Evaluating Transport Models under Controlled Conditions

(lessons and future directions of TMEP)

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(on behalf of TMEP collaboration)

EMMI workshop: Collective phenomena and the EOS of dense bayonic matter, GSI Garmstadt, Nov. 10-13, 2025

Outline:

- Motivation: Increase impact of HIC studies on determination of the EOS,
  in particular at suprasaturation densities

- TMEP: Comparison of transport codes under controlled conditions
- box calculations:
  test individual ingredients, comparison to exact results,
  discuss lessons learned: **fluctuations**

- HIC: open systems, much less agreement,
  explanations, but **difficult to quantify model dependence**
- future direction of TMEP
- addional box calculations: fluctuations, mom.dep. potentials, clusterization, SRC
- Heavy-ion collisions (HICs) **Uncertainty quantification:**
  Bayesian inference with many codes, BMA (Bayesian model averaging)

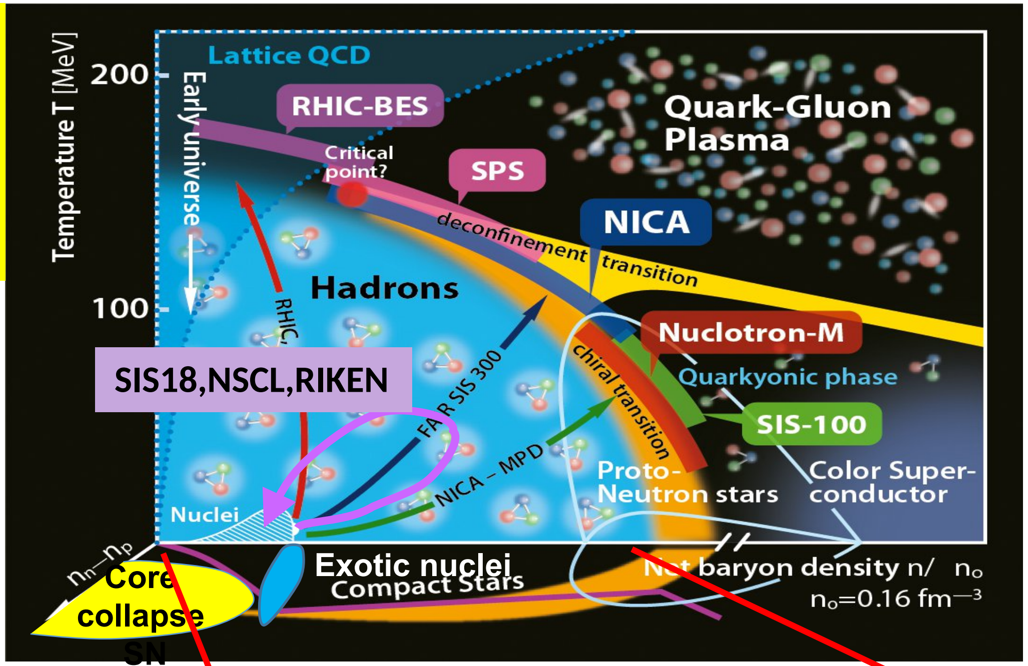
- summary and outlook

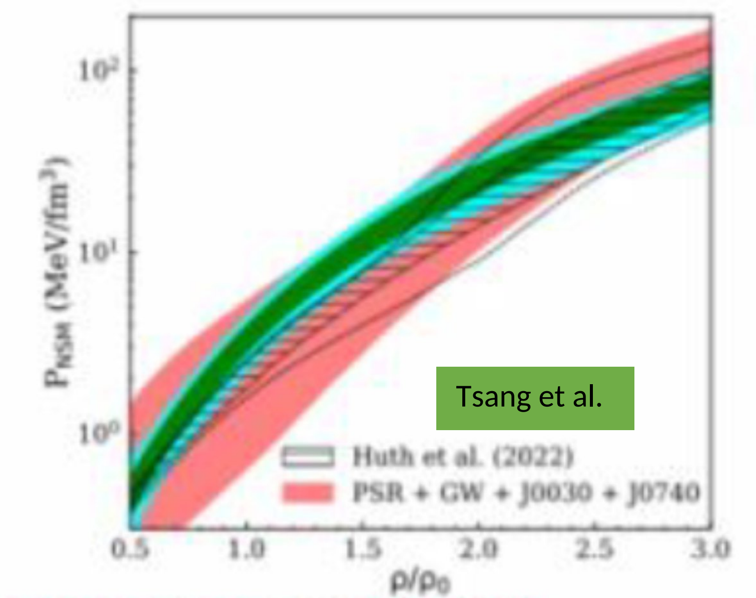
Motivation:
Importance of intermediate-energy heavy-ion collisions (HICs) for the exploration of equation-of-state (EOS)
as a fct. of density ✕, temp T, asymmetry 🚲

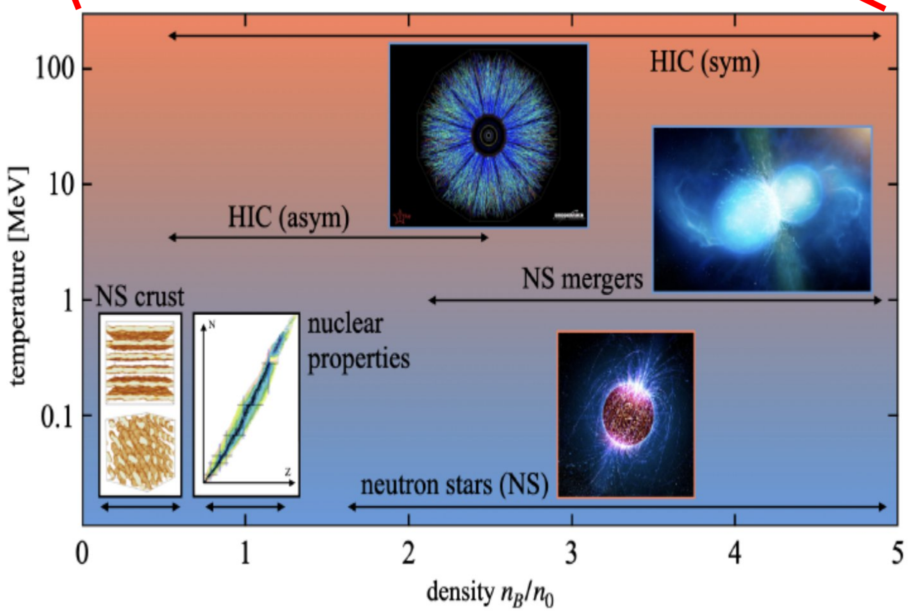
→ filling the gap between information from nuclear structure (✕ ≤ $✕_0$) and neutron star observations (✕ ⊛ 2.5 $✕_0$)

-> HICs can make a contribution to constrain EOS
 -> only astrophysics
 -> xEFT, Astro and HICs (Huth, et al., Nature 602 (22)
 -> structure, HICs and Astro (C.Y.Tsang, et al.,Nat.Astro 8 (24)

HICs are non-equilibrium processes, theoretical description complex,
but based on one code.
robustness of predictions is an issue.



