RUHR-UNIVERSITÄT BOCHUM

LOW-RESOLUTION EFT FOR NUCLEON-NUCLEON INTERACTIONS

Challenges in Effective Field Theory Descriptions of Nuclei

Hirschegg, January 19th 2026

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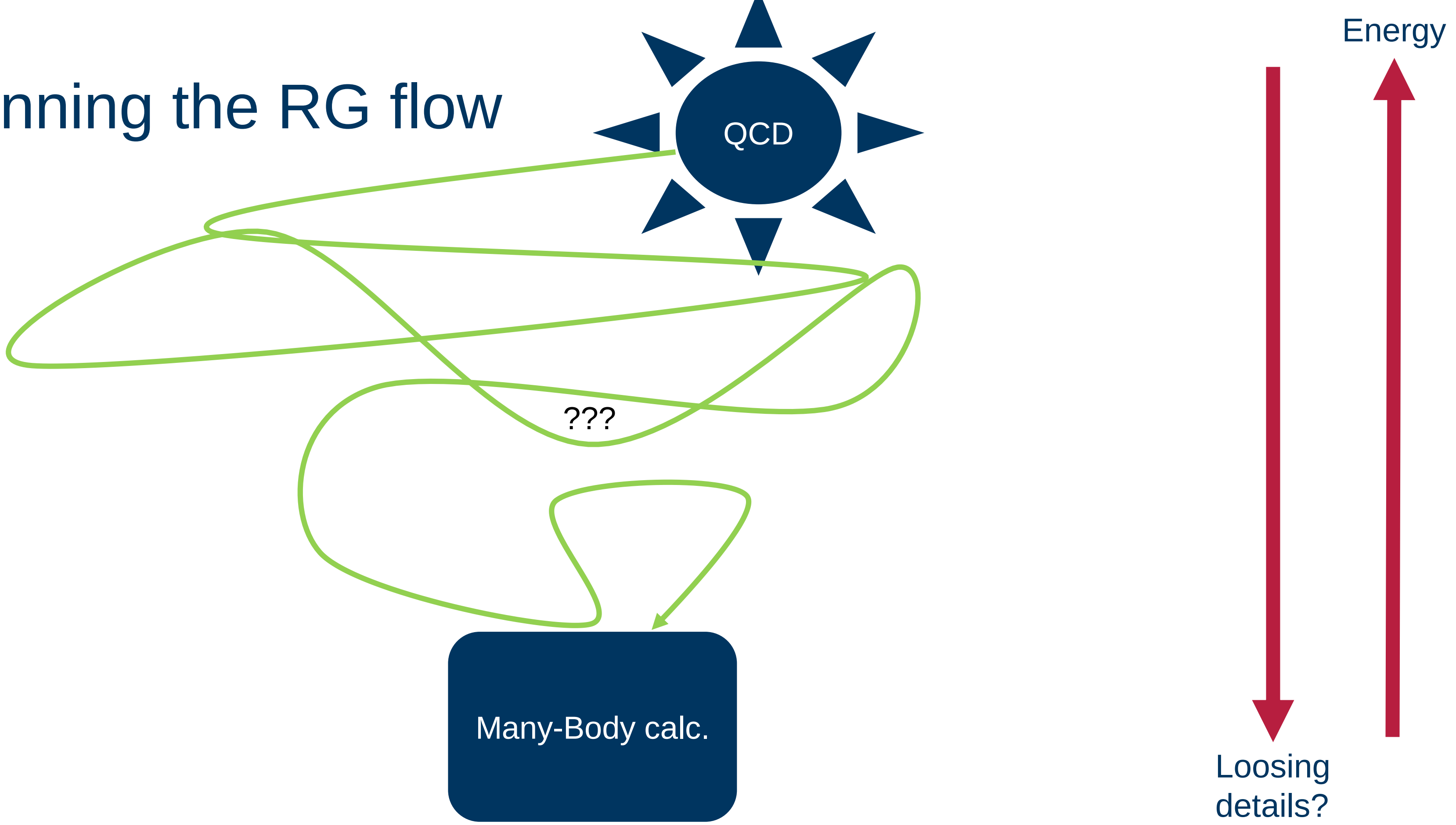
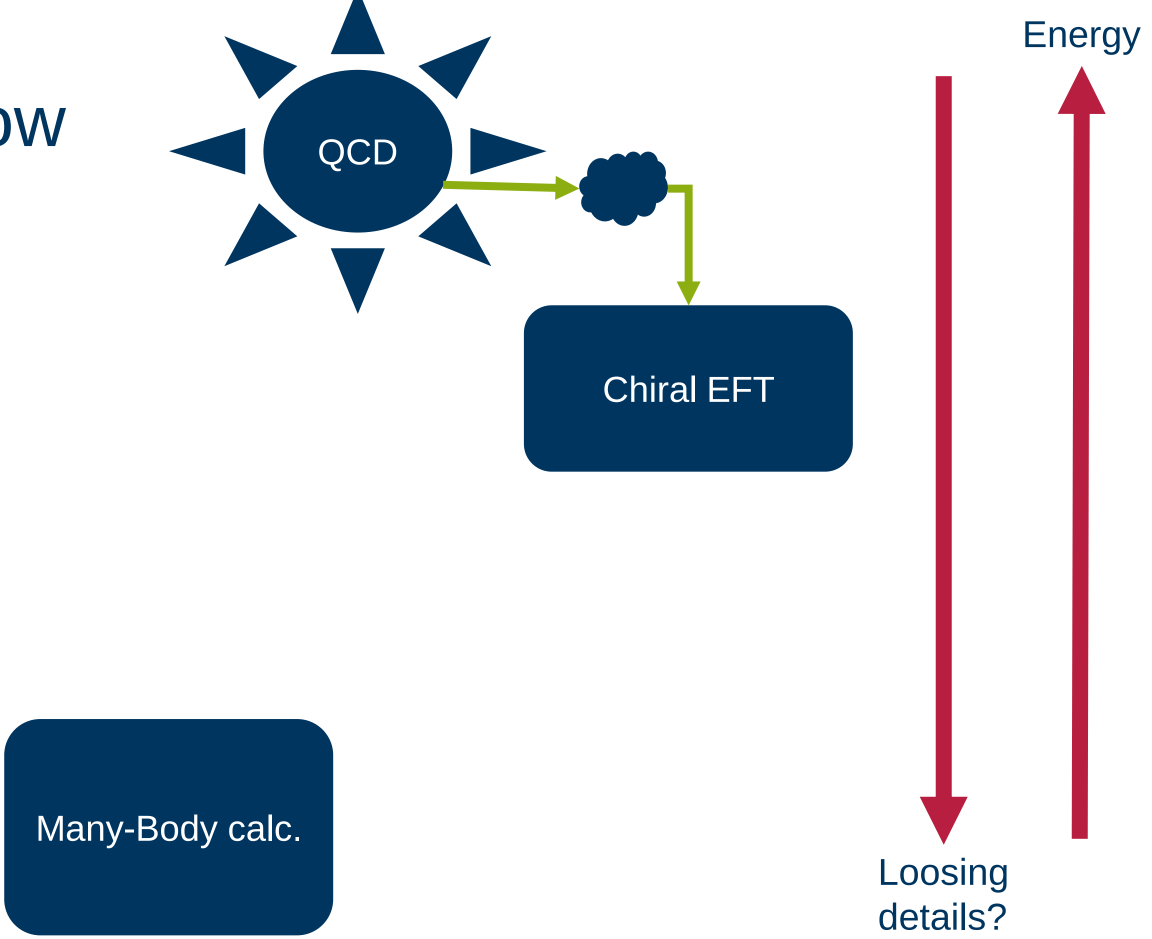
Running the RG flow



Running the RG flow



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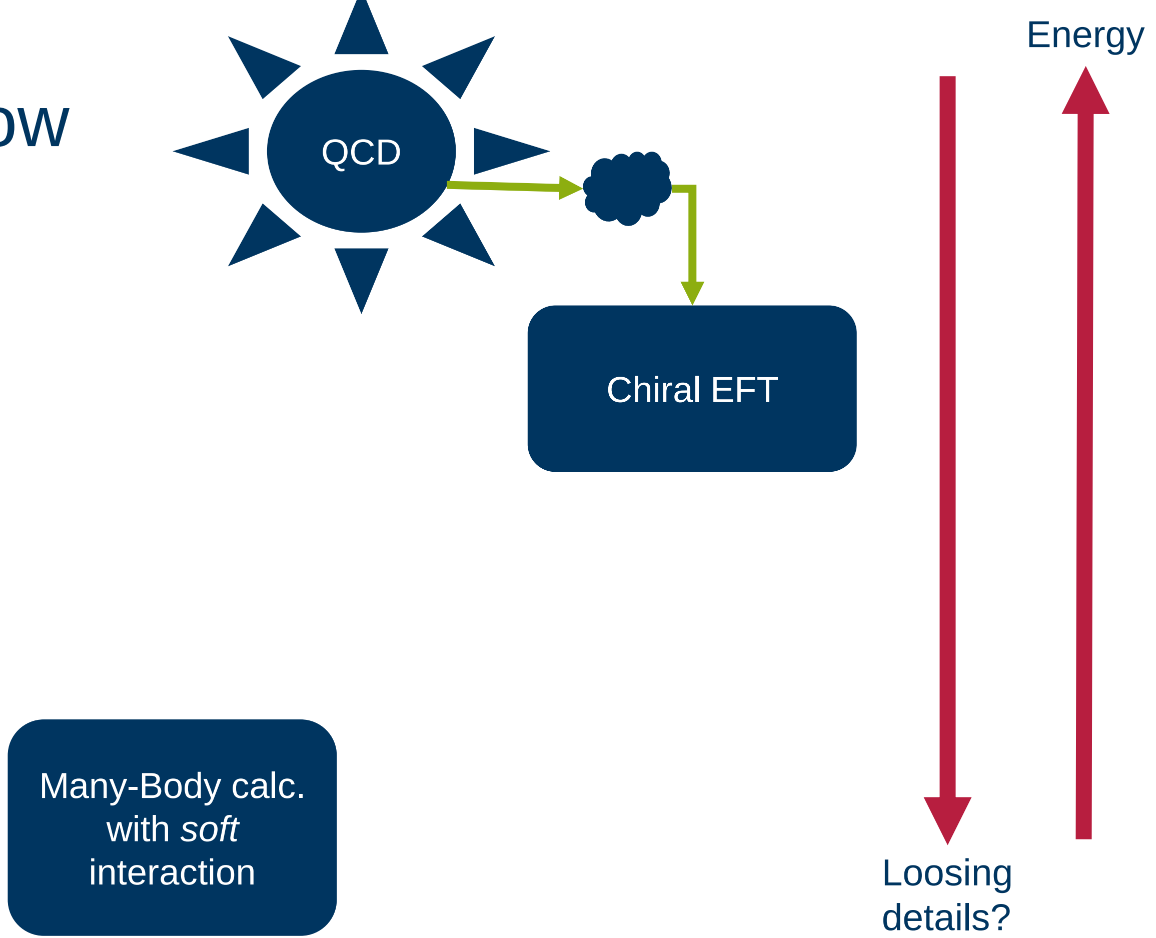

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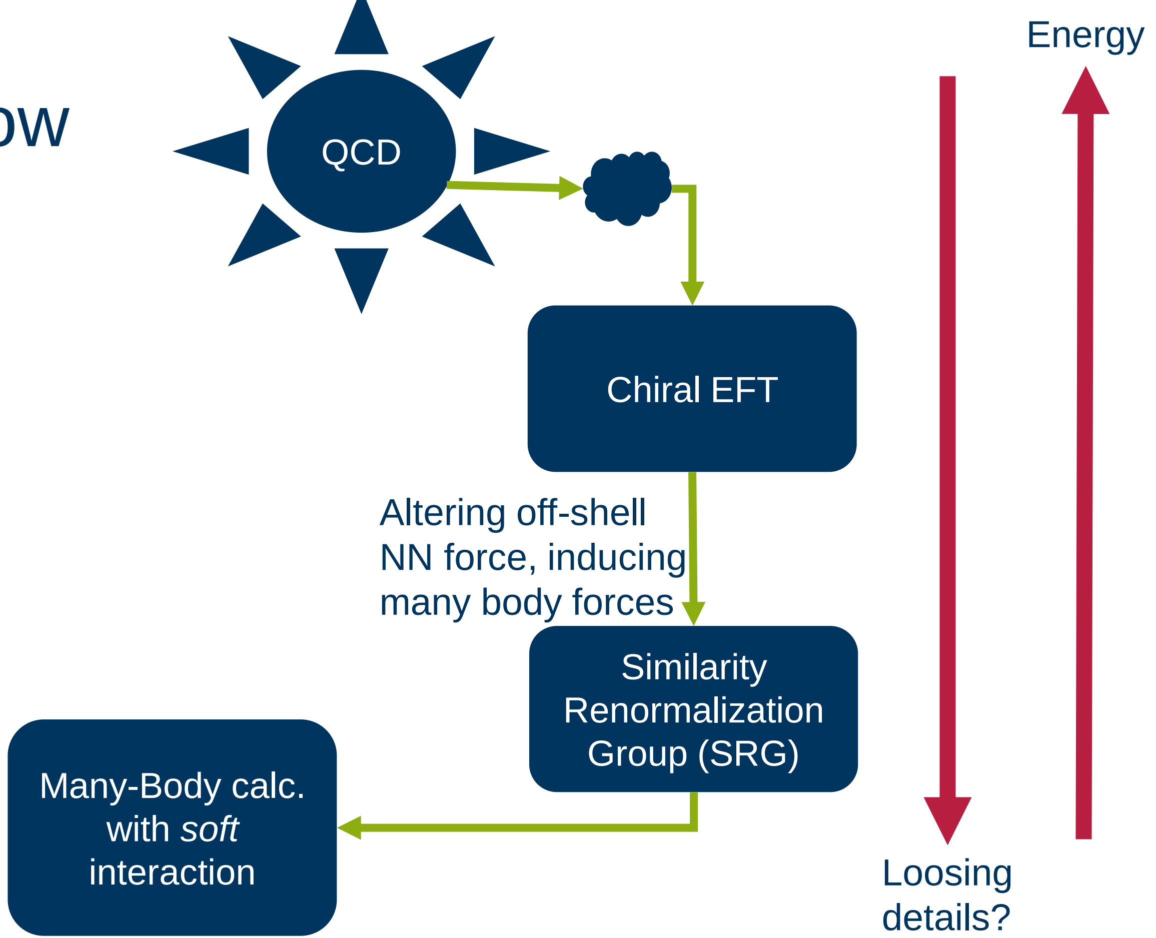

