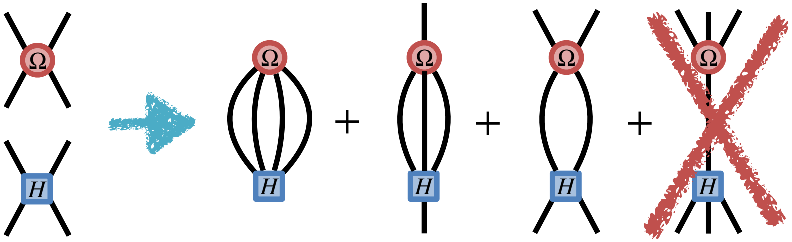
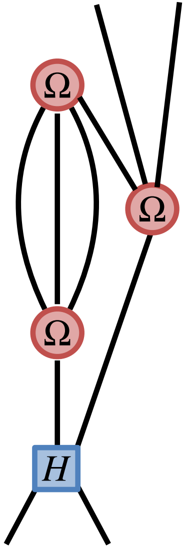
Towards an understanding of truncation errors in the IMSRG



Ragnar Stroberg

Challenges in Effective Field Theory
Descriptions of Nuclei
Hirschegg, Austria
January 18-24 2026



$$\begin{pmatrix} 0 & 1 & 3 & 0 \\ 1 & 0 & 0 & 1 \\ 3 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

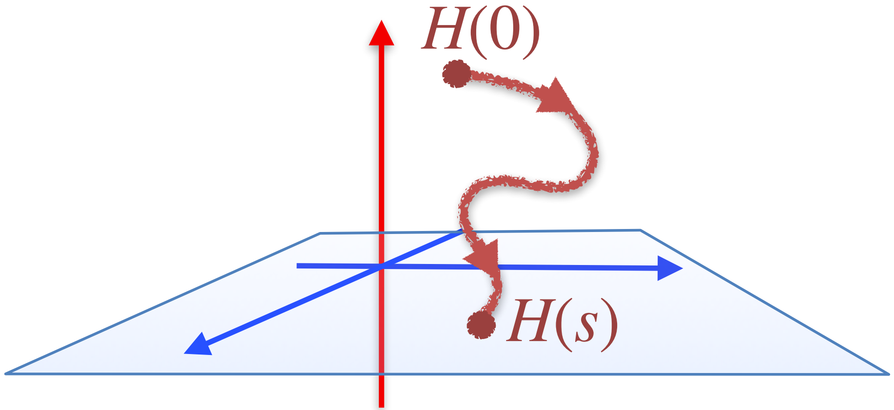
Work done with Bingcheng He, Andre Johnson, and Victor Vaida

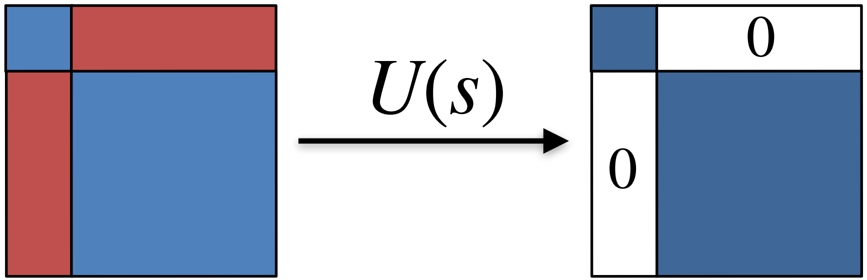
In-Medium Similarity Renormalization Group (IMSRG)

unitary transformation $$H(s) = U(s) H U^{\dagger}(s)$$

SRG flow equation $$\frac{dH}{ds} = [\eta(s), H(s)]$$

$$H = H^{\mathrm{d}} + H^{\mathrm{od}}$$





$$H^{\mathrm{od}}(s) \rightarrow 0$$

How do we estimate the truncation error?

Flow

$$\frac{dH}{ds} = [\eta, H]$$

⟷

Magnus

$$H(s) = e^{\Omega(s)} H e^{-\Omega(s)}$$

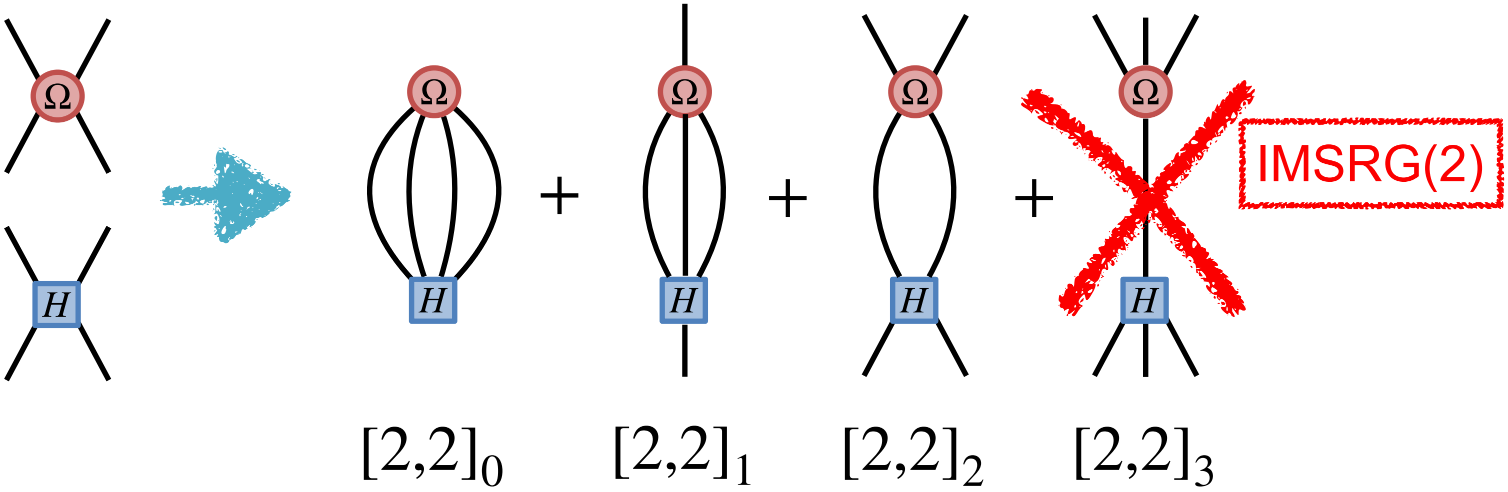
$$= H + [\Omega, H] + \frac{1}{2}[\Omega, [\Omega, H]] + \ldots$$

Assume Ω is given (and only 2-body).
How accurately are we evaluating $H(s)$?

Magnus IMSRG

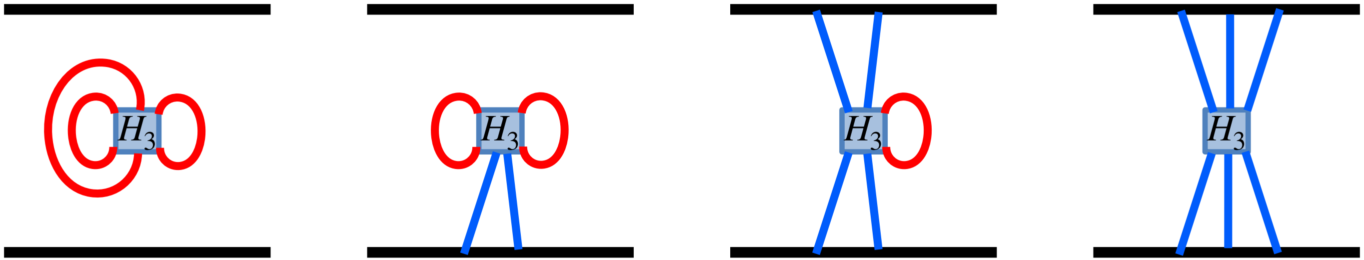
$$H(s) = e^{\Omega(s)} H e^{-\Omega(s)}$$

$$= H + [\Omega, H] + \frac{1}{2!}[\Omega, [\Omega, H]] + \frac{1}{3!}[\Omega, [\Omega, [\Omega, H]]] + \ldots$$



Why is it ok to throw away 3-body terms?