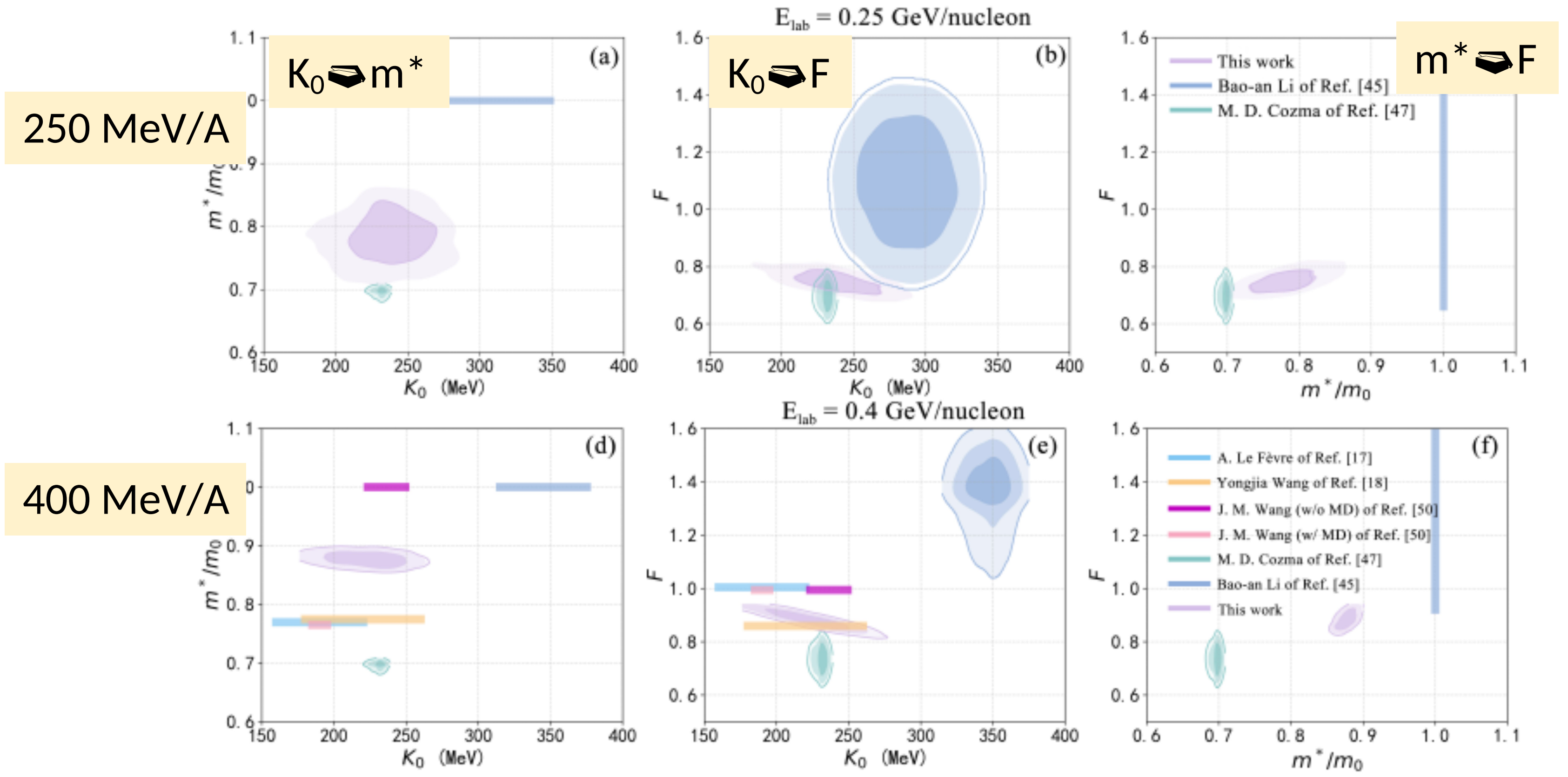




Model dependence of transport model results

non-equilibrium system-> non-equilibrium description, i.e. transport theory
Quantum transport theory (Kadanoff-Baym) complex -> semiclassical approx. (Boltzmann theory; molecular dynamics)

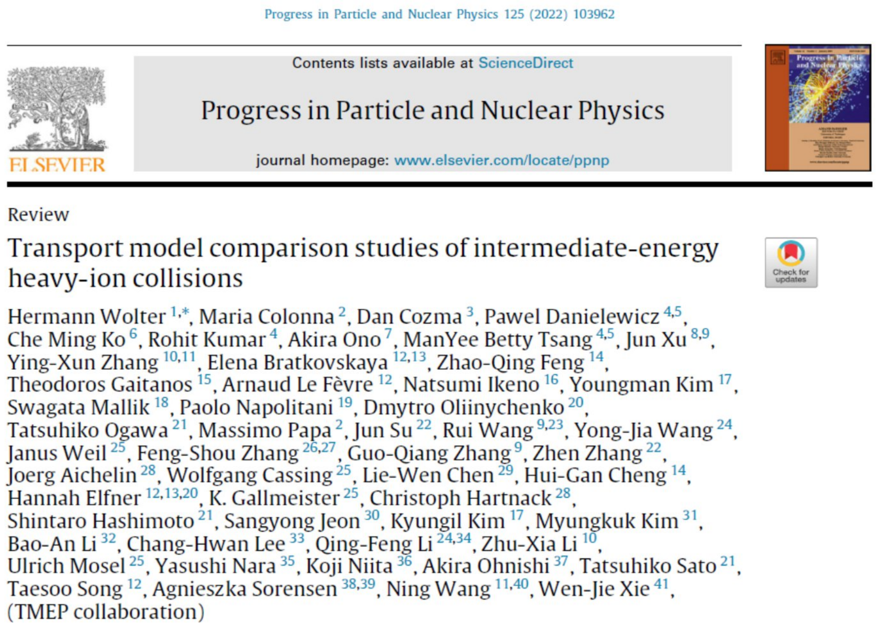
Example: Results of different codes on values and correlations between nuclear matter parameters
K_0, m^*/m, F (reduction of in-medium cross section) using the same (FOPI) data (G.J. Wei, et al, arxiv 2509.03406)



same data (FOPI, Au+Au),
regions of confidence
do not overlap

Increase robustness of conclusions: a) comparisons of transport codes in controlled conditions (TMEP)
b) not only compare but quantify uncertainty (Bayesian inference)

TMEP: compare different transport models und controlled conditions,
i.e. same physical model and similar calculational parameters .
but simplified physical model, no comp. to experiment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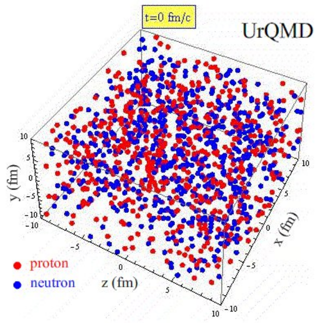
Review

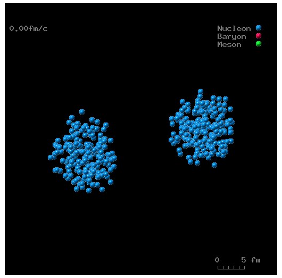
Transport model comparison studies of intermediate-energy heavy-ion collisions

Hermann Wolter [1,*], Maria Colonna [2], Dan Cozma [3], Pawel Danielewicz [4,5], Che Ming Ko [6], Rohit Kumar [4], Akira Ono [7], ManYee Betty Tsang [4,5], Jun Xu [8,9], Ying-Xun Zhang [10,11], Elena Bratkovskaya [12,13], Zhao-Qing Feng [14], Theodoros Gaitanos [15], Arnaud Le Fèvre [12], Natsumi Ikeno [16], Youngman Kim [17], Swagata Mallik [18], Paolo Napolitani [19], Dmytro Oliinychenko [20], Tatsuhiko Ogawa [21], Massimo Papa [2], Jun Su [22], Rui Wang [9,23], Yong-Jia Wang [24], Janus Weil [25], Feng-Shou Zhang [26,27], Guo-Qiang Zhang [9], Zhen Zhang [22], Joerg Aichelin [28], Wolfgang Cassing [25], Lie-Wen Chen [29], Hui-Gan Cheng [14], Hannah Elfner [12,13,20], K. Gallmeister [25], Christoph Hartnack [28], Shintaro Hashimoto [21], Sangyong Jeon [30], Kyungil Kim [17], Myungkuk Kim [31], Bao-An Li [32], Chang-Hwan Lee [33], Qing-Feng Li [24,34], Zhu-Xia Li [10], Ulrich Mosel [25], Yasushi Nara [35], Koji Niita [36], Akira Ohnishi [37], Tatsuhiko Sato [21], Taesoo Song [12], Agnieszka Sorensen [38,39], Ning Wang [11,40], Wen-Jie Xie [41], (TMEP collaboration)

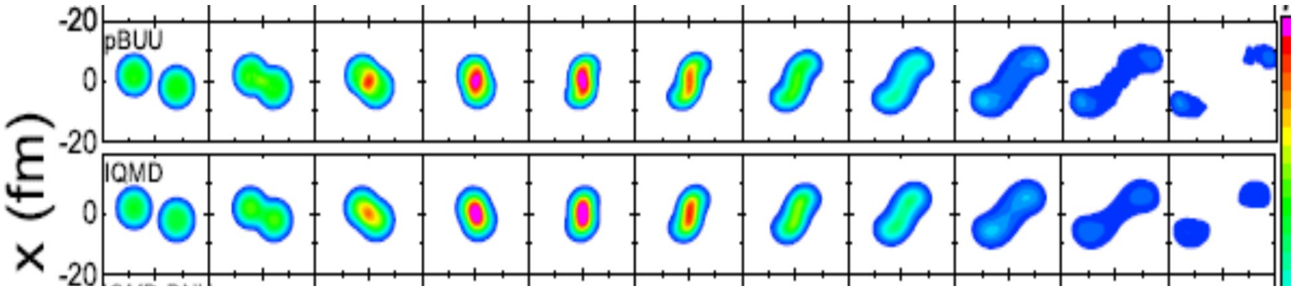


two modes of comparisons:

1. box calculations, periodic boundary conditions,
   - simulates infinite nuclear matter,
   - study individually different ingredients of physical model, compare to exact limits
   - understand important aspects of transport
   - always goes to equilibrium, may not say everything for dynamic HICs



2. Full heavy-ion collisions (HICs), compare codes between each other ,
   - substantial differences , explanations, but not sufficient convergence
   - systematic differences BUU-QMD
   - open system, small differences may cause large change in evolution



BUU
QMD

Box: Importance of Fluctuations:

transport equation (BUU)

$$\frac{Df(\vec{r},\vec{p};t)}{dt} = I_{coll}\left(\sigma^{in-med}_{12\to 34};1-f_i\right) + \delta I_{fluc}$$

