# Chiral EFT for nuclear modeling — on and off the beaten track

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***"Not a single uncertainty was properly quantified in this work."***

# Outline:

1. **$N^3LO_{Texas}$: A new chiral interaction for accurate nuclear modeling**
   (i) B. Hu, et al. *The neutron dripline in calcium isotopes from a chiral interaction.* arXiv:2512.11723 (2025).


2. **Nuclear modeling using renormalizable chiral EFT to high order**
   (i) O. Thim, A. Ekström, and **cf**. *Perturbative computations of neutron-proton scattering observables using renormalization-group invariant χEFT up to $N^3LO$,* Phys. Rev. C 109, 064001 (2024).
   (ii) O. Thim, A. Ekström, and **cf**. *Perturbative χEFT calculations of the deuteron and triton up to $N^2LO$,* Phys. Rev. C 112, 064008 (2025).
   (iii) O. Thim, A. Ekström, and **cf**. *Work in progress*
3. **Emergent symmetries of nuclei in chiral EFT**
   (i) A. Cavallin, O. Thim, and **cf**. *Entanglement and accidental symmetries in the nucleon-nucleon system, arXiv:2510.09466,* accepted in Phys. Rev. C.
   (ii) S.S. Li Muli, T.R. Djärv, **cf**, and D.R. Phillips. *The role of spin-isospin symmetries in nuclear β-decays, arXiv:2503.16372 (2025)*.

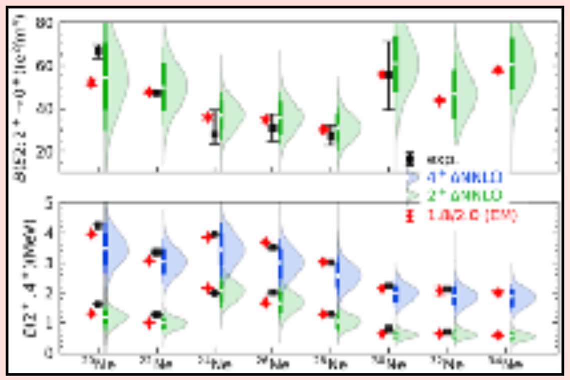
# $N^3LO_{Texas}$: A new chiral interaction for accurate nuclear modeling

# Accuracy: Some WPC interactions work “better” than others

## “Magic” utilize regulator and SRG off-shell freedom

K. Hebeler, et al. (2011)

$N^3LO$-NN (SRG evolved) $N^2LO$-3N fit LECs

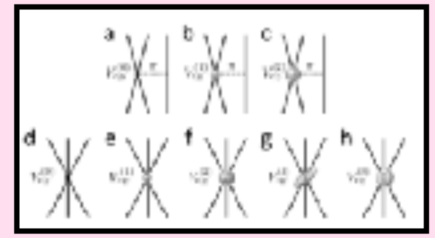


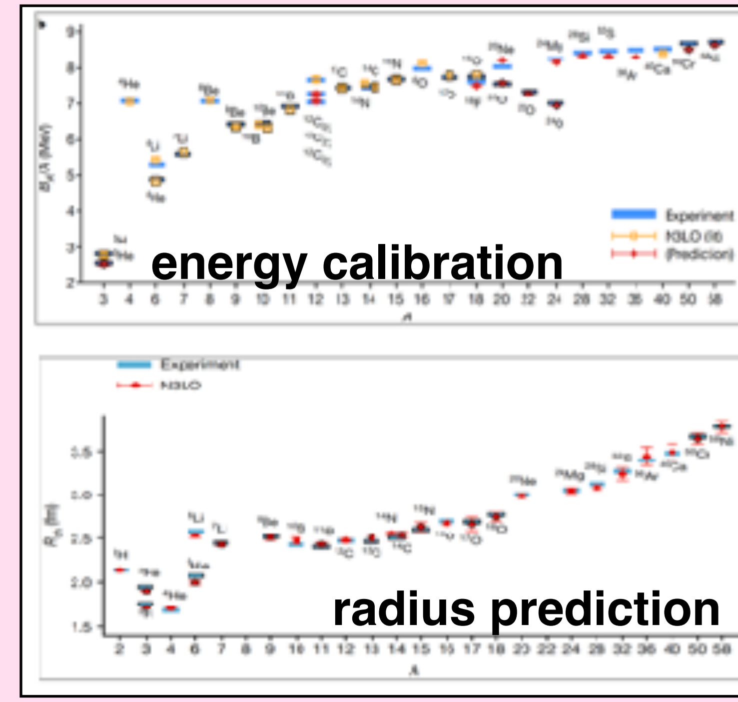
Accurate binding energies and spectra

Z. Sun, et al. (2025) and many more

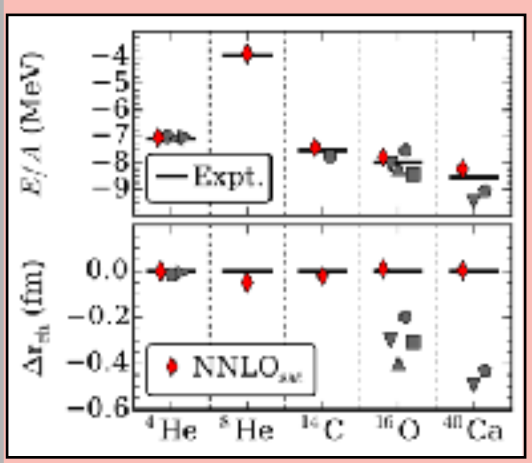
## Nuclear lattice EFT with promoted 3N terms

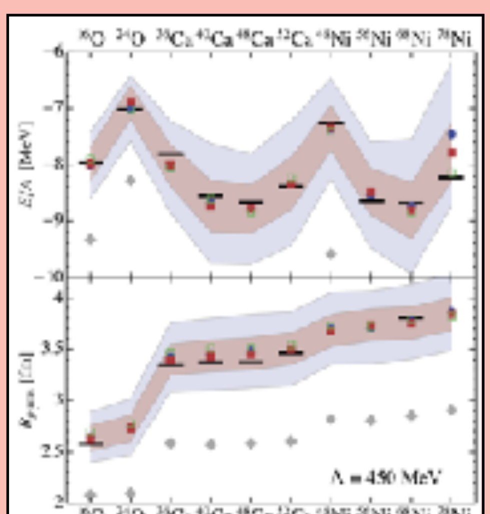
S. Elhatisari et al. (2024)