NO0b	NO1b	NO2b	NO3b

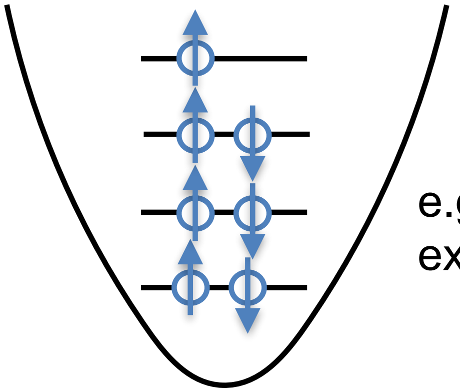


$\sim A^3 N_{qp}^0$	$\sim A^2 N_{qp}^1$	$\sim A^1 N_{qp}^2$	$\sim A^0 N_{qp}^3$

Systematic if $N_{qp} \ll A$

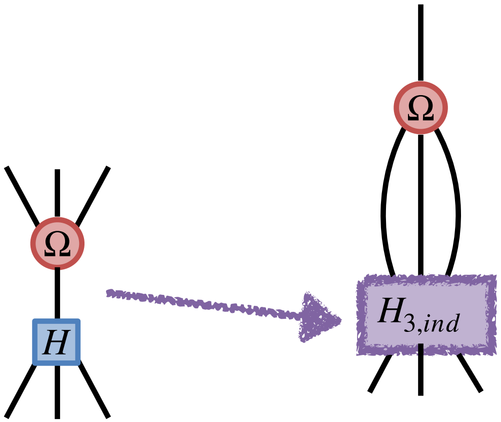
When doesn't that work?

1) Bulk contribution is not additive



e.g. magnetic moment, excitation energy

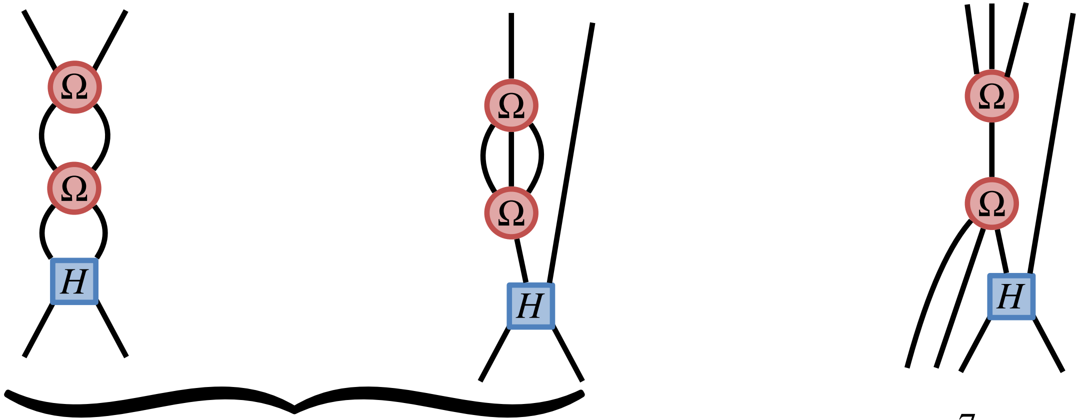
2) Flowing 3b feeds back into 0,1,2b



The IMSRG($3f_2$) approximation

$$[\Omega,[\Omega,H]] = \underbrace{[\Omega,[\Omega,H]_{1,2}]_{0,1,2}}_{\text{IMSRG(2)}} + \underbrace{[\Omega,[\Omega,H]_3]_{1,2}}_{\text{IMSRG}(3f_2)} + [\Omega,[\Omega,H]_3]_{3,4}$$

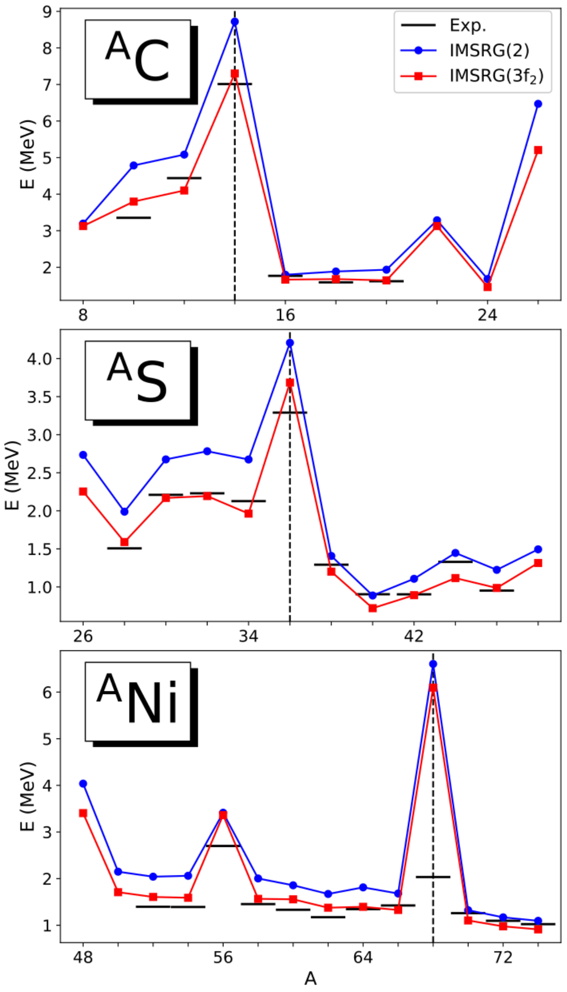
Bingcheng He

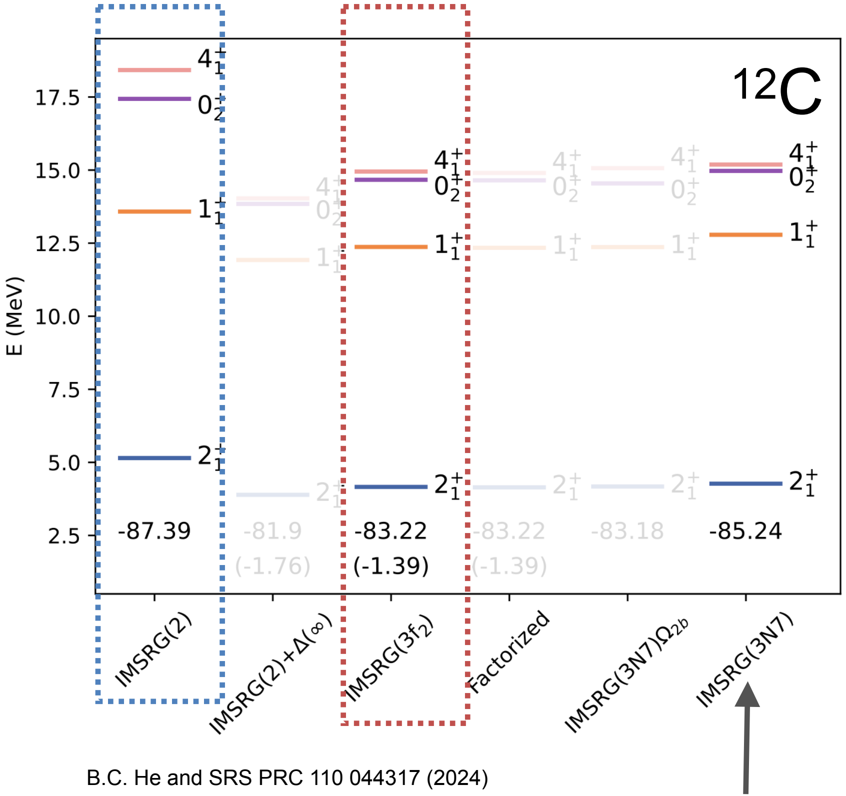


$\sim N^6$ scaling

$\geq N^7$ scaling

The IMSRG($3f_2$) approximation



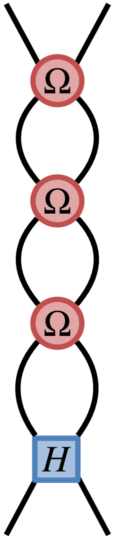


B.C. He and SRS PRC 110 044317 (2024)

