# Perspectives on determining the high density equation of state of nuclear matter using flow observables


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# Nucleonic Observables

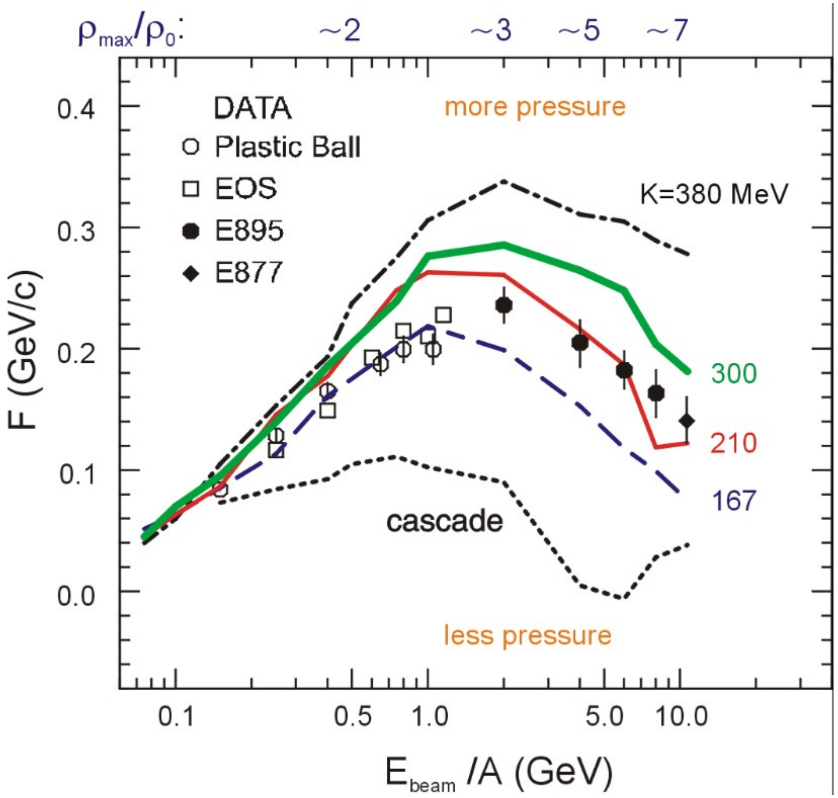
intermediate energy HIC collisions provide the opportunity to study three different aspects of the in-medium NN interaction:

- in-medium cross-sections
- momentum dependence of optical potential
- density dependence of EoS of cold nuclear matter

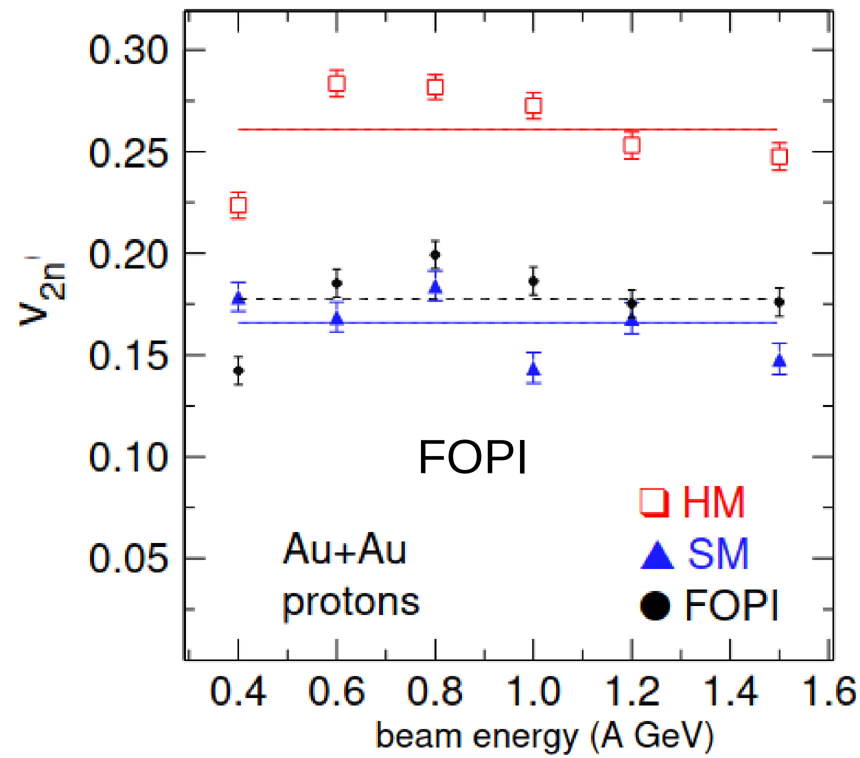
a divide-and-conquer approach has been often employed

arguably best suited for ratios of measured quantities: $K^+$ ratio (KaOS Coll): EoS of SNM

$\pi^-/\pi^+$ ratio (S$\pi$RIT Coll): symmetry energy




Danielewicz et al., Science 298, 1592 (2002)




Le Fevre et al., NPA 945, 112 (2016)

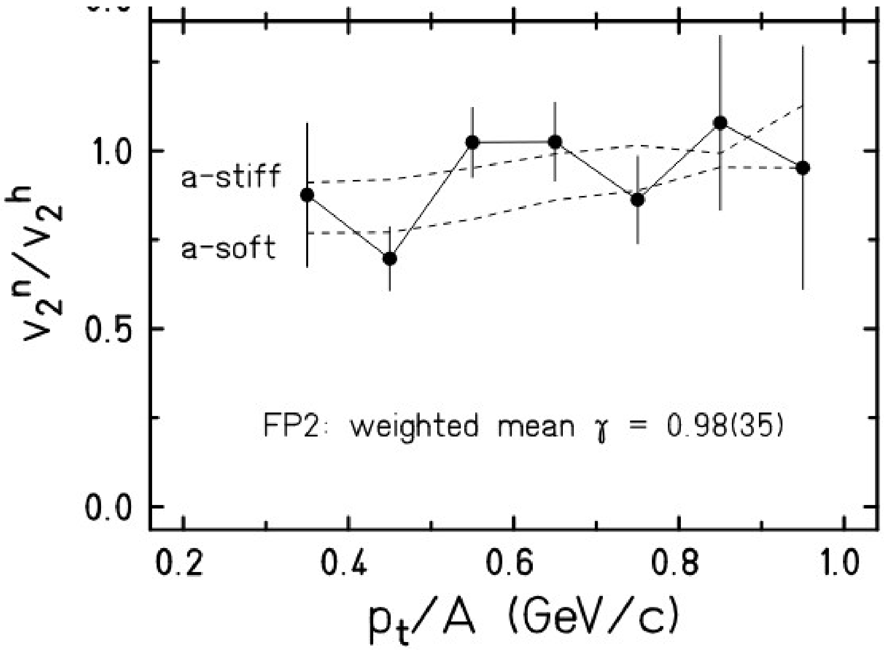
# Nucleonic Observables

extensive database of experimental data for nucleonic observables for intermediate energy HIC (FOPI, HADES, SπRIT, KaOS and others)

have not been used to their full potential

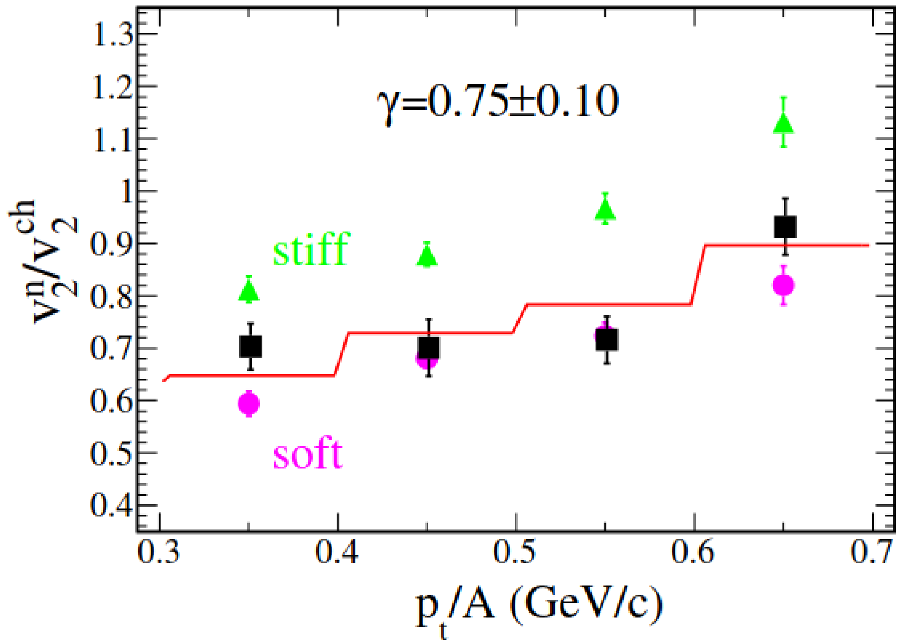
approach similar to that of Danielewicz et al. (Science 298, 2002), potential biases avoided by simultaneously determining in-medium cross-sections, optical potential and EoS

FOPI-LAND




P. Russotto et al., PLB 697, 471 (2011)

ASYEOS




P. Russotto et al., PRC 94, 034608 (2016)

SπRIT Collaboration: C.Y.Tsang et al. PLB 853, 138661 (2024)

# Interaction (nucleonic component)

**dcQMD** transport model: newest versions EPJA 57, 309 (2021); PRC 110, 064911 (2024)

momentum dependent potential **MDI**

Fix:
$U_\infty, K_0, J_0, m^*$ -isoscalar
$S(\tilde{u}), L, K_{sym}, \Delta m^*_{np}$ -isovector

$$\frac{E}{N}(\rho,\delta) = A_u(x,y)\frac{\rho(1-\delta^2)}{4\rho_0} + A_l(x,y)\frac{\rho(1+\delta^2)}{4\rho_0} + \frac{B}{\sigma+1}\frac{\rho^\sigma}{\rho_0^\sigma}(1-x\delta^2) + \frac{D}{3}\frac{\rho^2}{\rho_0^2}(1-y\delta^2) + \frac{1}{\rho\rho_0}\sum_{\tau,\tau'} C_{\tau\tau'}\iint d^3\vec{p}\,d^3\vec{p}'\frac{f_\tau(\vec{r},\vec{p})f_{\tau'}(\vec{r},\vec{p}')}{1+(\vec{p}-\vec{p}')^2/\Lambda^2}$$

momentum dependent part: similar with that of J. Xu et al. PRC 91, 014611 (2015)
(see also C. Hartnack, J. Aichelin PRC 49, 2801 (1994) )

independent part: extra term (vary L vs. $K_{sym}$ and also $J_0$ vs. K independently)

QMD EoM:

$$\frac{d\vec{p}_i}{dt} = \frac{-\partial\langle H\rangle}{\partial\vec{r}_i} \qquad \frac{d\vec{r}_i}{dt} = \frac{\partial\langle H\rangle}{\partial\vec{p}_i}$$

$$\langle H\rangle = \sum_i\sqrt{p_i^2+m_i^2} + \sum_{i,j,j>i}\left[\frac{A_u+A_l}{2} + \tilde{\tau}_i\tilde{\tau}_j\frac{A_l-A_u}{2}\right]u_{ij} + \sum_{i,j,j>i}\left[(C_l+C_u)+\tilde{\tau}_i\tilde{\tau}_j(C_l-C_u)\right]\frac{u_{ij}}{1+(\vec{p}_i-\vec{p}_j)^2/\Lambda^2} + \sum_i\frac{B}{\sigma+1}\left[1-x\tilde{\tau}_i\delta_i\right]u_i^\sigma + \sum_i\frac{D}{3}\left[1-y\tilde{\tau}_i\delta_i\right]u_i^2 + \sum_{i,j,j>i}U_{ij}^{Coul}$$