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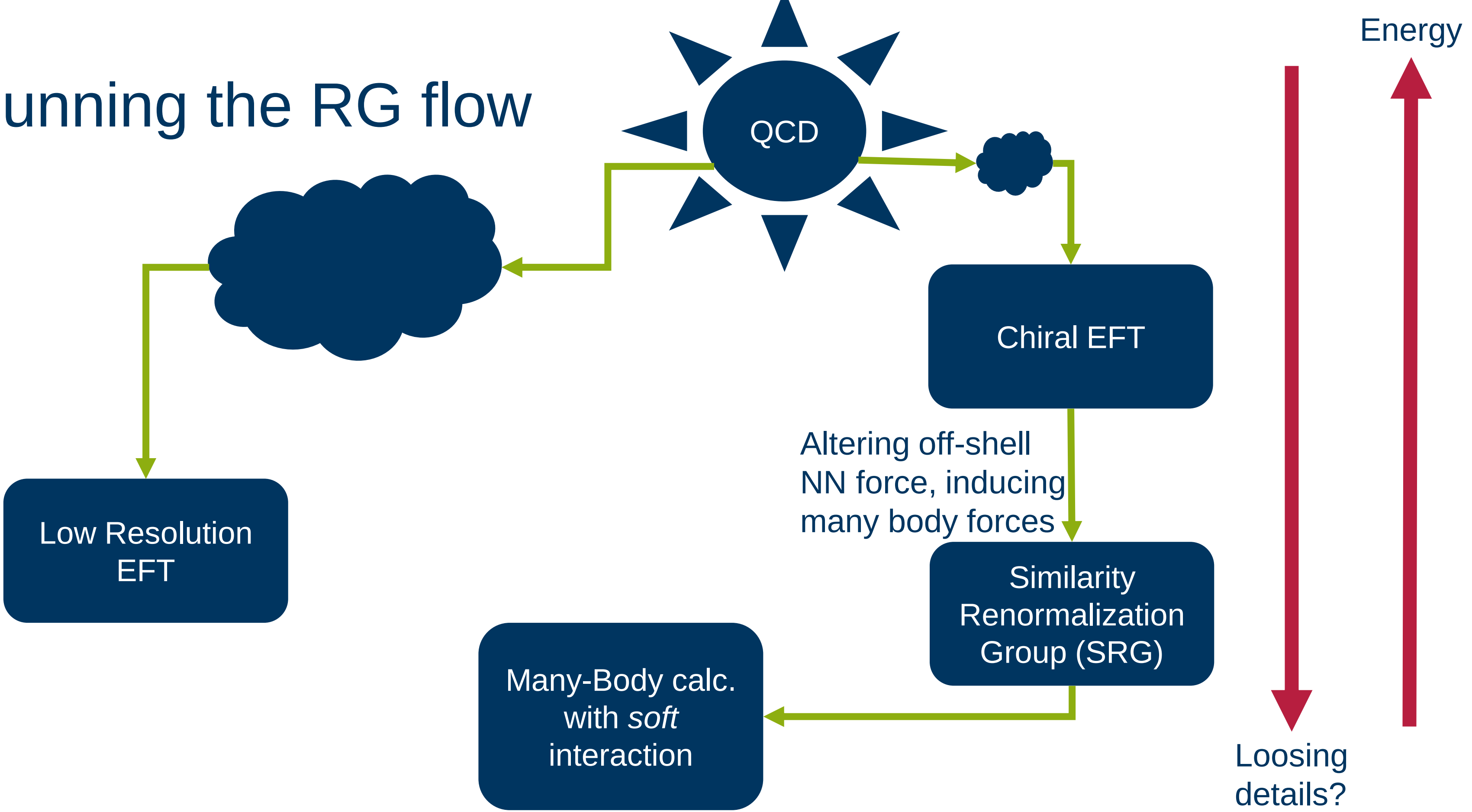

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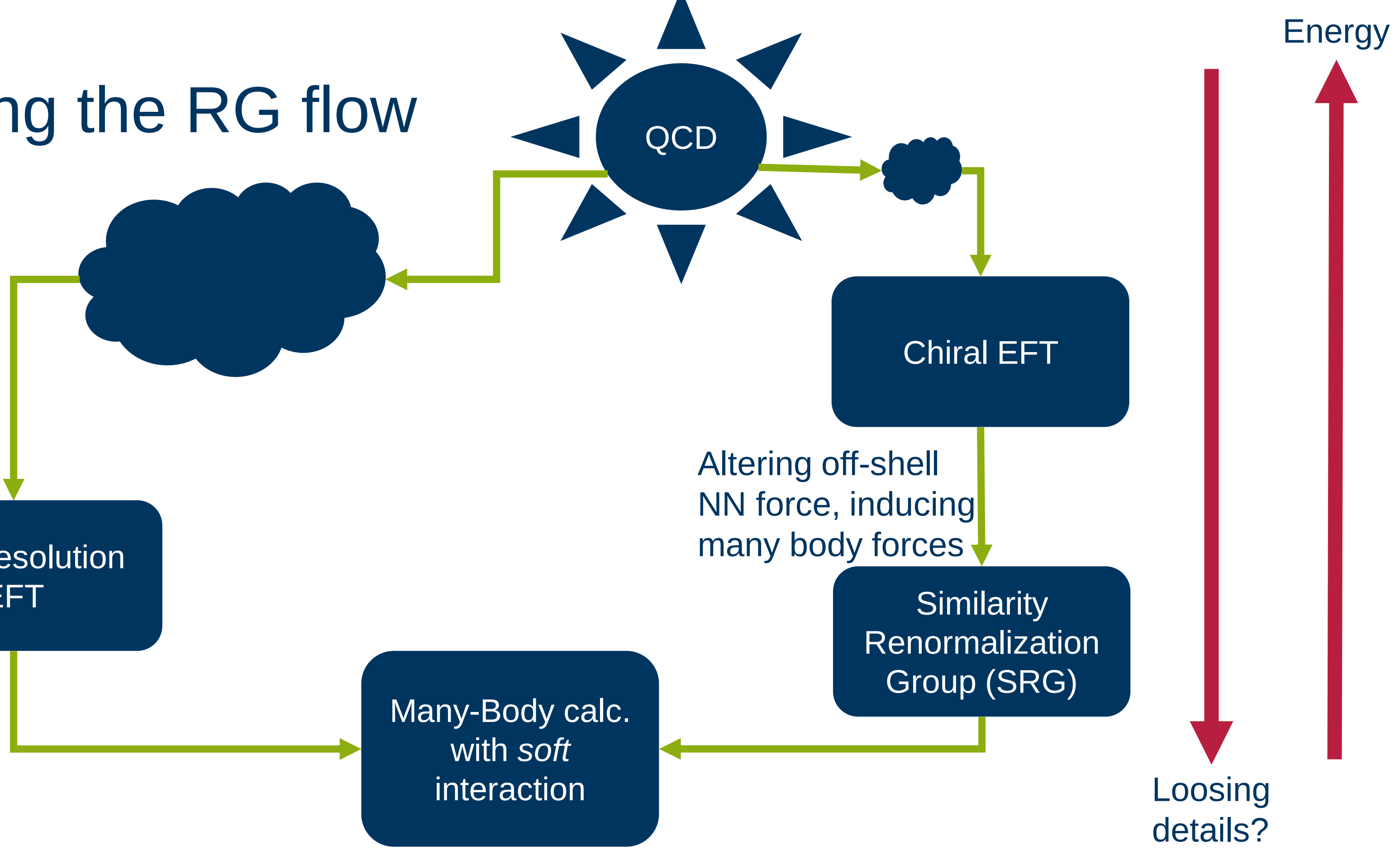

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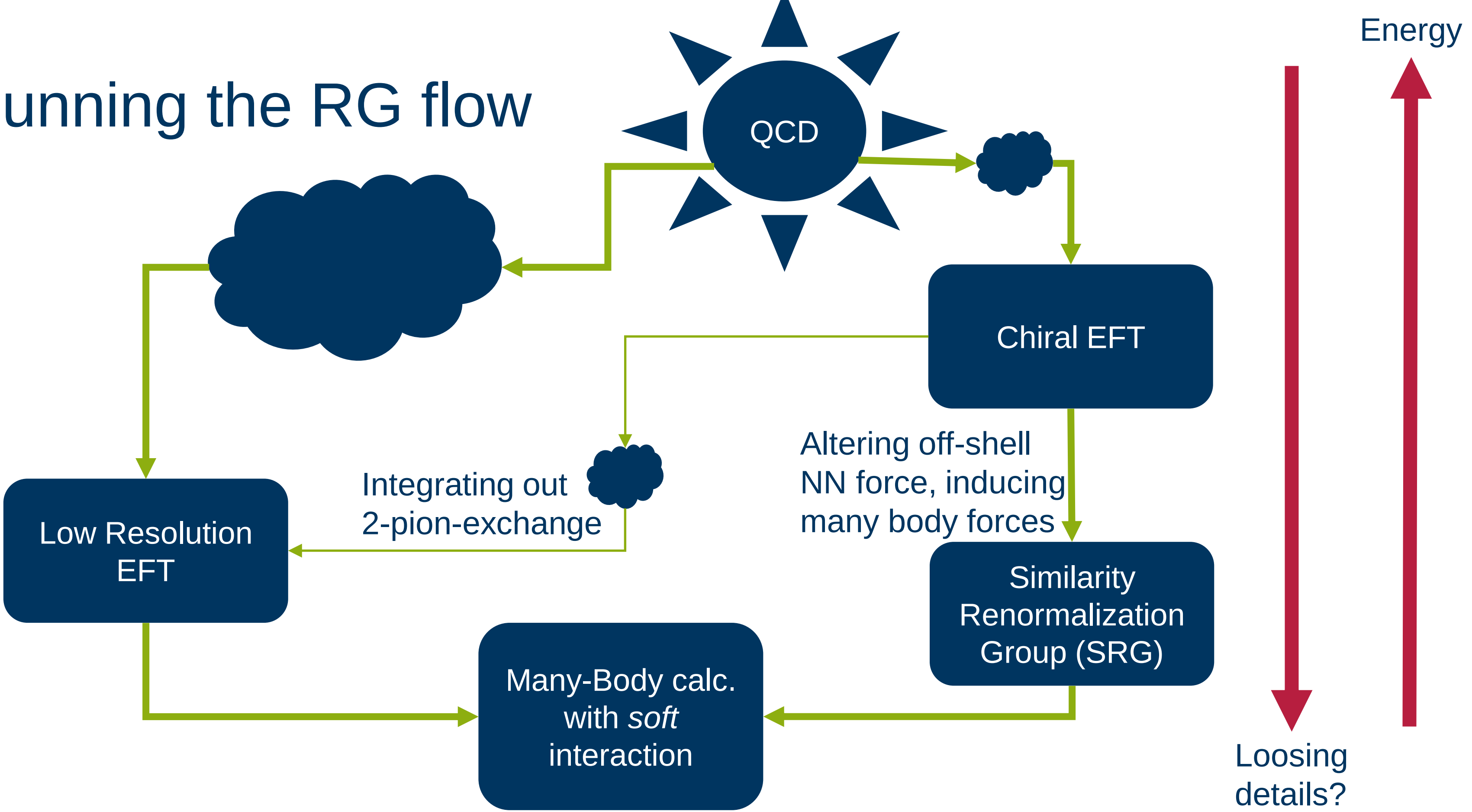


Construction of LowRes EFT

Power counting

- Expansion parameter of LowRes:

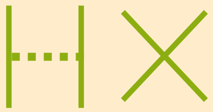
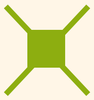
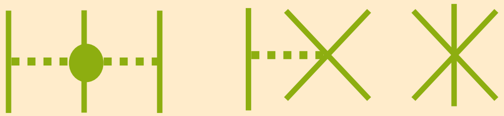
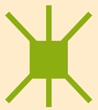
$$Q \sim \frac{p}{\Lambda_{\mathrm{B}}^{\mathrm{LowRes}}}$$

- Compared against chiral EFT:

$$Q \in \left(\frac{p}{\Lambda_{\mathrm{B}}^{\chi}}, \frac{M_{\pi}}{\Lambda_{\mathrm{B}}^{\chi}} \right) \sim \frac{1}{3}$$

- $\Lambda_{\mathrm{B}}^{\mathrm{LowRes}} \ll \Lambda_{\mathrm{B}}^{\chi}$

→ Naïvely LowRes has quicker convergence for low energies than chiral EFT, but slower at high energies

Order	Two-Nucleon-Force	Three-Nucleon-Force
LO (Q^0)		-
NLO (Q^2)		-
N2LO (Q^3)	-	
N3LO (Q^4)		?
Q^6		? 

