Soft Photon Theorem in QCD with Massless Quarks

EMMI Soft Photon Day, May 26, 2025

Yao Ma, GS, Aniruddha Venkata, 2311.06912, Phys. Rev. Lett. 2024

An overview, and where can 'soft' photons come from?

1. The context

2. Low's theorem and its decendents

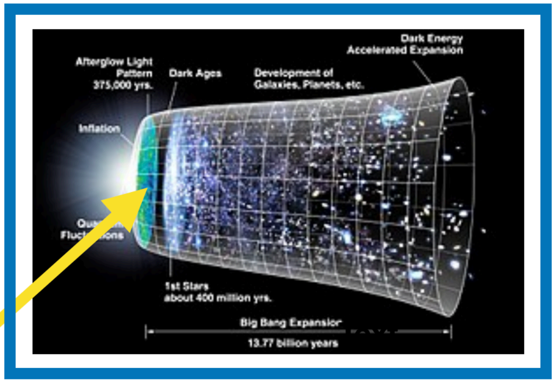
3. Beyond-Low soft photons?

4. A surprise at three loops (using QCD calculations)

5. Soft photons and correlations. What this might mean for real QCD.

1. The context

- **What our machines reveal:**



Very early on, quarks and gluons secluded themselves to a nearly vanishingly small, and ever-decreasing, proportion of space, occupying something like

one 10^{-45} th

of the volume of the observable universe.

- **From inside nuclei, the quarks speak to the outside world through the rest of the Standard Model. Nucleons and nuclei give electrons a reason to stick around and form the world we can see. We'll explore such "signals" in soft photon radiation.**

- **But the QCD degrees of freedom are still available, ready to lend a hand and work alongside the other degrees of freedom of the Standard Model, whenever enough energy arrives in the neighborhood.**

- **Among our standard tools for interpreting such phenomena . . .**

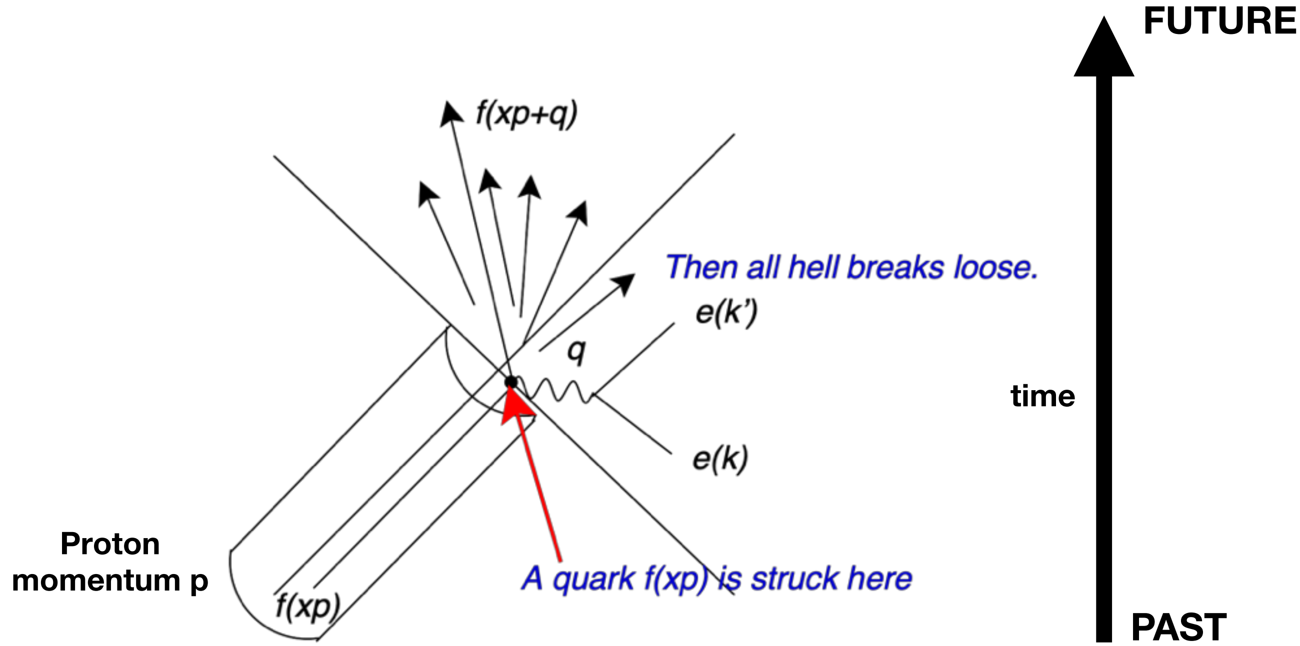
partonic process (perturbative)