(Vlasov) (dissipation) (fluctuation)

treatment of fluctuations:

1. BUU approach: Testparticles $f(\vec{r},\vec{p};t) = \sum_{i=1}^{A\,N_{TP}} g(\vec{r}_i-\vec{r}_i(t))\,\tilde{g}(\vec{p}-\vec{p}_i(t))$; g TP shape, zero or finite size

$N_{TP}\rightarrow$ □, deterministic, include flutuation term ● $I_{fluc,}$, ->**Boltzmann-Langevin** approach

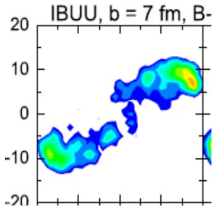
N_{TP} finite, numerical fluctuations, can be gauged to physical effects, e.g. damping of collective phenomena

2. QMD approach: molecular dynamics for nucleons with finite-size wave packets:

width parameter ***L*** regulates fluctuations

event fluctuations

$$f(\vec{r},\vec{p};t) = \left(\frac{\hbar}{\sqrt{L}}\right)^3 \sum_i \exp\left[-\frac{(\vec{r}-\vec{R}_i(t))^2}{2L}\right]\delta(\vec{p}-\vec{P}_i(t))$$

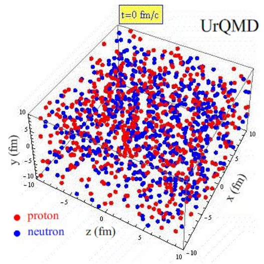


QMD event

different approximate approaches, influence results significantly

not clear, what is more realistic

in the end BUU and QMD should give the similar results for EOS from same data

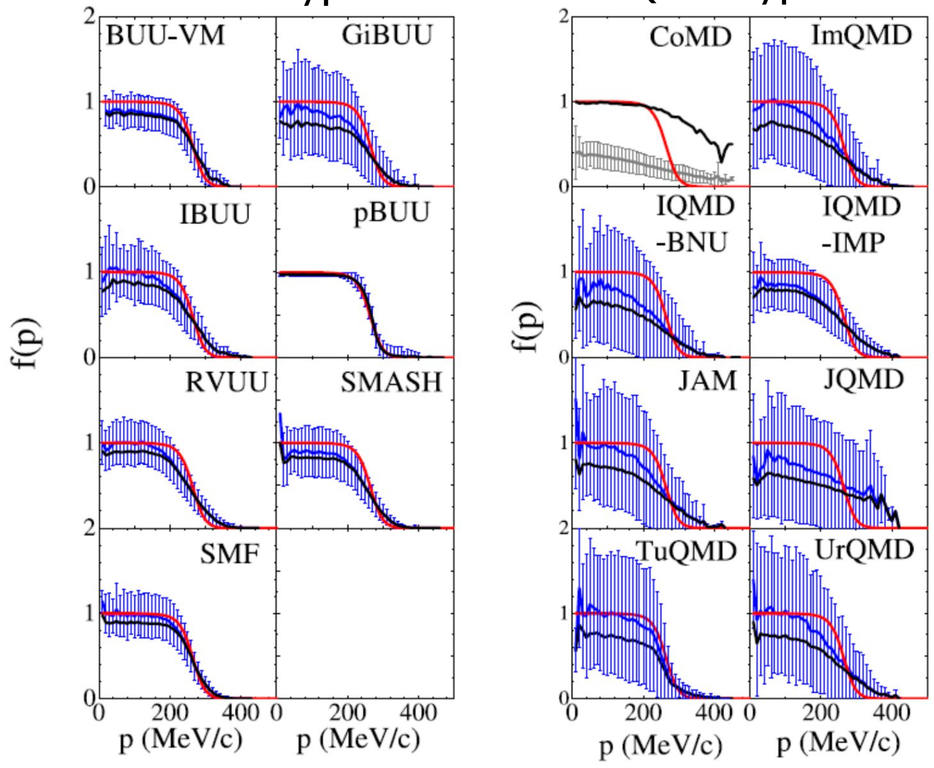


Box: Importance of Fluctuations: effect on blocking

Test of collision intergral w/o mean field (only nucleons, with Pauli blocking, at T=5 MeV
(YX. Zhang, et al., PRC 97 (2018))

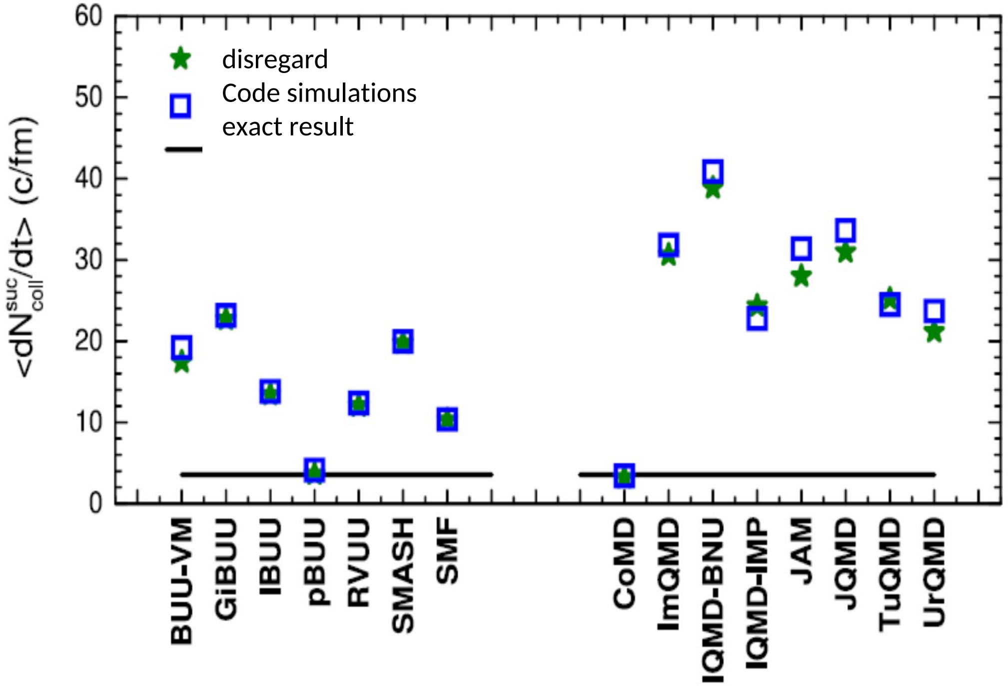
Fluctuations in occupation of final state occupation
exact: red
average: blue
effective (enforce f≤1): black

BUU-type QMD-type



larger fluctuations in QMD, smaller effective occupation

Collision rates, compared to exact result:



Reason: Fluctuations in Pauli blocking factor (1-f)
generally underblocking (black → red)
systematic diffrence in BUU and QMD

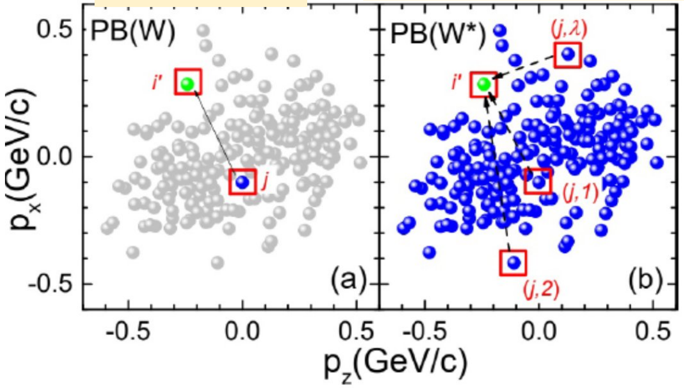
Box: importance of fluctuations: example of blocking term in QMD