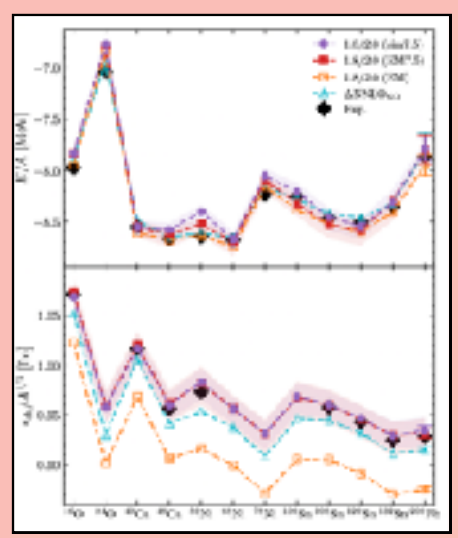


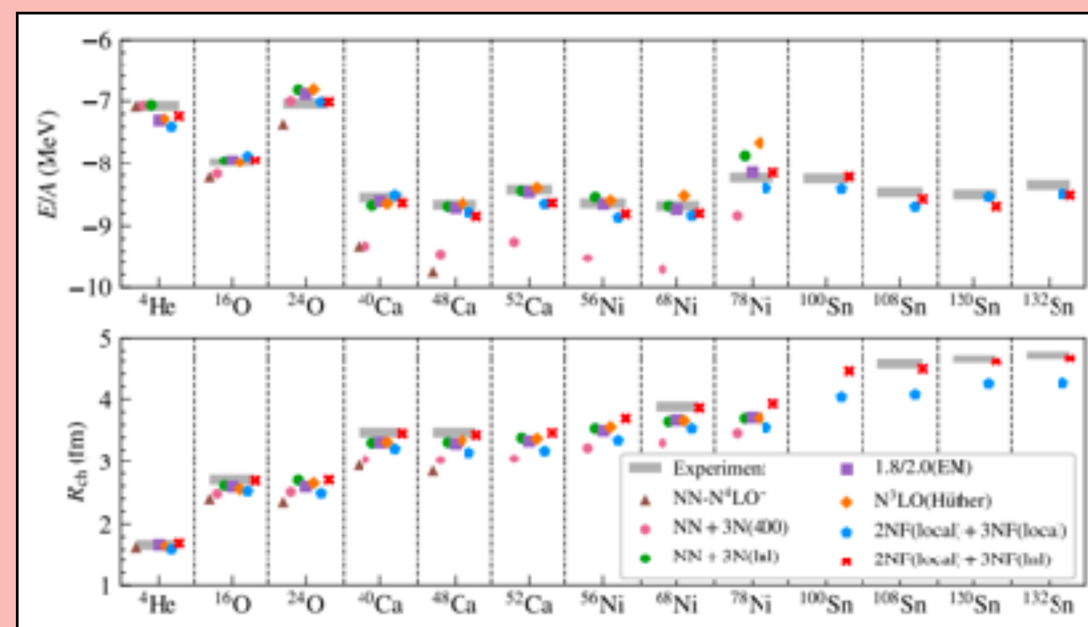



## Optimization with $^{16}$O energy (and radius)








**NNLOsat**
A. Ekström et al. (2015)

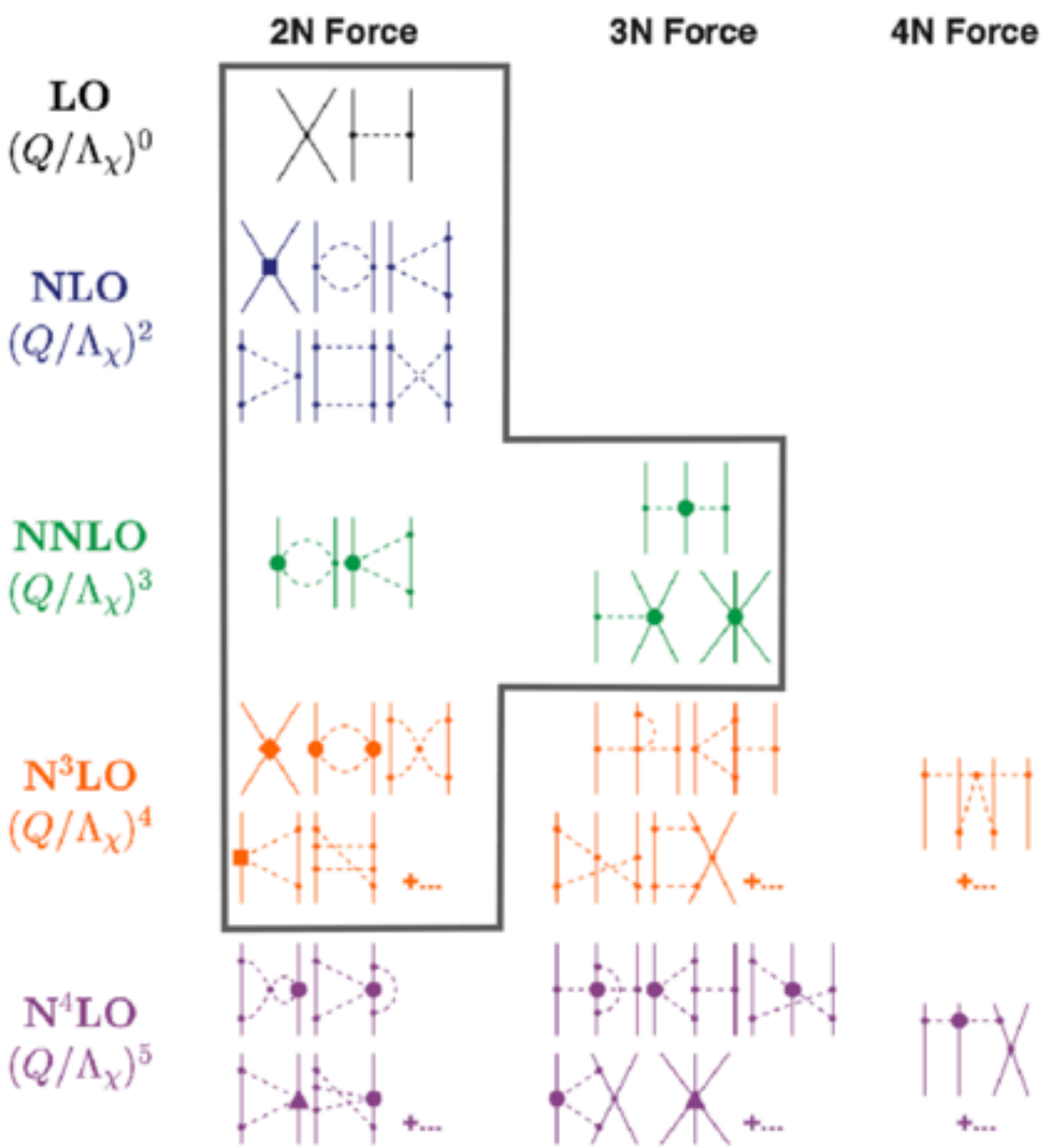
**Order-by-order family**
T. Hüther et al. (2020)

**“Magic interaction” family**
P. Arthuis, et al. (2024)

**Local-non-local regulators**
R.Z. Hu et al. (2026)
$N^3LO$-NN (local, SRG) $N^2LO$-3N fit LECs (lnl)

# N$^3$LO$_{\text{Texas}}$

B. Hu, et al.
arXiv:2512.11723.




See Entem, Machleidt, Nosyk (2017)

$$\Lambda = 2\,\text{fm}^{-1} = 394\,\text{MeV}$$

$$f_\Lambda(p) = \exp\left[-\left(p^2/\Lambda^2\right)^{\{n=4\}}\right]$$

$$f_\Lambda(p,q) = \exp\left[-\left(\left(p^2 + 3/4q^2\right)/\Lambda^2\right)^{\{n=4\}}\right]$$

**πN LECs according to Roy-Steiner analysis**

| | | | |
|---|---|---|---|
| $c_1\,[\text{GeV}^{-1}]$ | $-1.07 \pm 0.02$ | $\bar{d}_1 + \bar{d}_2\,[\text{GeV}^{-2}]$ | $1.04 \pm 0.06$ |
| $c_2\,[\text{GeV}^{-1}]$ | $3.20 \pm 0.03$ | $\bar{d}_3\,[\text{GeV}^{-2}]$ | $-0.48 \pm 0.02$ |
| $c_3\,[\text{GeV}^{-1}]$ | $-5.32 \pm 0.05$ | $\bar{d}_5\,[\text{GeV}^{-2}]$ | $0.14 \pm 0.05$ |
| $c_4\,[\text{GeV}^{-1}]$ | $3.56 \pm 0.03$ | $\bar{d}_{14} - \bar{d}_{15}\,[\text{GeV}^{-2}]$ | $-1.90 \pm 0.06$ |

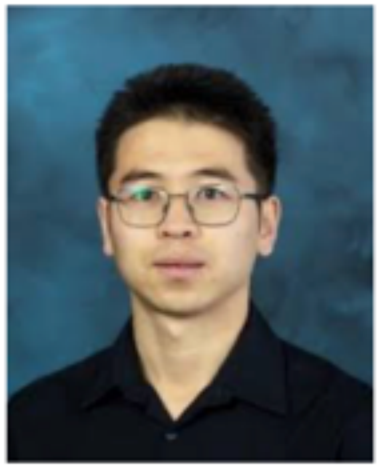
Hoferichter et al. (2015, 2016)

**N$^3$LO includes contacts** $\widehat{D}_{^1S_0}, \widehat{D}_{^3S_1}, \widehat{D}_{^3S_1-^3D_1}$

**N$^3$LO 2π-exchange and “relativistic corrections”**

$\frac{Q}{M_N} \sim \left(\frac{Q}{\Lambda_\chi}\right)^2$ We promote $V_{2\pi}^{(\text{N4LO})} \propto \frac{c_i}{M_N}$ ... to N3LO