Mesonic part: see Phys. Rev. C 95, 014601 (2017)

# Threshold Effects

- direct consequence of imposing (total) energy conservation in the medium

$$\sqrt{p_1^2+m_1^2}+U(p_1)+\sqrt{p_2^2+m_2^2}+U(p_2)=\sqrt{p'^2_1+m'^2_1}+U(p'_1)+\sqrt{p'^2_2+m'^2_2}+U(p'_2)$$

- only few transport models below 1 AGeV:

RBUU: G. Ferini et al. PRL 97, 202301 (2006), RVUU: T. Song, C.M. Ko PRC 91, 014901 (2015);
χBUU: Z. Zhang et al, PRC 98, 054614 (2018), AMD+JAM: N. Ikeno et al., PRC 108, 044601 (2023)

- required for thermodynamical consistency of the model

Z.Zhang et al, PRC 97, 014610 (2018)

- reactions: NN↔NN, NN↔NR, R↔Nπ (R↔Nππ not corrected)

DC, PLB 753, 166 (2016)

- assumptions (dcQMD):
  - two-body collisions are part of N-body one
  - in-medium two-body collisions modeled as a succession of bare (vacuum-like) collisions followed/preceded by energy exchanges with the fireball, while momentum is conserved
  - reaction with highest probability: corresponds to the one which included the bare collision of highest probability

Example: elastic NN

$$\frac{d\sigma^{(med)}}{d\Omega} = (2\pi)^4 \frac{m_1^* m_2^*}{k_i^* \sqrt{s_i^*}} |M_{fi}^{(med)}(\rho,\delta,\{\tau\})|^2 \frac{k_f^* m_{1'}^* m_{2'}^*}{\sqrt{s_f^*}}$$

$$|M_{fi}^{(med)}(\rho,\delta,\{\tau\})|^2 = \frac{1}{2}(|M_{fi}^{(vac)}(\tilde{s}_i)|^2 + |M_{fi}^{(vac)}(\tilde{s}_f)|^2) \longleftarrow \sqrt{\tilde{s}_{i,f}} - 2\,m_N = \sqrt{s_{i,f}^*} - \sqrt{s_{th}^*} + U_{i,f} - U_{th}$$

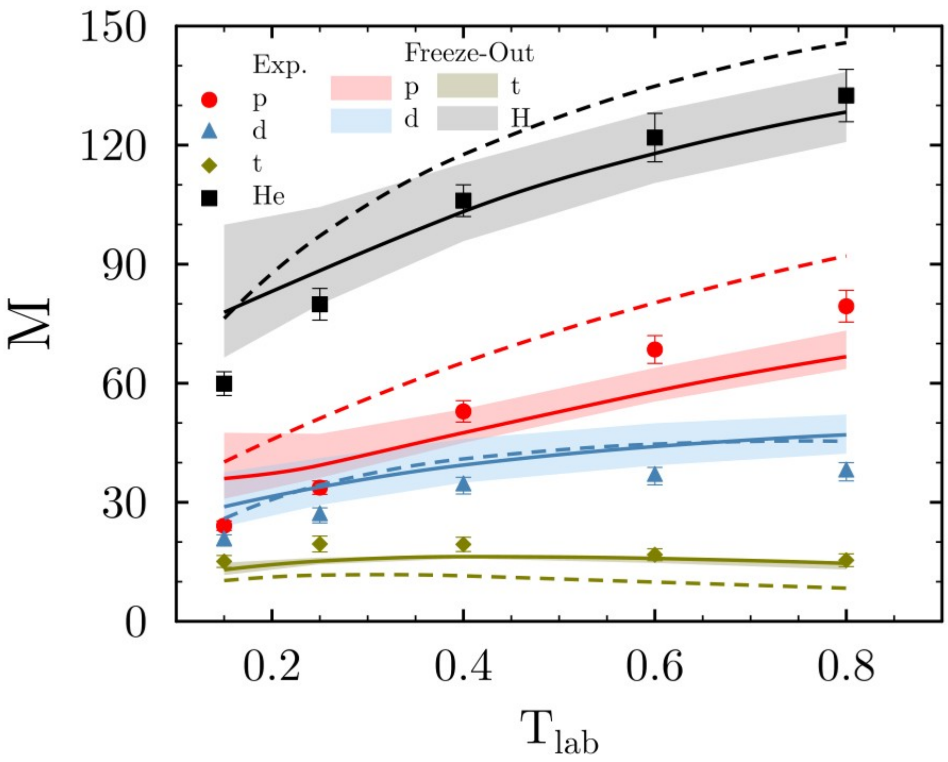
additional adhoc in-medium modification: $f(\rho,\delta)=\exp[\alpha\rho/\rho_0+\beta_1\delta^2\rho/\rho_0+\beta_2(\tau_1+\tau_2)\delta\rho/\rho_0]$

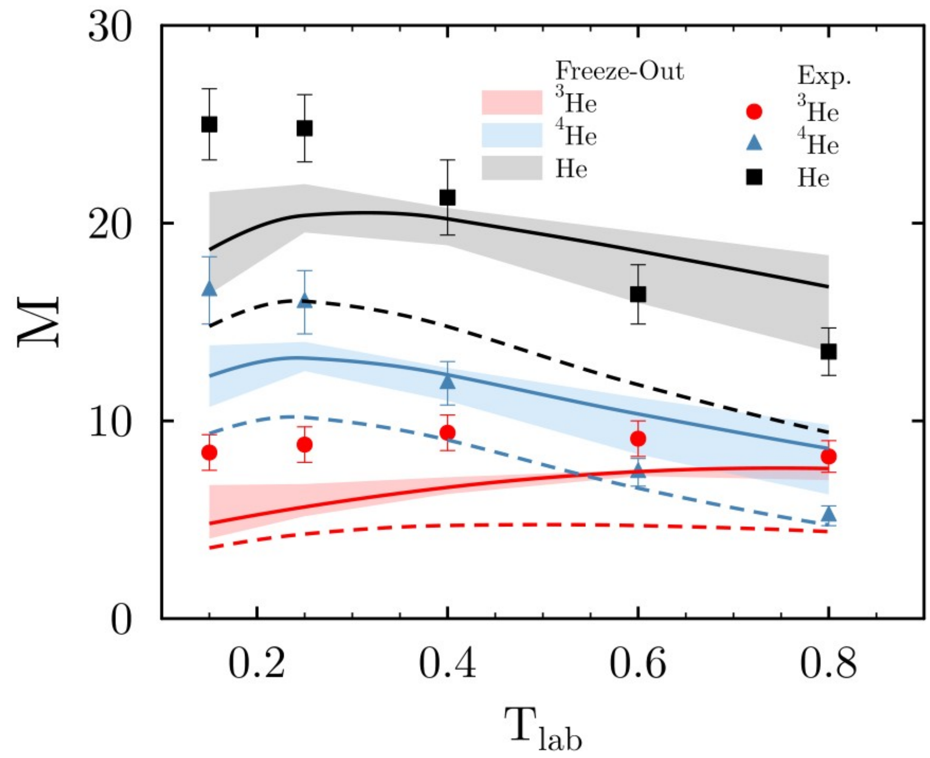
# Cluster multiplicities

- **final state:** MST algorithm all stable clusters (τ > 1ms ) with A≤15, 23 additional  A>15 (B,C,N,O)
- coalescence algorithm applied at local freeze-out time

- transport model parameters determined from a fit of $v_1$,$v_2$ and varxz experimental data
- δr=3.0-4.0 fm, δp=0.2-0.3 GeV/c (bands in figures below)

Hydrogen                central ($b_0$<0.15)AuAu HICs (FOPI)                Helium




dashed curves: results with the coalescence model applied at final time (150 fm/c)

# Probed Density (Free Protons)

- observables are functionals of the EoS
- sensitivity: functional derivatives w.r.t to EoS or d EoS/ dρ, etc.

$$\frac{d\,Obs}{d\,EoS}=\frac{\lim_{\epsilon\to 0} Obs[\frac{dEoS}{d\rho}(\rho)+\epsilon\delta(\rho-\tilde{\rho})]-Obs[\frac{dEoS}{d\rho}(\rho)-\epsilon\delta(\rho-\tilde{\rho})]}{2\epsilon}$$

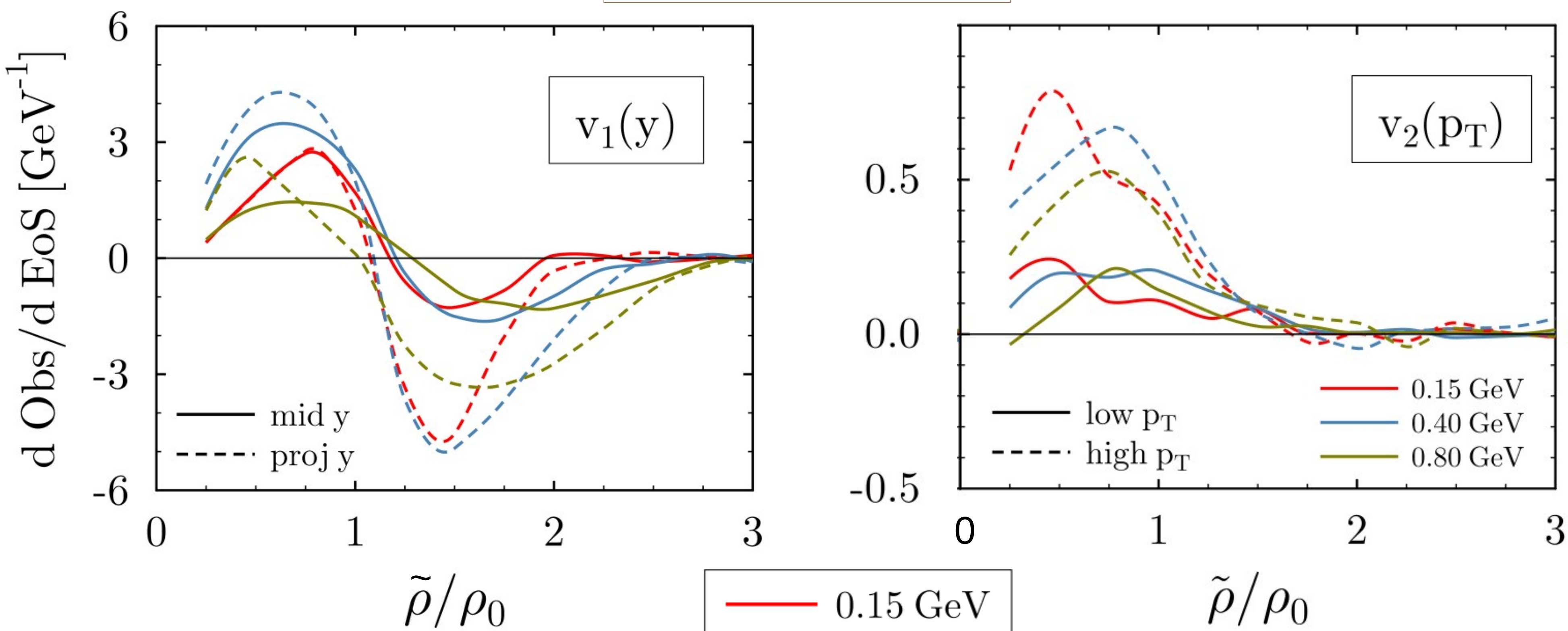
$$\Delta\langle H_{L_0}\rangle = \sum_{i=n,p}\frac{\varepsilon}{2}\left[Erf\left(\frac{\tilde{u}}{\eta}\right)-Erf\left(\frac{\tilde{u}-u_i}{\eta}\right)\right] \quad (1$$