Study of dependence of fluctuations in QMD

X. Chen,...,YX. Zhang, *A novel Pauli blocking method in quantum molecular dynamics type models*, Phys.Rev.C 109, L021604 (2024) and NuSYM2025

standard

new: av. over all momentum states

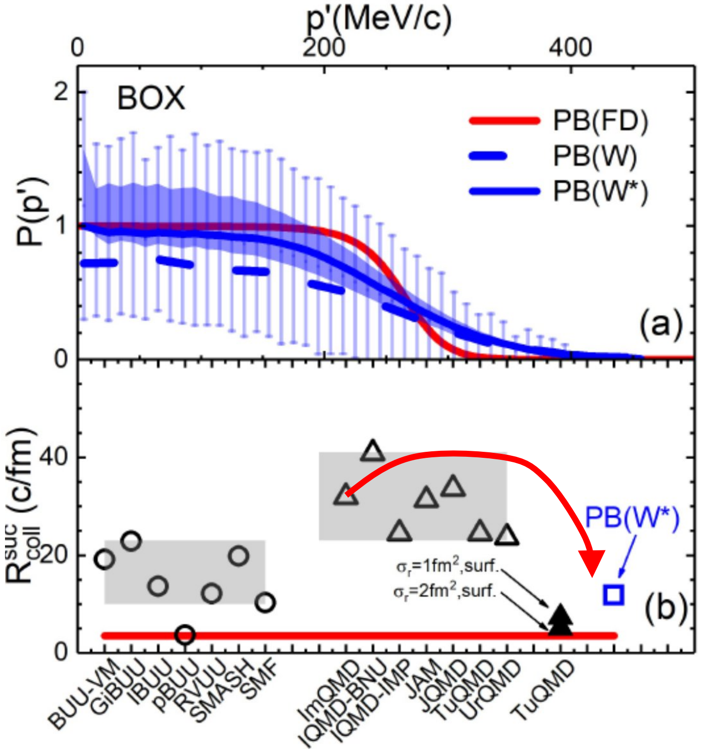


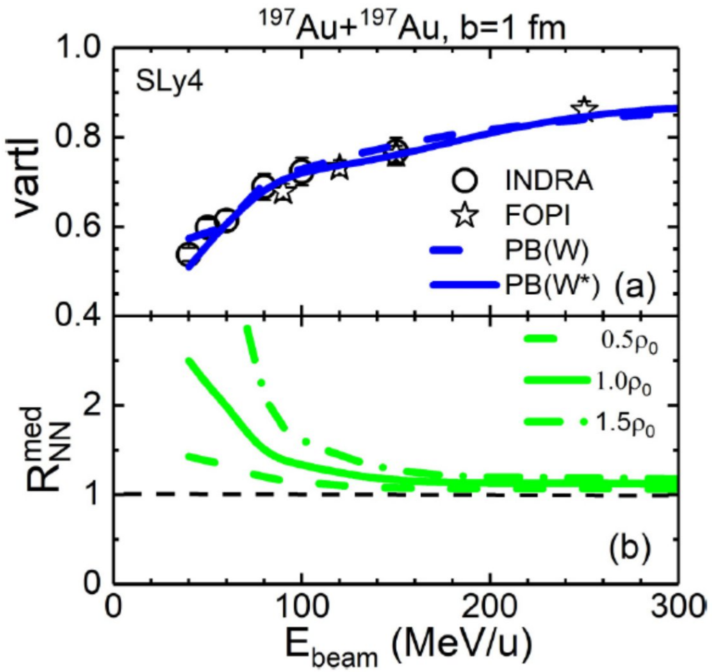
$$P_i = P(\mathbf{r}_i, \mathbf{p}'_i)$$

$$= \frac{1}{4/h^3} \frac{1}{(\pi\hbar)^3} \sum_{j=1, j\neq i}^{N} \exp\left[-\frac{(\mathbf{r}_i - \mathbf{R}_j)^2}{2\sigma_r^2}\right] c_j(\mathbf{p}'_i), \qquad (6)$$

where the factor $c_j(\mathbf{p}'_i)$ is calculated as

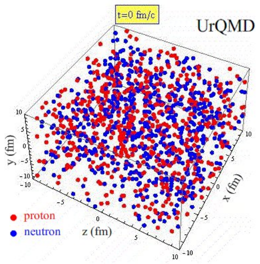
$$c_j(\mathbf{p}'_i) = \frac{1}{N} \sum_{\lambda=1}^{N} \exp\left[-\frac{(\mathbf{p}'_i - \mathbf{P}_{j,\lambda})^2}{2\sigma_p^2}\right]. \qquad (7)$$





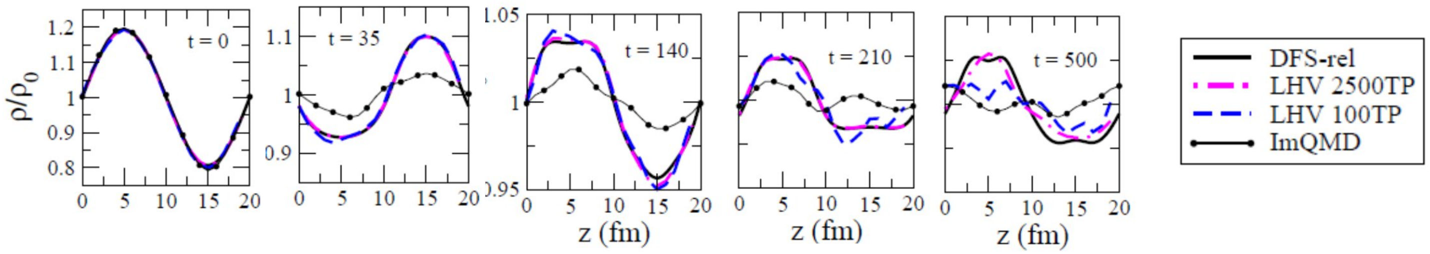
Increase of in-med Xsec
to fit **vartl** with new PB
-> up to factors of about 2!

Box: importance of fluctuations: influence on mean field propagation



Test of mean field evolution w/o collisions (M. Colonna, et al., PRC104 (2021))

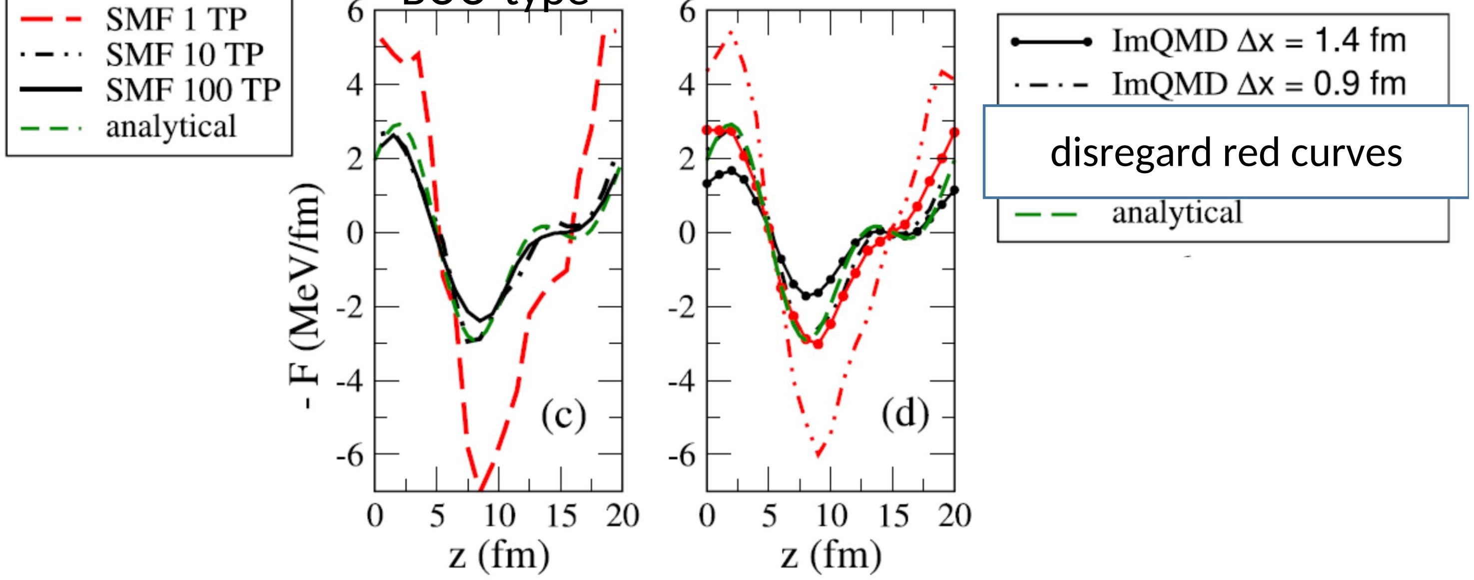
evolution of a standing wave



Force averaged in cells at initial time

SMF
BUU-type

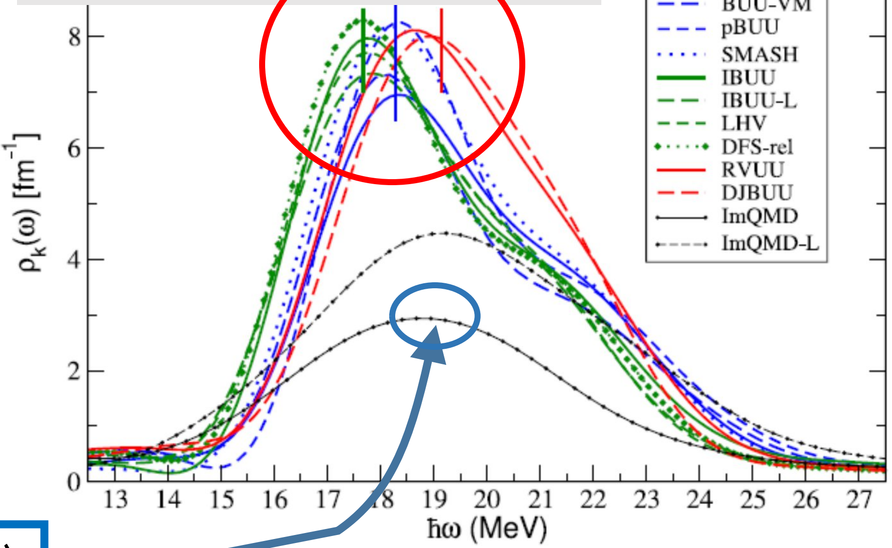
ImQMD



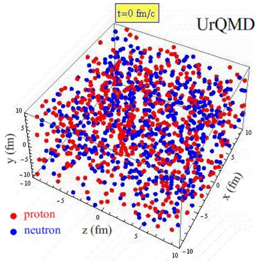
disregard red curves

strength function, power spectrum:

BUU type; horizontal lines exact results from Landau theory



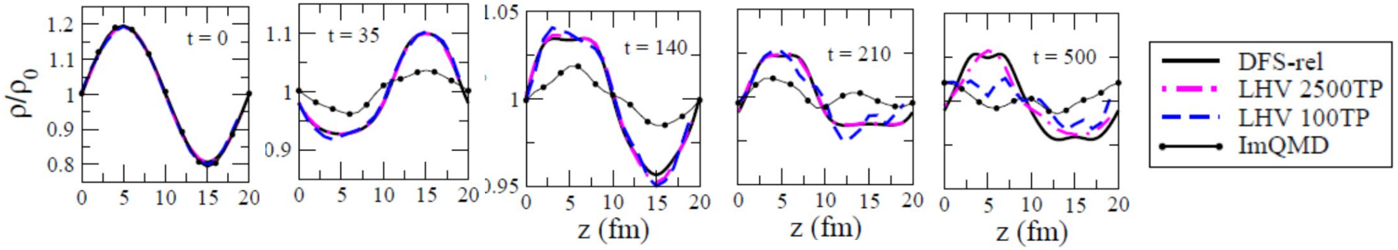
QMD codes: broadening (larger fluctuations)



Box: evaluation of mf terms: non-linear forces

Non-linear mean field evolution (M. Colonna, et al., PRC104 (2021))

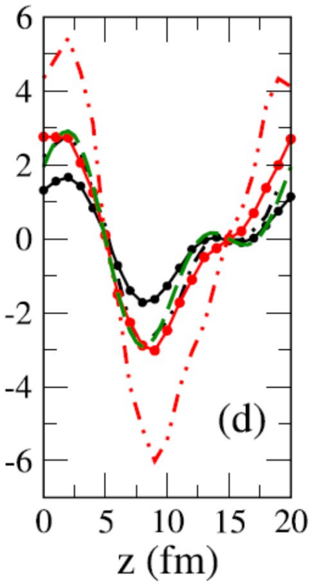
evolution of a standing wave

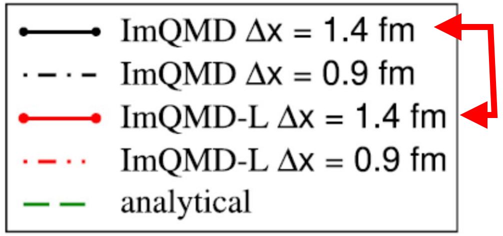


Force averaged in cells at initial time

ImQMD

non-linear force
$\langle\rho^{\gamma}\rangle$ $\langle\rho\rangle^{\gamma}$



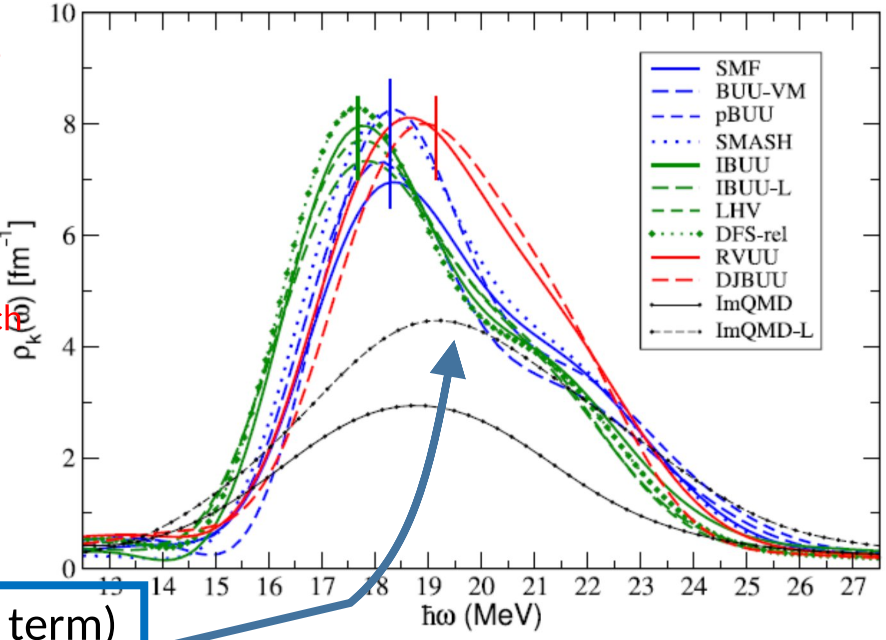


(black) approx. eval. of ρ^{γ} term

(red) better eval. of ρ^{γ} term

lattice Hamiltonian appraoch

strength function, power spectrum:
horizontal lines:exact results from Landau theory



QMD codes: shift (from approximation to ρ^{γ} term)
broadening (larger fluctuations)

Box: Collision Probability and Memory Effects

collision term (elastic)

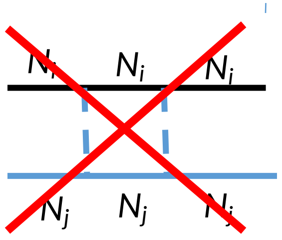
$$I_{coll} = \int d\vec{p}_2 d\vec{p}_{1'} d\vec{p}_{2'} \underbrace{v_{21}\,\sigma_{12}^{in-med}(\Omega)}_{\text{collision probablilty}} (2\pi)^3 \delta(p_1 + p_2 - p_{1'} - p_{2'}) \underbrace{\left[f_{1'} f_{2'} \bar{f}_1 \bar{f}_2 - f_1 f_2 \bar{f}_{1'} \bar{f}_{2'} \right]}_{\text{Pauli blocking (see above)}}$$

more general: $I_{coll} \rightarrow I_{el}(NN \leftrightarrow NN) + I_{inel}(NN \leftrightarrow N\Delta) + I_{decay}(\Delta \leftrightarrow N\pi)$

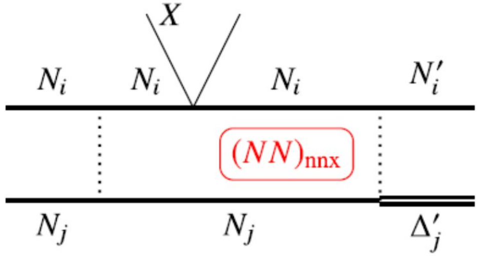
In **kinetic theory** collision term Markovian, i.e. collisions are not correlated.
In **simulations** the collision probability is treated by two methods

1. **geometric criterion**: two particles can collide if the distance is within the range given by the total cross section in a timestep. $\pi d_\perp^{*2} < \sigma \qquad |t^*_{\text{coll}} - t^*_0| < \frac{1}{2}\Delta t$

2. **statistical criterion:** assure that mean free path satisfies $\lambda = 1/\varrho\sigma$
   particles are chosen randomly in a cell and their collision proapility is $P_{ij}(\Delta t) \approx \varrho\sigma v_{ij}\Delta t$

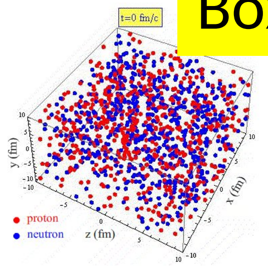


method 1 has to be supplemented to avoid **repeated collisions** by the same particles in subsequent steps. However, **higher order correlations** may occur, e.g. in an inelastic collisions with an intermediate interaction with a third particle.