# Calibrating $N^3LO_{Texas}$



Baishan Hu
Texas A&M

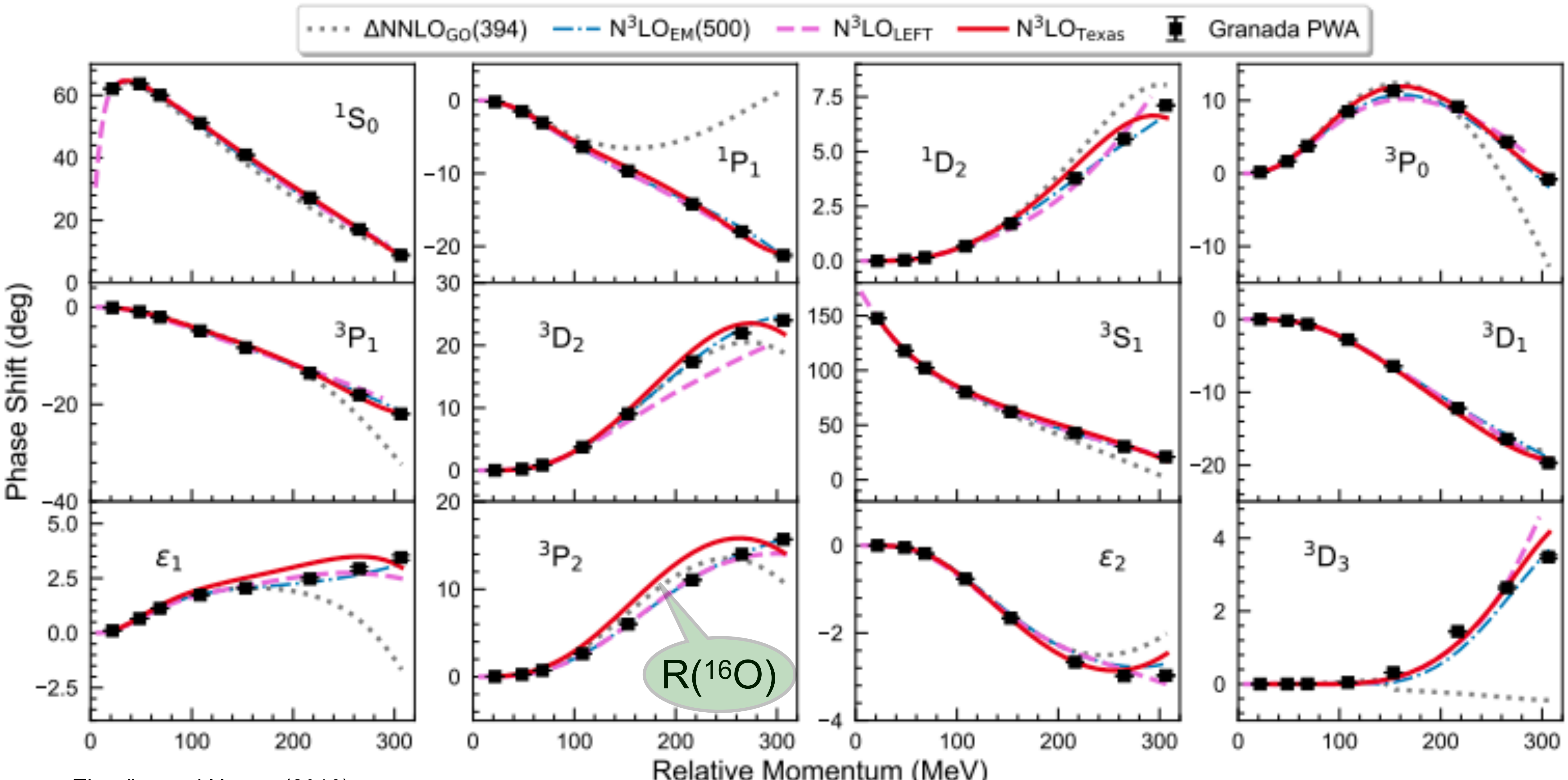
1. Setup emulators for np phase shifts, nn/pp ERE, and E/R($^2$H,$^4$He,$^{16}$O)
2. Generate 2000 random (LHS) points in space of 28 contact LECs (including $c_D$,$c_E$)
3. Optimize wrt L1 loss function (forgives outliers) for E<0 observables, heavy weight on R($^{16}$O). L2 loss function (penalizes outliers) for phase shifts.
4. Select subset of 20 interactions that yield good description of phase shifts and A=2,4,16 observables.
5. Validate with gs energies of $^{22,24}$O and $^{40,48}$Ca.
6. The "greatest one" is $N^3LO_{Texas}$!

**Matrix elements available via NuHamil**
import with zero tariff (at least until February 1st)

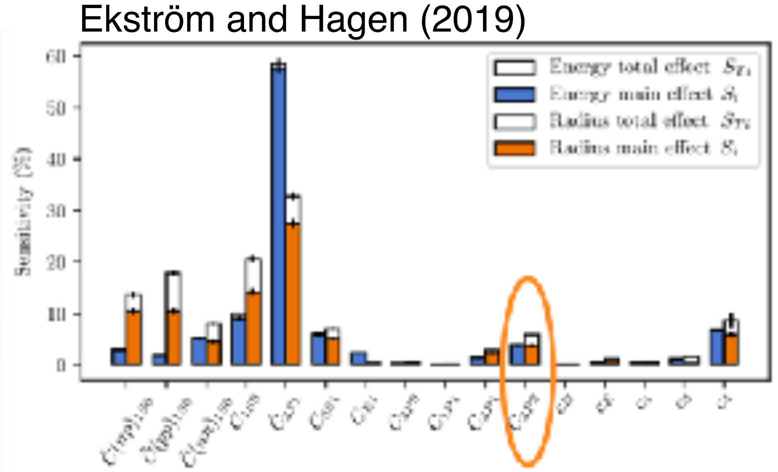
**Initial search domain**

| LEC | Min | Max |
|---|---|---|
| $\widetilde{C}^{pp}_{^1S_0}$ | $-0.30$ | $-0.10$ |
| $\widetilde{C}^{nn}_{^1S_0}$ | $-0.30$ | $-0.10$ |
| $C_{^1S_0}$ | 2.30 | 2.90 |
| $C_{^1P_1}$ | $-1.00$ | 1.00 |
| $C_{^3S_1}$ | 0.10 | 1.70 |
| $C_{^3P_2}$ | $-2.10$ | 2.40 |
| $D_{^1S_0}$ | $-28.00$ | $-18.00$ |
| $D_{^1P_1}$ | 8.50 | 18.00 |
| $\widehat{D}_{^3S_1}$ | $-9.70$ | 9.30 |
| $D_{^3D_1}$ | $-8.20$ | 4.50 |
| $D_{^3S_1-^3D_1}$ | $-7.40$ | 10.00 |
| $D_{^3D_2}$ | $-8.00$ | $-2.00$ |
| $D_{^3P_2-^3F_2}$ | $-2.60$ | 2.90 |
| $c_D$ | $-8.00$ | 8.00 |
| $\widetilde{C}^{np}_{^1S_0}$ | $-0.30$ | $-0.10$ |
| $\widetilde{C}_{^3S_1}$ | $-0.30$ | $-0.10$ |
| $C_{^3P_0}$ | 0.80 | 1.40 |
| $C_{^3P_1}$ | $-1.40$ | 1.50 |
| $C_{^3S_1-^3D_1}$ | 0.00 | 1.20 |
| $\widehat{D}_{^1S_0}$ | $-2.20$ | 4.50 |
| $D_{^3P_0}$ | 3.90 | 6.40 |
| $D_{^3P_1}$ | 2.00 | 10.50 |
| $D_{^3S_1}$ | $-40.60$ | 16.70 |
| $\widetilde{D}_{^3S_1-^3D_1}$ | $-10.00$ | 9.30 |
| $D_{^1D_2}$ | $-2.60$ | 1.10 |
| $D_{^3P_2}$ | $-3.00$ | 15.30 |
| $D_{^3D_3}$ | $-3.00$ | 1.00 |
| $c_E$ | $-8.00$ | 8.00 |

