}_{-131.16}$	$504.60^{+29.00}_{-37.42}$	$78.78^{+6.98}_{-11.25}$	$-36.11^{+5.00}_{-3.21}$
^{17}**O**	$-1844.90^{+138.28}_{-91.77}$	$570.92^{+30.10}_{-34.09}$	$74.41^{+6.86}_{-8.85}$	$-34.03^{+3.87}_{-3.15}$
^{18}**O**	$-1879.53^{+173.33}_{-60.50}$	$580.47^{+19.99}_{-41.44}$	$76.83^{+6.37}_{-9.91}$	$-35.10^{+3.95}_{-3.32}$

Ab-initio coupled-cluster theory

See talks by Gaute, Francesca, Joanna on coupled-cluster theory

LIT-CC can be applied also to muonic atoms (Lanczos sum rule method)



Ab-initio coupled-cluster theory

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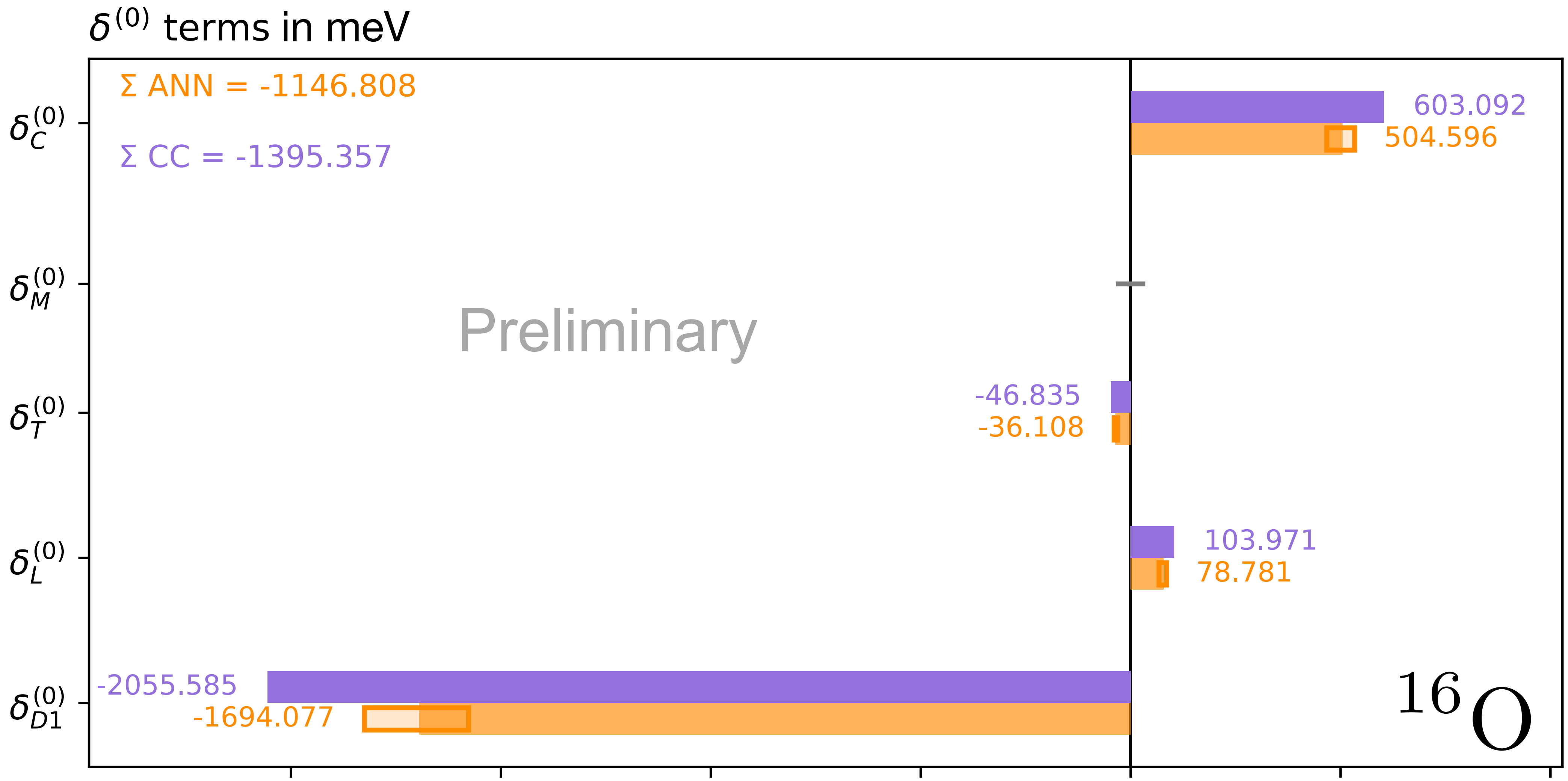