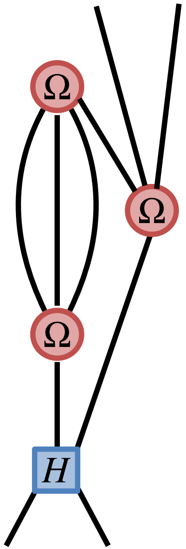
best calculation

Beyond 2 nested commutators: assessing importance

$$[\Omega, [\Omega, [\Omega, H]]]$$



$$[2,[2,[2,2]_2]_2]_2$$
$$\sim N^6$$



$$[2,[2,[2,2]_3]_4]_2$$
$$\sim N^9$$

Expensive! But is it negligible?

(Actually, it can be factorized to $N^5+N^5+N^6$, but there are **hundreds** of different diagrams.)

Features that may affect the importance a diagram

- Topological coherence (independent of H)
- Dynamical coherence (depends on H)
- Compatibility with form of Ω (e.g. $pphh$)
- (Approximate) symmetries like $SU(4)$ $\Rightarrow$loop enhancement

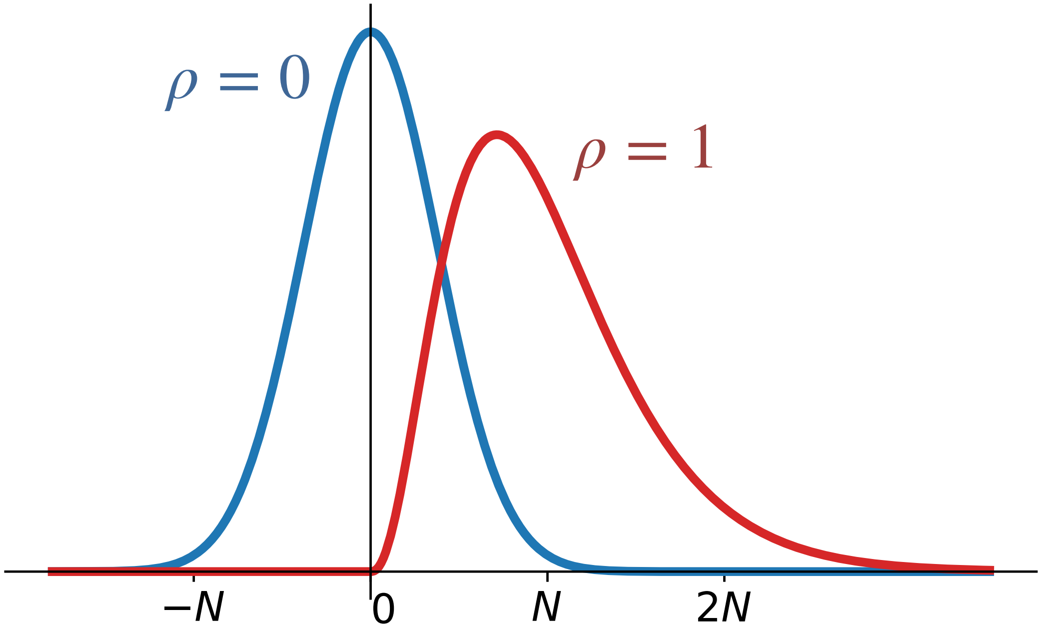
Coherent enhancement

x_i, y_i are Gaussian random variables with $\sigma^2 = 1$, covariance ρ

$$z = \sum_{i=1}^{N} x_i y_i$$

$$\langle z \rangle = \rho N$$

$$\sigma_z^2 = (1 + \rho^2) N$$



$$\langle z^2 \rangle = \langle z \rangle^2 + \sigma_z^2$$

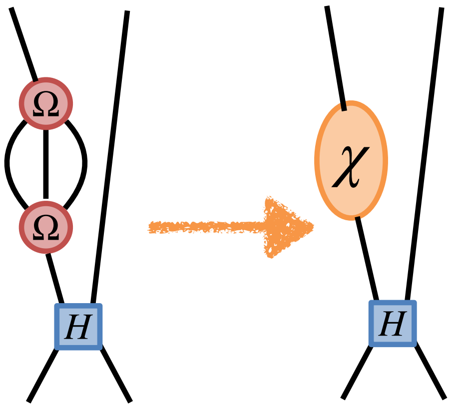
Expected size of z:

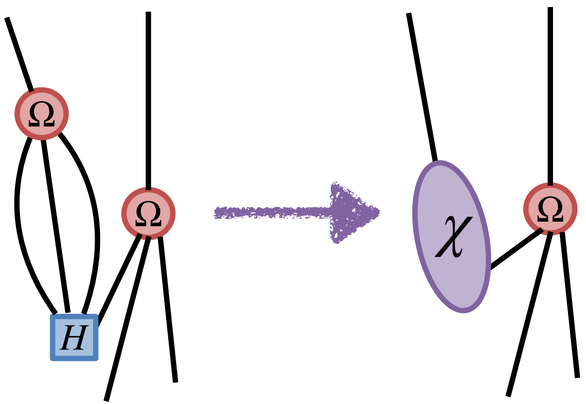
$$\langle z^2 \rangle^{1/2} = \begin{cases} \sqrt{N}, & \rho = 0 \\ \sim N, & \rho = 1 \end{cases}$$

$\Rightarrow$ enhancement by $\sqrt{N}$

Topological coherence (plus some symmetry)