$$\boxed{S = C \times F}$$

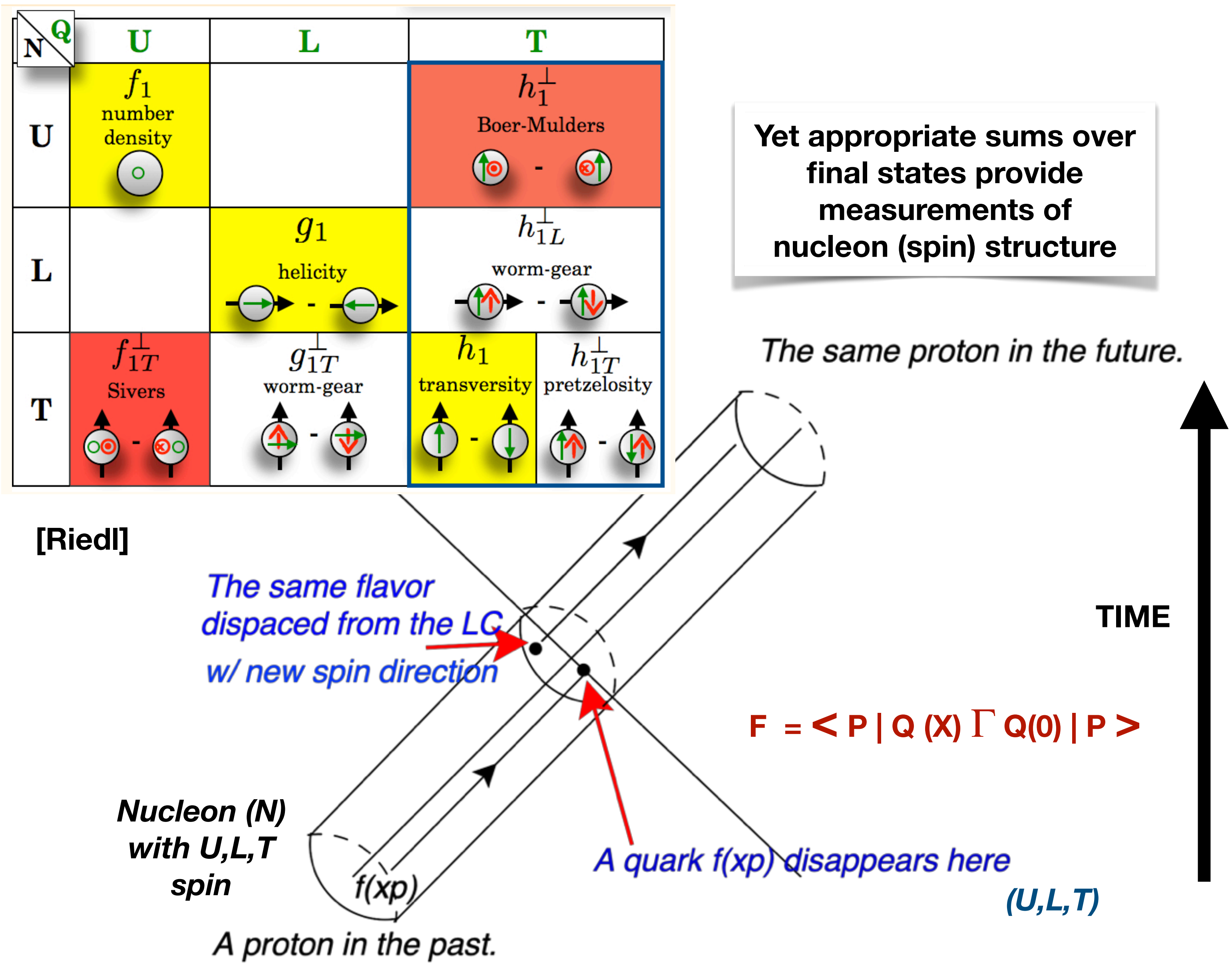
hadronic matrix element (nonperturbative)

The "universal" form of hard-scattering observables
"Factorization" for cross sections and amplitudes.