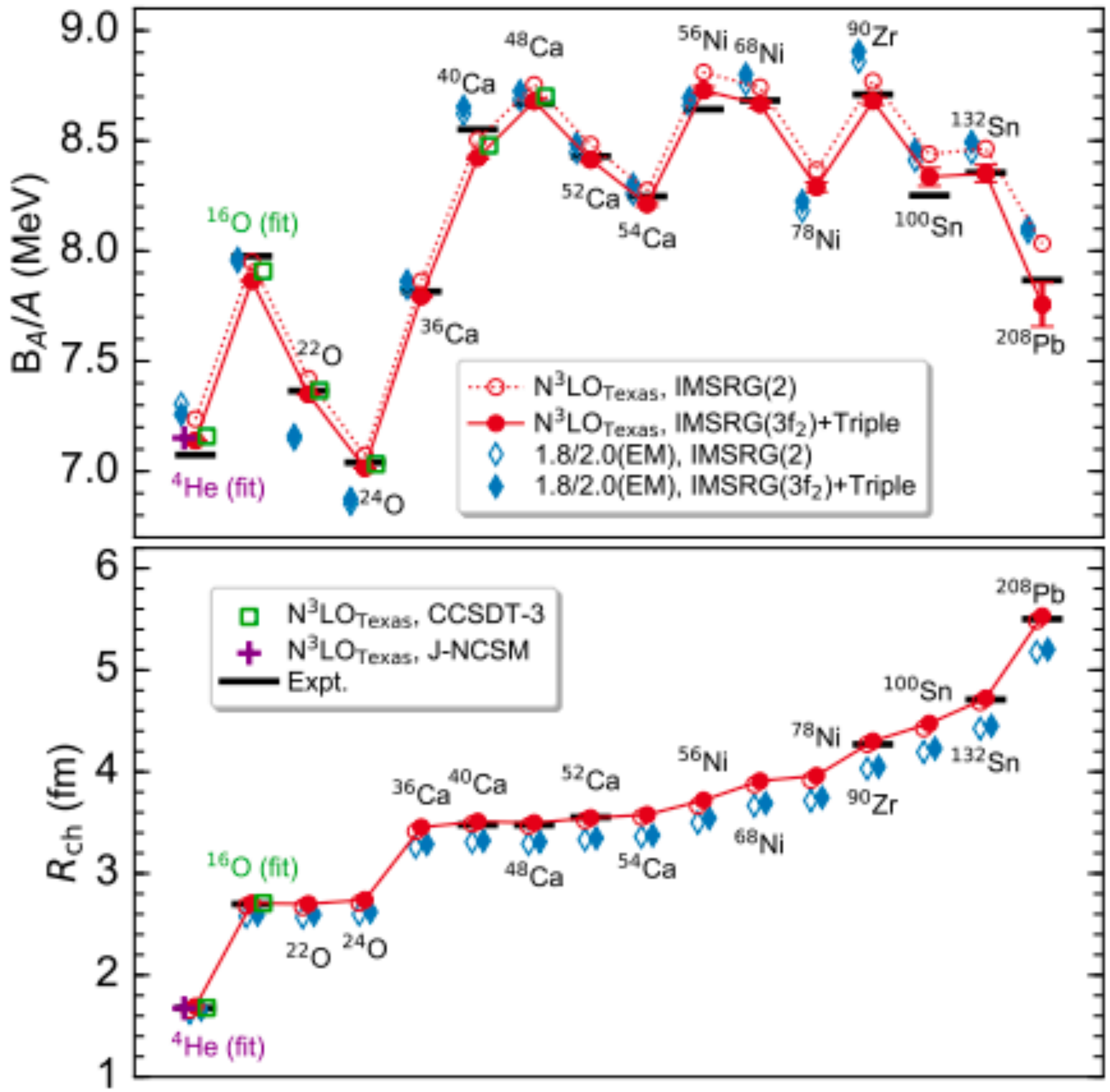
# $N^3LO_{Texas}$ accuracy: NN scattering




Ekström and Hagen (2019)

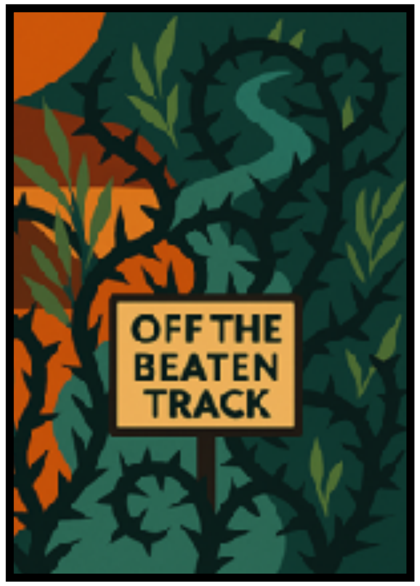


# $N^3LO_{Texas}$ accuracy: from light to heavy nuclei




**See G. Hagen's talk**

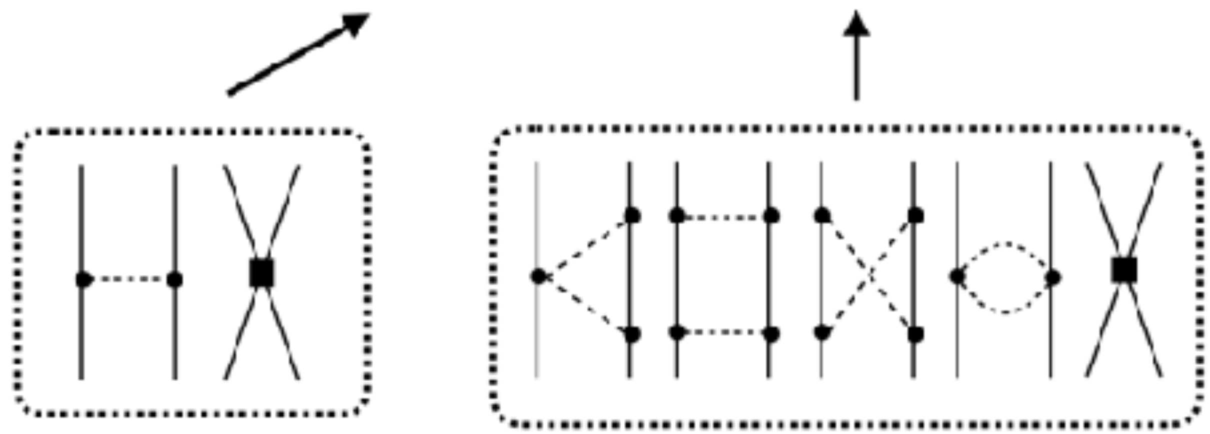
# Nuclear modeling using renormalizable chiral EFT to high order



# Weinberg power counting

## Construct nucleon-nucleon potentials

$$V = V_{\rm NN}^{(0)}(\boldsymbol{\alpha}^{(0)}) + V_{\rm NN}^{(2)}(\boldsymbol{\alpha}^{(2)}) \; + \; \ldots$$