$\Rightarrow$enhance triple connections





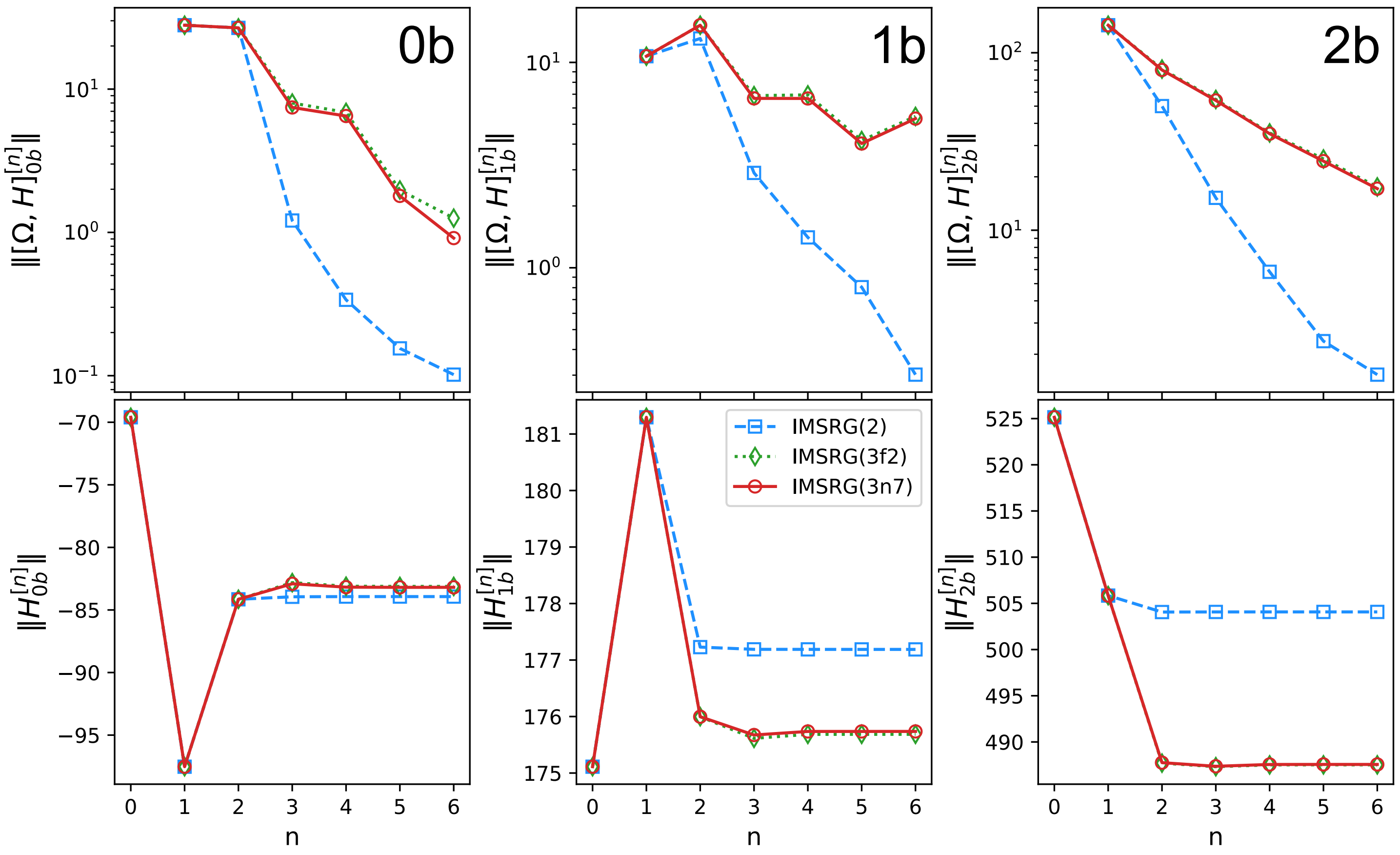
$$\chi_{ii} \sim \sum_{abc} \Omega_{iabc}\Omega_{bcai} = -\sum_{abc} |\Omega_{iabc}|^2$$

coherent $\sim \sqrt{n_{\text{sp}}^3} = n_{\text{sp}}^{3/2}$

$$\chi_{ii} \sim \sum_{abc} \Omega_{iabc}H_{bcai}$$

incoherent

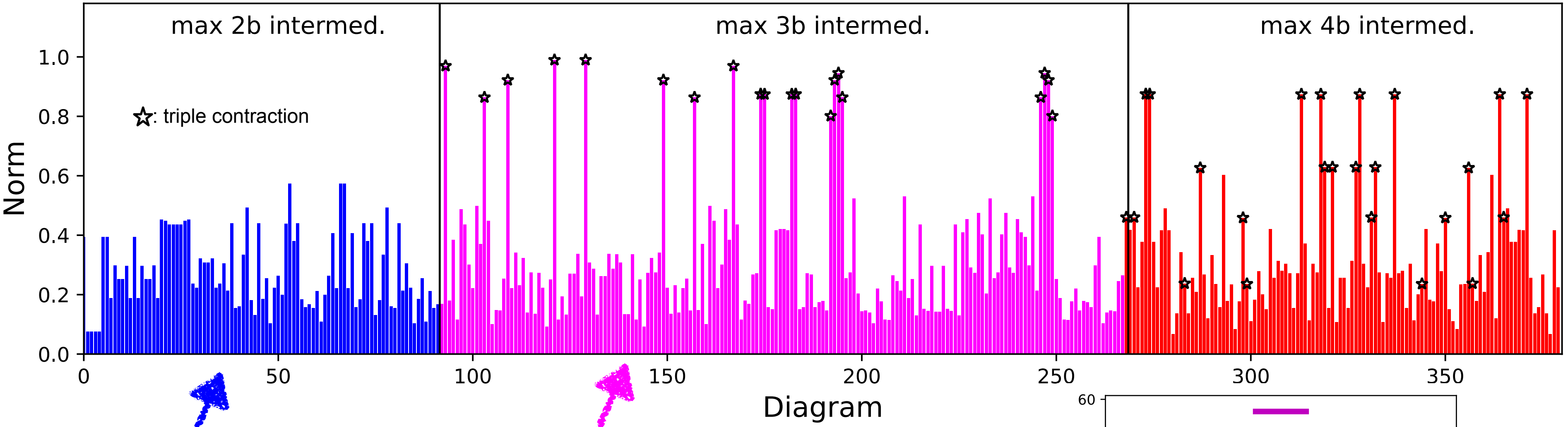
Nested commutators dominated by many-body intermediates



$$[\Omega, H]^{[n]} = \underbrace{[\Omega, \ldots [\Omega, H]]}_{n\text{-fold nested}}$$

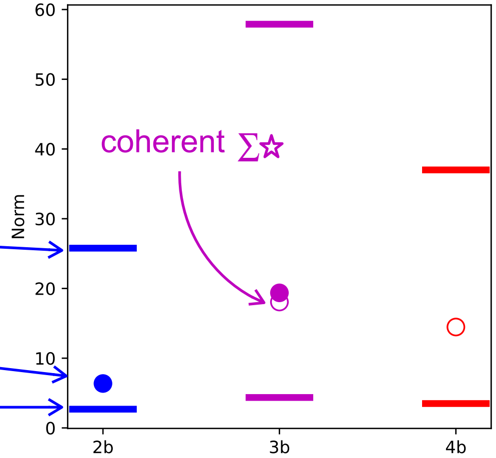
$$H^{[n]} = \sum_{m=0}^{n} \frac{1}{m!} [\Omega, H]^{[m]}$$

Contributions to $[\Omega, [\Omega, [\Omega, H]]]_{2b}$



Included at IMSRG(2)

Included at IMSRG($3f_2$)