Picturing a typical "deep inelastic" ep event, in the usual variables



- **But unitarity comes to the rescue, leaving us with parton distributions . . .**

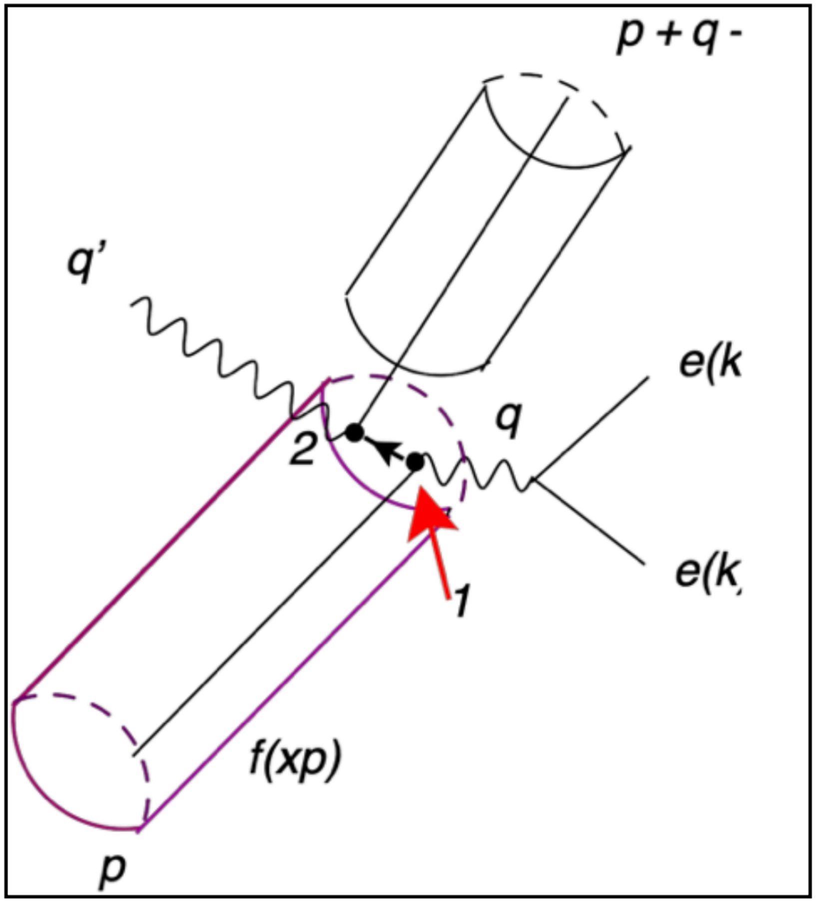


- More and more these distributions are amenable to independent lattice calculations.

Exclusive final states provide even more direct measurements.

If the quark is almost free between points 1 and 2 we measure

$\mathbf{F = \langle P\!+q \,|\, Q(x)\, \Gamma\, Q(0) \,|\, P \rangle}$



- **Off-forward matrix elements and form factors.**

- **Factorization depends in general on the choice of events**
  - **Very inclusive (DIS exemplary) washes out (in a well-defined way) potentially IR-sensitive corrections in the final state.**
  - **Very exclusive (Elastic form factors and DVCS exemplary) can have factorization "built in" for its external states**

- **In factorized cross sections or amplitudes, the dynamics of hadronization either cancels (inclusive) or itself factorizes (exclusive).**