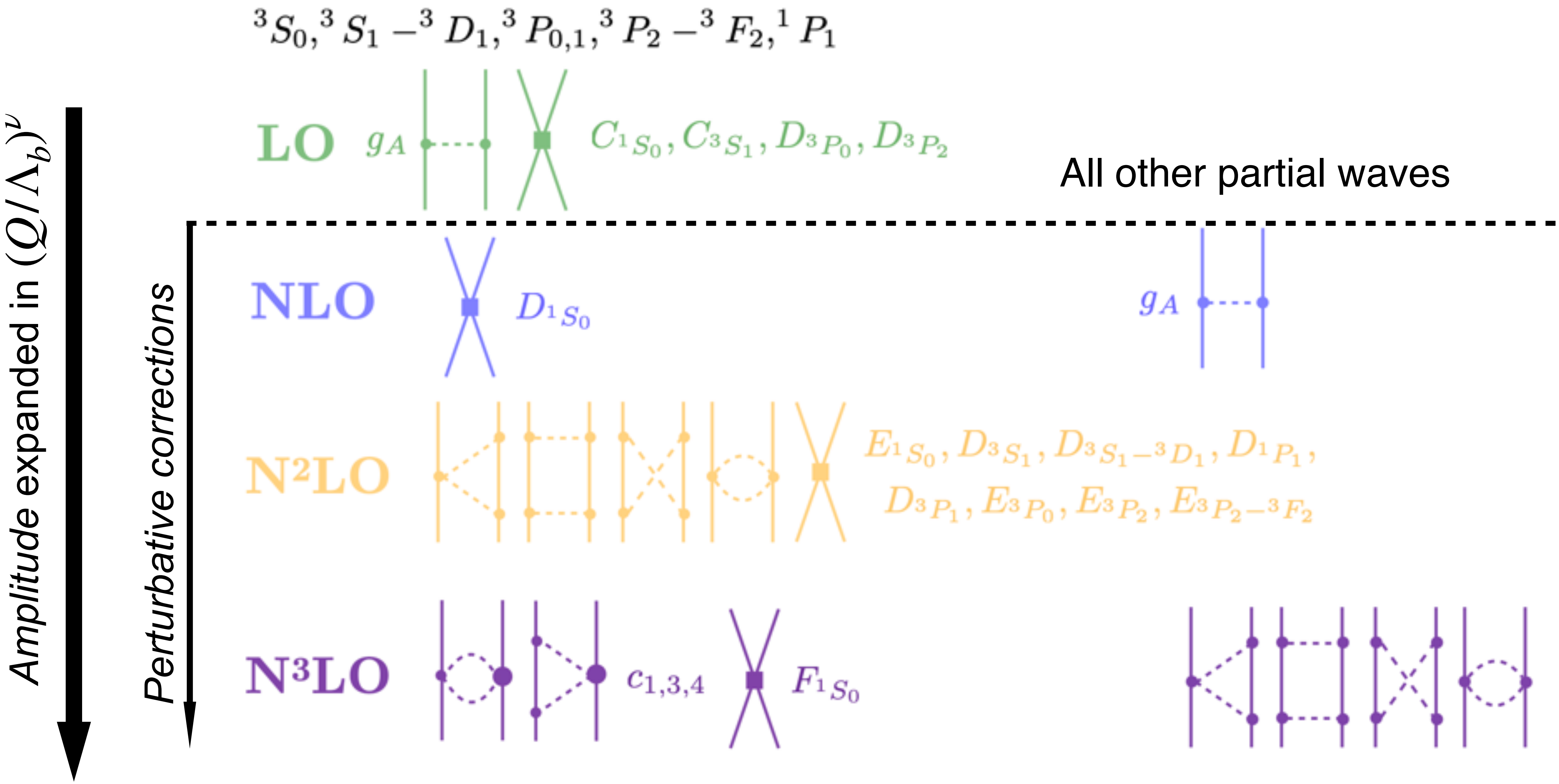
$\nu = 0$ $\nu = 2$

See, e.g., R. Machleidt and D. R. Entem, Phys. Rep. 503 (2011)

- Organize diagrams: $(Q/\Lambda_b)^\nu$ with $Q \sim m_\pi$, $\Lambda_b \sim 1$ GeV
- χEFT with WPC: Successful descriptions of two-and three-nucleon forces and interaction currents. Much employed in our community.
- Predictions of observables depend on regulator cutoff (= not RG invariant).

# RG-invariant χEFT: proposal by Long & Yang

## non-perturbative one-pion-exchange

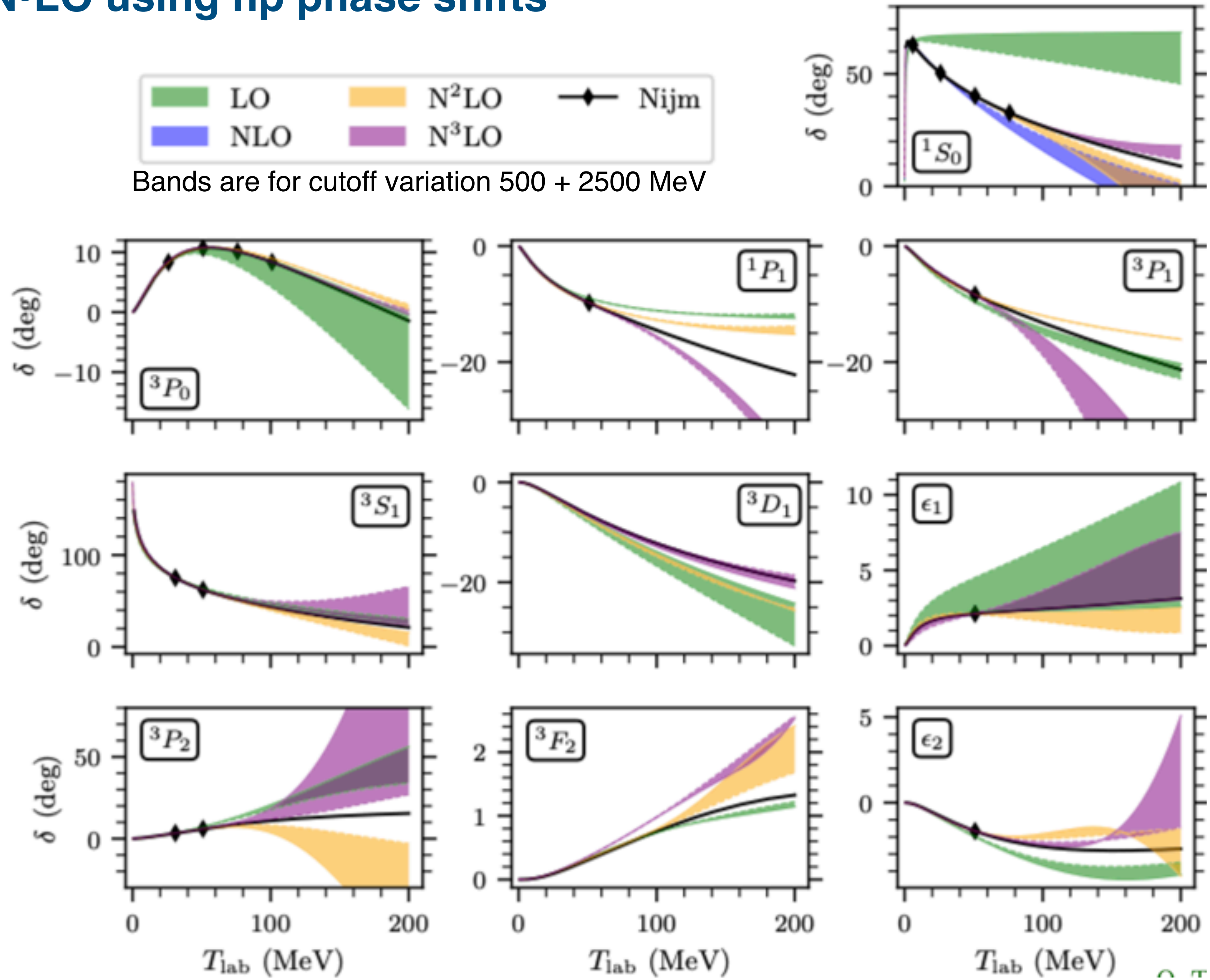



Long & Yang, (2011, 2012), Yang et al, (2023)

# RG-invariant χEFT: proposal by Long & Yang

## Calibrated up to N³LO using np phase shifts

Oliver Thim,
A. Ekström, **cf** (2024)