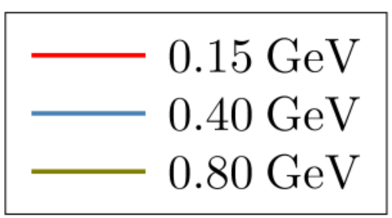
$$\Delta\langle H_{L}\rangle = \sum_{i=n,p}\frac{\varepsilon}{2}\tilde{\tau}_i\,\delta_i\left[Erf\left(\frac{\tilde{u}}{\eta}\right)-Erf\left(\frac{\tilde{u}-u_i}{\eta}\right)\right]$$

**Isoscalar EoS**

AuAu 3.35<b<6.0 fm

**Symmetry Energy**




mid y: $0.0<y/y_P<0.5$

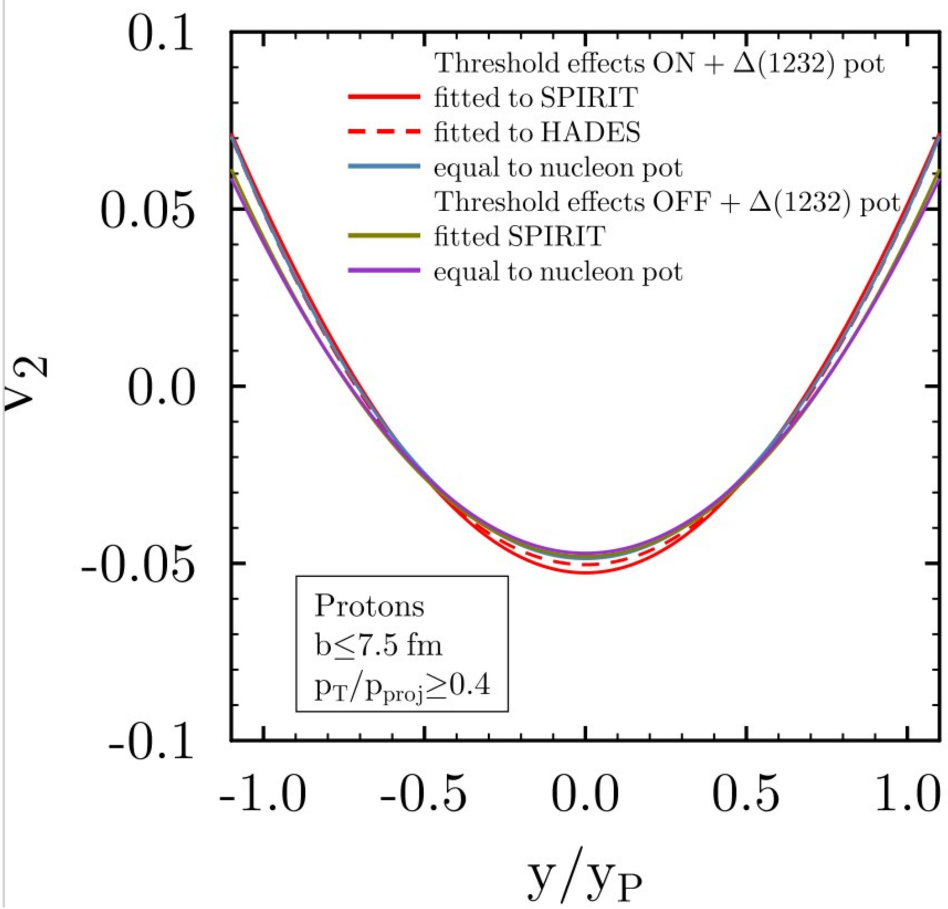
proj y: $0.5<y/y_P<1.0$

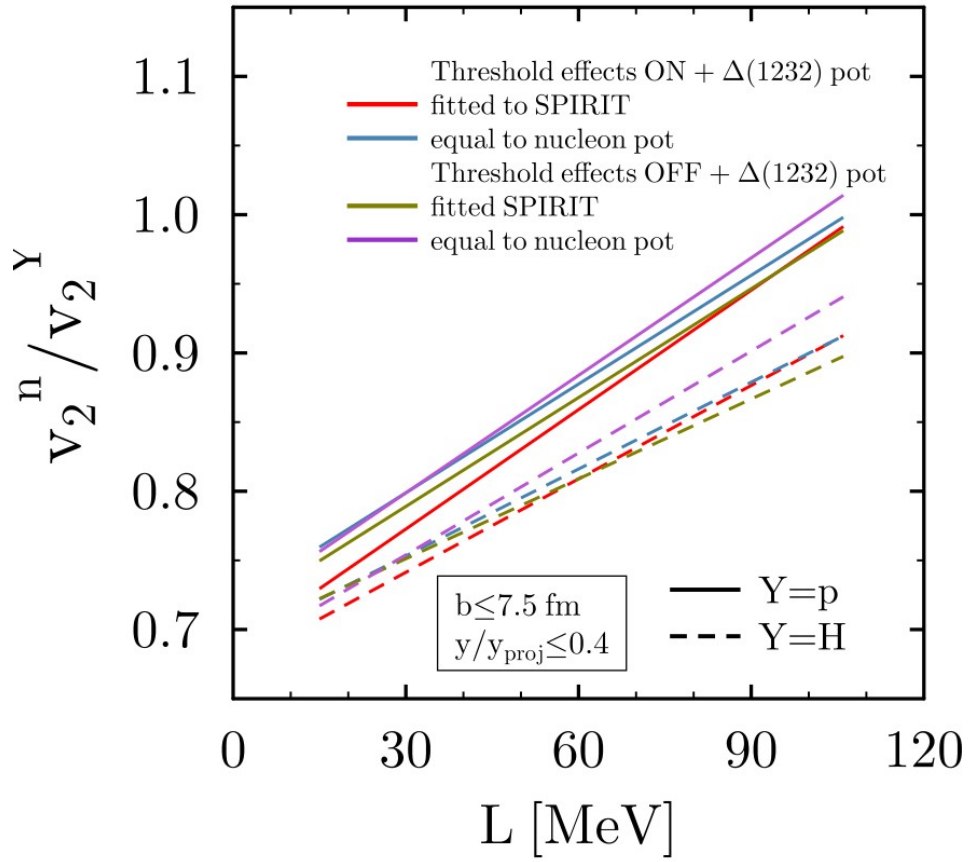
low $p_T$: $0.4<p_T/p_P<1.2$

high $p_T$: $1.2<p_T/p_P<2.0$

# Impact of Resonance Potentials on Flow

At 1.0 GeV/nucleon about 20% of nucleons excited into Δ(1232) at the highest density for central collisions (AuAu)




compressibility modulus: extracted from $v_{2n}=|v_{20}|+|v_{22}|$
impact of Δ(1232) potential on proton $v_{2n}$: ~ 10%
equivalent to $\delta K_0$=-25 MeV

impact on $v_2^n/v_2^Y$: ~5% equivalent to δL=10 MeV

A. Le Fevre et al., NPA 945, 112 (2016)

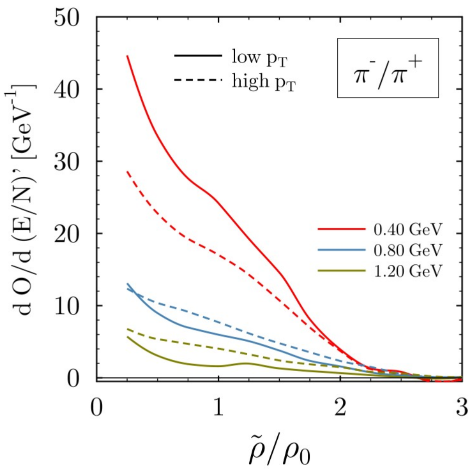
# Towards Higher Densities

**EoS of Symmetric Matter:**

1) Nucleonic Observables at impact energies below 0.8 GeV/nucleon: FOPI,SπRIT
2) Δ(1232) potentials: Pion production close to threshold (≤0.8 GeV/nucleon): FOPI,SπRIT
3) N(1440) potentials: Pion production at impact energies ≥ 1.0 GeV/nucleon: HADES
4) Nucleonic Observables at impact energies ≥ 1.0 GeV/ nucleon: HADES, FOPI

**Symmetry Energy**

1) Nucleonic Observables  at impact energies below 1.0 GeV/nucleon ($\rho \leq 1.5\ \rho_0$): ASYEOS,FOPI
2) Pion production ($\rho \leq 2.0\ \rho_0$): SπRIT, FOPI, HADES
3) Strangeness production ($\rho \leq 3.0\ \rho_0$ ??): FOPI, HADES




**Model Dependence**

TMEP Collaboration (see talk by H. Wolter on Monday)

# Nucleonic Observables

STEP 1

1) FOPI data set
- theoretical flow data away from mid-rapidity plagued by contamination from heavier clusters (most visible for proton $v_1(p_T)$ but also $v_1(y)$)
- exp data up 0.8 GeV/nucleon (impact of nucleon resonances on dynamics is small)

| Label | Observables | Reference |
|---|---|---|
| FOPI1 | $v_1(y)$, $\lvert y\rvert \leq 0.5$ for Z=1,2 clusters; M5 centrality<br>$^{197}$Au+$^{197}$Au, $^{129}$Xe+CsI, $^{58}$Ni+$^{58}$Ni (0.25, 0.40) | A. Andronic et al.<br>PRC 67, 034907(2003) |
| FOPI2 | $v_1(y)$, $\lvert y\rvert \leq 0.5$ for p, d, A=3 and $\alpha$ clusters; $0.25 \leq b_0 \leq 0.45$<br>$^{197}$Au+$^{197}$Au (0.25, 0.40, 0.60, 0.80)<br>$v_2(y)$, $\lvert y\rvert \leq 0.5$ for p, d and $\alpha$ clusters; $0.25 \leq b_0 \leq 0.45$<br>$^{197}$Au+$^{197}$Au (0.15, 0.25, 0.40, 0.60, 0.80)<br>$v_2(p_T)$, $\lvert y\rvert \leq 0.4$ for p, d and t clusters; $0.25 \leq b_0 \leq 0.45$<br>$^{197}$Au+$^{197}$Au (0.15, 0.25, 0.40, 0.60, 0.80) | W. Reisdorf et al.<br>NPA 876, 1 (2012) |
| varxz | varxz for p,d,t clusters; $b_0 \leq 0.15$<br>$^{40}$Ca+$^{40}$Ca (0.4, 1.0), $^{58}$Ni+$^{58}$Ni (0.15, 0.25), $^{96}$Ru+$^{96}$Ru (0.4, 1.0)<br>$^{96}$Zr+$^{96}$Zr (0.4), $^{129}$Xe+CsI (0.15, 0.25)<br>$^{197}$Au+$^{197}$Au (0.15, 0.25, 0.4, 0.6, 0.8) | W. Reisdorf et al.<br>NPA 848, 366 (2010) |
| spectra | longitudinal and transverse rapidity spectra for protons in $Z \leq 3$ clusters<br>$^{40}$Ca+$^{40}$Ca (0.4), $b_0 \leq 0.15$, $\lvert y_{L,T}/y_P\rvert \leq 1.25$ | W. Reisdorf et al.<br>PRL 92, 232301 (2004) |

results using above date set only: DC, Phys. Rev. C 110, 064911 (2024)