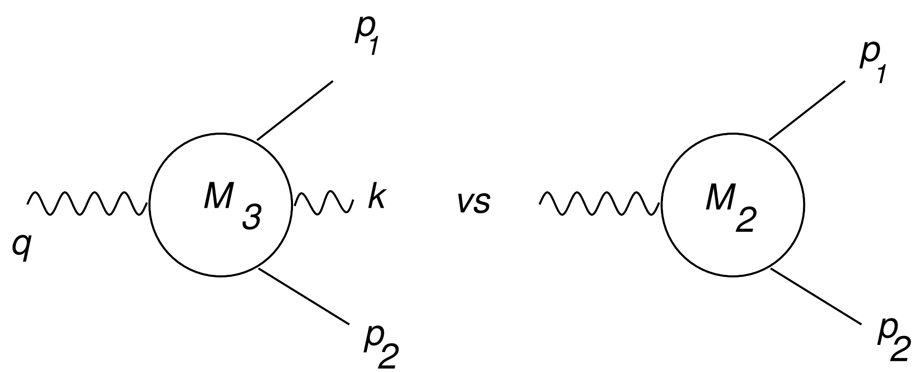
- **Is there any experimental access to the hadronization stage of evolution?**

- **Could QCD processes "signal" us through electromagnetic radiation?**

- **The soft photon anomaly comes to mind. Compared to . . .**

2. Low's theorem and its descendents (the classic results)

$$M_{a+1}(\{p_i\}, k, \epsilon(k)) \;=\; \sum_{i=1}^{a} \delta_i \, e_i \, \frac{p_i^\mu}{p_i \cdot k} \; \left[\epsilon_\mu(k) \;-\; (k_\mu \epsilon^\nu(k) - \epsilon_\mu(k) k^\nu) \; \frac{\partial}{\partial p_i^\nu}\right] M_a(\{p_i\})$$



- First term $\sim 1/\omega_k$, "soft photon theorem" (Identified in full generality by Weinberg 1965) [Follows from $k_\mu M_3^\mu = 0$; 4 independent constraints.]

- Second term: Low (1954), Burnett & Kroll (1968). "Lorentz force". This analysis required $\omega_k \ll m^2/\omega_{p_i}$.

Here, we'll mainly concern ourselves with the what the leading term predicts.

- **Low's theorem says that the soft radiative amplitude can be predicted directly from the non-radiative amplitude and its derivatives.**

- **Both the "leading power" ($p_i \cdot \epsilon / p_i \cdot k$)** *and* **the first correction (ω_k^0) are determined by correspondence to classical radiation. [Liénard-Weichert potential and Lorentz force recoil, respectively.]**

- **A beautiful result, but it says soft photons tell us "nothing new". Really, that's not surprising – arbitrarily soft photons can't resolve short times or distances.**

- **Still, if charges propagate independently of each other over time scale τ, they can radiate photons of energies $\omega_k > 1/\tau$. At energies $\omega_k < 1/\tau$, this propagation becomes invisible.**

- **There may be several time scales τ_i, corresponding to charged particles of different masses (quarks!).**

- **The soft photon theorem really literally applies only for energies $\omega_k < 1/\tau_{\rm min}$.**

3, Beyond-Low soft photons? What the data say, and what it suggests

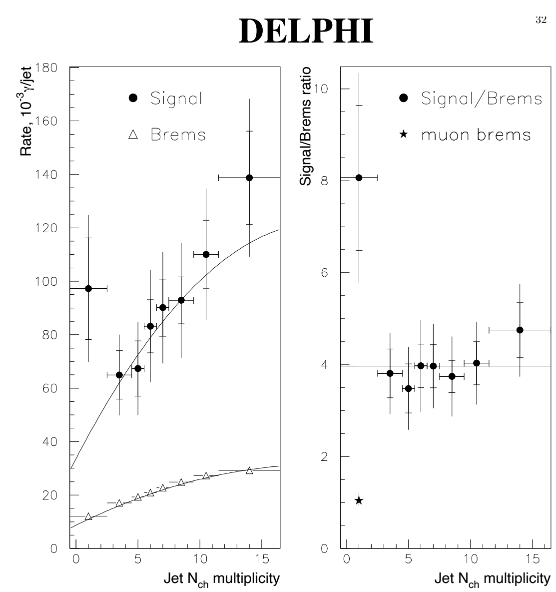
- **Low's analysis can be extended to multiparticle processes, and these predictions have been tested (for the leading power) in several high energy experiments involving hadrons at fixed-target energies, and by DELPHI at LEP1 [ex/0604038, 0901.4488, 1004.1587].** *Many* **have found an excess compared to the soft photon theorem.**