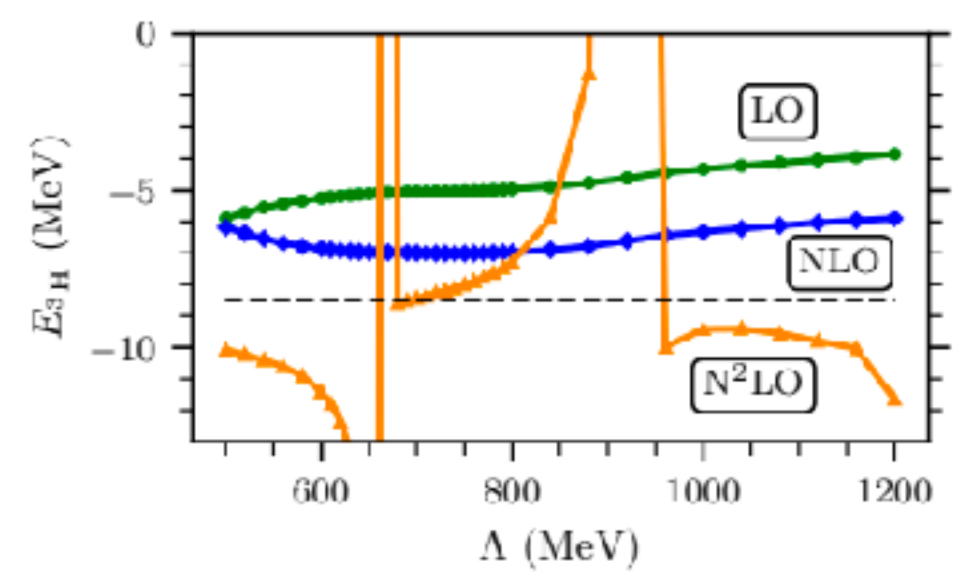

Bands are for cutoff variation 500 + 2500 MeV

# Predicting nuclei with RG-invariant χEFT

**Question:** Can an RG-invariant χEFT with partly perturbative pions describe nuclear observables in A>2 systems?

- Some studies of LO and NLO for A>2 observables
  Y.-H. Song, et al. (2017), C. J. Yang et al. (2021)
- We have developed the machinery to go up to N$^3$LO and are testing it in $^3$H, $^4$He, $^6$Li.
  - Option 1: Rayleigh-Schrödinger (difficult beyond A=3)
  - Option 2: Hellman-Feynman

Oliver Thim,
A. Ekström, **cf** (2025)




Approaching **exceptional cutoffs** yields ill-conditioned design matrix
A. M. Gasparyan and E. Epelbaum (2023)

Strategies to handle these have been proposed and partly tested. R. Peng et al. (2024)

**Rayleigh-Schrödinger**

LO: $H^{(0)}|\Psi_n^{(0)}\rangle = E_n^{(0)}|\Psi_n^{(0)}\rangle$

NLO: $E^{(1)} = \langle\Psi_0^{(0)}|V_{\rm NN}^{(1)}|\Psi_0^{(0)}\rangle$

N$^2$LO: $E^{(2)} = \langle\Psi_0^{(0)}|V_{\rm NN}^{(2)}|\Psi_0^{(0)}\rangle + \sum_{n\neq 0} \frac{|\langle\Psi_0^{(0)}|V_{\rm NN}^{(1)}|\Psi_n^{(0)}\rangle|^2}{E_0^{(0)} - E_n^{(0)}}$

N$^3$LO: ...

# Predicting nuclei with RG-invariant χEFT

**Preliminary results:** Applying RG-invariant χEFT for A>3 systems up to N3LO at low regulator cutoffs (below exceptional cutoffs)

**Hellman-Feynman**

$$V(\vec{x}) = V_{\rm NN}^{(0)} + x_1 \cdot V_{\rm NN}^{(1)} + x_2 \cdot V_{\rm NN}^{(2)} + x_3 \cdot V_{\rm NN}^{(3)}$$

LO:

$$H(\vec{x})|\Psi^{(0)}(\vec{x})\rangle = E^{(0)}(\vec{x})|\Psi^{(0)}(\vec{x})\rangle$$

$$E^{(0)}(\vec{x}) = \langle\Psi^{(0)}(\vec{x})|H(\vec{x})|\Psi^{(0)}(\vec{x})\rangle$$

NLO:

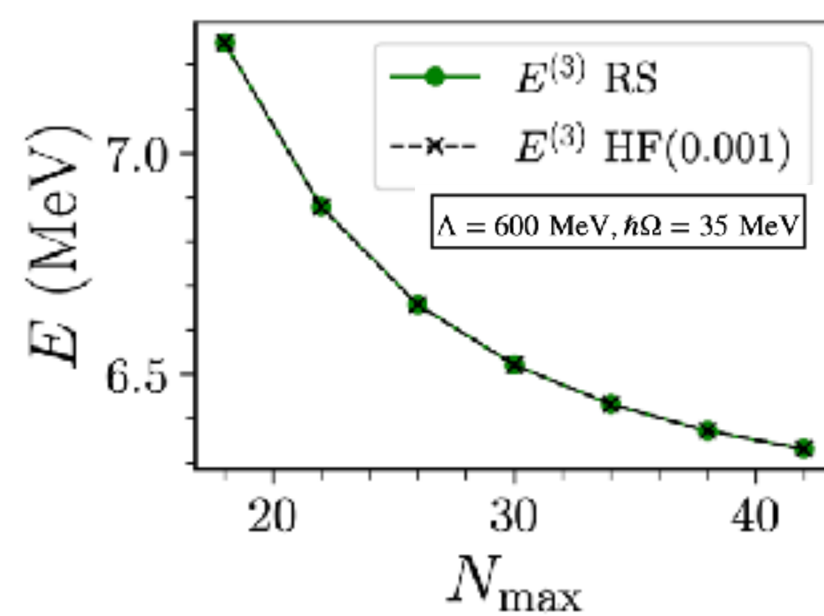
$$\frac{dE^{(0)}(\vec{x})}{dx_1} \overset{\text{H.F.}}{=} \langle\Psi^{(0)}(\vec{x})|\frac{dH(\vec{x})}{dx_1}|\Psi^{(0)}(\vec{x})\rangle$$
$$= \langle\Psi^{(0)}(\vec{x})|V_{\rm NN}^{(1)}|\Psi^{(0)}(\vec{x})\rangle|_{\vec{x}=0} = E^{(1)}$$

...

- All corrections are derivatives of

  $$E^{(0)}(\vec{x}) = \langle\Psi^{(0)}(\vec{x})|H(\vec{x})|\Psi^{(0)}(\vec{x})\rangle$$

- Can be computed numerically (but challenging beyond third order)