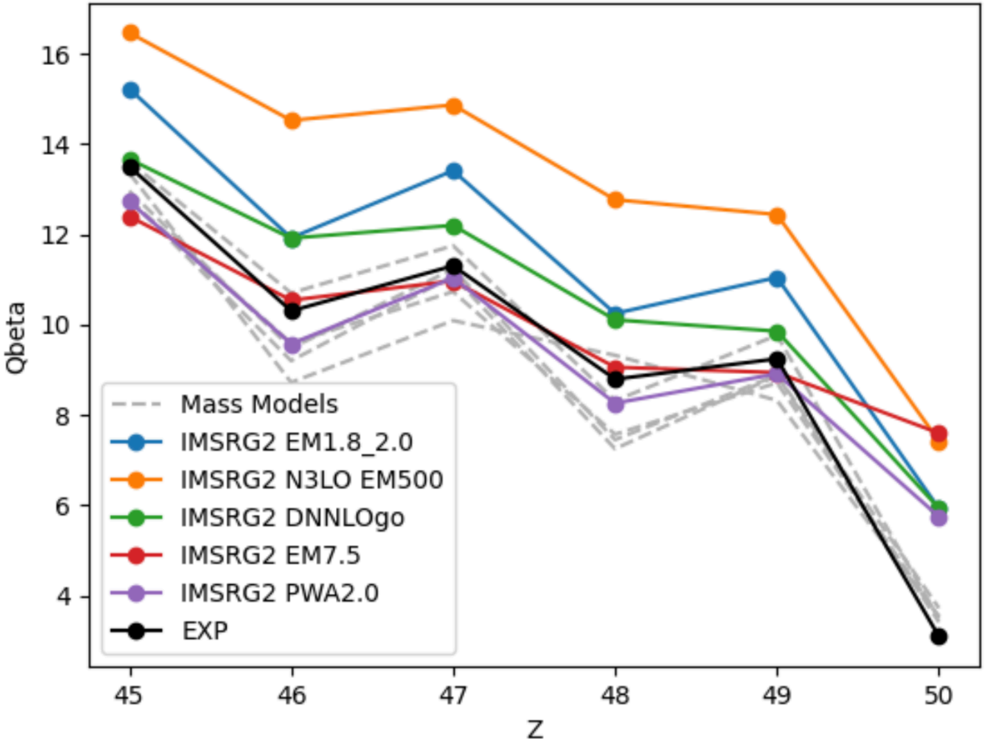
coherent sum

actual sum

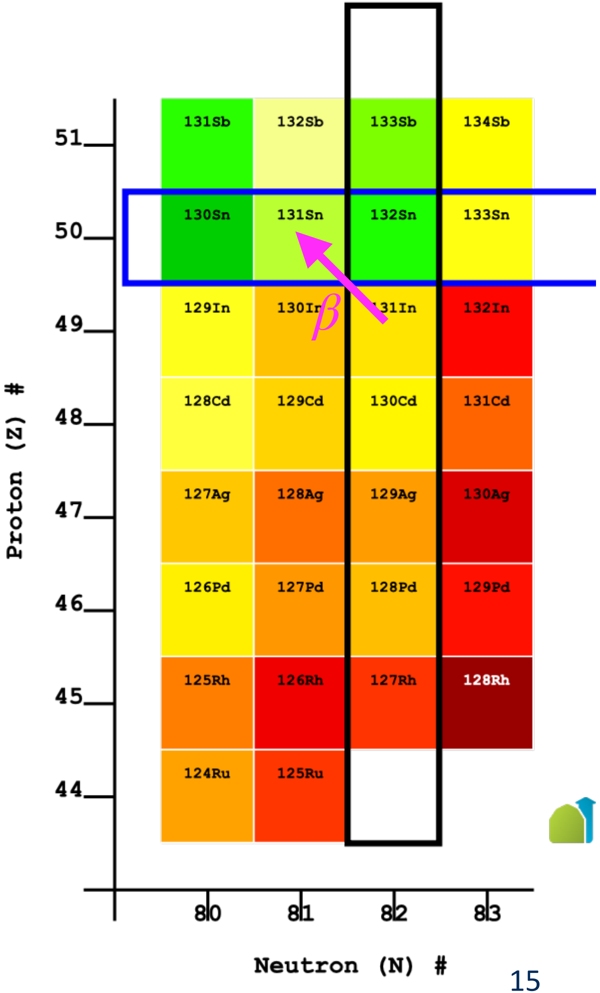
incoherent sum

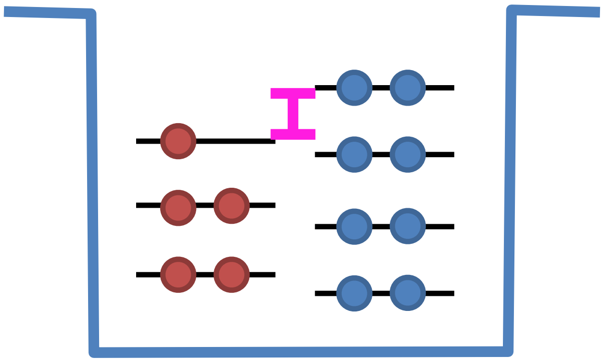
Calculations performed by ND undergraduate Victor Vaida