- **Cross section in the notation of DELPHI [ex/0604038] (for $e^+e^- \to Z^0 \to \mathrm{hadrons}+\gamma$):**

$$\frac{dN_\gamma}{d^3\vec{k}} = \frac{\alpha}{(2\pi)^2}\frac{1}{E_\gamma}\int d^3\vec{p}_1 ... d^3\vec{p}_N \sum_{i,j} \eta_i \eta_j \frac{-(P_i P_j)}{(P_i K)(P_j K)} \frac{dN_{hadrons}}{d^3\vec{p}_1 ... d^3\vec{p}_N}$$

- **The results, where "Brem" is the soft photon theorem expectation & "Signal" is data after subtracting hadronic decays, for various analyses show a factor of 3-4 above the prediction.**

	Selection conditions	Signal	Brems
1	General selection	1.170±0.062	0.340±0.001
2	General selection, DURHAM	1.060±0.067	0.351±0.001
3	General selection, JADE	1.070±0.074	0.332±0.001
4	The zero experiment	0.069±0.048	0.0750±0.0002
5	No rejection of jets containing e^+, e^-	1.170±0.061	0.339±0.001
6	No rejection of jets containing e^+, e^-, DURHAM	1.050±0.066	0.348±0.001
7	Strong rejection of jets with e^+, e^-	1.150±0.062	0.326±0.001
8	Strong rejection of jets with e^+, e^-, DURHAM	1.050±0.067	0.336±0.001
9	General selection + anti-B tag	1.240±0.167	0.363±0.002
10	General selection + B tag	1.390±0.159	0.326±0.002
	General selection, signal corrected for efficiency	69.1±4.5	17.10±0.01



- What's going on?

- DELPHI tested for soft photons for $e^+e^- \to Z^0 \to \mu^+\mu^- + \gamma$ and the theorem works fine [0901.4488]. So it "must" be due to the presence of hadrons.

- Returning to the question of "how soft is soft"? Low's original analysis applied for $p_i \cdot k \ll m^2 \to \omega_k \ll m^2/\sqrt{s}$, with $m \sim m_\pi$. For Low, scattering was relativistic, but not yet "ultra". For analyses at really high energy, we had to wait for:

  - Gribov (1967), who showed that the theorem applies at leading power in a much larger region, where $k_T \ll m$ (with ω_k in the large range: $\sqrt{s} \gg \omega_k \gg m^2/\sqrt{s}$). He emphasized that at high energy
  $$p \cdot \epsilon / p \cdot k \sim 1/k_T.$$
  - Then Del Duca (1990) generalized the next-to-leading part to the same region. For more recent analyses at high energy, see , e.g. van Beekveld et al. (2019), Balsach, Bonocore, Kulesza (2022 - 24).

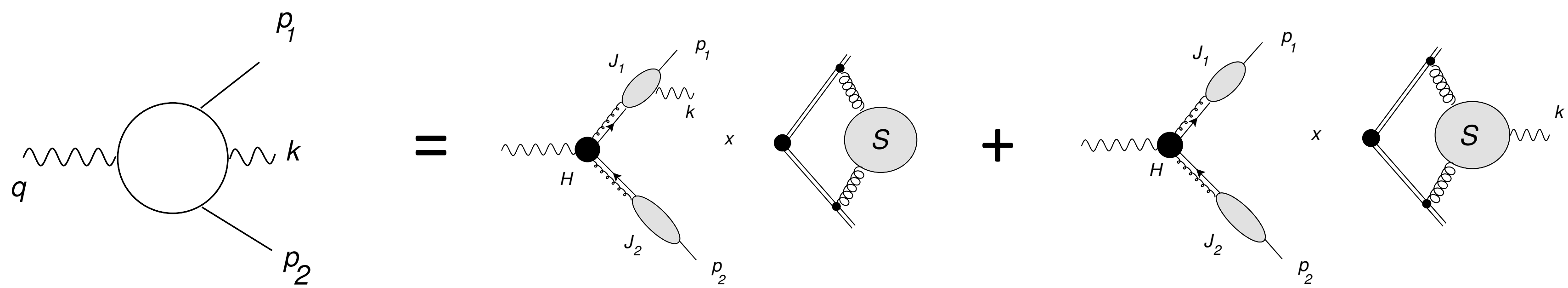
- All of this is still for $k_T \ll m$. In this regime, virtual fermion loops vanish like $(k_T/m)^2$, and contribute neither to leading nor to next-leading power in k. Then only the external lines (charges e_i) contribute.