# Predicting nuclei with RG-invariant χEFT

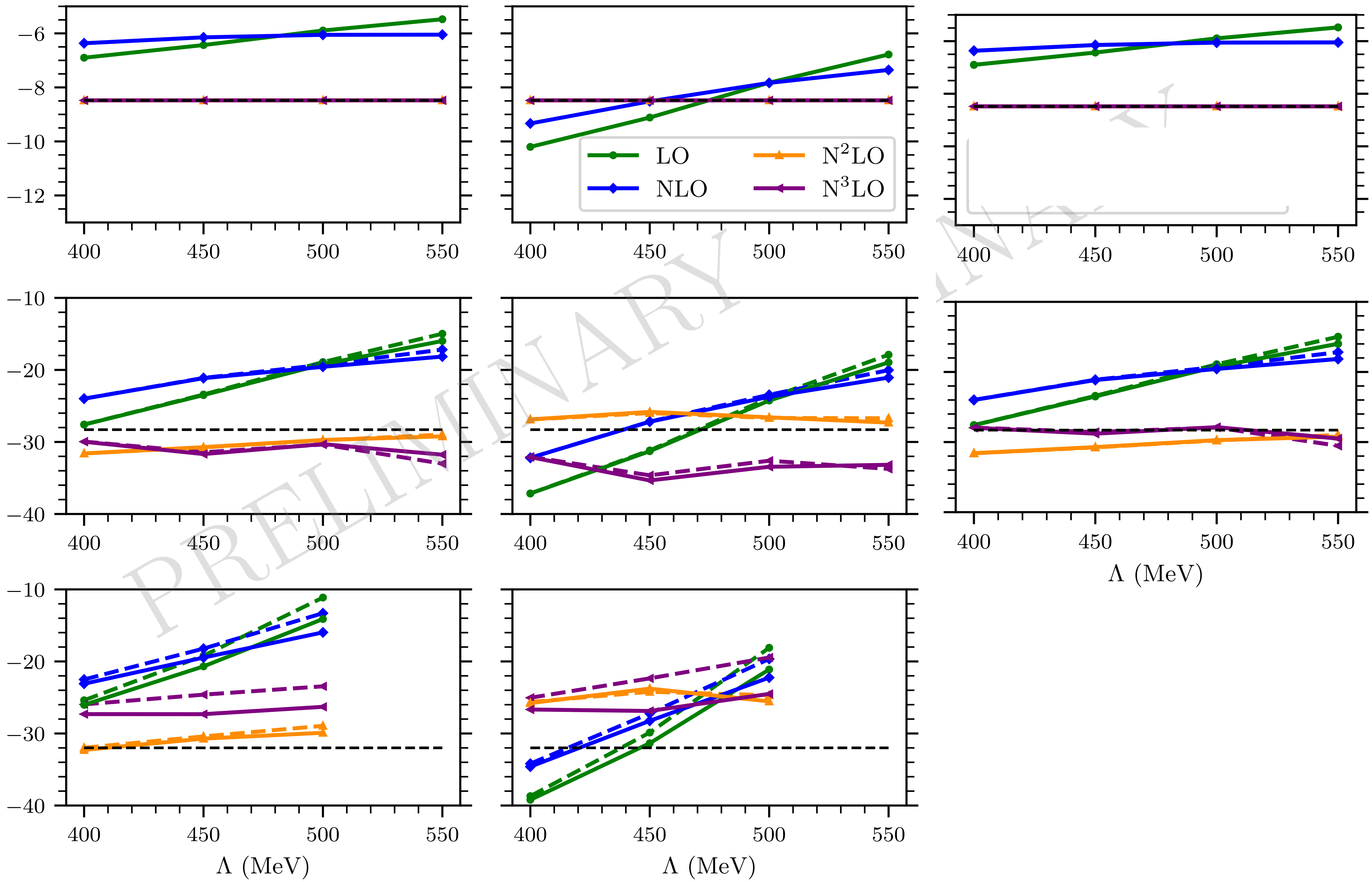
Work in progress

Fit to phase shifts | … also fit $E(^2H)$ | … tune $^3S\text{-}D_1$ channel to fit $E(^3H)$

$E(^3H)$ (MeV)

$E(^4He)$ (MeV)

$E(^6Li)$ (MeV)




Fit $E(^3H)$ at $N^2LO$,$N^3LO$

# Emergent symmetries of nuclei in chiral EFT

# Emergent symmetries

- Do atomic nuclei posses symmetries, that are not of QCD?
- How to use them to increase the predictive power of ab-initio nuclear theory?

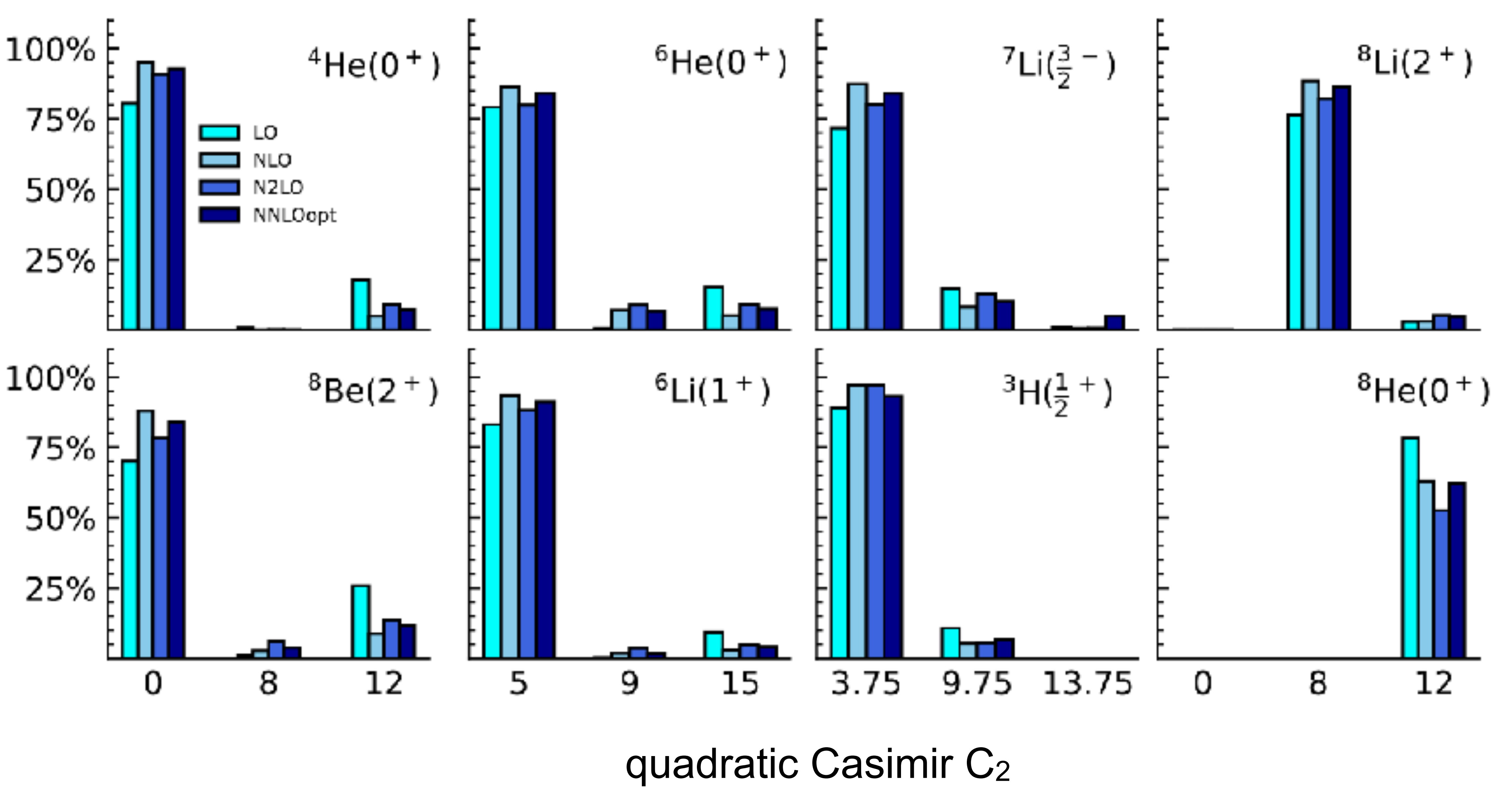
$$\begin{bmatrix} \uparrow \\ \downarrow \end{bmatrix}_S \otimes \begin{bmatrix} p \\ n \end{bmatrix}_T = \begin{bmatrix} \uparrow p \\ \uparrow n \\ \downarrow p \\ \downarrow n \end{bmatrix}_{ST}$$

E. Wigner (1937)

### Wigner SU(4) symmetry

- NN forces at LO in the **large number of colors** of are SU(4) symmetric for S-waves.
  D. B. Kaplan and M. J. Savage (1996)
- Proposed as a medium- and **long-distance symmetry** of the NN force. Broken at short-distances.
  A. Calle Cordon and E. Ruiz Arriola (2008)
- Argued that symmetry becomes evident in NN forces at a **particular resolution scale**.
  D. Lee et al. (2021)
- For SU(4) invariant Hamiltonians, nuclear states populate irreducible representations of the SU(4) group.

# Wigner SU(4) decomposition of light nuclei




Simone Li Muli,
T. Djärv, **cf**, D. Phillips (2025)

# Symmetry-protected observables

Being a generator of SU(4), the Gamow-Teller operator is unable to transition states to different irreps.

| Transition | Within irrep | Eq. (4) | This work (NCSM) LO | NLO | NNLOopt | $N_{\max}$ | VMC [7] |
|---|---|---|---|---|---|---|---|
| $^{3}\mathrm{H}(\frac{1}{2}^{+}) \longrightarrow {}^{3}\mathrm{He}(\frac{1}{2}^{+})$ | ✓ | 2.449 | 2.267 | 2.332 | 2.313 | 18 | – |
| $^{6}\mathrm{He}(0^{+}) \longrightarrow {}^{6}\mathrm{Li}(1^{+})$ | ✓ | 2.449 | 2.178 | 2.259 | 2.260 | 14 | 2.200 |
| $^{7}\mathrm{Be}(\frac{3}{2}^{-}) \longrightarrow {}^{7}\mathrm{Li}(\frac{3}{2}^{-})$ | ✓ | 2.582 | 2.301 | 2.375 | 2.357 | 12 | 2.317 |
| $^{7}\mathrm{Be}(\frac{3}{2}^{-}) \longrightarrow {}^{7}\mathrm{Li}(\frac{1}{2}^{-})$ | ✓ | 2.309 | 2.086 | 2.178 | 2.175 | 12 | 2.157 |
| $^{8}\mathrm{Li}(2^{+}) \longrightarrow {}^{8}\mathrm{Be}(2^{+})$ | X | 0.0 | 0.018 | 0.081 | 0.093 | 10 | 0.147 |
| $^{8}\mathrm{He}(0^{+}) \longrightarrow {}^{8}\mathrm{Li}(1^{+})$ | X | 0.0 | 0.066 | 0.364 | 0.335 | 10 | 0.386 |