# Nucleonic Observables

2) FOPI-LAND, ASYEOS – dedicated symmetry energy measurement

3) SPIRIT – dedicated effective masses and symmetry energy campaign

| Label | Observables | Reference |
|---|---|---|
| FOPI-LAND | $v_2(p_T)$, 0.25$\leq$ y $\leq$ 0.75 for n,Z=1; b $\leq$ 7.5 fm<br>$^{197}$Au+$^{197}$Au (0.40) | P. Russotto et al.<br>PLB 697, 471 (2011) |
| ASYEOS | $v_2(p_T)$, 0.3$\leq$ y $\leq$ 0.7 for n,Z>0; b $\leq$ 7.5 fm<br>$^{197}$Au+$^{197}$Au (0.40) | P. Russotto et al.<br>PRC 94, 034608 (2016) |
| SPIRIT | varxz for p,d,t clusters; $b \leq 1.6$ fm<br>$^{112}$Sn+$^{124}$Sn (0.27)<br>$v_1(p_T), v_1(y), v_2(y)$ for p in Z=1,2 clusters; $3.6 \leq b \leq 6.3$ fm<br>$^{132}$Sn+$^{124}$Sn (0.27)<br>$v_1(p_T), v_1(y), v_2(y)$ for p in Z=1,2 clusters; $4.0 \leq b \leq 6.0$ fm<br>$^{108}$Sn+$^{112}$Sn (0.27) | C.Y. Tsang et al.<br>PLB 853, 138661 (2024) |

# Theoretical Simulations

**Model parameters:**

| | | |
|---|---|---|
| m* | [0.6, 0.9] | isoscalar effective mass |
| $V_\infty$ | [25, 125] MeV | isoscalar potential $p \to \infty$ |
| $K_0$ | [ 165, 355 ] MeV | compressibility modulus |
| α | [-0.4, 0.8] | in-medium $\sigma_{NN}$, δ=0.0 |
| $\Delta m^*_{np}$ | [-0.25,0.25] at ($\rho=\rho_0$, δ=0.5) | n-p effective mass diff. |
| L | [15, 145 ] MeV | slope symmetry energy |
| $\beta_1$ | [-0.5,3.5] | in-medium $\sigma_{NN}$, δ<>0.0 |
| $\beta_2$ | [-1.5, 2.5] | $\sigma_{nn}$<>$\sigma_{pp}$, δ<>0.0 |

**Enforced correlations:**

$J_0$=-600+($K_0$-165)*3.125

$K_{sym}$=-488+L*6.728

in units of [MeV]

Input:

E/N($\rho_0$)=-16.0 MeV

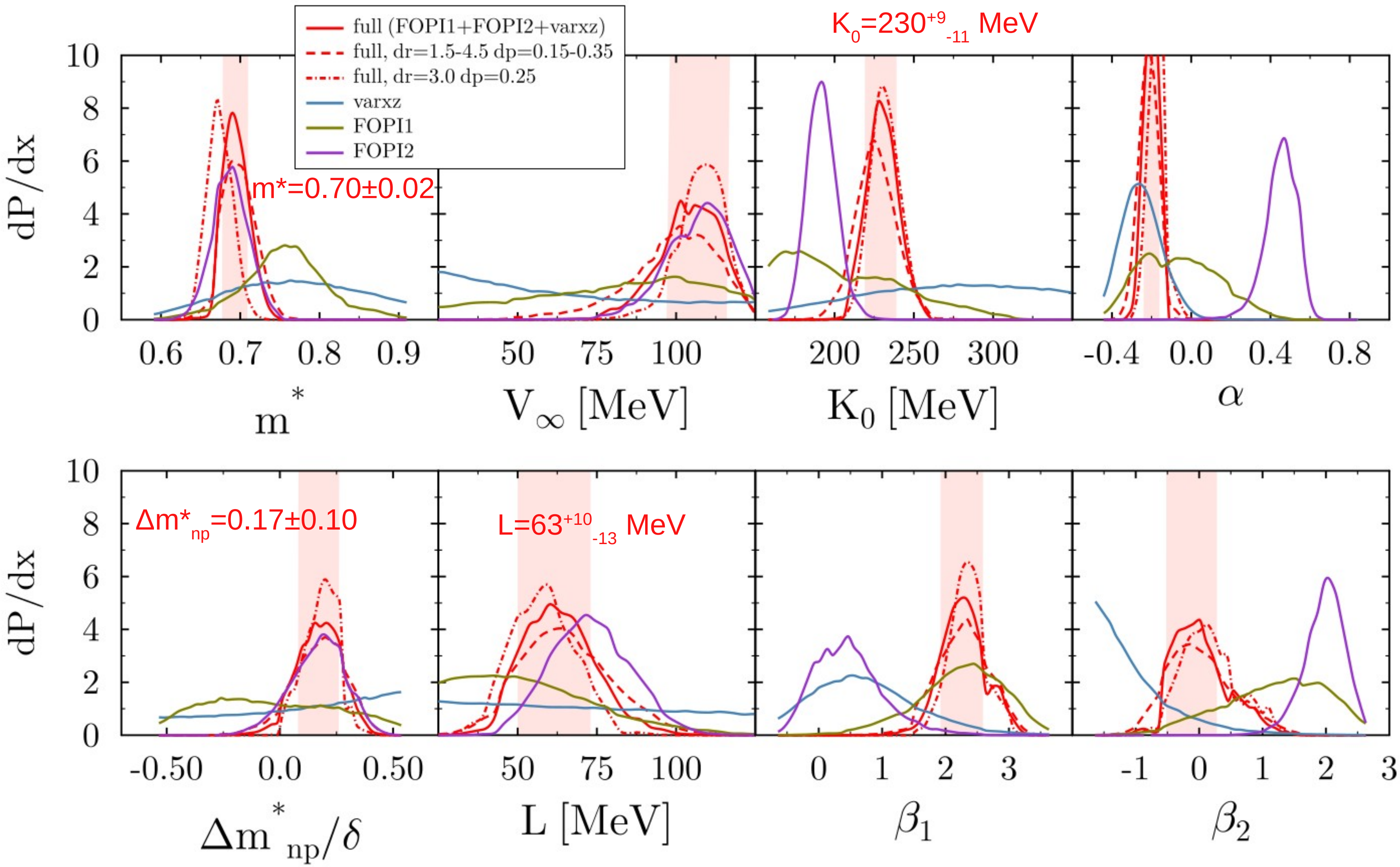
S(0.62$\rho_0$)=25.5 MeV

properties of doubly-magic nuclei (binding energies, rms charge radii, single particle energies)
Brown, PRL 111, 232502 (2013)

Model uncertainty: statistical+ systematical (δr=3.0-4.0 fm, δp=0.2-0.3 GeV/c ) added in quadrature

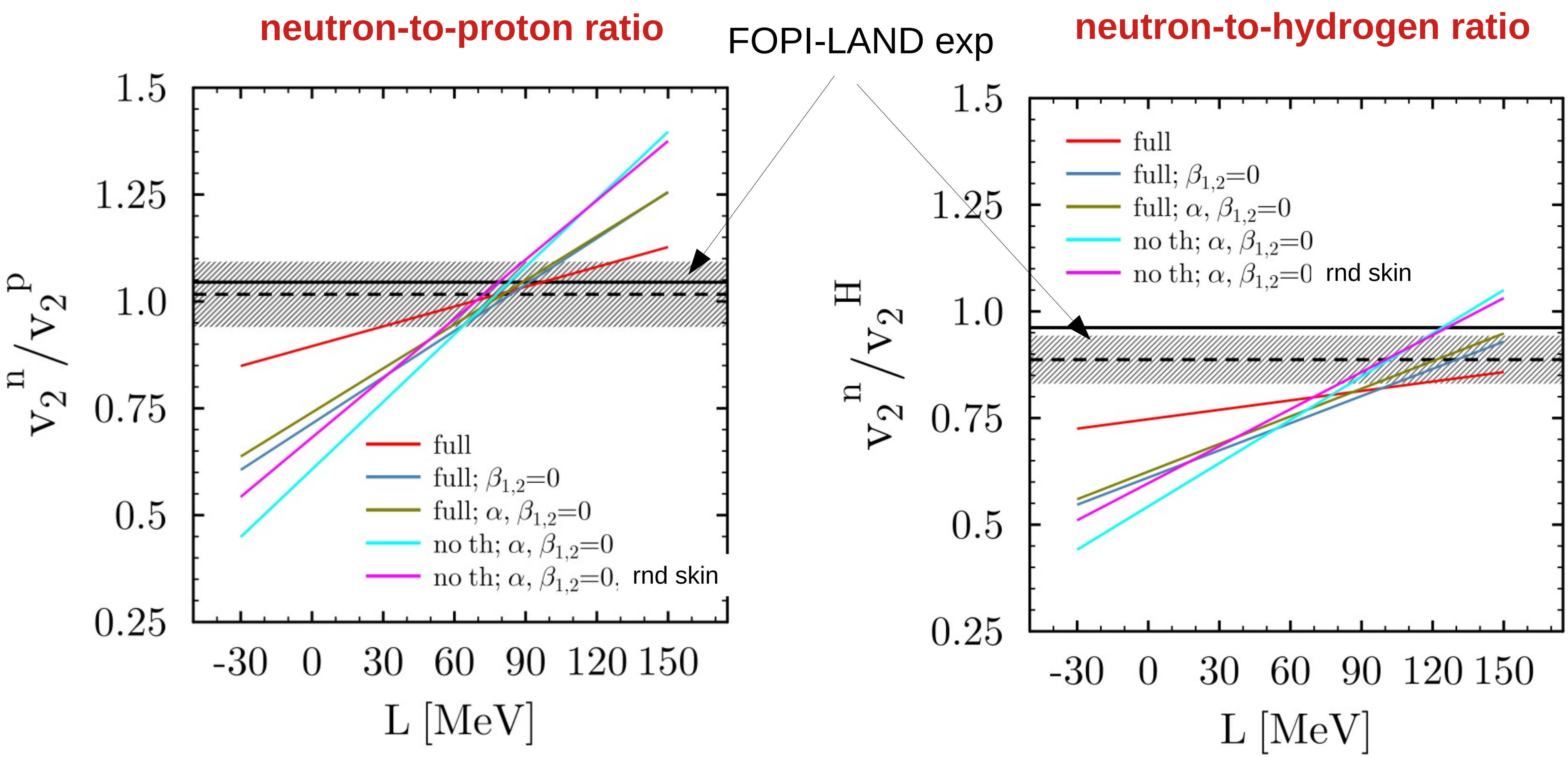
Model emulator: sum of monomials of degree ≤ 2; checked robustness LOO-CV method

# Constraints




- exp data for isospin symmetric systems crucial for breaking degeneracy of isospin dep. and isospin indep. in-medium effects on cross-sections