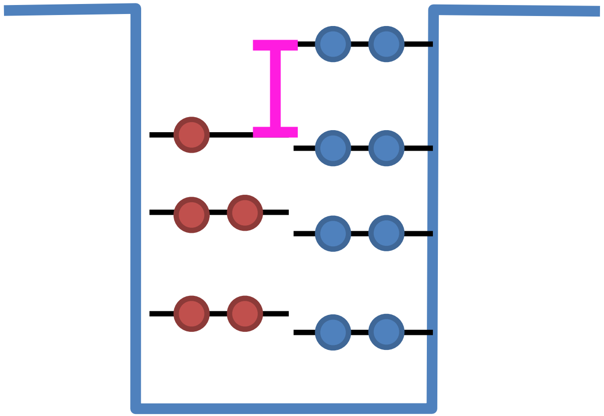
Q_β values for nucleosynthesis

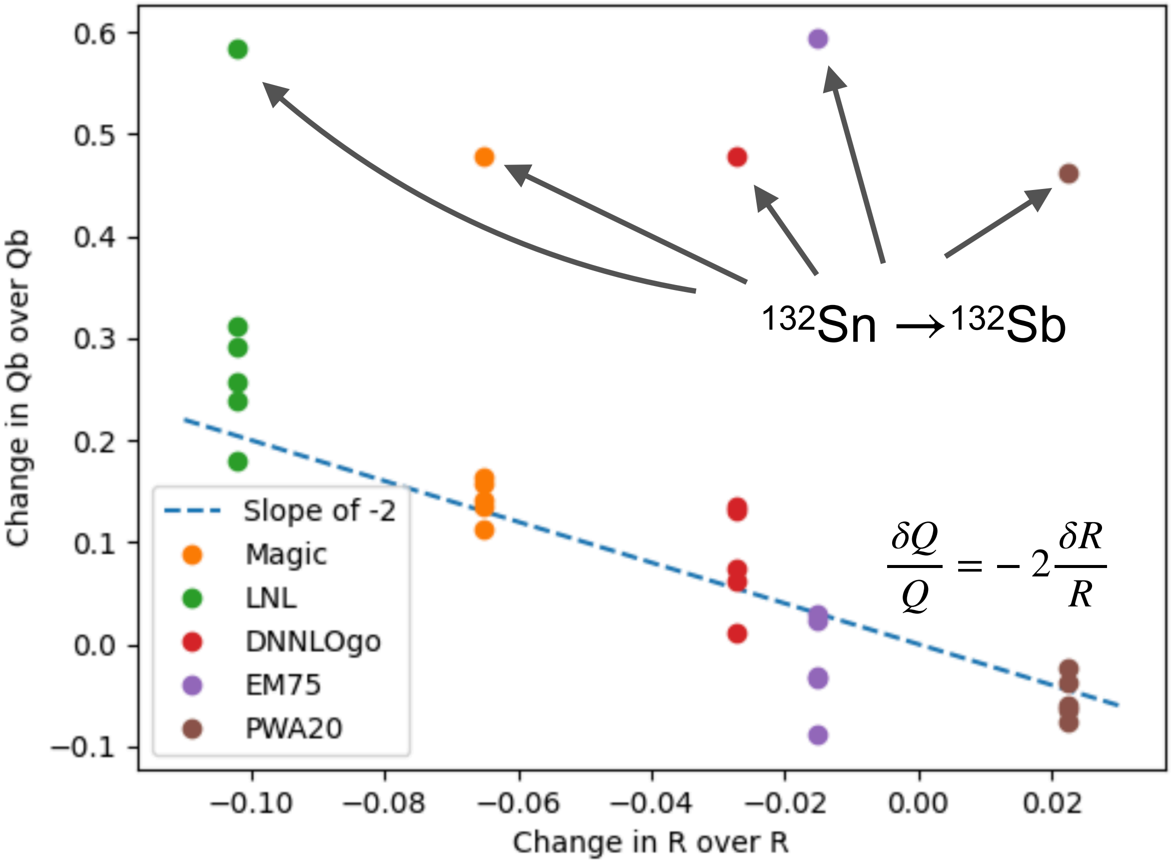


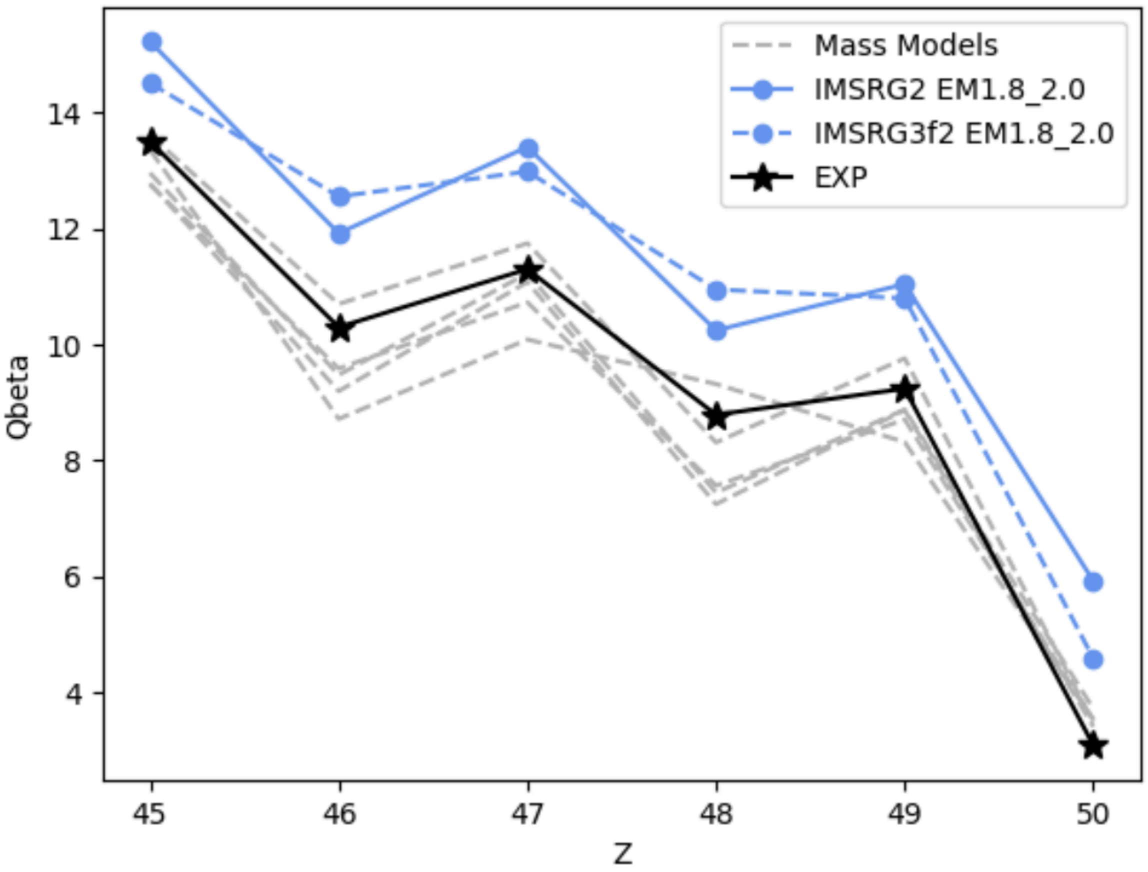
Andre Johnson








0.6
0.5
0.4
0.3
0.2
0.1
0.0
−0.1
Change in Qb over Qb
−0.10
−0.08
−0.06
−0.04
−0.02
0.00
0.02
Change in R over R
Slope of -2
Magic
LNL
DNNLOgo
EM75
PWA20
$^{132}Sn \rightarrow ^{132}Sb$
$\frac{\delta Q}{Q} = -2\frac{\delta R}{R}$

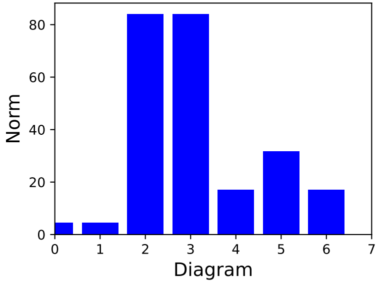


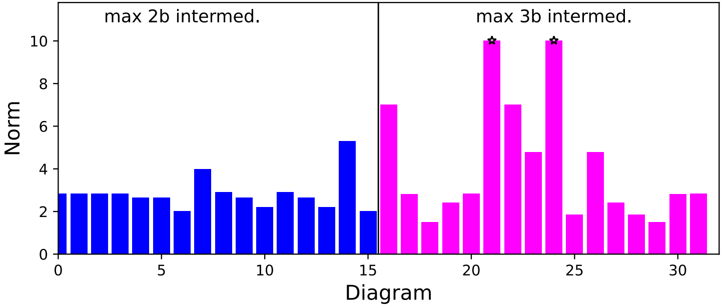
IMSRG(3f2) correction kills pairing?

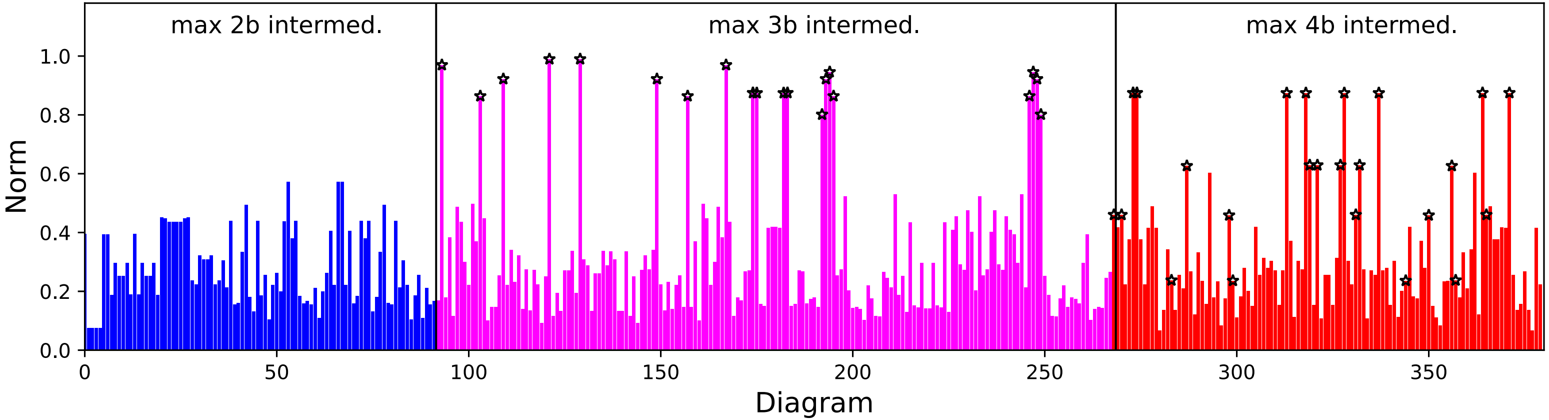
Summary

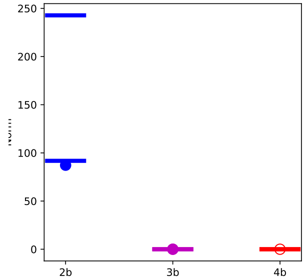
- Truncating nested commutators does not seem to be an especially efficient approximation
- Triply-connected operators, including terms nominally part of IMSRG(4), appear to also be important at 3 nested commutators.
- Ω_{3b} can usually be treated perturbatively, once we have it. But obtaining it may require nonperturbative evaluation.
- Systematic behavior of Q_β values exhibit a strong dependence on the nuclear radius, which can be understood from a simple square well.