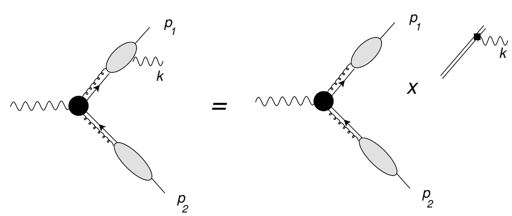
- But isn't $k_T \gg m_{u,d}^{\mathrm{current}}$ when $k_T \sim$ MeV? How can we get a better idea?

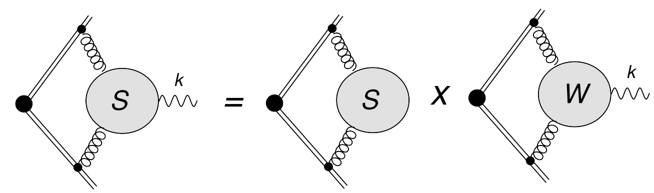
- How about looking at massless QCD"?

4. A surprise at three loops (Yao Ma, GS, Aniruddha Venkata [2311.06912], PRL.)

- **Pair production for quark f with charge e_f in QCD with n_0 massless fermions.**
- **Schematic steps in the analysis (separate radiation from external lines (in J) and internal loops (in S)):**





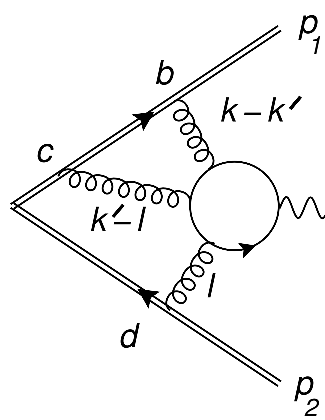


- **The result at leading power (Ma, GS, Venkata):**

$$M_3^{(f)}(\{p_i\}, k, \epsilon(k)) \;=\; \left[e_f \;+\; \Gamma_{\mathrm{EM}}^{(f)}(\alpha_s) \left(\sum_{n=1}^{n_0} e_n \right) \right] \left[\frac{p_1 \cdot \epsilon(k)}{p_1 \cdot k} - \frac{p_2 \cdot \epsilon(k)}{p_2 \cdot k} \right] M_2^{(f)}(\{p_i\})\,.$$

- **Where the new quantity, Γ_{EM} is finite and real to all orders, even though individual diagrams are IR divergent. It's multiplied by the charges in the loops.**
- **It's gauge invariant, as it must be, and behaves like $1/k_T$.**

- **The very lowest order for $\Gamma_{\rm EM}$ is at α_s^3, and is given by the sum of diagrams with three gluons and one photon connected to a quark loop, like:**



- **Each diagram behaves like $1/(4-D)^5$ in dimensional regularization, but all the poles cancel in the sum. A strong confirmation of the all-orders claim.**
- **The result in massless QCD can be abstracted from the calculations for gluon emission ("soft gluon current") in Chen et al. [23309.03832] & Herzog et al. [2309.07884].**

$$\Gamma_{\rm EM}^{(F)} \;=\; -\left(\frac{\alpha_s}{\pi}\right)^3 C_F^{(3)}\,\left(-\frac{\zeta_2}{2}+\frac{\zeta_3}{6}+\frac{5\zeta_5}{6}\right)\;\sim\;\left(\frac{\alpha_s}{\pi}\right)^3 C_F^{(3)}\;\times 0.2$$

- $C_F^{(3)} = 10/9$ **is the "cubic Casimir", replaced by 4 in massless QED.**

5. Soft Photons and Correlations: What this might mean for real QCD

- For k_T "low enough" ($\omega_k < m^2/\sqrt{s}$), we should still expect the "pure soft photon theorem". But experiments to date may not have been "low enough" in photon energy ($\sim 0.2 - 1GeV$), given the masses of light quarks.