Degrees of freedom of LowRes EFT long-range

- Long-range: **only** 1-π**-**exchange, using local, Gaussian regulator and *without* subtractions

$$V_{1\pi} = -\frac{g_{\mathrm{A}}^2}{4F_\pi^2}\left(\frac{\vec{\sigma}_1 \cdot \vec{q}\vec{\sigma}_2 \cdot \vec{q}}{q^2 + M_\pi^2}\right) e^{-\frac{q^2+M_\pi^2}{\Lambda^2}}$$

- With pion mass M_π, pion decay constant F_π, pion-nucleon coupling constant g_{A}, nucleon-spin matrices σ_1, σ_2, cutoff value Λ, momentum transfer q of nucleons

- Same at all orders

→ No (explicit) need for chiral symmetry

Degrees of freedom of LowRes EFT short-range

- Short-range: contact forces (same as in chiral EFT), nonlocal Gaussian regulator: $\exp((p^2 - p'^2)/\Lambda^2)$
- In PWD form, up to Q^4:

$$
\begin{aligned}
\langle i_S,\, p'|V_{\text{cont}}|i_S,\, p\rangle &= \tilde{C}_{i_S} + C_{i_S}(p^2 + p'^2) + D_{i_S} p^2 p'^2 \,, \\
\langle i_P,\, p'|V_{\text{cont}}|i_P,\, p\rangle &= C_{i_P} pp' + D_{i_P} pp'(p^2 + p'^2) \,, \\
\langle i_D,\, p'|V_{\text{cont}}|i_D,\, p\rangle &= D_{i_D} p^2 p'^2 \,, \\
\langle {}^3S_1,\, p'|V_{\text{cont}}|{}^3D_1,\, p\rangle &= C_{\epsilon 1} p^2 + D_{\epsilon 1} p^2 p'^2 \,, \\
\langle {}^3P_2,\, p'|V_{\text{cont}}|{}^3F_2,\, p\rangle &= D_{\epsilon 2} p^3 p' \,,
\end{aligned}
$$

- A-priori **LECs are unkown**
- At each even power of Q a new set of contact operators appears:
  - At Q^0 3(+1), at Q^2 10(+1), at Q^4 22(+1),at Q^6 39(+1) LECs in the 2N system ((+x), x is LECs only in nn system)
- LECs are bare constants, i.e. dependent on scheme!

From Lagrangians to Observables: In a nutshell

- Finite set of diagrams can be summed into a QM potential $V = V(p, p')$

- Lippmann-Schwinger-equation (actually an integral equation)

$$T = V + VG_0T \Rightarrow (1 - VG_0)T = V$$

- With G_0 as free propagator and T as transition matrix (directly connected to observables)
- Solved in momentum space and partial-wave decomposed, all(!) calculations are **non-relativistic**

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- What about LECs? Fitted to experimental data later on! ← **inverse problem**
- 3NF LECs have to be fitted to 3N experimental data → inverse problem with Faddeev equation

Methodology
Fitting

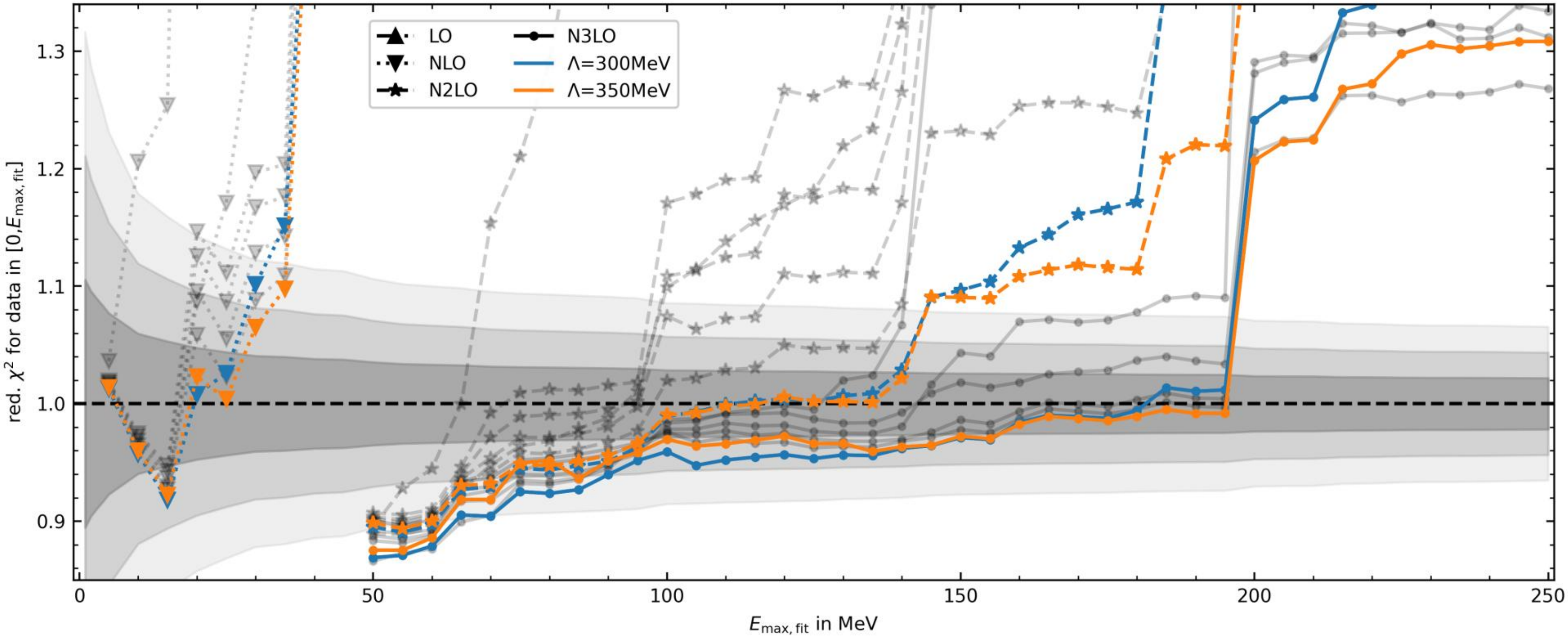
How to determine the Fit range?

- LECs are tuned to experimental data → Which experimental data should we use?
  - Bochum Database (very similar to Granada Database) (see PhD thesis of P. Reinert,2022)
  - Most important parameter $E_{\text{max,fit}}$ (exp. data up to which energy are used for the fit?)

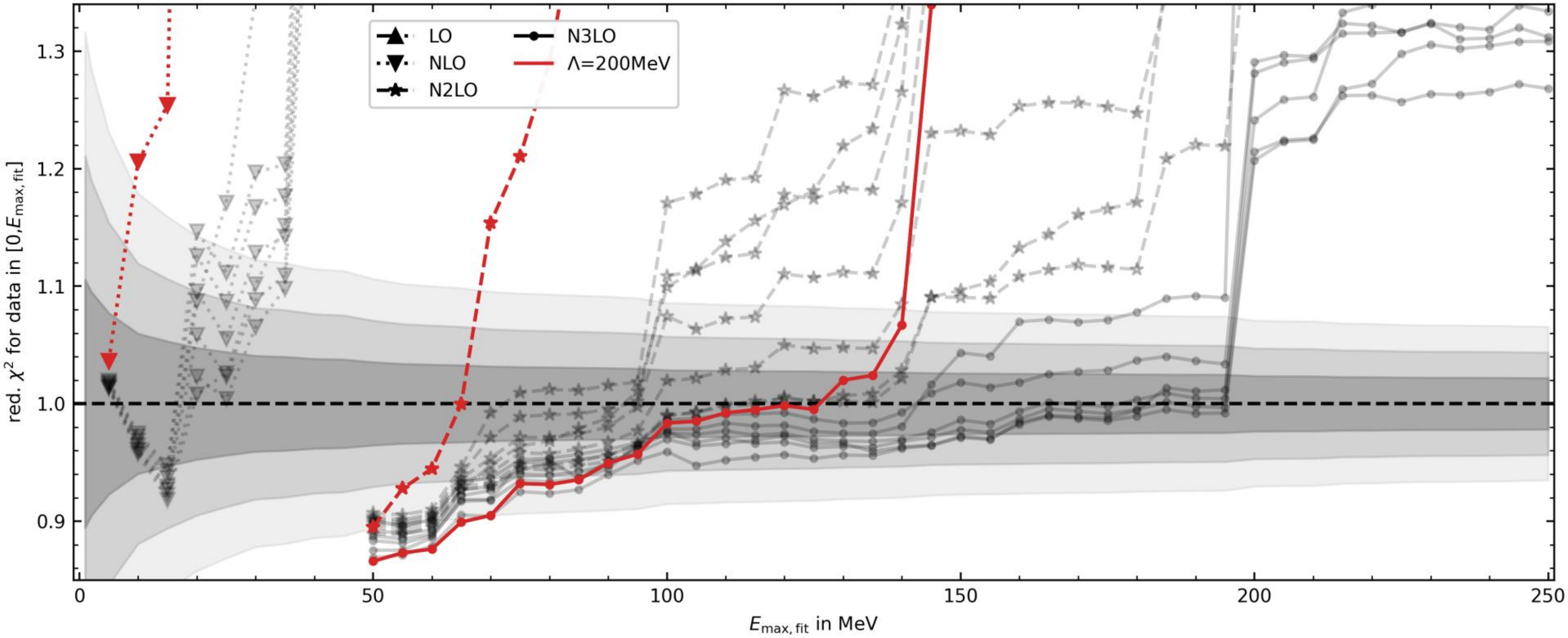
- Used data and details change for every order (and cutoff)
  - Q^0 : Fit up to $E_{\text{lab}} = 10\ \text{MeV}$ ← self consistency not possible
  - Q^2, Q^4 : Fit to maximal energy, such that final fit is **statistically self-consistent**
  - Q^4 : **Constrain** fit by Deuteron BE and coherent scattering length
  - At Q^0, Q^2 relative inv. norms of exp. data are fixed with Bochums Chiral EFT SMS N4LO+ 450MeV

- 1 LEC only appearing in neutron-neutron interaction is tuned to $a_{nn} = -18.9\ \text{fm}$