# FOPI-LAND Experiment Revisited

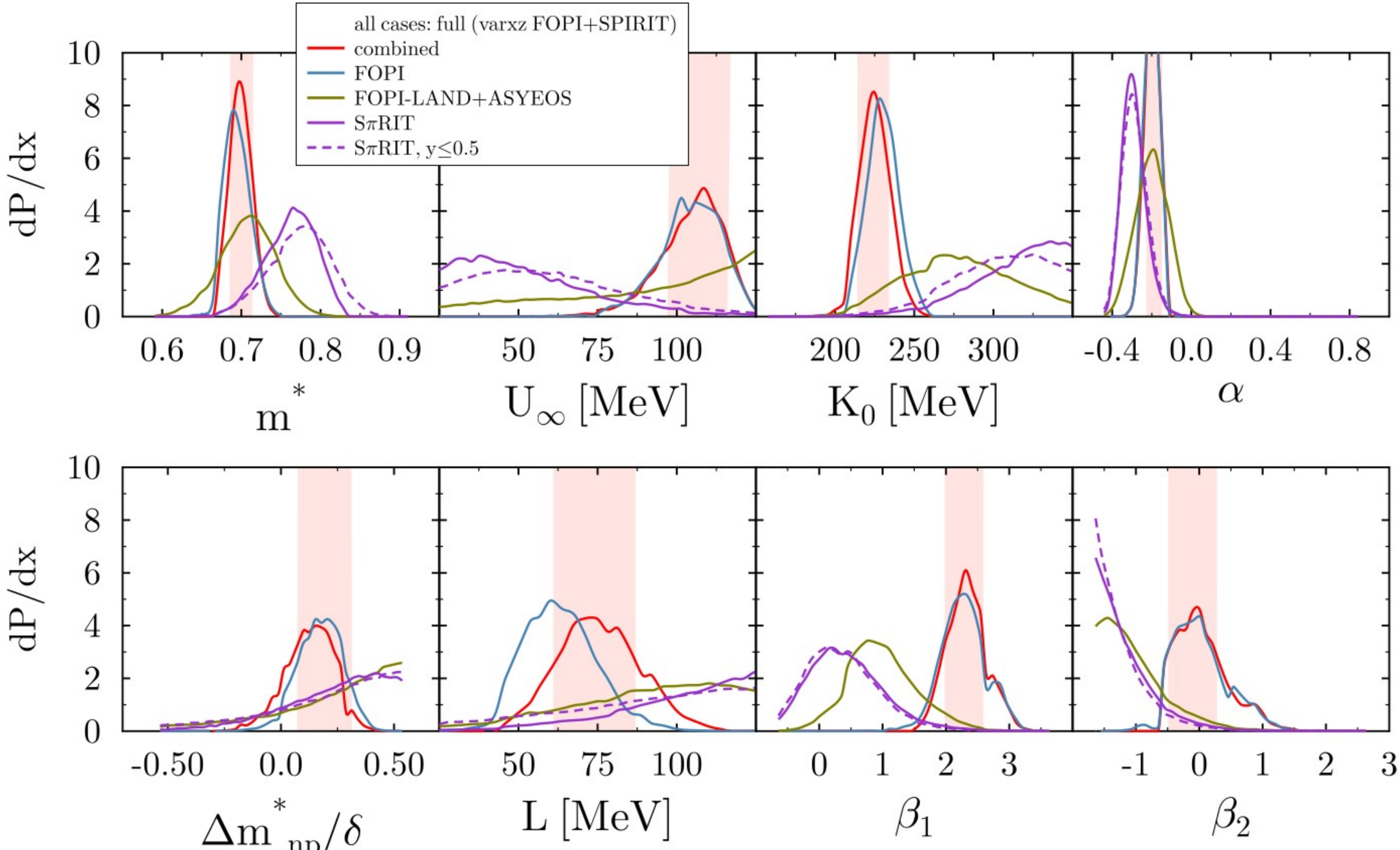



- identical results for neutron-to-charged particles ratio (ASYEOS experiment)

- future ASYEOS2 measurement (2025) aims at determining n/p with greater accuracy

- compatible with previous analysis (DC Eur. Phys. J A 54, 40 (2018))

# Combined Constraint




| | FOPI | combined |
|---|---|---|
| $K_0$ [MeV] | 230 ± 10 | 225 ± 10 |
| L [MeV] | 63 ±11 | 74 ± 13 |

# Pionic Observables

STEP 2

- $\pi^-$ and $\pi^+$ multiplicities in central collisions
- impact energy smaller than 800 MeV/nucleon
- limited contributions from resonances heavier than Δ(1232)

| Label | Observables | Reference |
| --- | --- | --- |
| SPIRIT | $M(\pi^-)$, $M(\pi^+)$; b<3.0 fm<br><br>$^{108}Sn+^{112}Sn$ (0.27)<br>$^{112}Sn+^{124}Sn$ (0.27)<br>$^{132}Sn+^{124}Sn$ (0.27) | G. Jhang et al.,<br>Phys. Lett. B 813, 136016 (2021) |
| FOPI | $M(\pi^-)$, $M(\pi^+)$; $b_0<0.15$<br><br>$^{96}Ru+^{96}Ru$ (0.40)<br>$^{96}Zr+^{96}Zr$ (0.40)<br>$^{40}Ca+^{40}Ca$ (0.40, 0.60, 0.80)<br>$^{197}Au+^{197}Au$ (0.40, 0.60, 0.80) | W. Reisdorf et al.,<br>Nucl. Phys. A 848, 366 (2010) |

# Δ(1232) Potential

nucleonic resonance of isospin T and isospin projection τ

$$U_R^{\tau}(\rho,\beta,p)=\frac{1}{2}(1-\tau/T)U_{-1/2}(\rho,\beta,p)+\frac{1}{2}(1+\tau/T)U_{1/2}(\rho,\beta,p)$$

particular case of a T=3/2 resonance

$$\begin{aligned}
U_{\Delta^-} &= U_{-\frac{1}{2}} &&= U_{is} + U_{iv}\\
U_{\Delta^0} &= \tfrac{2}{3}U_{-\frac{1}{2}} + \tfrac{1}{3}U_{\frac{1}{2}} &&= U_{is} + \tfrac{1}{3}U_{iv}\\
U_{\Delta^+} &= \tfrac{1}{3}U_{-\frac{1}{2}} + \tfrac{2}{3}U_{\frac{1}{2}} &&= U_{is} - \tfrac{1}{3}U_{iv}\\
U_{\Delta^{++}} &= U_{\frac{1}{2}} &&= U_{is} - U_{iv}
\end{aligned}$$

Δ(1232) parameters for

$$U^{\Delta}_{is/iv} \approx U^{nucleon}_{is/iv}$$

| Input | | Parameters | |
|---|---|---|---|
| $m^*_\Delta$ | 0.65 | $\Lambda$ | 700.98 |
| $U_0^\Delta(\rho_0, p=0)$ | -67.0 | $C_l + C_u$ | -153.82 |
| $U_0^\Delta(2\rho_0, p=0)$ | -55.0 | $A_l + A_u$ | -26.15 |
| $U_0^\Delta(\rho_0, p=\infty)$ | +75.0 | $B$ | 88.08 |
| | . | $D$ (fixed) | 0.0 |
| | . | $\sigma$ (fixed) | 1.465 |
| $\delta m^*_\Delta$ | 0.175 | $C_l - C_u$ | 125.50 |
| $U_{1,sym}^\Delta(\rho_0, p=0)$ | +45.0 | $A_l - A_u$ | -109.97 |
| $U_{1,sym}^\Delta(2\rho_0, p=0)$ | +67.5 | $x$ | 0.140 |
| | . | $y$ (fixed) | 0.0 |

expressions for the isoscalar and isovector potentials

$$U_{is}(\rho,\beta,p)=\frac{A_u+A_l}{2}\frac{\rho}{\rho_0}+B\left(\frac{\rho}{\rho_0}\right)^{\sigma}(1-x\beta^2)+D\left(\frac{\rho}{\rho_0}\right)^{2}(1-y\beta^2)+\frac{C_l+C_u}{\rho_0}[I(p,p_F^n)+I(p,p_F^p)]$$

$$U_{iv}(\rho,\beta,p)=\frac{A_l-A_u}{2}\frac{\rho}{\rho_0}\beta-2x\frac{B}{\sigma+1}\left(\frac{\rho}{\rho_0}\right)^{\sigma}\beta-\frac{2y}{3}\left(\frac{\rho}{\rho_0}\right)^{2}\beta+\frac{C_l-C_u}{\rho_0}[I(p,p_F^n)-I(p,p_F^p)]$$

$$\text{with } I(p,p_F^{\tau})=\int d^3p'\frac{f_\tau(r,p')}{1+(p-p')^2/\Lambda^2}$$

$$U_0(\rho,p) = U_{is}(\rho,\beta=0,p),$$

$$U_{sym,1}(\rho,p) = \lim_{\beta\to 0}\frac{U_{iv}(\rho,\beta,p)}{\beta}$$

# Theoretical Simulations

## Varied Model parameters:

| | | |
|---|---|---|
| $\alpha_R$ | [-3.0, 1.5] | in-medium $\sigma_{NN\to NR}$, $\delta$=0.0 |
| $\beta_R$ | [-3.0, 3.0] | in-medium $\sigma_{NN\to NR}$, $\delta$<>0.0 |
| $\Delta m^*_{np}$ | [-0.25,0.25] at ($\rho=\rho_0$, $\delta$=0.5) | n-p effective mass difference |
| L | [15, 140] MeV | slope of symmetry energy at $\rho=\rho_0$ |
| $U^\Delta_0$ | [-100, 0] MeV | Isoscalar Δ potential ($\rho=\rho_0$, p=0) |
| $m^*_\Delta$ | [0.4, 0.9] | Isoscalar Δ effective mass ($\rho=\rho_0$, $p=p_F$) |
| $U^\Delta_{sym,1}$ | [20, 110] MeV | Isovector Δ potential ($\rho=\rho_0$, p=0) |
| $\Delta m^*_\Delta$ | [-0.25, 0.25] at ($\rho=\rho_0$, $\delta$=0.5) | Isovector Δ effective mass splitting |

all other model parameters related to nucleonic degrees of freedom were set to values determined from study of nucleonic observables (stopping, $v_1$, $v_2$) Phys. Rev. C 110, 064911 (2024)

**Enforced correlations:**

$K_{sym}$=-488+L*6.728

in units of [MeV]

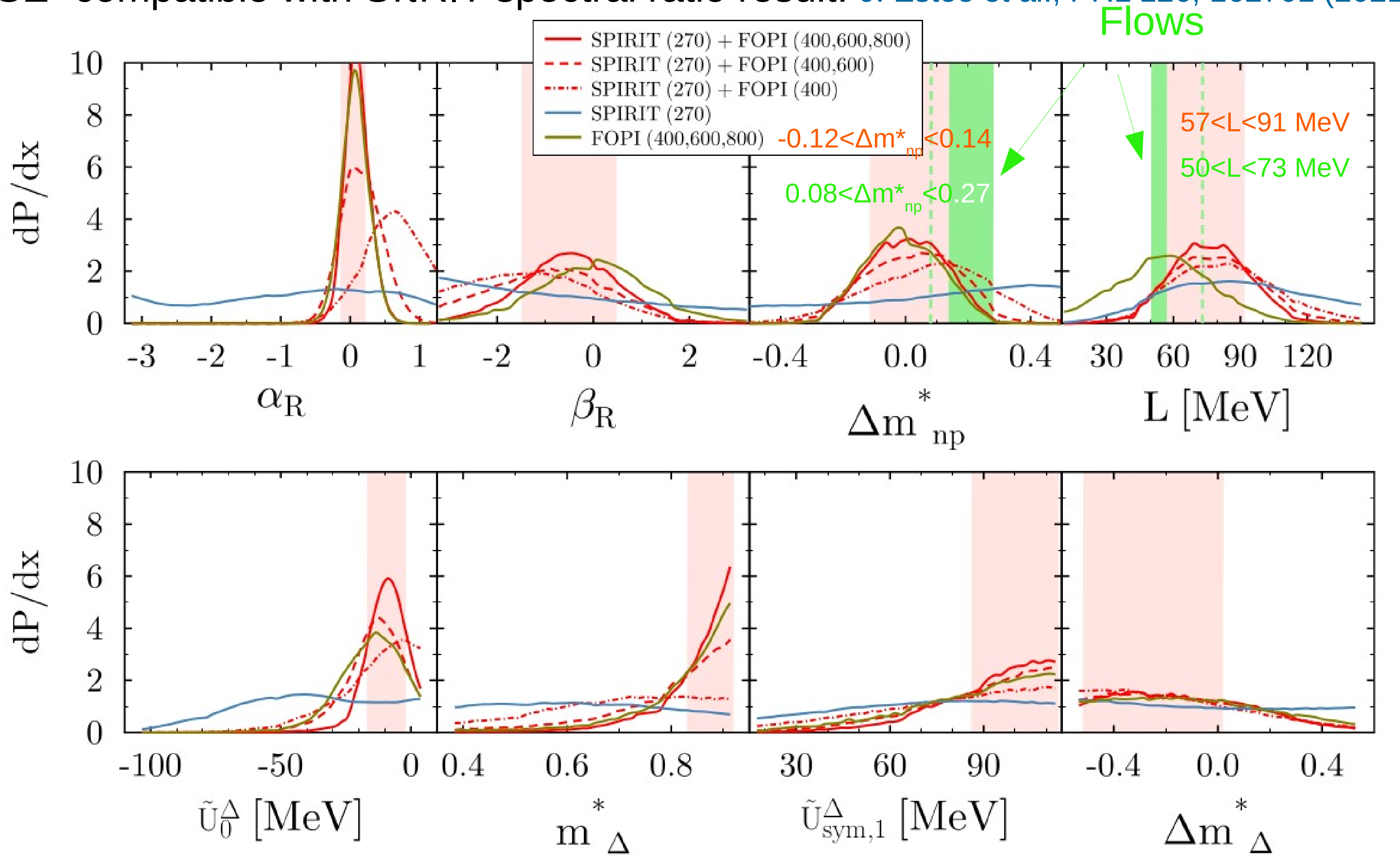
Input:
$S(0.62\rho_0)$=25.5 MeV

properties of doubly-magic nuclei (binding energies, rms charge radii, single particle energies)
Brown, PRL 111, 232502 (2013)