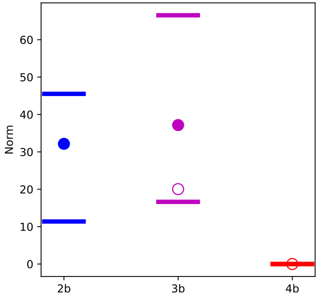
Additional slides

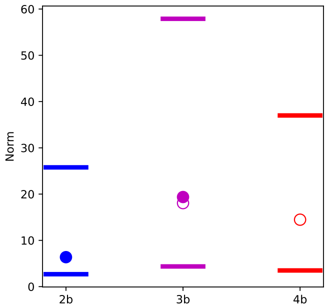

Norm
Diagram


max 2b intermed.
max 3b intermed.
Norm
Diagram

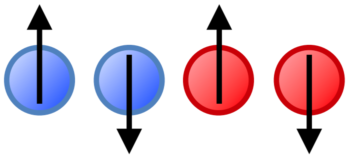

max 2b intermed.
max 3b intermed.
max 4b intermed.
Norm
Diagram


2b
3b
4b


Norm
2b
3b
4b

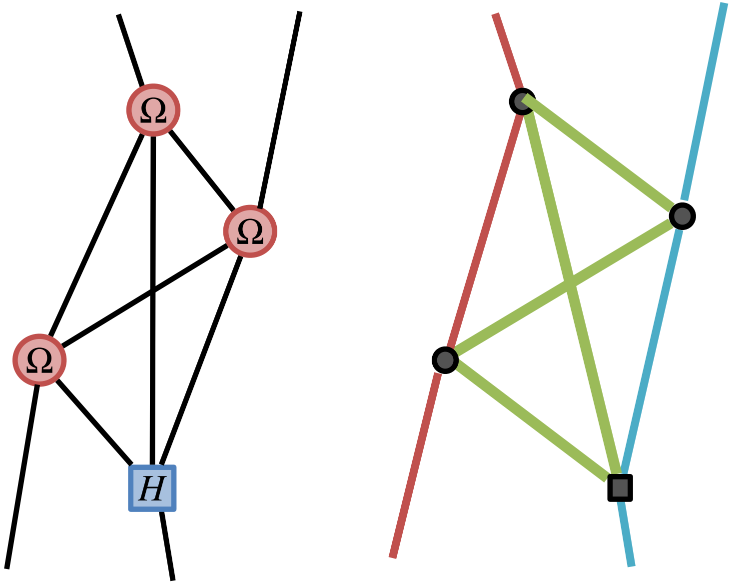

Norm
2b
3b
4b

Analogy with large N_c?

Wigner SU(4): 

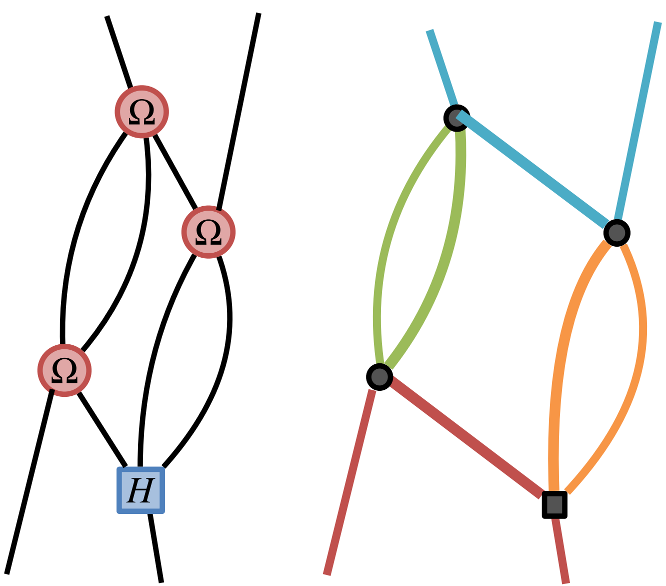
$N_c = 4$

non-planar diagram



$\sim N_c$

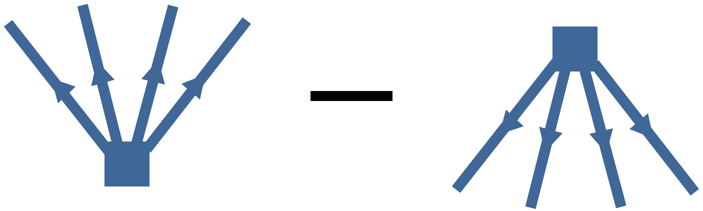
planar diagram



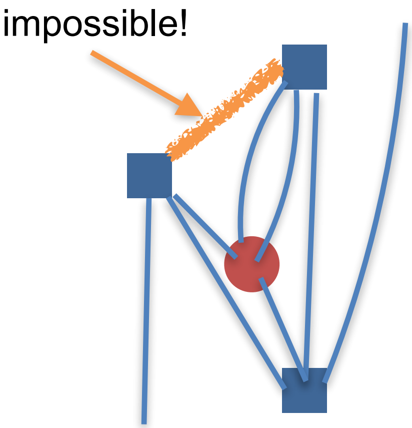
$\sim N_c^2$

Compatibility with form of Ω

$\Omega \sim$

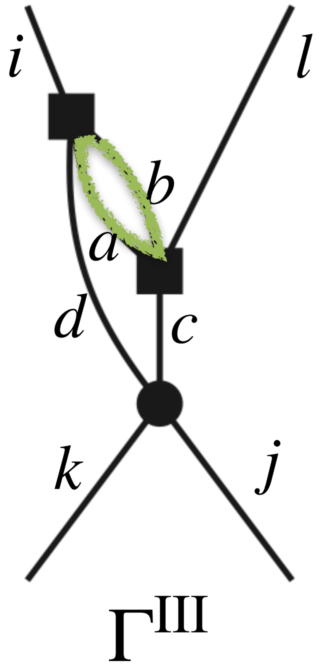


(single reference)



Beyond 2 nested commutators, need some of the Ωs to have 0 connections. For k nested commutators, need $\sim k/2$ disconnected pairs $\Rightarrow$ 0s in the adjacency matrix.

Dynamical coherence (approximate separability)



$$\Gamma^{\mathrm{III}}_{ijkl} \sim \sum_{abcd} \Omega_{idab}\Omega_{abcl}\Gamma_{cjkd}$$

if Ω is separable, i.e. $\Omega_{abcd} = v_{ab}v_{cd}$ (or $v_{ac}v_{bd}$)

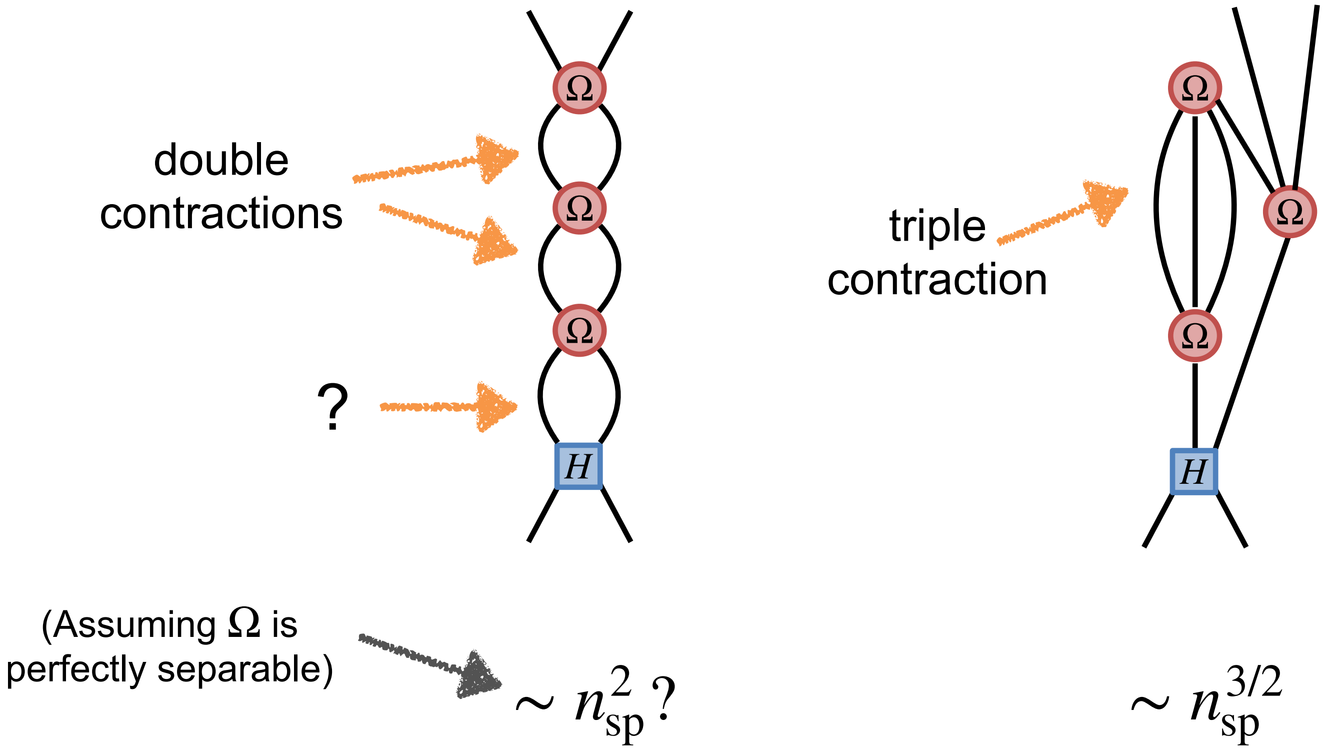
$$\Gamma^{\mathrm{III}}_{ijkl} \sim \sum_{abcd} v_{id}v_{ab}v_{ab}v_{cl}\Gamma_{cjkd}$$