Determination of $E_{\mathrm{max,fit}}$ - order by order



Determination of $E_{\mathrm{max,fit}}$ - small cutoffs



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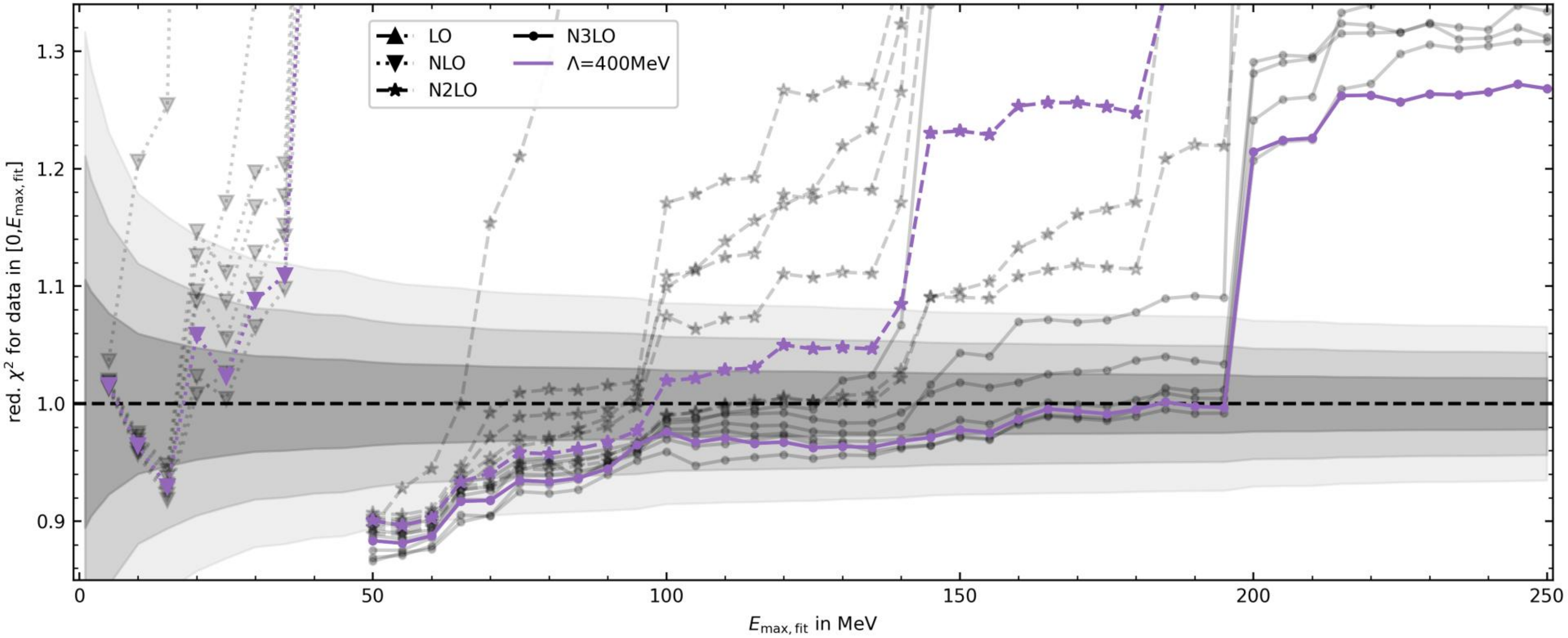
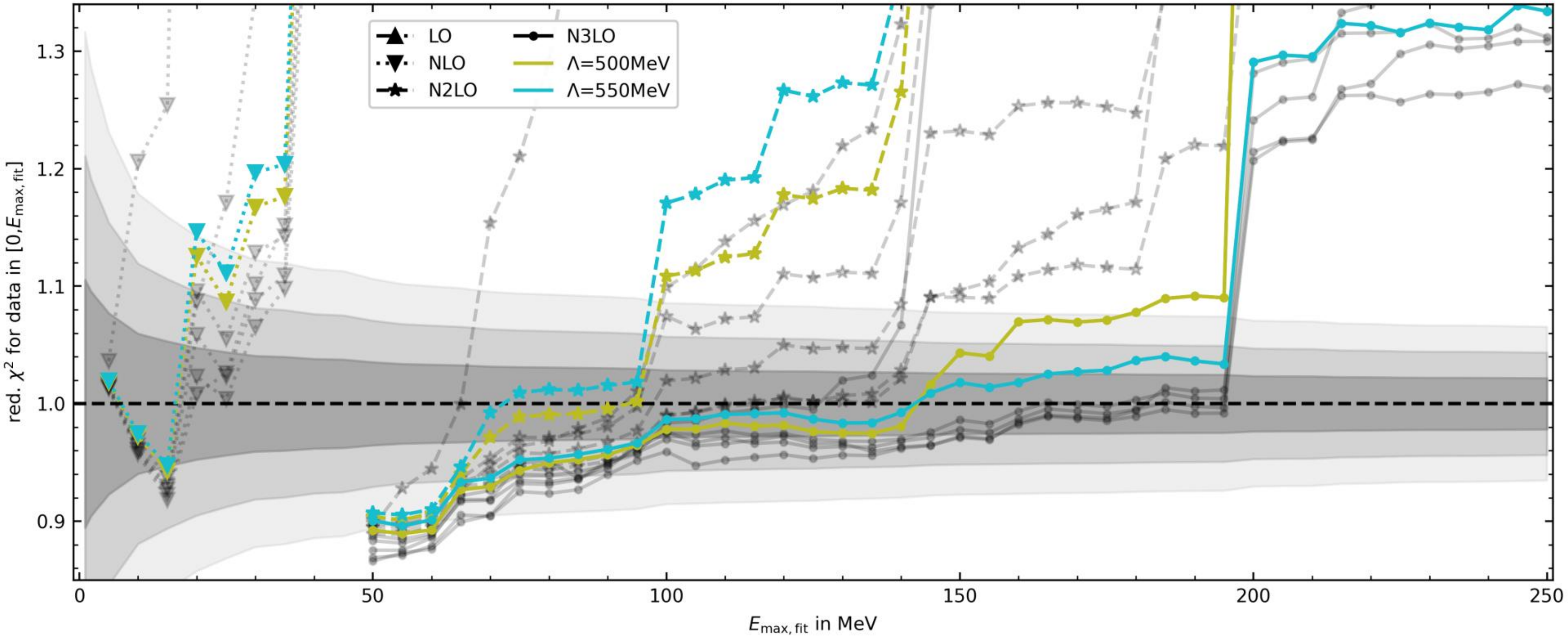
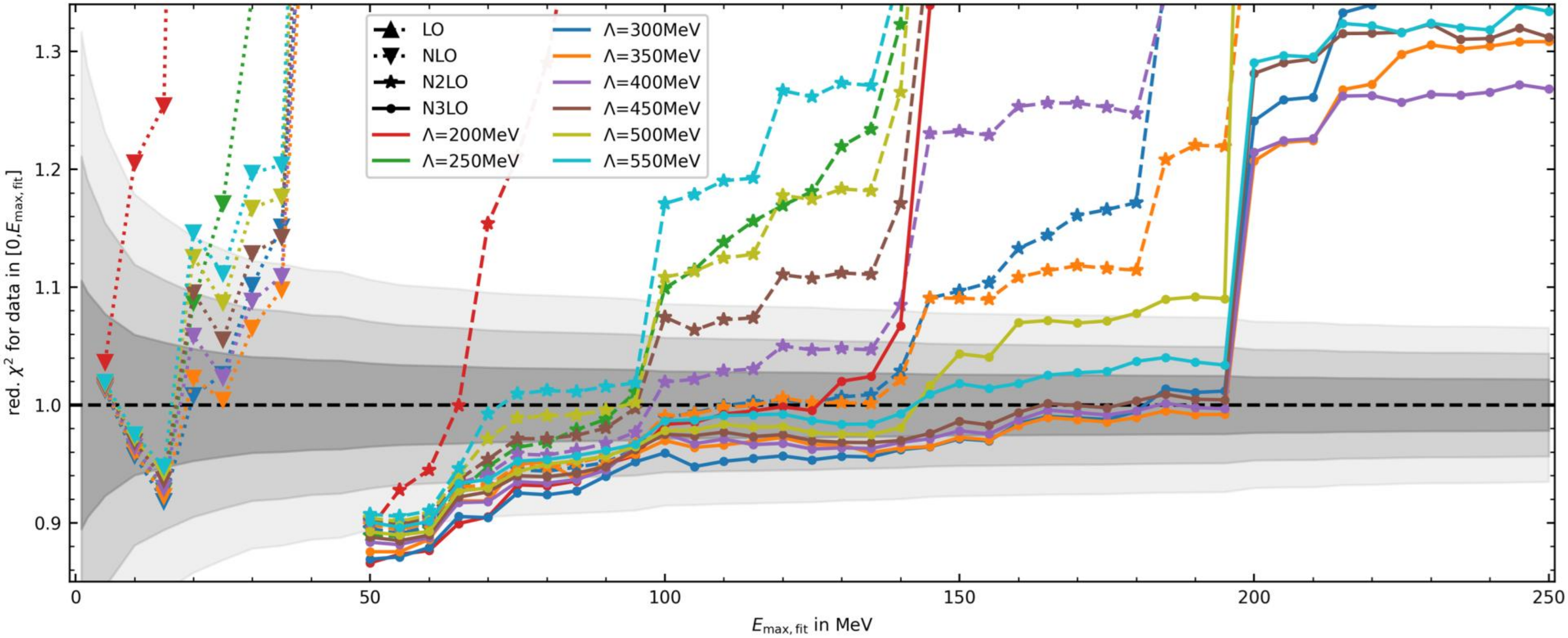
Determination of $E_{\mathrm{max,fit}}$ - reasonable large cutoffs



Determination of $E_{\mathrm{max,fit}}$ - very large cutoffs



Determination of $E_{\mathrm{max,fit}}$ - full picture



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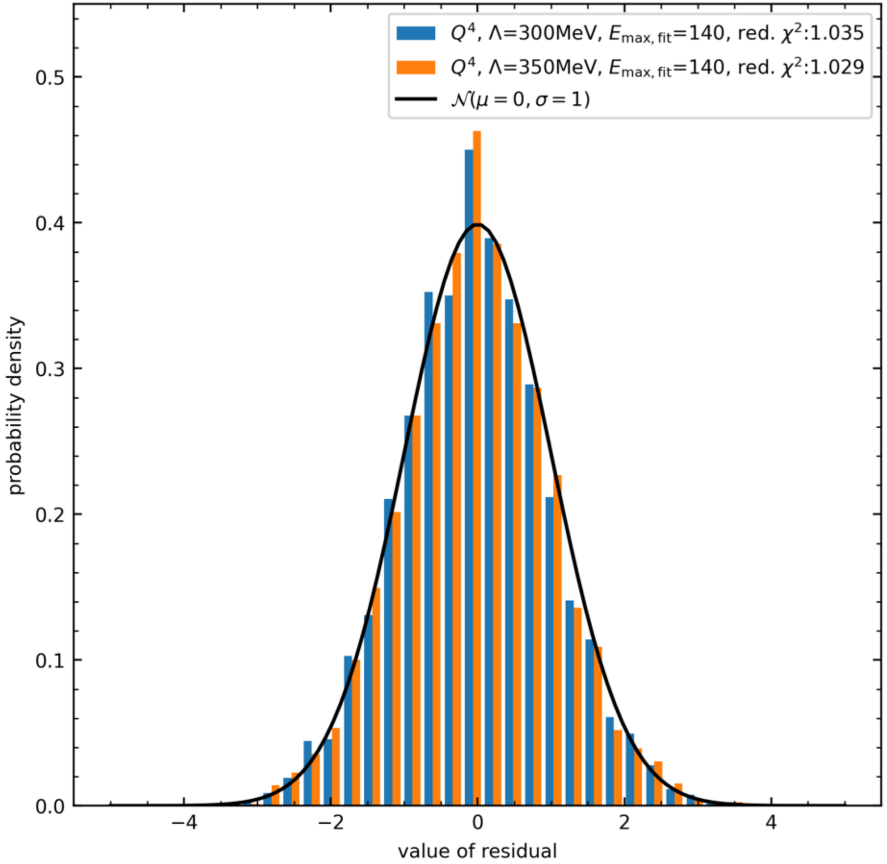
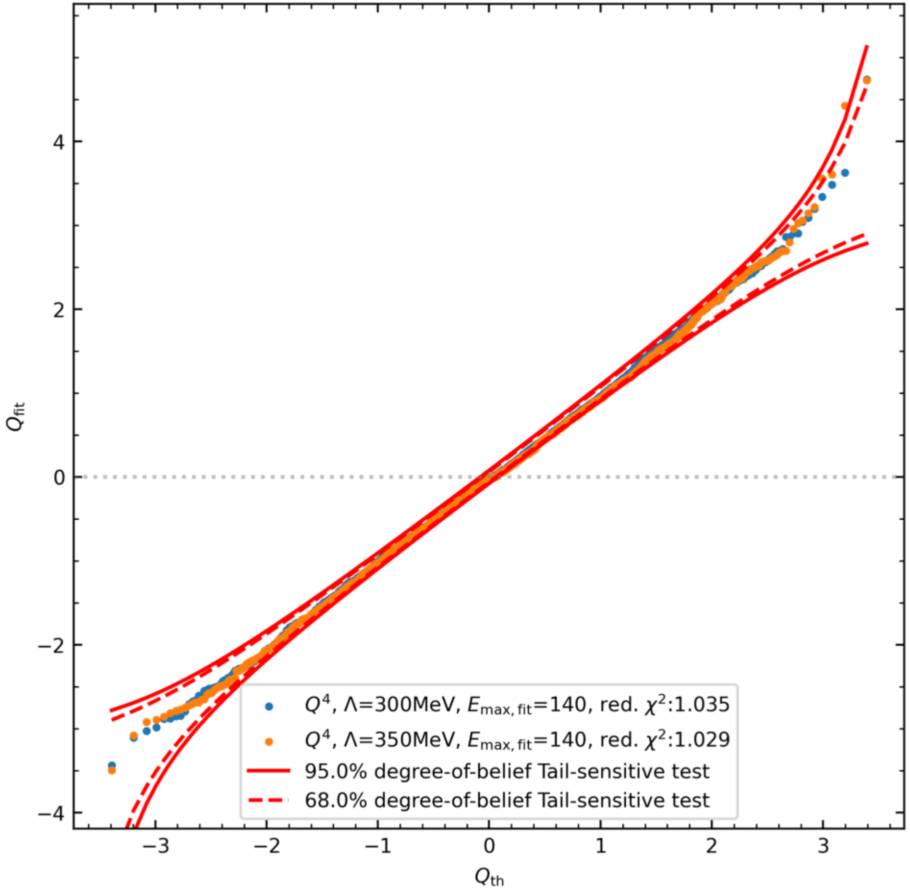
Statistical verification from histogram

- red. $\chi^2 = \Sigma_i R_i^2 / N_{\mathrm{dat}}$ is just one number
- We need to verify that the residuals of the fit follow a normal distribution with $\mu = 0, \sigma = 1$
- Compare Histogram of residuals with normal distribution



Statistical verification from QQ-Plots

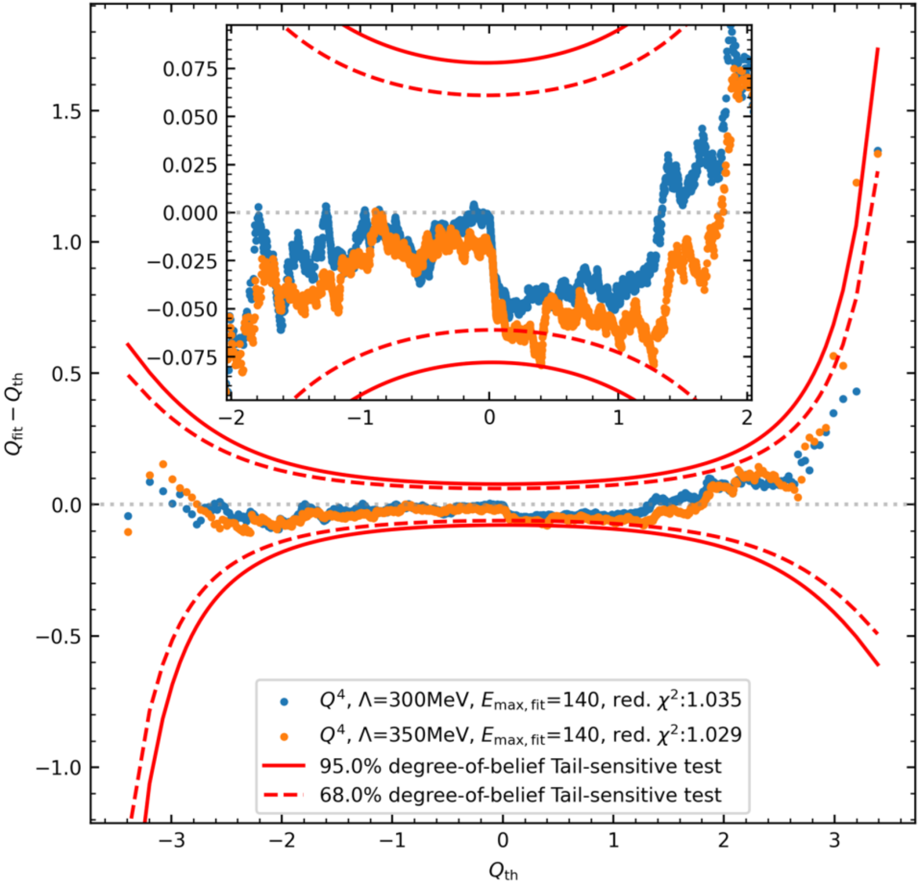
- red. $\chi^2 = \Sigma_i R_i^2 / N_{\mathrm{dat}}$ is just one number
- We need to verify that the residuals of the fit follow a normal distribution with $\mu = 0, \sigma = 1$
- Make a quantile-quantile plot



Statistical verification from QQ-Plots

- red. $\chi^2 = \Sigma_i R_i^2 / N_{\text{dat}}$ is just one number
- We need to verify that the residuals of the fit follow a normal distribution with $\mu = 0, \sigma = 1$
- Make a quantile-quantile plot
  - Rotate it
  - Compare against Tail-sensitive test

→ Verify that potential actually describes data, to which it is fitted!