Version used here: includes non-resonant pion production channels (preliminary)

# Constraints

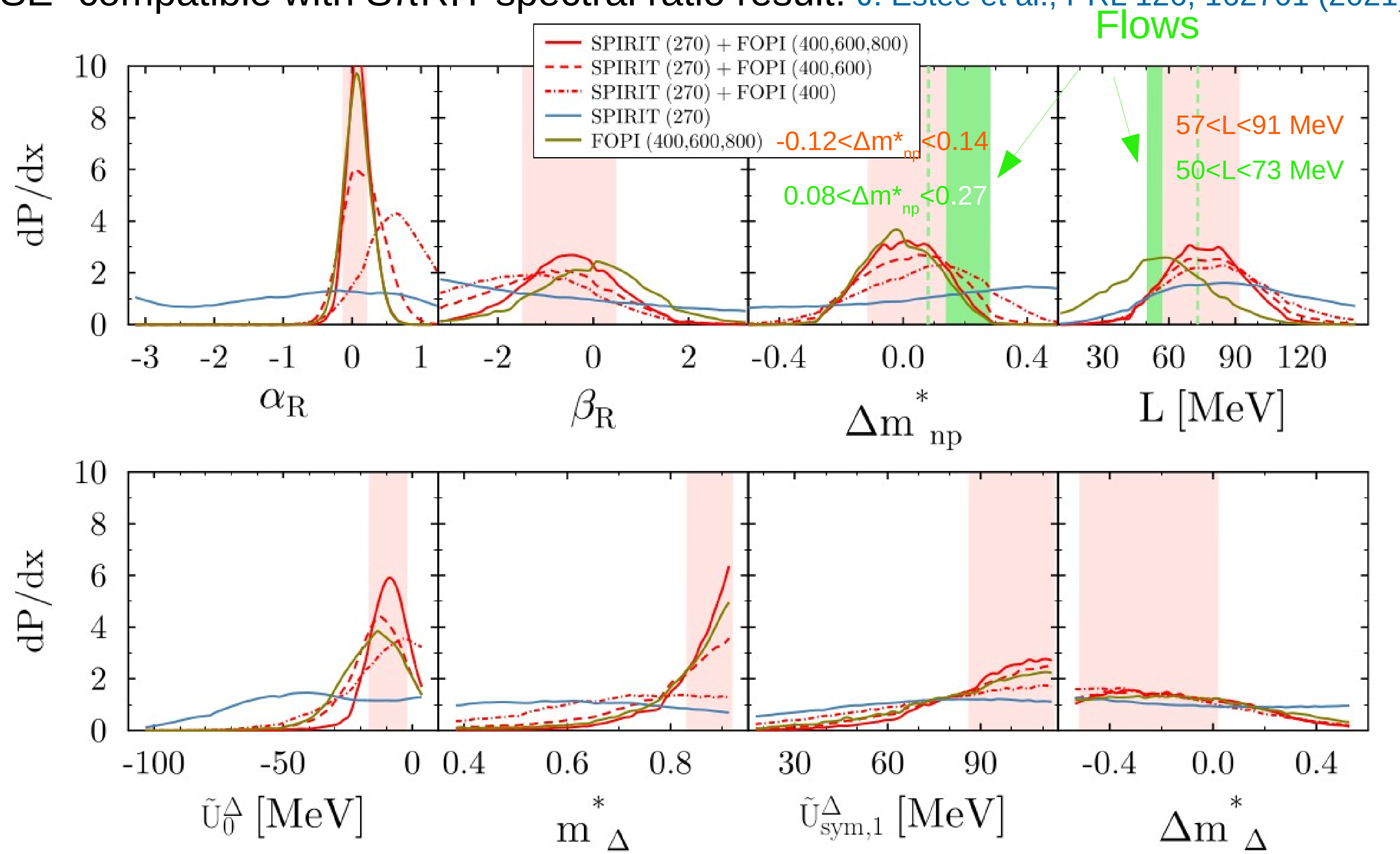
SE -compatible with SπRIT spectral ratio result: J. Estee et al., PRL 126, 162701 (2021)




- exp data for isospin symmetric systems (CaCa) significant impact on isospin asymmetry dependence of in-medium cross-sections: $\beta_R \approx 2.0$ if omitted ($\approx -2.0$ if AuAu systems removed)
- similar observation for $U^{\Delta}_0$ and to a lesser extent $\alpha_R$
- If Δ(1232) potential fixed to that of nucleons => strong suppression of transition matrix elements required (see K. Godbey et al., Phys. Lett. B 829, 137134 (2022) )

# Constraints

SE -compatible with SπRIT spectral ratio result: J. Estee et al., PRL 126, 162701 (2021)




Ab initio calculations - Argonne $v_{28}$ interaction (BBG)=> repulsive Δ potential

$U^{\Delta}_{0}$ - 3D reduction of Bethe-Salpeter equation similar (DB)

- strong dominant repulsive contribution from T=2 sector

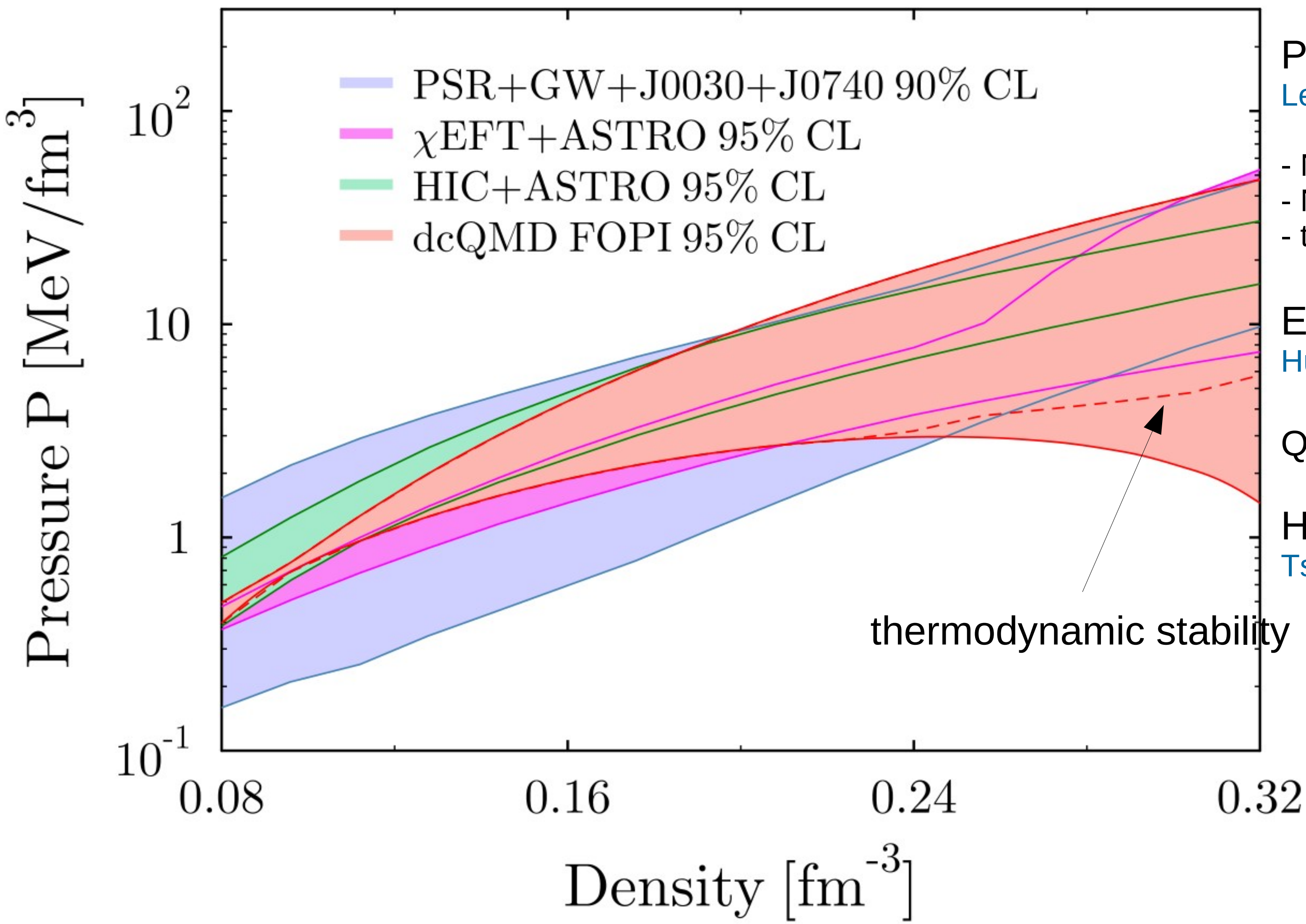
Baldo, Ferreira NPA 569, 645 (1994)

Malfliet, de Jong PRC 46, 2567 (1992)

# Pressure

Neutron star matter (δ=0.93)

Input: $S(0.62\rho_0)$=25.5 MeV




Pure Astro:
Legred et al PRD 104, 063003

- NS radii
- NS maximum mass
- tidal deformatibility (GW)

EFT:
Huth et al Nature 606, 276

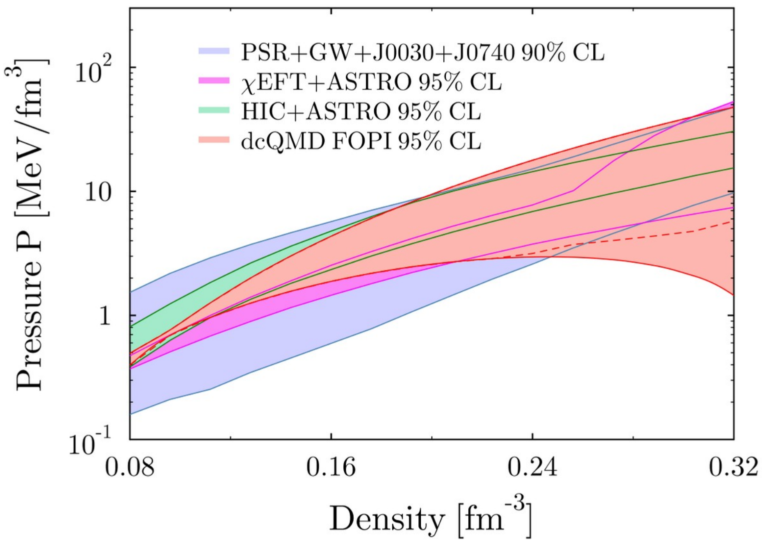
QMC -2 and 3 body forces
($N^2LO$)