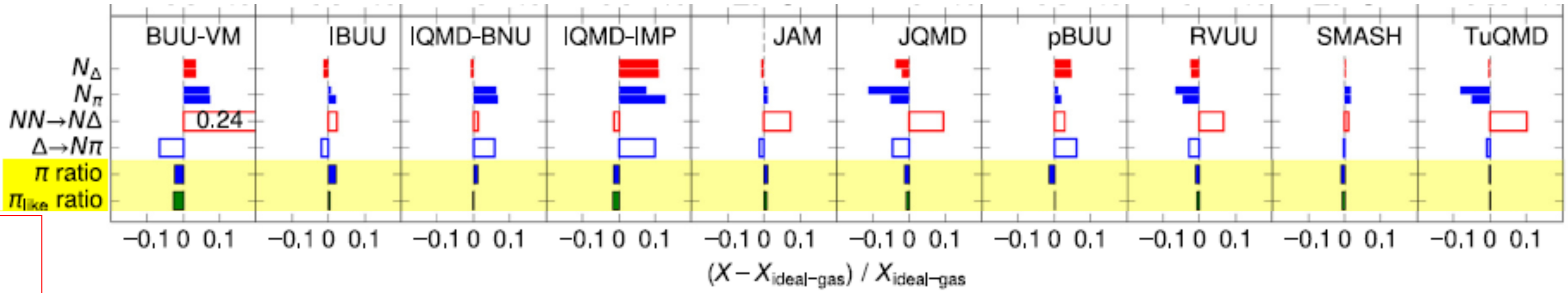


other effects may be due to the **order** in which elastic, inelastic and decay processes are treated. This was found to influence and multiplicities

Box 2: collision probability and memory effects



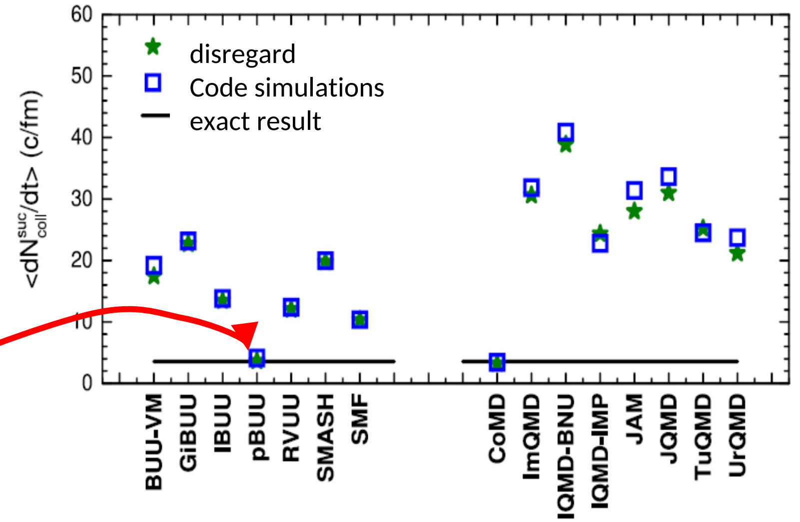
test of pion production in a box w/o Pauli blocking, cascade (A. Ono, et al., PRC 100 (2019))
extrapolation to time step zero
multiplicities and multiplicity ratios (relative difference to exact result)



determines
π–/π+ ratio
in a HIC

Understanding differences:

- correlations between collisions (non-Markovian behavior)
- geometric criterion induces correlations
- strategies in handling sequence of elastic and inelastic collisions
- Cancel rather well in ratios in a box
- statistical prescription (e.g. pBUU) works very well in box



Box calculations: lessons

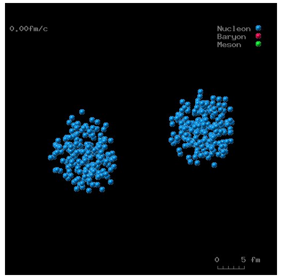
fluctuations affect evolution of HIC substantially:
less effective blocking, force calculation in mf propagation

memory effects
Simulations using geometrical criteria for the collision probility induce correlations,
cannot be completely eliminated, and are very dependent on strategies in the collision term
Have an effect on multiplicities of secondary particles
but perhaps strict Markovian behavior is an idealization?

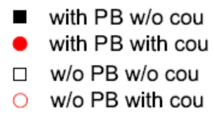
other effects (not yet studied in box calculation):
fluctuations are seeds for cluster correlations and light cluster production
(also for fragment formation)

-> important insights into transport smulations from box calculations
concern major open problem in treatment of HICs

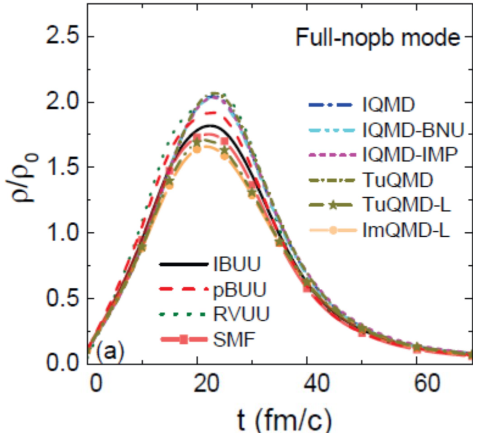
Comparison of HICs: open system



Sn+Sn@270 MeV/A (J. Xu, et al., PRC 109, 044609 (2024))
controlled input: common initializ., simple mom.-indep. EOS, σ_{el}=const, NN→NΔ, Δ→ Nπ

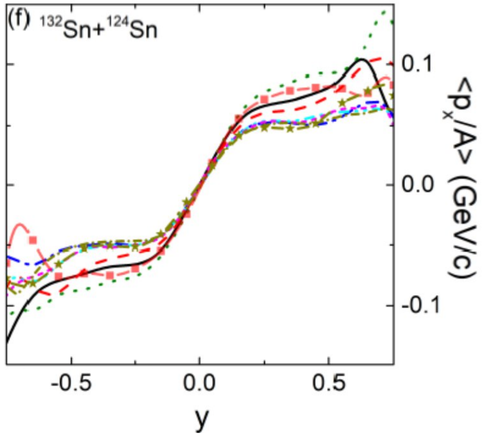


density evolution



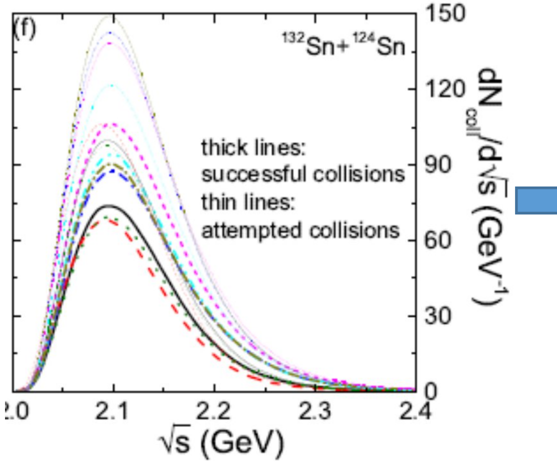
nucleon evolution not identical,
BUU codes have lower density fluctuations, approx. in non-linear term in QMD, weaker repulsion

side-ward flow



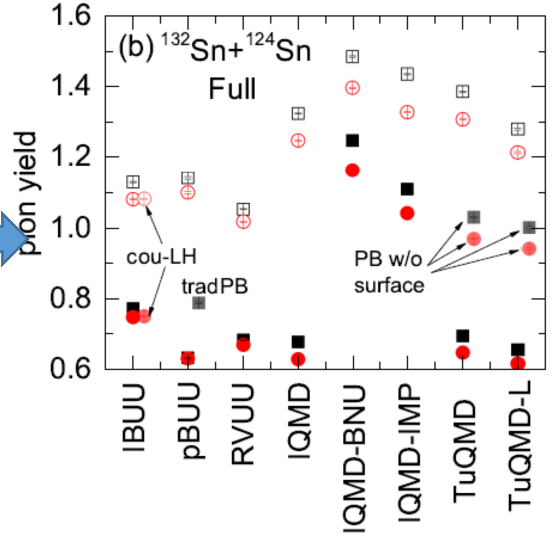
correspondingly different stoppîng and flow
BUU codes have stronger flow

inelastic reaction rates NN→NΔ



inelastic reaction rates are correspondingly weaker in BUU

pion multiplicities



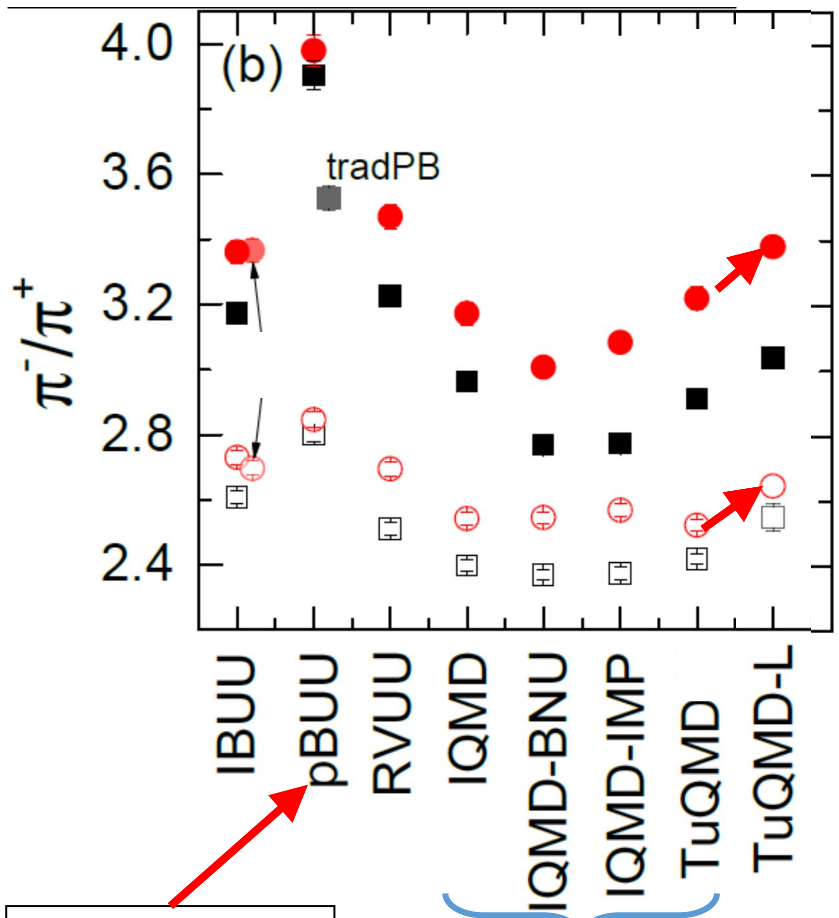
pion multiplicities
(w and w/o Pauli and Coulomb)
generally lower for BUU