Correlation matrix

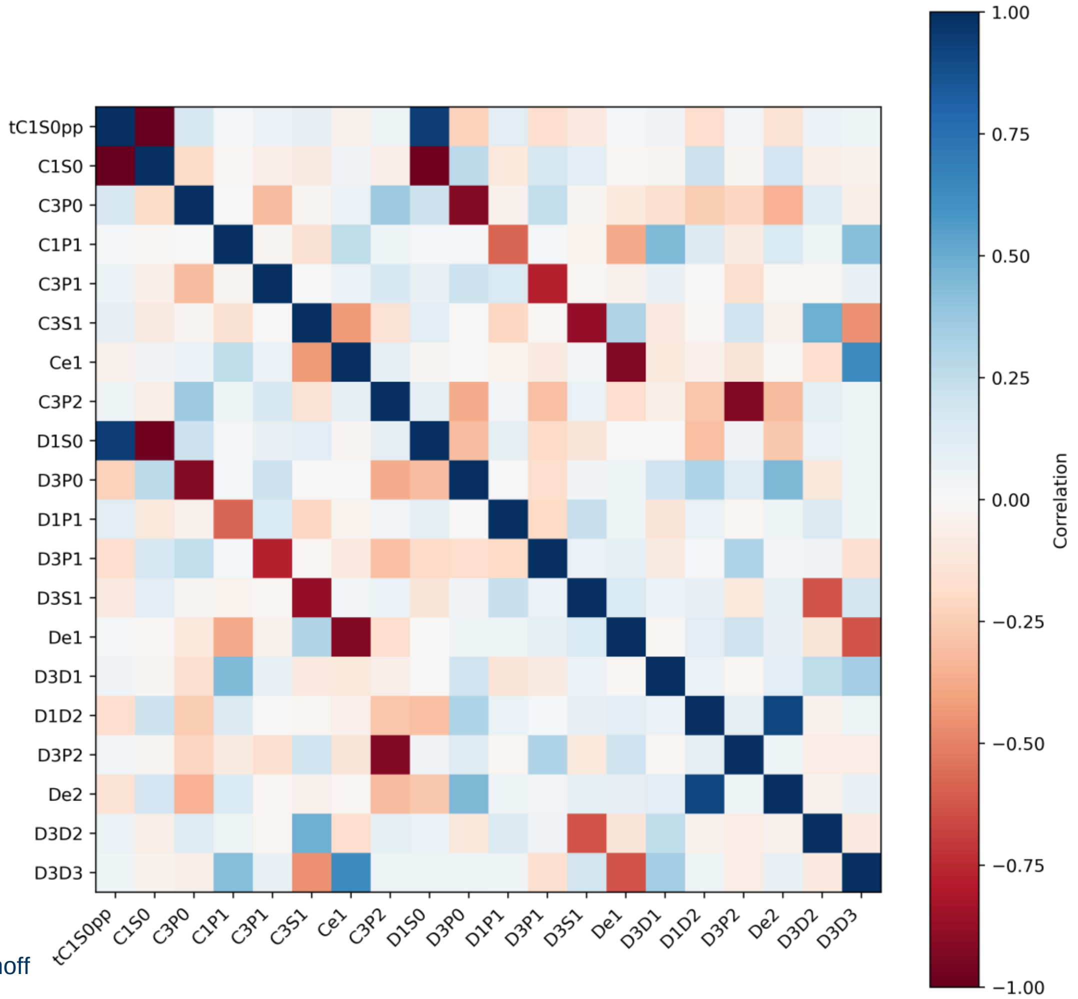
- Started each fit several times with different starting values

→ Always end up in same minimum

→ … or very, very large red. χ^2

- $\tilde{C}_{3S1}$ and $\tilde{C}_{1S0}^{np}$ are eliminated from constraints (Deuteron BE an coh. scatt length)

- Correlation Matrix for N2LO, 300MeV



Determination of $E_{\mathrm{max,fit}}$ - Conclusions

- Higher Order in Expansion → Fit up to higher energies possible
  - This comes at the **high price of many more contact interactions**
  - Q^0 : Fit up to $E_{\mathrm{lab}} = 10\ \mathrm{MeV}$ ← self consistency not possible
  - Q^2 : Fit up to $E_{\mathrm{lab}} \sim 30\ \mathrm{MeV}$ Q^4 : Fit up to $E_{\mathrm{lab}} \sim 140\ \mathrm{MeV}$ Q^6 : Fit up to $E_{\mathrm{lab}} \sim 195\ \mathrm{MeV}$

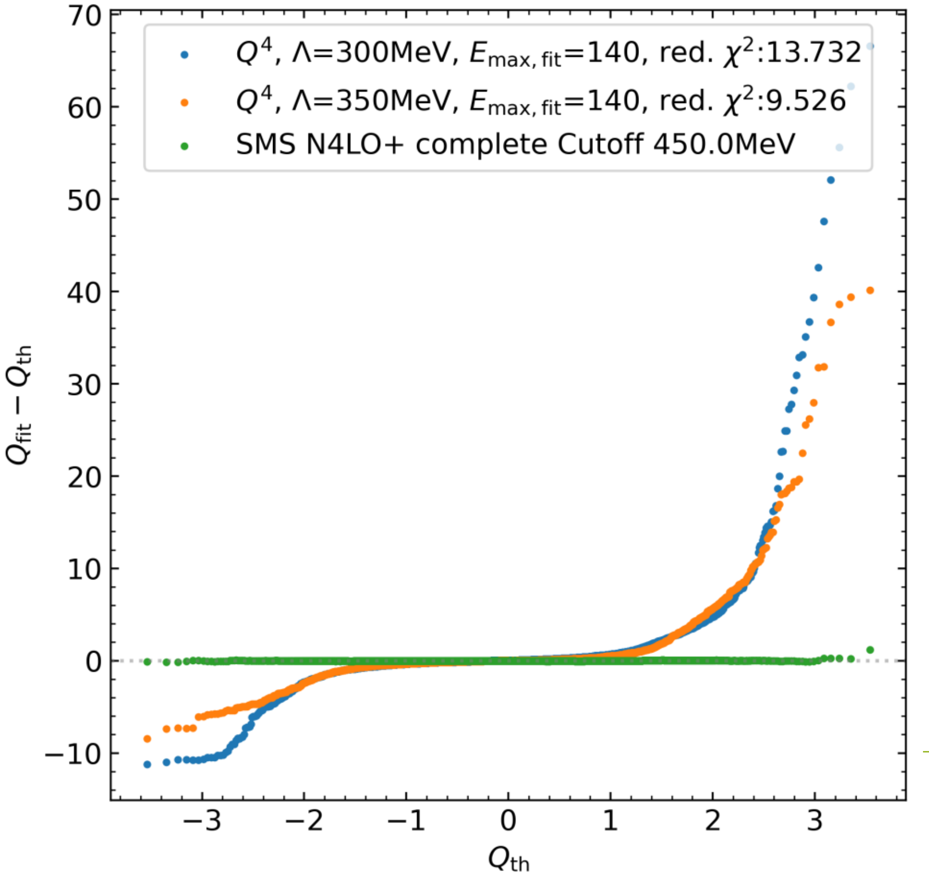
- **Cutoff around 250-400MeV** leads to a good description of data
  - Higher and smaller cutoffs worsen the description (drastically)

- Comparisons with **chiral EFT**
  - SMS potential with 27 LECs, describes NN data up to 280MeV with red. $\chi^2 \sim 1$ (P. Reinert PhD thesis, 2022)
  - Norfolk potentials at N(3)LO yield red. $\chi^2 \sim 1$ up to 125MeV (Phys. Rev. C 94, 054007) – with 26 (instead of 21(+2)) parameters including Delta, TPE, etc.

Why then care about chiral symmetry?

- **Low-Resolution potential** at Q^4 has **22 LECs**, describes all NN scattering data up to 280MeV with **red.** $\boldsymbol{\chi^2 \sim 10}$
- Bochums **SMS chiral EFT** at N4LO+ has **27 LECs**, describes all NN scattering data up 280MeV with **red.** $\boldsymbol{\chi^2 = 1.022}$

→ This is the effect of the 2-pion exchange**,** i.e. **parameter-free prediction of chiral symmetry**



Methodology
Uncertainty Quantification

Three important sources of uncertainties

- Uncertainties of the fitting protocol of the LECs
  - Numerical approximations (especially in EM contributions)
  - Fitting algorithm
  - Physical constants
  - Data selection

- Statistical and systematical uncertainties of the data itself

- **Theory uncertainty**
  - Impact of neglecting higher orders of chiral /low-resolution EFT

In this talk

- **Aim**: probability distribution $\boldsymbol{p}\left(\boldsymbol{X}^{(\boldsymbol{k})}\right)$ of observable X at some order k of low-resolution EFT.

Method

- How to obtain $p\left(X^{(k)}\right)$?

  1.) Include contact diagrams of next higher order $\mathrm{k}+1$ to potential

  2.) Set LECs $\vec{a}^{(k+1)}$ to natural values

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  3.) Refit all lower order LECs $\vec{\mathrm{a}}^{(i\,\epsilon\,[0,\mathrm{k}])}$

  4.) Repeat m times for different LECs of next higher order

  → m different values for $X^{(k)}$ ~ sample of $p\left(X^{(k)}\right)$

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- Mathematically: Explicitly integrating out the diagrams of order $\mathrm{k}+1$

$$p\left(X^{(k)}\right) = \int \mathrm{d}\,\vec{a}^{(k+1)} \cdot \delta\left(X^{(k+1)}\left(\vec{\mathrm{a}}^{(i\,\epsilon\,[0,\mathrm{k}+1])}\right) - X^{(k)}\right) \cdot p\left(\vec{\mathrm{a}}^{(i\,\epsilon\,[0,\mathrm{k}+1])}\right)$$

- Uncertainty: width of $p\left(X^{(k)}\right)$

$p\left(X^{(k)}\right)$ is not only truncation uncertainty

Theory

- Uncertainty of next-higher order contact forces

Data

- Stat. and sys. uncertainties
- Distribution of data among observables and energy ranges

Beliefs

- Natural LECs
- No inclusion of theory‘s precision predicition in fit
- Fit protocol
- …

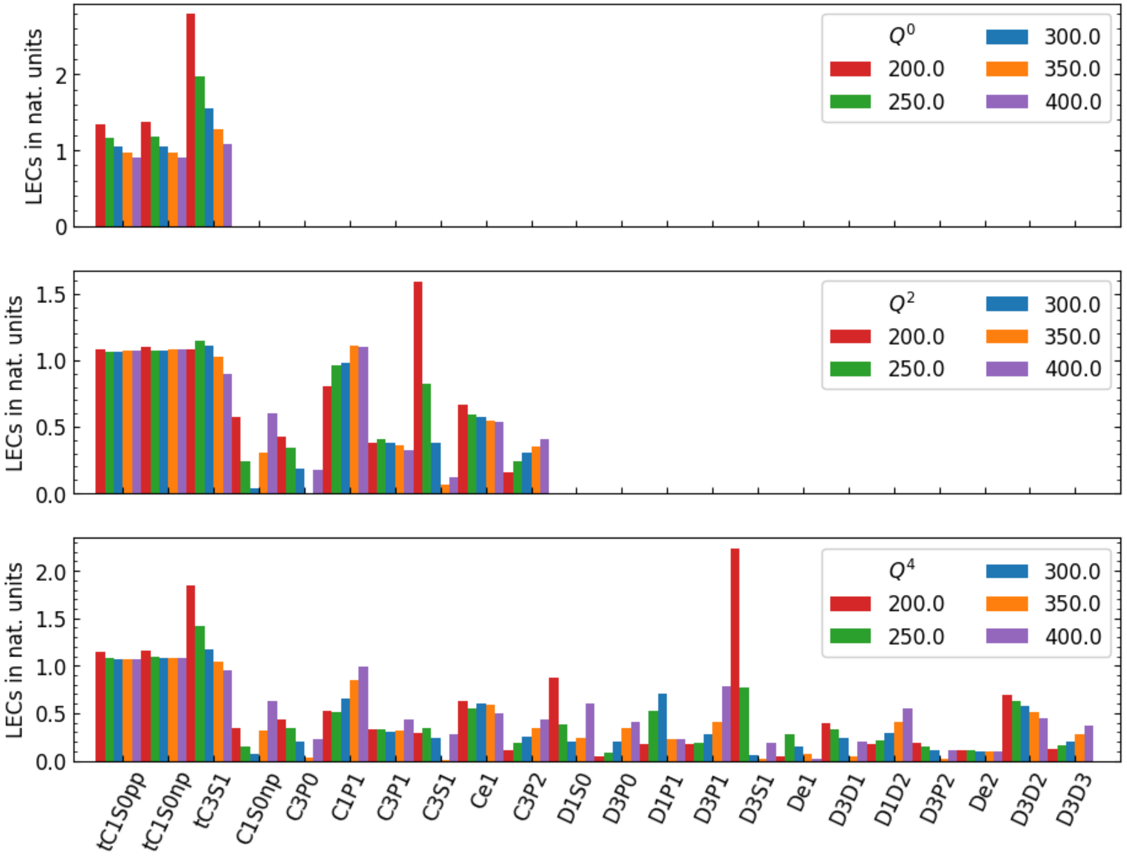
$$p\left(X^{(k)} \mid \ldots \right)$$

Preliminary Results
2-Body system

Naturalness of the Low-Resolution EFT

- Naturalness: Every physical parameter should be in the order of 1
  - Compared to a reasonable physical scale
  - No proof, but generally accepted idea in physics

- We estimate, LECs are of size:
  - $c \sim 4\pi/(F_\pi^2 \Lambda^n)$, where n is the Q^n at which LEC appears first