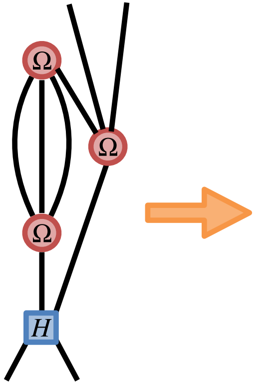
Expect an enhancement

$$\sqrt{N} \sim \sqrt{n_{\mathrm{sp}}^2} = n_{\mathrm{sp}}$$

per double connection

$$[\Omega, [\Omega, [\Omega, H]]]$$

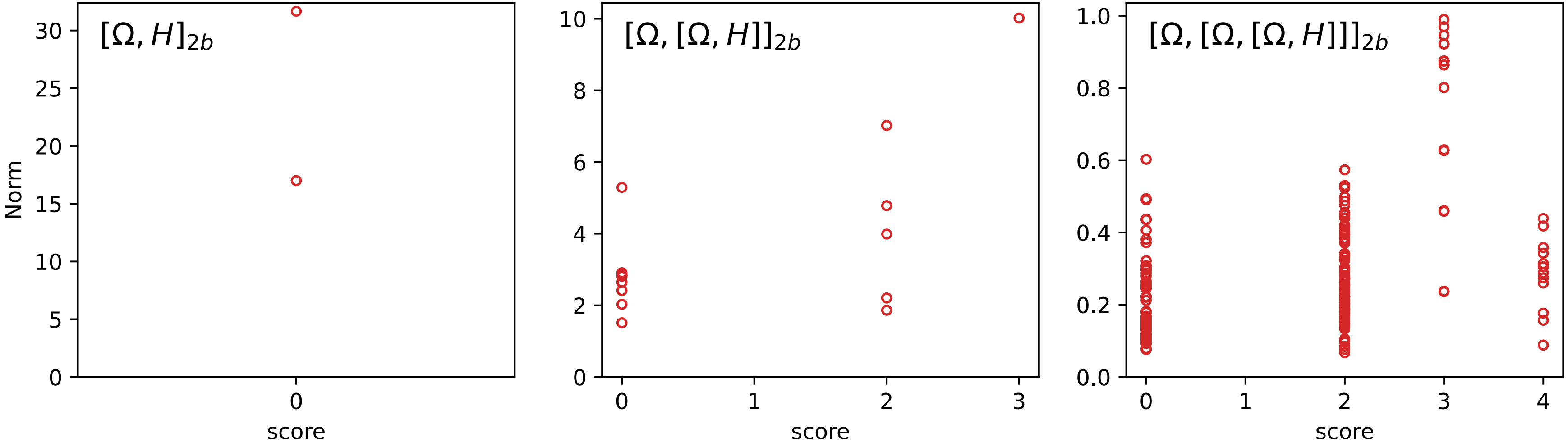




$$\begin{pmatrix} 0 & 1 & 3 & 0 \\ 1 & 0 & 0 & 1 \\ 3 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

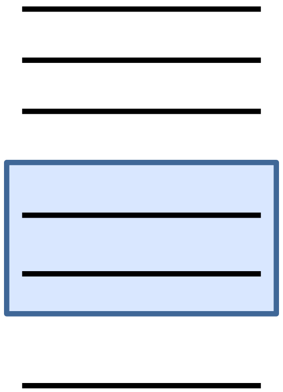
$$\text{score} = \frac{1}{2} \sum (\text{all entries} > 1)$$

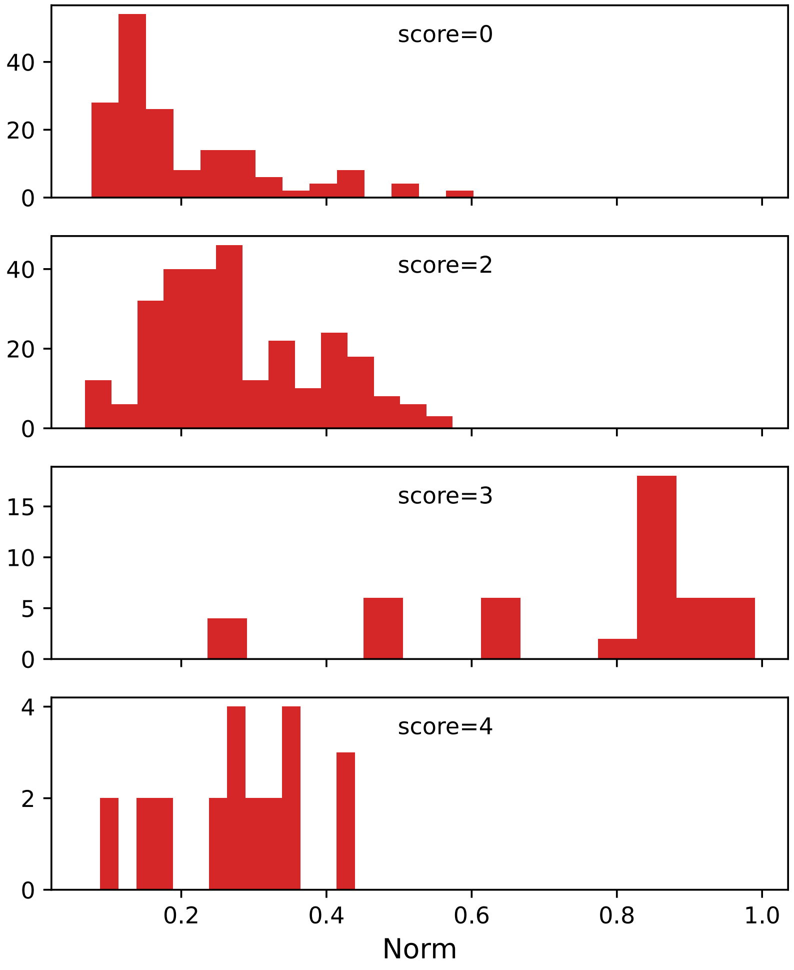
Each dot represents a directed graph



$e_{max} = 3$

p-shell
decoupling





$$[\Omega, [\Omega, [\Omega, H]]]_{2b}$$