Analytic Gamow-Teller matrix elements in the SU(4) limit

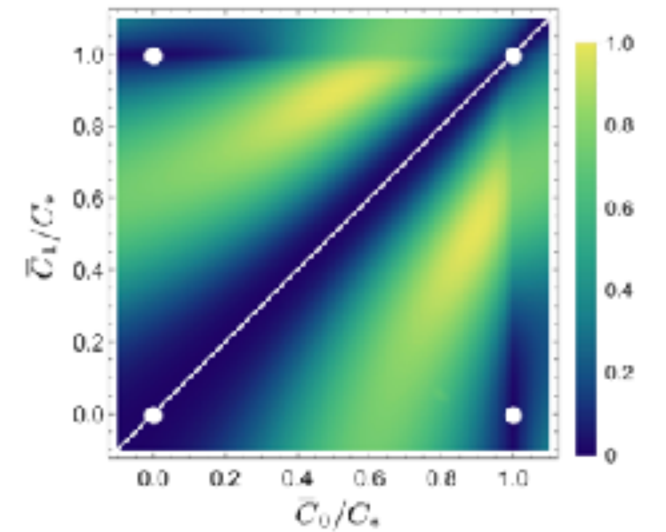
Consequently, the observable does not follow the expected chiral EFT convergence

$$y_k = y_{\mathrm{ref}} \sum_{n=0}^{k} c_n Q^n$$

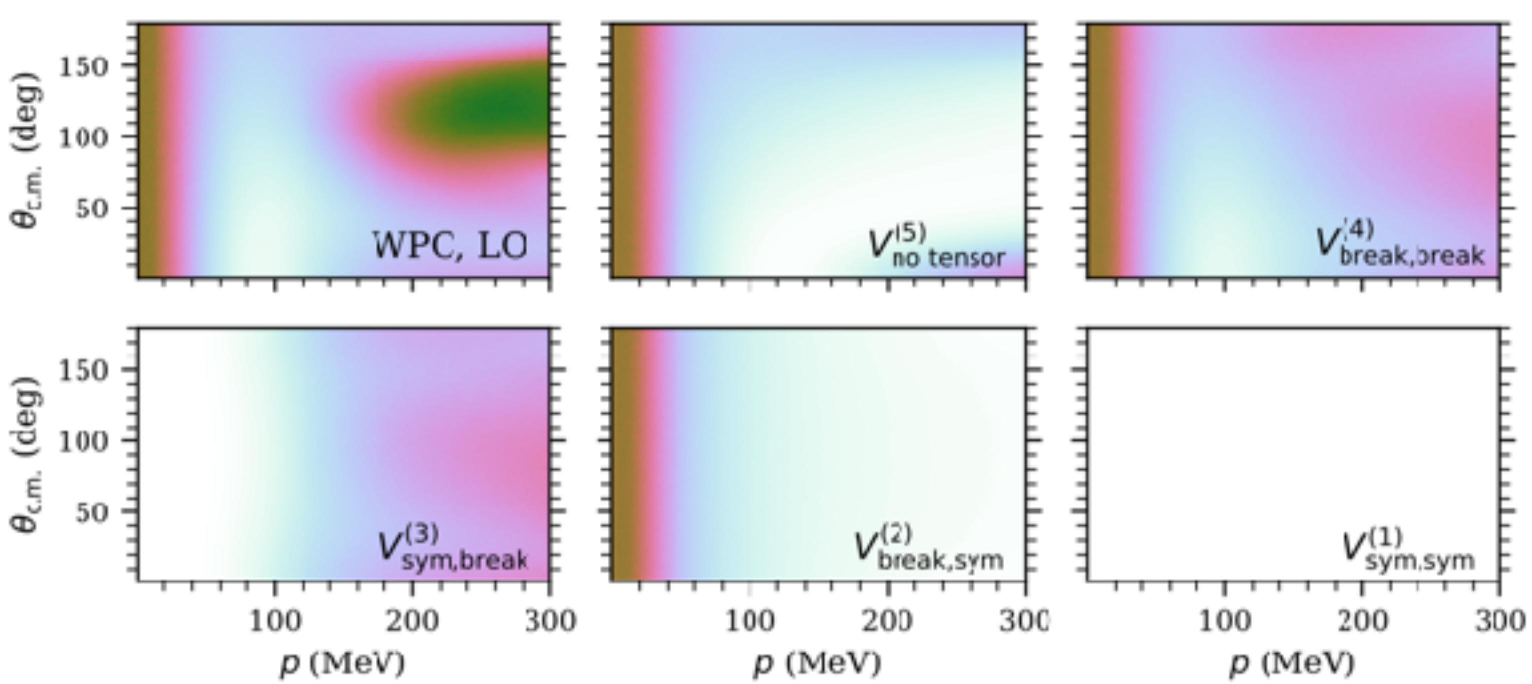
# Spin-entanglement suppression and Wigner SU(4) symmetry

Beane et al. (2019) identified a striking connection between spin-entanglement suppression in S-wave np scattering and Wigner SU(4) symmetry for (low-energy) nuclear forces.

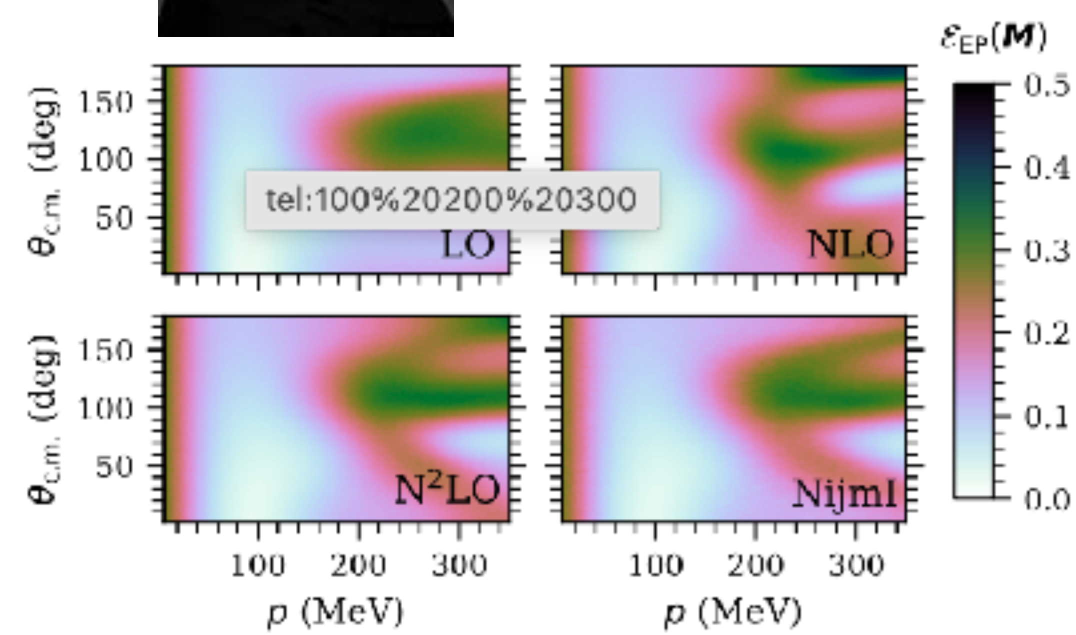
**Entanglement power of the S matrix (S waves only)**




## Entanglement power of the M matrix for "symmetry-enhanced" LO potentials




Alma Cavallin, O. Thim, **cf** (2025)



# Outlook

- High-accuracy nuclear potentials from chiral EFT
  - Revisit regulator dependence and fitting protocol.
  - Quantify the **precision** of nuclear modeling with $N^3LO_{Texas}$.
- Nuclear predictions in RG-invariant schemes
  - Calibration and EFT truncation errors
  - MB calculations in **perturbative schemes.**
- Emerging symmetries
  - Error models for symmetry-protected observables.
  - Entanglement-guided power-counting schemes