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- We estimate, LECs are of size:
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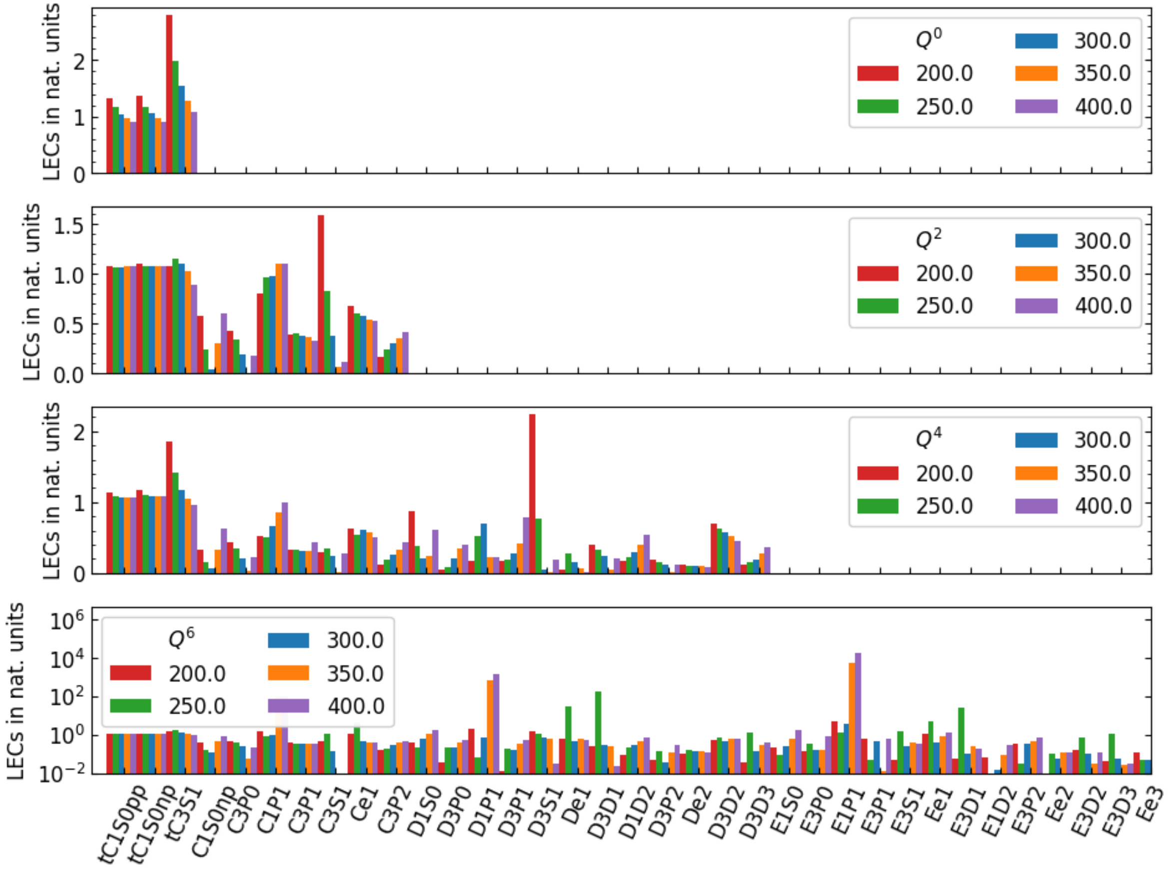
- LECs of Q^6 are not natural!
- LECs of larger cutoff are not natural!



Infer the natural scale for the LECs

- Needed for explicit uncertainty estimation to set values for higher order LECs

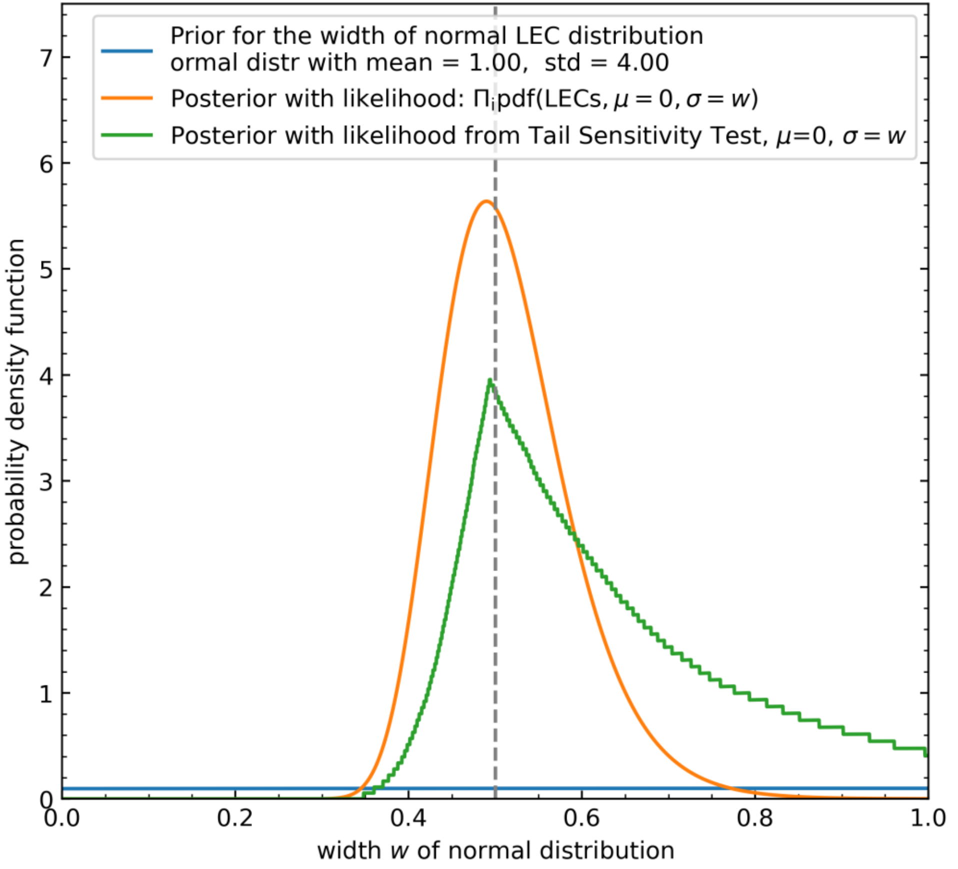
  Bayes theorem:

  $$p(w \mid LECs) \propto p(LECs \mid w) \cdot p(w)$$

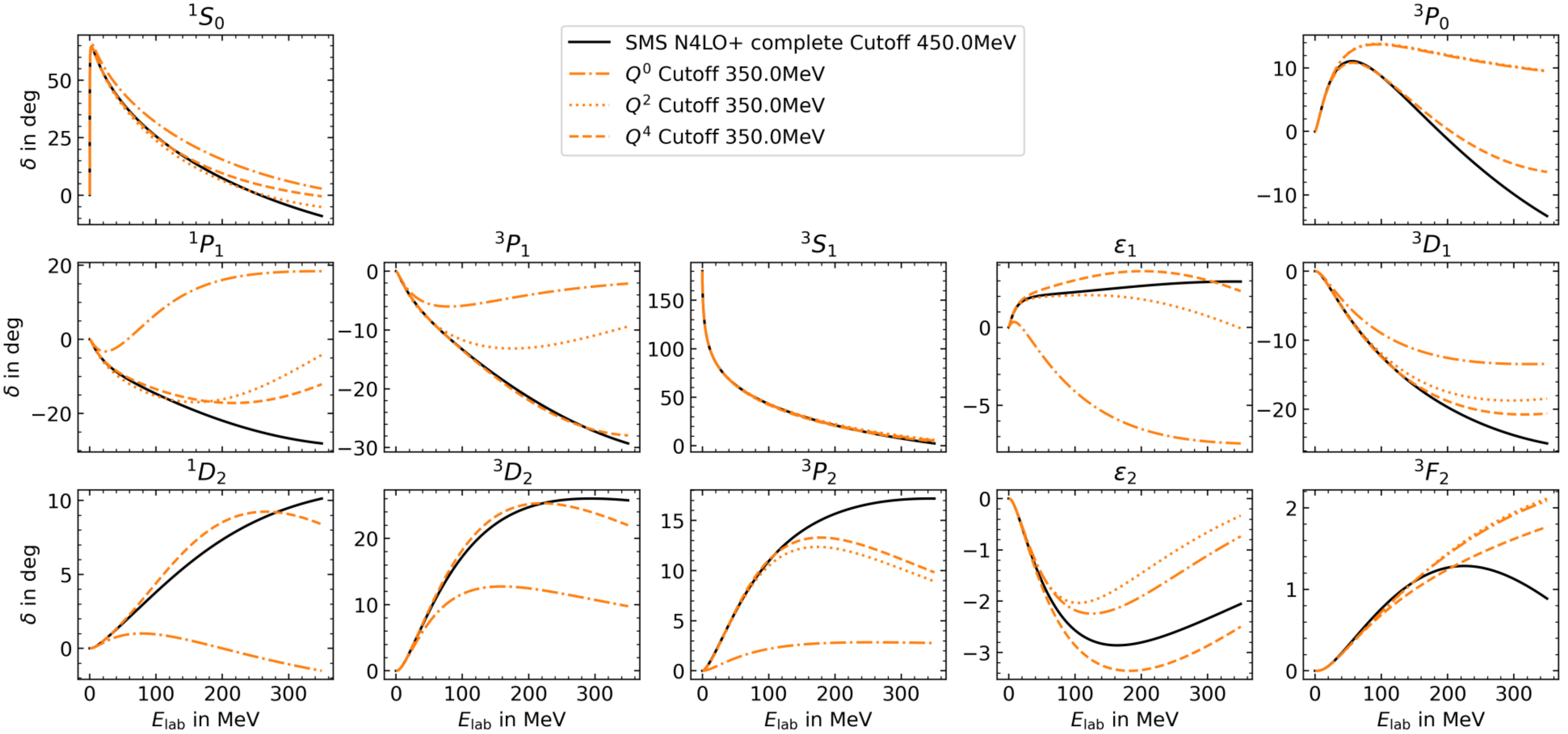
  w is the width of normal distribution

- If assumptions are fulfilled, the **LECs** are indeed drawn from a **Gaussian distribution with $\boldsymbol{\sigma \sim 1}$**