Conclusion: differences in the evolution of the system, leads to difference in pion observables

HIC are open systems: small differences can propagate and lead to larger final results, ingrediens interact (unlike in box)

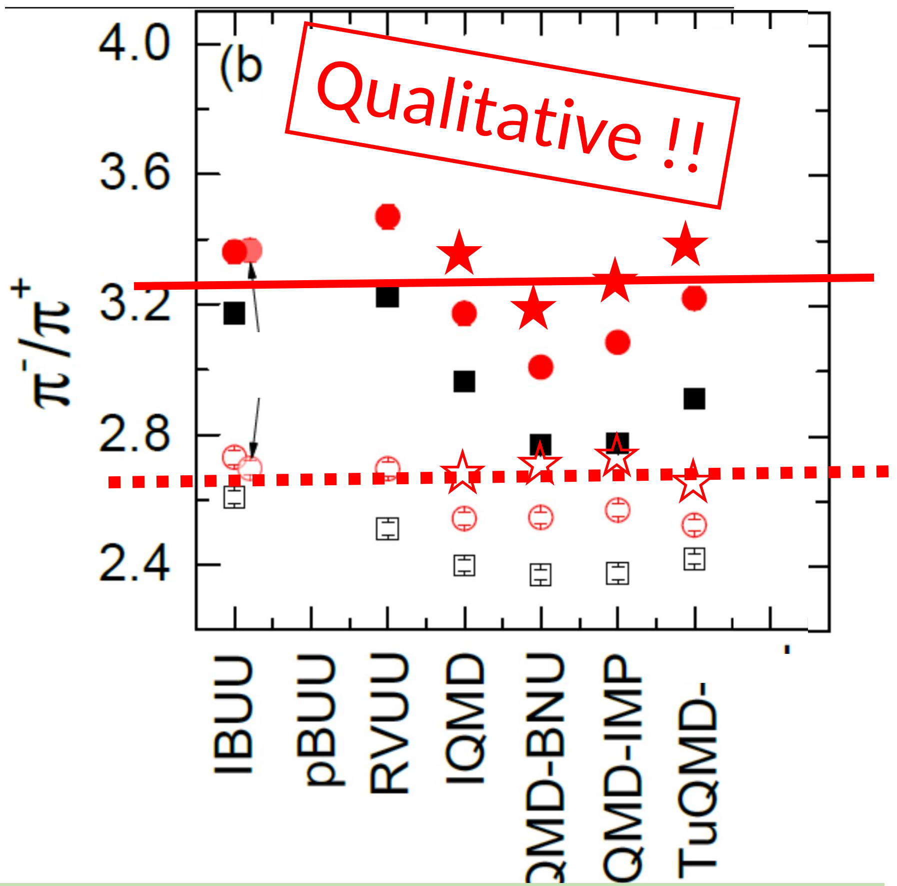
But differences can be understood

Look closer at charged pion ratio, assumed to be a good probe for the symmetry energy



■ with PB w/o cou
● with PB with cou
□ w/o PB w/o cou
○ w/o PB with cou

correct approximately for this effect (stars)



, feel symmetry energy not comparble

QMD codes, approx. for
✕■
weaker repulsion,
reduced pion ratio,
correct by TuQMD diff.

agreement w/o Pauli-blocking very good
with Pauli-blocking differences ■ **10%**
(remaining differences due to Pauli blocking surface correction)

Thus, can explain differences, but how to quantify uncertainties

Intermediate conclusion for code comparisons:

box calculation:

- we learn a lot about simulations of transport codes
- in particular we discussed the importance of fluctuations and of memory effects
- we can compare to exact limits, and in most cases satisfy these or understand deviations
- box calculations test equilibrium conditions, but not dynamic properties

HICs:

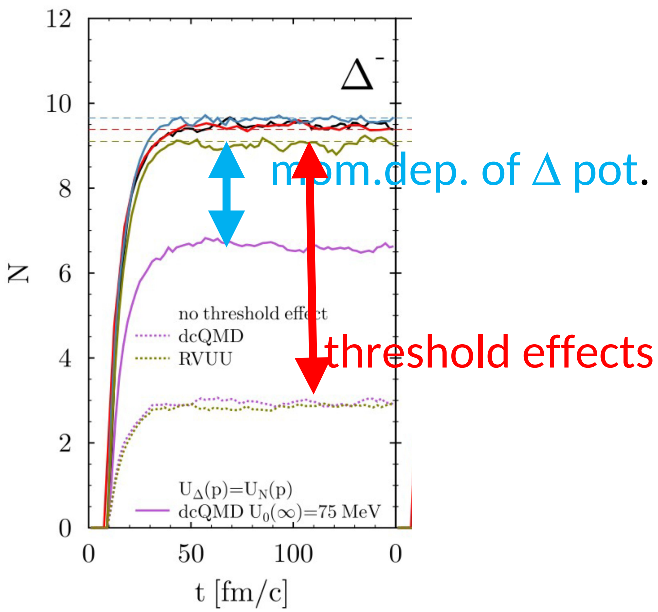
- we find considerable differences, which we can mostly explain. residual differences are < 10%
- but we cannot eliminate them, and it is not clear, what is best.
- open systems, surface effects
- uncertainty estimates between code are difficult
- one probably should go beyond mere comparisons of codes

ideas of future projects for TMEP:

- further box calculations to test some more critical ingredients of transport
- HIC: uncertainty quantification

Further **box comparisons:**

1. **momentum dependence of the potentials**, pre-requisite for a realistic description of HIC
   on-going study (Dan Cozma) including pion degrees-of-freedom
   compare to thermal model (equilibrium)
   results: mom.-dependence changes results substantially, threshold effects
   non-resonant pion production important at low pion energies



2. **fluctuations in a box**.
   exact limits from Laundau theory in the small amplitude limit
   insights on the different approaches of BUU and QMD on fluctuations

Fluctuation-dissipation theorem

- variance $\sigma_k^q = T/F^q(k)$, where $F^q(k) = (\frac{\partial U_k^q}{\partial \rho^q} + 1/\mathcal{N})$

U_k ≡ *Fourier transform of the mean-field potential* $\mathcal{N} = -\frac{4}{(2\pi)^3} \int dp \frac{\partial f_0}{\partial \epsilon}$

3. clusterization
   compare perturbative (coalescence) and dynamic (light cluster as explicit degrees of freedom) approaches
   medium effects on clusters (Mott effect)

$$\langle f_\tau \rangle_\nu(\vec{P}) \equiv \int \tilde{f}_\tau\left(\frac{\vec{P}}{A} + \vec{q}\right) |\phi_\nu(\vec{q})|^2 d\vec{q} \leqslant F_A^{cut}$$

A larger F_A^{cut} corresponds to a weaker Mott effect

=p,n
d,t,h,

Momentum distribution of the constituent nucleons, related to the light-nuclei internal wave function

Heavy-ion collisions: Uncertainty quantification of transport results

- go beyond mere comparisons
- try to quantify uncertainty of results not only from a single code, but from several codes,
- no exact results, use experiment as a reference

$\rightarrow$ comparative Bayesian inference

sketch possible steps and requirements

Bayesian inference with realistic models: steps

1. **Collect codes, that have all the physical ingredients for a realistic description of intermediate energy HIC,**
- mom.-dependence, symmetry energy, effective masses and n/p mass differences, medium modifications of Xsecs
- physical models not necessarily identical parametrization,but should allow to vary the physical properties
- BUU and QMD codes (if possible also BL and AMD),
- not necessarily very many, but willing to make modifications