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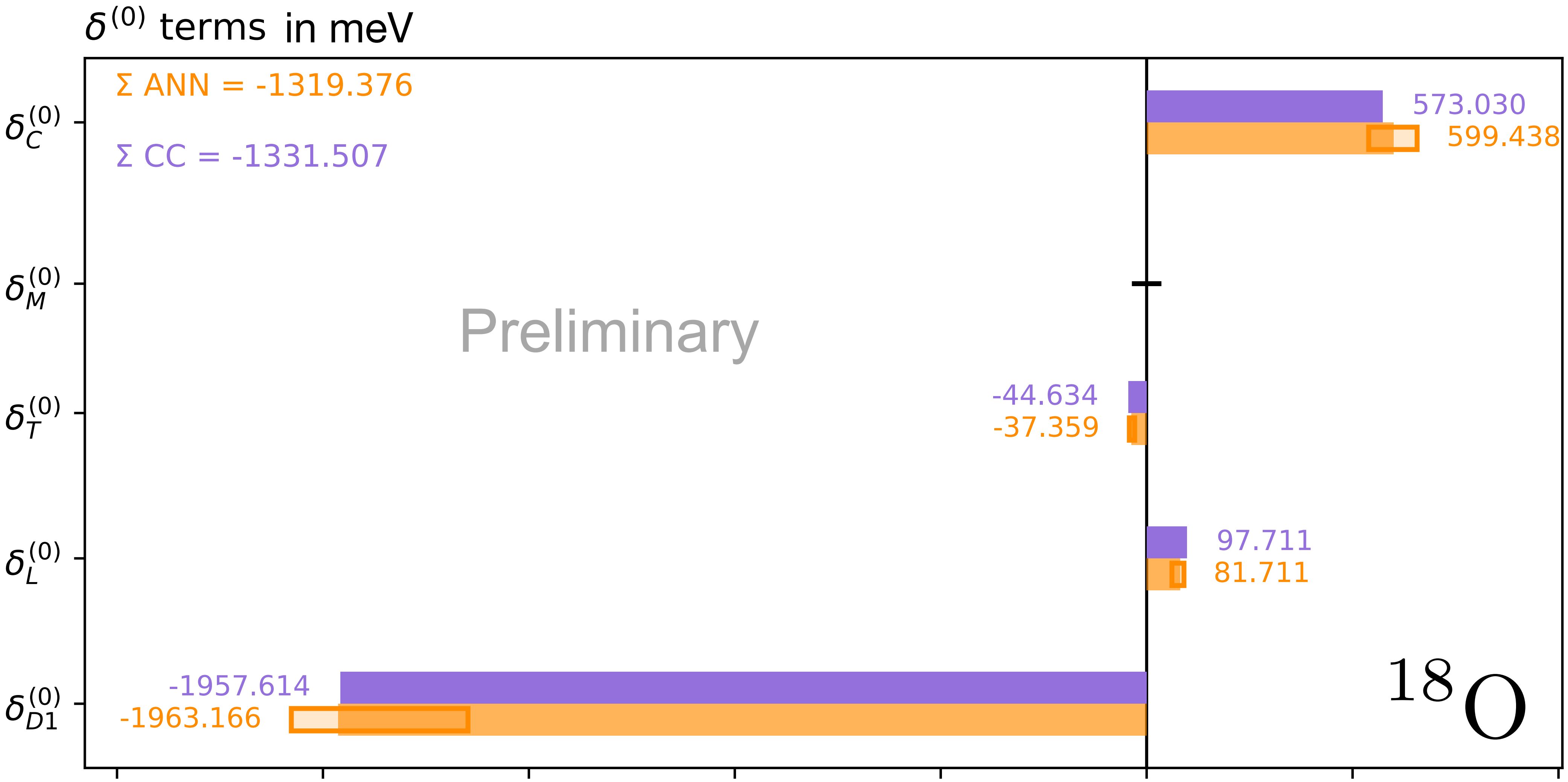

Outlook

- Ab-initio nuclear theory has allowed a strong reduction of uncertainties in the evaluation of TPE for the Lamb shift.

- We can extend ab-initio calculations to heavier systems (work in progress)

- It is interesting to develop simpler phenomenological approaches (work in progress).

Thanks to my collaborators

Tim Egert, Weiguang Jiang, Francesco Marino

Simone Li Muli, Thomas Richardson

Nir Barnea, Chen Ji

THANK YOU