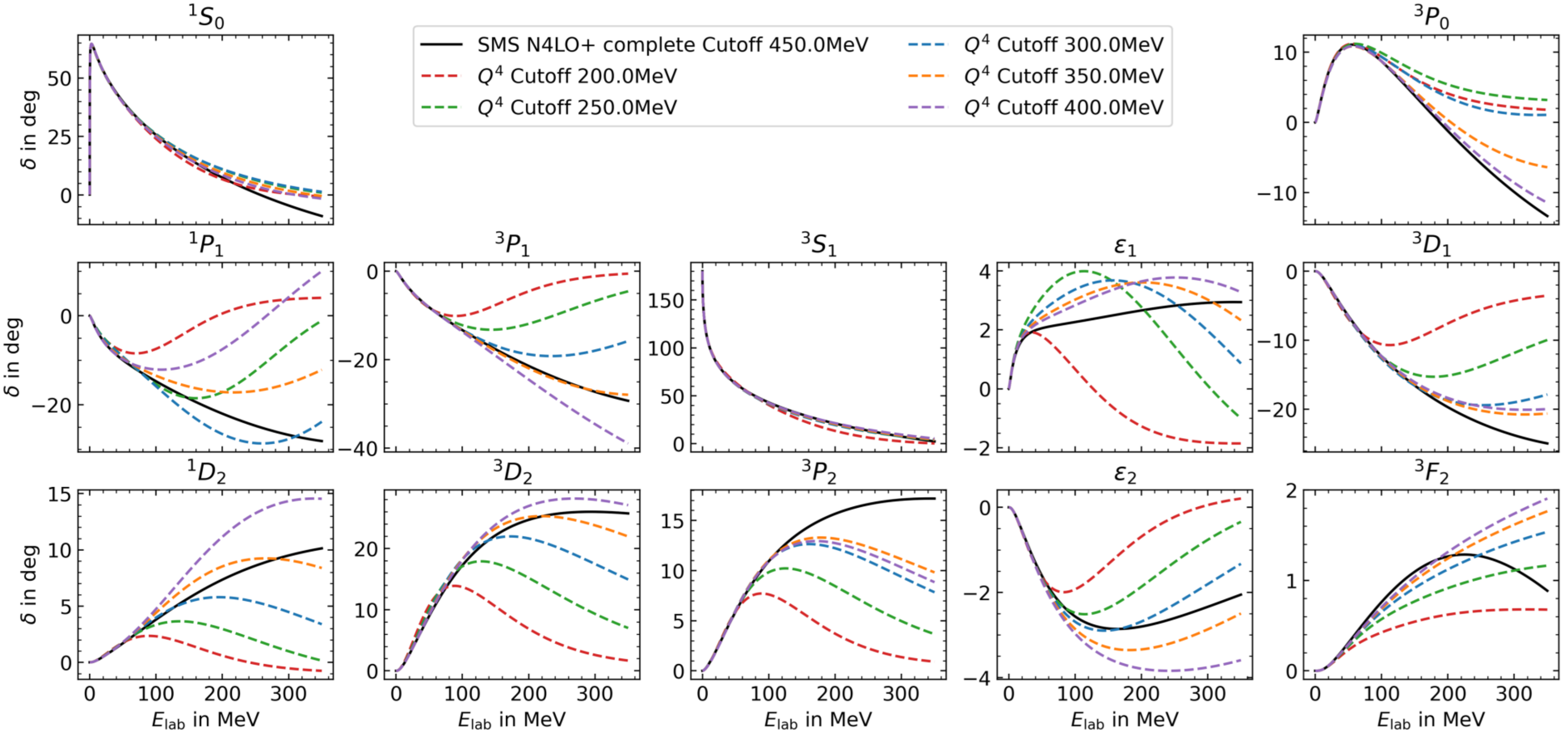
- Here LECs of Q^4 300MeV



Phaseshifts – order by order



Phaseshifts – cutoff variation Q^4



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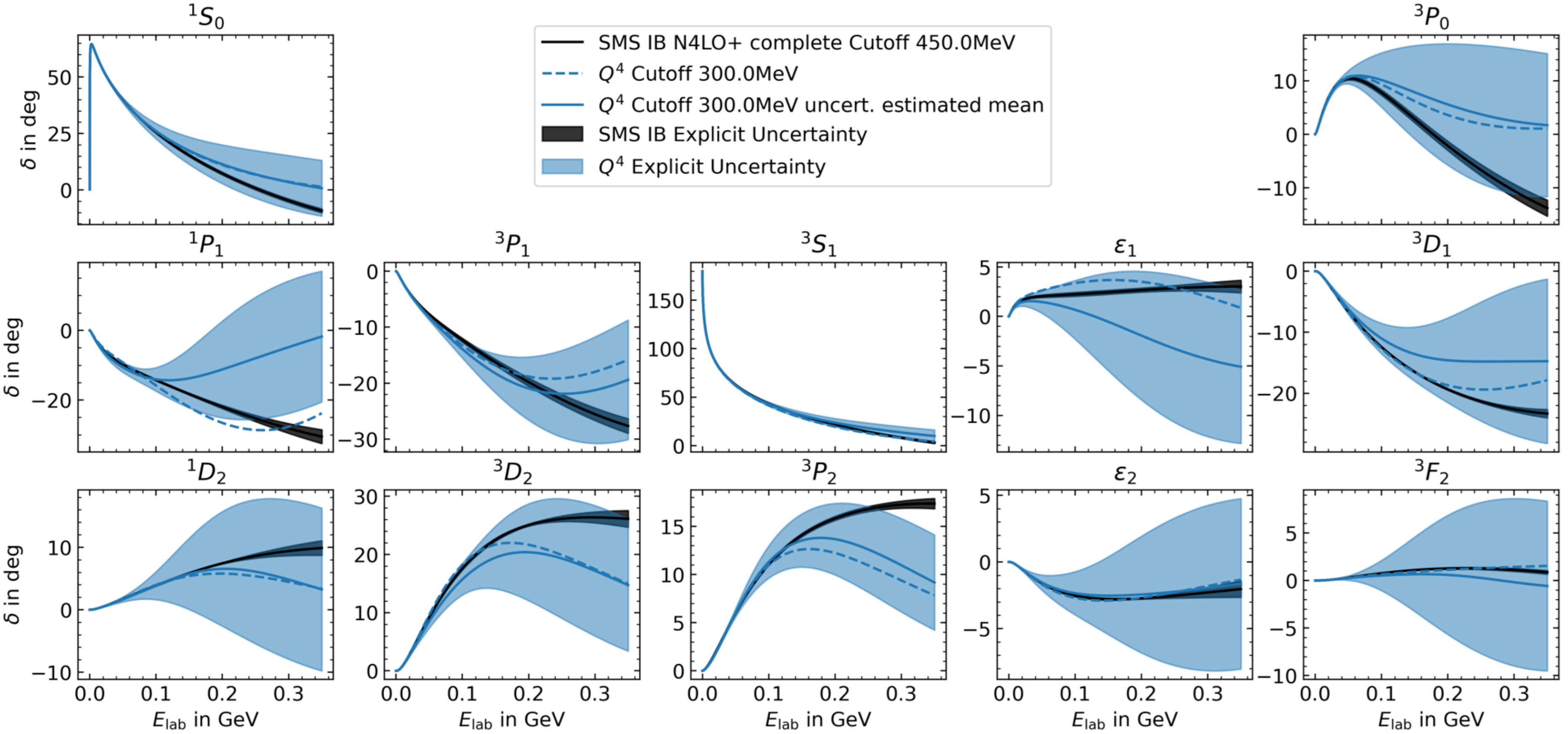
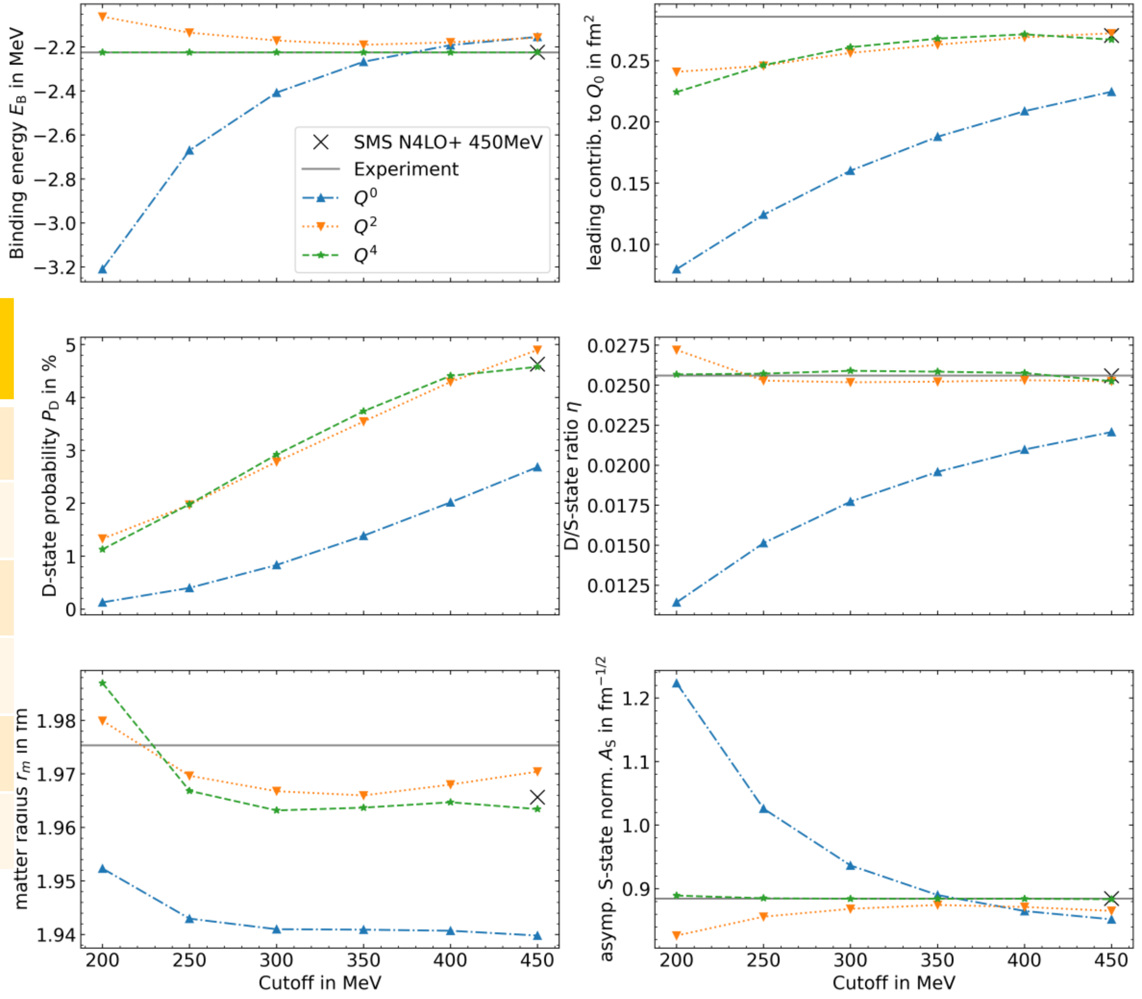
Phaseshifts – Uncertainty bands



Deuteron properties

- P_D is **non-observable**!
- Full Q needs scheme-dep. additions
- Estimated explicit uncertainties:

	Q^4 **LowRes** Λ **=300MeV**	**N4LO+ SMS IB** Λ **=450MeV**
E_B	2.2225 ± 0 MeV	2.2225 ± 0 MeV
Q_0	$0.252 \pm 0.021\mathrm{fm}^2$	$0.27502 \pm 0.0029\mathrm{fm}^2$
r_m	1.961 ± 0.004 fm	1.9662 ± 0.0011 fm
η	0.0252 ± 0.0018	0.02606 ± 0.00022
A_S	0.88631 ± 0.0010	0.88454 ± 0.00020
P_D	2.7 ± 0.7 %	$4.79 \pm 0.18\%$

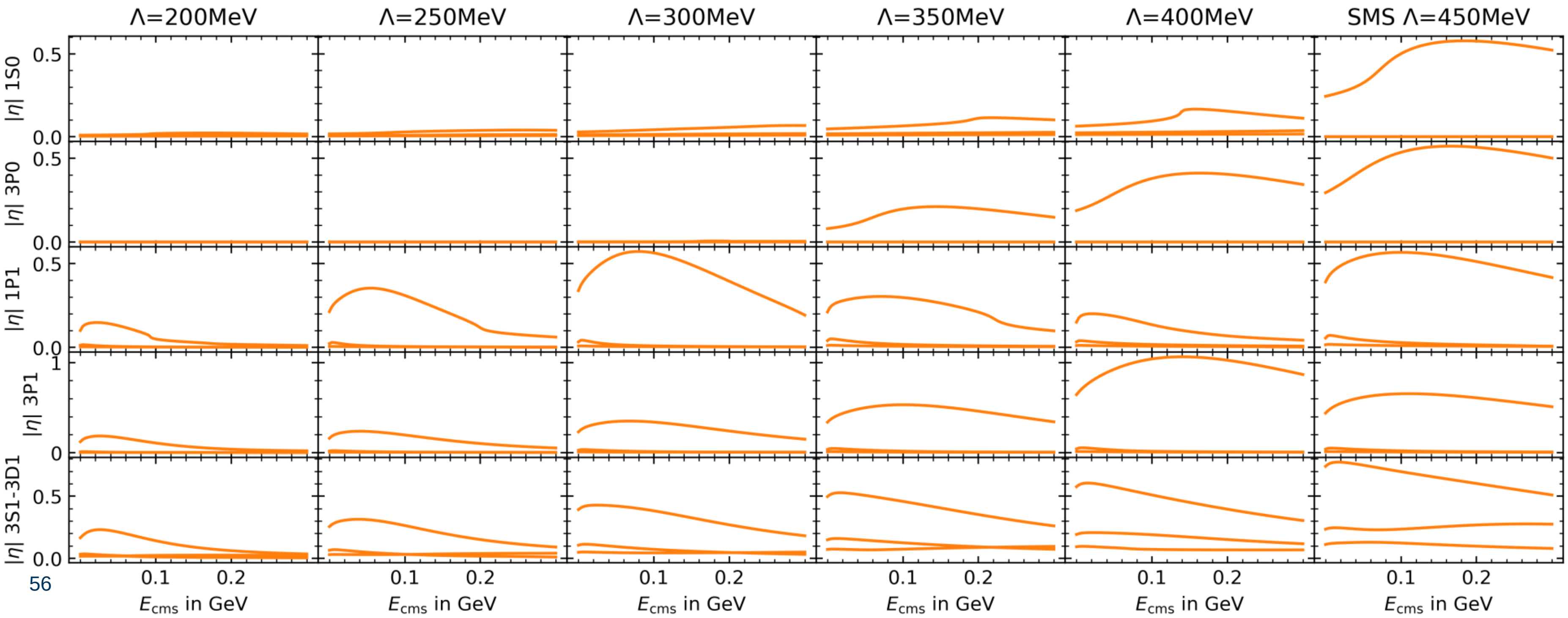


Weinbergs eigenvalues of Q^4

Lippmann-Schwinger eq.
$T = V + VG_0T$

- Due to **lower cutoff** values, potentials are very soft
- Softness is characterized by **smallness of repulsive eigenvalues** η of $V\mathrm{G}_0$(“Weinbergs eigenvalues”)

→ Determines how quickly Neumann series converges (iterative sol. of Lippmann-Schwinger equation)

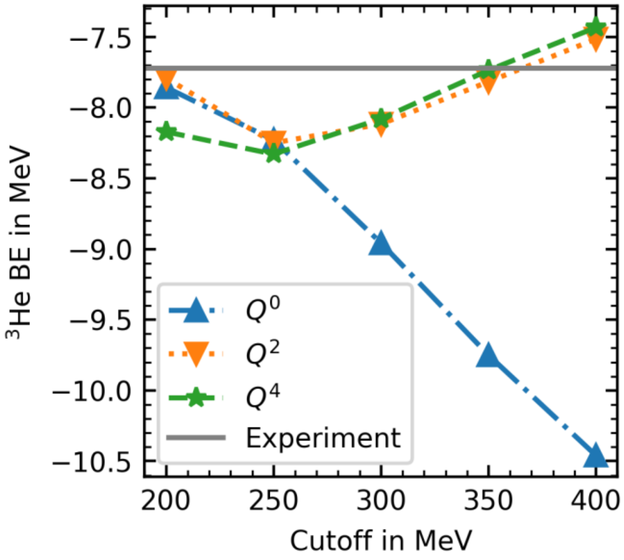


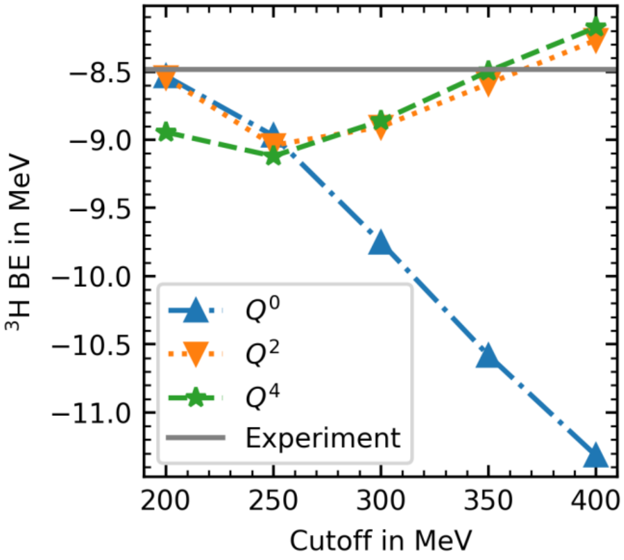
Preliminary Results
3-Body system and beyond