- For $k_T \gg m_f$ we do expect radiation from virtual quark EM currents involving flavor f.

- We haven't yet generalized the very simple example above to real QCD experiments, but the example is strong evidence that soft photons from virtual currents can be important in addition to those from external particles in the original soft photon theorem.

- If this is the case, the soft photons seen in DELPHI and previous experiments could have been messengers from the era of hadronization (Kharzeev & Loshaj (2013), Wong (2014)), or products of the influence of vaccum fluctuations (Botz, Haberal, Nachtman (1994)). The massless QCD analysis above provides a perturbative analog, perhaps a step toward a unified picture.

- The experimental future: ALICE 3. Perhaps JLab and EIC.

- To quantify the correlations of soft photons with hadrons of any (including no) charge, we might perhaps use a variant of QCD energy flow operators:

$$\mathcal{E}\left(\hat{n}\right)\,|X\rangle \;=\; \sum_{p_i\in X} p_i^0\,\delta^2\left(\Omega_{\vec{k}_i}-\hat{n}\right)\,|X\rangle$$

Correlations based on this energy-weighted operator have has the nice property of infrared safety; insensitivity to the nonperturbative hadronization.

- For a photon, we don't have IR problems at lowest order if the energy is nonzero, and we can define a "number" operator at fixed momentum without energy weighting:

$$\mathcal{G}_\gamma\left(\hat{n}'\right)\,|X,k_\gamma\rangle \;=\; \sum_{\gamma\in X} \delta^2\left(\Omega_{\vec{k}_\gamma}-\hat{n}'\right)\,|X,k_\gamma\rangle$$

- Putting these together in a cross section, we get a set of QCD-energy flow/electromagnetic radiation correlations:

$$\begin{aligned}\sigma_a(p,p',\hat{n},\hat{n}') \;&\equiv\; \sum_X \int \frac{d^4k_\gamma}{(2\pi)^3}\,\delta_+(k_\gamma^2)\,\delta^2\left(\Omega_{\vec{k}_\gamma}-\hat{n}'\right)\;\omega_k^a \\ &\quad\times \sum_{i\in X} p_i^0\,\delta^2\left(\Omega_{\vec{p}_i}-\hat{n}\right)\,|\langle X,\gamma|p,p'\rangle|^2\;(2\pi)^4\delta^4(p+p'-p_X-k_\gamma)\end{aligned}$$

- The usual summing over final states with translation invariance and unitarity gives

$$\sigma_a(p,p',\hat{n},\hat{n}') = \int \frac{d^4k_\gamma}{(2\pi)^3}\, \delta_+(k_\gamma^2)\, \delta^2\left(\Omega_{\vec{k}_\gamma} - \hat{n}'\right)\, \omega_k^a$$
$$\times \int d^4y\, e^{-ik_\gamma\cdot y}\, \langle p,p'|\, j_{\rm em}(y)\cdot\epsilon_\gamma\, \mathcal{E}(\hat{n})\, j_{\rm em}(0)\cdot\epsilon_\gamma^*\, |p,p'\rangle$$

- This could apply to any initial state $|a,b>$.

- For photon energies of the order of $\sqrt{s}$, $\sigma_a(p,p',\hat{n}.\hat{n}')$ is IR safe (if factorized). The observation of a soft photon, however, could lead to sensitivity to long-distance effects, which may (or may not) be predicted by the soft photon theorem using only the charged particles in the initial and final states.

- This is just an example. Perhaps data on such correlations can provide benchmarks against which models of hadronization and vacuum scattering can be tested.