HIC:
Tsang et al Nat. Astron. 8, 328

SNM: - elliptic flow
- GMR

ANM: - dipole polarizability,
- neutron skin
- nuclear masses
- iso-diff, n/p ratio, flow ratio, pion ratio

# Summary & Conclusions




**Study of EoS, effective masses and σ* using nucleonic observables** in AuAu collisions of intermediate impact energy (0.15-0.80 GeV/nucleon)

- in-medium effects on elastic collisions that depend on density, isospin asymmetry and isospin projection
- clusterization algorithm applied at local freeze-out time
- systematic uncertainty due to coalescence parameters;
- constraints extracted from FOPI experimental data for $v_1(y)$, $v_2(y)$, $v_2(p_T)$ and varxz
- model dependence: threshold effects and isospin asymmetry dependence of σ* have significant impact

**Study of SE using pionic observables**

- full agreement (in contradiction with the situation ~10 years ago)

**68% CL Result**

$m^*=0.70 \pm 0.02$ $V_\infty=104 \pm 8$ MeV

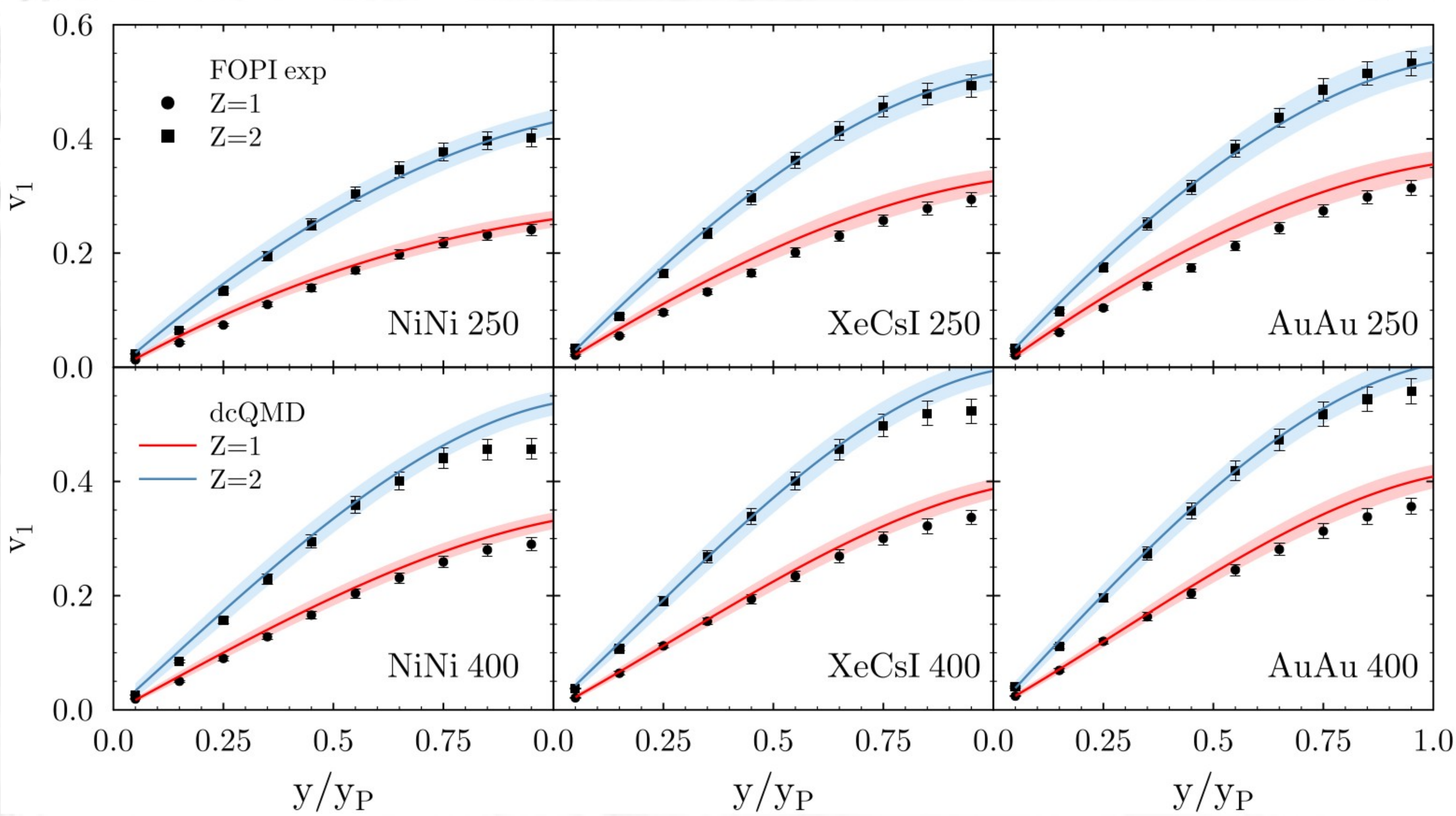
$\Delta m^*_{np}=(0.17 \pm 0.10)\delta$

$K_0=230 \pm 10$ MeV

$L=63 \pm 12$ MeV

**Perspectives:** - remove imposed correlation between $K_0 \leftrightarrow J_0$ and $L \leftrightarrow K_{sym}$; allow for a variation of $E/N(\rho_0)$ and $S(0.1\ fm^{-3})$.
- include explicit cluster degrees of freedom to be able to use experimental data sets to their full potential
- improve model to use more accurate part of pion spectra and access higher densities with HIC above 1.0 GeV/nucleon
- estimate model dependence (TMEP)

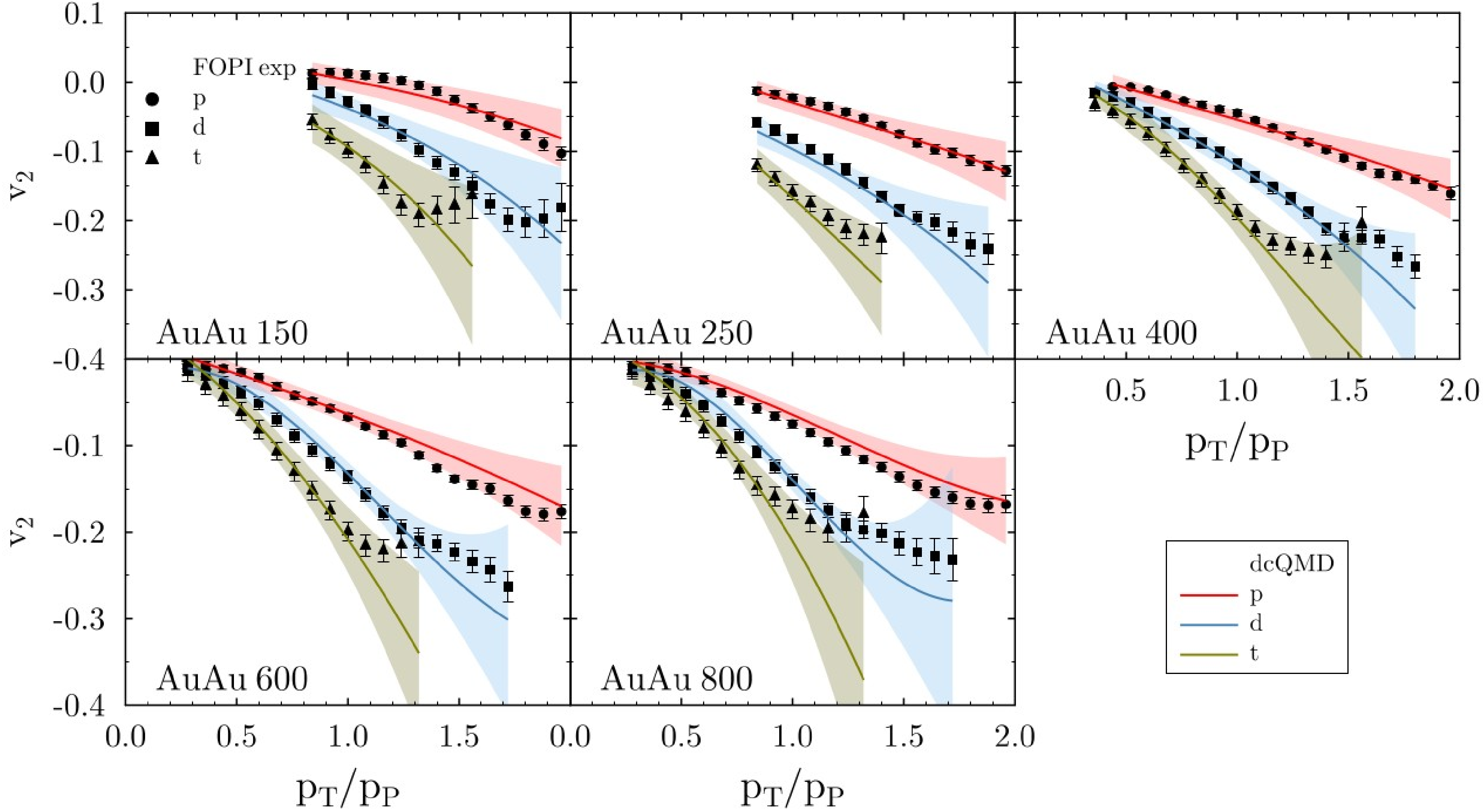
# FOPI1: $v_1(y)$




- M5 centrality
- only data with $y/y_P$<0.5 included in the fit

Full comparison to experimental data set: see D.C. Phys. Rev. C 110, 064911 (2024)

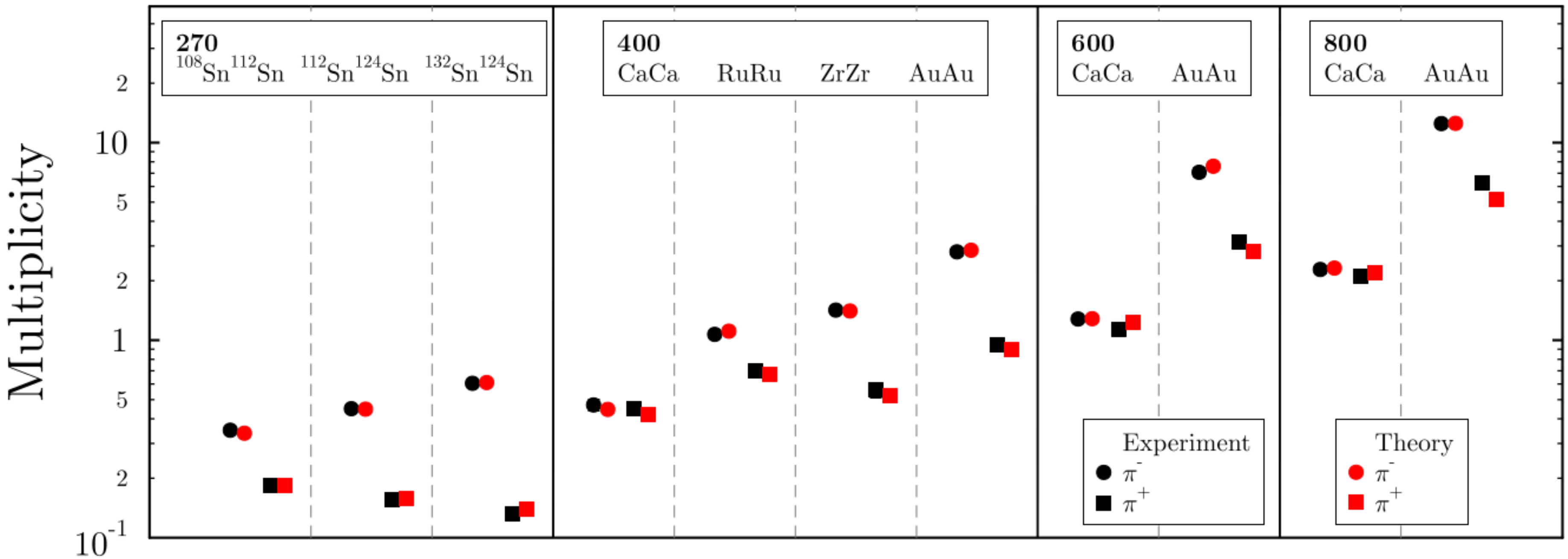
# FOPI: $v_2(p_T)$




$0.25<b_0<0.45$, $|y/y_P|<0.4$

# Pions: Theory vs Experiment

- most predictions within one standard deviation from experimental values
- one exception: $\pi^+$ multiplicity for AuAu collisions at 800 MeV/nucleon (≈ 2σ)