2. **codes should satisfy benchmark calculations in box simulations**

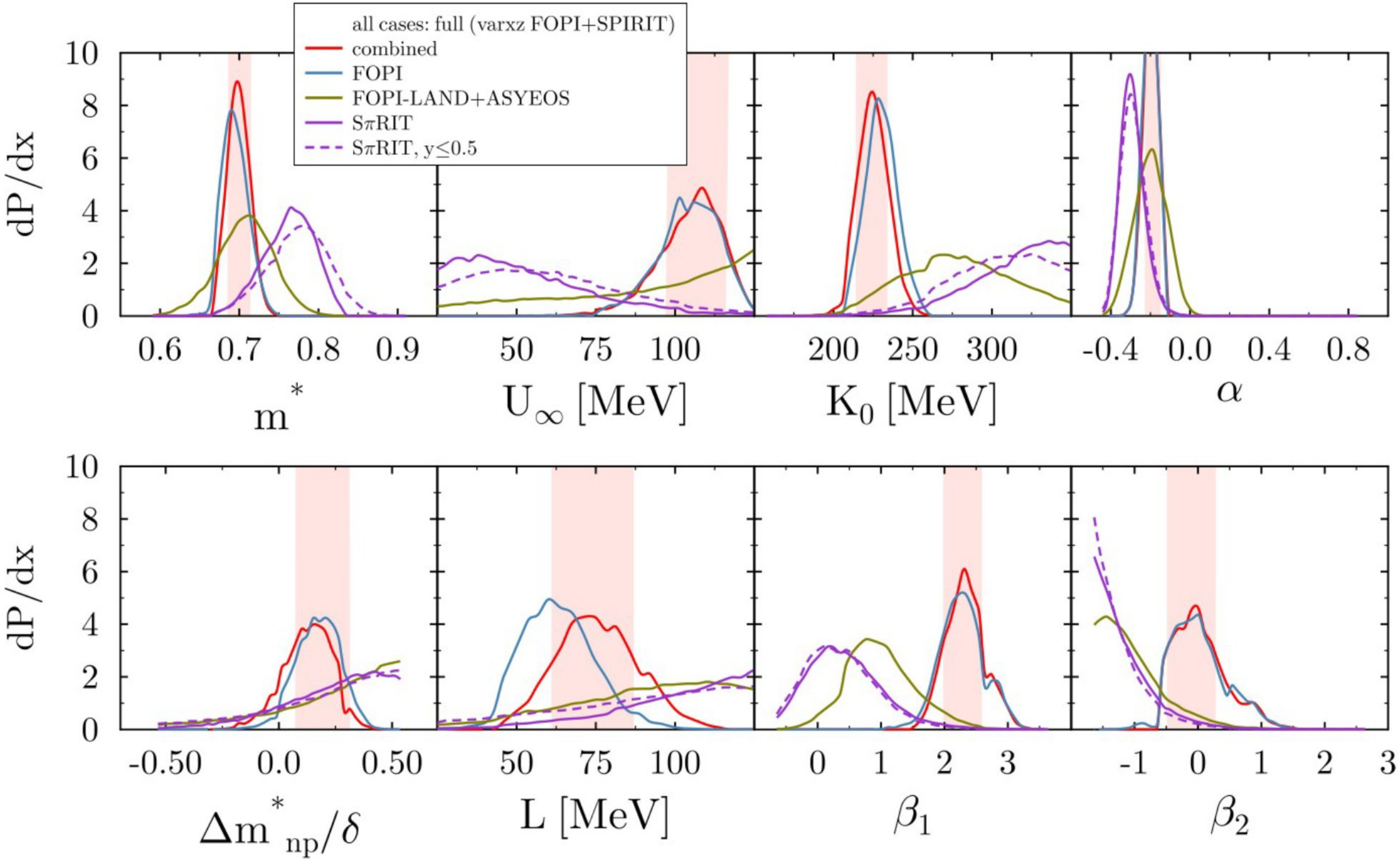
3. **Select observables sensitive to the EOS and dynamical properties**
- sufficiently complete data should exist, which include stopping and flow as functions of (y,p_T)
  for a suffiently large range of energies to allow to asses a range of densities and several systems (asymmetry effects)
- one could start with a subset of data and with a smaller set of parameters and expand subsequently,

4. Each code makes a **Bayesian inference** on the given set of data.
   obtains **probability density distributions (PDFs)** for the chosen EOS properties.
   sensitivity of the chosen EOS properties to data given by width of marginalized PDFs
   could be published individually for each code

Bayesian Inference: posterior Probability Distributions Functions (PDFs)

Example: Extensive Bayesian inference by Dan Cozma with the code dcQMD using FOPI, FOPI-LAND, ASY-EOS, and SpiRIT data on flow (v_1, v_2, varxz, y and p_T-spectra, and stopping: D . Cozma, arXiv 2407.16411, PRC 110 (2024)

PDFs for EOS parameters **using different data sets**



	FOPI	combined
K_0 [MeV]	230 ± 10	225 ± 10
L [MeV]	63 ±10	74 ± 13

The PDFs are strongy influenced by the choice of the data.

The width of the PDF give the intra-model uncertainty of the parameters by the given data.

Narrow PDFs signify a strongly constraining observable for the respective parameter, thus the **sensitivity** of an observable.

This is important information, but is derived from only one code, i.e. is model dependent.

Comparative Baysian inference (Bayesian Model Averaging, BMA):

- PDFs of different codes are compared
- they are combined in a Bayesian model averaging (BMA) with weights,
- which depend on the goodness of the optimal fit
- the averaged PDFs give the result of transport anlyses on the EOS as a whole and its uncertainty

Bayesian model averaging

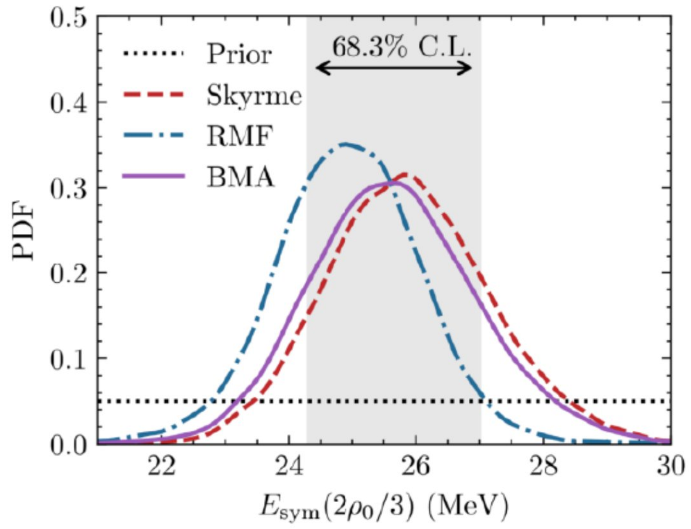
- Model prior $\pi(M_i)$: our preference on M_i before seeing the data.
- Model likelihood: (pseudo-)evidence $Z_i = p(\mathbf{y}\,|\,M_i)$
- Model posterior:

$$p(M_i\,|\,\mathbf{y}) = \frac{p(\mathbf{y}\,|\,M_i)\pi(M_i)}{\sum_j p(\mathbf{y}\,|\,M_j)\pi(M_j)}$$

- Model averaging for observable $\mathcal{O}$:

$$p(\mathcal{O}\,|\,\mathbf{y}) = \sum_i p(\mathcal{O}\,|\,\mathbf{y}, M_i)p(M_i\,|\,\mathbf{y})$$

Example from nuclear structure (M. Qiu,..,Z.Zhang, PLB849, 183435 (24))
BMA for the inference from data of proton-neutron chemical potential differences $\Delta\mu_{pn}{}^*$ on the symmetry energy at density $E_{sym}(2/3\ \rho_0)$ for two models Skyrme and RMF



M. Qiu, B.-J. Cai, L.-W. Chen, C.X.Yuan and **ZZ**, PLB 849, 138435 (2024)

width of BMA gives uncertainty of $E_{sym}(2/3\rho_0)$ taking into account two structure models

Comparative Baysian analysis: Observables

- **primary observables**, that determine the bulk properties of matter: nucleonic (n,p,LC)
  complete: flow, stopping (y, p_T; asymetry , density)
- understand and control first

secondary observables: pionic (,), strange (K,) depend strongly on bulk description,
reasonably complete similar as nucleonic data,
avoid ratios or double ratios, reduce sensitivity
issues:

- , potentials, momentum dependent
- resonant, non-resonant production mechanism (Cozma)
- anti-strange mesons are good messenger of the dense phase

Summary and outlook

TMEP project to evaluate codes by comparison of results with unified input

1. **box calculations,** comparison to exact equilibrium results
   - learn much about critical aspects of transport simulations
   - particular important: role of fluctuations
   - future box studies: fluctuations, momentum dependence, clusterization, short-range correlations

2. **HICs,** comparison between codes
   - differences in open systems
   - possible to explain difference, **residual difference estimated <10%**
   - not possible to quantify model dependence

3. one possible way to go: **Comparative Bayesian inference**
   - realistic models, **„complete“ as possible set of data**, Bayesian inference for each code
   - Baysian model averaging with weight, depending on quality of universal fit

   ☾ **uncertainty due to model dependence**

   other possible way: develop common modular code in a community effort (EMMI?)

Establish HICs as an indispensible way to constrain the EOS

Thank you for your attention