3H and 3He Binding energies

- Expected size of 3NFs ~ few hundred keVs, cutoff 300MeV at Q^4 gets BE almost perfect right

  → Comparable to leading 3NF in chiral EFT





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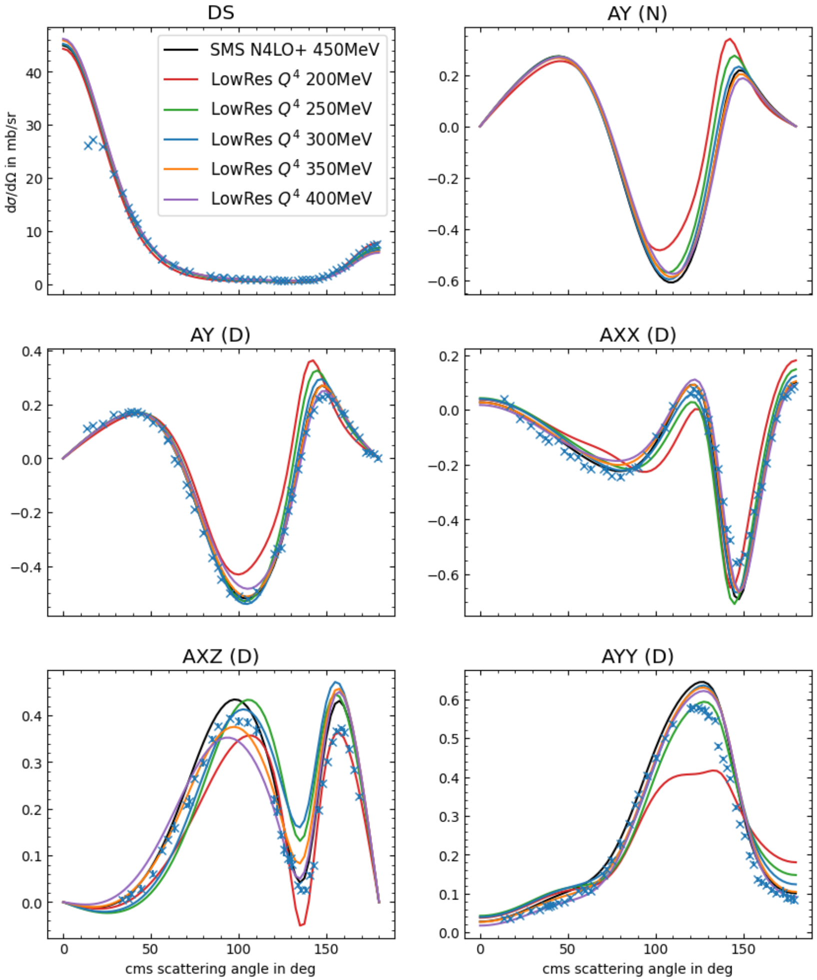
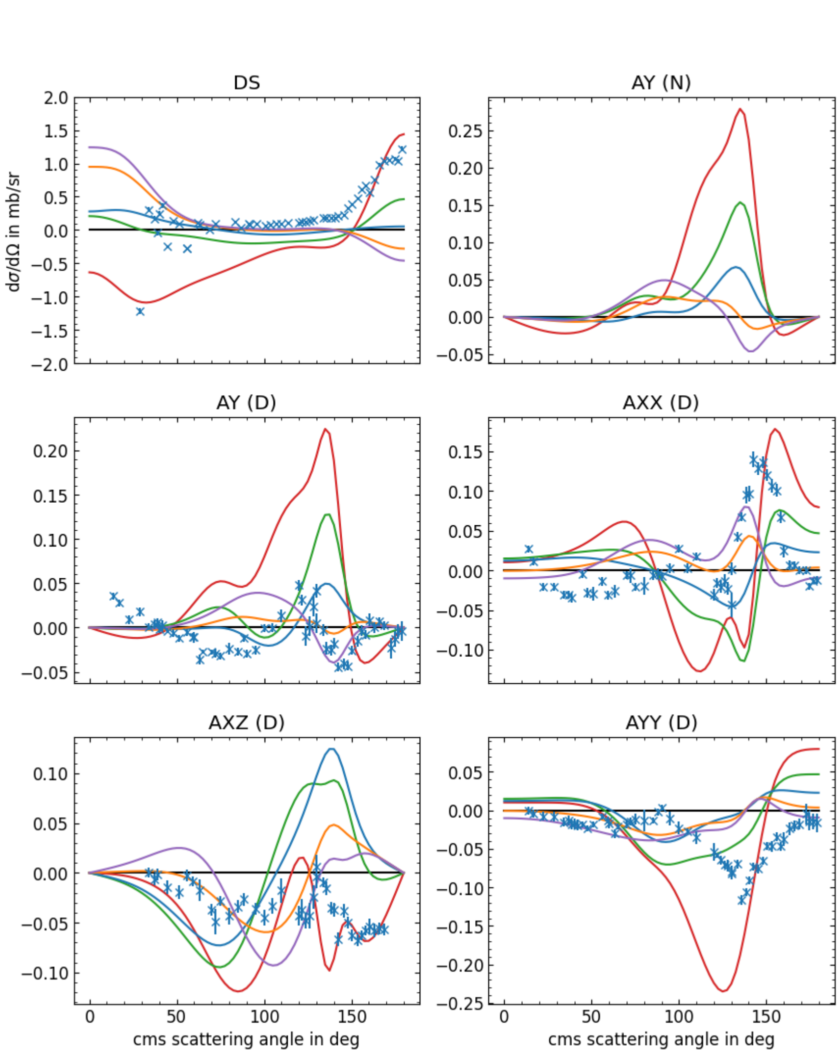
Nd Scattering at 70MeV



Subtract SMS N4LO+ 450MeV from everything



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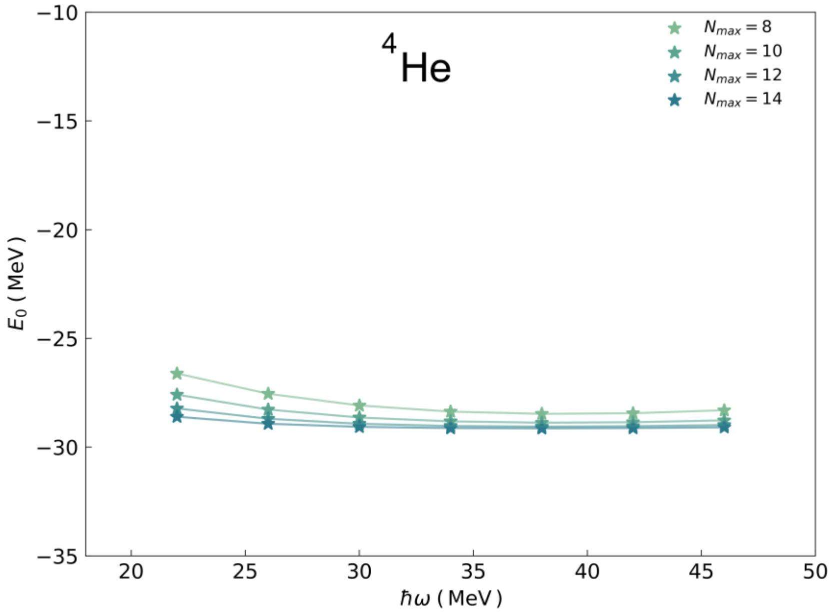
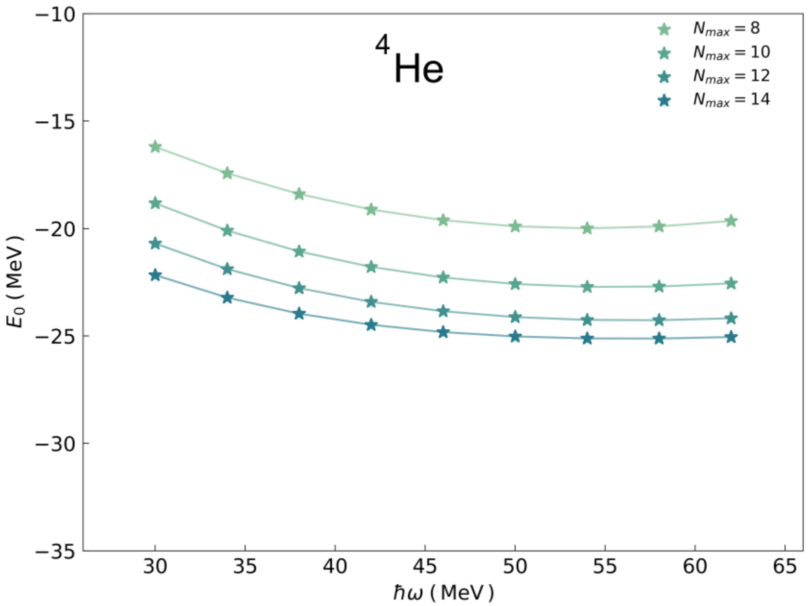
- Many thanks to 商浩宇 (Haoyu Shang)!

No-core-shell-model (NCSM) used



Low-Resolution Potential Cutoff:350MeV



Bochums Chiral EFT SMS Cutoff:500MeV

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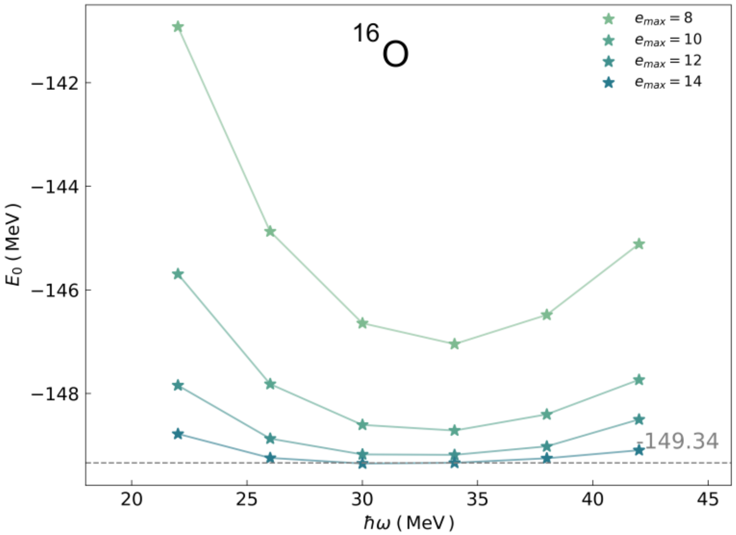
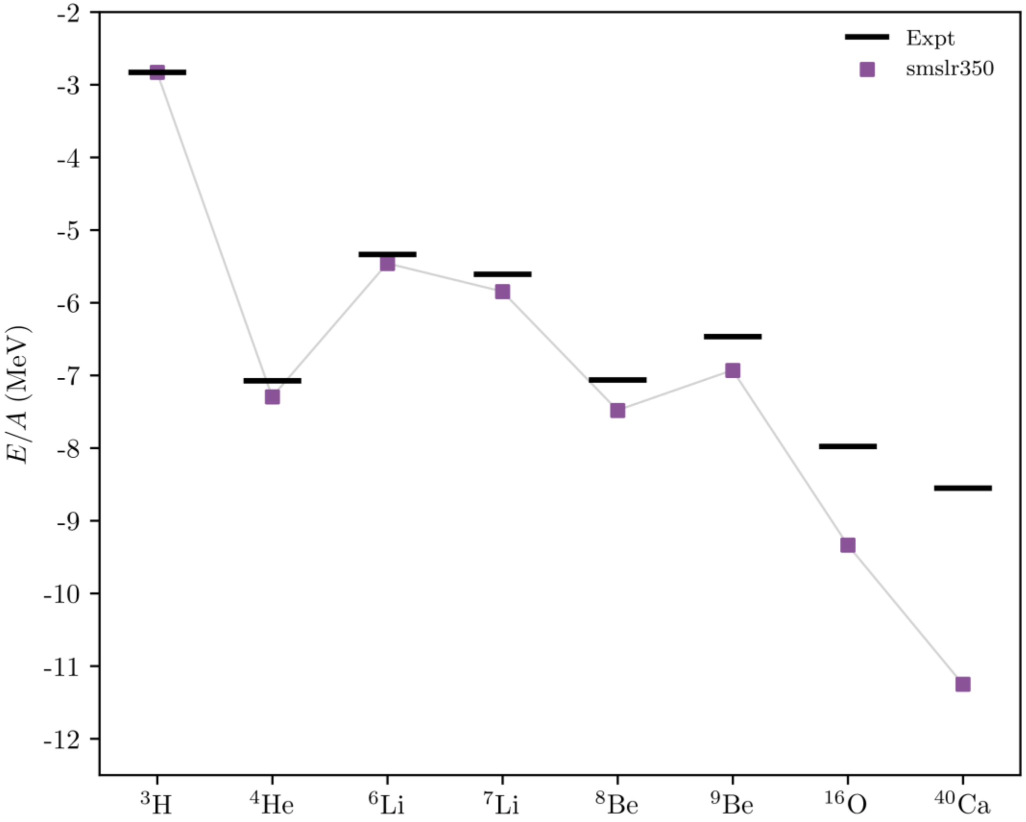
- Many thanks to 商浩宇 (Haoyu Shang)!
- NCSM for smaller, IMSRG for larger nuclei
- 3NF enough or TPE needed?





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Conclusion and Outlook

- Low-Resolution EFT is **EFT between pionless EFT and chiral EFT** and very **soft** (Cutoff 200-400MeV)
- “Unexpectedly" well working (good description of low-energy few-body observables), **why?**
- Next: **3N-Forces** and their LECs, many body calculations (cheaper, because of softness ?), determination of Λ_{B} from convergence of the EFT, uncertainty estimation in many-body system and all orders/cutoffs
- What are *‘low-energy’* observables?

- Take-away-messages

1. **High-precision, High-energy** observables

→ No way around Chiral EFT, including chiral symmetry

2. Less precise, Low-energy, but **low computational cost**, i.e. less detailed picture

→ Low-Resolution potentials