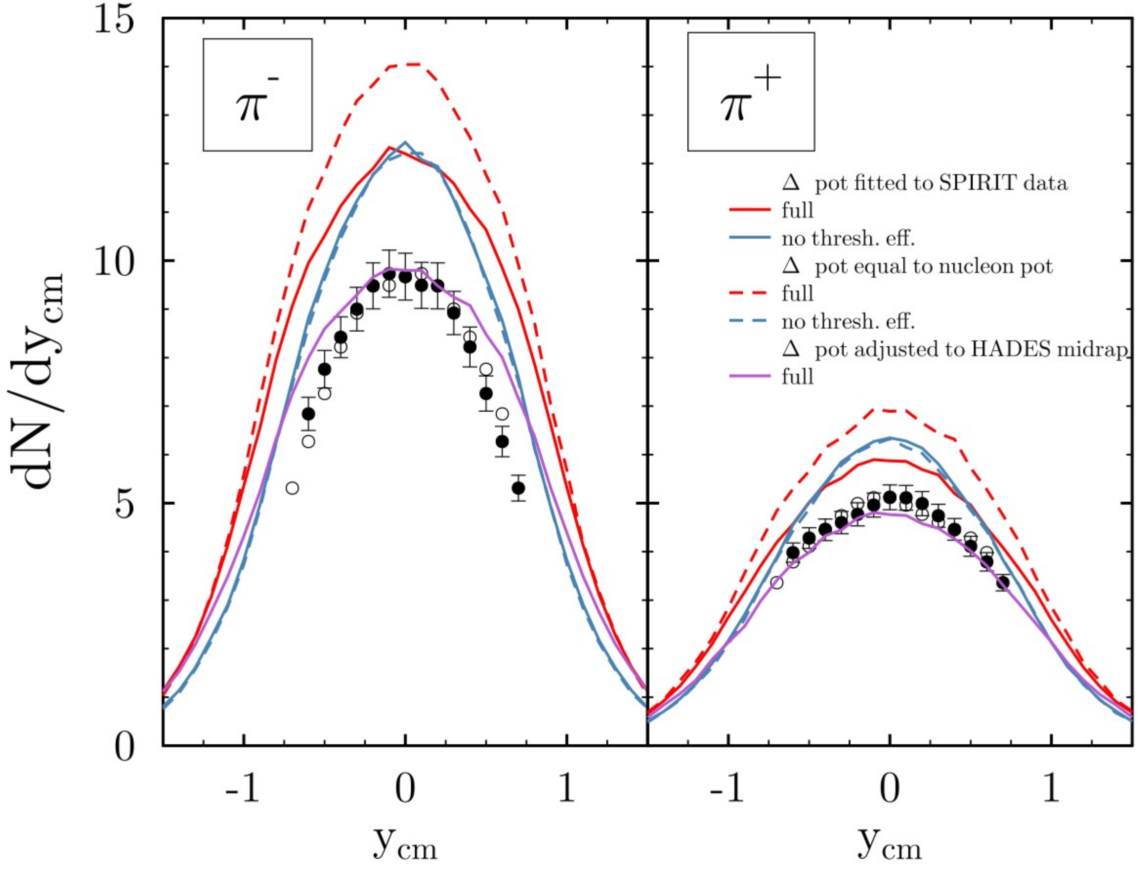


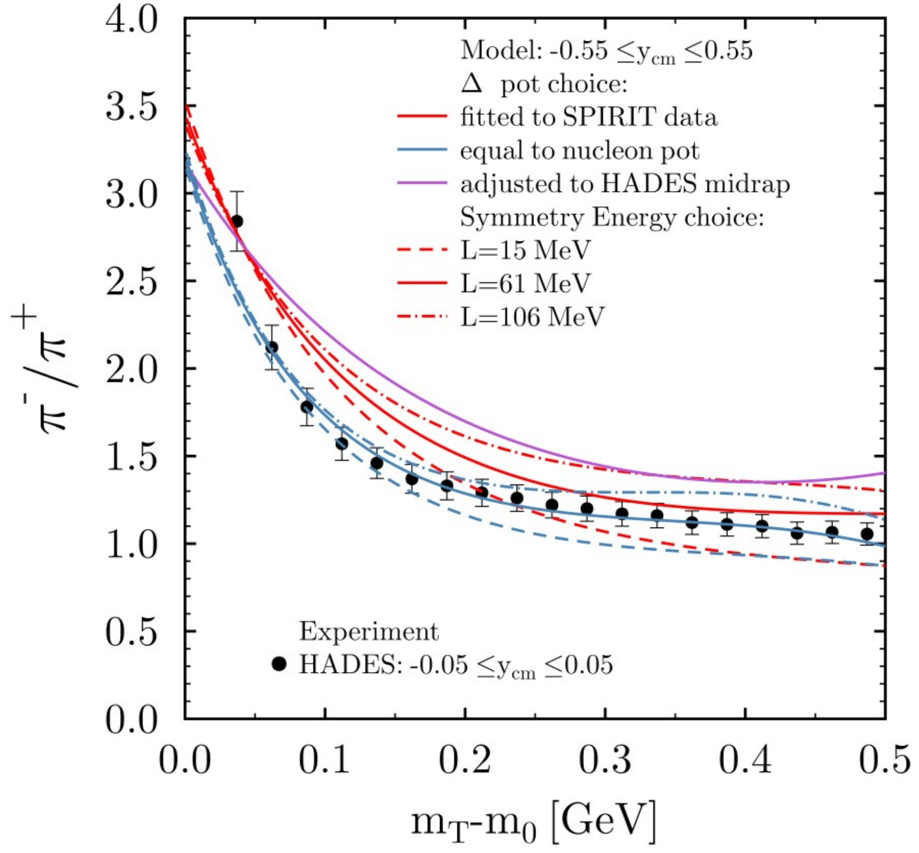
# Pion production in AuAu at 1.23 AGeV

J. Adamczewski-Musch et al. (HADES), EPJA 56, 259 (2020)

Note: HADES pion total yield are larger then FOPI yields at 1.2 GeV by about 30%

very preliminary calculation addressing the feasibility of studying the symmetry energy



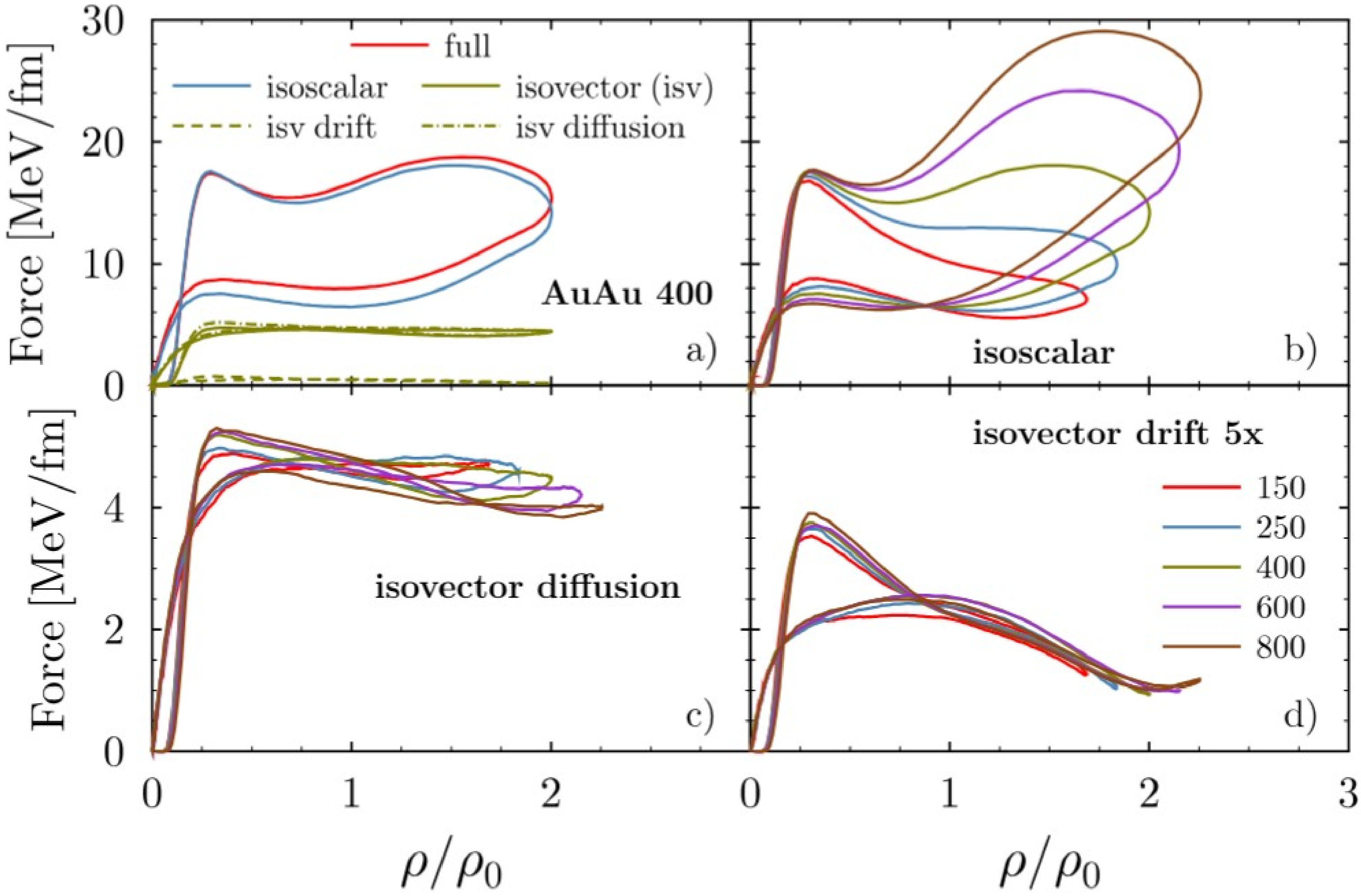



# Average Forces

-trajectories of HIC events in the force vs. density plane (clockwise)

-average magnitudes in a sphere of R=3 fm centered on origin of axes

-model settings: agreement with FOPI data



# Average Isospin Asymmetry, Gradients

-trajectories of HIC events in the force vs. density plane (clockwise)

-average magnitudes in a sphere of R=3 fm centered on origin of axes

-model settings: agreement with FOPI data